Kaon Decay, Regeneration and Cascade Decay of K_S, and Violation of CP-Symmetry within a Finite time Path QFT

I. Dadić[1] research done in a collaboration with D. Klabučar[2]

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Abstract

Abstract: We demonstrate that the finite-time-path field theory (FTPFT), is adequate tool for the calculation of kaon oscillation and decay. We apply a theory with mass-mixing Lagrangian by using the Gell-Mann - Pais like mixing matrix. The Dyson-Schwinger equations contribute to a pair K^0 and $\bar{K}^0$ an another pair of CP symmetric $K_S \leftrightarrow K_L$ kaons with different masses. This leads to K^0 and $\bar{K}^0$ oscillations. To the mixing matrix we add self-energies connected to 2π and 3π decays of K^S and K^L kaons,respectively. We calculate single particle distribution of π_0, π_+, and π_- as a function of time, as it emerges from K^S and K^L decay to 2π.The pions decay further as $\pi_0 \to 2\gamma$, or $\pi_+(-) \to \mu, \nu$. These pion decays reflects in the time dependence of pion distribution. We don't need artificial tools like non-hermiticity of the hamiltonian hypotesis, neither the on-shell hypotesis.

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2 Content

References

Kaon Decay Parameters

Mean lifetime for

$\tau_{\pm} = (1.23800.0020)10^{-8}s$

$\tau_S = (8.9540.004)10^{-11}s = .136 \times 10^{12}\bar{h}MeV^{-1}$

$\tau_L = (5.1160.021)10^{-8}s = .778 \times 10^{14}\bar{h}MeV^{-1}$

$\tau_{\pm} = 2.603 \times 10^{-8}s$

$\tau_0 = 0.83 \times 10^{-16}s$

$1/\tau_S = 7.35 \times 10^{-12}MeV$

$1/\tau_L = 1.29 \times 10^{-14}MeV$

$m_{K_0} = 497.611MeV$.

$\Delta = m_L - m_S = 3.484(6) \times 10^{-12}MeV$.

A hierarchy

$m_K >> 1/\tau_0\sqrt{(M_K\Delta)} >> 1/\tau_0 >> 1/\tau_S > \Delta >> 1/\tau_L >> 1/\tau_{\pm}$.

Gell-Mann-Pais theory

Murray Gell-Mann and Abraham Pais :

$$\psi(t) = U(t)\psi(0) = e^{iHt}\begin{pmatrix} a \\ b \end{pmatrix}, \quad H = \begin{pmatrix} M & \Delta \\ \Delta & M \end{pmatrix} + \begin{pmatrix} i\Gamma_{11} & i\Gamma_{12} \\ i\Gamma_{12} & i\Gamma_{22} \end{pmatrix}$$

$$U_{K_S,K_0} = \frac{1}{2^{1/2}} = U_{K_S,\bar{K}_0} = U_{K_L,K_0} = -U_{K_L,\bar{K}_0}$$

$$|K_S >= \frac{1}{2^{1/2}}(|K_0 > +|\bar{K}_0 >), \quad |K_L >= \frac{1}{2^{1/2}}(|(K_0 > -|\bar{K}_0 >)$$

$$|K_L^0 >= (p|K_0 > -q|\bar{K}_0 >), \quad |K_S^0 >= (p|K^0 > +q|\bar{K}^0 >$$

$$|K_0 >= g_+(t)|K_0 > -\frac{q}{p}g_-(t)|\bar{K}_0 > , \quad |\bar{K}_0 >= -\frac{p}{q}g_-(t)|K_0 > +g_+(t)|\bar{K}_0 >$$

$$\frac{q}{p} = \frac{1-\varepsilon}{1+\varepsilon}., \quad |\varepsilon| = (2.228 \pm 0.011)10^{-3} \tag{2.1}$$

To get decays and CP-violation add complex valued widths to Δ
Direct and indirect CP-violation (ε and ε')

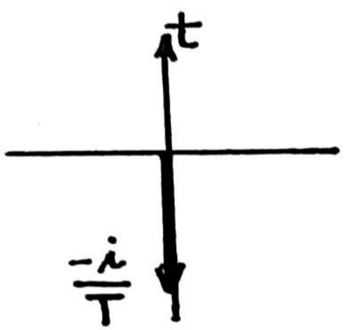
Serious drawbecks:

1. it requires hamiltonian to be non-hermitian, to deal with CP nonconservation.
2. The model uses wave functions describing partcles on mass-shell.

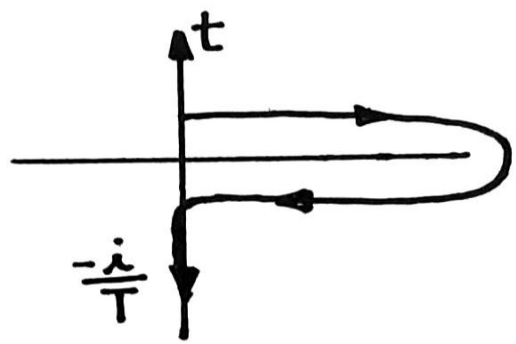
Interest for non-hermitian hamiltonian . Our approach does not require non-hermiticity.

Finite Time path and Two Point Functions

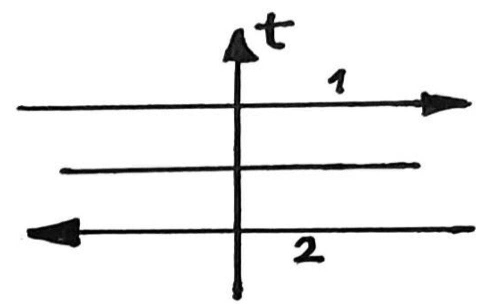
Thermodynamic   Thermal Field Theory



Imaginary time



Real time



Keldysh

Evolution of the Product of Two One-Point Operators

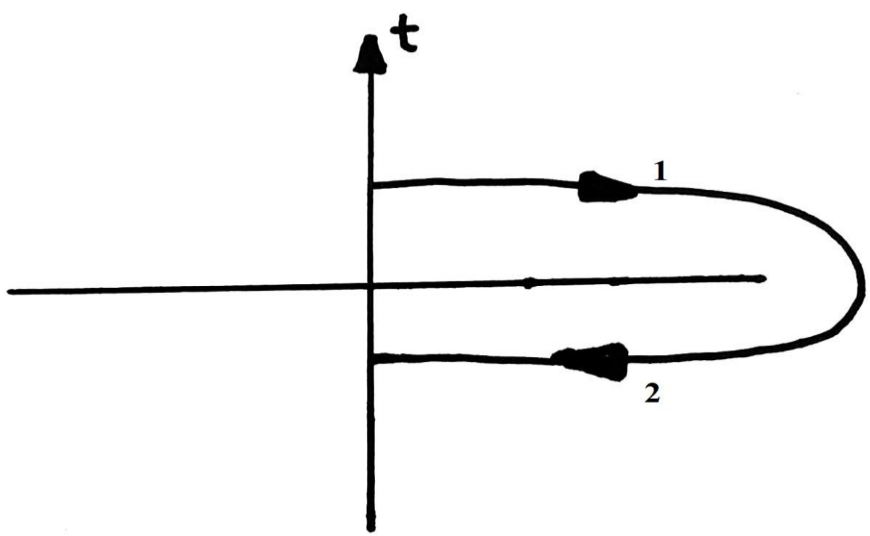
in Heisenberg representation

$$< \psi_H|O_2(t_2)O_1(t_1)|\psi_H >$$

$$=< \psi_H|e^{i\bar{h}Ht_2}O_2(0)e^{-i\bar{h}Ht_2}e^{i\bar{h}Ht_1}O_1(0)e^{-i\bar{h}Ht_1}|\psi_H >$$

$$=< \psi_H|U(0,t_2)O_2(0)U(t_2,t_1)O_1(0)U(t_1,0)|\psi_H >$$

Example $O_2(t_2)O_1(t_1) = a^+(t_2)a(t_1)$,
$=$ number operator when $t_2 \to t_1$



Perturbation Expansion

Perturbation expansion is performed by expanding evolution operator $U(t_1, t_2)$ in powers of $H_I(t)$. If t belongs to path 2 it acquires extra factor (-1) for each power of H_I, because of time-reversed path.

The extended Wick theorem (see Fetter-Walecka [?] for details) then transforms individual terms into the products of two-point functions. the important difference is that the not-contracted a^+ and a operators, act on initial density matrix and produce additional terms depending on initial single-particle distribution functions. Two point functions take matrix form e.g. D_{11} is obtained by both points taken from path 1, while for D_{12} and D_{21} one point is from path 1 and the other point is from path 2, and finally for D_{22} both points are from path 2 (For better transparency, in further text, we shall replace index $_2$ by $_{-1}$).

Finite Time Path and Wigner transforms

$$F(x,y), \quad 0 \le x_0, y_0 \le t$$

$$F_{X_0}(p_o,\vec{p}) = \int_{-2X_0}^{2X_0} ds_0 \int d^3 s e^{-i(s_0 p_0 - \vec{s}\vec{p})} F(x,y)$$

$$X = \frac{x+y}{2},\ s = x - y,$$

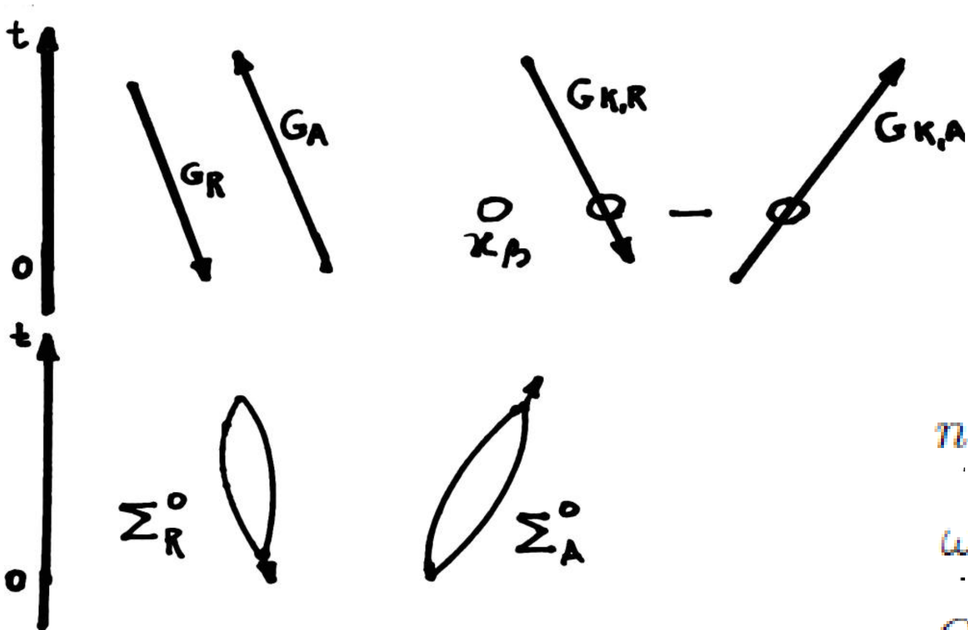
Shift to ∞

If $F_\infty(p_0',\vec{p})$ exists then

$$F_{X_0}(p_0,\vec{p}) = \int_{-\infty}^{\infty} dp_0' P_{X_0}(p_0,p_0') F_\infty(p_0',\vec{p}),$$

$$P_{X_0}(p_0,p_0') = \frac{1}{\pi}\Theta(X_0)\frac{\sin\left(2X_0(p_0-p_0')\right)}{p_0-p_0'}, \quad \lim_{X_0\to\infty} P_{X_0}(p_0,p_0') = \delta(p_0-p_0'),$$

Bare Propagators (1 for kaons , 2 for antikaons)



$$G_{\infty,1(2),R(A)}(p,m) = \frac{-i}{p^2 - m^2 \pm 2ip_0\epsilon}$$

$$G_{\infty,1,K}(p,m) = G_{\infty,1,K,R}(p) - G_{\infty,1,K,A}(p)$$

$$= 2\pi\delta(p^2 - m^2)(1 + 2n(p_0,\vec{p}))$$

$$n(p_0,\vec{p}) = \Theta(p_0)n_1(\vec{p}) + \Theta(-p_0)n_2(-\vec{p}),$$

$$\omega_p = \sqrt{\vec{p}^2 + m^2}.$$

$$G_{\infty,1,K,R(A)}(p) = -\{[1 + 2n_1(\omega_p)]\frac{p_0 + \omega_p}{2\omega_p}$$

$$+ [1 + 2n_2(\omega_p)]\frac{p_0 - \omega_p}{2\omega_p} G_{\infty,1,R(A)}(p,m)$$

$n_1(\omega_p)$ is initial kaon distribution function,

$n_2(\omega_p)$ is initial antikaon distribution function

Feynman propagator

$$G_F(p) = \frac{1}{2}[G_{\infty,1,R}(p)$$

$$+ G_{\infty,1,A}(p) - G_{\infty,1,K,n_1=n_2=0}(p)]$$

Convolution Product of two-point functions

$$C = A * B \Leftrightarrow C(x,y) = \int_0^t dz_0 \int d^3z A(x,z)B(z,y)$$

$$C_{X_0}(p_0,\vec{p}) = \int dp_{01}dp_{02}P_{X_0}(p_0,\frac{p_{01}+p_{02}}{2})\frac{1}{2\pi}\frac{ie^{-iX_0(p_{01}-p_{02}+i\epsilon)}}{p_{01}-p_{02}+i\epsilon}A_\infty(p_{01},\vec{p})B_\infty(p_{02},\vec{p}).$$

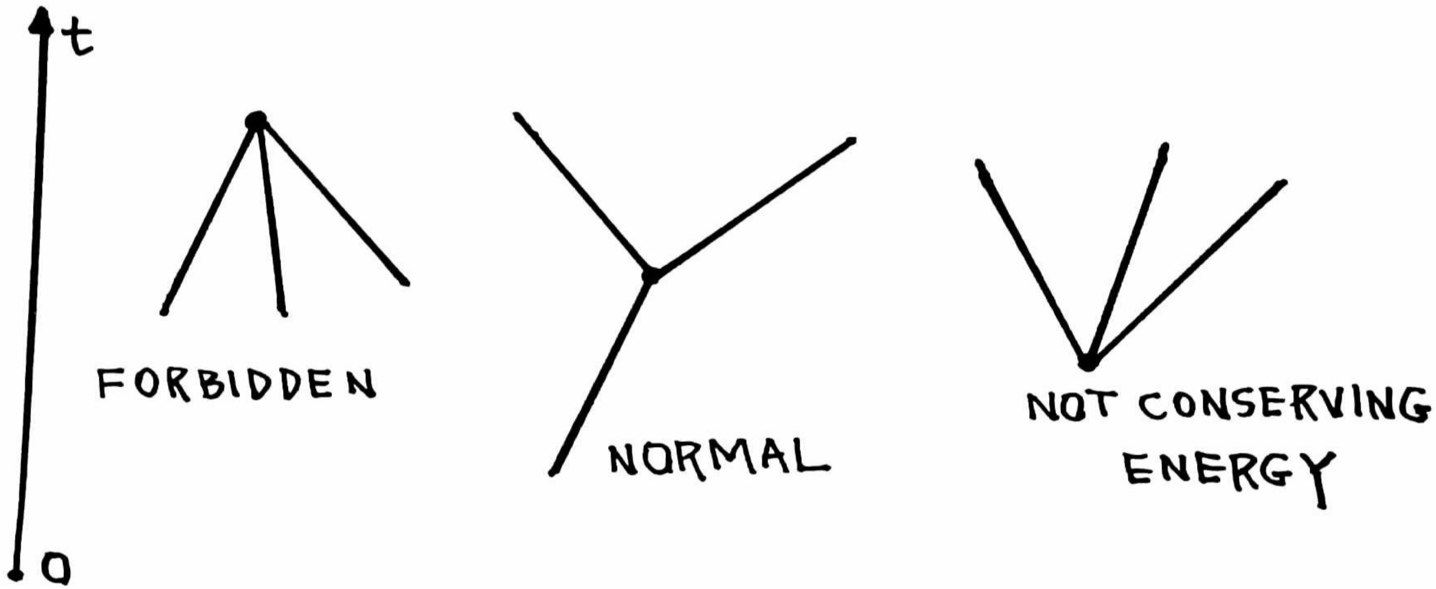
Definition of the retarded (advanced) functions: (1) the function of p_0 is analytic above (below) the real axis, (2) the function goes to zero as $|p_0|$ approaches infinity in the upper (lower) semiplane. The choice above (below) and upper (lower) refers to R (A) components .

Under the assumption that A or B satisfies (1) and (2) (A as advanced or B as retarded) we obtain

$$C_{X_0}(p_0,\vec{p}) = \int dp'_0 P_{X_0}(p_0,p'_0)A_\infty(p'_0,\vec{p})B_\infty(p'_0,\vec{p}).$$

$$C_\infty(p_0,\vec{p}) = A_\infty(p'_0,\vec{p})B_\infty(p'_0,\vec{p}).$$

Vertex Function



The $\delta^3(\sum_i \lambda_i \vec{p}_i)$ -3-momentum conserved
Energy integrand (all propagators retarded)

$$EI = \frac{i \exp[-it(\sum_{i=1,n} p_{0,i} - \sum_{j=n+1,l} p_{0,j})] - 1}{2\pi(\sum_{i=1,n} p_{0,i} - \sum_j p_{0,j}) \Pi_{i=1,n} \Pi_{j=n+1,l} (p_{0,i}^2 - \omega_{p_i}^2 - i\epsilon p_{0,i})(p_{0,j}^2 - \omega_{p_j}^2 + i\epsilon p_{0,j})}$$

i - incoming, o - outgoing propagator

A) The times of the second end of all propagators are below the vertex time. Contribution of vertex is cancelled

B) At least one time of the end of the propagator is below the vertex time. The integral over this propagators energy gives 1.

In the renormalized theory the regularization does not spoil this conclusion.

C)The times of the second end of all propagators are below the vertex time. The integral can capture singularities of propagators. Energy is not conserved.

Spontaneous regularization of the energy not conserving vertex.

$$I_{nc} = i \int \frac{dp_{0,l} \exp[-it(\sum_{i=1,n} p_{0,i} - \sum_{j=n+1,l} p_{0,j})] - 1}{2\pi(\sum_{i=1,n} p_{0,i} - \sum_{j==n+1,l} p_{0,j})}$$

The subtracted constant can be integrated out.

The above recipe does not apply to the vertex with both ends of the propagator attached to it. These vertices do not conserve energy. Instead they are defined as an integral in which the energy is smeared.

The exponential time dependence "takes care" of energy - time uncertainty relations.

The Dyson-Schwinger equations -General

$$\tilde{D}_R = D_R + iD_R * \Sigma_R * \tilde{D}_R$$

$$\tilde{D}_A = D_A + iD_A * \Sigma_A * \tilde{D}_A$$

$$\tilde{D}_K = D_K + i[D_R * \Sigma_A * \tilde{D}_A + D_R * \Sigma_K * \tilde{D}_A + D_R * \Sigma_R * \tilde{D}_A]$$

$$\Sigma_R = \sum_{n=0} \Sigma_R^n$$

$$\Sigma_K = \sum_{n=1} \Sigma_K^n$$

where $*$-symbol is convolution product.

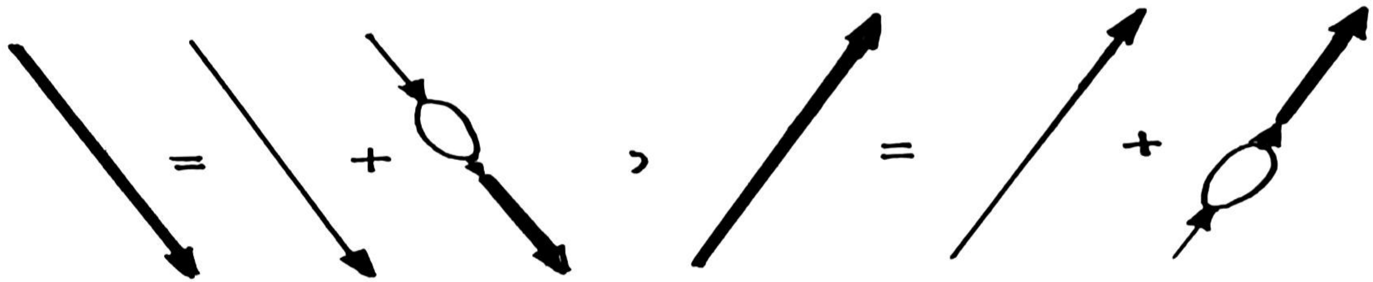
Σ_R^0 - multiplies algebraically. possible partial resummation of D-S equarion

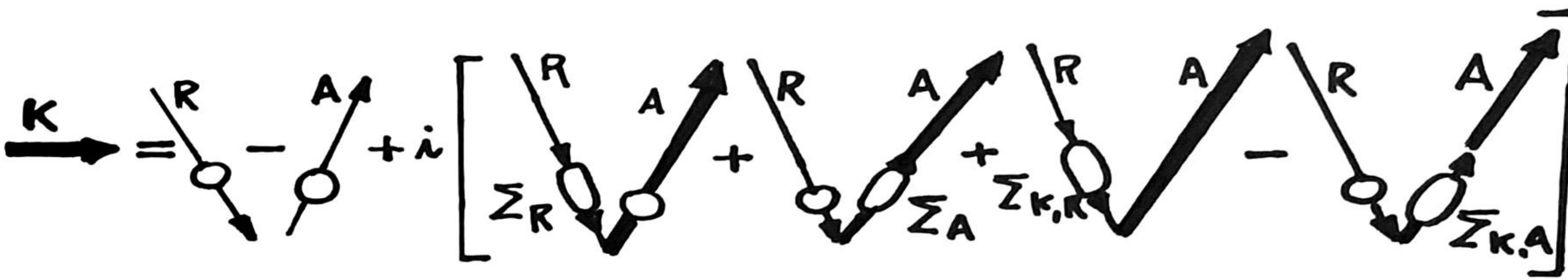
$\Sigma_R^0 = R + iI$ at the pole of D_R correction to MASS and LIFE TIME enters the propagator.

This is certain SHIFT of PARADIGMA: spectrum of resummed propagators are not determined by Σ_F as in S-matrix formalism but by Σ_R^0.

Already noticed in QCD

constituent quarks vs. current quarks




K
R
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A
ΣR
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A
ΣA
R
A
ΣK,R
R
A
ΣK,A

Particle Number as an Average of Equal Time Limit of G_K

$$< N_{\vec{p}}(t) >= (2\pi)^3 d\mathcal{N}/(d^3x d^3p),$$

After the time t, the number of kaons (antikaon) is expressed through the AVERAGE of EQUAL-TIME LIMITS (AETL) of the resummed Keldysh component $G_{\alpha,\alpha,K}$ of the Wigner transform of kaon propagator taken from above ($\delta > 0$) and from below ($\delta < 0$):

$$1 + \delta_{1,\alpha}\langle N_{K^0,\vec{p}}(t)\rangle + \delta_{2,\alpha}\langle N_{\bar{K}^0,\vec{p}}(t)\rangle$$

$$= \frac{\omega_p}{2\pi}\left[\lim_{0<\delta\to 0} + \lim_{0>\delta\to 0}\right]\int dp_0\, e^{-ip_0\delta}\,\tilde{G}_{\alpha,\alpha,K,t}(p)\,,$$

where $\delta = s_{01} - s_{02}$ and $t = X_0 = (s_{01} + s_{02})/2$.

AVERAGE - owing to the complex poles of G_K
the particle number can be known only approximately
other definitions of a particle number in the literature. more or less equivalent to our.
Our definition of particle number has some good properties :
1. It reproduces correctly the lowest order contribution (from $\S_K$).
2. for rising t it mildly approachess S-matrix theory results (e.g.for Compton scattering).

2.1 Lagrangian and Self-Energies

The Lagrangian consists of five important pieces:

1. Free Lagrangian L_0 describes kaons K^0 and $\bar{K}^0$ as free ,stable particles of the mass m_{K^0}.

2. mass mixing Lagrangian (2.1), which mixes $CP = 1$ (K_S) and $CP = -1$ (K_L) kaon states into the observed K^0 and $\bar{K}^0$ kaons.

3. Couplings $K_L \leftrightarrow 3\pi$ and other couplings of K_L, which cause 3π decay and other decays which are associated to $CP = -1$ state K_L. They generate self-energy $\Sigma_{L,R(A)} = \Sigma_{1,R(A)}$.

4. Couplings $K_S \leftrightarrow 2\pi$ and other couplings of K_S, which cause 2π decay and other decays which are associated to $CP = 1$ state K_S. They generate self-energy $\Sigma_{S,R(A)} = \Sigma_{2,R(A)}$.

5. Cp violating Lagrangian, which causes K_L to decay into 2π. to this term, three selfenergies are associated:

self-energy $\Sigma^{\neq}_{1,R(A)}$ which describes the process $K_L \to 2\pi \to K_L$

hybrid selfenergy $\Sigma_{K_L \to K_S}\Sigma^{\neq}_{21,R(A)}$ which is associated to precesses where $\Sigma^{\neq}_{\infty,12,R(A)}$ in which first process is CP violating, while the second one $2\pi \to K_S$ is CP conserving

hybrid selfenergy $\Sigma^{\neq}_{21,R(A)}$, which describes the reversed process $K_S \to 2\pi \to K_L$.

$$\mathcal{L}(x) = \mathcal{L}^0(x) + \mathcal{L}^{mix}(x) + \mathcal{L}^{K_L \leftrightarrow 3\pi}(x) + \mathcal{L}^{K_S \leftrightarrow 2\pi}(x)$$

$$\mathcal{L}^0(x) = \sum_{\mu,\alpha} (\partial_\mu \Psi_\alpha)^*(x)(\partial^\mu \Psi_\alpha)(x) - \sum_\alpha \Psi_\alpha^* \Psi_\alpha(x)$$

$$\mathcal{L}^{mix}(x) = \sum_{\alpha,i} \bar{\Psi}_\alpha(x) U_{\alpha,i}^* M_{ij} U_{\beta,j} \Psi_\beta(x)$$

$$U_{K_S,K_0} = \frac{1}{2^{1/2}} = U_{K_S,\bar{K}_0} = U_{K_L,K_0} = -U_{K_L,\bar{K}_0}$$

$$M_{ij} = \delta_{ij}\Delta_i, \quad i,j = 1,2$$

$$M = \begin{pmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{pmatrix}$$

Where $U\alpha, i$ is 2×2 matrix analogous to PMNS-matrix. Here, α is index for equal mass (m_K) kaons-$\alpha = 1$ forK_0 and $\alpha = 2$ for $\bar{K}_0$,and i is index for CP eigenstate kaons $i = 1$ is choosen for K_L, while $i = 2$ for K_S.

Dyson-Schwinger Equations and Solutions

The dynamics of kaon oscillation, decay, and CP violation is now determined by Dyson-Schwinger equations (DSE) satisfied by the Wigner transforms of bare and resummed kaon propagators:

In DSE there is $\tilde{M}_{R(A)}$ which is natrix M extended through retarded or advanced self-energies

$$\tilde{M}_{\infty,R(A)} = \begin{pmatrix} \Delta_1 + \Sigma_{1,\infty,R(A)} + \Sigma^{\neq}_{\infty,1,R(A)} & \Sigma^{\neq}_{\infty,12,R(A)} \\ \Sigma^{\neq}_{\infty,21,R(A)} & \Delta_2 + \Sigma_{\infty,2,R(A)} \end{pmatrix}$$

the self-energies $\Sigma^{\neq}$ are so small that they should be taken into account only linearly.

The self-energy for DSE is

$$\tilde{\Sigma}_{\infty,\beta,\alpha,R(A)} = \sum_i U^*_{\beta,i}\tilde{M}_{\infty,ij,R(A)}(p)U_{\alpha,i}$$

The Dyson-Schwinger equations for “oscillating” kaons are

$$\tilde{G}_{\beta,\eta,R} = G_{\beta,R}\delta_{\beta,\eta} + iG_{\beta,R} * \tilde{\Sigma}_{\beta,\alpha,R} * \tilde{G}_{\alpha,\eta,R}$$

$$\tilde{G}_{\beta,\eta,A} = G_{\beta,A}\delta_{\beta,\eta} + iG_{\beta,R} * \tilde{\Sigma}_{\beta,\alpha,A} * \tilde{G}_{\alpha,\eta,A}$$

$$\tilde{G}_{\beta,\eta,K} = G_{\beta,K}\delta_{\beta,\eta} + i[G_{\beta,R} * \tilde{\Sigma}_{\beta,\alpha,R} * \tilde{G}_{\alpha,\eta,K}$$

$$+G_{\beta,K} * \tilde{\Sigma}_{\beta,\alpha,A} * \tilde{G}_{\alpha,\eta,A} + G_{\beta,R} * \tilde{\Sigma}_{\beta,\alpha,K} * \tilde{G}_{\alpha,\eta,A}]$$

The formal solution is

$$\tilde{G}_{\beta,\eta,R} = \sum_i U^*_{\beta,i}[1 - i\tilde{\Sigma}_R G_R]^{-1} G_R U_{\eta,i}$$

$$\tilde{G}_{\beta,\eta,A} == \sum_i U^*_{\beta,i} G_A [1 - i\tilde{\Sigma}_A G_A]^{-1} U_{\eta,i}$$

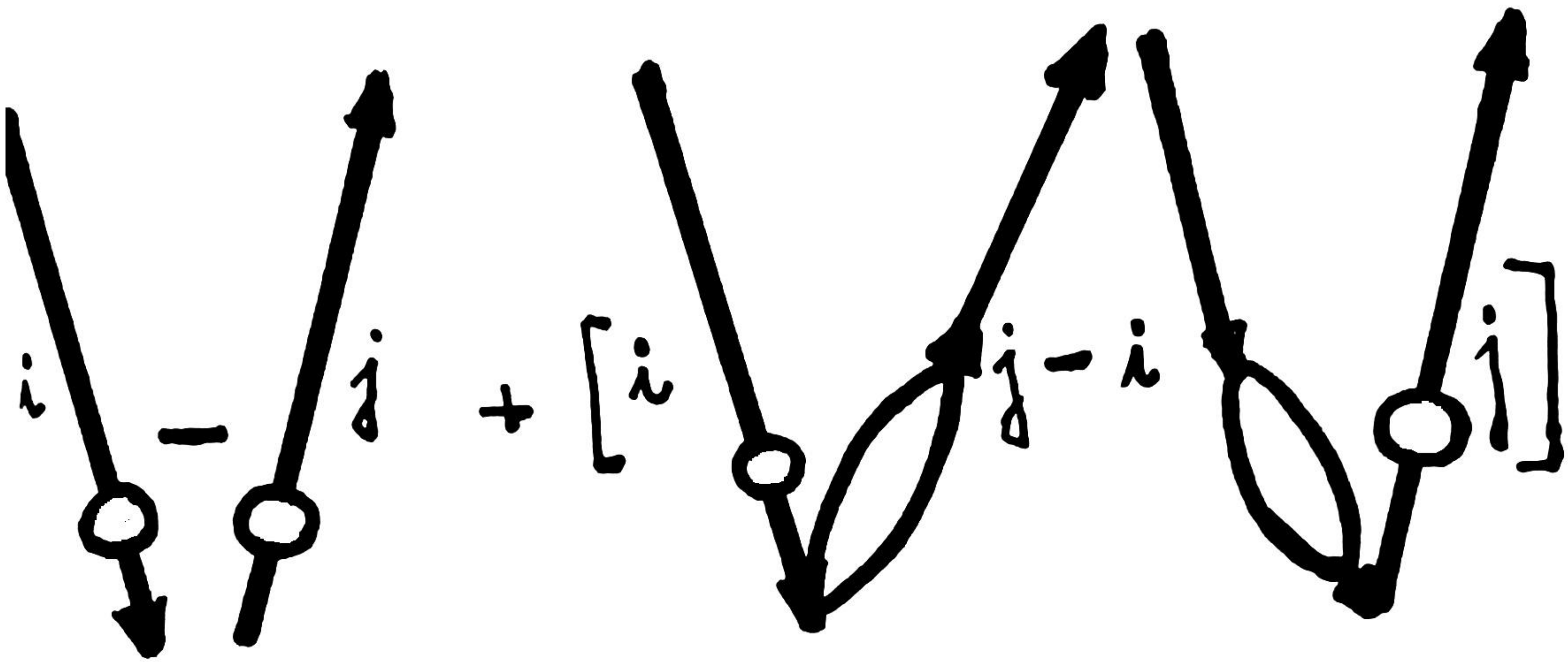
$$\tilde{G}_{\beta,\eta,K} = \tilde{G}_{\beta,\eta,K,alg} + \tilde{G}_{\beta,\eta,K,conv}$$

$$\tilde{G}_{\beta,\eta,K,alg} = -\sum_i U^*_{\beta,i} U_{\eta,i} G_{K,A} [1 - i\tilde{\Sigma}_{i,A} G_A]^{-1}$$

$$+\sum_i U^*_{\beta,i} U_{\eta,i} [1 - i\tilde{\Sigma}_{i,R} G_R]^{-1} G_{K,R}$$

$$\tilde{G}_{\beta,\eta,K,conv} = i \sum_{ii} U^*_{\beta,i} U \eta_i U^*_{\eta,j} U_{\beta,j}$$

$$[1 - i\tilde{\Sigma}_{i,R} G_R]^{-1} [G_{K,R} * \tilde{\Sigma}_{j,A} G_A - G_R \tilde{\Sigma}_{i,R} * G_{K,A}] [1 - i\tilde{\Sigma}_{j,A} G_A]^{-1}.$$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Oscillating Kaons - Singularities of the Resummed $\tilde{G}_{\beta,\alpha,R}$

To proceed further we search for the singularities of $\tilde{G}_{i,R}(p_0)$.
The symmetries of the self-energies are imortant

$$\tilde{\Sigma}_{K,R}(-p_0,\vec{p}) = -\tilde{\Sigma}^*_{K.R}(p_0,\vec{p}), \quad \tilde{\Sigma}_{R}(-p_0,\vec{p}) = \tilde{\Sigma}^*_{R}(p_0,\vec{p})$$

POLE APPROX.: REPLACE $\tilde{\Sigma}_{i,R}(p_0)$ by a constant $\tilde{\Sigma}_{i,R}(\bar{p}_0)$ taken at the $\bar{p}_0 = \pm\omega_p$.

Define $\tilde{\Sigma}_{i,R}(\pm\omega_p) = R_i \mp I_i$ dictated by the symmetry, negative I_i violates causality.

The approximation is valid in vicinity of the corresponding zero of $G^{-1}_{i,R}$.

The equation: $p_0^2 - \omega_p^2 - \mathcal{R}_i + i\mathcal{I}_i = 0$ with $\omega_p = (\vec{p}^2 + m^2)^{1/2}$, poses two solutions $p_{0,1,2} = \pm[\omega_p^2 + \mathcal{R}_i - i\mathcal{I}_i]^{1/2}$, but only one (with + sign) represents the pole near $+\omega_p$, the otner is near $-\omega_p$ where imaginary part of $\tilde{\Sigma}_{i,R}(p_0)$ has opposite sign, thus it is not the pole. The pole near $-\omega_p$ is obtained from the equation containing $-i\mathcal{I}_i$.

$$\omega_{i,\lambda_i} = \lambda_i[\omega_p^2 + \mathcal{R}_i - i\lambda_i\mathcal{I}_i]^{1/2}, \quad \lambda_i = \pm 1$$

$$\omega_{i,\lambda_i} \approx \lambda_i(\omega_p + \frac{\mathcal{R}_i}{2\omega_p}) - i\frac{\mathcal{I}_i}{2\omega_p} = \lambda_i\omega_i - \frac{im_i\Gamma_i}{\omega_i},$$

$$\omega_i^2 = \vec{p}^2 + m_i^2 - \Gamma_i^2, \quad m >> |\mathcal{R}_i|, \ |\mathcal{I}_i|$$

Near the poles the propagator is approximated as:

$$\tilde{G}_{i,R} \approx \frac{-i}{2\omega_{i,\lambda_i}(p_0 - \omega_{i,\lambda_i} + i\epsilon)}, \quad |p_0 - \omega_{i,\lambda_i}| << \omega_p$$

The Propagator of Decayng Pions

Pion propagator with self-energy containing the information about pion interactions satisfies Dyson-Schwinger equations

$$\tilde{D}_{l,R} = D_{l,R} + iD_{l,R} * \Sigma_{l,R} * \tilde{D}_{l,R}$$

$$\tilde{D}_{l,A} = D_{l,A} + iD_{l,R} * \Sigma_{l,A} * \tilde{D}_{l,A}$$

$$\tilde{D}_{l,K} = D_{l,K} + i[D_{l,R} * \Sigma_{l,R} * \tilde{D}_{l,K}$$

$$+ D_{l,K} * \Sigma_{l,A} * \tilde{D}_{l,A} + D_{l,R} * \Sigma_{l,K} * \tilde{D}_{l,A}]$$

The self-energies $\Sigma_{l,R}(q)$ and $\Sigma_{l,A}(q)$ are complicated analytic functions, mostly unknown. We shall treat them in analogy with the kaon self-energy $\tilde{\Sigma}_{i,R}(p)$. The symmetries of the self-energies $\Sigma_{l,R(A)}(q)$ and $\Sigma_{l,K,R(A)}(q)$ are

$$\Sigma_{l,K,R}(-|p_0|,\vec{p}) = -\Sigma^*_{l,K.R}(|p_0|,\vec{p}) = \Sigma^*_{l,K,A}(-|p_0|,\vec{p}) = -\Sigma_{l,K,A}(|p_0|,\vec{p}),$$

$$\Sigma_{l,R}(-|p_0|,\vec{p}) = \Sigma^*_{l,R}(|p_0|,\vec{p}) = \Sigma^*_{l,A}(-|p_0|,\vec{p}) = \Sigma_{l,A}(|p_0|,\vec{p})$$

The formal solution with $\Sigma_{l,R(A)}$ replaced by $\Sigma^0_{l,R(A)}$ is

$$\tilde{D}_{l,R} = [1 - iD_{l,R}\Sigma^0_{l,R}]^{-1}D_{l,R} = D_{l,R}[1 - i\Sigma^0_R D_{l,R}]^{-1},$$

$$\tilde{D}_{l,A} = [1 - iD_{l,A}\Sigma^0_{l,A}]^{-1}D_{l,A} = D_{l,A}[1 - i\Sigma^0_A D_{l,A}]^{-1},$$

$$\tilde{D}_{l,K} = -D_{l,K,A}(1 - i\Sigma^0_{l,A}D_{l,A})^{-1} + (1 - iD_R\Sigma^0_{l,R})^{-1}D_{l,K,R}$$

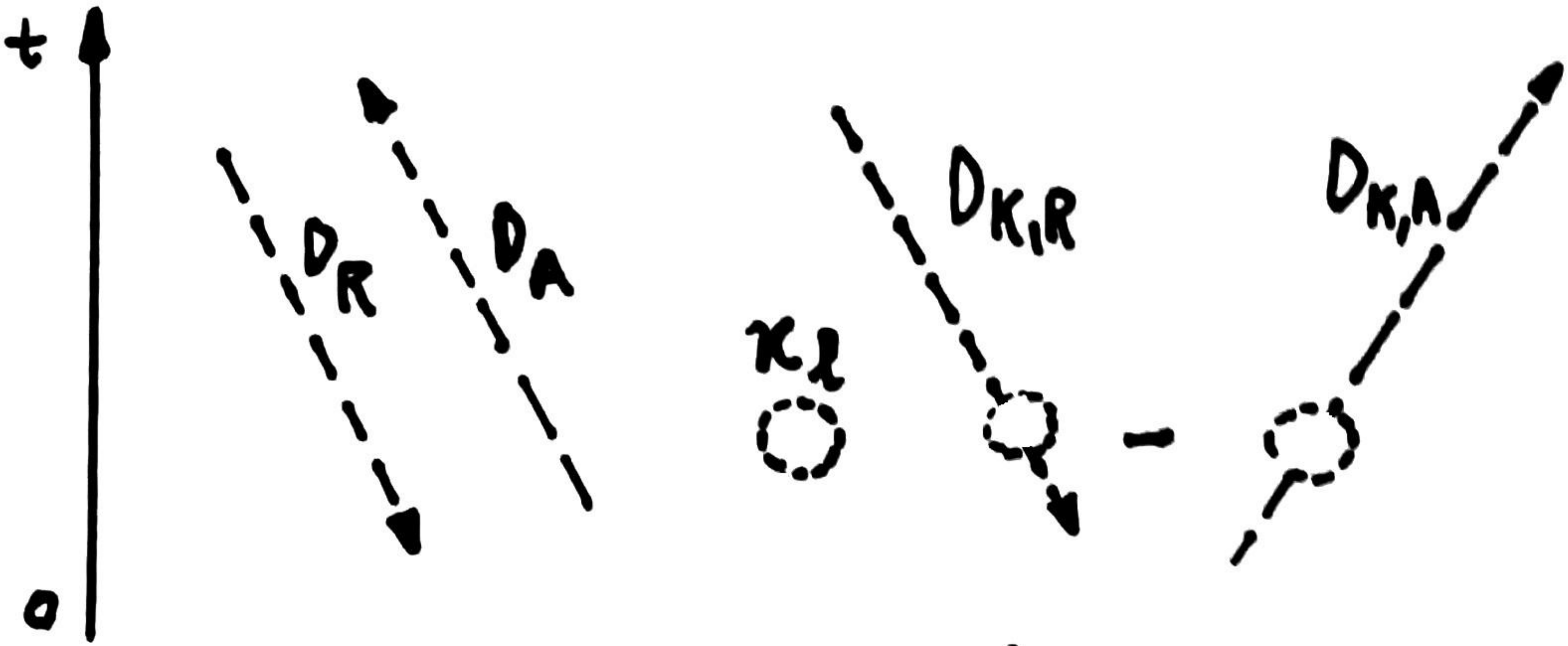
$$+i(1 - iD_R\Sigma^0_R)^{-1}[D_{l,R} * \Sigma_{l,K} * D_{l,A} - D_{l,R}\Sigma^0_{l,R} * D_{l,K,A}$$

$$+D_{l,K,R} * \Sigma^0_{l,A}D_{l,A}](1 - i\Sigma^0_{l,A}D_{l,A})^{-1}.$$

The self-energy $\Sigma^0_{l,R(A)}$ multiplies the propagators as an algebraic function and we can perform partial resummation of Dyson-Schwinger series which involve it. As we don't know the analytic form of $\Sigma^0_{l,R}(q_0)$, we rely on the expected properties: vanishing at large $|q_0|$ and behaviour near the poles of the bare propagator $\bar{q}_{l,0} = \pm\omega_{l,q}$. Therefore

$$\Sigma^0_{l,R}(\pm\omega_{l,q}) = R(\omega_{l,q}) \mp I(\omega_{l,q})$$

with the interpretation related to mass shift and the decaty rate.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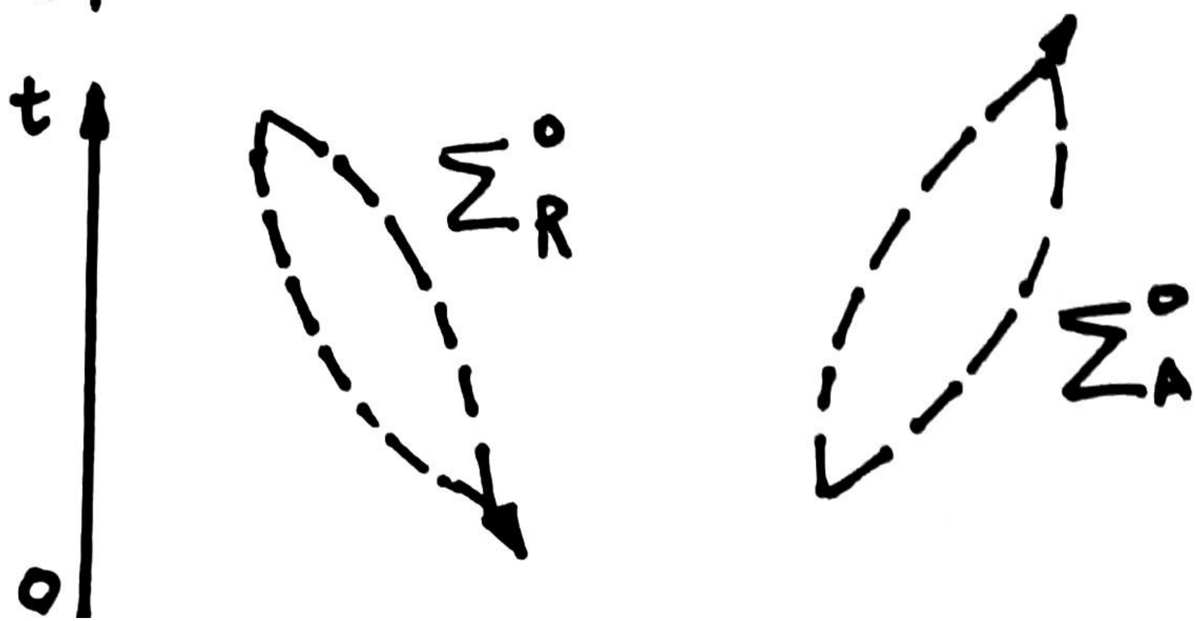
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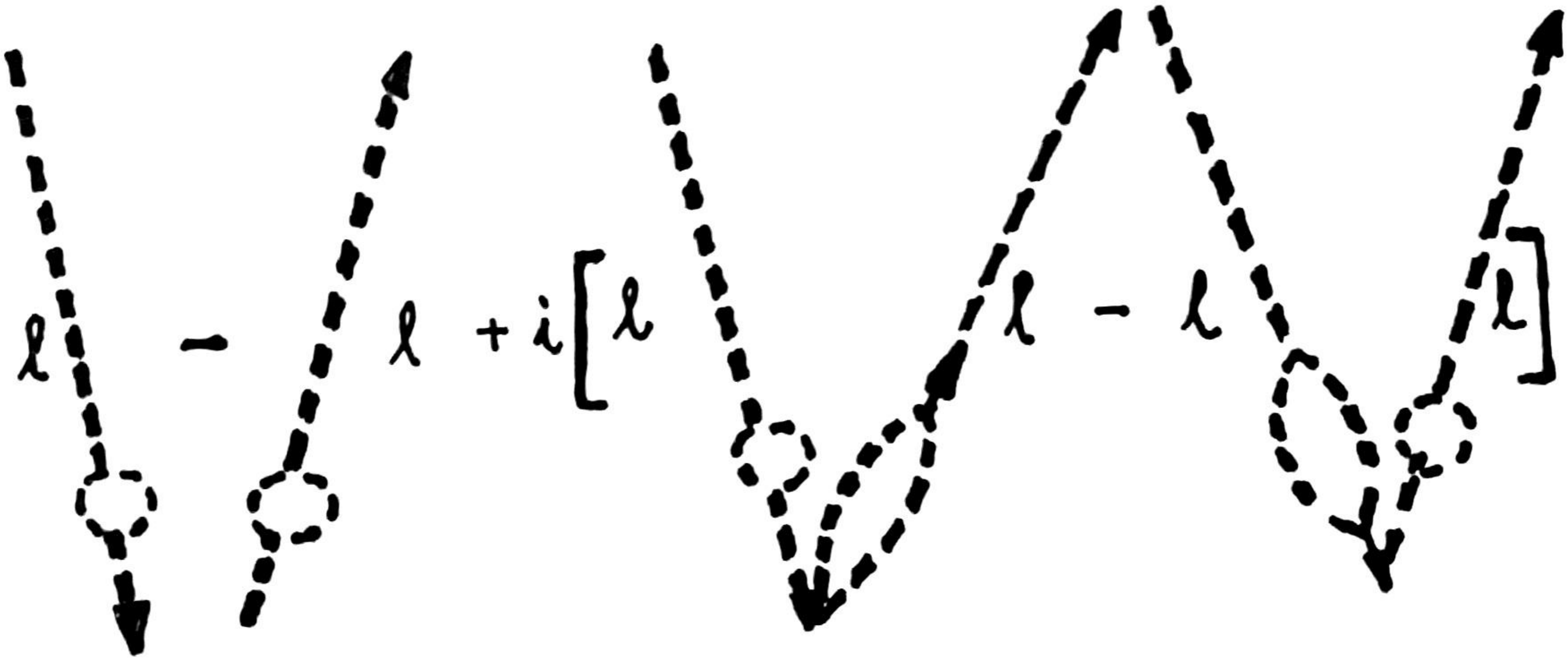
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$$\omega_{l,\lambda_l,p} = \lambda_l[\omega_{l,p}^2 + R(\omega_{l,q}) - i\lambda_l I(\omega_{l,q})]^{1/2}, \quad \lambda_l = \pm 1$$

$$\omega_{l,\lambda_l} \approx \lambda_l \omega_{l,p} + \frac{R(\omega_{l,q})}{2\omega_{l,pl}} - i\frac{I(\omega_{l,q})}{2\omega_{l,p}}$$

where we have assumed inequalities $m_l >> |R(\omega_{l,q})|, \ |I(\omega_{l,q})|$.

Notice that we had obtained, taken naively, four poles. But, only two of them are situated in the vicinity of bare propagator poles. The other two correspond to forbided extrapolations of $\Sigma^0_{R(A)}$, (i.e. from ω_p basin of solutions to $-\omega_p$ basin). These false solutions woud violate the causality (wrong sign of imaginary part). This is very similar to the case of partial resummation of Dyson-Schwinger equations for kaons. We reparametrize the above poles in a more "familiar" way (with self-evident translation)

$$\omega_{l,\lambda_l,p} = \lambda_l[\vec{p}^2 + (m_l - i\lambda_l\Gamma_l)^2]^{1/2}, \quad \lambda_l = \pm 1,$$
$$\omega_{l,\lambda_l,p} \approx \lambda_l\omega_{l,p} - \frac{i m_l \Gamma_l}{\omega_{l,p}}, \quad \omega_{l,p}^2 = \vec{p}^2 + m_l^2 - \Gamma_l^2$$

The resummed pion propagators in the vicinity of the poles of bare propagators are

$$[1 - iD_{l,R}\Sigma^0_{l,R})]^{-1}D_{l,R} \approx \frac{-i}{2\omega_{l,\lambda_l,p}(p_0 - \omega_{l,\lambda_l,p} + i\epsilon)}, \ |p_0 - \omega_{l,\lambda_l,p}| << \omega_{l,p}$$

Number of Kaons as a Function of Time

The propagator (3.21) carries the information, which, after the equal time limit, gives the number and momentum-distribution of both types of kaons measured at the time t. The kaon (antikaon) number is defined as

$$< N_{\vec{p}}(t) >= (2\pi)^3 dN/(d^3x d^3p),$$

After the time t, the number of kaons (antikaon) is expressed through the average of equal-time limits (AETL) of the resummed Keldysh component $\tilde{G}_{\beta,\alpha,K}$ $(\beta = \alpha)$ of the Wigner transform of kaon propagator taken from above $(\delta > 0)$ and from below $(\delta < 0)$:

$$1 + \delta^1_\alpha \langle\, N_{K^0,\vec{p}}(t)\,\rangle \; + \delta^2_\alpha \langle\, N_{\bar{K}^0,\vec{p}}(t)\,\rangle$$

$$= \frac{\omega_p}{2\pi}\,[\lim_{0<\delta\to 0} + \lim_{0>\delta\to 0}]\int dp_0\, e^{-ip_0\delta}\,\tilde{G}_{\alpha,\alpha,K,t}(p)\,,$$

where $\delta = s_{01} - s_{02}$ and $t = (s_{01} + s_{02})/2$.

As we do not know the true eigenvectors of the full hamiltonian, the particle number can be known only approximately.

Kaon Antikaon Oscillation

Initial Kaon stream: $t=0 \quad n_1(\omega_p), \quad n_2(\omega_p)=0,$

Kaon number; $\quad \alpha=\beta=1,$

$N_{K_0}(t)=\frac{n_{\alpha+}(|\vec{p}|,\vec{p})}{4}[e^{-\frac{2m_1\Gamma_1}{\omega_p}t}+e^{-\frac{2m_2\Gamma_2}{\omega_p}t}+2e^{-\frac{m_1\Gamma_1+m_2\Gamma_2}{\omega_p}t}\cos\frac{m_1^2-m_2^2}{2\omega_p}t]$

Antikaon number $\quad \alpha=\beta=2.$

$N_{\bar{K}_0}(t)=\frac{n_{\alpha+}(|\vec{p}|,\vec{p})}{4}[e^{-\frac{2m_1\Gamma_1}{\omega_p}))t}+e^{-\frac{2m_2\Gamma_2}{\omega_p}))t}-2e^{-\frac{m_1\Gamma_1+m_2\Gamma_2}{\omega_p}))t}\cos\frac{m_1^2-m_2^2}{2\omega_p}t]$

At rest $\omega_p=M\approx m_1\approx m_2$

$$N_{K_0}(t) = \frac{n_{\alpha+}(|\vec{p}|,\vec{p})}{4}[e^{-2\Gamma_1 t}+e^{-2\Gamma_2 t}+2e^{-(\Gamma_1+\Gamma_2)t}\cos\Delta t]$$

$$N_{\bar{K}_0}(t) = \frac{n_{\alpha+}(|\vec{p}|,\vec{p})}{4}[e^{-2\Gamma_1 t}+e^{-2\Gamma_2 t}-2e^{-(\Gamma_1+\Gamma_2)t}\cos\Delta t]$$

In the absence of CP violation, there is strict symmetry between K_0 and $\bar{K}_0$ decays.

Kaon Decays and CP violation

For inclusion of $\Sigma^{\neq}_{12}$ and $\Sigma^{\neq}_{21}$ one has to sum the D-S for retarded propagator:

$$\tilde{G}_{\beta,\eta,R} = \sum_i \sum_i U^*_{\beta,i} U_{\eta,i} [1 - i\Sigma_{i,R} G_R]^{-1} G_R$$

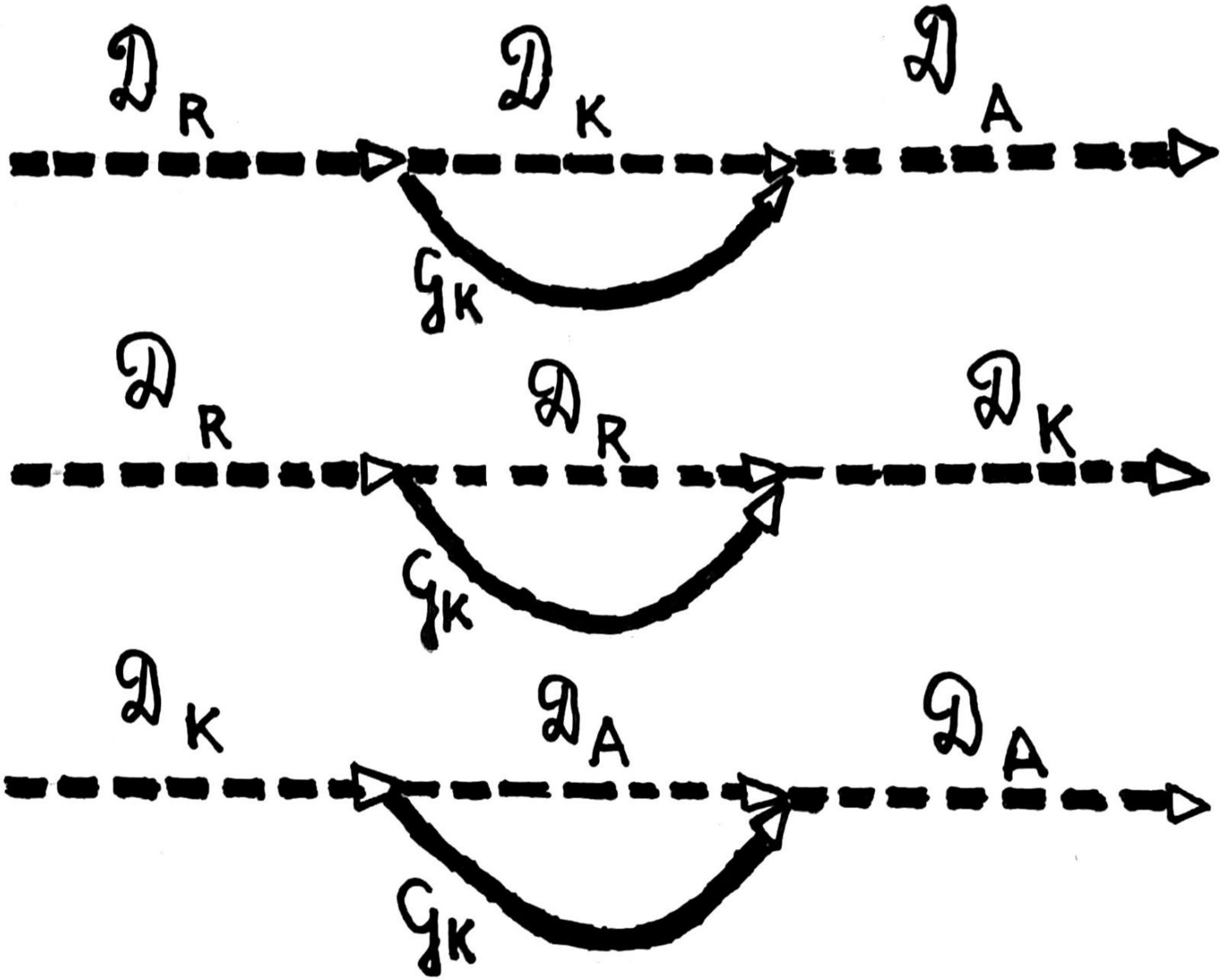
$$+U^*_{\beta,2} U_{\eta,1} [1 - i\Sigma_{1,R} G_R]^{-1} G_R i\Sigma^{\neq}_{12,R} [1 - i\Sigma_{2,R} G_R]^{-1} G_R$$

$$+U^*_{\beta,1} U_{\eta,2} [1 - i\Sigma_{2,R} G_R]^{-1} G_R i\Sigma^{\neq}_{21,R} [1 - i\Sigma_{1,R} G_R]^{-1} G_R$$

$$\tilde{G}_{\beta,\eta,A} = \sum_i \sum_i U^*_{\beta,i} U_{\eta,i} [1 - i\Sigma_{i,A} G_A]^{-1} G_A$$

$$+U^*_{\beta,1} U_{\eta,2} [1 - i\Sigma_{1,A} G_A]^{-1} G_A i\Sigma^{\neq}_{12,A} [1 - i\Sigma_{2,A} G_A]^{-1} G_A$$

$$+U^*_{\beta,2} U_{\eta,1} [1 - i\Sigma_{2,A} G_A]^{-1} G_A i\Sigma^{\neq}_{21,A} [1 - i\Sigma_{1,A} G_A]^{-1} G_A$$



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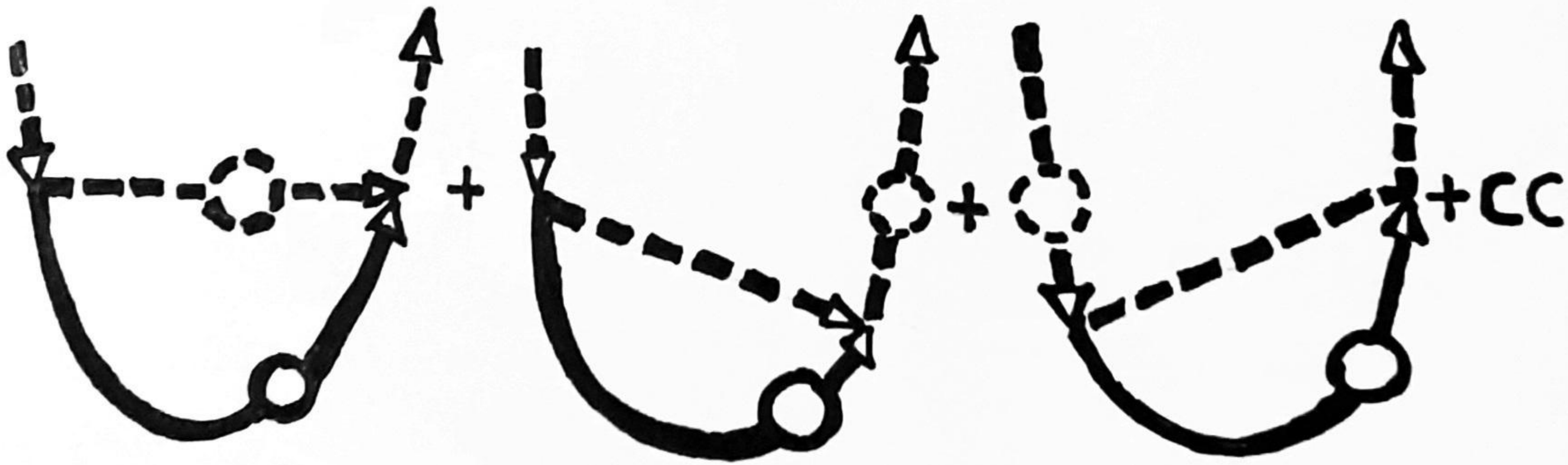
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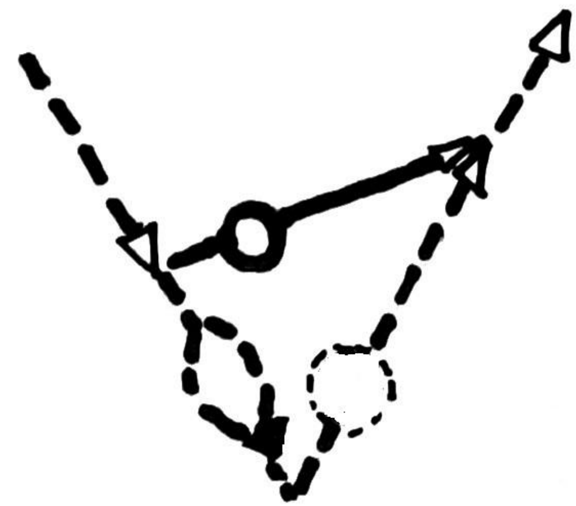
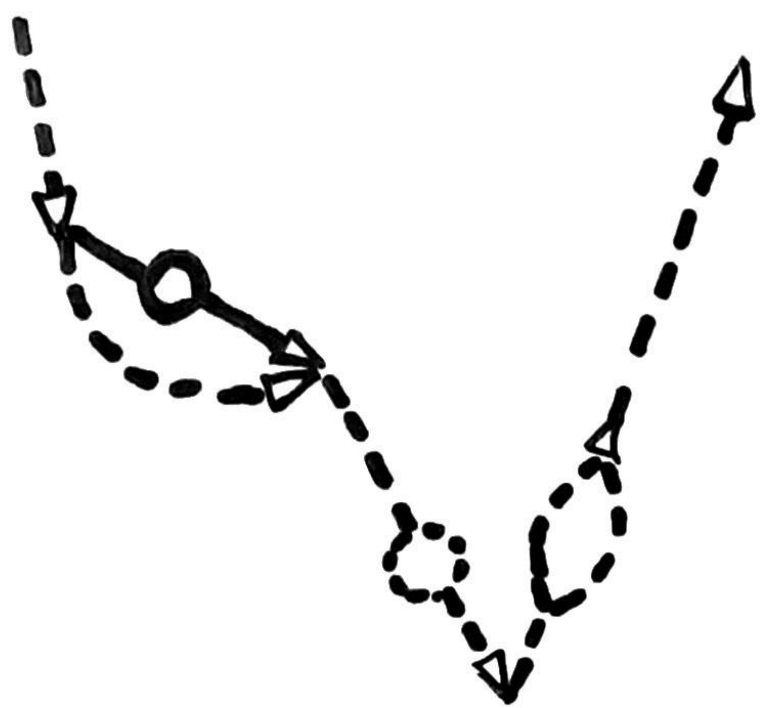
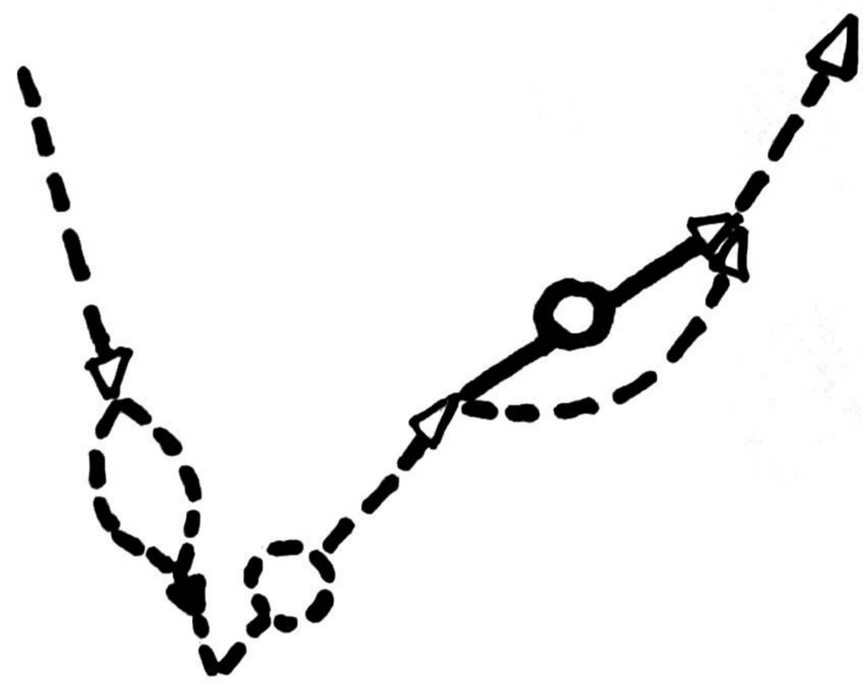

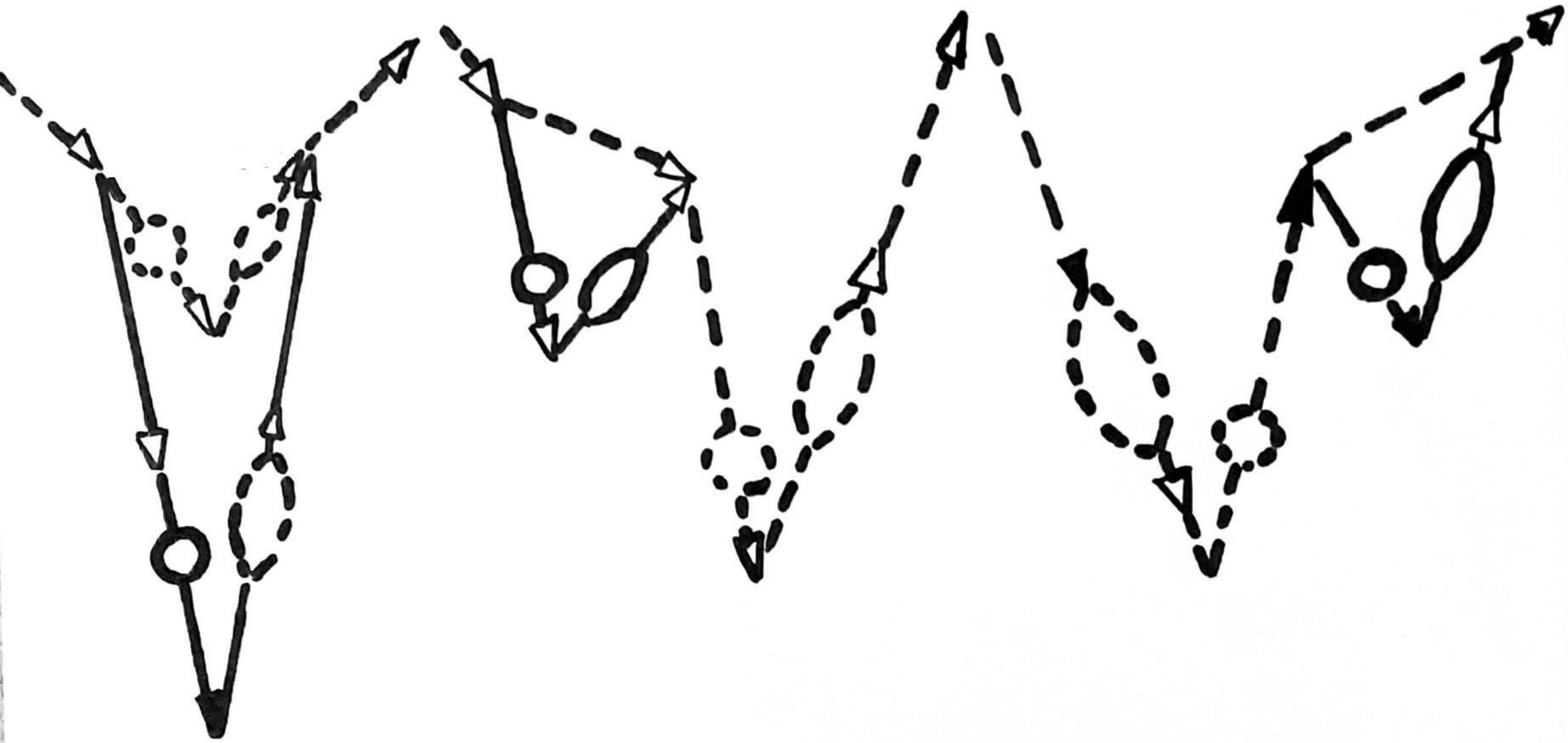

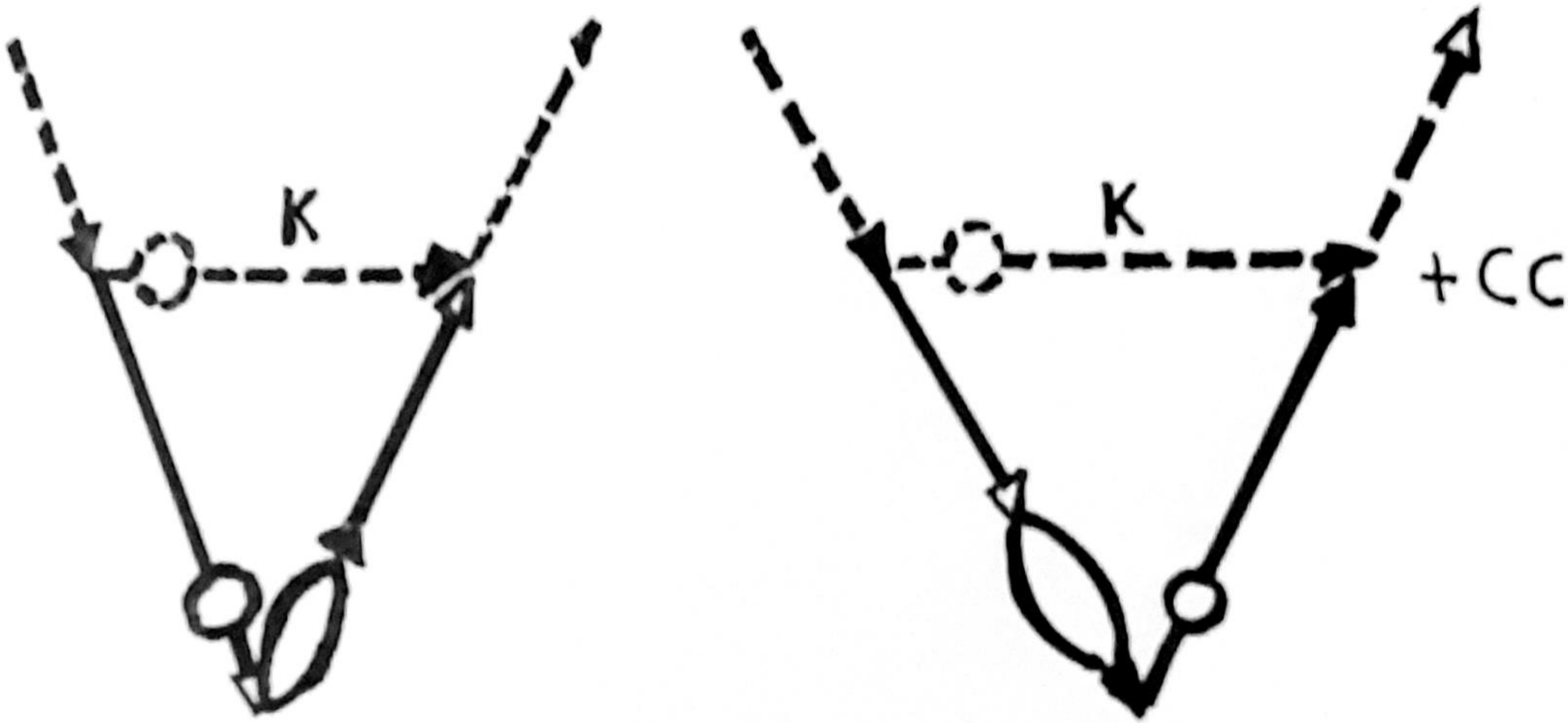
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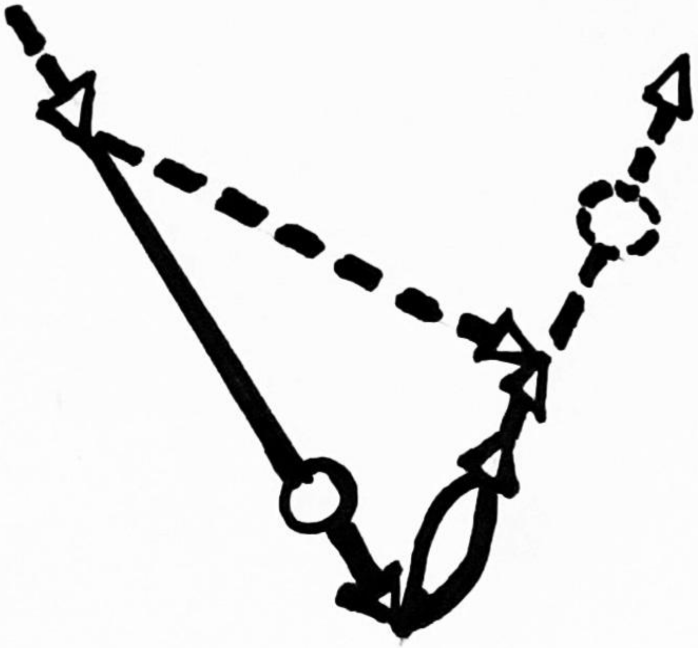

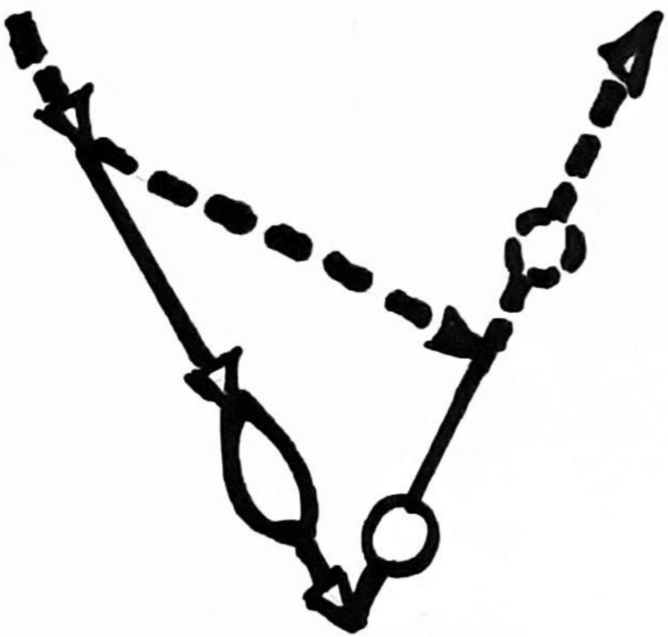
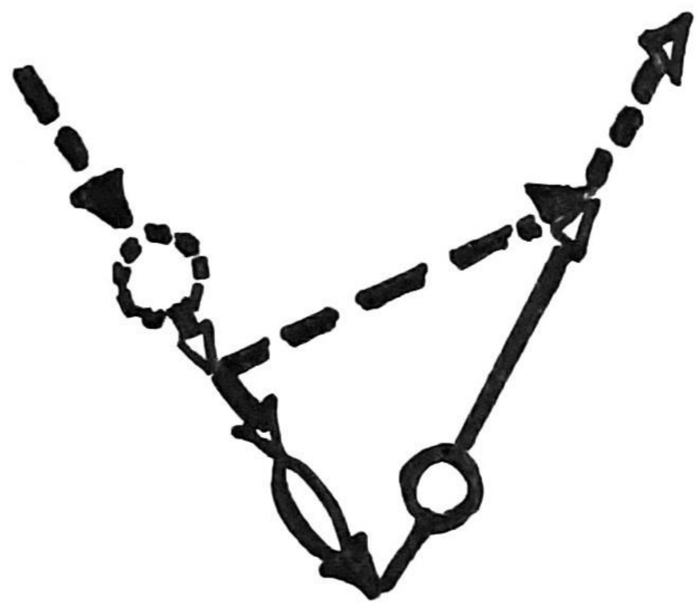

Our calculation method

There are contributions with four and others with five singular propagators

four energy integrals $\frac{dq_{01}dq_{02}dp_{01}dE_l}{2^3\pi^4}$,

representing two external pion energies, one kaon energy and one "mass-shell" pion energy $d|\vec{q}|^2 = dE_l^2, \quad E_l^2 = m_l^2 + |\vec{q}|^2$.

The first three integrations should, each, pick-up some singularity, otherwise the whole contribution vanishes.

The $\int_0^\infty d|\vec{q}|$ integral consists of the pole contribution and principal value integral.

The principal value integral shoud be ignored.

In doing so we are led by three types of arguments:

1. the total contribution should be real, and we are led by an analogy with S-matrix calculation of imaginary part of some diagram and the corresponding Cutkosky rules.

2. In the close vicinity of singularity the contributions from lower values of $|\vec{q}|$ and the contributions from higher values of $|\vec{q}|$cancel.

3. Finally, we know the value of the self-energy only in a single point, so we couldn't do principal value integral, anyway.

The calculation is simplified by choosing the kaon rest frame $n(\beta, \vec{p}) = \delta^3(\vec{p})\delta^1_\beta$. If not all four integrations catch the singularity the contribution will be ignored.

Among those terms, only terms with the convolution in $\tilde{G}_K$ will describe oscillations.

Large The Contribution from Kaon Minimal-Time Vertex

Average of equal time limit (AETL)

$$1+ < N_{\vec{q}}(t) >_l + < N_{\vec{q}}(t) >_{-l}= \frac{\omega_{ql}}{2\pi} [\lim_{0<\delta\to 0} + \lim_{0>\delta\to 0}] \int dq_0 \, e^{-iq_0\delta} \mathcal{D}_{t,\beta,\beta,l,K,(q_0,\vec{q})*}$$

$$= \frac{\omega_{ql}}{2\pi} [\lim_{0<\delta\to 0} + \lim_{0>\delta\to 0}] \int dq_0 \, e^{-iq_0\delta} \int \frac{dp_{01} d^3p}{(2\pi)^4} \int dq_{01} dq_{02} P_t(q_0, \frac{q_{01}+q_{02}}{2})$$

$$\frac{i}{2\pi} \frac{e^{-it(q_{01}-q_{02}+i\epsilon)}-1}{q_{01}-q_{02}+i\epsilon} \; i \sum_{ij,\eta} T_{ij} U^*_{\beta,i} U_{\eta,i} U^*_{\eta,j} U_{\beta,j}$$

$$[1 - i\tilde{\Sigma}_{i,R} G_R]^{-1} [G_{\beta,K,R}(p_1) [D_{l,R}(q_1) D_{l,R}(q_1-p_1) D_{l,A}(q_2) [\frac{q_{02}}{\omega_{l\pi q}} - \frac{q_{01}-p_{01}}{\omega_{l\pi q-p}}]$$

$$+ D_{l,R}(q_1) D_{l,A}(q_2-p_2) D_{l,A}(q_2) [-\frac{q_{01}}{\omega_{l\pi q}} + \frac{q_{02}-p_{02}}{\omega_{l\pi q-p}}]] \tilde{\Sigma}_{j,A}(p_2) G_A(p_2) [1 - i\tilde{\Sigma}_{j,A} G_A]^{-1}(p_2)]$$

$$+ [1 - i\tilde{\Sigma}_{i,R} G_R]^{-1}(p_1) [G_R \tilde{\Sigma}_{i,R}(p_1) [D_{l,R}(q_1) D_{l,R}(q_1-p_1) D_{l,A}(q_2) [\frac{q_{02}}{\omega_{l\pi q}} - \frac{q_{01}-p_{01}}{\omega_{l\pi q-p}}]$$

$$+ D_{l,R}(q_1) D_{l,A}(q_2-p_2) D_{l,A}(q_2) [-\frac{q_{01}}{\omega_{l\pi q}} + \frac{q_{02}-p_{02}}{\omega_{l\pi q-p}}]] [1 - i\tilde{\Sigma}_{j,A} G_A]^{-1}(p_2)] G_{\beta,K,A}(p_2).$$

We can integrate over dq_0 to obtain

$$1+ < N_{\vec{q}}(t) >_l + < N_{\vec{q}}(t) >_{-l} = \int \frac{dp_{01} d^3p}{(2\pi)^4} \frac{\omega_{ql}}{2\pi} \int dq_{01} dq_{02}$$

$$\frac{i}{2\pi} \frac{e^{-it(q_{01}-q_{02}+i\epsilon)} - 1}{q_{01} - q_{02} + i\epsilon} i \sum_{ij,\eta} T_{ij} U^*_{\beta,i} U_{\eta,i} U^*_{\eta,j} U_{\beta,j}$$

$$[1 - i\tilde{\Sigma}_{i,R} G_R]^{-1} [G_R(p_1) G_R(p_1) [D_{l,R}(q_1) D_{l,R}(q_1 - p_1) D_{l,A}(q_2) [\frac{q_{02}}{\omega_{l\pi q}} - \frac{q_{01} - p_{01}}{\omega_{l\pi q-p}}]$$

$$+ D_{l,R}(q_1) D_{l,A}(q_2 - p_2) D_{l,A}(q_2) [-\frac{q_{01}}{\omega_{l\pi q}} + \frac{q_{02} - p_{02}}{\omega_{l\pi q-p}}]] [1 - i\tilde{\Sigma}_{j,A} G_A]^{-1}(p_2)] G_A(p_2)$$

$$[-\tilde{\Sigma}_{i,R}(p_1) \kappa_\beta(p_2) + \kappa_\beta(p_1) \tilde{\Sigma}_{j,A}(p_2)].$$

As there are only K_0 kaons (i.e. no $\bar{K}_0$ kaons and no pions) in the beginning, the contribution is dominated by the choise $\lambda_{p_1} = \lambda_{p_2} = \lambda_{q_1} = \lambda_{q_2} = 1$ and $\lambda_{q-p} = -1$.

PRELIMINARY

Upon taking into account the results of the quadruple integration (next subsection), we obtain the main result

$$< N_{\vec{q}}(t) >_l + < N_{\vec{q}}(t) >_{-l}$$

$$\propto |T_1|^2 \frac{[e^{-2t\Gamma_1} - e^{-2tm_l\Gamma_l/\omega_{l,q})}]}{\Gamma_1 - m_l\Gamma_l/\omega_{l,q})} + |T_2|^2 \frac{[e^{-2t\Gamma_2} - e^{-2tm_l\Gamma_l/\omega_{l,q})}]}{\Gamma_2 - m_l\Gamma_l/\omega_{l,q})}$$

$$-T_1T_2^2 \frac{[e^{t[-i\Delta-(\Gamma_1+\Gamma_2)]} - e^{-2tm_l\Gamma_l/\omega_{l,q})}][\Delta - i(\Gamma_1 + \Gamma_2)]}{\Delta - i(\Gamma_1 + \Gamma_2 - m_l\Gamma_l/\omega_{l,q})}$$

$$-T_1^*T_2 \frac{[e^{t[i\Delta-(\Gamma_1+\Gamma_2)]} - e^{-2tm_l\Gamma_l/\omega_{l,q})}][\Delta + i(\Gamma_1 + \Gamma_2)]}{\Delta + i(+\Gamma_2 - m_l\Gamma_l/\omega_{l,q})}.$$

This expression describes both π_0 and π_l, $l = \pm 1$ decays. But as $m_0\Gamma_0/\omega_{0,q} >> \Gamma_2 >> \Gamma_1 >> m_l\Gamma_l/\omega_{l,q}$, in the $\pi_+\pi_-$ decay we find both oscillation and decay, while in $2\pi_0$ case oscillation term is completely negligible.

Kaon decay and subsequent Pion decay not statistically independent!

No chance to verify it experimentally. the reproduction of $2\pi_0$ decay vertices with enough precission should not be expected.

Conclusions

We demonstrate that the Finite-Time-Path TFT is appropriate tool for the treatment of kaon decay, oscillation and CP-symmetry violation. Without any modification, just by the use of appropriate input parameters the results are applicable also to B^0 and D^0 decays and oscillations.

Within a very simple model with Interaction Lagrangian built as mass mixing matrix , we calculate kaon decay, oscillation and CP-simmetry violations. Caskading processes $K_0 \to \pi_+ + \pi_-$ and $K_0 \to \pi_0 + \pi_0$ are calculated under the condition that pions decay further

The time dependent single particle distribution of pions show clear oscillation patern for $\pi_\pm$. The K_S regeneration is strongly suresse in π_0 case.

Model kann predict ,at least in principle, some features like

1. Oscillation and decay parameters
2. Energy difference $m_{K_S} - m_{K_L}$
3. It suggest the search for a process in which the suppresion of K_S regeneration can be experimentally verified.

Drawbacks of the model:

1. These predictions rely heavily on the available calculations of various self-energies Σ^0. While there is quite an amount of calculations of decay rates , but the calculations are done with Σ_F, i.e. not Σ^0_R, mass shifts are almost never calculated.

2.Self-energies are complicated analytic functions . They involve poles and cuts in the complex plane. In this calculation these contirbutions(residues) are ignored. We use the value of self-energy at a single point, and combine it with symmetry properties and the behaviour near the edge of complex plane.

3. for four-point functions the formalism requires further development. Fortunatelly, to calculate scattering processes, one can identify the contributions with integrated paricles degrees of freedom , as it is usual when dealing with inclusive processes.

Further research can proceed along two main lines:

1. fitting the data and work on improving the model by taking into account the new data.

2. apply the formalism to the other oscillating and decay processes decay of positronium, damping rates, in out of equilibrium many body problems, etc.

3.We have applied the method to the case of neutrino oscillations and obtained predictions equivalent to Pontecorvo-Maki-Nakagawa-Sakata formula.

4. Intriguing possibility that eventual heavy neutrino decay may induce the time dependent mixpure of neutrinos.