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Quantum Phase of Matter: A Qutrit Perspective and Novel Order Parameter Discovery

Enrique Rico Ortega
Friday, 22/11/2024

Academia-Industry Matching Event (AIME24)

Nov 21 – 22, 2024
Mercure Budapest Castle Hill Hotel
Europe/Budapest timezone

Quantum Phase Transitions: A Qutrit Perspective and Novel Order Parameter Discovery

Enrique Rico Ortega

Mercure Budapest Castle Hill Hotel

09:45 - 10:30

**HUN
REN**



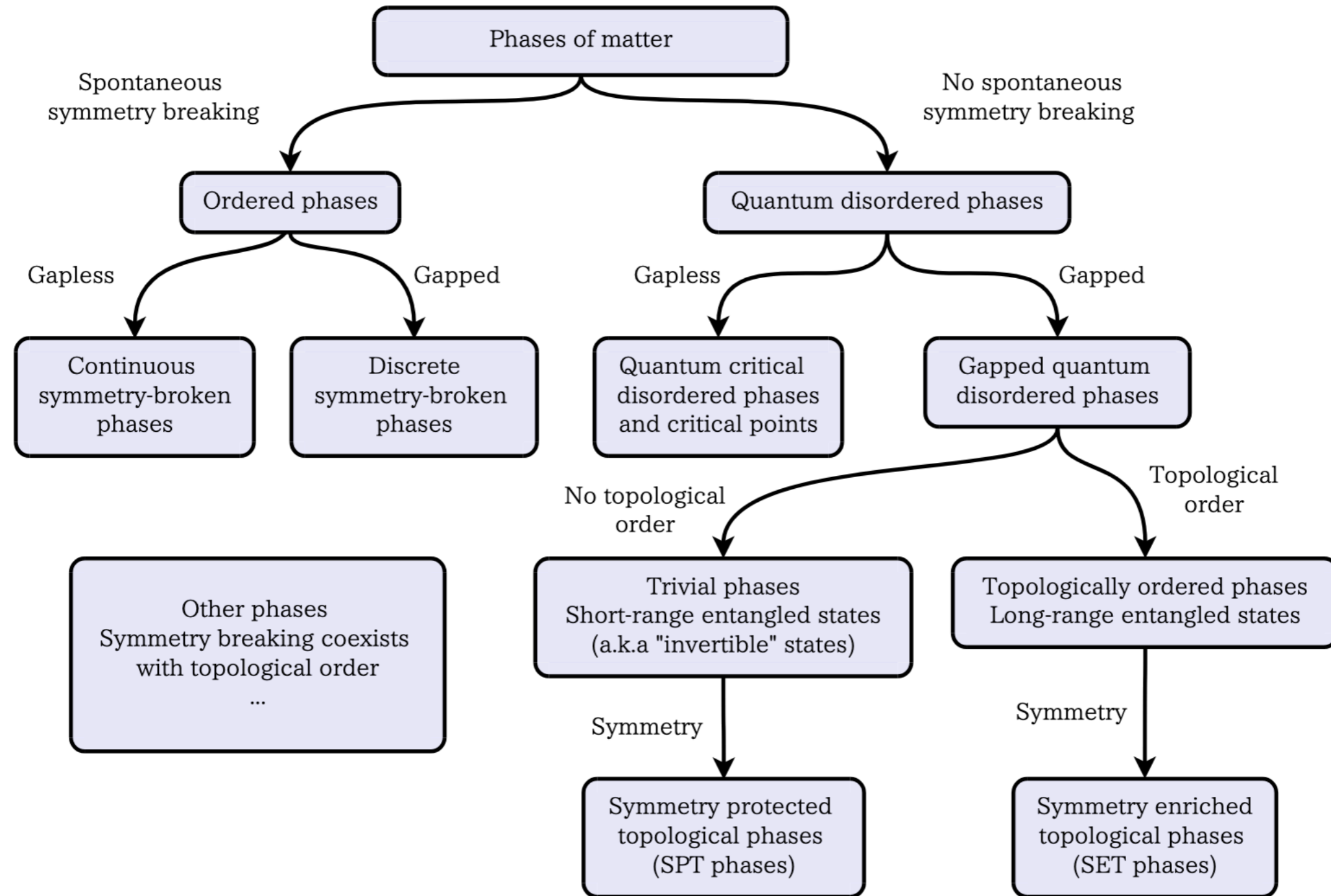
HUN-REN Wigner Research Centre for Physics



MTA
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Phases of matter



Talk given on August 23, 2021 at GGI.

"Topological phases of matter and quantum entanglement"

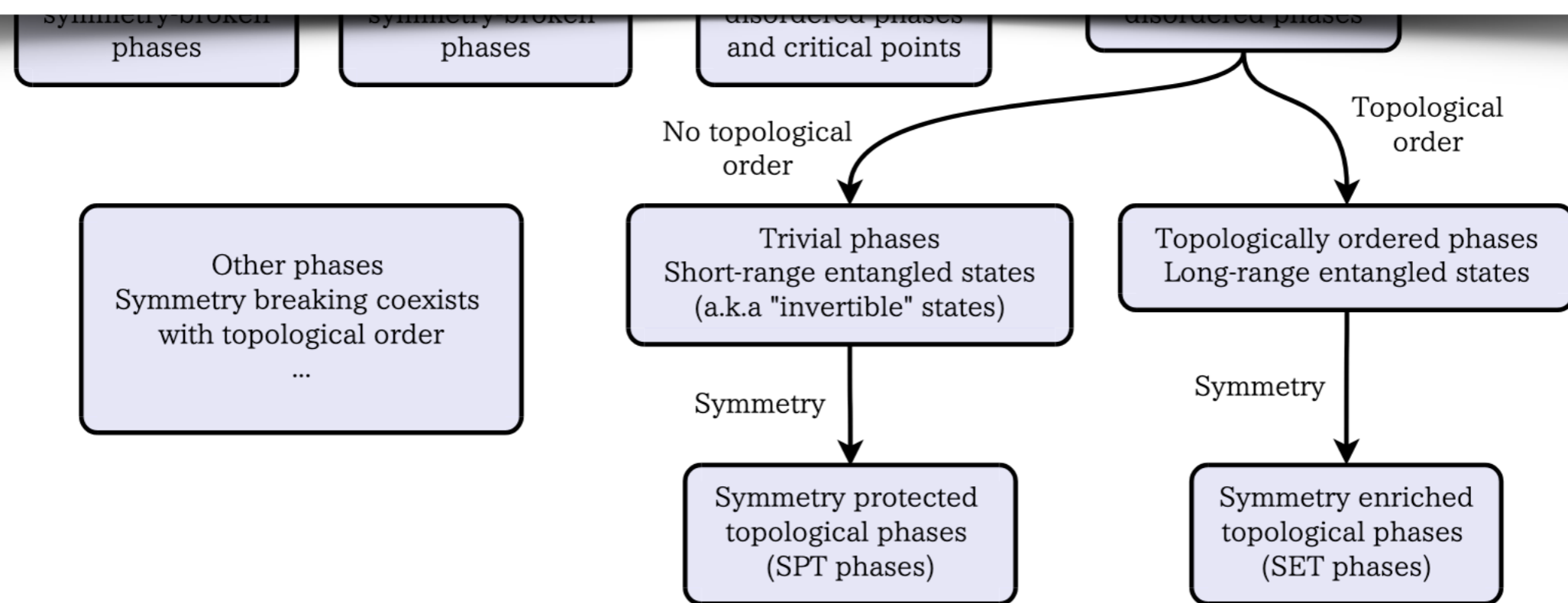
Shinsei Ryu (Princeton University)

Phases of matter

Two main questions:

How to engineer in the lab new Quantum Phases of Matter

How to characterise these new Quantum Phases



Phases of matter

Two main questions:

How to engineer in the lab new Quantum Phases of Matter

How to characterise these new Quantum Phases

symmetry broken
phases

symmetry broken
phases

disordered phases
and critical points

disordered phases

Topological

New tools from Quantum Technologies:

Quantum Simulators and Quantum Computers

Quantum Information ideas and measures



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Constructing the spin-1 Haldane phase on a qudit quantum processor

C. L. Edmunds,^{1,*} E. Rico,^{2,3,4,†} I. Arrazola,⁵ G. K. Brennen,⁶ M. Meth,¹ R. Blatt,^{1,7,8} and M. Ringbauer¹

 > quant-ph > arXiv:2408.04702

Constructing the spin-1 Haldane phase on a qudit quantum processor

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arXiv > quant-ph > arXiv:2408.04702

Order Parameter Discovery for Quantum Many-Body Systems

Nicola Mariella^{+,1,*} Tara Murphy^{+,1,2,†} Francesco Di Marcantonio,^{3,‡} Khadijeh Najafi,^{4,§} Sofia Vallecorsa,^{5,¶} Sergiy Zhuk,^{1,**} and Enrique Rico^{3,6,7,††}

arXiv > quant-ph > arXiv:2408.01400

What is the Haldane phase?

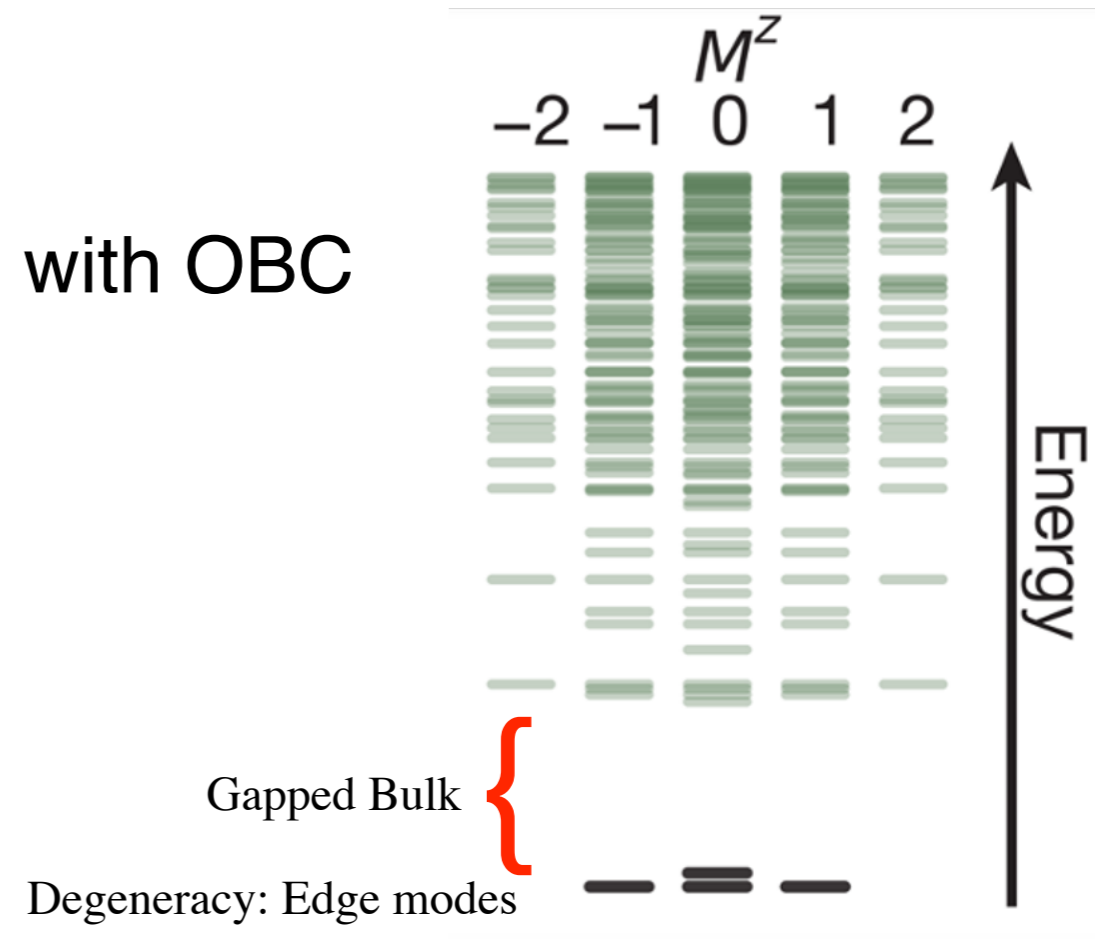
Symmetry Protected Topological Phases

$$\hat{H} = + \sum_i \vec{S}_i \vec{S}_{i+1}$$

- Gapped spectrum to next excited state
- Four-fold (nearly) degenerate ground state with OBC
- Protected by a $Z_2 \times Z_2$ spin symmetry

→ First example of a SPT phase

→ Against all known evidence at the time



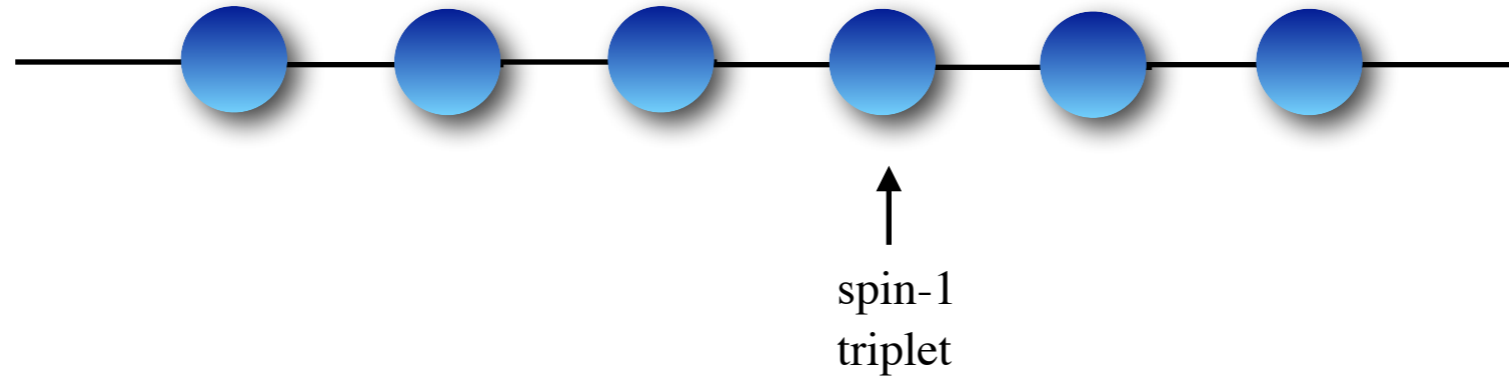
What is the Haldane phase?

Emerging experimental evidence

- Prior heuristic experimental evidence seemed to suggest that isotropic (rotationally symmetric) Hamiltonians were always gapless
- “Natural” spin-one crystals were made, and all evidence controversially pointed towards an energy gap
CsNiCl₃ (1985)
Ni(C₂H₈N₂)₂NO₂(ClO₄) (1990)
- Analog simulation in an ultra-cold atom Fermi-Hubbard ladder (2021)
- We are now performing the first digital simulation of Haldane using trapped ions

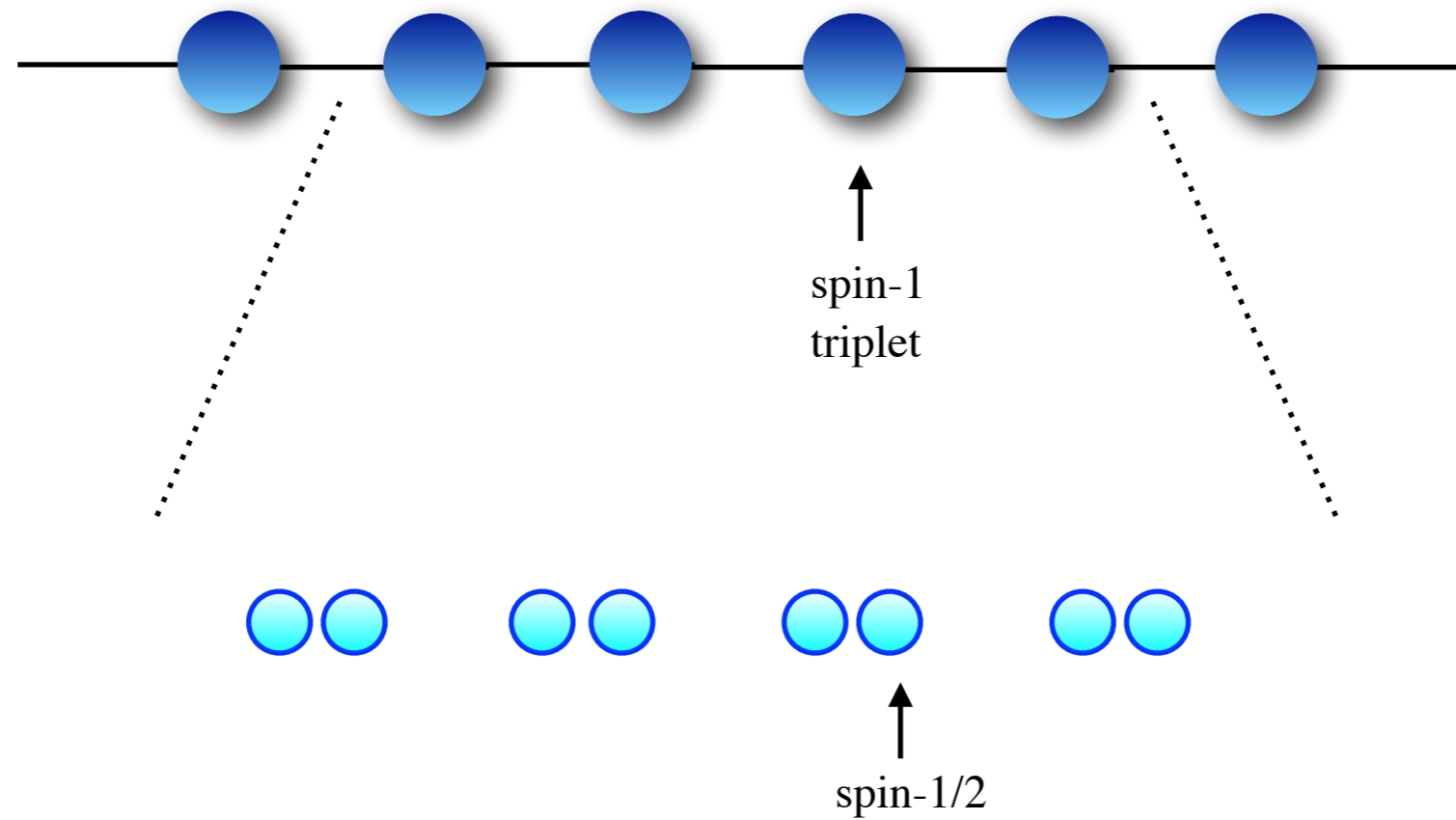
Simulating the Haldane phase ground state

$$\hat{H}_{\text{AKLT}} = \sum_i \left[\vec{S}_i \vec{S}_{i+1} + \frac{1}{3} \left(\vec{S}_i \vec{S}_{i+1} \right)^2 \right] \propto \sum_i P_{S=2} \left(\vec{S}_i + \vec{S}_{i+1} \right)$$



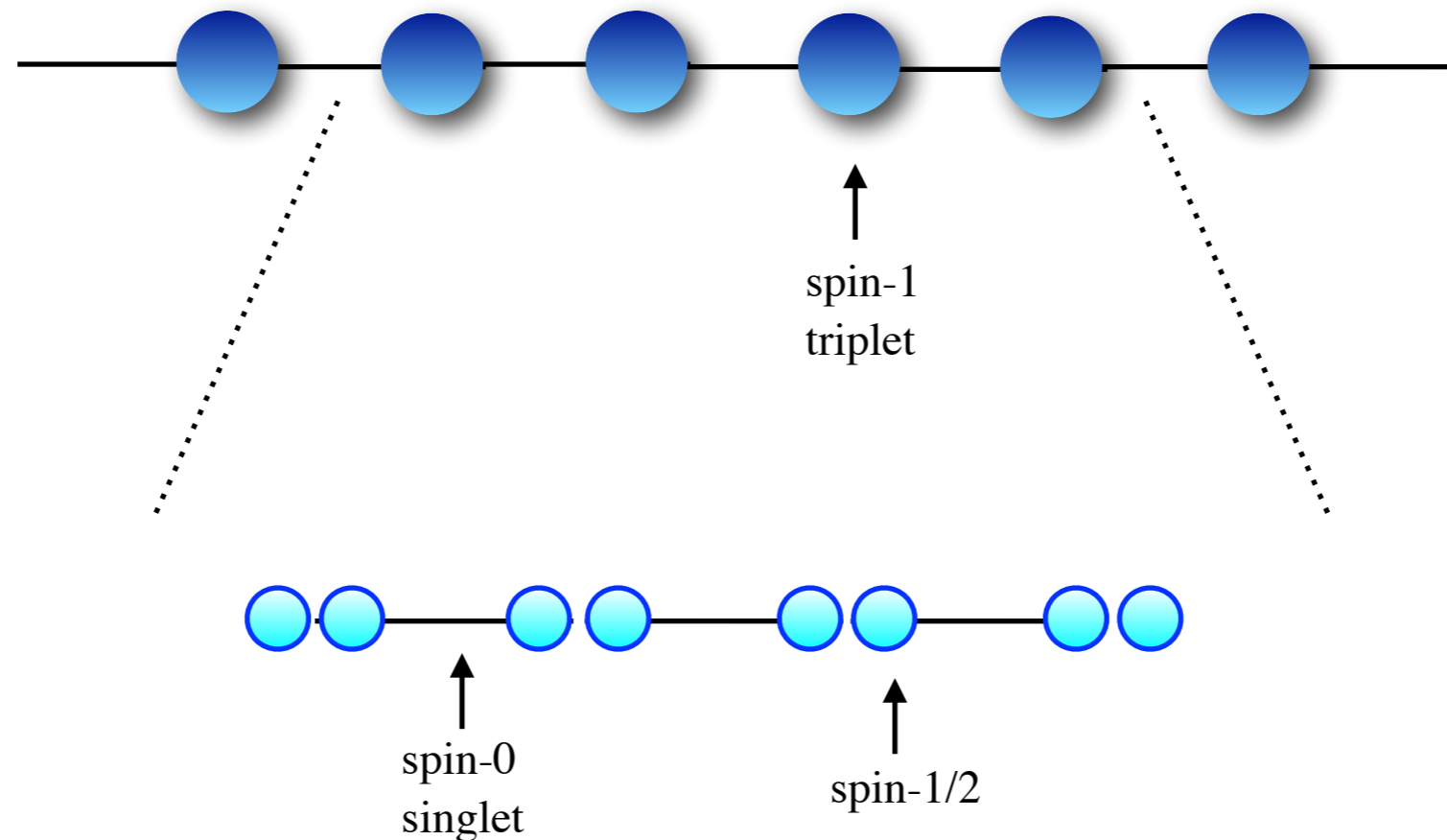
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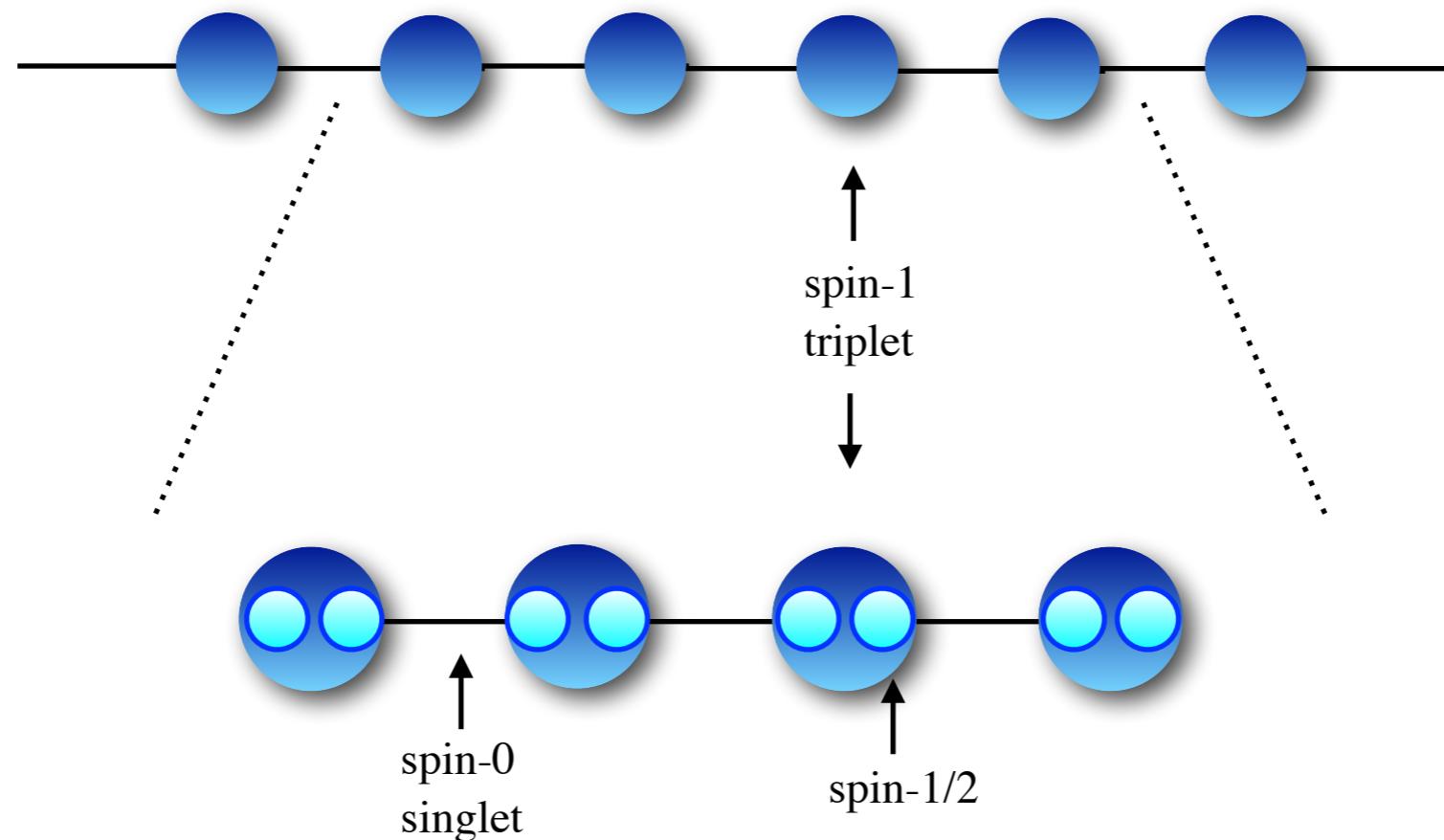
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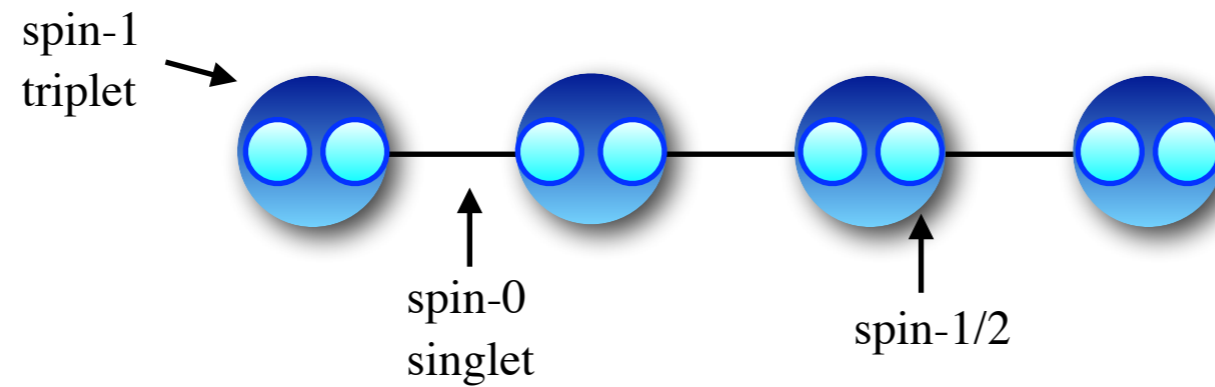
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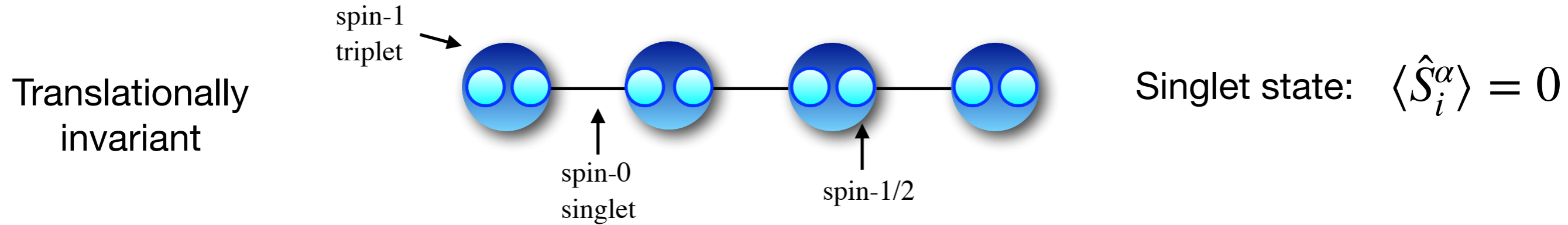
Simulating the Haldane phase ground state

Translationally
invariant



Singlet state: $\langle \hat{S}_i^\alpha \rangle = 0$

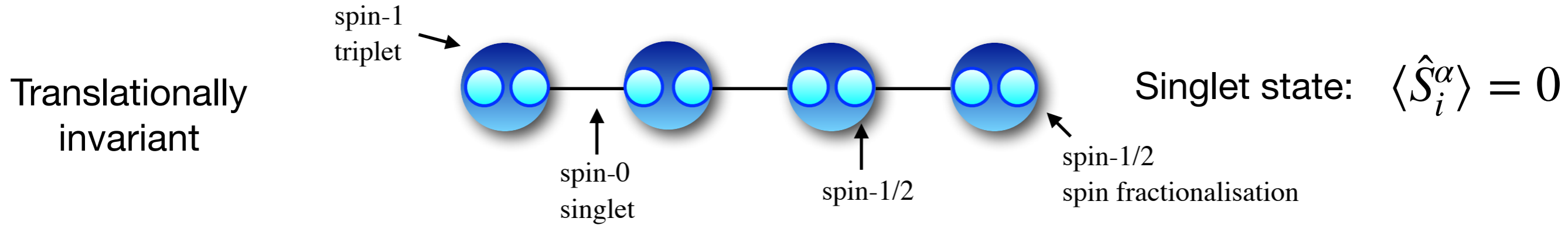
Simulating the Haldane phase ground state



No long-range order: Finite correlation length.
Exponential decay of two point correlation function.

$$\langle \hat{S}_i^\alpha \hat{S}_j^\beta \rangle = \delta_{\alpha\beta} (-1)^{i+j} e^{-|i-j|/\xi}$$

Simulating the Haldane phase ground state



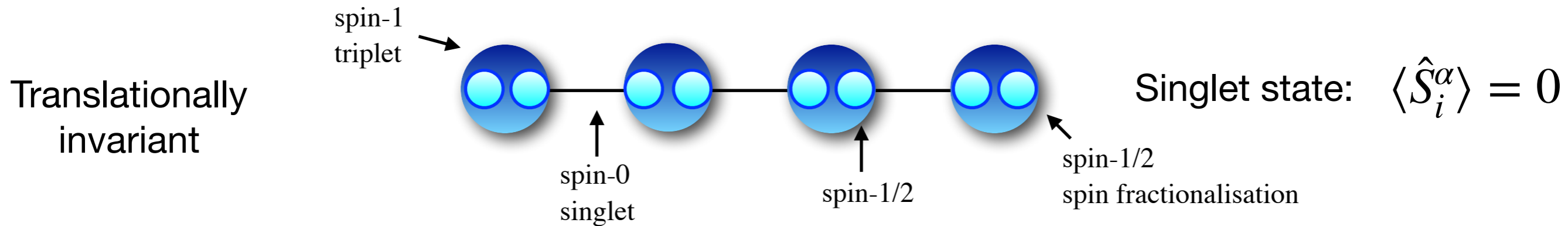
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Composition rules: PBC = Unique state; OBC = 4-fold degeneracy

$$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1 \quad 0 \otimes \frac{1}{2} = \frac{1}{2} \quad \frac{1}{2} \otimes 1 = \frac{1}{2} \Rightarrow SU(2)_2$$

Simulating the Haldane phase ground state



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Non-local order parameter.- String order parameter and entanglement length
den Nijs, Rommelse (1989)

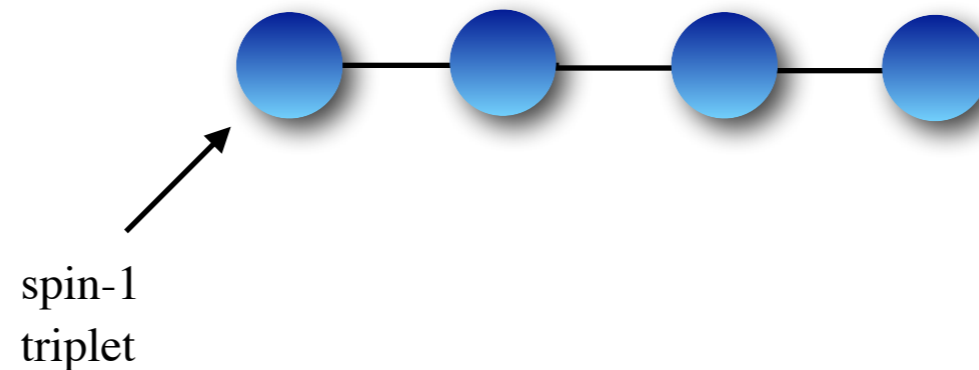
Cirac, Martin-Delgado, Popp, Verstraete (2005)

$$O_{\text{str}}^\alpha = \lim_{|i-j| \rightarrow \infty} \langle \hat{S}_i^\alpha e^{i\pi \sum_{k=i+1}^{j-1} \hat{S}_k^\alpha} \hat{S}_j^\alpha \rangle$$

Simulating the Haldane phase ground state

Non-local order parameter.- Entanglement length

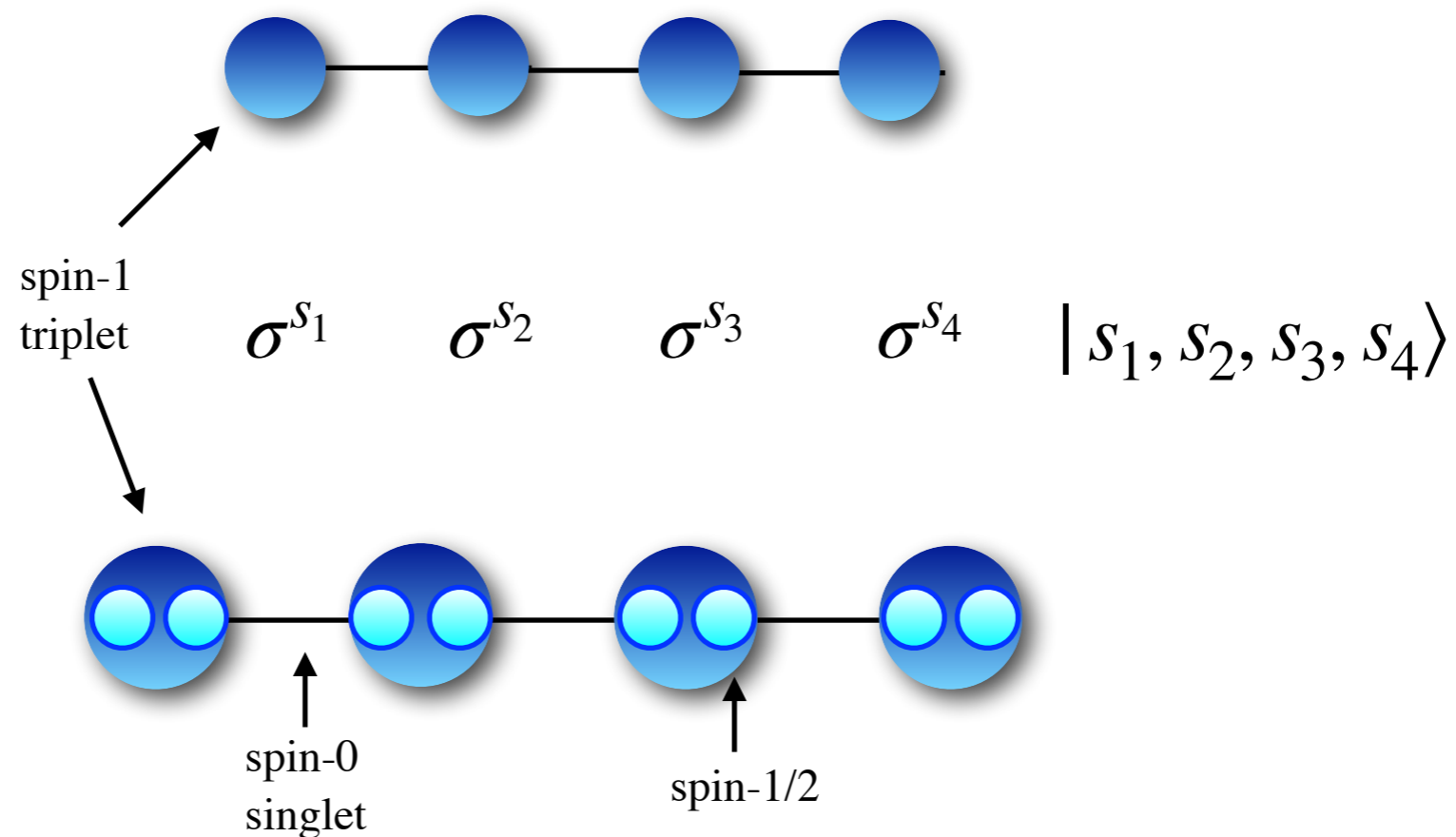
$$|AKLT\rangle = \sum_{\{s\}} A^{s_1} A^{s_2} \dots A^{s_N} |s_1, s_2, \dots, s_N\rangle$$



Simulating the Haldane phase ground state

Non-local order parameter.- Entanglement length

$$|AKLT\rangle = \sum_{\{s\}} A^{s_1} A^{s_2} \dots A^{s_N} |s_1, s_2, \dots, s_N\rangle$$



Simulating the Haldane phase ground state

Non-local order parameter.- String order parameter

$$e^{i\theta S_{\text{tot}}^\alpha} = e^{i\theta S_1^\alpha} e^{i\theta S_2^\alpha} e^{i\theta S_3^\alpha} e^{i\theta S_4^\alpha}$$

spin-1
triplet



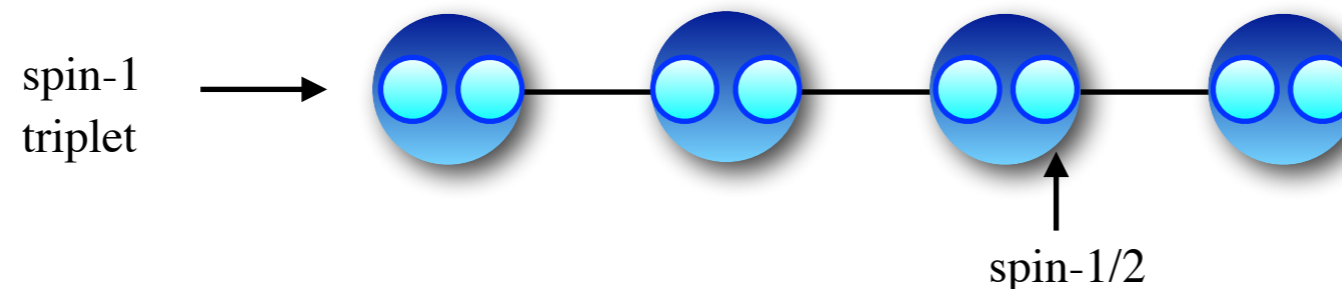
Simulating the Haldane phase ground state

Non-local order parameter.- String order parameter

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$$e^{i\theta S_{\text{tot}}^\alpha} \sim e^{i\theta \sigma_1^\alpha / 2} e^{i\theta \sigma_2^\alpha / 2} e^{i\theta \sigma_3^\alpha / 2} e^{i\theta \sigma_4^\alpha / 2} e^{i\theta \sigma_5^\alpha / 2} e^{i\theta \sigma_6^\alpha / 2} e^{i\theta \sigma_7^\alpha / 2} e^{i\theta \sigma_8^\alpha / 2}$$



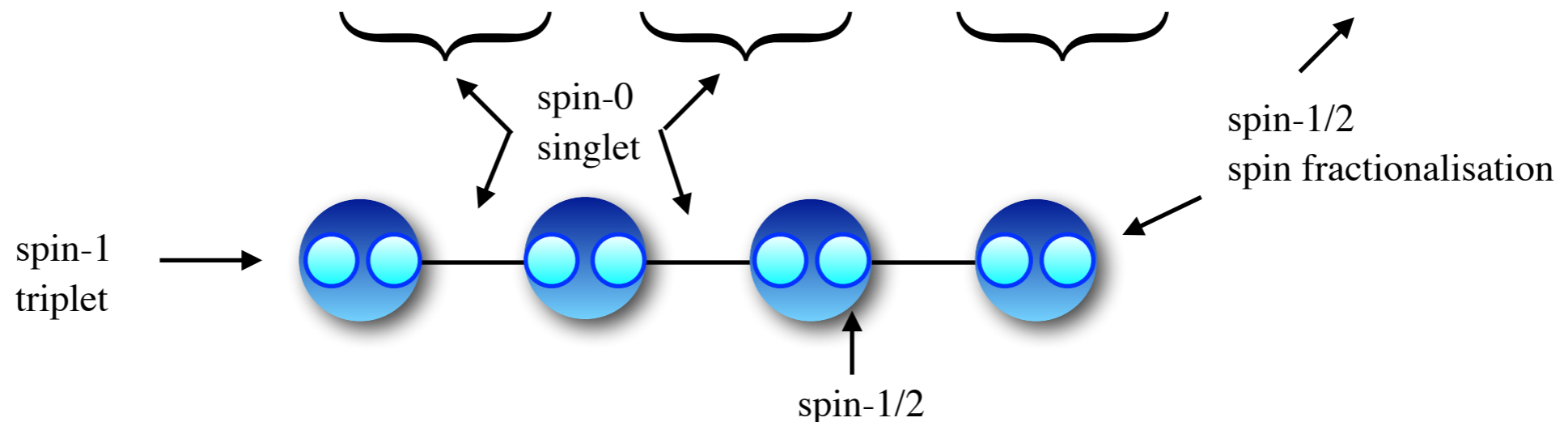
Simulating the Haldane phase ground state

Non-local order parameter.- String order parameter

$$e^{i\theta S_{\text{tot}}^\alpha} = e^{i\theta S_1^\alpha} e^{i\theta S_2^\alpha} e^{i\theta S_3^\alpha} e^{i\theta S_4^\alpha}$$



$$e^{i\theta S_{\text{tot}}^\alpha} \sim e^{i\theta \sigma_1^\alpha / 2} e^{i\theta \sigma_2^\alpha / 2} e^{i\theta \sigma_3^\alpha / 2} e^{i\theta \sigma_4^\alpha / 2} e^{i\theta \sigma_5^\alpha / 2} e^{i\theta \sigma_6^\alpha / 2} e^{i\theta \sigma_7^\alpha / 2} e^{i\theta \sigma_8^\alpha / 2}$$



Simulating the Haldane phase ground state

Non-local order parameter.- String order parameter

$$\hat{H}_{\text{cluster}} = - \sum_i \hat{\sigma}_{i-1}^z \hat{\sigma}_i^x \hat{\sigma}_{i+1}^z$$

$$[\hat{\sigma}_{i-1}^z \hat{\sigma}_i^x \hat{\sigma}_{i+1}^z, \hat{\sigma}_{j-1}^z \hat{\sigma}_j^x \hat{\sigma}_{j+1}^z] = 0 \quad \hat{\sigma}_{i-1}^z \hat{\sigma}_i^x \hat{\sigma}_{i+1}^z |gs\rangle = |gs\rangle \quad \text{Stabiliser-State}$$

$$Z_2 \times Z_2 \text{ symmetry:} \quad \hat{X}_{\text{even}} = \hat{\sigma}_2^x \hat{\sigma}_4^x \cdots \hat{\sigma}_{2N}^x \quad \hat{X}_{\text{odd}} = \hat{\sigma}_1^x \hat{\sigma}_3^x \cdots \hat{\sigma}_{2N-1}^x$$

Simulating the Haldane phase ground state

Non-local order parameter.- String order parameter

$$\hat{H}_{\text{cluster}} = - \sum_i \hat{\sigma}_{i-1}^z \hat{\sigma}_i^x \hat{\sigma}_{i+1}^z$$

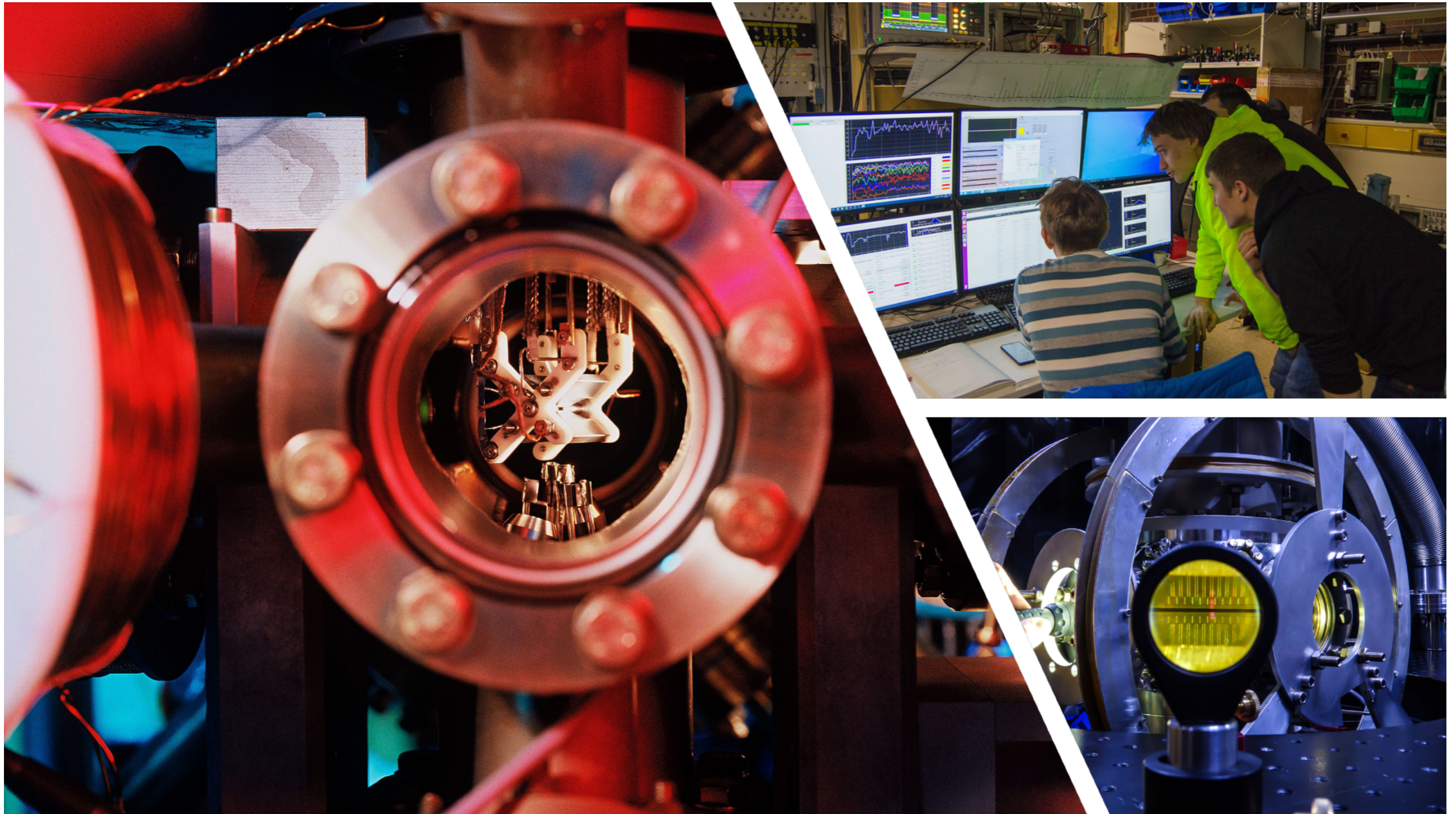
$$[\hat{\sigma}_{i-1}^z \hat{\sigma}_i^x \hat{\sigma}_{i+1}^z, \hat{\sigma}_{j-1}^z \hat{\sigma}_j^x \hat{\sigma}_{j+1}^z] = 0 \quad \hat{\sigma}_{i-1}^z \hat{\sigma}_i^x \hat{\sigma}_{i+1}^z | \text{gs} \rangle = | \text{gs} \rangle \quad \text{Stabiliser-State}$$

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$$\left(\hat{\sigma}_{2N}^z \right) \left(\hat{\sigma}_1^x \hat{\sigma}_2^z \right) \Big|_{\text{g.s.}} = \hat{X}_{\text{odd}} \quad \left(\hat{\sigma}_{2N-1}^z \hat{\sigma}_{2N}^x \right) \left(\hat{\sigma}_1^z \right) \Big|_{\text{g.s.}} = \hat{X}_{\text{even}}$$

$$\text{Projective symmetry representation at the boundary: } \{ \hat{\sigma}_1^x \hat{\sigma}_2^z, \hat{\sigma}_1^z \} = 0$$

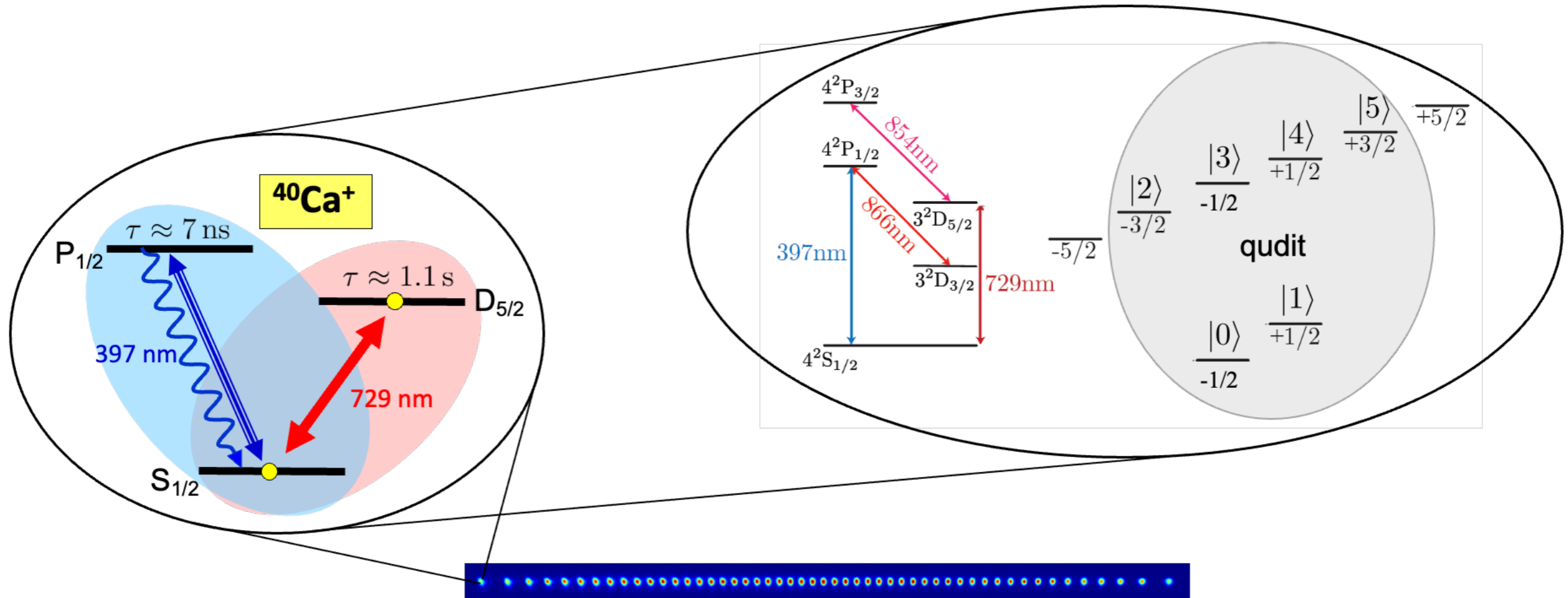
Trapped $^{40}\text{Ca}^+$ ion “qudits”



M. Ringbauer et al., arxiv:2109.06903 (2021)

Y. Chi et al., Nature Communications 13, 1166 (2022)

Trapped $^{40}\text{Ca}^+$ ion “qudits”



Directly simulate non-spin- $1/2$ systems in nature by utilising extra levels in ions

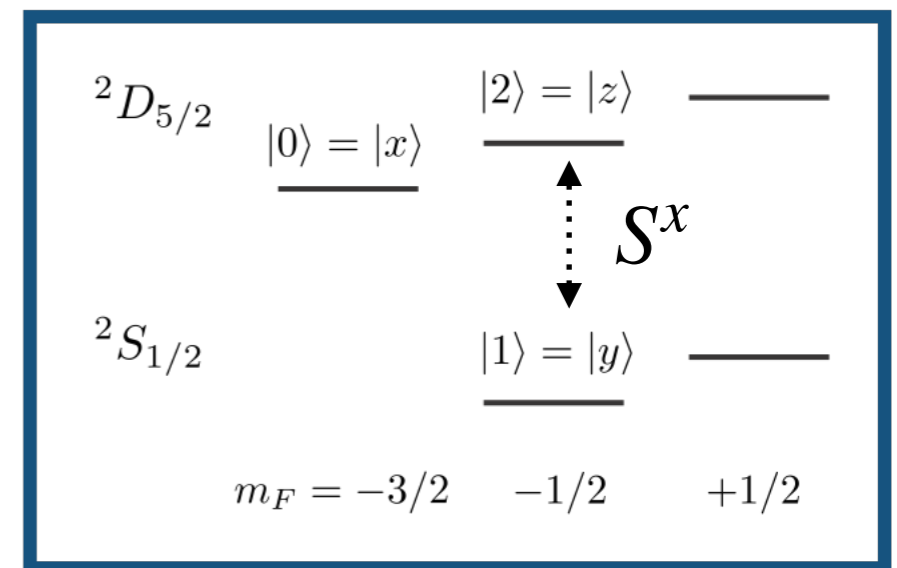
Creating the Spin-1 AKLT chain

AKLT Spin-1 Basis Definition

$$\hat{H}_{\text{AKLT}} = \sum_i \left[\vec{S}_i \vec{S}_{i+1} + \frac{1}{3} \left(\vec{S}_i \vec{S}_{i+1} \right)^2 \right]$$

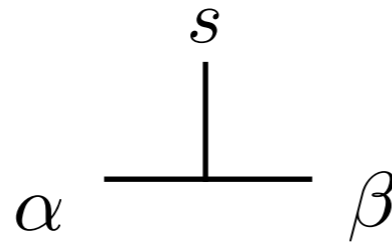
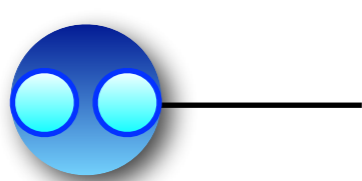
$$S^\alpha |\beta\rangle = i\epsilon^{\alpha\beta\gamma} |\gamma\rangle, \quad S^x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$^{40}\text{Ca}^+$ energy level encoding



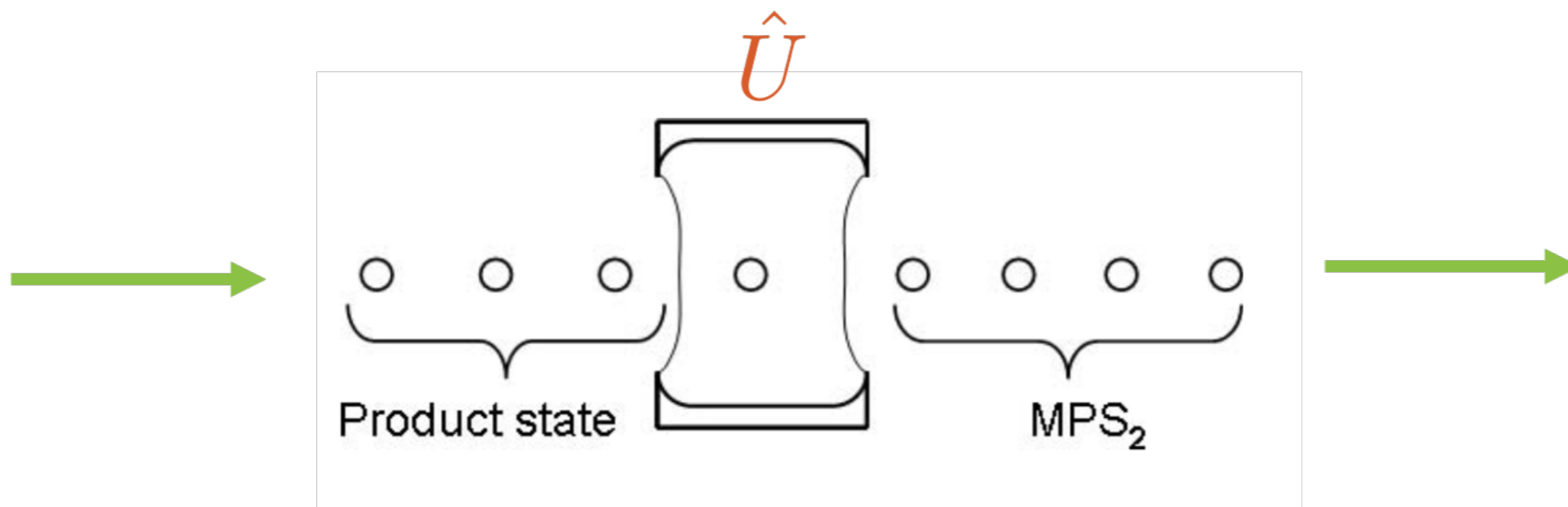
The state $|\alpha\rangle$ is the zero-projection eigenvectors of the operator S^α

Sequential Preparation of the AKLT ground state

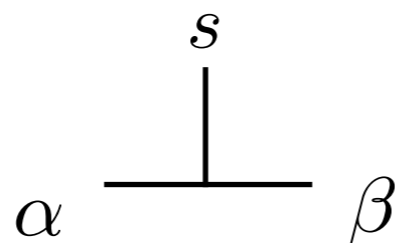
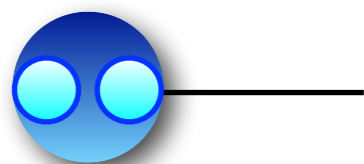


$$|\alpha\rangle = \sum_{s,\beta} \sigma_{\alpha\beta}^s |s, \beta\rangle$$

- The AKLT ground state is a matrix product state \rightarrow equivalent to a sequentially generated state
- Uncorrelated atoms pass sequentially through a “cavity” and couple to the cavity mode
- Choosing an appropriate coupling unitary will generate the AKLT state



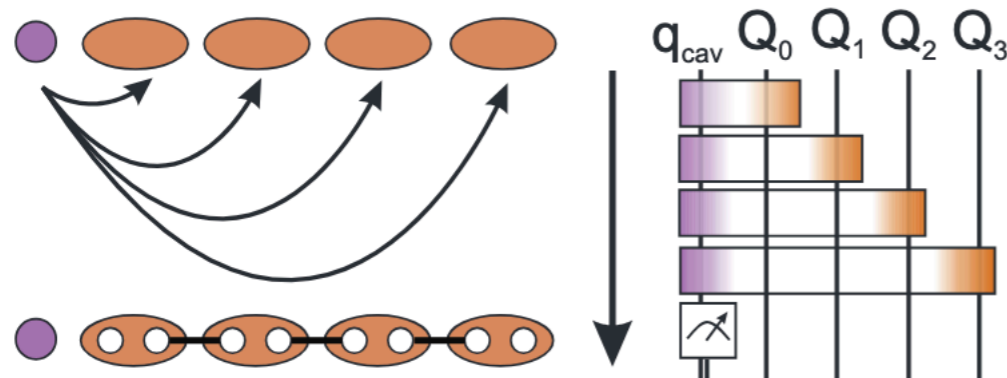
MPS Coupling Gate



$$|\alpha\rangle = \sum_{s,\beta} \sigma_{\alpha\beta}^s |s, \beta\rangle$$

Control-unitary: spin-1/2 unitary controlled by spin-1

$$|\beta\rangle \sum_s |s\rangle \mapsto |\alpha\rangle = \sum_{s,\beta} \sigma_{\alpha\beta}^s |s, \beta\rangle$$



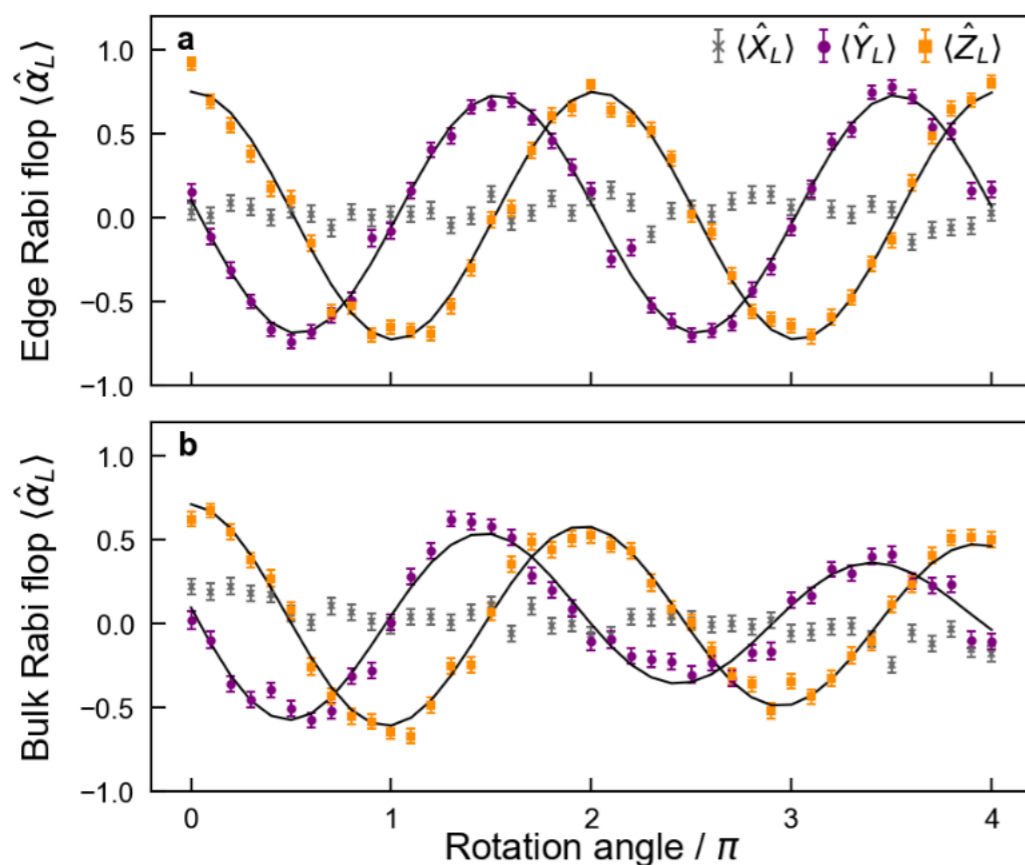
45 local gates (fidelity ~ 0.999)
2 entangling gate (fidelity ~ 0.985)

$$\text{Fidelity} \sim (0.99945 * 0.9852)^N$$

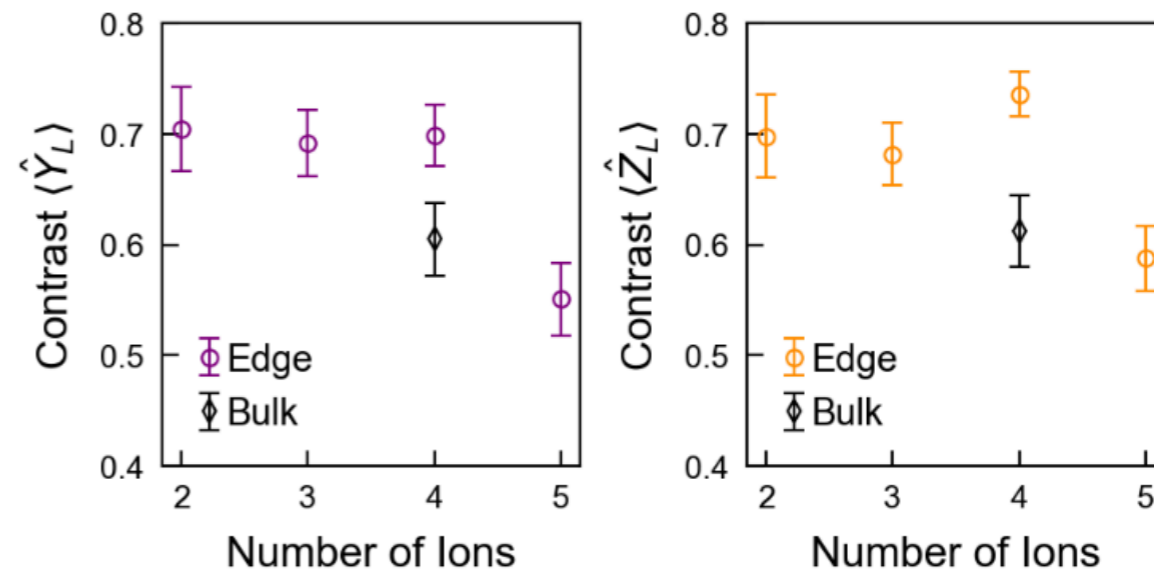
$$= \{0.86, 0.80, 0.74, 0.69\} \text{ for } N = \{2, 3, 4, 5\}$$

Bulk - edge correspondence

Rabi flops of the Fractionalized edge qubit



Rabi flop contrast



Summary and outlook

- Using qutrits, we can directly create the ground state of a spin-1 AKLT chain
- We observe the critical features of an SPT, in particular, spin fractionalisation of the spin-1 chain into two qubits on the boundaries
- The AKLT state is a perfect quantum repeater and has interesting properties for one-way quantum computation
- Looking at ways to explore more complex SPTs, such as higher dimensional spin-1 lattices using qudits

The Team



 Federal Ministry
Republic of Austria
Education, Science
and Research



I. Arrazola.- PostDoc researcher at IFT-UAM

G. K. Brennen.- Professor at Macquarie University

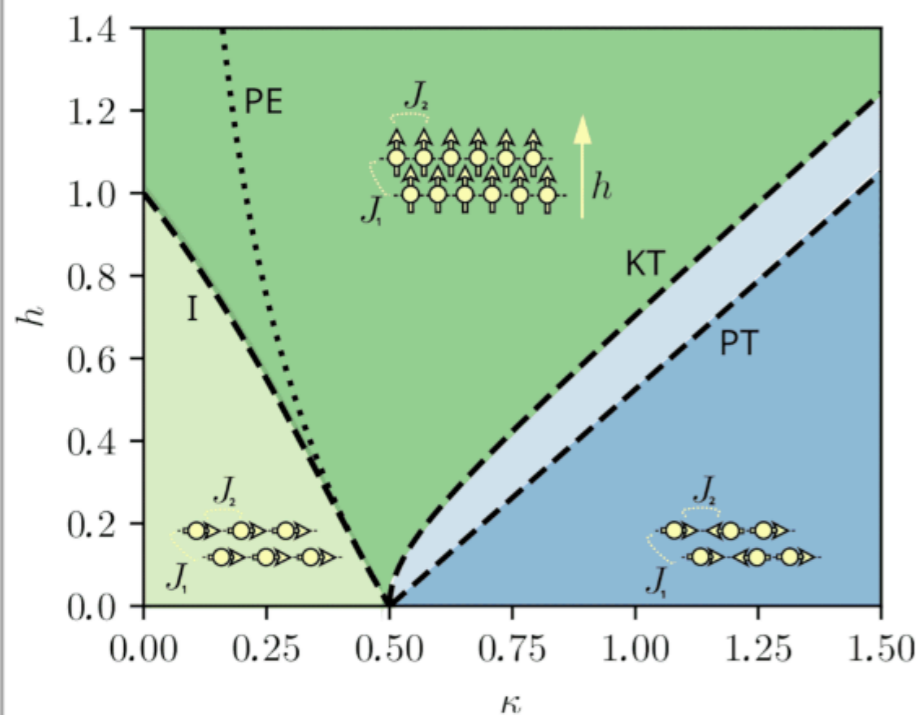
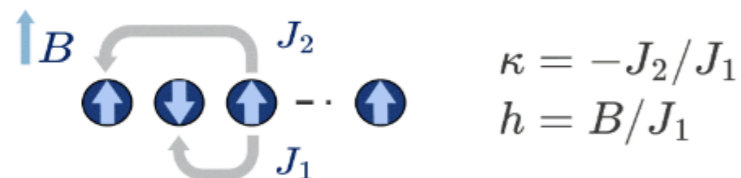
C. L. Edmunds, M. Meth, R. Blatt, M. Ringbauer.- Qudit group at trapped ion experiments in Innsbruck

The problem:

Consider some Hamiltonian depending on some coupling parameters

$$H(\kappa, h) = - \sum_i S_i^x S_{i+1}^x + \kappa \sum_i S_i^x S_{i+2}^x + h \sum_i S_i^z$$

ANNNI model



Objective: design a mathematical tool
for classifying different phases of matter
(classification)

for detecting the phase transitions
(anomaly detection)

The problem:

Phase transition happens when a “smooth variation” of a Hamiltonian as a function of coupling parameters leads to an abrupt “change” of a ground state.

Phases of matter are usually classified using so called order parameters: e.g. averaged magnetisation is the order parameter for transverse field Ising model.

With the general behaviour: ordered phase (order parameter is different from zero) and disordered phase (order parameter equals to zero). Phase transition at the boundary between phases.

How to classify phases and detect phase transitions of a quantum system without tailored “order parameters”?

Visualisation of the uses of the reduced fidelity susceptibility

Possible way-out
fidelity susceptibility:

$$\langle \psi(\lambda) | \psi(\lambda + \nu) \rangle$$

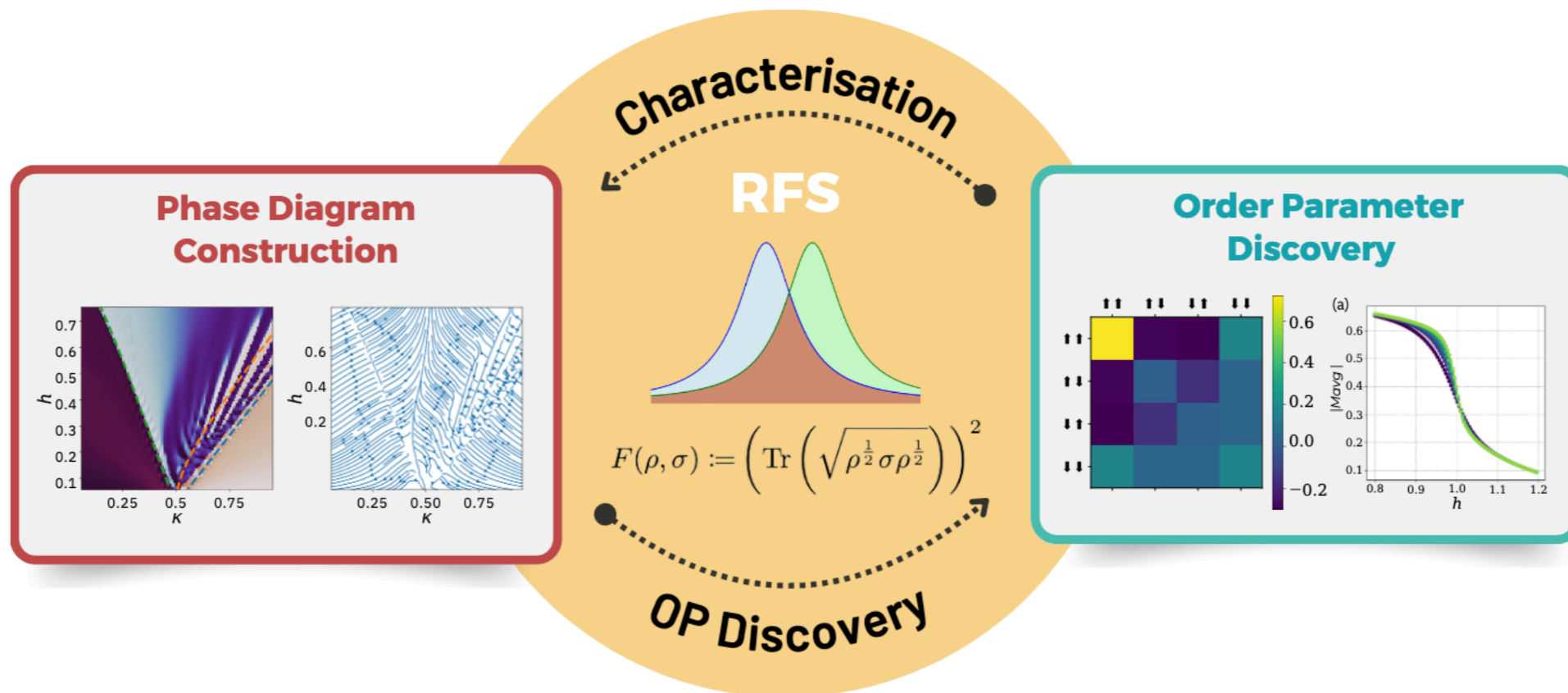
untrainable in a many-body
situation: diagonal dominance,
orthogonality catastrophe

Visualisation of the uses of the reduced fidelity susceptibility

Possible way-out fidelity susceptibility:

$$\langle \psi(\lambda) | \psi(\lambda + \nu) \rangle$$

untrainable in a many-body situation: diagonal dominance, orthogonality catastrophe



$$F(\rho, \sigma) = \left[\text{Tr} \left(\sqrt{\rho^{1/2} \sigma \rho^{1/2}} \right) \right]^2$$

The tools:



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The Uhlmann-Jozsa fidelity for density matrices:

$$F(\rho, \sigma) = \left[\text{Tr} \left(\sqrt{\rho^{1/2} \sigma \rho^{1/2}} \right) \right]^2$$

and let:

$$f(\lambda, \nu) = \sqrt{F(\rho(\lambda), \rho(\lambda + \nu))} \rightarrow 1 - \frac{\partial^2}{\partial \nu^2} f \Big|_{\nu=0} \frac{\nu^2}{2}$$

The tools:



The Uhlmann-Jozsa fidelity for density matrices:

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we define:

$$g(\lambda) = - \sum_i \frac{\partial^2}{\partial \nu_i^2} f \Big|_{\nu=0}$$

and the gradient field:

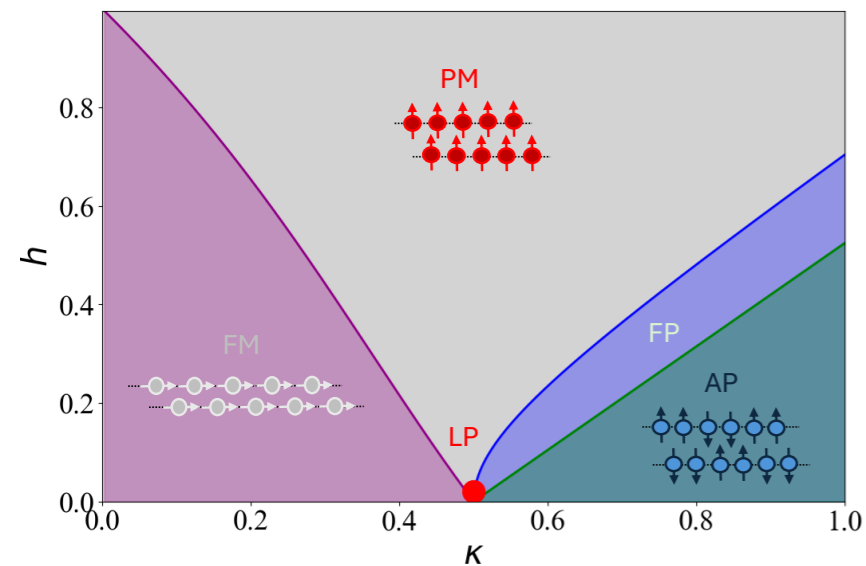
$$P_i(\lambda) = \frac{\partial}{\partial \lambda_i} g(\lambda)$$

First results:

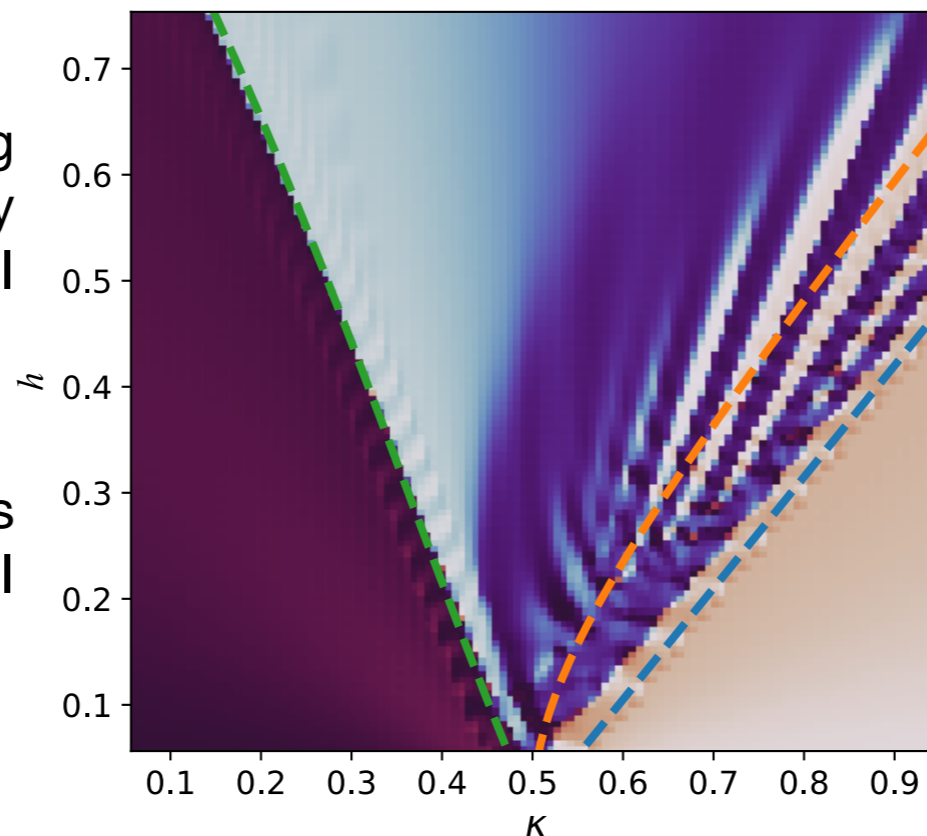
In the reduced fidelity susceptibility vector field:

Sources are phase transitions and

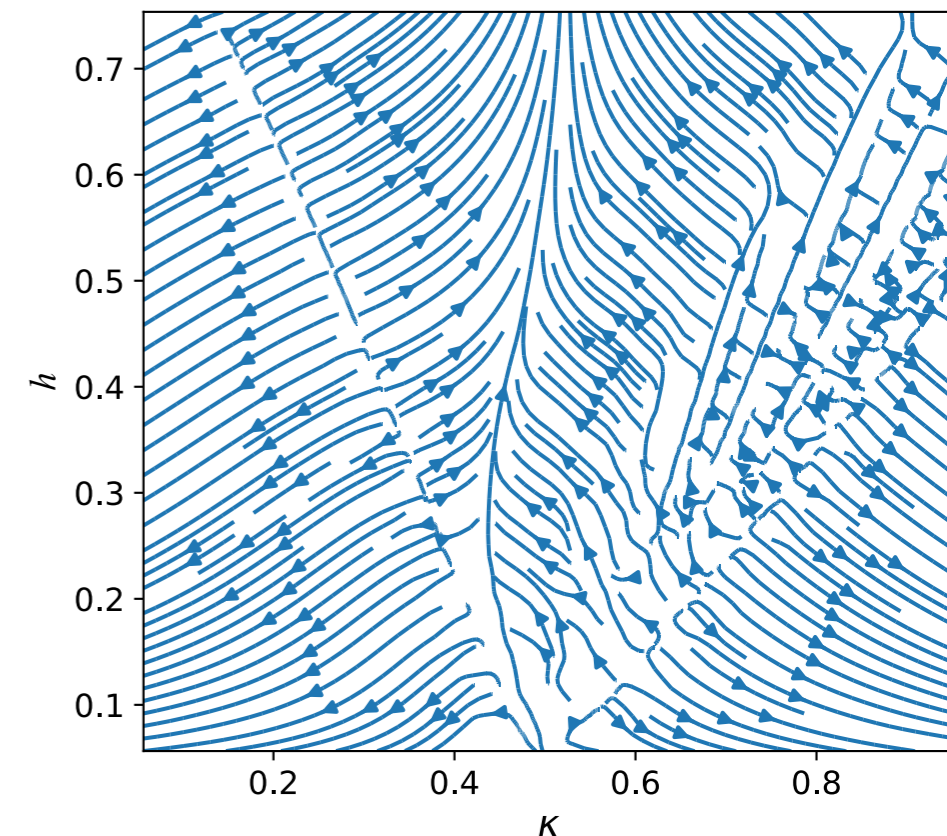
Sinks are representative ground states for the corresponding phase.



(a)



(b)



Phase Diagram obtained using the reduced fidelity susceptibility of the one-dimensional ANNNI Model

The plots are given by the angles of the vector field for the ANNNI model with 50 spins (DMRG)

Order parameter discovery:



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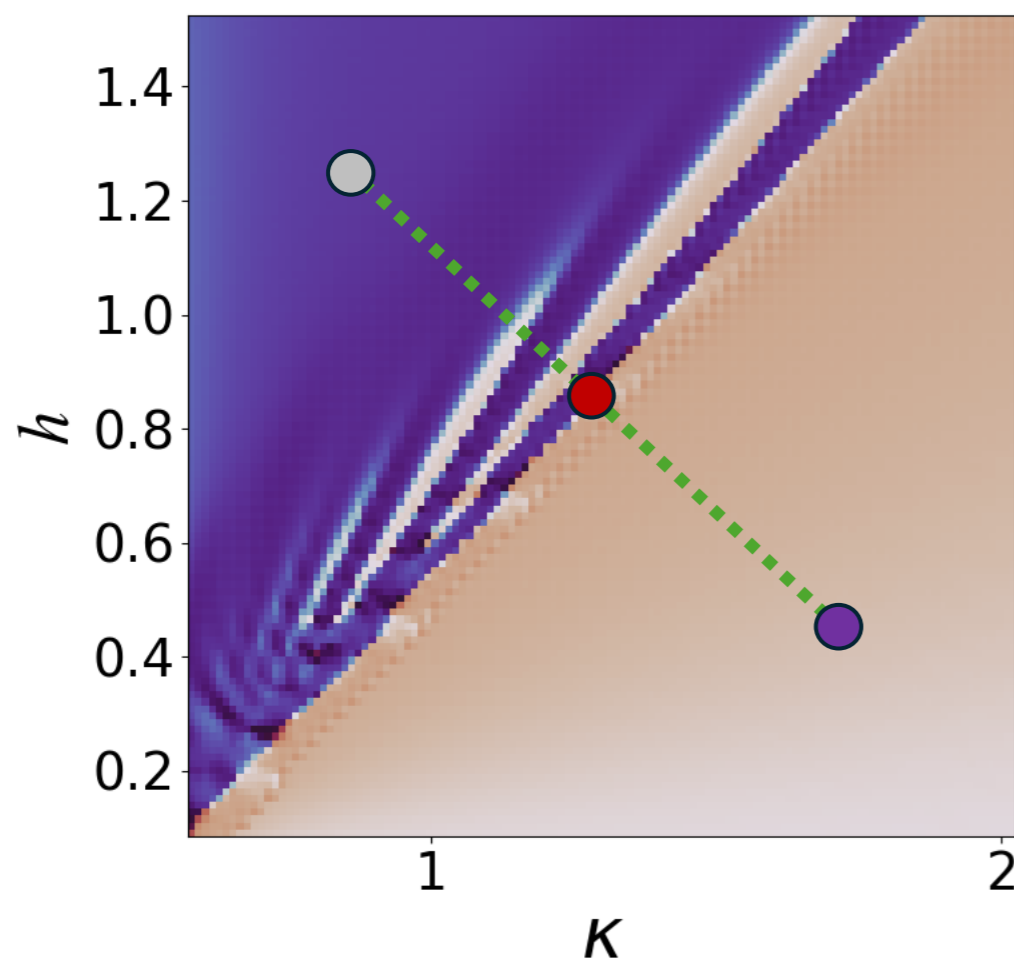


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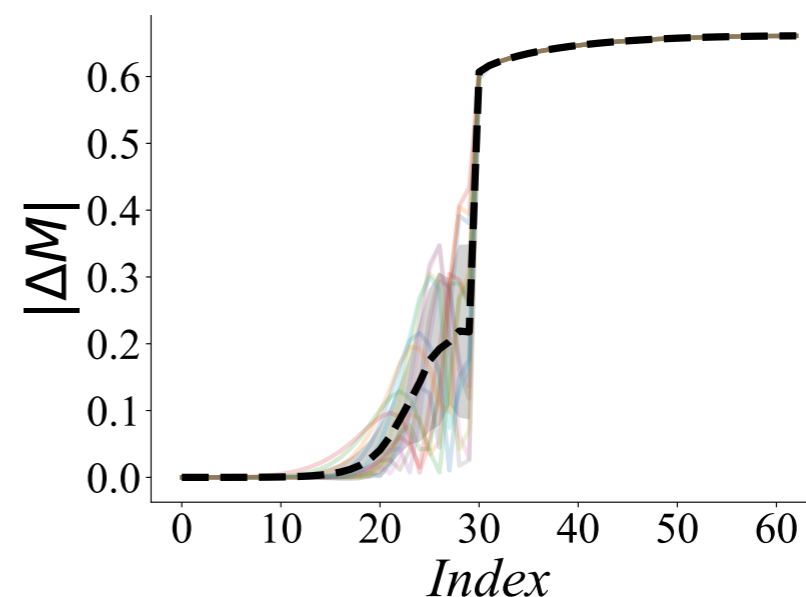
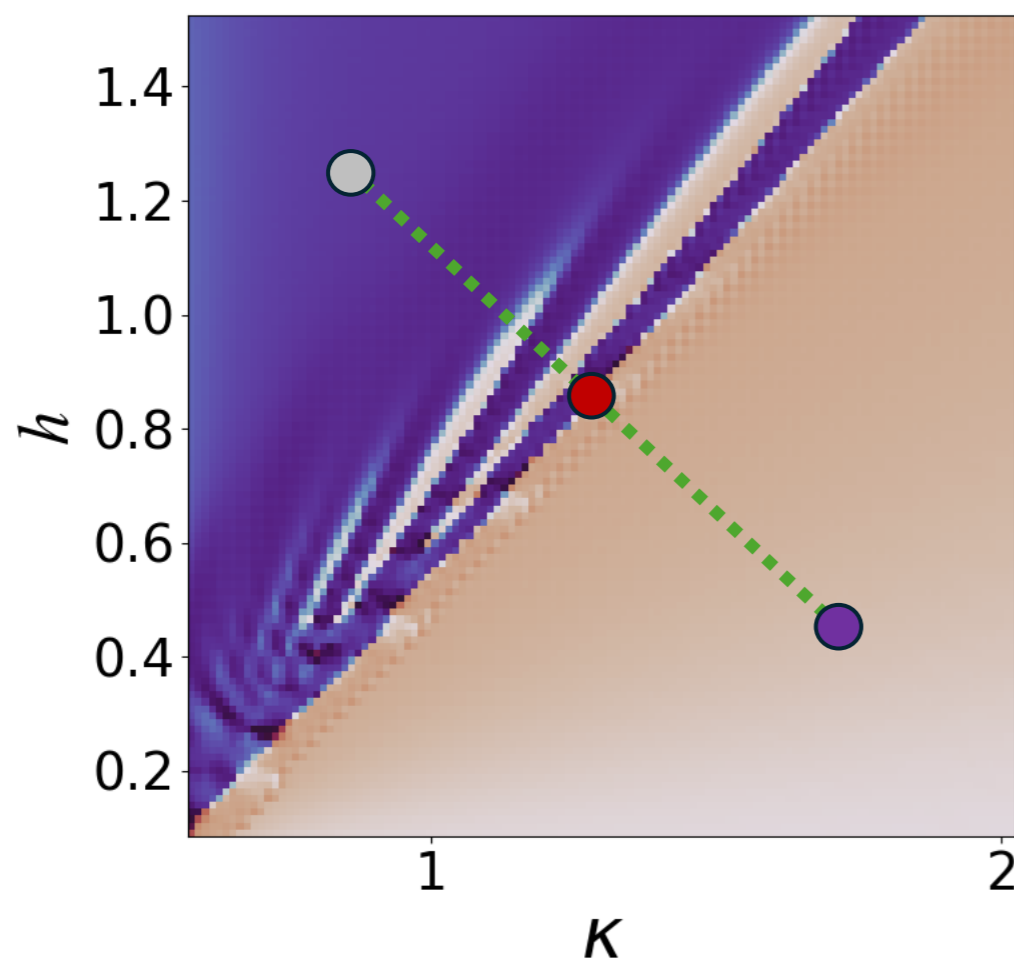
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The order parameter for a given system is not unique; any thermodynamic variable that is zero in the un-ordered phase and non-zero in an adjacent (on the phase diagram), usually ordered phase, is a possible choice for an order parameter.



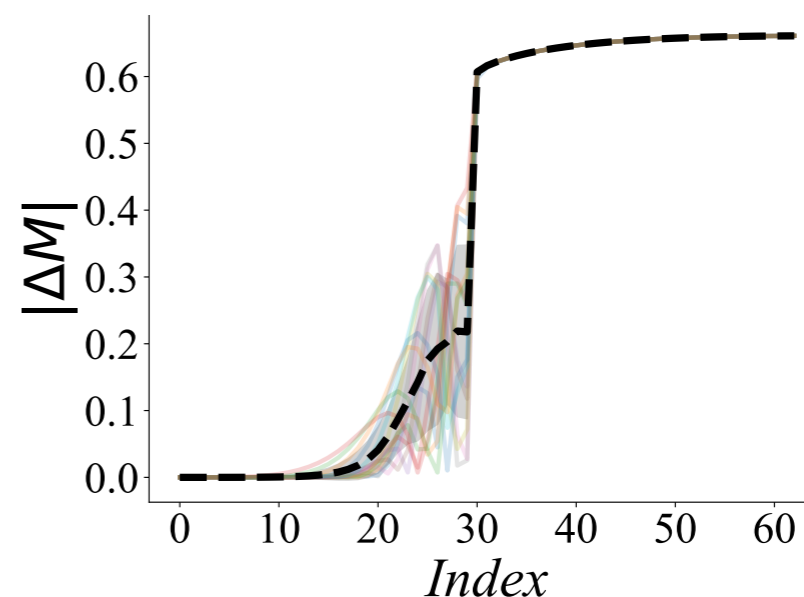
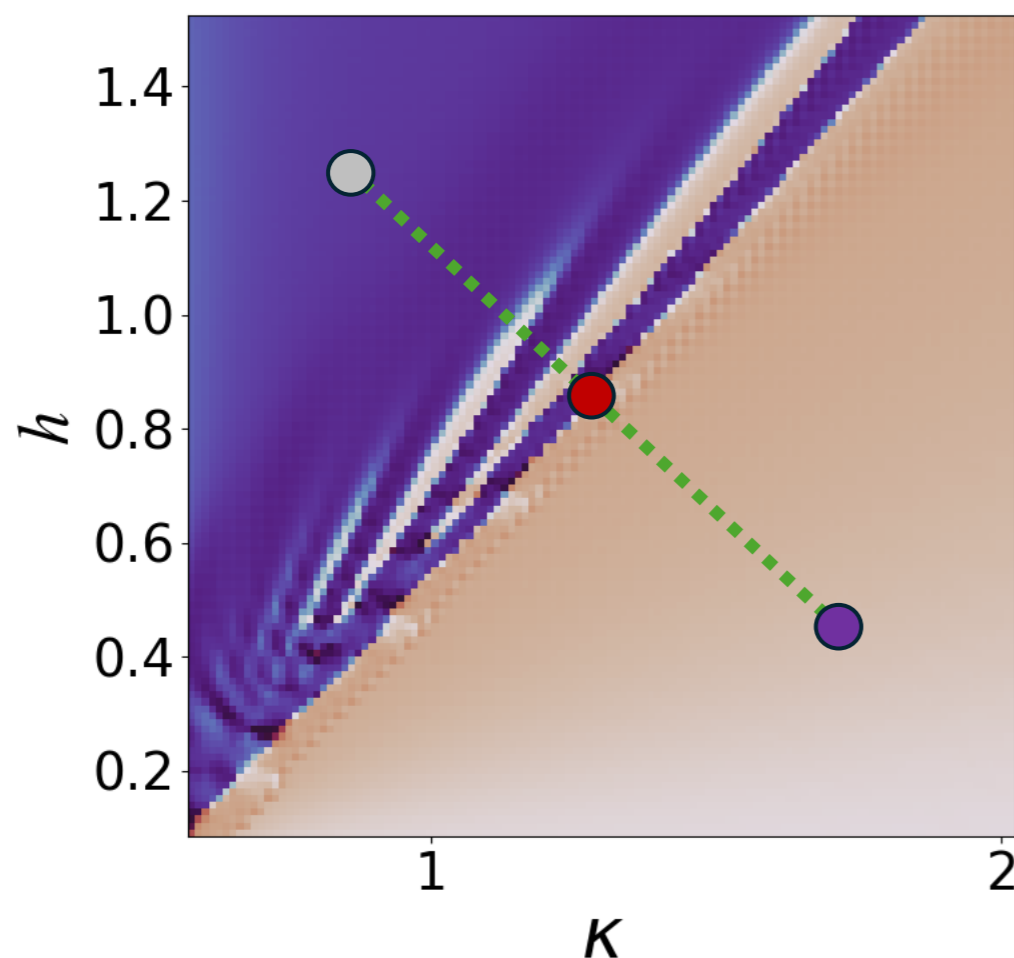
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$$\min_M - \frac{1}{|I^+|} \sum_{i \in I^+} \langle M \rangle_i^2 + \frac{\gamma}{|I^-|} \sum_{j \in I^-} \langle M \rangle_j^2$$

Order parameter discovery:



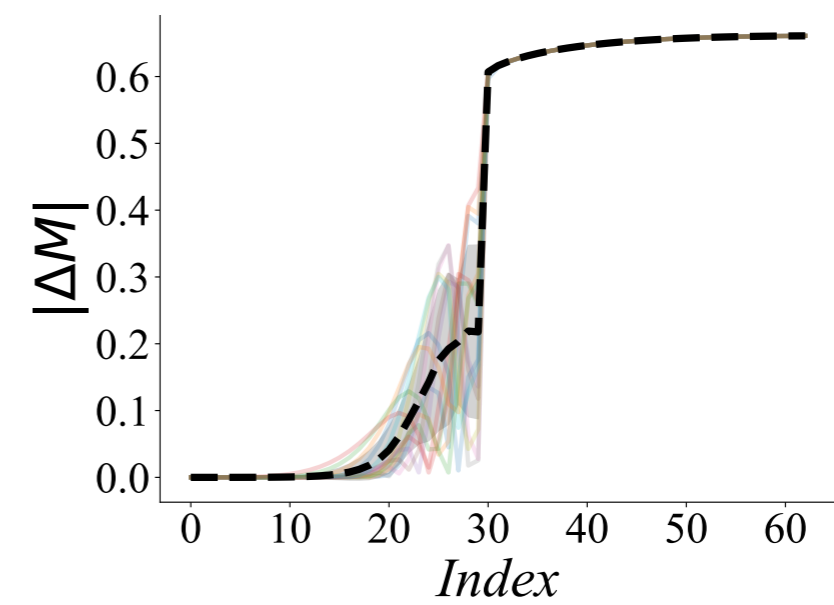
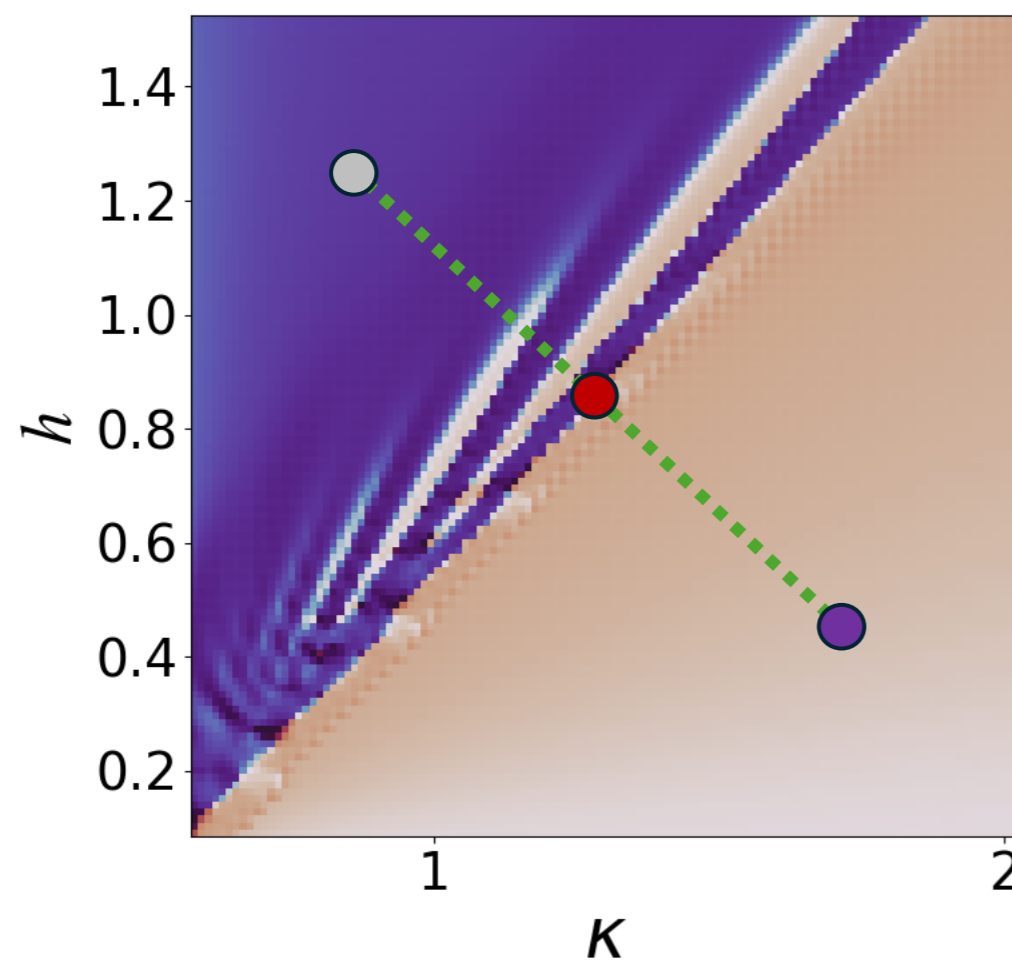
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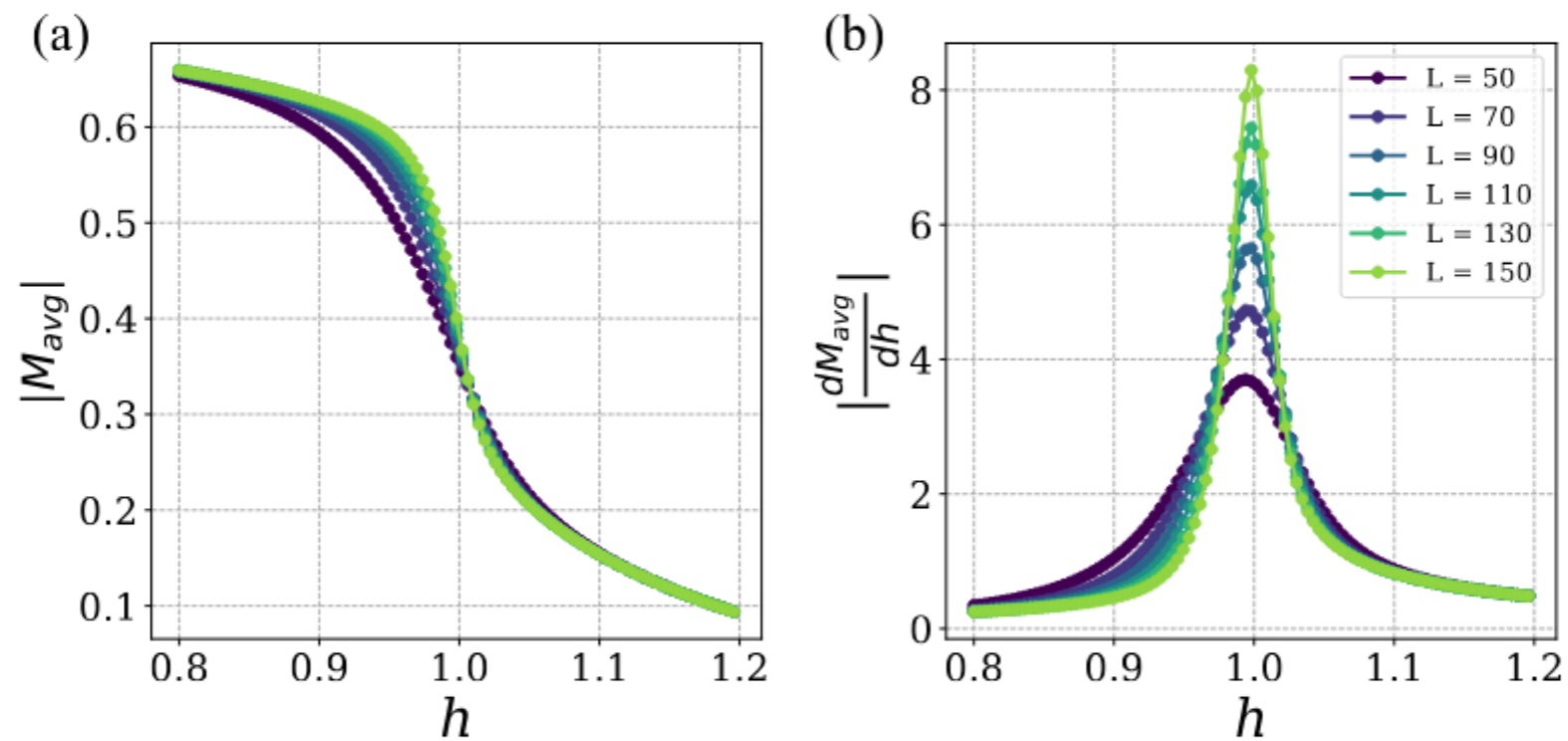
quadratically constrained quadratic program

$$\min_M -\frac{1}{|I^+|} \sum_{i \in I^+} \langle M \rangle_i^2 + \frac{\gamma}{|I^-|} \sum_{j \in I^-} \langle M \rangle_j^2$$

$$\Rightarrow \min_{\mathbf{x} \in \mathbb{C}^{m^2}} \mathbf{x}^\dagger A \mathbf{x} \quad |\mathbf{x}|_2^2 \leq 1$$

Order parameter discovery:

The two-site observable obtained using the order parameter discovery framework



Calculations with O(150) sites

FSS for the ANNNI Model



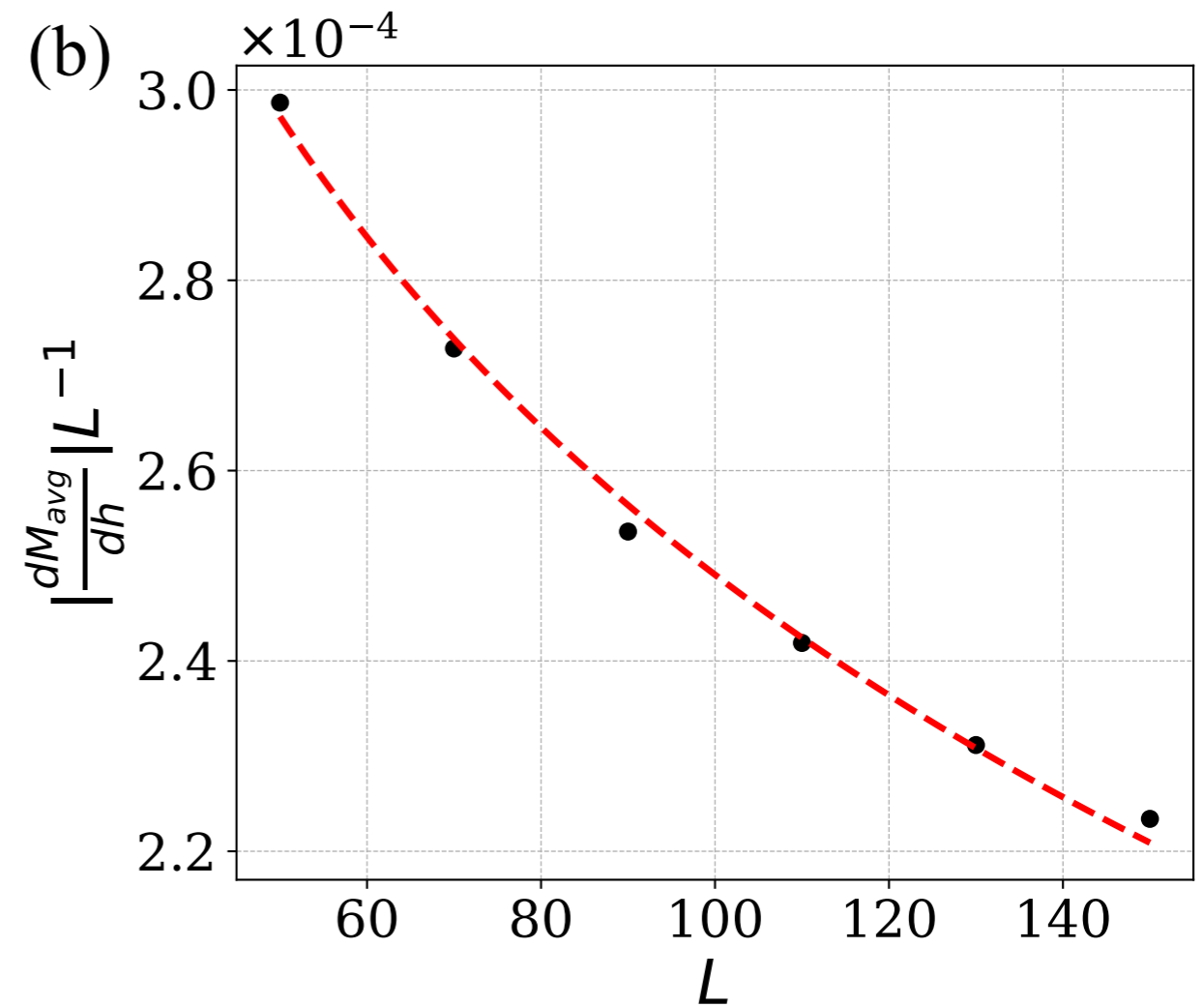
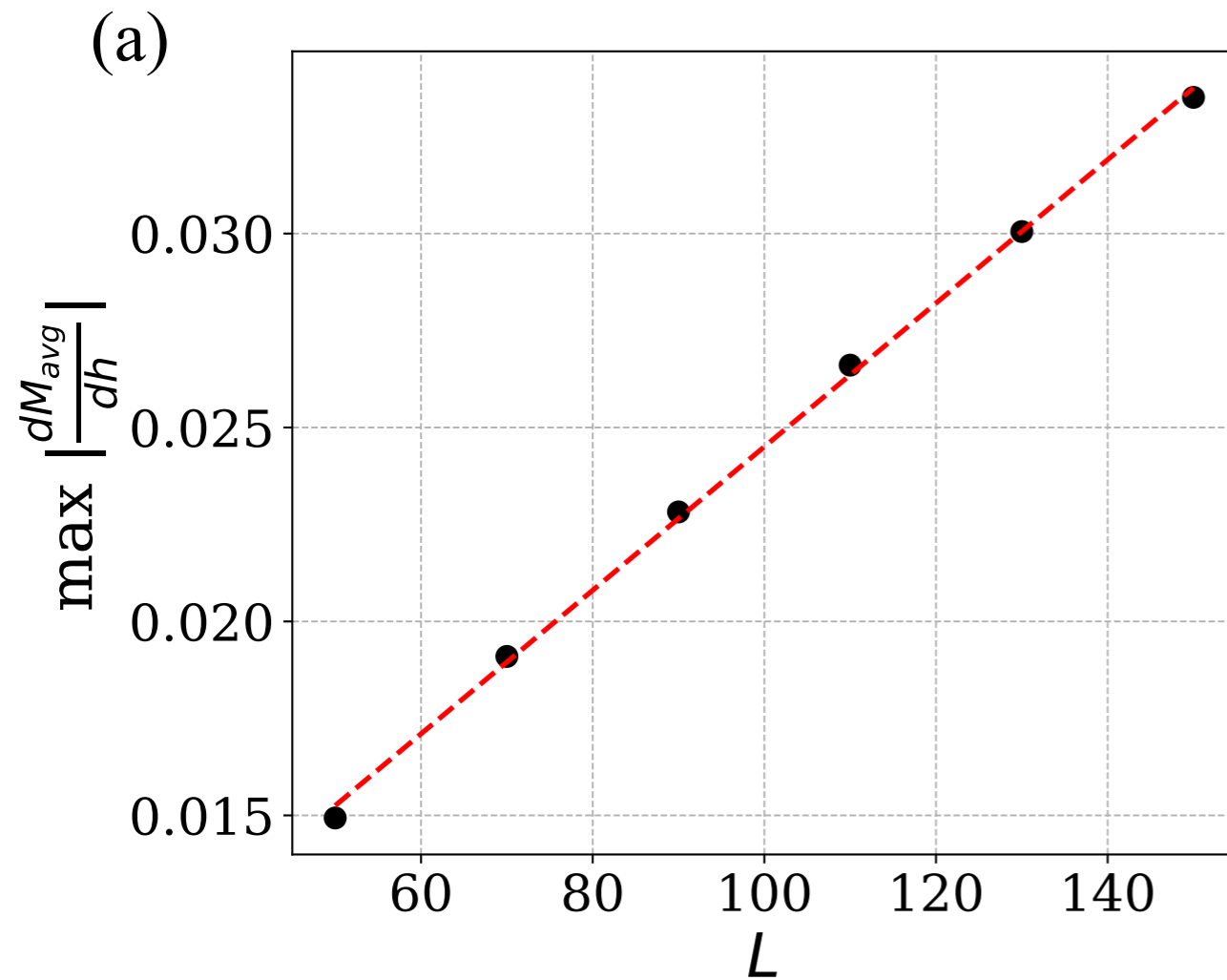
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Fit critical exponent...



$$\max_h \left\{ \frac{\partial}{\partial h} \langle M \rangle \right\} = aL^{1/\nu} (1 + bL^{-\theta/\nu})$$

Summary and outlook

Results: design a mathematical tool

for classifying different phases of matter
(classification)

We use the ***reduce density fidelity susceptibility*** for
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Summary and outlook

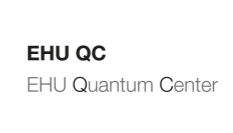
Results: design a mathematical tool

for classifying different phases of matter
(classification)

We use the ***reduce density fidelity susceptibility*** for
the qualitative classification of the different phases

for detecting the phase transitions (anomaly detection)

We define method for ***order parameter discovery*** which
allows to have a quantitative description of the critical point



The team:

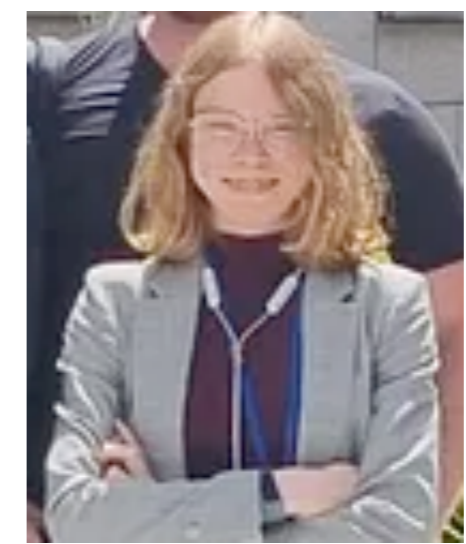
IBM Quantum



Francesco Di Marcantonio.- PhD student at UPV/EHU

Sofia Vallecorsa.- Permanent researcher at CERN

Nicola Mariella, Tara Murphy, Sergiy Zhuk.- Researchers at IBM Dublin



Constructing the spin-1 Haldane phase on a qudit quantum processor

C. L. Edmunds,^{1,*} E. Rico,^{2,3,4,†} I. Arrazola,⁵ G. K. Brennen,⁶ M. Meth,¹ R. Blatt,^{1,7,8} and M. Ringbauer¹

arXiv > quant-ph > arXiv:2408.04702

Order Parameter Discovery for Quantum Many-Body Systems

Nicola Mariella^{+,1,*} Tara Murphy^{+,1,2,†} Francesco Di Marcantonio,^{3,‡} Khadijeh Najafi,^{4,§} Sofia Vallecorsa,^{5,¶} Sergiy Zhuk,^{1,**} and Enrique Rico^{3,6,7,††}

arXiv > quant-ph > arXiv:2408.01400