Correspondence between Modified Gravity and Generalized Uncertainty Principle

Aneta Wojnar

Complutense University of Madrid

HUN-REN Wigner Research Centre for Physics

Budapest

Motivation and plan of the talk

Motivation:

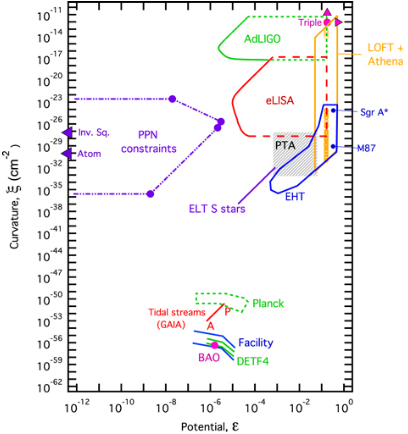
- To understand effects of gravity on thermodynamic systems
- To constrain theories of modified and quantum gravity

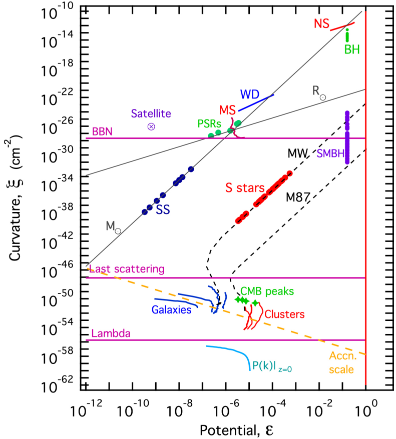
Plan of the talk:

- Description of thermodynamic systems in the presence of gravity
- GR case: choosing an observer and coordinate system
- Modified Gravity in the lab
- Seismology as a tool to test fundamental interactions:

  Cost Action FuSe: *Testing Fundamental Physics with Seismology*

The stellar and galaxy curvature regime not considered too much in MG...





Untested regime in the **the galaxy and stellar physics regime**. It could potentially hide the onset of corrections to GR (T. Baker et al 2015 ApJ 802, 63).

Curvature $\xi = (R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta})^{\frac{1}{2}} = \sqrt{48}\frac{GM}{r^3c^2}$

Potential $\varepsilon = \frac{GM}{rc^2}$

Gravity vs matter: motivation based on a number of indications

- Effective quantities: opacity[1], ...
- Modifications introduced by modified gravity to pressure[2]
- Chemical reactions rates depend on gravity[3]
- Specific heat and crystallization depend on modified gravity[4]
- Chemical potential depends on gravity[5]
- Elementary particle interactions modified by modified gravity (dependence of the metric on the local energy-momentum distributions[6]
- EoS depends on relativistic effects introduced by GR[7]
- Thermonuclear processes...?[8]
- Fermi and Bose equations of state depend on (modified/quantum) gravity[9]

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[1] J. Sakstein, PRD 92 (2015) 124045; ...
[2] H-Ch. Kim, PRD 89 (2014) 064001
[3] P. Lecca, J. Phys.: Conf. Ser. 2090 (2021) 012034
[4] S. Kalita, L. Sarmah, AW, PRD 107 (2023) 4, 044072
[5] I.K. Kulikov, P.I. Pronin, Int. J. Theor. Phys. 34, (1995) 9
[6] A.D.I Latorre, G.J. Olmo, M. Ronco, PRB 780, 294 (2018)
[7] G.M. Hossain, S. Mandal, JCAP 02 (2021) 026; PRD 104 (2021) 123005
[8] J. Sakstein, PRD 92 (2015) 124045; AW, PRD 103 (2021) 4, 044037; M. Guerrero, AW, in preparation
[9] AW, PRD 107 (2023) 4, 044025; A. Pachol, AW, Class.Quant.Grav. 40 (2023) 19, 195021; AW, PRD 109 (2024) 2, 024011; AW PRD 109 (2024) 124031

Quantum gravity[10]

Observation 1:

Modifies Heisenberg uncertainty principle (GUP, EUP)

$$\Delta x_i \Delta p_i \geq \frac{\hbar}{2}\Big(1 + \text{modification}\Big)$$

or/and dispersion relation

$$E^2 + p^2\Big(1 + \text{modification}\Big) = m^2$$

[10] LQG, Doubly Special Relativity, String Theory, Noncommunative geometry,...

Observation 2:

The weighted phase space volume is modified (D - dim of the phase space).

$$\frac{d^D\mathbf{x}d^D\mathbf{p}}{1+\text{modification}}$$

Consequence: modified partition function ($z=e^{\mu/k_BT}$)

$$\ln\mathcal{Z}=\frac{V}{(2\pi\hbar)^3}\frac{g}{\pm1}\int\ln\left(1\pm ze^{-E/k_BT}\right)\frac{d^3p}{1+\text{modification}}$$

Conclusion: Quantum Gravity modifies equations of state since

$$P=k_BT\frac{\partial}{\partial V}\ln\mathcal{Z},$$
$$n=k_BT\frac{\partial}{\partial\mu}\ln\mathcal{Z}\mid_{T,V},$$
$$U=k_BT^2\frac{\partial}{\partial T}\ln\mathcal{Z}\mid_{z,V}$$

Observation 3: MG as an effective theory derived from QG

GR case[11]

Let us consider a non-relativistic massive particle in the gravitational field of the Earth

$$S = -\int m\, d\, s = -\int m \sqrt{-g_{\mu\nu}(x)\dot{x}^{\mu}\dot{x}^{\nu}}\, d\, \tau$$

Choosing a **non-rotating** observer, we can write down the metric as (3+1 split)

$$ds^2 = -N^2 \left(dx^0\right)^2 + h_{ij}(N^i dx^0 + dx^i)(N^j dx^0 + dx^j),$$

while the Lagrangian reads ($h_{ij} \equiv g_{ij}$)

$$L = -m\sqrt{N^2 \left(\dot{x}^0\right)^2 - \left(\dot{x}^0 N^i + \dot{x}^i\right)\left(\dot{x}^0 N^j + \dot{x}^j\right) h_{ij}}.$$

We will consider the non-relativistic part of the above Lagrangian ($G_{ij} = h_{ij}/N$)

$$L_{NR} = \frac{m}{2}\left(N^i + \dot{x}^i\right)\left(N^j + \dot{x}^j\right) G_{ij} - mN.$$

[11] L. Petruzziello, F. Wagner, PRD 103 (2021) 104061

Quantum treatment

Non-relativistic Hamiltonian of our particle with $p_i \equiv \pi_i - mN^j G_{ij}$

$$H_{NR} = \frac{1}{2m} p_i p_j G^{ij} + m \left(N - \frac{N^i N^j G_{ij}}{2} \right).$$

- Hilbert space construction: proper measure $d\mu = \sqrt{G} d^3 x$
- Position and momentum operators $[\hat{x}^i, \hat{p}_j] = i\hbar\delta^i_j$:

$$\hat{x}^i \psi = x^i \psi \qquad \hat{p}_i \psi = -\left[i\hbar \left(\partial_i + \frac{1}{2}\Gamma^j_{ij}(G) \right) + mN^j G_{ij} \right] \psi$$

- Momentum uncertainty $\sigma_p \equiv \sqrt{\hat{p}^2 - \hat{p}^i \hat{p}_i}$
- Uncertainty relation (ρ - measure of position uncertainty):

$$\sigma_p \rho \gtrsim \pi\hbar \left[1 - \frac{\rho^2 R|_{p_0}}{12\pi^2} + \xi \frac{\rho^4}{\lambda_C^2} \nabla_j N_i \nabla^j N^i |_{p_0} \right].$$

Let's take classical limit $\frac{1}{i\hbar}[\hat{A}, \hat{B}] \to \{A, B\}$ and get the time evolutions of the coordinates and momenta:

$$\dot{x}_i = \{x_i, H\}, \quad \dot{p}_i = \{p_i, H\}$$

- Non-trivial commutation relations:

$$[\hat{x}_i, \hat{x}_j] = 0 \text{ and } [\hat{p}_i, \hat{p}_j] \sim R_{ijk}^{\ \ \ k}(G(\phi))$$

- Get the weighted phase space volume which is invariant under time evolution:

$$dx'^3 dp'^3 = \frac{dx^3 dp^3}{\text{modification}}, \quad \text{here: modification is } \phi(x) \text{ dependent!}$$

- Get your thermodynamics and go to the lab (or talk to Aneta).

[12] C. Pfeifer, AW, in preparation

Palatini $f(R)$ and EiBI gravity[13] - effective approach

It turns out that Palatini-like gravity in the weak limit corresponds to linear GUP

Poisson equation - the additional term can be interpreted as a modification to the matter fluid

$$\nabla^2\phi = \frac{\kappa}{2}\left(\rho + \bar{\alpha}\nabla^2\rho\right)$$

The partition function in the grand-canonical ensemble:

$$\ln Z = \frac{V}{(2\pi\hbar)^3}\frac{g}{a}\int f(E)\frac{d^3p}{(1-\sigma p)^b}$$

So the deformation of the phase space is

$$\frac{1}{(2\pi\hbar)^3}\int\frac{d^3x d^3p}{(1-\sigma p)^d},$$

$\rightarrow$ linear GUP with $b = 1$.

The covariant form of linear GUP which may correspond to the Palatini-like gravity could take the following form:

$$[x_\mu, p_\nu] = i\hbar\left[g_{\mu\nu} - \alpha\left(p g_{\mu\nu} + \frac{p_\mu p_\nu}{p}\right)\right].$$

[13] AW, PRD 109 (2024) 2, 024011; A Farag Ali, AW, CGQ 41 (2024) 10, 105001

Modified Gravity and tabletop experiments[14] - liquid helium

The non-interacting Bose-Einstein condensate imposes $-10^{12} \lesssim \sigma \lesssim 3\times 10^{24}$ s/kg m for the linear GUP and $-10^{-1} \lesssim \bar\beta \lesssim 10^{11}$ m^2 for Palatini gravity.

Landau model (in An Introduction to the Theory of Superfluidity (CRC Press, 2018) pp. 185-204.)

$$\hbar\omega = \begin{cases} \hbar c k & \text{if } k << k_0, \\ \Delta + \frac{\hbar^2 (k-k_0)^2}{2\gamma} & \text{if } k \approx k_0, \end{cases}$$

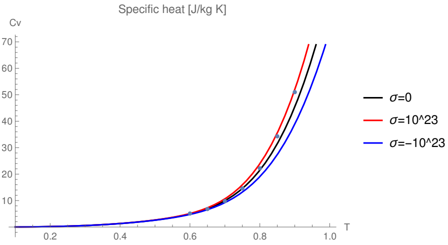
The quantum states of He^4 close to the ground state $\rightarrow$ the states of a non-interacting gas with energy levels

$$U = E_0 + \frac{V}{2\pi^2}\int_0^\infty \frac{k^2\hbar\omega_k}{e^{\beta\hbar\omega_k}-1}\frac{dk}{(1-\sigma\hbar k)}.$$

Total specific heat $C_V = \frac{\partial U}{\partial T}\,|_V$ (in Jkg^{-1}K^{-1})

$$C_{He^4} = 20.7T^3 + \frac{387\times 10^3}{T^{3/2}}e^{-8.85/T} + \sigma\left(5.73\times 10^{-24}T^4 + \frac{7.83\times 10^{-19}}{T^{3/2}}e^{-8.85/T}\right)$$

$-10^{23} \lesssim \sigma \lesssim 10^{23}$ s/kg m and $-10^{9} \lesssim \bar\beta \lesssim 10^{9}$ m^2



Specific heat of liquid helium in low temperatures. The data points taken from H. Kramers, in Progress in Low Temperature Physics, Vol. 2 (Elsevier, 1957) pp. 59-82.

[14] AW, PRD 109 (2024) 12, 124031

Non-relativistic equations of modified and quantum gravity

Modified Poisson equation

$$\nabla^2\Phi \approx \frac{1}{2}(\rho + \text{modification})$$

For spherical-symmetric spacetime the gravitational potential the hydrostatic equilibrium equation

$$\frac{d\Phi}{dr} = -\rho^{-1}\frac{dP}{dr},$$
$$M = \int 4\pi' \tilde{r}^2 \rho(\tilde{r}) d\tilde{r},$$

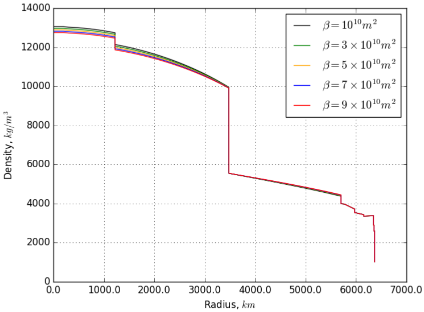
+ matter description (EoS or **seismic data**, temperature dependence,...)
+ eventual equations for additional fields

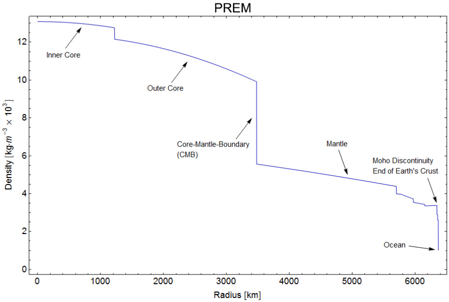
A new method of testing theories of gravity proposed[15]

[15] A. Kozak, AW, Phys.Rev.D 104 (2021) 8, 084097

Terrestrial planets - seismology vs gravity I [17]

The Earth's density profile (inner and outer core, mantle + outer layers given by the Birch law)





On the RHS: Palatini gravity ($\Delta\rho = 600$, $\rho_m = 5550$); on the left: Preliminary Reference Earth Model (PREM) A. M. Dziewonski, D. L. Anderson, Phys. Earth Plan. Int. 25 (1981) 297.

Exoplanets properties: central values and CMB are affected by modified gravity[16]

[16] A. Kozak, AW, Universe 8 (2021) 1, 3

[17] A. Kozak, AW, PRD 104 (2021) 8, 084097; IJGMMP 19 (2022) Supp01, 2250157; Phys. Rev. D 108 (2023) 4, 044055

Terrestrial planets - seismology vs gravity II [18]

- No exchange of heat between different layers (adiabatic compression)
- The planet is a spherical-symmetric ball in hydrostatic equilibrium
- The planet consists of radially symmetric shells with the given density jump between the inner and outer core $\Delta\rho = 600$, central density $\rho_c = 13050$ and density at the mantle's base $\rho_m = 5563$ (in kg/m^3) - PREM
- Mass $M = 4\pi \int_0^R r^2 \rho(r) dr$ and moment of inertia $I = \frac{8}{3}\pi \int_0^R r^4 \rho(r) dr$ where R is Earth's radius, play a role of the constraints (given by observations with a high accuracy)
- The outer layers' density profile described by Birch law $\rho = a + bv_p$

v_p is the longitudinal elastic wave. It contributes, together with the transverse elastic wave v_s, to the seismic parameter Φ_s and the elastic properties of an isotropic material

$$\Phi_s = v_p^2 - \frac{4}{3}v_s^2 = \frac{K}{\rho}, \quad K = \frac{dP}{d\ln\rho}$$

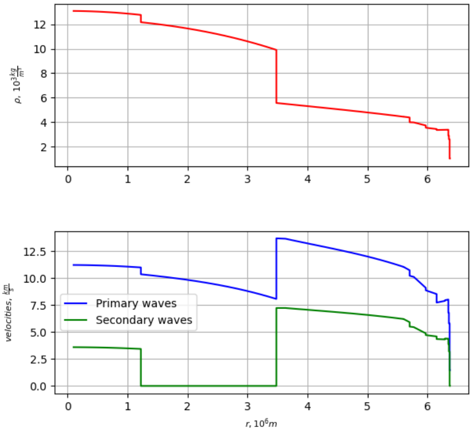
The hydrostatic equilibrium equation in MG:

$$\frac{d\rho}{dr} = -\rho\left(\frac{GM(r)}{r^2} + \text{modification}\right)\Phi_s^{-1},$$

[18] A. Kozak, AW, Phys.Rev.D 108 (2023) 4, 044055

The density profile given by the PREM (Newtonian)

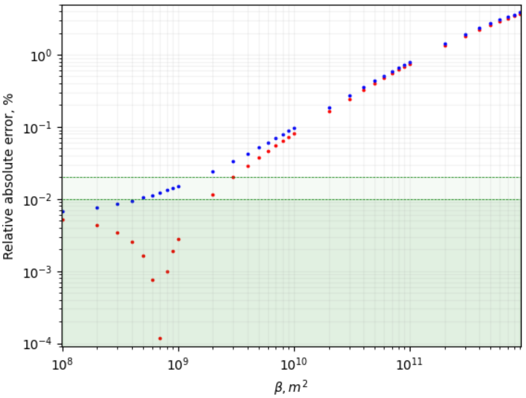
- The density profile given by the preliminary reference Earth model in which Newtonian gravity is assumed.
- The velocities' plots are obtained from data without using any theory of gravity.
- The primary waves are the same as the longitudinal waves, while the secondary waves are transverse in nature.
- The units are in km/s for velocities, while the densities are in kg/m^3.



Terrestrial planets - seismology vs gravity III [19]

Constraining theory (moment of inertia $I = 8.01736 \pm 0.00097 \times 10^{37}$kg m^2 and mass $M = 5.9722 \pm 0.0006 \times 10^{24}$kg)

- Relative absolute error for the mass and the moment of inertia of Earth. Red dots represent errors for the moment of inertia, while blue ones correspond to the mass.
- The dark green stripe represents a 1σ region for both quantities, while the light green denotes a 2σ region.
- The green region denotes the uncertainties for both mass and moment of inertia because, for either of them, the ratio of σ to the mean value is similar ($\approx 0.01\%$).
- The values of $(\rho_m, \rho_c, \Delta\rho)$ chosen for numerical calculations are $(5563, 13050, 600)\text{kg/m}^3$, respectively.



[19] A. Kozak, AW, Phys.Rev.D 108 (2023) 4, 044055

Theories of gravity constrained so far

Modified Poisson equation

$$\nabla^2\phi(\mathbf{x}) = 4\pi G\Big(\rho(\mathbf{x}) + \nabla^2\alpha\big(\mathbf{x},\rho(\mathbf{x})\big)\Big),$$

- Palatini $f(R)$ and Eddington-inspired Born-Infeld gravity (Ricci-based)[20]: $\alpha(r,\rho) = \epsilon/2\rho(r)$, and $\epsilon = 4\beta$

$$-2\times 10^9 \lesssim \beta \lesssim 10^9 \mathrm{m}^2 \text{ for Palatini },\ -8\times 10^9 \lesssim \epsilon \lesssim 4\times 10^9 \mathrm{m}^2 \text{ for EiBI}$$

- DHOST theories $\alpha(r,\rho) = \frac{\Upsilon}{4} r^2\rho(r)$

$$-10^{-3} \lesssim \Upsilon \lesssim 10^{-3}$$

- Quantum gravity: Snyder and qGUP ($\beta_0 := \beta M_P^2 c^2$): $\beta_0 < 4.67\times 10^{44}$
- Quantum gravity: linear GUP: $-6\times 10^{22} \lesssim \sigma \lesssim 3\times 10^{22}$ s/kg m

[20] New cosmological data provides bounds $|\beta| < 10^{49}$ m^2, Aguiar Gomes+, JCAP 01 (2024) 011

The uncertainties for the models' parameters I

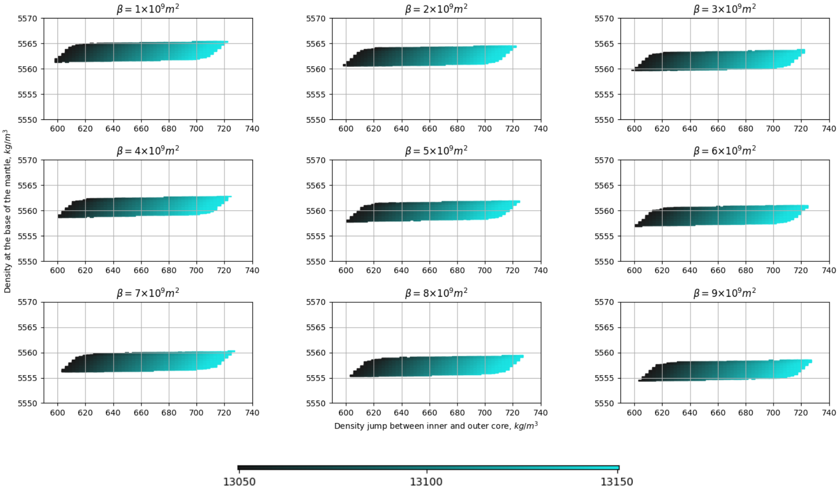


Figure: 1σ confidence regions of the theory parameters $(\rho_c, \rho_m, \Delta\rho)$ for different values of the β parameter, being of order 10^9 m^2. The darker color corresponds to lower values of the central density, while the brighter one - to higher. The range of the central density is shown in the color bar below the figures. The units are kg/m^3.

The uncertainties for the models' parameters II

- There always exists a region for a given value of the theory parameter for which all three density parameters result in a good agreement with experimental measurements
- $\Delta\rho$ and ρ_c admit much wider ranges of their values, not taking out of the 1σ region.
- ρ_m can differ by no more than $2-3$ kg m^{-3} from the value assumed in our calculations in order to remain within the 1σ region
- To incorporate bigger uncertainty of ρ_m, increase in the range of ρ_m and $\Delta\rho$, and/or the range of β would be necessary
- Large uncertainty in the determination of ρ_m is related to a bigger range of β parameter's allowed values
- Example: for $\beta = 10^9$m^2, deviations from the PREM ρ_m $(\beta = 0)$ leading to the same values of M and I, is 0.02% while, in the worst case, for the uncertainty of the PREM model 50 kg m^{-3}, is 0.9% ($\Delta\rho$ and ρ_c unchanged). It increases the bound to 10^{11}m^2.

Astrophysical bounds on Generalized Uncertainty Principle[21]

Our bound when more realistic physics taken into account

$$\beta_0 \leq 1.36 \times 10^{48}$$ from low-mass stars (A. Pachol, AW, Eur.Phys.J.C 83 (2023) 12, 1097)

$$\beta_0 < 4.67 \times 10^{44}$$ from Earthquakes (A. Kozak, A. Pachol, AW, arXiv:2310.00913)

Experiment	Reference	Upper bound on β
Perihelion precession (Solar System, 1)	[328, 330]	10^{34}
Time-of-flight measurements	[327]	10^{16}
Equivalence principle (pendula)	[246]	10^{73}
Gravitational bar detectors	[406, 407]	10^{93}
Equivalence principle (atoms)	[408]	10^{45}
Low-mass stars	[409]	10^{48}
LIV in torsion pendulum	[171]	10^{51}
Perihelion precession (Solar System, 2)	[129, 161]	10^{69}
Perihelion precession (pulsars)	[129]	10^{71}
Gravitational redshift	[161]	10^{76}
Black hole quasi normal modes	[257]	10^{77}
Light deflection	[129, 161]	10^{78}
Time delay of light	[161]	10^{81}
Black hole shadow	[253]	10^{90}
Black hole shadow	[257, 265]	10^{90}

21 See review by Bosso+ 2023 Class. Quantum Grav. 40 195014

Tabletop experiment bounds on Generalized Uncertainty Principle[22]

Our bound when more realistic physics taken into account

$\beta_0 \leq 1.36 \times 10^{48}$ from low-mass stars (A. Pachol, AW, Eur.Phys.J.C 83 (2023) 12, 1097)

$\beta_0 < 4.67 \times 10^{44}$ from Earthquakes (A. Kozak, A. Pachol, AW, arXiv:2310.00913)

Experiment	Reference	Upper bound on β
Phonon cavity	[399]	10^{46}
Harmonic oscillators	[400, 401]	10^{60}
LIV in hydrogen atom	[304]	10^{30}
Scanning tunneling microscope	[65, 373]	10^{33}
μ anomalous magnetic moment	[65, 402]	10^{33}
Hydrogen atom	[54, 57, 95, 395, 396]	10^{34}
Lamb shift	[65, 403]	10^{36}
^{87}Rb interferometry	[404, 405]	10^{39}
Kratzer potential	[90]	10^{46}
Stimulated emission	[110]	10^{46}
Landau levels	[65, 69, 403]	10^{50}
Quantum noise	[112]	10^{57}

22 See review by Bosso+ 2023 Class. Quantum Grav. 40 195014

Improving the method and future constraints

- Spherical-symmetric 1-dim Earth with adiabatic compression:
  - to introduce the complexities of Earth's true geometry (it rotates)
  - to estimate the equatorial moment of inertia relative to the polar moment by applying travel time ellipticity corrections to PREM[23]
  - to recognize the imperfections of layers and accounting for variable density jumps
  - to take into account a temperature variation with depth.
- Core description:
  - PREM does not describe well the boundaries of the outer and inner core
  - to use a more precise model like AK135-F[24] - it incorporates the complexities of core waves
  - to use equations of state for modeling core density and bulk moduli[25] (improving the uncertainties in density jumps at the inner and outer core boundaries).
- Birch law - a probable reevaluation when dealing with seismic data from Mars (the coefficients obtained experimentally).

[23] B. L. N. Kennett, O. Gudmundsson, Geophysical Journal International 127.1 (1996): 40-48.

[24] B. L. N. Kennett, E. R. Engdahl, R. Buland, Geophysical Journal International 122.1 (1995): 108-124.

[25] J. C. E. Irving, S. Cottaar, V Lekic, Science advances 4.6 (2018): eaar2538.

Improving the thermodynamic description and future plans

- To consider gravity effects in the elasti moduli and lattice description of the Earth's materials - corrections to the thermal energy (in progress)
- To take into account gravity effects in equations of state, melting and transport properties (in progress)
- To consider modified dispersion relation in the above calculations

I am spearheading an application, along with a number of colleagues, for COST Action 2024, with the goal of bringing together researchers in the areas of (quantum) gravity, seismology, and solid-state physics.

awojnar@ucm.es

CA FuSe: Testing Fundamental Physics with Seismology

Institutional distribution of Network of Proposers

Based on institutional affiliation deemed as most relevant to the Proposal by each Proposer.

85.1% Higher education, research-performing & associated organisations
6.4% Governmental organisation or agency (national, regional or local)
6.4% Business enterprise (small- or medium sized; SME)
2.1% European RTD organisation

Gender Distribution

59.6% Males
40.4% Females

Number of Young Researchers and Innovators

This figure takes into account only those Proposers who were under the age of 40 at the date of submission of this Proposal.

19

Core Expertise of Proposers: Distribution by Sub-Field of Science

The Core Expertise is defined by each Proposer at registration and it is the sub-field of science corresponding to the first resea selected.

46.8% Physical Sciences
23.4% Earth and related Environmental sciences
10.6% Computer and Information Sciences
10.6% Electrical engineering, electronic engineering, Information engineering
2.1% Mathematics
4.2% Other
2.1% Unspecified

Distribution of Affiliations

COST Full Member (21) : Austria, Croatia, Cyprus, Czech Republic, Estonia, France, Greece, Hungary, Italy, Lithuania, Malta, North Macedonia, Norway, Poland, Portugal, Romania, Serbia, Spain, Switzerland, Türkiye, United Kingdom
COST Cooperating Member (0)
COST Partner Member (0)
Near Neighbour Country (1) : Egypt
International Partner (1) : Brazil
European RTD Organisation (1)
EU Institutions, Bodies, Offices and Agencies (EC/EU) (0)
International Organisation (0)

COST Inclusiveness Target Countries

66.7 %

Number of Proposers

47

Summary and conclusions

- Tests of gravity with the use of stars and substellar objects (BD, (exo)-planets, seismology)
- We must be consistent in describing physical systems in different scales
- We should consider more realistic models on both sides: gravity and matter - rotating bodies, magnetic fields, ..., opacities (atmosphere), mirophysics description - to obtained better bounds and understand the gravity effects
- More research on matter properties in the MG and QG frameworks is necessary

Thanks!

awojnar@ucm.es

Palatini gravity in a nutshell

$$S = S_{\rm g} + S_{\rm m} = \frac{1}{2\kappa}\int \sqrt{-g} f(\hat{R}) d^4x + S_{\rm m}(g_{\mu\nu}, \psi_m),$$

where $\hat{R} = \hat{R}^{\mu\nu}(\hat{\Gamma})g_{\mu\nu}$. Modified field equations wrt $g_{\mu\nu}$ and $\hat{\Gamma}$ are

$$\begin{aligned} f'(\hat{R})\hat{R}_{\mu\nu} - \frac{1}{2}f(\hat{R})g_{\mu\nu} =& \kappa T_{\mu\nu}, \\ \hat{\nabla}_\beta(\sqrt{-g}f'(\hat{R})g^{\mu\nu}) =& 0 \quad \to \quad h_{\mu\nu} = f'(\hat{R})g_{\mu\nu}. \end{aligned}$$

The trace of the first MFE wrt $g_{\mu\nu}$ gives the structural equation

$$f'(\hat{R})\hat{R} - 2f(\hat{R}) = \kappa \mathcal{T},$$

where $\mathcal{T}$ is a trace of e-m tensor $T_{\mu\nu}$ wrt $g_{\mu\nu}$, provides $\hat{R} = \hat{R}(\mathcal{T})$.

- Non-linear system of a second order PDE.
- $f(\hat{R}) = \hat{R} - 2\Lambda$ is fully equivalent to the Einstein $R - 2\Lambda$.
- Any $f(\hat{R})$ vacuum solution $\to$ Einstein vacuum solution with the cosmological constant.
- Modifies non- and relativistic stellar structure equations[26].

[26] K. Kainulainen et al, PRD. 76 (2007) 043503; **AW**, EPJC 78 (2018) 421; **AW** EPJC 79 (2019) 51; A. Sergyeyev, **AW**, EPJC 80