

Probing hadron structure through exclusive photon-meson photoproduction

Nikola Crnković

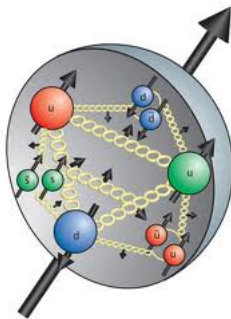
Rudjer Bošković Institute, Croatia

ACHT2025, May 5, 2025

- Introduction - Structure (Form Factors, PDFs)
- Generalized Parton Distributions (GPDs)
through deeply virtual processes
- Photon-meson photoproduction
- Preliminary numerical results
- Summary and outlook

work in progress in collaboration with K. Passek-K., G. Duplančić, S. Nabeebaccus,
B. Pire, L. Szymanowski, S. Wallon

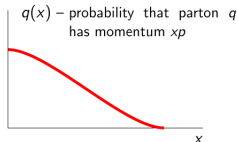
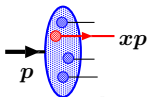
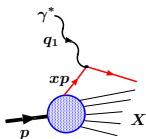
Structure of Hadrons



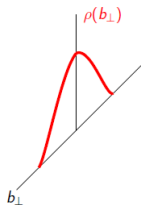
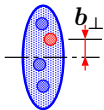
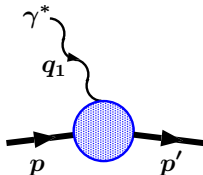
- distribution of momentum?
- positions of partons?
- ...

Nuclear structure from EMFFs and PDFs

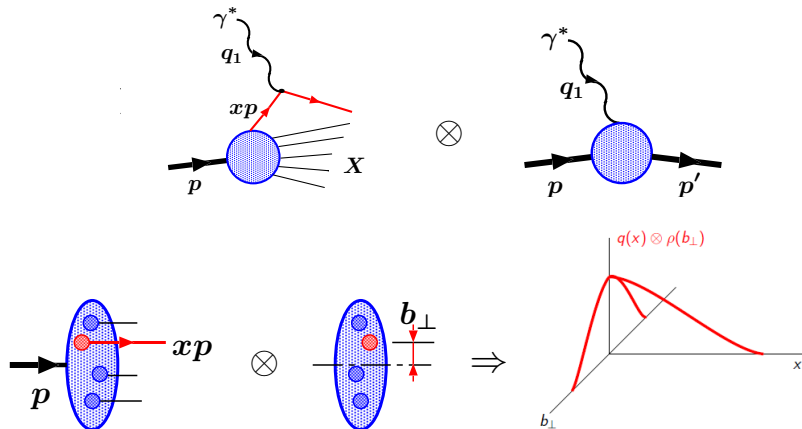
- Deeply inelastic scattering $\boxed{-q_1^2 \equiv Q^2 \rightarrow \infty, x_{BJ} \equiv \frac{-q_1^2}{2p \cdot q_1} \rightarrow \text{cte.}} \quad (\text{Bjorken limit})$



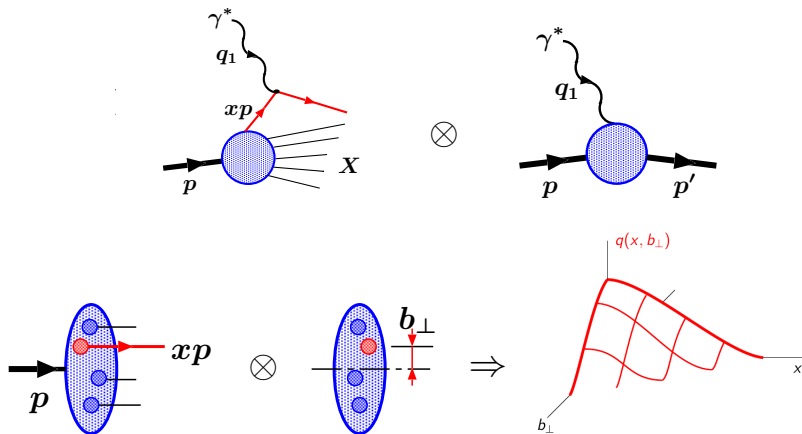
- Form factors $\rightarrow$ charge distribution $\boxed{\rho(b_\perp) \sim \int dq_1 e^{-iq_1 \cdot b_\perp} F_1(t = q_1^2)}$



Nuclear structure from EMFFs and PDFs



Nuclear structure from EMFFs and PDFs



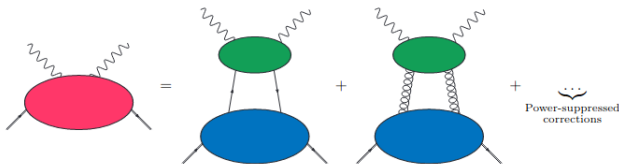
Exclusive processes and factorization

exclusive hard-scattering: $\exists$ large scale(s) $\rightarrow$ factorization:

$$\text{hard scattering amplitude} = \text{elementary hard-scattering amplitude} \otimes \text{hadron wave functions (GPDs, DAs)}$$

pQCD

evolution, input

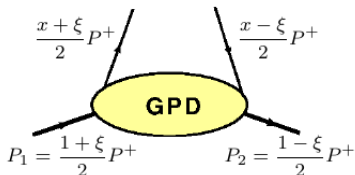


[Mezrag 22']

DA ... distribution amplitude (meson, nucleon)

GPD ... generalized parton distribution

Generalized parton distributions (GPDs)



$$P = P_1 + P_2$$

$$\Delta = P_2 - P_1$$

[Müller '92, et al. '94, Ji, Radyushkin '96]

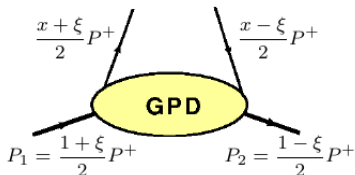
$$a^\mu = (a^0, a^1, a^2, a^3) \rightarrow a^\pm = a^0 \pm a^3$$

$$F^a(x, \xi, t = \Delta^2; \mu) = \int \frac{d\tau^-}{2\pi} e^{ixP^+\tau^-} \langle N(P_2) | \mathcal{O}^a(\tau) | N(P_1) \rangle \Big|_{\tau^+=0, \tau_\perp=0}$$

$a \in q, g, \mu \dots$ factorization scale

- x : parton's average longitudinal momentum fraction, $-1 \leq x \leq 1$
- $\xi = -\frac{\Delta^+}{P^+}$: longitudinal momentum transfer (skewness), $\xi = \frac{x_B}{2-x_B}$
- $t = \Delta^2$: momentum transfer squared (Mandelstam variable)

Generalized parton distributions (GPDs)



$$P = P_1 + P_2$$

$$\Delta = P_2 - P_1$$

[Müller '92, et al. '94, Ji, Radyushkin '96]

$$a^\mu = (a^0, a^1, a^2, a^3) \rightarrow a^\pm = a^0 \pm a^3$$

$$F^a(x, \xi, t = \Delta^2; \mu) = \int \frac{d\tau^-}{2\pi} e^{i x P^+ \tau^-} \langle N(P_2) | \mathcal{O}^a(\tau) | N(P_1) \rangle \Big|_{\tau^+ = 0, \tau_\perp = 0}$$

$a \in q, g, \mu \dots$ factorization scale

- vector (H^a, E^a) and axial vector GPDs ($\tilde{H}^a, \tilde{E}^a$)

→ chiral-even

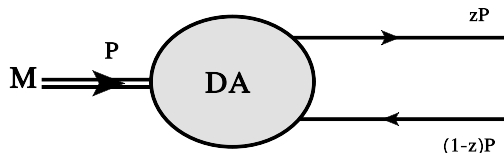
$$\mathcal{O}^q(\tau) = \bar{q}(\tau) \gamma^+ (\gamma^+ \gamma_5) q(-\tau)$$

- transversity GPDs ($H_T^a, E_T^a, \tilde{H}_T^a, \tilde{E}_T^a$)

→ chiral-odd

$$\mathcal{O}^q(\tau) = \bar{q}(\tau) i \sigma^{+i} q(-\tau)$$

Meson distribution amplitudes (DAs)



[Efremov, Radyushkin '80, Lapage, Brodsky '79,'80]

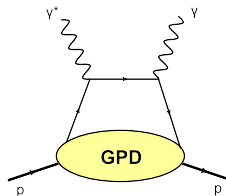
$$\phi_M^a(z; \mu) = \int \frac{d\tau^-}{2\pi} e^{i(2z-1)P^+\tau^-} \langle 0 | \mathcal{O}^a(\tau) | M(P) \rangle \Big|_{\tau^+=0, \tau_\perp=0}$$

$a \in q, g, \mu \dots$ factorization scale

- z : parton's longitudinal momentum fraction, $0 \leq z \leq 1$
- same operators as in GPD

Selected exclusive processes of interest

Deeply virtual Compton scattering (DVCS)



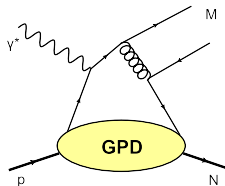
$$\gamma^* p \rightarrow \gamma p$$

factorization: [Collins, Freund '99]

$$H^q, E^q, \tilde{H}^q, \tilde{E}^q$$

$$H^G, E^G, \tilde{H}^G, \tilde{E}^G \text{ (NLO)}$$

Deeply virtual meson production (DVMP)



$$\gamma^* p \rightarrow MN$$

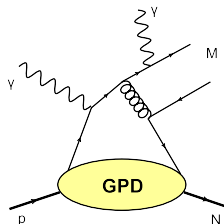
factorization:

[Collins, Frankfurt, Strikman '97]

$$H^{qi}, E^{qi}; H^G, E^G \text{ (V}_L\text{)}$$

$$\tilde{H}^{qi}, \tilde{E}^{qi} \text{ (PS)}$$

Photon-meson photoproduction



$$\gamma p \rightarrow \gamma MN$$

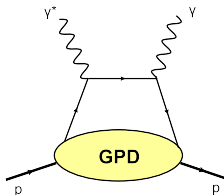
factorization: [Qiu, Yu '22,'23]

$$H^a, E^a, \tilde{H}^a, \tilde{E}^a$$

$$H_T^a, E_T^a, \tilde{H}_T^a, \tilde{E}_T^a$$

Selected exclusive processes of interest

Deeply virtual
Compton scattering
(DVCS)



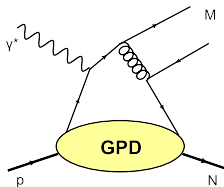
$$\gamma^* p \rightarrow \gamma p$$

$$2 \rightarrow 2$$

$$\frac{1}{x - \xi} \otimes GPD(x, \xi, t)$$

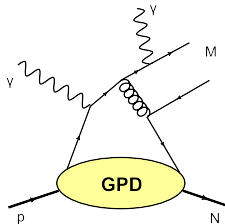
(LO)

Deeply virtual meson
production
(DVMP)



$$\gamma^* p \rightarrow MN$$

Photon-meson
photoproduction



$$\gamma p \rightarrow \gamma MN$$

$$2 \rightarrow 3$$

$$f(x, \xi, \alpha) \otimes GPD(x, \xi, t)$$

(LO)

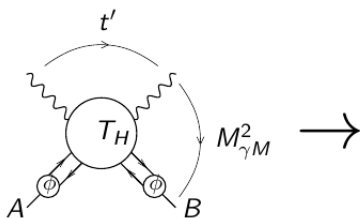
- The richer kinematics of the $2 \rightarrow 3$ processes allow for an efficient probing of the sensitivity of the GPDs x dependence [Qiu,Yu,'23]
→ two large scales similarly as in DDVCS
- transversity GPDs accessible

In this work:

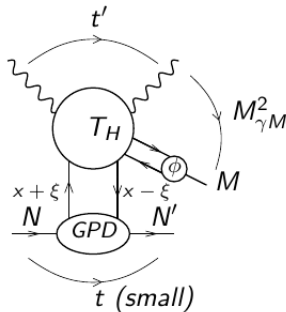
- ▶ completing and systematizing the results for all channels
- ▶ fast numerical code for different DA models with evolution included
- ▶ investigate the photoproduction of neutral pseudoscalar mesons

Photon-meson photoproduction: Factorization

$$\gamma + N \rightarrow \gamma + M(\pi, \eta, \rho, \dots) + N'$$



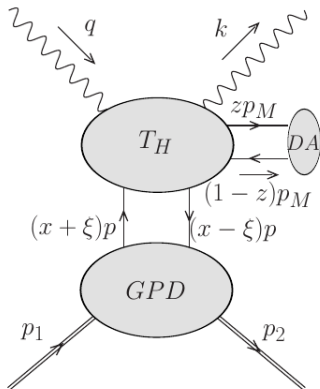
large angle factorisation
à la Brodsky Lepage



(à la "time-like" DVCS)

Photon-meson photoproduction: Kinematics

$$\gamma^*(q) + N(p_1) \rightarrow \gamma(k) + M(p_M) + N'(p_2)$$



$$u' = (p_M - q)^2 \gg$$

$$t' = (k - q)^2 \gg$$

$$s' = M_{\gamma M}^2 = (k + p_M)^2 \gg$$

$$t = (p_2 - p_1)^2 \ll$$

$$s = S_{\gamma N}^2 = (q + p_1)^2$$

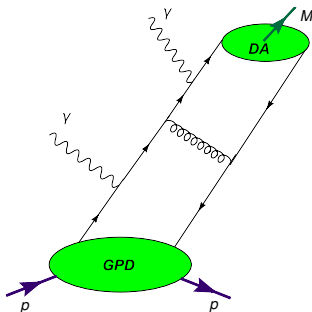
$$\xi = \frac{\tau}{2 - \tau}, \quad \tau = \frac{M_{\gamma M}^2}{S_{\gamma N}^2 - m_N^2}$$

• amplitudes depend on ξ , $-t$, s , $\alpha = -u'/s'$

• convenient to introduce $y = \frac{\xi+x}{2\xi}$, $\bar{y} = 1 - y = \frac{\xi-x}{2\xi}$, $\frac{\xi-1}{2\xi} \leq y \leq \frac{\xi+1}{2\xi}$

Photon-meson photoproduction

$$\gamma q \rightarrow \gamma q(q\bar{q})$$



LO ρ mesons: [Boussarie, Pire, Szymanowski, Wallon '16]

LO $\pi^\pm$ mesons: [Duplanić, Passek-K, Pire, Szymanowski, Wallon '18]

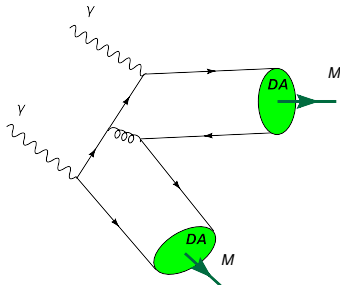
$$\pi^\pm : \tilde{H}^q, \tilde{E}^q, H^q, E^q$$

$$\rho_L^0 : H^q, E^q, \tilde{E}^q, \tilde{H}^q$$

$$\rho_T^0 : H_T^q, E_T^q, \tilde{E}_T^q, \tilde{H}_T^q$$

Meson pair production

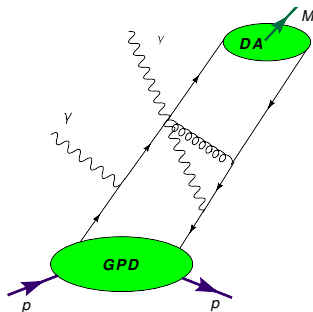
$$\gamma\gamma \rightarrow (q\bar{q})(q\bar{q})$$



NLO: [Nižić '87, Duplanić, Nižić '06]

Photon- π^0 photoproduction

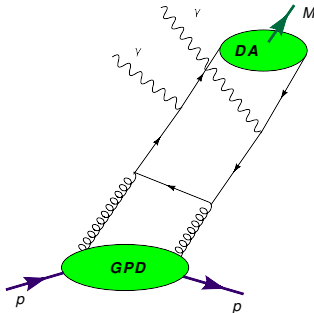
$$\gamma q \rightarrow \gamma(q\bar{q})q$$



$$\tilde{H}^q, \tilde{E}^q$$

$$H^q, E^q$$

$$\gamma g \rightarrow \gamma(q\bar{q})g$$



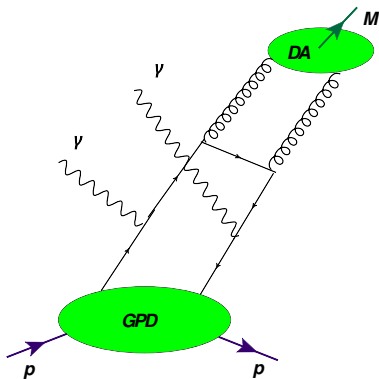
$$\tilde{H}^G, \tilde{E}^G$$

$$H^G, E^G$$

$$F_T^G, \tilde{F}_T^G$$

Photon- η, η' photoproduction

additionally $\gamma q \rightarrow \gamma(gg)q$



Valence Fock components of $M = \eta, \eta'$:

$$|q\bar{q}_8\rangle = |(u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}\rangle \quad (\text{flavour-octet})$$

$$\begin{cases} |q\bar{q}_1\rangle = |(u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}\rangle & (\text{flavour-singlet}) \\ |gg\rangle \end{cases}$$

Novel features:

① flavour-mixing (singlet-octet mixing)

② $|gg\rangle$ states contribute

$\Rightarrow$ mixing of $q\bar{q}_1$ and gg DAs under evolution

Factorization

$$^a\mathcal{F} = \int_{-1}^1 dx \int_0^1 dz \ T^a(x, \xi, z, \mu_F, \mu_\varphi; s, \alpha) \ F^a(x, \xi, t, \mu_F) \ \phi_M(z, \mu_\varphi)$$

$a = q, g$ $\mu_F, \mu_\varphi \dots$ factorization scales

- T^a : subprocess hard-scattering amplitudes $\rightarrow$ pQCD

more complicated expressions than in the case of DVCS and DVMP
 $\Rightarrow$ more demanding integrations with F^a and ϕ_M

- F^a : GPD

$\rightarrow$ different models based on [Radyushkin '98],[Goloskokov, Kroll '10]

- ϕ_M : meson distribution amplitude (DA)

$\phi(z) = 6z(1-z) \dots$ asymptotic DA

$$\phi_q^M(z, \mu_\varphi) = 6z(1-z) \left[1 + \sum_{n=2}^{\infty} 'a_n^M(\mu_\varphi) C_n^{3/2}(2z-1) \right]$$

$$\phi_g(z, \mu_\varphi) = 30z^2(1-z)^2 \left[\sum_{n=2}^{\infty} 'a_n^g(\mu_\varphi) C_{n-1}^{5/2}(2z-1) \right]$$

$\dots$ expansion in Gegenbauer pol. $\Rightarrow$ evolution

- generic building blocks

$$y = \frac{\xi + x}{2\xi}, z \dots y \in \mathbb{R}, z \in [0, 1]$$

$$\alpha = -u'/s' = \frac{1 + \cos \theta'}{2}$$

$$\bar{a} \equiv 1 - a$$

- $\frac{1}{z(y \pm i\epsilon)}, \frac{1}{z(\bar{y} \pm i\epsilon)} \rightarrow$ usual "moment" type terms
- $\frac{1}{z(y\bar{z} - \alpha z\bar{y} + i\epsilon)}, \dots \rightarrow$ terms that provide better sensitivity to GPD and DA form

$\gamma q \rightarrow (q\bar{q})q$ contribution

- no problem with integrations (factorization proven [Qiu, Yu '22, '23])
- ▷ obtained simpler closed expression suited for improved numerical integration (inclusion of different DA models, evolution taken into account, faster code)
- ▷ PV formalism enables efficient treatment of poles:
example, mixed term

$$\frac{1}{z\bar{z} (y\bar{z} - \alpha z\bar{y} + i\epsilon)} \rightarrow \frac{1}{\alpha z\bar{z} (\alpha + \bar{\alpha} y)} \left(\text{PV} \frac{1}{z - \frac{y}{\alpha + \bar{\alpha} y}} + i\pi \delta \left(z - \frac{y}{\alpha + \bar{\alpha} y} \right) \right)$$

$\gamma g \rightarrow (q\bar{q})g$ contributions (π^0 production)

- gluon GPDs: $\mathcal{O}^g(\tau) = G^{+\mu}(-\tau) \tilde{G}_\mu^+(\tau)$

$$\langle N(P_2) | A^\mu A^\nu | N(P_1) \rangle \rightarrow \frac{1}{(y \pm i0)(\bar{y} \pm i0)} (g^{\mu\nu} F^g - i\epsilon_\perp^{\mu\nu} \tilde{F}^g + \dots)$$

- due to gg projector with factor $\frac{1}{(y+i\epsilon)(\bar{y}+i\epsilon)}$, contributions

$$\frac{1}{(y+i\epsilon)(\bar{y}+i\epsilon)} \frac{N(y, z, \alpha)}{z\bar{z} (y-i\epsilon)(\bar{y}-i\epsilon)(y\bar{z} - \alpha z\bar{y} + i\epsilon)} \otimes^y F^g \otimes^z \phi_\pi$$

$$N(0, 0, \alpha) = N(1, 1, \alpha) = 0; \phi_\pi(z) \sim z\bar{z}$$

demand additional attention

$$(F^g, \tilde{F}^g(x = \pm\xi) \neq 0)$$

$\Rightarrow$ additional regularization? ($k_\perp$), breakdown of factorization?

[Nabeebaccus, Schoenleber, Szymanowski, Wallon '23]

$\gamma q \rightarrow (gg)q$ contributions (η, η' production)

- gluon DAs: $\mathcal{O}^g(\tau) = G^{+\mu}(-\tau) \tilde{G}_\mu^+(\tau)$

$$\langle 0 | A^\mu A^\nu | M(P) \rangle \rightarrow \frac{1}{z \bar{z}} i \varepsilon^{\mu\nu} \phi_g(z)$$

- gg projector $\sim \frac{1}{z \bar{z}}$, $\phi_g \sim z^2 \bar{z}^2$

$$\frac{1}{z \bar{z}} \frac{N(z, y, \alpha)}{z \bar{z} (y - i\epsilon)(\bar{y} - i\epsilon)(\bar{y} z - \alpha \bar{z} y + i\epsilon)} \otimes^y F^q \otimes^z \phi_g$$

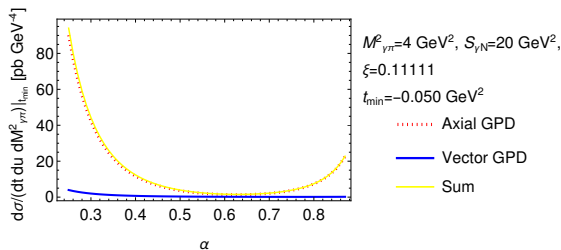
- ▷ shown that there is no problem with factorization
- ▷ closed expressions obtained
- ▷ numerical analysis underway

Numerics (preliminary!): $\gamma\pi$

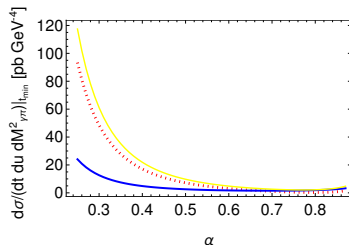
$$\frac{d\sigma}{dt d\alpha dM_{\gamma\pi}^2} = \frac{|\overline{\mathcal{M}}|_{t_{\min}}^2}{32S_{\gamma N}^2(2\pi)^3}$$

$$\alpha = -u'/s' = \frac{1 + \cos\theta'}{2}$$

$\gamma\pi^+$ photoproduction



$\gamma\pi^0$ photoproduction

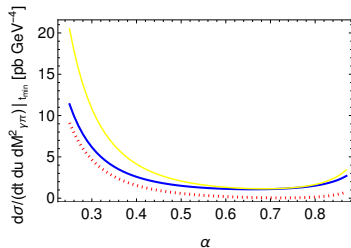


- contributions of axial GPDs ($\tilde{H}^q, \tilde{E}^q$) dominate
→ probe for axial GPDs

Numerics (preliminary!): $\gamma\eta, \gamma\eta'$

$$\alpha = -u'/s' = \frac{1 + \cos \theta'}{2}$$

$\gamma\eta$ photoproduction

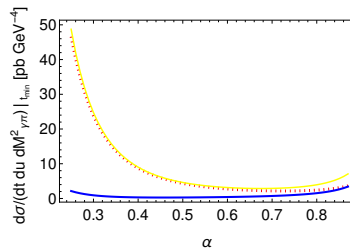


$M^2_{\gamma\pi}=4 \text{ GeV}^2, S_{\gamma N}=20 \text{ GeV}^2,$
 $\xi=0.11111$

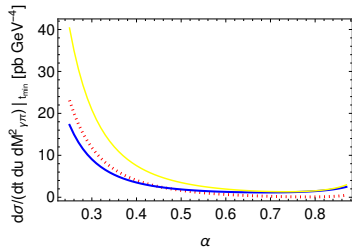
$t_{\min}=-0.050 \text{ GeV}^2$

..... Axial GPD
 — Vector GPD
 — Sum

$\gamma\eta'$ photoproduction



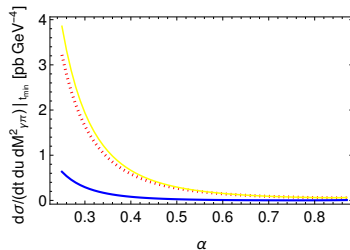
$\gamma\eta$: only $(q\bar{q}) \rightarrow (q\bar{q})$



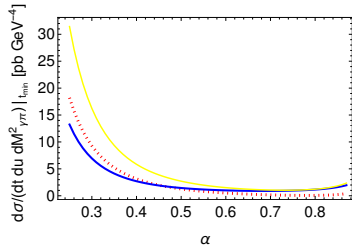
$M_{\gamma\pi}^2 = 4 \text{ GeV}^2$, $S_{\gamma N} = 20 \text{ GeV}^2$,
 $\xi = 0.11111$
 $t_{\min} = -0.050 \text{ GeV}^2$

..... Axial GPD
 — Vector GPD
 — Sum

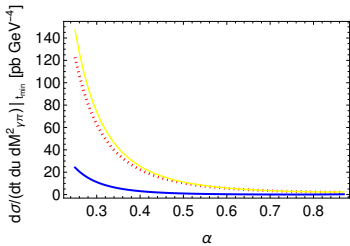
$\gamma\eta$: only $(q\bar{q}) \rightarrow (gg)$



$\gamma\eta'$: only $(q\bar{q}) \rightarrow (q\bar{q})$



$\gamma\eta'$: only $(q\bar{q}) \rightarrow (gg)$



▷ (gg) Fock state in η' important

Summary: Photon-meson photoproduction

- Hard-exclusive processes described in terms of GPDs offer challenging but promising tool for resolving hadron structure.
- γ -meson photoproduction proposed: provides greater sensitivity to x dependence and access to transversity GPDs at LO.
- (γM) : efficient closed analytical forms found, development of faster numerical code with evolution included.
- $(\gamma\eta, \eta')$: contributions with 2-gluon DA evaluated
- A vast amount of experimental data (HERA, JLab, COMPASS) and more is yet to come (EIC in 2030 or proposed LHeC, EicC).

Summary: Photon-meson photoproduction

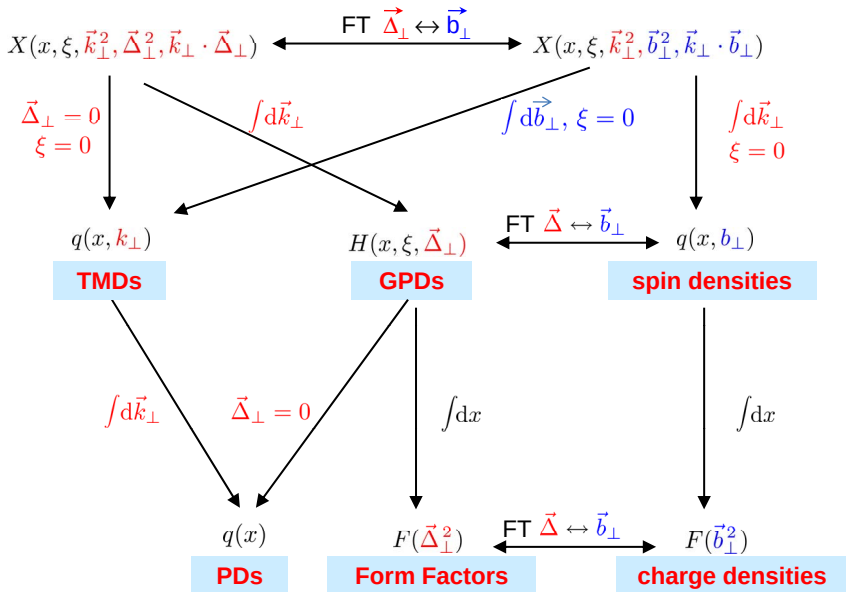
- Hard-exclusive processes described in terms of GPDs offer challenging but promising tool for resolving hadron structure.
- γ -meson photoproduction proposed: provides greater sensitivity to x dependence and access to transversity GPDs at LO.
- (γM) : efficient closed analytical forms found, development of faster numerical code with evolution included.
- $(\gamma\eta, \eta')$: contributions with 2-gluon DA evaluated
- A vast amount of experimental data (HERA, JLab, COMPASS) and more is yet to come (EIC in 2030 or proposed LHeC, EicC).

Thank you!

Additional slides

GTMDs

Wigner-Ds



Definition of GPDs

- without helicity flip (chiral-even Γ matrices): 8 chiral-even GPDs:

$$F^q(x, \eta, t = \Delta^2) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P_2 | \bar{q}(-z) \gamma^+ q(z) | P_1 \rangle \Big|_{z^+=0, z_\perp=0}$$

$$F^g(x, \eta, t = \Delta^2) = \frac{2}{P^+} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P_2 | G_a^{+\mu}(-z) G_{a\mu}^+(z) | P_1 \rangle \Big|_{\dots}$$

$$a^\mu = (a^0, a^1, a^2, a^3) \rightarrow a^\pm = a^0 \pm a^3$$

- Lorentz decomposition:

$$F^a = \frac{\bar{u}(P_2) \gamma^+ u(P_1)}{P^+} H^a + \frac{\bar{u}(P_2) i \sigma^{+\nu} u(P_1) \Delta_\nu}{2MP^+} E^a \quad a = q, g$$

[Müller '92, et al. '94, Ji, Radyushkin '96]

Definition of GPDs

- without helicity flip (chiral-even Γ matrices): 8 chiral-even GPDs:

$$\tilde{F}^q(x, \eta, t = \Delta^2) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P_2 | \bar{q}(-z) \gamma^+ \gamma_5 q(z) | P_1 \rangle \Big|_{z^+=0, z_\perp=0}$$

$$\tilde{F}^g(x, \eta, t = \Delta^2) = \frac{4}{P^+} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P_2 | G^{+\mu}(-z) \tilde{G}_\mu^+(z) | P_1 \rangle \Big|_{\dots}$$

- Lorentz decomposition:

$$\tilde{F}^a = \frac{\bar{u}(P_2) \gamma^+ \gamma_5 u(P_1)}{P^+} \tilde{H}^a + \frac{\bar{u}(P_2) i\sigma^{+\nu} \gamma_5 u(P_1) \Delta_\nu}{2MP^+} \tilde{E}^a \quad a = q, g$$

[Müller '92, et al. '94, Ji, Radyushkin '96]

Definition of GPDs

- With helicity flip (chiral-odd Γ matrices): 8 chiral-odd GPDs (transversity GPD):

$$F_T^q(x, \eta, t = \Delta^2) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P_2 | \bar{q}(-z) i\sigma^{+i} q(z) | P_1 \rangle \Big|_{z^+=0, z_\perp=0}$$

- Lorentz decomposition:

$$\begin{aligned} F_T^q = & \frac{\bar{u}(P_2) i\sigma^{+i} u(P_1)}{P^+} H_T^q + \frac{\bar{u}(P_2) (P^+ \Delta^i - \Delta^+ P^i) u(P_1)}{M^2 P^+} \tilde{H}_T^q + \\ & + \frac{\bar{u}(P_2) (\gamma^+ \Delta^i - \Delta^+ \gamma^i) u(P_1)}{2MP^+} E_T^q + \frac{\bar{u}(P_2) (\gamma^+ P^i - P^+ \gamma^i) u(P_1)}{MP^+} \tilde{E}_T^q \end{aligned}$$

[Müller '92, et al. '94, Ji, Radyushkin '96]

Definition of GPDs

- With helicity flip (chiral-odd Γ matrices): 8 chiral-odd GPDs (transversity GPD):

$$F_T^g(x, \eta, t = \Delta^2) = \frac{1}{P^+} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P_2 | \mathbf{S} G^{+i}(-z) G^{j+}(z) | P_1 \rangle \Big|_{z^+=0},$$

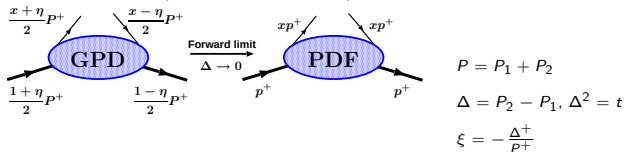
- Lorentz decomposition:

$$F_T^g = \mathbf{S} \frac{(P^+ \Delta^j - \Delta^+ P^j)}{2MP^+} \frac{\bar{u}(P_2)}{P^+} \left[i\sigma^{+i} H_T^g + \frac{(P^+ \Delta^i - \Delta^+ P^i)}{M^2} \tilde{H}_T^g + \right. \\ \left. + \frac{(\gamma^+ \Delta^i - \Delta^+ \gamma^i)}{2M} E_T^g + \frac{(\gamma^+ P^i - P^+ \gamma^i)}{M} \tilde{E}_T^g \right] u(P_1)$$

[Müller '92, et al. '94, Ji, Radyushkin '96]

Properties of GPDs

- Forward limit ($\Delta \rightarrow 0, \xi \rightarrow 0$): H -GPDs $\rightarrow$ PDFs



- Sum rules: GPD $\rightarrow$ form factors

$$\sum_{q=u,d} Q_q \int_{-1}^1 dx \begin{Bmatrix} H^q(x, \xi, t) \\ E^q(x, \xi, t) \end{Bmatrix} = \begin{Bmatrix} F_1(t) \\ F_2(t) \end{Bmatrix}$$

- Polynomiality and positivity constraints
- Possibility of solution of proton spin problem

$$\frac{1}{2} \int_{-1}^1 dx x \left[H^q(x, \xi, t) + E^q(x, \xi, t) \right] = J^q(t) \quad [\text{Ji '96}]$$

Polynomiality and positivity constraints of GDP

- Polynomiality

Lorentz covariance $\Rightarrow \int_{-1}^1 dx x^m H^q(x, \xi, t) = \sum_{k=0}^m \xi^k A_{m,k}^q(t)$

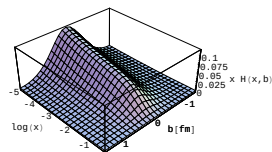
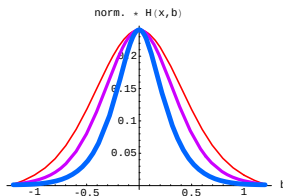
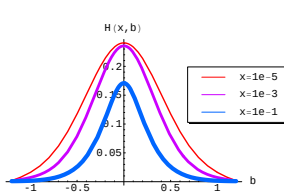
- Positivity

Positivity of Hilbert space norm $\Rightarrow H^q(x, \xi, t) \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}$

Three-dimensional image of a proton

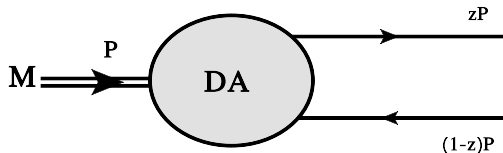
- Fourier transform of GPD for $\xi = 0$ can be interpreted as probability density depending on x and transversal distance b [Burkardt '00, '02]

$$q(x, \vec{b}) = \int \frac{d^2 \vec{\Delta}}{(2\pi)^2} e^{-i\vec{b} \cdot \vec{\Delta}} H(x, \xi = 0, \Delta^2 = -\vec{\Delta}^2)$$



[Kumerički, Müller, Passek-K. '08]

Meson distribution amplitudes (DAs)



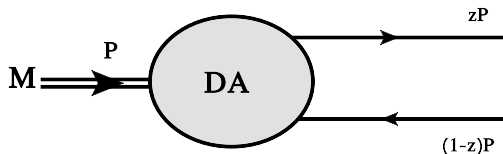
[Efremov, Radyushkin '80, Lapage, Brodsky '79,'80]

$$\phi_M^a(z; \mu) = \int \frac{d\tau^-}{2\pi} e^{i(2z-1)P^+\tau^-} \langle 0 | \mathcal{O}^a(\tau) | M(P) \rangle \Big|_{\tau^+=0, \tau_\perp=0}$$

$a \in q, g, \mu \dots$ factorization scale

- z : parton's longitudinal momentum fraction, $0 \leq z \leq 1$

Meson distribution amplitudes (DAs)



[Efremov, Radyushkin '80, Lapage, Brodsky '79,'80]

$$\phi_M^a(z; \mu) = \int \frac{d\tau^-}{2\pi} e^{i(2z-1)P^+\tau^-} \langle 0 | \mathcal{O}^a(\tau) | M(P) \rangle \Big|_{\tau^+=0, \tau_\perp=0}$$

$a \in q, g$, $\mu \dots$ factorization scale

- pseudoscalar mesons ($\pi, \eta, \dots$)

$$\mathcal{O}^q(\tau) = \bar{q}(\tau) \gamma^+ \gamma_5 q(-\tau)$$

- longitudinally polarized vector mesons (ρ_L)

$$\mathcal{O}^q(\tau) = \bar{q}(\tau) \gamma_5 q(-\tau)$$

- transversely polarized vector mesons (ρ_T)

$$\mathcal{O}^q(\tau) = \bar{q}(\tau) i\sigma^{+i} q(-\tau)$$

$$\phi_{\pi}(x, \mu_F) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle 0 | \bar{q}(-z) \gamma^+ \gamma_5 q(z) | \pi(P) \rangle \Big|_{z^+=0, z_{\perp}=0}$$

$$\phi_{\pi}(x, \mu_F) = 6x(1-x) \left[1 + \sum_{n=2,4,\dots} a_{\pi,n}(\mu_F) C_n^{3/2}(2x-1) \right]$$

$$a_{M,n}(\mu_F) = a_{M,n}^{LO}(\mu_F) + \frac{\alpha_S(\mu_F)}{4\pi} a_{M,n}^{NLO}(\mu_F)$$

$$a_{M,n}^{LO}(\mu_F) = \left(\frac{\alpha_S(\mu_O)}{\alpha_S(\mu_F)} \right)^{\gamma_n/\beta_0} a_{M,n}^{LO}(\mu_O) \quad (\leq a_{M,n}^{LO}(\mu_O))$$

① flavour-mixing

[review Feldman '00]



simplest possibility: to take particle dependence and the flavour-mixing to be solely embedded in the decay constants f_M^i

- $\Phi_{Mi} = f_M^i \phi_i \quad i \in \{1, 8\}$
- the decay constants parametrized as

$$f_\eta^8 = f_8 \cos \theta_8 \quad f_\eta^1 = -f_1 \sin \theta_1$$

$$f_{\eta'}^8 = f_8 \sin \theta_8 \quad f_{\eta'}^1 = f_1 \cos \theta_1$$

[Leutwyler '98, Felmann, Kroll, Stech, '98,'99]

② $|gg\rangle$ states contribute

$\Rightarrow$ mixing of $q\bar{q}_1$ and gg DAs under evolution

$$(\Phi_{M1} \equiv \Phi_{Mq}) \quad (\Phi_{Mg})$$

$\downarrow$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \begin{pmatrix} \Phi_{Mq} \\ \Phi_{Mg} \end{pmatrix} = \begin{pmatrix} V_{qq} & V_{qg} \\ V_{gq} & V_{gg} \end{pmatrix} \otimes \begin{pmatrix} \Phi_{Mq} \\ \Phi_{Mg} \end{pmatrix}$$

Factorization

- Scattering amplitude $\mathcal{M}$ is expressed in terms of form factors $\mathcal{F}$ analogous to CFFs in DVCS

$$^a\mathcal{F} = \int_{-1}^1 dx \int_0^1 dz \, T^a(x, \xi, z, \mu_R, \mu_F, \mu_\varphi; s, \alpha) F^a(x, \xi, t, \mu_F) \phi_M(z, \mu_\varphi)$$

$a = q, g$ $\mu_F, \mu_\varphi \dots$ factorization scales
 $\mu_R \dots$ renormalization scale

- T^a : subprocess hard-scattering amplitudes $\rightarrow$ pQCD
 - depend on two scales: $s, \alpha = -u'/s'$
 - more complicated expressions than in the case of DVCS and DVMP
 $\Rightarrow$ more demanding integrations with F^a and ϕ_M
- F^a : GPD
- ϕ_M : meson DA

- F^a : GPD

→ GK model [Radyushkin '98],[Goloskokov, Kroll '10] for valence quarks

- ϕ_M : meson DA

→ models:

$\phi(z) = 6z(1-z) \dots$ asymptotic DA

$\phi(z) = \frac{8}{\pi} \sqrt{z(1-z)} \dots$ holographic DA [Brodsky, de Teramond '06]

$$\phi_q^M(z, \mu_\varphi) = 6z(1-z) \left[1 + \sum_{n=2}^{\infty} 'a_n^M(\mu_\varphi) C_n^{3/2}(2z-1) \right]$$

$$\phi_g(z, \mu_\varphi) = 30z^2(1-z)^2 \left[\sum_{n=2}^{\infty} 'a_n^g(\mu_\varphi) C_{n-1}^{5/2}(2z-1) \right]$$

$\dots$ expansion in Gegenbauer pol. $\Rightarrow$ evolution

$$a_2^\pi = 0.1364, \mu_0 = 2 \text{ GeV [Bali, '19]},$$

$$a_2^1 = -0.12, a_2^g = 0.63, a_2^8 = -0.05, \mu_0 = 1 \text{ GeV [Kroll, Passek-K., '12]}$$

Factorization II

- Scattering amplitude $\mathcal{M}$ is expressed in terms of form factors $\mathcal{H}$, $\mathcal{E}$, $\mathcal{H}$, $\mathcal{E}$ analogous to CFFs in DVCS

$$\mathcal{M}_\pi = \frac{1}{n \cdot p} \bar{u}(p_2, \lambda') [\hat{n} \mathcal{H}_\pi + \frac{i \sigma^{+\alpha} \Delta_\alpha}{2M} \mathcal{E}_\pi + \hat{n} \gamma_5 \tilde{\mathcal{H}}_\pi + \frac{n \cdot \Delta}{2\pi} \gamma_5 \tilde{\mathcal{E}}_\pi] u(p_1, \lambda)$$

$$TA = \varepsilon_{q\perp} \cdot \varepsilon_{q\perp}^*,$$

$$TB = (\varepsilon_{q\perp} \cdot p_\perp)(\varepsilon_{q\perp}^* \cdot p_\perp)$$

$$TA5 = (\varepsilon_{k\perp}^* \cdot p_\perp) \epsilon^{np \varepsilon_{q\perp} p_\perp}$$

$$TB5 = -(\varepsilon_{q\perp} \cdot p_\perp) \epsilon^{np \varepsilon_{k\perp}^* p_\perp}$$

$\gamma q \rightarrow (gg)q$ contributions (η, η' production)

- gluon DAs: $\mathcal{O}^g(\tau) = G^{+\mu}(-\tau) \tilde{G}_\mu^+(\tau)$

$$\langle 0 | A^\mu A^\nu | M(P) \rangle \rightarrow \frac{1}{z \bar{z}} i \varepsilon^{\mu\nu} \phi_g(z)$$

- gg projector $\sim \frac{1}{z \bar{z}}$, $\phi_g \sim z^2 \bar{z}^2$

$$\frac{1}{z \bar{z}} \frac{N(z, y, \alpha)}{z \bar{z} (y - i\epsilon)(\bar{y} - i\epsilon)(\bar{y} z - \alpha \bar{z} y + i\epsilon)} \otimes^y F^g \otimes^z \phi_g$$

- ▷ shown that there is no problem with factorization
- ▷ closed expressions obtained
- ▷ numerical analysis underway

Recent work: $\gamma\pi^0$ photoproduction

quark contributions

- obtained simpler closed expression for sum of diagrams suited for improved numerical integrations and inclusion of different DA models
- PV formalism enables efficient treatment of poles;
example, mixed term:

$$\frac{1}{z\bar{z}(y\bar{z} - \alpha z\bar{y} + i\epsilon)} \rightarrow \frac{1}{\alpha z\bar{z}(\alpha + \bar{\alpha}y)} \left(\text{PV} \frac{1}{z - \frac{y}{\alpha + \bar{\alpha}y}} + i\pi \delta\left(z - \frac{y}{\alpha + \bar{\alpha}y}\right) \right)$$

$y = \frac{\xi + x}{2\xi}, z \dots$ partons momentum fractions, $\alpha = -u'/s'$, and $\bar{\alpha} \equiv 1 - \alpha$

gluon contributions

- determined and closed expressions for the sum of diagrams obtained
- due gg projector with factor $\frac{1}{(y + i\epsilon)(\bar{y} + i\epsilon)}$, contributions

$$\frac{1}{(y + i\epsilon)(\bar{y} + i\epsilon)} \frac{N(y, z, \alpha)}{z\bar{z}(y - i\epsilon)(\bar{y} - i\epsilon)(y\bar{z} - \alpha z\bar{y} + i\epsilon)}$$

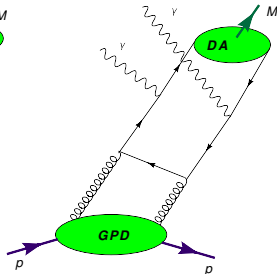
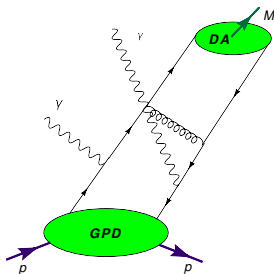
demand additional attention

$\Rightarrow$ additional regularization? ($k_{\perp}$), breakdown of factorization?

$\rightarrow$ work in progress

Photon- π^0 photoproduction

$$\gamma q \rightarrow \gamma(q\bar{q})q, \quad \gamma g \rightarrow \gamma(q\bar{q})g$$



$$\tilde{H}^q, \tilde{E}^q$$

$$H^q, E^q$$

$$\tilde{H}^G, \tilde{E}^G$$

$$H^G, E^G$$

$$F_T^G, \tilde{F}_T^G$$

(M) π^0 photoproduction

$$\gamma\gamma \rightarrow (q\bar{q})(q\bar{q})$$

$$\gamma\gamma \rightarrow (gg)(q\bar{q})$$

$$\gamma\gamma \rightarrow (PS)\pi^0 \rightarrow \tilde{H}^q, \tilde{E}^q$$

$$\gamma\gamma \rightarrow (S)\pi^0 \rightarrow H^q, E^q$$

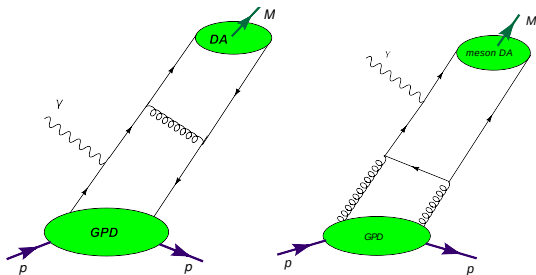
$$\gamma\gamma \rightarrow (PS)_g\pi^0 \rightarrow \tilde{H}^G, \tilde{E}^G$$

$$\gamma\gamma \rightarrow (S)_g\pi^0 \rightarrow H^G, E^G$$

$$\gamma\gamma \rightarrow (T)_g\pi^0 \rightarrow F_T^G, \tilde{F}_T^G$$

LO: [Bayer, Grozin '85]

$$\gamma^* q \rightarrow (q\bar{q})q, \gamma^* g \rightarrow (q\bar{q})g$$

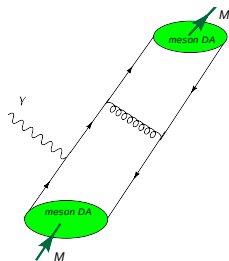


NLO DV PS^+ prod.: [Belitsky and Müller '01]

NLO DV V_L prod.: [Ivanov et al '04,]

NLO DV V_L (corr.), PS , (S , PV_L) prod.: [Duplanić, Müller, Passek-K. '17]

$$\gamma^*(q\bar{q}) \rightarrow (q\bar{q})$$



NLO: [..., Melić et al '99]

$$\gamma_L^*(M^\pm) \rightarrow M^\pm,$$

$$\gamma_L^*(S) \rightarrow V_L, \gamma_L^*(V_L) \rightarrow S$$

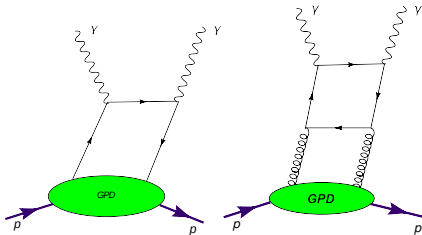
$$\gamma_L^*(PV_L) \rightarrow PS, \gamma_L^*(PS) \rightarrow PV_L$$

$\Rightarrow$ DVMP

(D)DVCS

$$\gamma^* q \rightarrow \gamma^{(*)} q, \gamma^* g \rightarrow \gamma^{(*)} g$$

25



NLO: [Ji, Belitsky et al, Mankiewicz et al, '97]
[Pire, Szymanowski, Wagner '11]

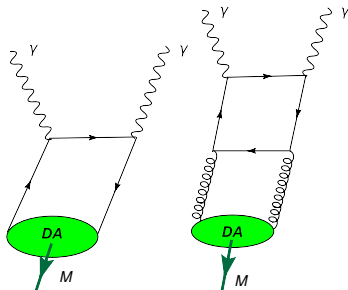
β_0 proportional NNLO: [Belitsky, Schäfer '98]

NNLO from conf. sym: [Müller '05, Kumerički, Müller, Passek-K '07,]

NNLO: [Braun, Manashov, Moch, Schoehleber '20, '21, Braun, Ji, Schoehleber '22]

Meson transition form factor

$$\gamma^* \gamma^{(*)} \rightarrow (q\bar{q}), \gamma^* \gamma^{(*)} \rightarrow (gg)$$



NLO: [..., Kroll, Passek-K '02] [Kroll, Passek-K '19]

β_0 proportional NNLO: [Melić, Nijžić, Passek '01]

NNLO from conf. sym: [Melić, Müller, Passek '02]

NNLO [Braun, Manashov, Moch, Schoehleber '21,

Gau, Huber, Ji, Wang '22]

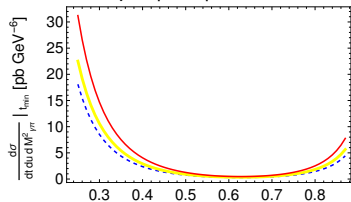
- experiment
 - DVCS, vector (ρ , J/Ψ , ϕ) and pseudoscalar (π , η) meson production measured by H1, ZEUS, HERMES (HERA, DESY), COMPASS (SPS, CERN), CLAS, Hall-A,C (JLab) ...
 - LHeC, EicC proposed
 - EIC (Electron Ion Collider at Brookhaven, 2030) under construction (luminosity 100-1000 times HERA)

Numerics (preliminary!)

$$\frac{d\sigma}{dt du dM_{\gamma\pi}^2} = \frac{|\overline{\mathcal{M}}|_{t_{\min}}^2}{32 S_{\gamma N}^2 M_{\gamma\pi}^2 (2\pi)^3}$$

$$\alpha = -u'/s' = \frac{1 + \cos \theta'}{2}$$

$\gamma\pi^+$ photoproduction



$$M_{\gamma\pi}^2 = 4 \text{ GeV}^2, S_{\gamma N} = 20 \text{ GeV}^2,$$

$$\xi = 0.11111$$

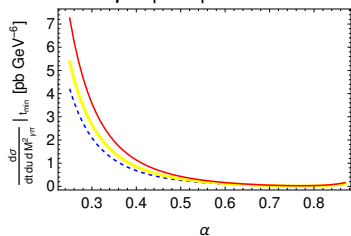
$$t_{\min} = -0.050 \text{ GeV}^2$$

--- asymptotic DA

— Gegenbauer DA; $a_2 = 0.1364$

— holographic DA

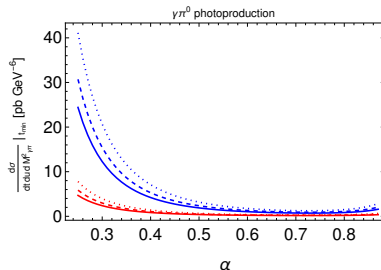
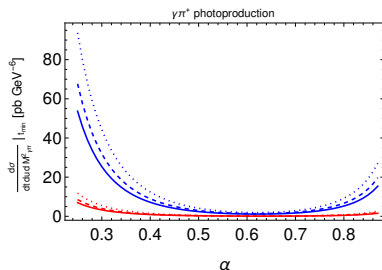
α
 $\gamma\pi^0$ photoproduction



Numerics (preliminary!)

$$\frac{d\sigma}{dt du dM_{\gamma\pi}^2} = \frac{|\overline{\mathcal{M}}|_{t_{min}}^2}{32 S_{\gamma N}^2 M_{\gamma\pi}^2 (2\pi)^3}$$

$$\alpha = -u'/s' = \frac{1 + \cos \theta'}{2}$$



- $S_{\gamma N} = 20 \text{ GeV}^2$
- $s' = M_{\gamma\pi}^2 = 3 \text{ GeV}^2$, $\xi = 0.081081$
- $s' = M_{\gamma\pi}^2 = 5 \text{ GeV}^2$, $\xi = 0.14286$
- DAs: Asymptotic (straight line), Gegenbauer $a_2 = 0.1364$ (dashed), Holographic (dotted)

- The richer kinematics of the $2 \rightarrow 3$ processes allow for an efficient probing of the sensitivity of the GPDs x dependence [Qiu, Yu, '23]
→ two large scales similarly as in DDVCS
- transversity GPDs accessible

$2 \rightarrow 3$ processes:

- $\gamma N \rightarrow (MM)N'$ [El Beiyad, Enberg, Ivanov, Pire, Segond, Szymanowski, Teryaev, Wallon: hep-ph/0209300, hep-ph/0601138, 1001.4491]
- $\gamma N \rightarrow (\gamma M)N'$ [Boussarie, Duplančić, Nabeebaccus, Passek-K., Pire, Szymanowski, Wallon: 1609.03830, 1809.08104, 2212.00655, 2302.12026]
- $\gamma N \rightarrow (\gamma\gamma)N'$ [Grocholski, Pedrak, Pire, Sznajder, Szymanowski, Wagner: 1708.01043, 2003.03263, 2110.00048, 2204.00396]
- $\pi N \rightarrow \gamma\gamma N'$ [Qiu, Yu: 2205.07846]

- The richer kinematics of the $2 \rightarrow 3$ processes allow for an efficient probing of the sensitivity of the GPDs x dependence [Qiu,Yu,'23]
→ two large scales similarly as in DDVCS
- transversity GPDs accessible

$$\underline{\gamma N \rightarrow (\gamma M) N'}$$

- ▷ completing and systematizing the results for all channels
- ▷ fast numerical code for different DA models with evolution included
- ▷ investigate the photoproduction of neutral pseudoscalar mesons