On hard exclusive processes, meson production, and hadron tomography

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Outline

- Introduction:

  from DIS and PDFs to hard exclusive processes

  and generalized parton distributions (GPDs)

- Deeply virtual hard exclusive processes:

  Compton scattering (DVCS), meson production (DVMP)

  - DVMP at higher order in α_S: NLO global DIS+DVCS+DVρ^0P fits

    [Čuić, Duplančić, Kumerički, P-K. '23]

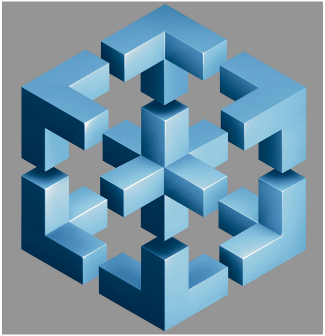
  - DVMP at higher-twist: DVπ^0P at twist-3

    [Duplančić, Kroll, P-K., Szymanowski '24]

- Summary and outlook

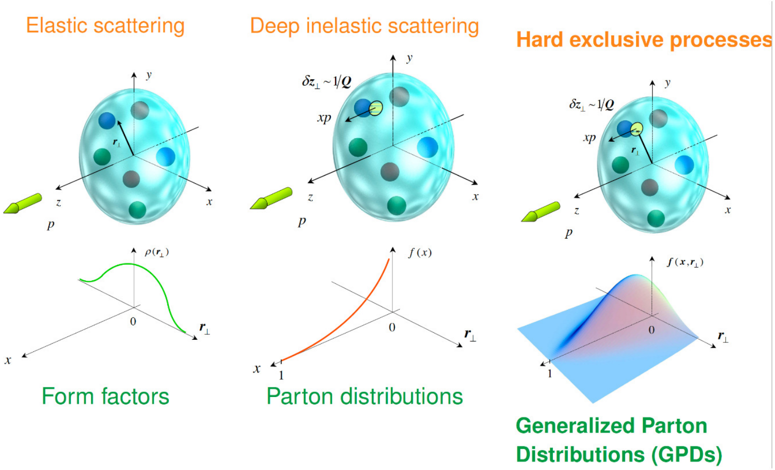
Introduction

Hadron structure ?

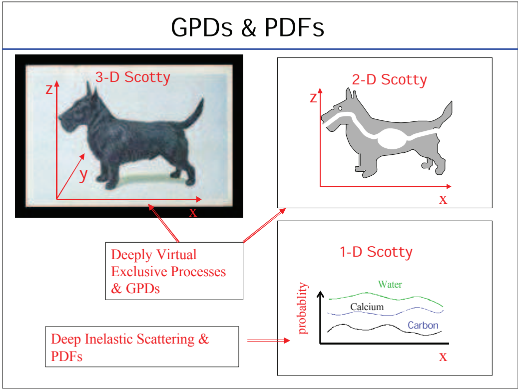


(Escher 3D, Al Borge)

Introduction

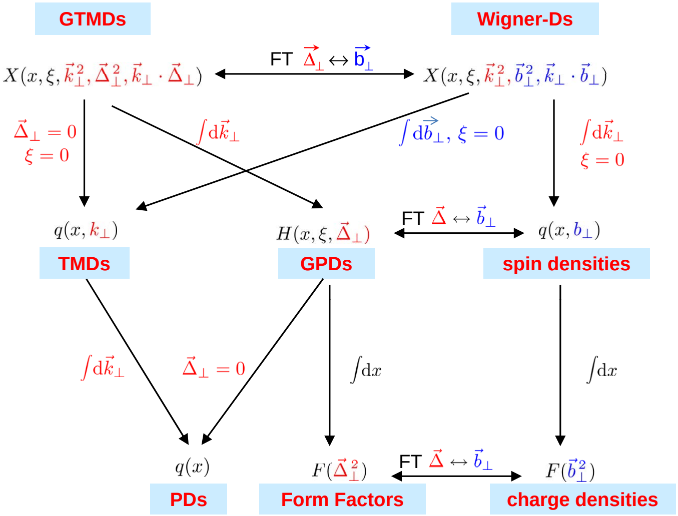


Introduction



[V. D. Burkert, 2006]

Introduction



[B. Pasquini, 2010]

(→ see also Y. Hagiwara talk)

Hard exclusive processes

Prerequisites

$\exists$ large scale(s)

$\Rightarrow$ 1/scale (twist) expansion and α_S expansion

$\rightarrow$ factorization:

hard scattering amplitude ($\mathcal{M}$)	$=$	elementary hard-scattering amplitude ($\mathcal{H}$)	$\otimes$	hadron wave functions (DAs, GPDs)

(via partons) $\Uparrow$ pQCD

evolution $\Uparrow$ pQCD

form $\uparrow$ input

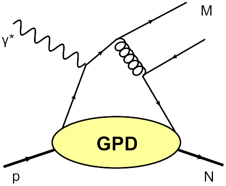
$+$ power corrections/higher-twists

(factorization?)

DA . . . distribution amplitude (meson, baryon)
GPD . . . generalized parton distributions

Handbag factorization: meson production

$\boxed{2 \to 2}$



$\boxed{\gamma^{(*)} N \to M N'}$

DEEPLY VIRTUAL	WIDE ANGLE
$Q^2 >>,\ -t <<$	$-t,\ -u,\ s >>$
DVMP	WAMP
[Collins, Frankfurt, Strikman '97]	[Huang, Kroll '00]

DVMP:

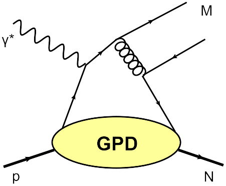
- factorization $\boxed{\mathcal{H}^a \otimes GPD \otimes DA}$
- GPDs at small $(-t)$

WAMP:

- arguments for fact.
- GPDs at large $(-t)$

Handbag factorization: meson production

$\boxed{2 \to 2}$



$\boxed{\gamma^{(*)} N \to M N'}$

DEEPLY VIRTUAL

$Q^2 >>, -t <<$

DVMP

[Collins, Frankfurt, Strikman '97]

- factorization
  $\boxed{\mathcal{H}^a \otimes GPD \otimes DA}$
- GPDs at small $(-t)$
- tw2: γ_L^*, tw3: γ_T^*

WIDE ANGLE

$-t, -u, s >>$

WAMP

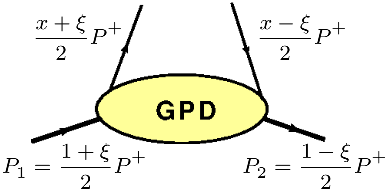
[Huang, Kroll '00]

- arguments for fact.
- GPDs at large $(-t)$

large scale $\mathcal{Q}^2$ ($\mathcal{Q}^2$, s or ...)

- twist expansion: $\langle \mathcal{H} \rangle^{tw2} + \frac{\langle \mathcal{H} \rangle^{tw3}}{\mathcal{Q}} + \ldots$
- α_S expansion for each twist: $\alpha_S(\mathcal{Q}) \langle \mathcal{H} \rangle^{LO} + \alpha_S^2(\mathcal{Q}) \langle \mathcal{H} \rangle^{NLO}$

Generalized Parton Distributions



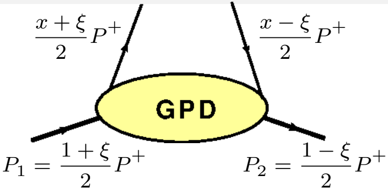
$P = P_1 + P_2$

$\Delta = P_2 - P_1$

GPDs: $\boxed{F^a(x, \xi, t; \mu),\ a \in \{q, G\}}$ μ. . . factorization scale

x	parton's "average" longitudinal momentum fraction
$\xi = -\frac{\Delta^+}{P^+}$	longitudinal momentum transfer (skewness)
$\Delta^2 = t$	momentum transfer

Generalized Parton Distributions



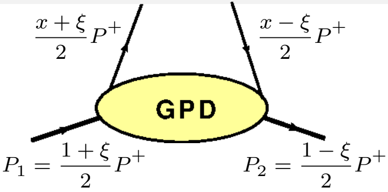
$P = P_1 + P_2$

$\Delta = P_2 - P_1$

GPDs: $\boxed{F^a(x,\xi,t;\mu),\ a \in \{q,G\}}$ μ... factorization scale

- vector (H^a, E^a) and axial-vector GPDs ($\widetilde{H}^a$, $\widetilde{E}^a$)
- transversity GPDs (H_T^a, E_T^a, $\widetilde{H}_T^a$, $\widetilde{E}_T^a$)

Generalized Parton Distributions



$P = P_1 + P_2$

$\Delta = P_2 - P_1$

GPDs: $\boxed{F^a(x, \xi, t; \mu),\ a \in \{q, G\}}$ $\mu\ldots$factorization scale

- vector $(H^a,\ E^a)$ and axial-vector GPDs $(\widetilde{H}^a,\ \widetilde{E}^a)$
- transversity GPDs $(H_T^a,\ E_T^a,\ \widetilde{H}_T^a,\ \widetilde{E}_T^a)$

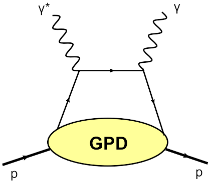
▷ forward limit: $H^a,\ \widetilde{H}^a,\ H_T^q \overset{\xi=0,t=0}{\longrightarrow}$ PDFs

▷ sum rules: $\int_{-1}^{1} dx\, H^q, E^q \to$ form factors

▷ higher moments: $\int_{-1}^{1} dx\ x\ H, E \to$ angular momentum, EMT
$\to$ proton spin problem, pressure inside the proton

GPDs from hard exclusive processes

DVCS



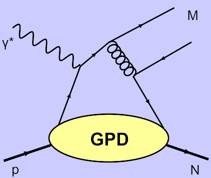
$$\gamma^* N \to \gamma N$$

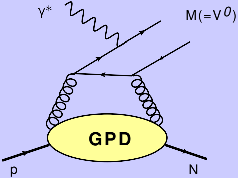
H^q, E^q, $\widetilde{H}^q$, $\widetilde{E}^q$

NLO:

H^G, E^G, $\widetilde{H}^G$, $\widetilde{E}^G$, F_T^G

DVMP





$$\gamma_L^* N \to M N'$$

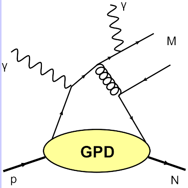
$M = V_L$: H^{q_i}, E^{q_i}; H^G, E^G

$M = PS$: $\widetilde{H}^{q_i}$, $\widetilde{E}^{q_i}$

$$\gamma_T^* N \to M N'$$

$M_{\text{twist-3}} \Rightarrow F_T^q$

(γM) P



$$\gamma N \to (\gamma M) N'$$

F^a, F_T^a

→ see N. Crnković talk

Meson production status

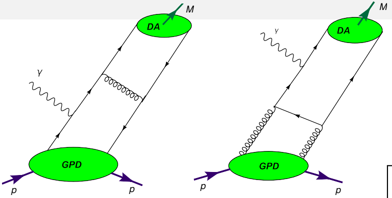
- experimental data from HERA, JLab and COMPASS (CERN), expected from EIC (Brookhaven, 2030)
- DVρ^0P:
  - data show importance of γ_L^* contributions $(Q^2 < 100$ GeV$^2)$

    $\rightarrow$ twist-2 predictions describe σ_L
  - global DIS+DVCS+DVV_LP fits at NLO [Čuić, Duplančić, Kumerički, P-K. '23]
- DVπP:
  - data show suppression of γ_L^* contributions $(Q^2 < 10$ GeV$^2)$

    $\rightarrow$ twist-3 predictions describing γ_T^* contributions needed
  - twist-3 contributions analyzed [Duplančić, Kroll, P-K., Szymanowski '24]

DVMP at twist-2 NLO

DVMP to NLO



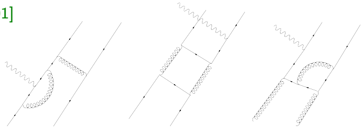
$$\sim \alpha_S(Q)\, \langle \mathcal{H} \rangle^{LO}$$

NLO DV PS$^+$ prod.: [Belitsky and Müller '01]

NLO DV V_L prod.: [Ivanov et al '04,]

NLO DV V_L (corr.), PS, (S, PV_L) prod.:

[Duplančić, Müller, P-K. '17]



$$\sim \alpha_S^2(Q)\, \langle \mathcal{H} \rangle^{NLO}$$

- only few DVMP phenomenological analysis to NLO
- NLO corrections important: large NLO corrections,
  reduction of dependence on the scales and schemes
- GPDs universal ⇒ NLO global DIS+DVCS+DVMP fits needed

From x space to conformal momentum space

$$
{}^{a}\mathcal{M}(\xi, t, Q^2) = \int_{-1}^{1} \mathrm{d}x \int_{0}^{1} \mathrm{d}\tau \, T^{a}(x, \xi, \tau, \mu^2)) \, F^{a}(x, \xi, t, \mu^2) \, \phi_M(\tau, \mu^2)
$$

$a{=}q,G$

T^a...subprocess hard-scattering amplitude

F...GPDs, ϕ_M...meson DA

conformal moments (analogous to Mellin moments in DIS $x^n \to C_n^{3/2}(x), C_n^{5/2}(x)$)

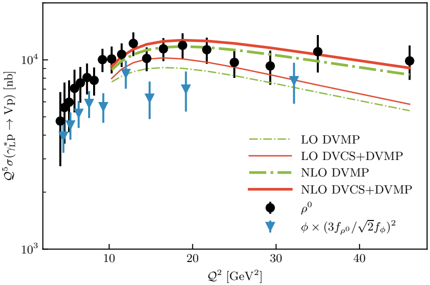
[Müller, Lautenschläger, P-K., Schäfer 2014] [Duplančić, Müller, P-K. 2017]

$$
\begin{aligned}
{}^{a}\mathcal{M}(\xi, t, Q^2) &= \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \left[i \pm \left\{ \begin{array}{c} \tan \\ \cot \end{array} \right\} \left(\frac{\pi j}{2} \right) \right] \xi^{-j-1} \\
&\times \left[\mathrm{T}_{jk}(Q^2/\mu^2) \overset{k}{\otimes} \phi_{\mathrm{M},k}(\mu^2) \right] F_j^{\mathsf{a}}(\xi, t, \mu^2)
\end{aligned}
$$

→ advantages: easy evolution, interesting GPD modeling, moments accessible on lattice, stable numerics and efficient fitting

- small-x global fits to HERA collider data (DIS, DVCS, DVρ^0_LP)

[Čuić, Duplančić, Kumerički, P-K. '23]



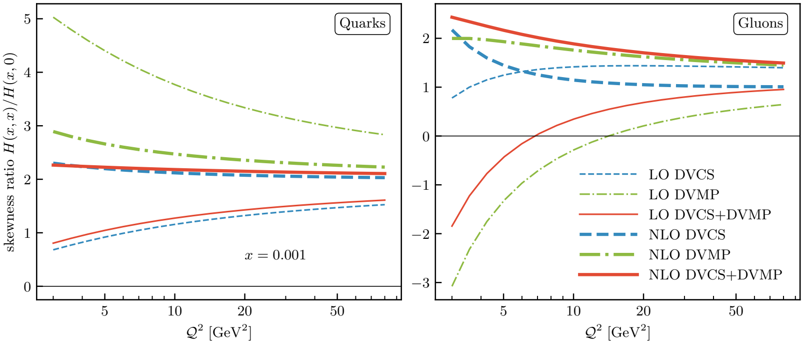
σ_L asymptoticaly: $\dfrac{1}{Q^6}$

experimental data for fixed x_B: $\approx \dfrac{1}{Q^4}$, for fixed W: $\approx \dfrac{1}{Q^5}$

$\to$ successful description of Q^2 dependence

Global NLO fits (DIS+DVCS+DVV_LP)

Skewness ratio $r = \dfrac{H(x,x)}{H(x,0)}$



- r measures goodness of GPD extraction $\Rightarrow$ NLO fit successful

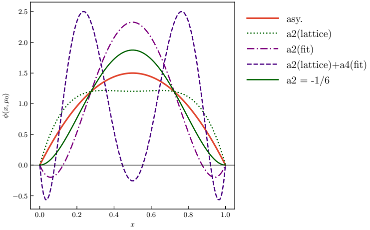
Concluding remarks: DVρ_L^0P at twist-2 NLO

- Global DIS+DVCS+DVMP fits show importance of NLO
  $\Rightarrow$ universal GPDs

- DVρ_L^0P can only be described at NLO

- Meson DA additional nontrivial nonperturbative input
  $\Rightarrow$ end-point supressed ρ_L^0 DA favoured?

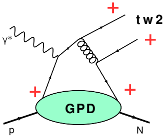
$\rightarrow$ work in progress with Raj Kishore, K. Kumerički



DVπP at twist-3

π production to twist-3

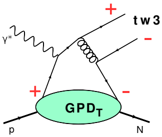
$\mathcal{H}^{\pi}_{0\lambda,\mu\lambda}$ . . . non-flip subprocess amplitudes (twist-2)

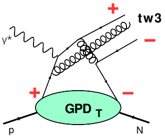


$\widetilde{H},\ \widetilde{E}$

$\mathcal{H}^{\pi}_{0-\lambda,\mu\lambda}$ . . . flip subprocess amplitudes (twist-3) $\boxed{\sim \mu_\pi/Q}$ ($\mu_\pi = 2$ GeV)

just meson DA tw-3 contributions





$H_T,\ \bar{E}_T = 2\widetilde{H}_T + E_T,\ \ldots$

distribution amplitudes (DAs):

twist-2 ($q\bar{q}$) : ϕ_π

2-body ($q\bar{q}$) twist-3 $\phi_{\pi p},\ \phi_{\pi\sigma}$ 3-body ($q\bar{q}g$) twist-3 $\phi_{3\pi}$

$\rightarrow$ connected by equations of motion (EOMs)

Subprocess amplitudes: twist-3

[Duplančić, Kroll, P-K., Szymanowski '24]

- 2- ($q\bar{q}$) and 3-body ($q\bar{q}g$) contributions necessary for gauge invariance

- end-point singularities:

$$\int_0^1 \frac{d\tau}{\tau}\phi_{\pi p}(\tau)\ \frac{1}{(x+\xi+i\epsilon)^2} \overset{x}{\otimes} H_T(\bar{E}_T)$$

$$\phi_{\pi p}(\tau) = 1 + \omega_{\pi p}\, C_2^{1/2}(2\tau - 1) + \ldots$$

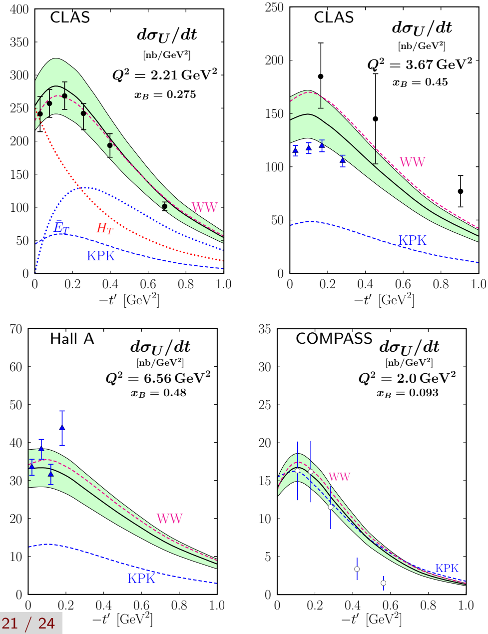
τ ... quark long. momentum fraction

$\rightarrow$ factorization broken $\Rightarrow$ regularization:

- modified perturbative approach (MPA)
  (with $k_\perp$ quark transverse momenta) as in [Goloskov, Kroll, '10])
- pure collinear picture with effective m_g^2

$$\int_0^1 d\tau \phi_{\pi p}(\tau) \frac{1}{((x-\xi)\bar{\tau} - m_g^2(2\xi)/Q^2 + i\epsilon)} \frac{1}{(x-\xi+i\epsilon)} \overset{x}{\otimes} H_T(\bar{E}_T)$$

Modified perturbative approach (MPA): $d\sigma_U$



solid curves: set mod

dashed curves: set KPK, WW

exp data:

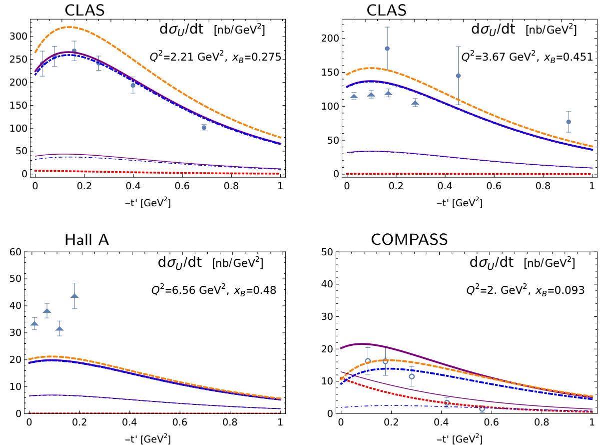
- full circles [CLAS '14]
- triangles [Hall A '20]
- open circles [COMPASS '19]

$$\frac{d\sigma_U}{dt} = \frac{d\sigma_T}{dt} + \epsilon \frac{d\sigma_L}{dt}$$

$$\frac{d\sigma_T}{dt} : H_T, \bar{E}_T \qquad \frac{d\sigma_L}{dt} : \widetilde{H}, \widetilde{E}$$

- σ_L negligible except for COMPASS kin. (40%)

Collinear approach with m_g^2: $d\sigma_U$



set mod: purple thick

set KPK: purple thin

WW: orange dashed

red curves: tw2

blue curves: tw3

exp data:

full circles [CLAS '14]

triangles [Hall A '20]

open circles [COMPASS '19]

- tw2 (σ_L) significant for COMPASS kinematics (small x_B)
- Q^2 dependence challenging

Concluding remarks: DVπP at twist-3

- Improved twist-3 analysis includes 2- and 3-body contributions:

  $\rightarrow$ twist-3 dominates at accessible energies, except for COMPASS kinematics.

- NLO corrections to twist-2 may be important for COMPASS kinematics.

  $\rightarrow$ new data available (2412.19923)

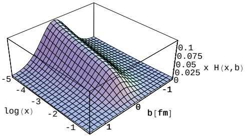
- Next steps: GPD fits (MPA and collinear), DA fits or both

To conclude...

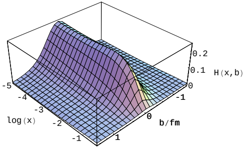
Hard exclusive processes and GPDs → framework timeline:

- Exploration (DONE)
  - parton-model theory, factorization
  - first measurements
- Consolidation (HERE WE ARE)
  - model developement (GPDs, DAs)
  - many consistent measurements
- Precision (FIRST STEPS....)
  - full-fledged global analysis
  - precision measurements

quarks:



gluons:



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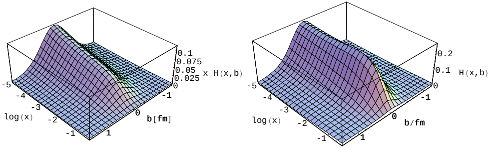
quarks: gluons:



Thank you!

Appendix

Generic questions

- is factorization proven/assumed? broken?
- pQCD calculation:
  - are accessible energies high enough for perturbative treatment?
  - which are theoretical uncertainties of finite order predictions?
  - are higher-orders included?
  - are power corrections/higher-twist effects important/to be included at experimentally accessible energies?
- nonperturbative input:
  - form/modeling of DAs/GPDs

From momentum fraction to CPaW formalism

DVCS: Compton form factors

$$\mathcal{F}^a(\xi,t,Q^2) = \int \mathrm{d}x \, T^a(x,\xi,Q,\mu_F;\mu_R) \, F^a(x,\xi,t,\mu_F)$$

$a = q, G$ or NS,S

DVMP: Transition form factors

$$\mathcal{F}^a_M(\xi,t,Q^2) = \int \mathrm{d}x \int \mathrm{d}u \, T^{M,a}(x,\xi,u,\ldots) \, F^a(x,\xi,t,\mu_F) \, \phi_M(u,\mu_\varphi)$$

F^a ... GPD, ϕ_M ... DA, T^a ... hard-scattering amplitude

- conformal partial wave expansion: $C_n^{3/2}(x)$ (quarks), $C_n^{5/2}(x)$ (gluons)

$$F^q_j(\xi,\ldots) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x \, \xi^{j-1} C_j^{3/2}(x/\xi) F^q(x,\xi,\ldots), \ \ldots, \ T^a_j, \ T^{M,a}_{j,k}$$

- series summed using Mellin-Barnes integral over complex j

$$\int_{-1}^{1} \frac{dx}{2\xi} \to 2\sum_{j=0}^{\infty} \xi^{-j-1} \to \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \, \xi^{-j-1} \left[i \pm \begin{Bmatrix} \tan \\ \cot \end{Bmatrix} \left(\frac{\pi j}{2} \right) \right] \equiv \overset{j}{\otimes}$$

[Müller '06, Müller, Schäfer '06]

Global NLO fits (DIS+DVCS+DVV_LP)

small-x global fits to HERA collider data (ρ_0)

- only NLO predecessor: [Lautenschlager, Müller, Schäfer '13 unpublished]
- hard scattering amplitude corrected [Duplančić, Müller, P-K. '17]
- new NLO fit [Čuić, Duplančić, Kumerički, P-K. '23]: improved treatment of experimental data

GPD model: [Kumerički, Müller, P-K., Schäfer '07, Kumerički, Müller '10]

- $$H_j^a(\xi,t) = q_j^a \frac{1+j-\alpha_0^a}{1+j-\alpha_0^a-\alpha'^a t}\left(1-\frac{t}{m_a^2}\right)^{-2}\left(1+s_2^a\,\xi^2+s_4^a\,\xi^4\right)$$

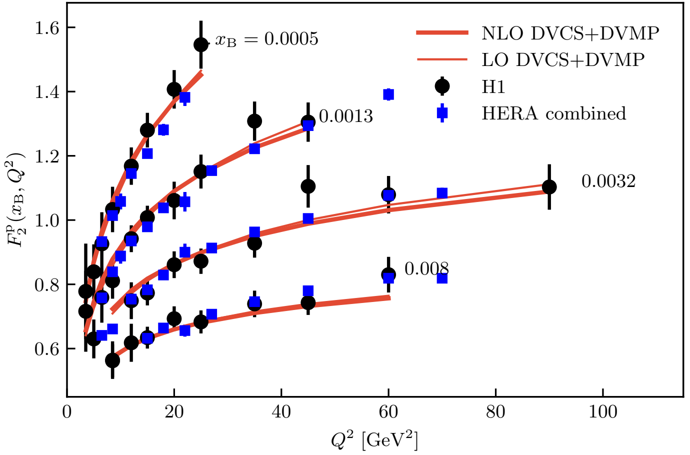
$$q_j^a = N_a \frac{B(1-\alpha_0^a+j,\beta^a+1)}{B(2-\alpha_0^a,\beta^a+1)}$$

- small-x kinematics $\Rightarrow a \in \{\mathsf{sea}, \mathsf{G}\}$, only dominant H GPD

Fit parameters:

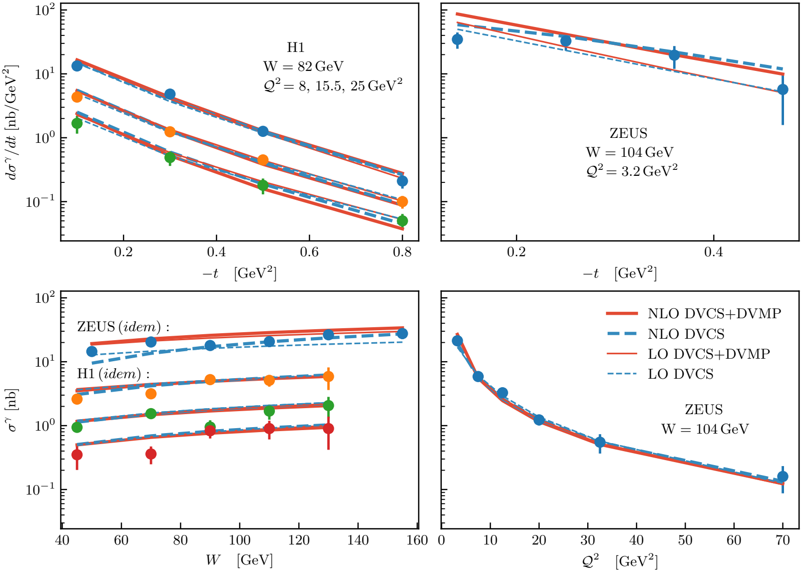
- DIS: $\{N_{\mathsf{sea}},\, \alpha_0^{\mathsf{sea}},\, \alpha_0^{\mathsf{G}}\}$
- DVCS+DVMP: $\{\alpha'^{\mathsf{sea}},\, \alpha'^{\mathsf{G}},\, m_{\mathsf{sea}}^2,\, m_{\mathsf{G}}^2,\, s_2^{\mathsf{sea}},\, s_2^{\mathsf{G}},\, s_4^{\mathsf{sea}},\, s_4^{\mathsf{G}}\}$

Global NLO fits (DIS+DVCS+DVV_LP)



- may seem trivial, but not all popular models describe DIS

Global NLO fits (DIS+DVCS+DVV_LP)



Global NLO fits (DIS+DVCS+DVV$_L$P)

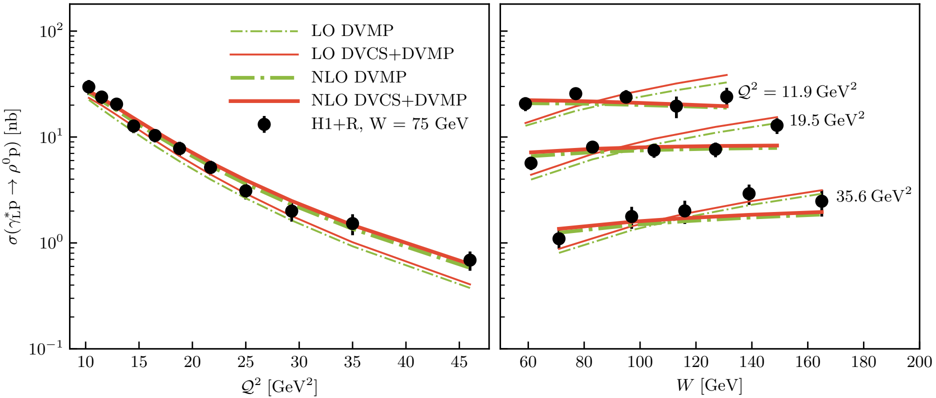
Dataset	Refs.	n_{pts}	LO-			NLO-		
			DVCS	DVMP	DVCS-DVMP	DVCS	DVMP	DVCS-DVMP
DIS	[90]	85	0.6	0.6	0.6	0.8	0.8	0.8
DVCS	[92–95]	27	0.4	$\gg 1$	0.6	0.6	$\gg 1$	0.8
DVMP	[88, 89]	45	$\gg 1$	3.1	3.3	$\gg 1$	1.5	1.8
Total		157	$\gg 1$	$\gg 1$	1.4	3.7	$\gg 1$	1.1

Table 3. Values of χ^2/n_{pts} for each LO or NLO model (columns) for the total DIS + DVCS + DVMP dataset and for subsets corresponding to different processes (rows). (The values denoted by $\gg 1$ are greater than 10.).

- NLO DVCS-DVMP fit describes the data well

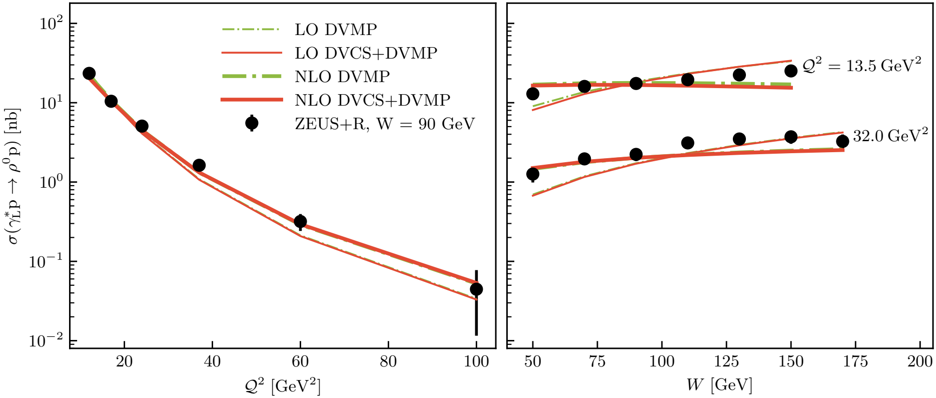
Global NLO fits (DIS+DVCS+DVV$_L$P)

[Čuić, Duplančić, Kumerički, P-K. '23]

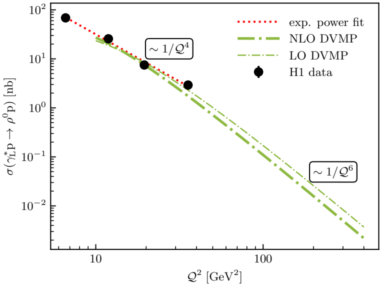


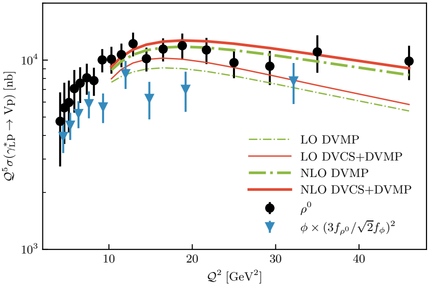
$$\boxed{R \equiv \frac{\sigma_{\mathsf{L}}^{\rho^0}}{\sigma_{\mathsf{T}}^{\rho^0}}} \to R(W, \mathcal{Q}^2) = \frac{\mathcal{Q}^2}{m_{\rho^0}^2}\left(1 + a\frac{\mathcal{Q}^2}{m_{\rho^0}^2}\right)^{-p}\left(1 + b\frac{\mathcal{Q}^2}{W}\right) \text{ fit}$$

Global NLO fits (DIS+DVCS+DVV$_L$P)



Global NLO fits (DIS+DVCS+DVV$_L$P)





σ_L asymptoticaly: $\dfrac{1}{Q^6}$

experimental data for fixed x_B: $\approx \dfrac{1}{Q^4}$, for fixed W: $\approx \dfrac{1}{Q^5}$

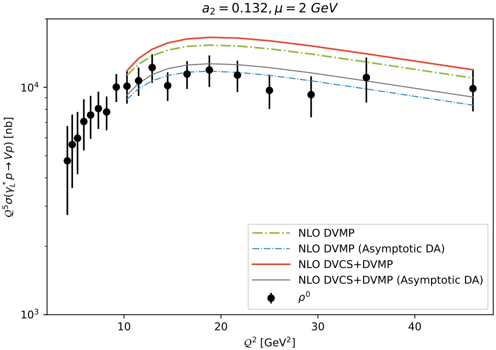
- successful description of Q^2 dependence

Improving DA description

→ work in progress with Raj Kishore, K. Kumerički

$$\phi^{\rho}(u,\mu_F) = 6u(1-u)\left[1 + a_2(\mu_F)\, C_2^{3/2}(2u-1) + \dots\right]$$

$a_2(\mu_0) = 0.132,\ \mu_0 = 2$ GeV [Braun et al. '16]



⇒ significant impact of the DA form

Improving DA description

→ work in progress with Raj Kishore, K. Kumerički

- new GPD fit:

Dataset	Refs.	n_{pts}	LO	NLO			
			asy.	asy.	a_2 (lat.)	a_2 (fit)	a_2 (lat.) $+a_4$ (fit)
DIS	[5]	85	0.6	0.8	0.8	0.8	0.8
DVCS	[6, 7, 8, 9]	27	0.6	0.8	0.9	0.9	0.9
DVMP	[1, 10]	45	3.3	1.8	2.1	0.9	0.7
Total		157	1.4	1.1	1.2	0.9	0.8

- a_2(fit)$= -0.369$
- $a_{2\,\text{[lattice: Braun et al. '16]}} = 0.132$, a_4(fit)$= -0.519$

⇒ dominant influence of LO $\sim \sum a_n$

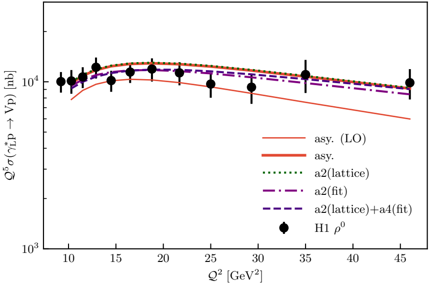
⇒ indication of end-point supressed ρ_L DA [Liu, Shuryak, Zahed '24]

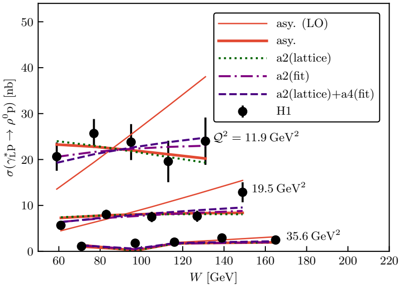
⇒ GPDs not changed much

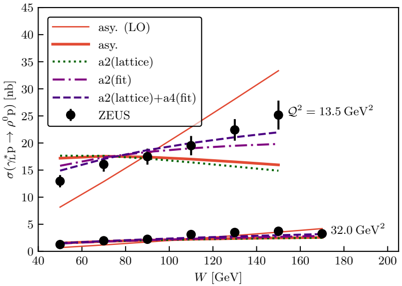
(similar skewness ratio, W dependence improved)

Improving DA description

→ work in progress with Raj Kishore, K. Kumerički

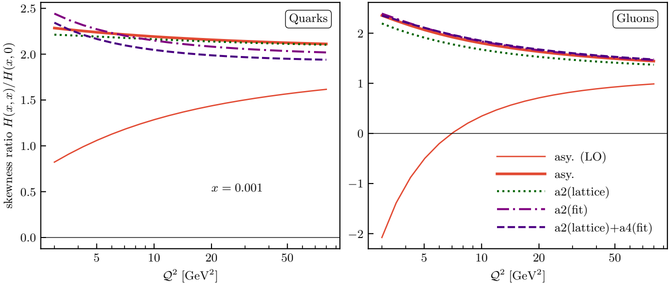


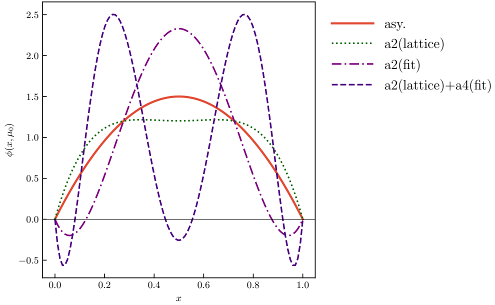




Improving DA description

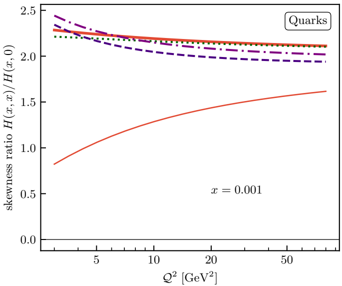
→ work in progress with Raj Kishore, K. Kumerički

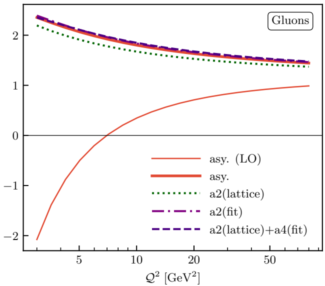


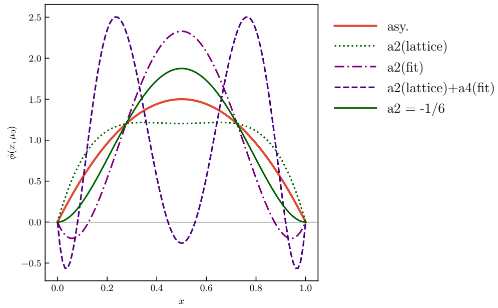


Improving DA description

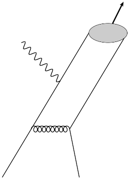
→ work in progress with Raj Kishore, K. Kumerički







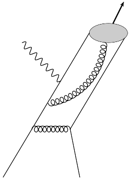
Subprocess amplitudes $\mathcal{H}$: projectors



$q\bar{q} \to \pi$ projector [Beneke, Feldmann '00]

$$(\tau q' + k_\perp) + (\bar{\tau} q' - k_\perp) = q'$$

$$\mathcal{P}_2^\pi \sim f_\pi \Big\{ \gamma_5 \not{q}' \phi_\pi(\tau, \mu_F) + \mu_\pi(\mu_F) \Big[\gamma_5 \, \phi_{\pi p}(\tau, \mu_F) - \frac{i}{6} \gamma_5 \, \sigma_{\mu\nu} \frac{q'^\mu n^\nu}{q' \cdot n} \phi'_{\pi\sigma}(\tau, \mu_F) + \frac{i}{6} \gamma_5 \, \sigma_{\mu\nu} \, q'^\mu \phi_{\pi\sigma}(\tau, \mu_F) \frac{\partial}{\partial k_{\perp\nu}} \Big] \Big\}_{k_\perp \to 0}$$



$q\bar{q}g \to \pi$ projector [Kroll, P-K '18]

$$\tau_a q' + \tau_b q' + \tau_g q' = q'$$

$$\mathcal{P}_3^\pi \sim f_{3\pi}(\mu_F) \, \frac{i}{g} \gamma_5 \, \sigma_{\mu\nu} q'^\mu g_\perp^{\nu\rho} \, \frac{\phi_{3\pi}(\tau_a, \tau_b, \tau_g, \mu_F)}{\tau_g} , \quad f_{3\pi} \sim \mu_\pi$$

$\mu_\pi = m_\pi^2/(m_u + m_d) \cong 2$ GeV

DAs and EOMs

$$
\begin{aligned}
\tau\phi_{\pi p}(\tau) + \frac{\tau}{6}\,\phi'_{\pi\sigma}(\tau) - \frac{1}{3}\,\phi_{\pi\sigma}(\tau) &= \phi_\pi^{EOM}(\bar\tau) \\
\bar\tau\phi_{\pi p}(\tau) - \frac{\bar\tau}{6}\phi'_{\pi\sigma}(\tau) - \frac{1}{3}\phi_{\pi\sigma}(\tau) &= \phi_\pi^{EOM}(\tau)
\end{aligned}
$$

$$
\phi_\pi^{EOM}(\tau) = 2\frac{f_{3\pi}}{f_\pi\mu_\pi}\int_0^{\bar\tau}\frac{d\tau_g}{\tau_g}\,\phi_{3\pi}(\tau,\bar\tau-\tau_g,\tau_g)
$$

- EOMs and symmetry properties
  $\Rightarrow$ the subprocess amplitudes in terms of two twist-3 DAs
  and 2- and 3-body contributions combined
- combined EOMs $\rightarrow$ first order differential equation $\Rightarrow$ from known form of $\phi_{3\pi}$ [Braun, Filyanov '90] one determines $\phi_{\pi p}$ (and $\phi_{\pi\sigma}$)

Note: $q\bar{q}g$ projector and EOMs were derived using light-cone gauge for constituent gluon

Subprocess amplitudes: twist-3

General structure:

$$\begin{aligned}\mathcal{H}^{\pi,tw3} &= \mathcal{H}^{\pi,tw3,q\bar{q}} + \mathcal{H}^{\pi,tw3,q\bar{q}g} \\ &= \big(\mathcal{H}^{\pi,\phi_{\pi p}} + \underbrace{\mathcal{H}^{\pi,\phi_\pi^{EOM}}\big) + \big(\mathcal{H}^{\pi,q\bar{q}g,C_F}} + \mathcal{H}^{\pi,q\bar{q}g,C_G}\big) \\ &= \mathcal{H}^{\pi,\phi_{\pi p}} \quad + \qquad \mathcal{H}^{\pi,\phi_{3\pi},C_F} \qquad + \mathcal{H}^{\pi,\phi_{3\pi},C_G}\end{aligned}$$

- 2- and 3-body contributions necessary for gauge invariance
- WAMP
  - photoproduction ($Q \to 0$): $\mathcal{H}^{\pi,\phi_{\pi p}} = 0$
  - no end-point singularities for $\hat{t} \neq 0$!

Subprocess amplitudes: twist-3

General structure:

$$\begin{aligned}
\mathcal{H}^{\pi,tw3} &= \mathcal{H}^{\pi,tw3,q\bar{q}} + \mathcal{H}^{\pi,tw3,q\bar{q}g} \\
&= \left(\mathcal{H}^{\pi,\phi_{\pi p}} + \underbrace{\mathcal{H}^{\pi,\phi_\pi^{EOM}}\right) + \left(\mathcal{H}^{\pi,q\bar{q}g,C_F}} + \mathcal{H}^{\pi,q\bar{q}g,C_G}\right) \\
&= \mathcal{H}^{\pi,\phi_{\pi p}} \quad + \qquad \mathcal{H}^{\pi,\phi_{3\pi},C_F} \qquad + \mathcal{H}^{\pi,\phi_{3\pi},C_G}
\end{aligned}$$

- 2- and 3-body contributions necessary for gauge invariance
- DVMP ($\hat{t} \to 0$):
  - end-point singularities in $\mathcal{H}^{\pi,\phi_{\pi p}}$
  - $\mathcal{H}^{\pi,\phi_\pi^{EOM}} = 0$

Subprocess amplitudes: twist-3

$$
\begin{aligned}
\mathcal{H}^{\pi,tw3} &= \mathcal{H}^{\pi,tw3,q\bar{q}} + \mathcal{H}^{\pi,tw3,q\bar{q}g} \\
&= \left(\mathcal{H}^{\pi,\phi_{\pi p}} + \underbrace{\mathcal{H}^{\pi,\phi_\pi^{EOM}}\right) + \left(\mathcal{H}^{\pi,q\bar{q}g,C_F}}\, + \mathcal{H}^{\pi,q\bar{q}g,C_G}\right) \\
&= \mathcal{H}^{\pi,\phi_{\pi p}} \quad + \quad \mathcal{H}^{\pi,\phi_{3\pi},C_F} \quad + \mathcal{H}^{\pi,\phi_{3\pi},C_G}
\end{aligned}
$$

- DVMP ($\hat{t} \to 0$): $\hat{s} = -\frac{\xi - x}{2\xi} Q^2, \hat{u} = -\frac{\xi + x}{2\xi} Q^2$

$$
\mathcal{H}^{\pi,\phi_{\pi p}}_{0-\lambda,\mu\lambda} \sim (2\lambda+\mu)\, f_\pi \mu_\pi C_F \alpha_S(\mu_R) \left(\frac{e_a}{\hat{s}^2} + \frac{e_b}{\hat{u}^2}\right) \boxed{\int_0^1 \frac{d\tau}{\bar{\tau}} \phi_{\pi p}(\tau)}
$$

$$
\mathcal{H}^{\pi,\phi_{3\pi},C_F}_{0-\lambda,\mu\lambda} \sim -(2\lambda+\mu)\, f_{3\pi}\, C_F \alpha_S(\mu_R) \left(\frac{e_a}{\hat{s}^2} + \frac{e_b}{\hat{u}^2}\right)
\times \boxed{\int_0^1 \frac{d\tau}{\bar{\tau}^2} \int_0^{\bar{\tau}} \frac{d\tau_g}{\tau_g(\bar{\tau}-\tau_g)} \phi_{3\pi}(\tau, \bar{\tau}-\tau_g, \tau_g)}
$$

$$
\mathcal{H}^{P,\phi_{3\pi},C_G}_{0-\lambda,\mu\lambda} \sim -(2\lambda+\mu)\, f_{3\pi}\, C_G \alpha_S(\mu_R) \left(\frac{e_a}{\hat{s}^2} + \frac{e_b}{\hat{u}^2} + \frac{e_a + e_b}{\hat{s}\hat{u}}\right)
\times \boxed{\int_0^1 \frac{d\tau}{\bar{\tau}} \int_0^{\bar{\tau}} \frac{d\tau_g}{\tau_g(\bar{\tau}-\tau_g)} \phi_{3\pi}(\tau, \bar{\tau}-\tau_g, \tau_g)}
$$

Pion distribution amplitudes

Twist-2 DA: $\phi_\pi(\tau,\mu_F) = 6\tau\bar{\tau}\left[1 + a_2(\mu_F)\, C_2^{3/2}(2\tau-1)\right]$

Twist-3 DAs:

$$\begin{aligned}\phi_{3\pi}(\tau_a,\tau_b,\tau_g,\mu_F) \quad = \quad & 360\tau_a\tau_b\tau_g^2\Big[1 + \omega_{1,0}(\mu_F)\,\frac{1}{2}(7\tau_g-3) \\ & +\omega_{2,0}(\mu_F)\,(2-4\tau_a\tau_b-8\tau_g+8\tau_g^2) \\ & +\omega_{1,1}(\mu_F)\,(3\tau_a\tau_b-2\tau_g+3\tau_g^2)\Big]\end{aligned}$$

[Braun, Filyanov '90]

using EOMs [Kroll, P-K '18]:

$$\begin{aligned}\phi_{\pi p}(\tau,\mu_F) \quad = \quad & 1 + \frac{1}{7}\,\frac{f_{3\pi}(\mu_F)}{f_\pi\mu_\pi(\mu_F)}\Big(7\,\omega_{1,0}(\mu_F) - 2\,\omega_{2,0}(\mu_F) - \omega_{1,1}(\mu_F)\Big) \\ & \times\Big(10\,C_2^{1/2}(2\tau-1) - 3\,C_4^{1/2}(2\tau-1)\Big)\,, \quad \phi_{\pi\sigma}(\tau) = \ldots\end{aligned}$$

Parameters:

- $a_2(\mu_0) = 0.1364 \pm 0.0213$ at $\mu_0 = 2$ GeV [Braun et al '15] (lattice)
- $\omega_{10}(\mu_0) = -2.55\,, \omega_{10}(\mu_0) = 0.0$ and $f_{3\pi}(\mu_0) = 0.004\ \mathrm{GeV}^2$. [Ball '99]
- $\omega_{20}(\mu_0) = 8.0$ [Kroll, P-K '18] fit to π^0 photoproduction data [CLAS '17]

Evolution of the decay constants and DA parameters taken into account.

Form factors and GPDs at large t

$R_i \ldots 1/x$ moment of $\xi = 0$ GPD (K_i)

- $R_V(\leftarrow H)$, $R_T(\leftarrow E)$ from nucleon form factor analysis [Diehl, Kroll '13]
- $R_A(\leftarrow \tilde{H})$ form factor analysis and WACS KLL asymmetry [Kroll '17]
- $S_T(\leftarrow H_T)$, $\bar{S}_T(\leftarrow \bar{E}_T)$ low $-t$ from DVMP analysis [Goloskokov, Kroll '11]
- $S_S(\leftarrow \tilde{H}_T) \cong \bar{S}_T/2$ ($\bar{E}_T = 2\tilde{H}_T + E_T$)

GPD parameterization [Diehl, Feldmann, Jakob, Kroll '04, Diehl, Kroll '13]

$$K_j^a(x, \xi = 0, t) = k_j^a(x) \exp\left[t\, f_j^a(x)\right]$$

$$f_j^a(x) = \left(B_j^a - {\alpha'}_i^a \ln x\right)(1-x)^3 + A_j^a x(1-x)^2$$

- strong $x - t$ correlation
- power behaviour for large $(-t)$
- choice for transversity GPDs $A = 0.5$ GeV^{-2}

Parameterization of GPDs at small t

double distribution representation [Müller '94, Radyushkin '99]

$$K_j^a(x,\xi,t) = \int_{-1}^{1} d\rho \int_{-1+|\rho|}^{1-|\rho|} d\eta\; \delta(\rho+\xi\eta-x)\, K_j^a(\rho,\xi=0,t)\, w_j^a(\rho,\eta)$$

- weight function $w_j^a \to$ generates ξ dependence
- zero-skewness GPD:

$$K_j^a(x,\xi=0,t) = k_j^a(x)\;\exp\left[(b_j^a - {\alpha'}_j^a \ln x)\, t\right]$$

  - H - GPDs: $k_j^a(x)$ from PDFs (q, Δq, δq)
  - E - GPDs: $k_j^a(x) = N_j^a x^{-\alpha_j^a(0)}(1-x)^{\beta_j^a}$

- double-distribution integral analytically evaluated [Goloskokov, Kroll '08]

Parameters:

- $\{\ N_j^a,\ b_j^a,\ {\alpha'}_j^a,\ \alpha_j^a(0),\ \beta_j^a\ \}$ [Goloskokov, Kroll '11, '14] [Duplančić, Kroll, P-K., Szymanowski '24]

- moments of H_T and $\bar{E}_T$ compared to lattice results

Soft physics input

GPDs

- double distribution representation [Müller '94, Radyushkin '99], double-distribution integral analytically evaluated [Goloskokov, Kroll '08]

DAs

$$\begin{aligned}\phi_{3\pi}(\tau_a, \tau_b, \tau_g, \mu_F) \quad = \quad & 360\tau_a\tau_b\tau_g^2\Big[1 + \omega_{1,0}(\mu_F)\,\frac{1}{2}(7\tau_g - 3) \\ & +\omega_{2,0}(\mu_F)\,(2 - 4\tau_a\tau_b - 8\tau_g + 8\tau_g^2) \\ & +\omega_{1,1}(\mu_F)\,(3\tau_a\tau_b - 2\tau_g + 3\tau_g^2)\Big]\end{aligned}$$

[Braun, Filyanov '90]

$\rightarrow \phi_{\pi p}$ using EOMs [Kroll, P-K '18]

evolution taken into account

Treatment of end-point singularities: MPA

⇒ Modified perturbative approach (MPA) [Goloskov, Kroll, '10]

- $k_\perp$ quark transverse momenta in pion

$$\frac{1}{((x+\xi)\tau - k_T^2/Q^2(2\xi) + i\epsilon)} \frac{1}{(x+\xi+i\epsilon)}$$

- $\phi_\pi \to$ light-cone wave function $\Psi_\pi \sim \phi_\pi \exp\left[-a_\pi^2 k_\perp^2\right]$
- $\int_0^1 d\tau \quad \to \int d^2\mathbf{k}_T \int_0^1 d\tau \quad \overset{\text{FT}}{\to} \int d^2\mathbf{b} \int_0^1 d\tau$
- Sudakov form factor $\exp\left[-S(\tau, \mathbf{b}, Q^2)\right]$

- consistently treated 2- and 3-body tw3 contributions, as well as tw2
- involved multidimensional integrations
- calculation of NLO corrections would be complicated

Treatment of end-point singularities: m_g^2

$\Rightarrow$ pure collinear picture with effective gluon mass m_g^2

[Schwinger '62, Cornwall '82, ..., Shuryak, Zahed '21]

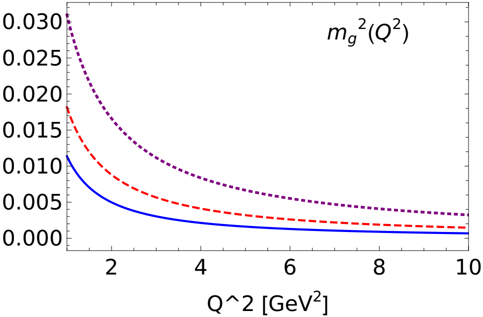
$$\int_0^1 d\tau \phi_{\pi p}(\tau) \frac{1}{((x+\xi)\tau - m_g^2/Q^2(2\xi) + i\epsilon)} \frac{1}{(x-\xi+i\epsilon)} \overset{x}{\otimes} H_T(\bar{E}_T)$$

$$m_g^2(Q^2) = \frac{m_0^2}{1+(Q^2/M^2)^{1+p}}$$ [Aguilar, Binosi, Papavassiliou '14]

$m_g^2(0) = 0.01$ GeV^2

- proof of concept
- suitable for faster fitting
- easier determination of NLO corrections (already available for tw2)

Parameterization of effective gluon mass



$$m_g^2(Q^2) = \frac{m_0^2}{1 + (Q^2/M^2)^{1+p}}$$ [Aguilar, Binosi, Papavassiliou '14]

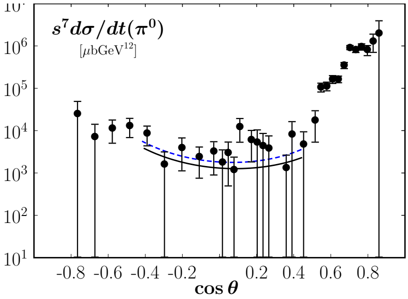
solid, dashed, dotted $\rightarrow (M, p) \in \{(381\text{MeV}, 0.26), (436\text{MeV}, 0.15), (557\text{MeV}, 0.08)\}$

Results from photoproduction (π)

- complete tw-3 prediction for π_0 photoproduction fitted to CLAS data

  $\Rightarrow \phi_{3\pi}$ coefficients $\omega_{1,0}$, $\omega_{2,0}$, $\omega_{1,1}$ (set KPK)

[Kroll, P-K '21]



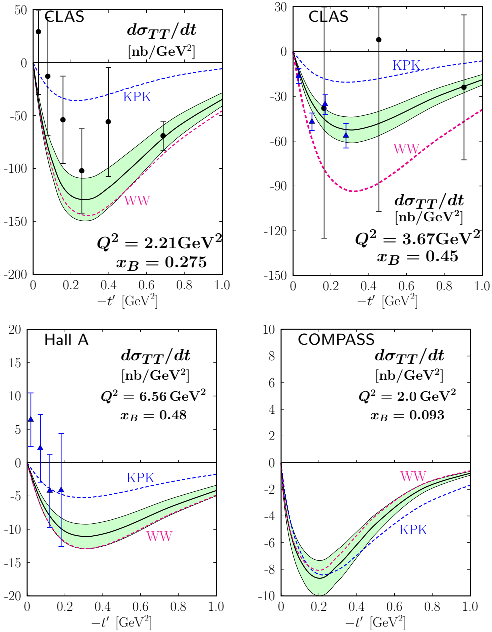
solid curve: set mod (DVπ^0P)

dashed curve: set KPK

exp data:

full circles [CLAS '18]

Modified perturbative approach (MPA): $d\sigma_{TT}$



solid curves: set mod

dashed curves: set KPK, WW

exp data:

full circles [CLAS '14]

triangles [Hall A '20]

$$\frac{d\sigma_{TT}}{dt} : \bar{E}_T, \qquad \left|\frac{d\sigma_{TT}}{dt}\right| \leq \frac{d\sigma_T}{dt}$$

- $d\sigma_{TT}$ large
- good description with set mod
- strong dependence on DA

DVMP differential cross-sections

$$\frac{d^4\sigma}{dW^2dQ^2dtd\varphi} = \frac{\alpha_{em}(W^2-m_N^2)}{16\pi^2E_L^2m_N^2Q^2(1-\varepsilon)}\left(\frac{d\sigma_T}{dt}+\varepsilon\frac{d\sigma_L}{dt}\right.$$

$$\left.+\varepsilon\cos(2\varphi)\frac{d\sigma_{TT}}{dt}+\sqrt{2\varepsilon(1+\varepsilon)}\cos\varphi\frac{d\sigma_{LT}}{dt}\right)$$

$$\frac{d\sigma_U}{dt}=\frac{d\sigma_T}{dt}+\epsilon\frac{d\sigma_L}{dt}$$

$$\frac{d\sigma_L}{dt}:\widetilde{H},\widetilde{E} \qquad \frac{d\sigma_T}{dt}:H_T,\bar{E}_T \qquad \frac{d\sigma_{TT}}{dt}:\bar{E}_T \qquad \frac{d\sigma_{LT}}{dt}:\widetilde{E},H_T$$

$$\left|\frac{d\sigma_{TT}}{dt}\right|\leq\frac{d\sigma_T}{dt}$$

To conclude...

- hard-exclusive processes offer challenging but promising tool for resolving hadron structure
- a vast amount of experimental data has been published in the last decades and more is yet to come (JLab, COMPASS), new dedicated machines are approved (EIC) or proposed (LHeC)
- a lot of data to explain and great opportunities for the proposals for the new measurements
- from the theoretical side there are questions to be resolved and phenomenological tools to be developed

Summary

- DVρ_L^0P
  - Twist-2 NLO contributions can describe the data.
  - Global DIS+DVCS+DVMP fits show importance of NLO.
  - DVMP can only be described at NLO.
- DVπ^0P
  - The improved twist-3 analysis (2- and 3-body meson Fock states included) shows that twist-3 dominates except for COMPASS kinematics (small x_B).
- Meson production promising in accessing information about GPDs.
- Meson DA additional nontrivial nonperturbative input.