Color Superconductivity in 2-Flavor QCD and its Role in Neutron Stars

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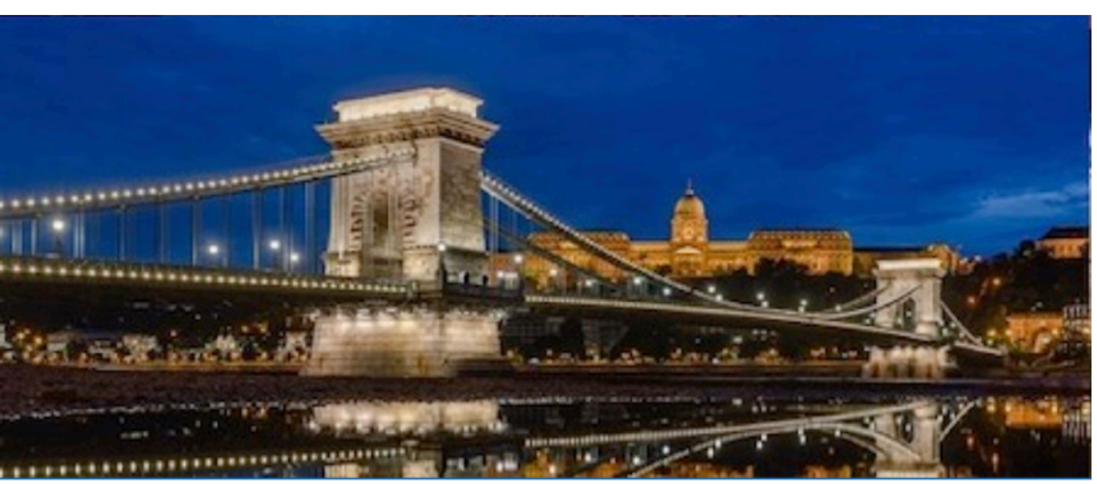
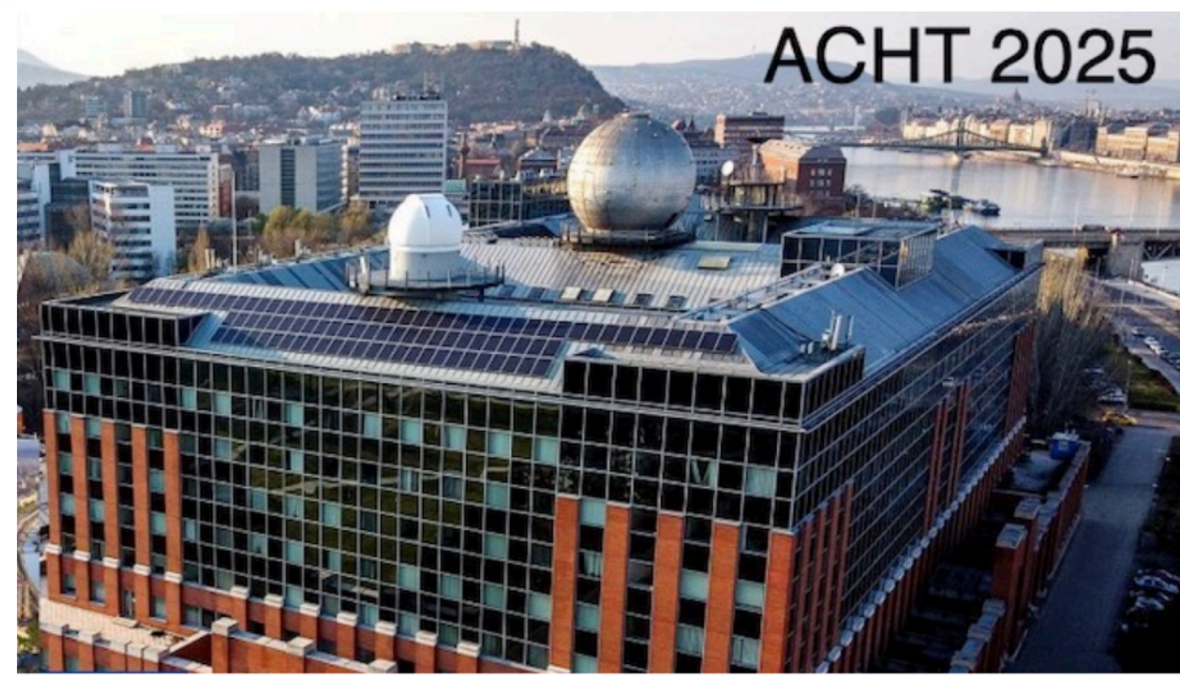


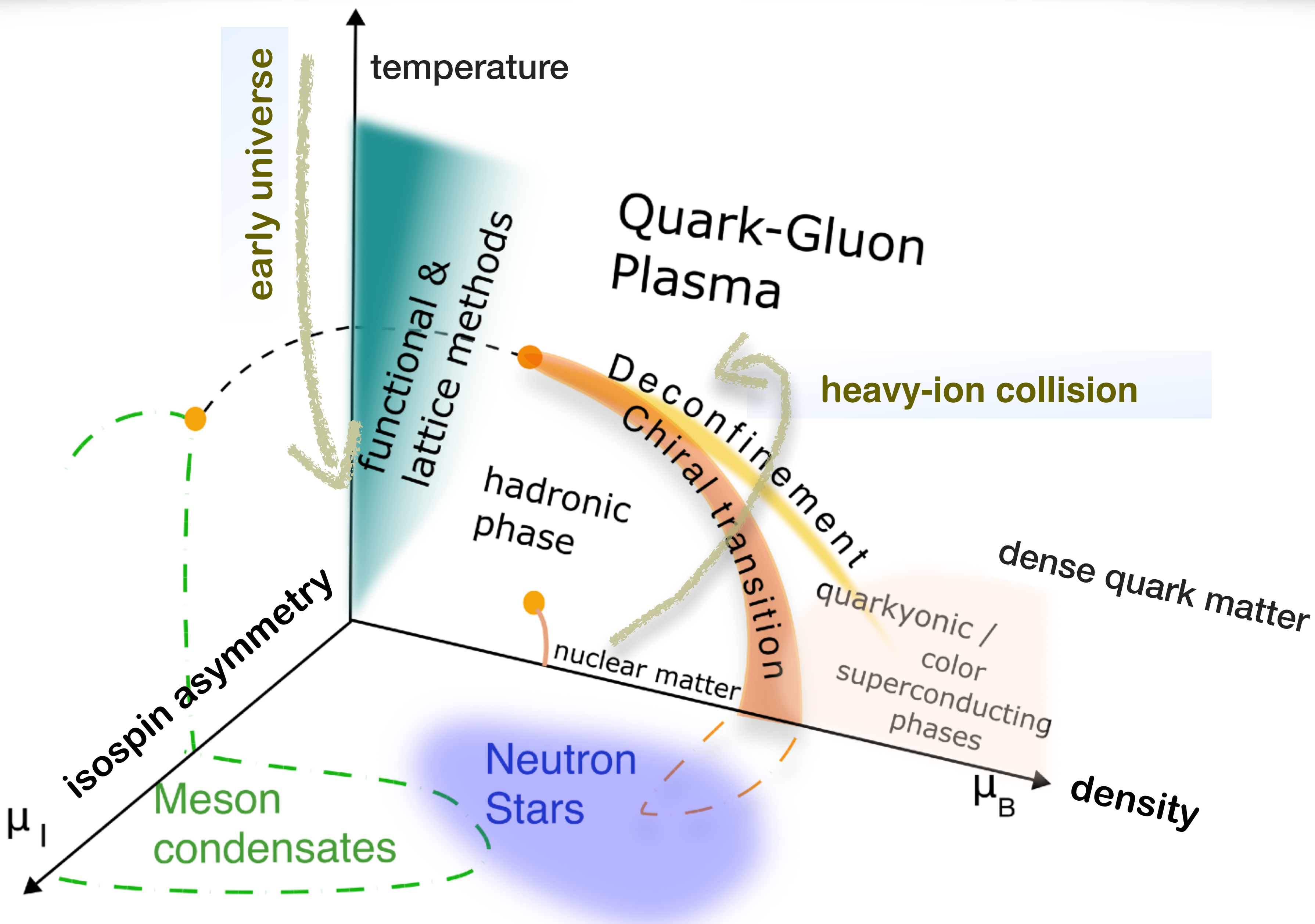
May 5th, 2025



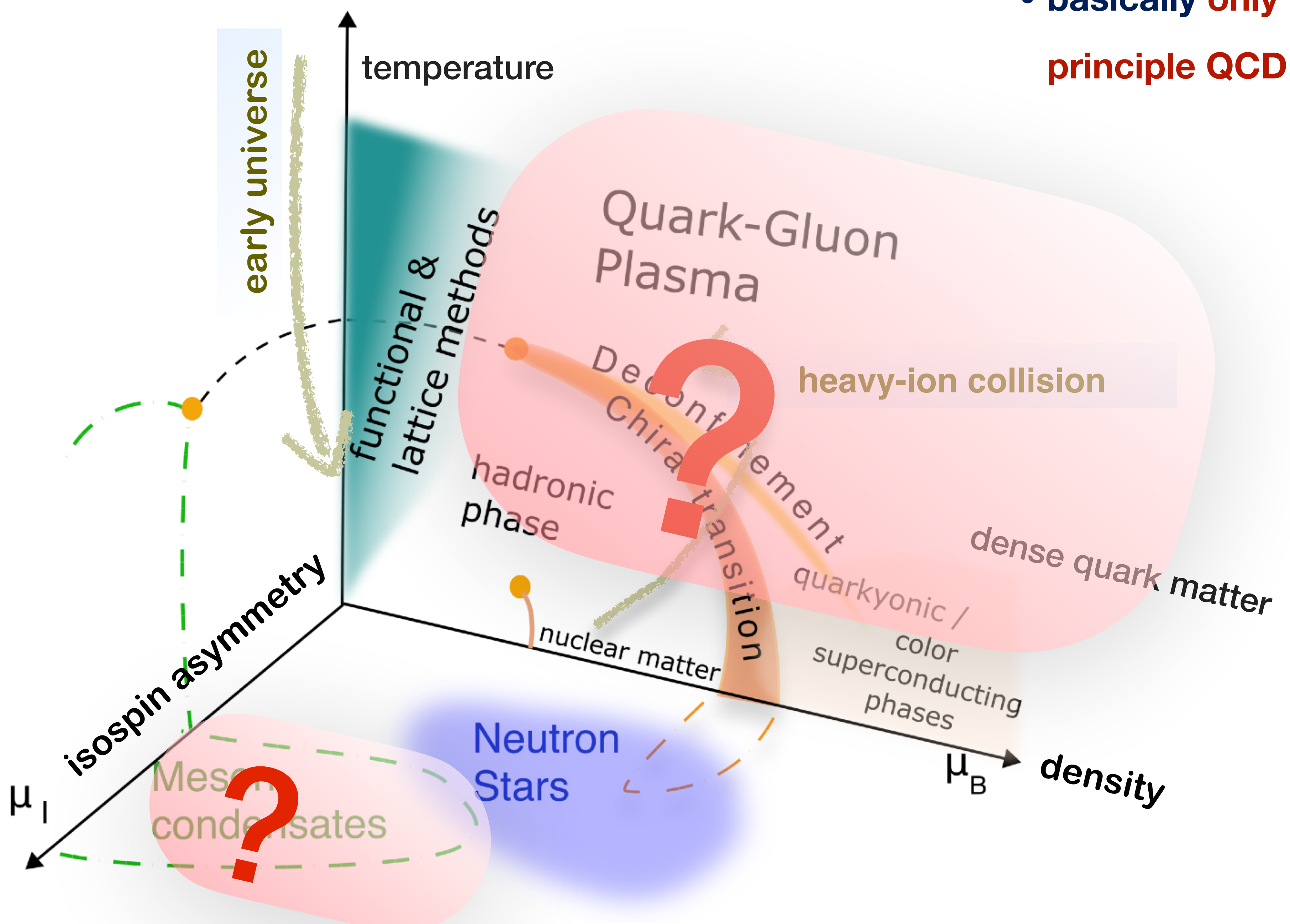
ACHT 2025: Non-perturbative methods in strongly interacting quantum many-body systems 5–7 May 2025 Lágymányos Campus, Eötvös Loránd University, Budapest

QCD under extreme conditions

ultimate goal: microscopic understanding of strong interacting matter by first principle QCD

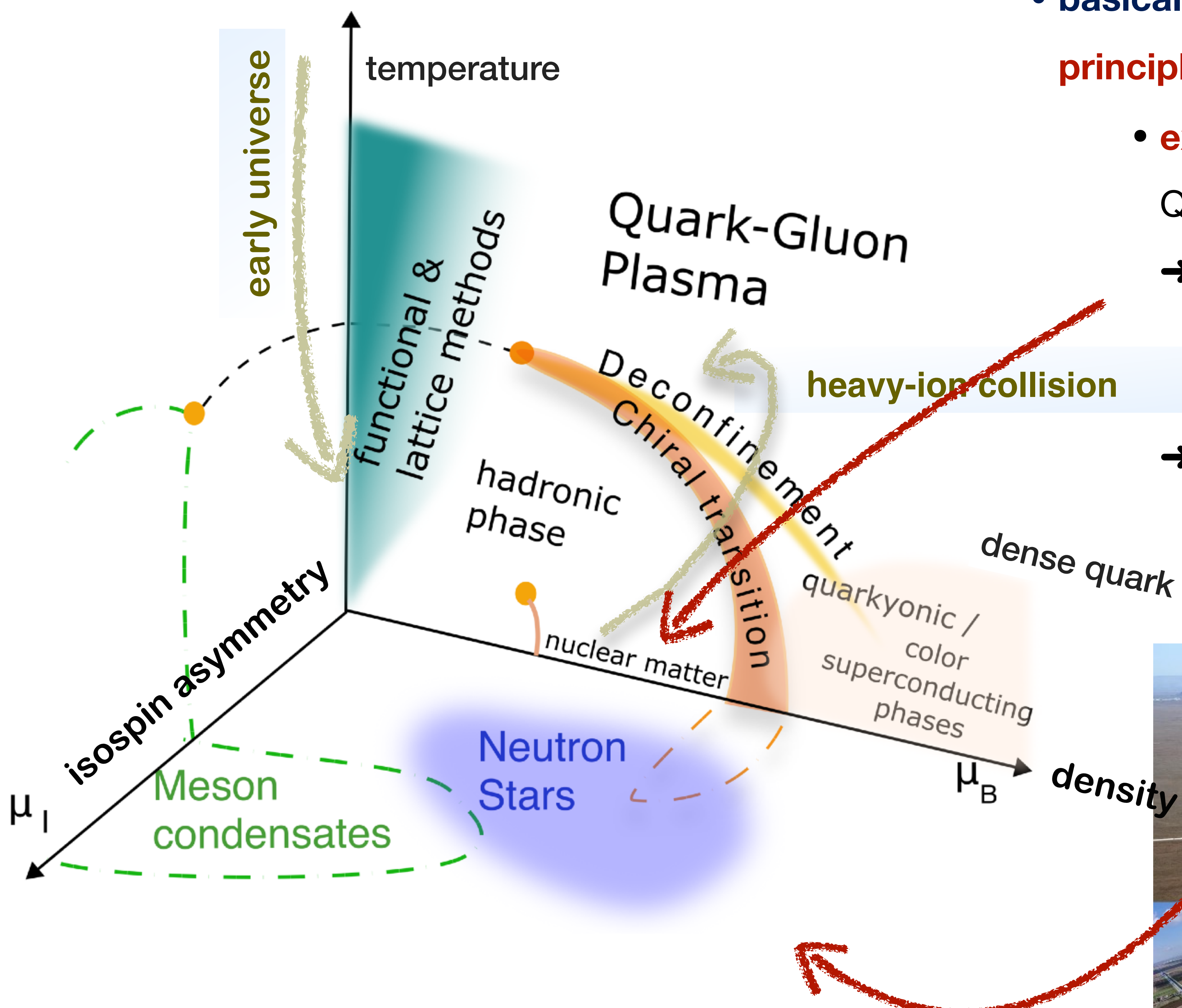


QCD under extreme conditions



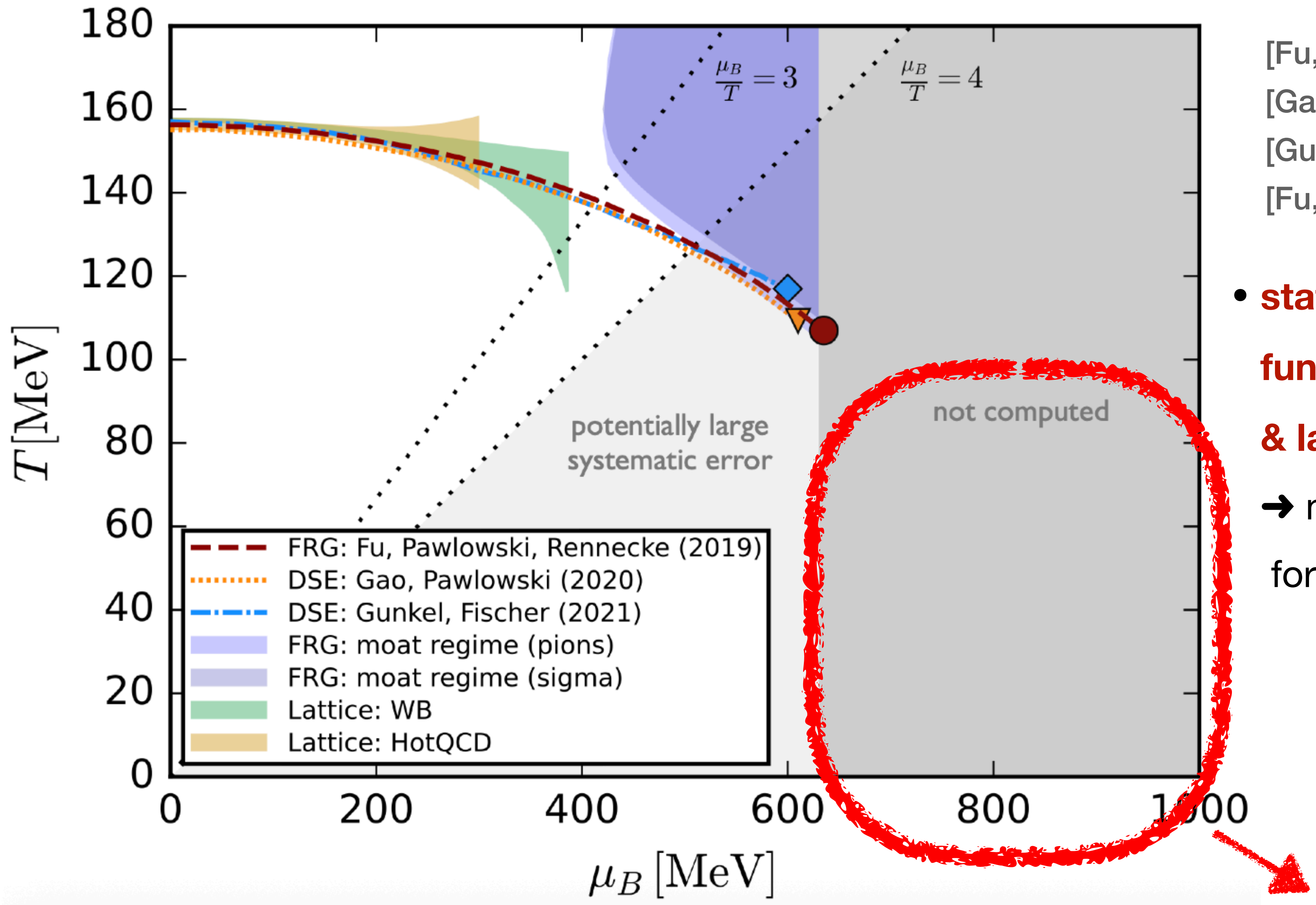
- **basically only corners known from first principle QCD**

QCD under extreme conditions



- **basically only corners known from first principle QCD**
  - **experiments to** probe cold dense QCD matter
    ➜ **heavy-ion colisions**
    (e.g. BNL, FAIR etc)
    ➜ **neutron star physics**
    (e.g. NICER,
    aLIGO, aVirgo,KAGRA, Geo600)

status first-principle QCD



[Fu, Pawlowski, Rennecke 2019]
[Gao, Pawlowski 2020]
[Gunkel, Fischer 2021]
[Fu, Pawlowski, Pisarski et al. 2025]

- **status first-principle QCD: functional methods (FRG/DSE) & lattice simulations**

➜ need improvements / input for $\frac{\mu_B}{T} > 4$

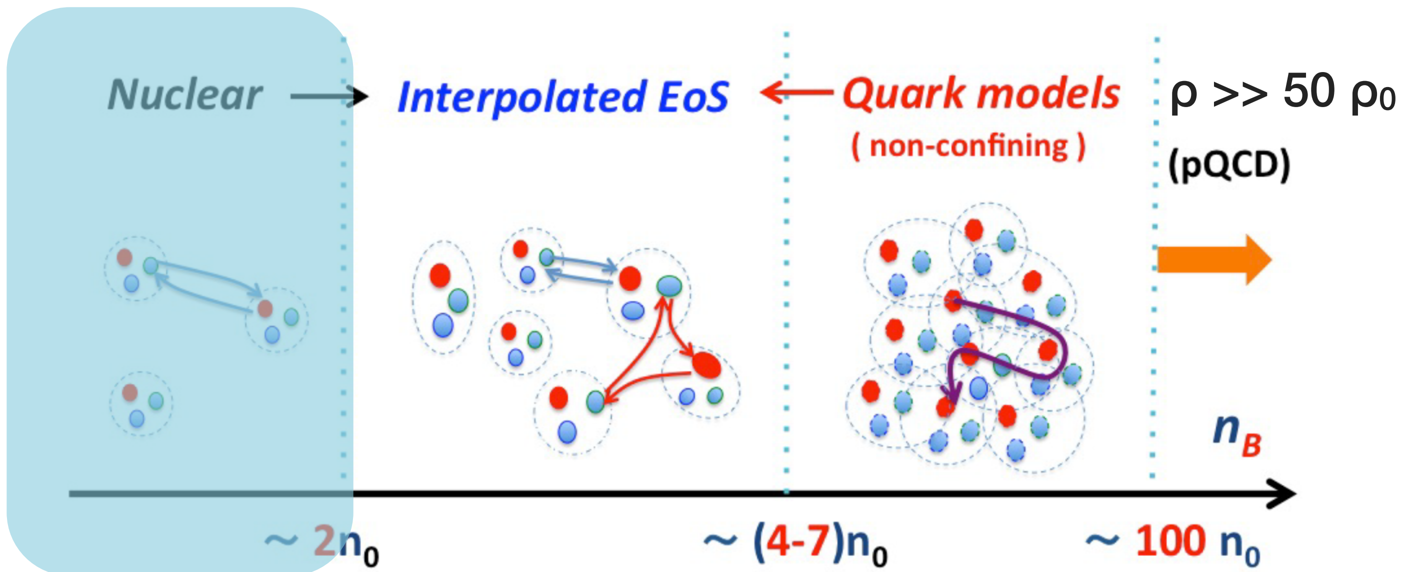
model-based extrapolations

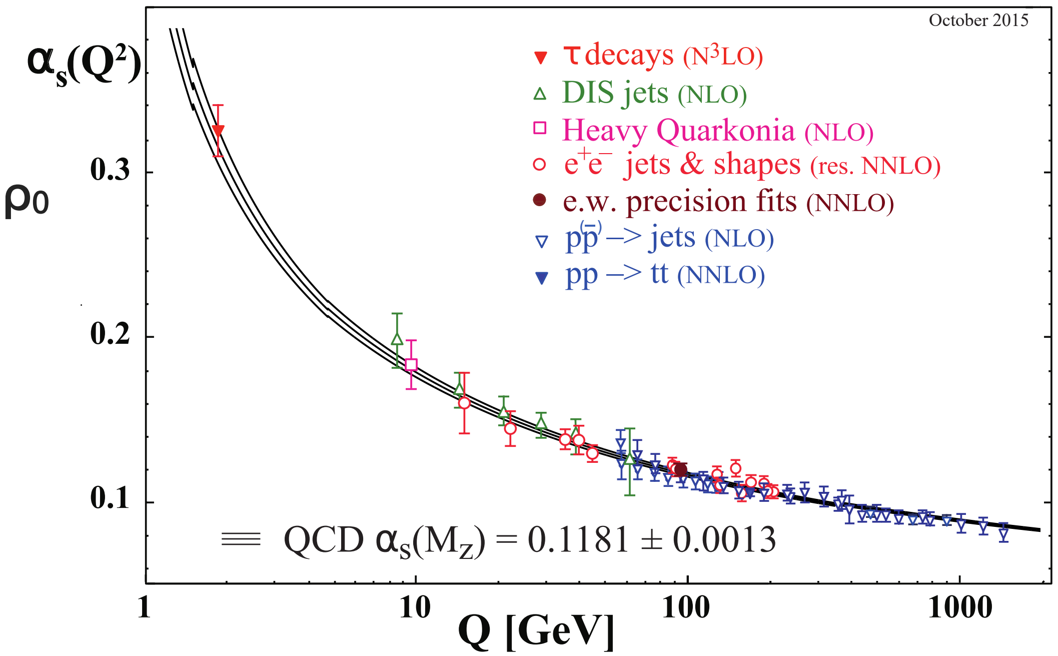
or

QCD-assisted truncations

low-T phases of dense matter

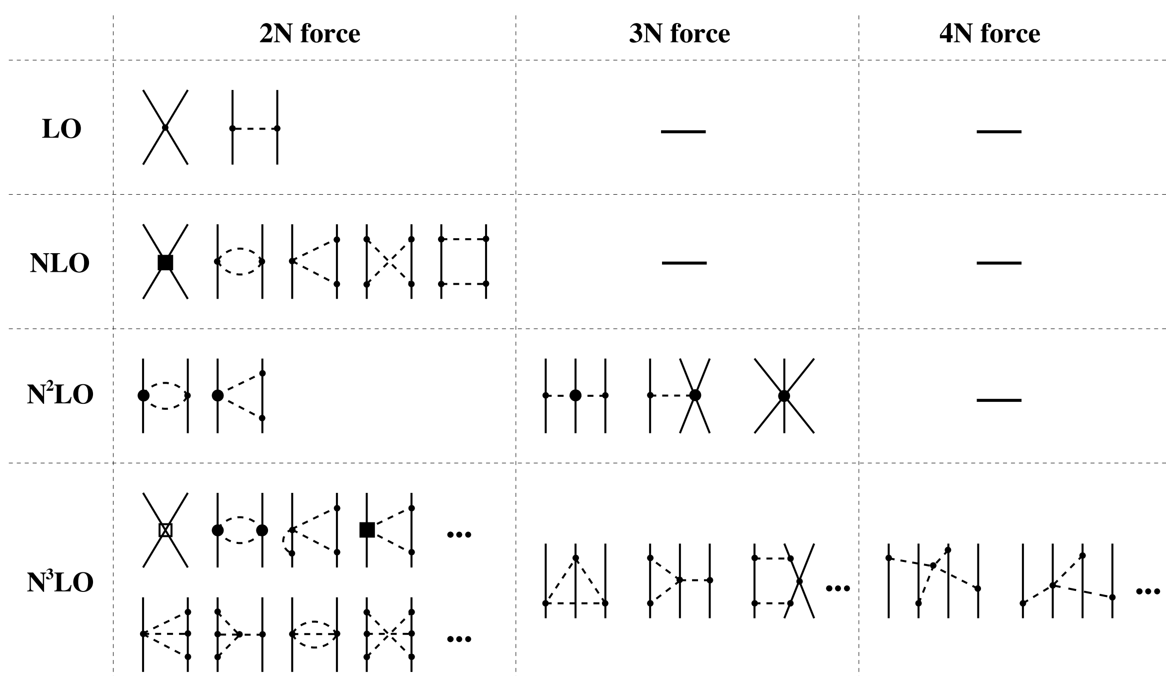
[Baym, Kojo et al 2018] ’3-window’ model of dense matter





Nuclear phase:
1-2 meson/quark exchanges

EoS from **nuclear physics**
ρ < 2ρ0 χEFT



[Beane, Bedaque, Epelbaum, **Hebeler**, Kaplan, Machleidt, Meisner, Phillips, Savage, **Schwenk**, van Kolck, Wise …]

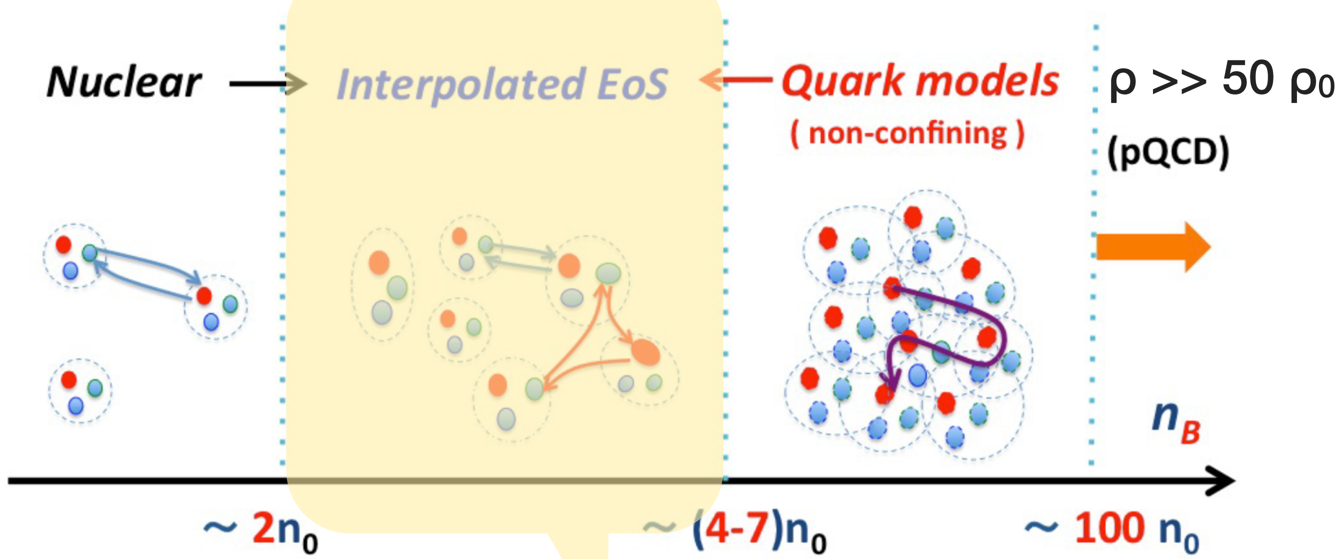
χEFT Hamiltonian organized by Q/Λ

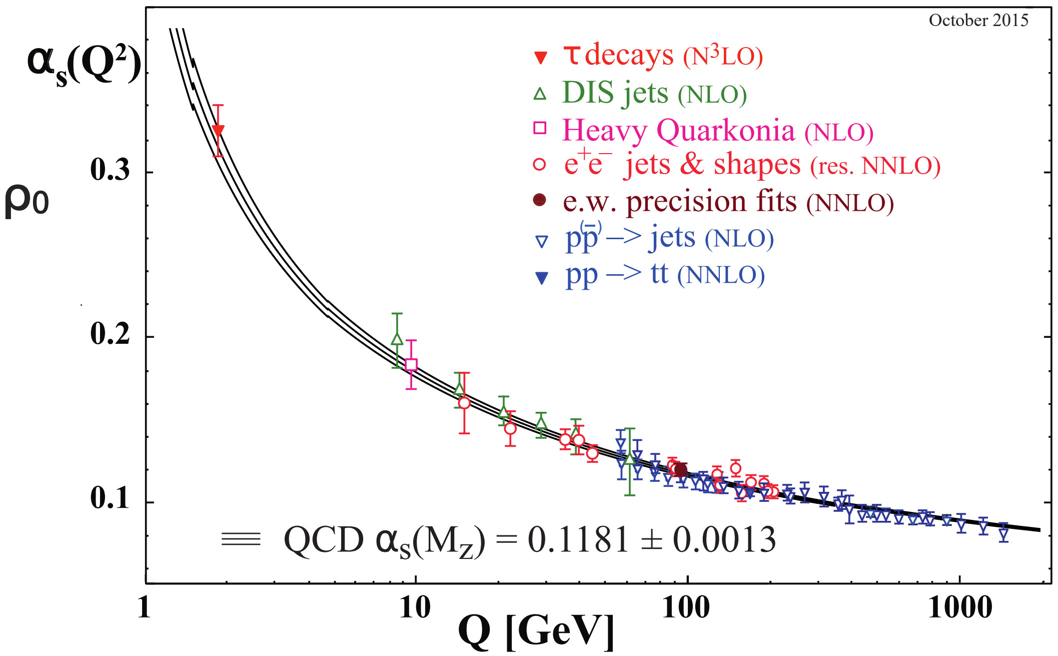
intrinsic breakdown scale
Λ ~ 500 - 600 MeV

Q: rel momenta between nucleons

low-T phases of dense matter

[Baym, Kojo et al 2018] '3-window' model of dense matter





Nuclear phase:
1-2 meson/quark exchanges

EoS from
nuclear physics
$\rho < 2\rho_0$ χEFT

interpolated EoS
many meson/quark exchanges

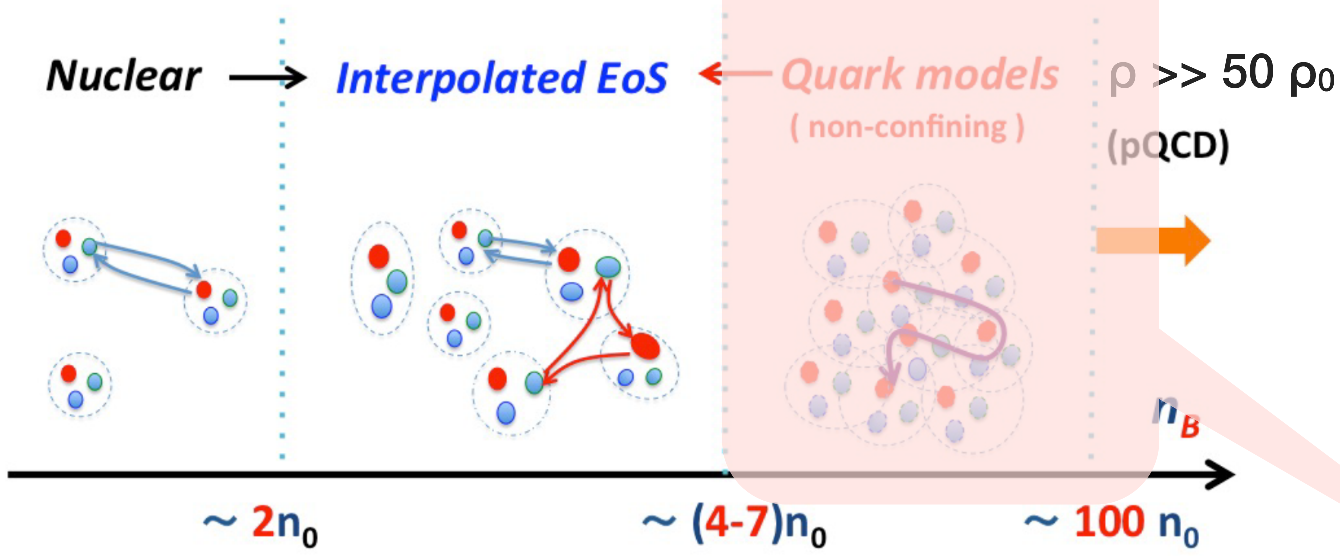
system gradually changes
from hadronic to quark matter
- diquarks, colored quarks virtually …

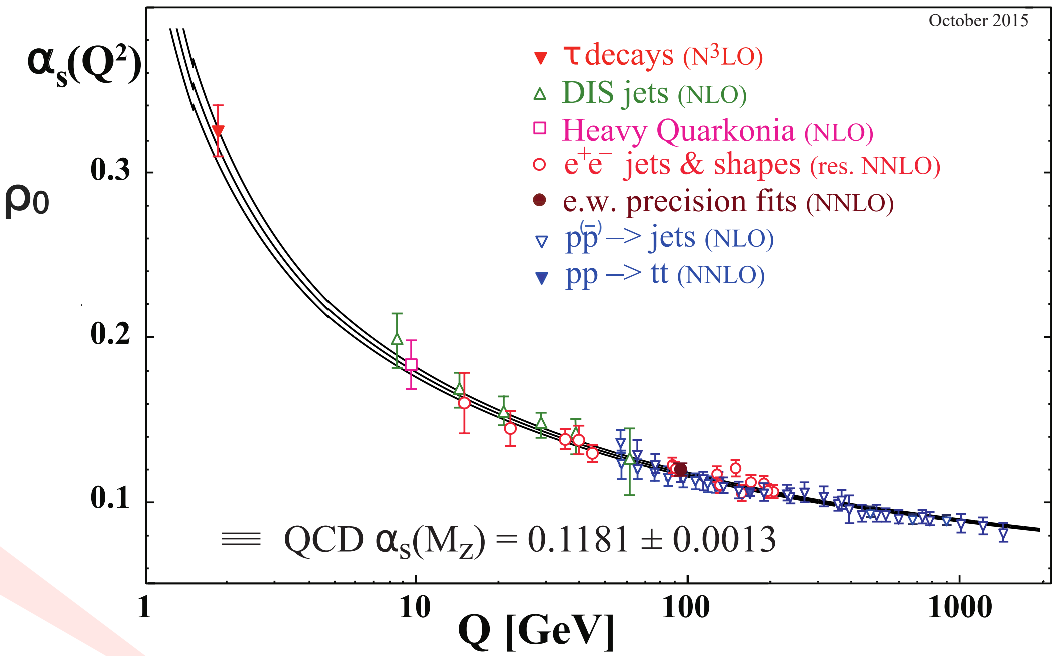
- role of strangeness / hyperons

$2\rho_0 < \rho < 7\rho_0$ Neutron stars

low-T phases of dense matter

[Baym, Kojo et al 2018] '3-window' model of dense matter





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$2\rho_0 < \rho < 7\rho_0$ Neutron stars

[Blaschke, Fischer, Oertel, Schaffner-Bielich, ...]

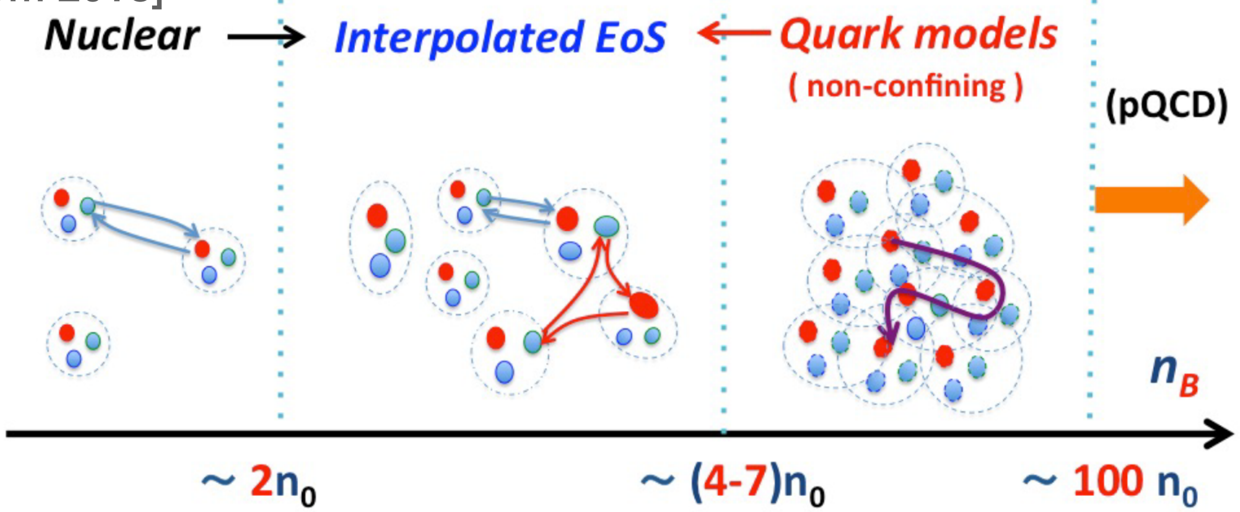
Quark phase:
quarks no longer specific to baryons

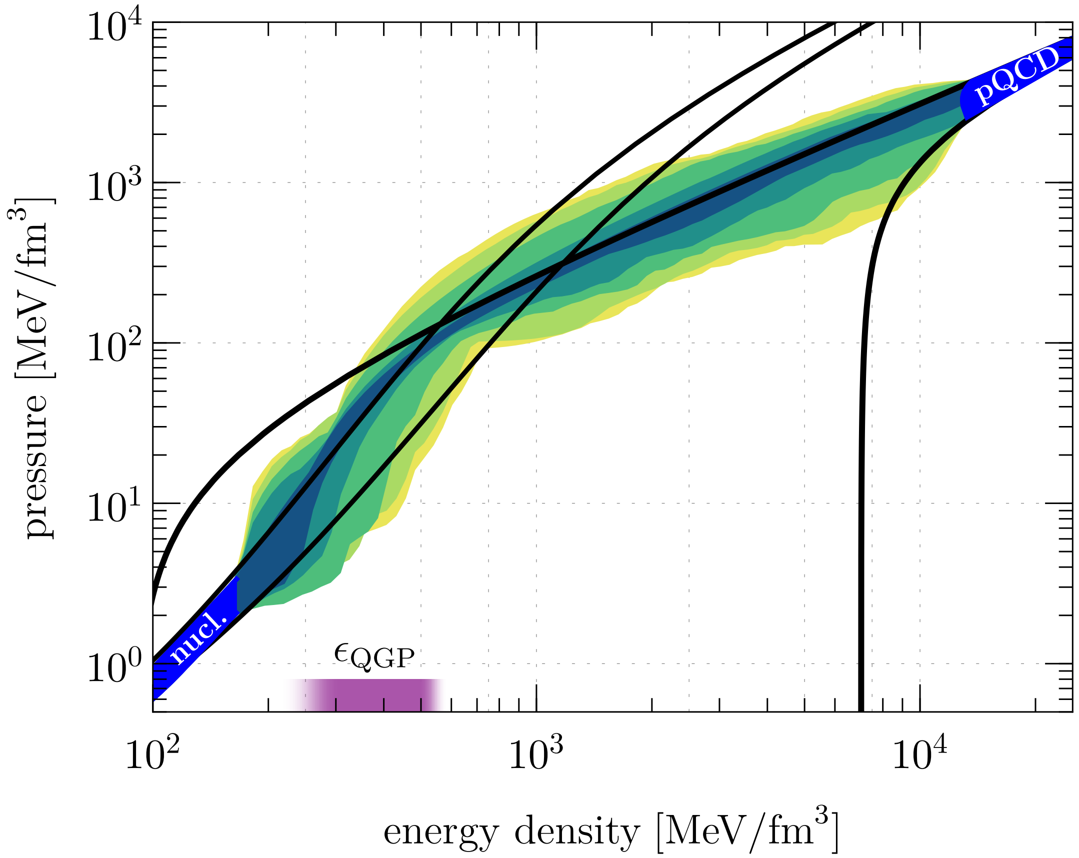
mostly mean-field investigations like NJL-type or phenomenological models

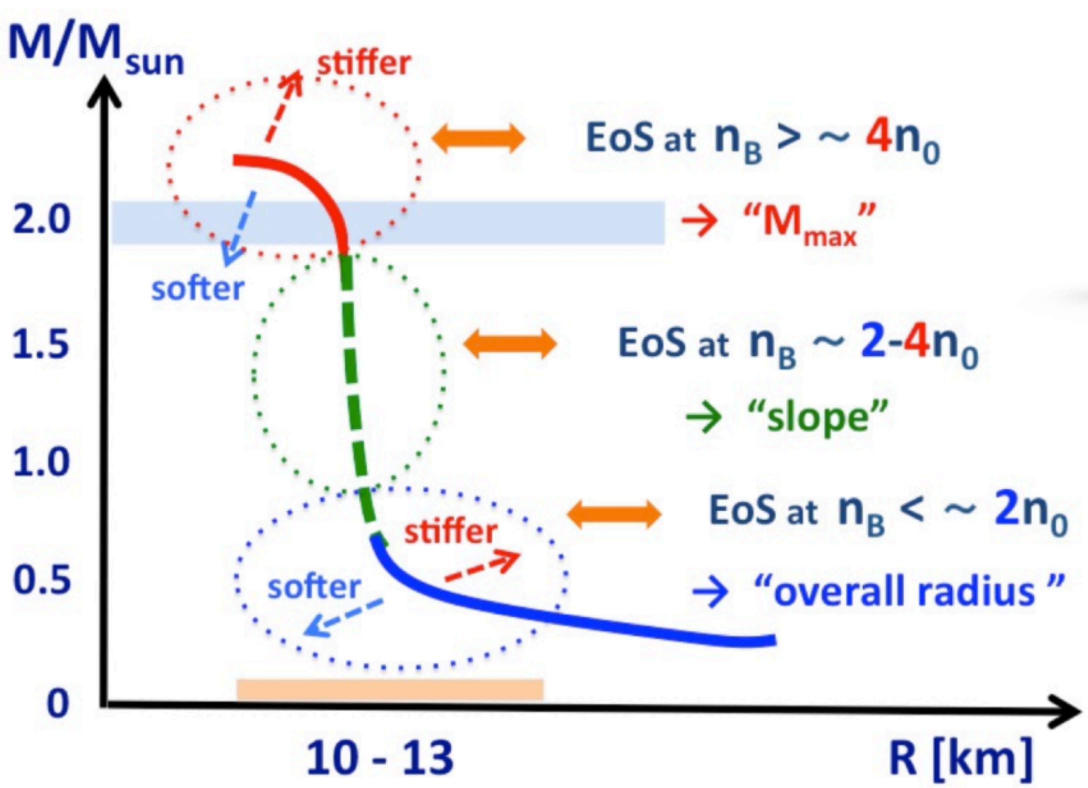
➜ upgrade with FRG methods

conflicting constraints on EoS

[Baym 2018]







EoS ↔ TOV equation ↔ M-R relation (observables)

<u>three constraints on the EoS:</u>

1. stiff enough (@high density) ➜ $2M_\odot$
2. soft enough (@low density) ➜ Radius
3. speed of sound < 1

the ultimate goal

... solving first principle QCD

Connect low-energy models to first principle QCD

Functional Renormalization Group

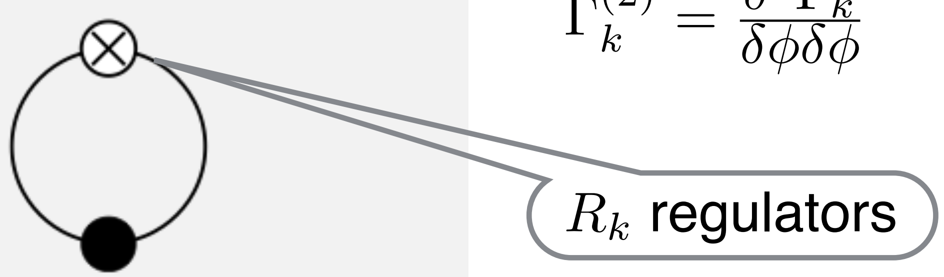
■ Wetterich Equation (average effective action)

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\, \partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right)$$

$$t = \ln(k/\Lambda)$$

$$\Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

$$k\partial_k \Gamma_k[\phi] \sim \frac{1}{2}$$



[Wetterich 1993]

■ truncation: e.g. Quark-meson type approximation

$$\Gamma_k = \int d^4x \bar{q}[i\gamma_\mu\partial^\mu - g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + V_k(\phi^2)$$

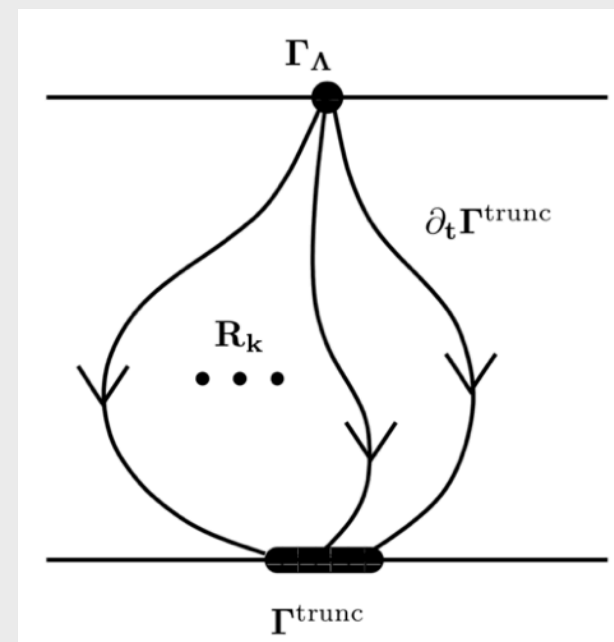
arbitrary potential

$$V_{k=\Lambda}(\phi^2) = \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

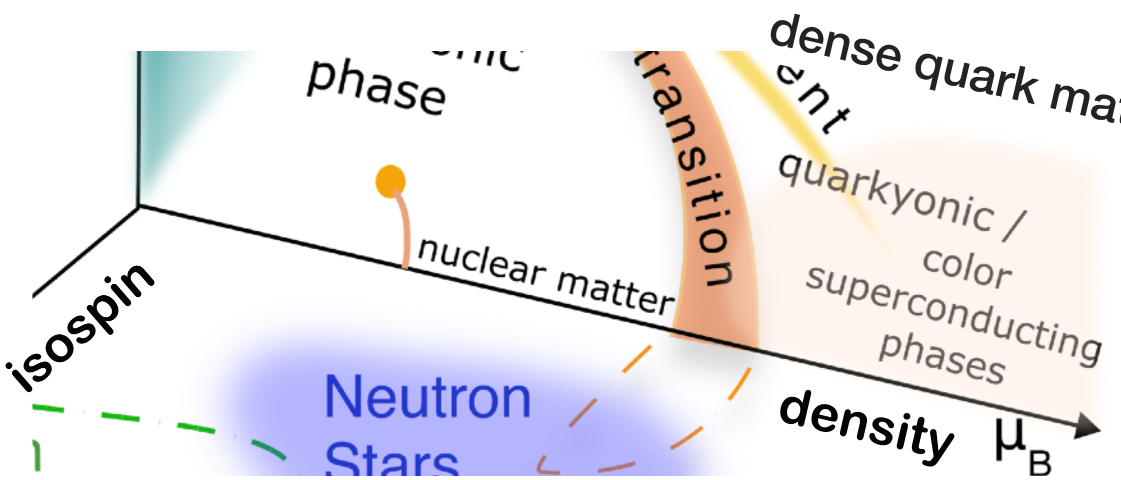
regulator conditions:

$$R_k(p^2) = p^2 r(p^2/k^2)$$

- $\lim_{p^2/k^2 \to \infty} R_k(p^2) = 0$
- $\lim_{p^2/k^2 \to 0} R_k(p^2) > 0 \; (= k^2)$
- $\lim_{k \to \infty} R_k(p^2) \to \infty$



QCD at finite density



- pQCD: @O(100 GeV) (deep high-energy perturbative region)
  n ~ 50-100 n_0
- large densities: m_q negligible ↔ μ

  CFL: diquark pairing (p=0, $J^P=0^+$)
  always in QCD by gluon-exchange
- smaller densities: $m_{strange} > m_{light}$ ➜ smaller p_F
  CFL unstable due to different p_F
- 2SC or/and quaryonic phases

here: 2SC phases: 2 quark flavor with 2 color pair

$$SU(2)_L \times SU(2)_R \times SU(3)_c \to SU(2)_L \times SU(2)_R \times SU(2)_c$$

via Higgs mechanism: 8-3 gluons become massive
(no Goldstone bosons)

EoS from QCD

- QCD procedure: start @O(100 GeV) (deep high-energy perturbative region)

[Braun et al. 2012++]

Λ ~ O(10GeV) quarks, gluons

- symmetric regime

quarks, gluons ➜ mesons

k_Φ

- symmetry-broken regime

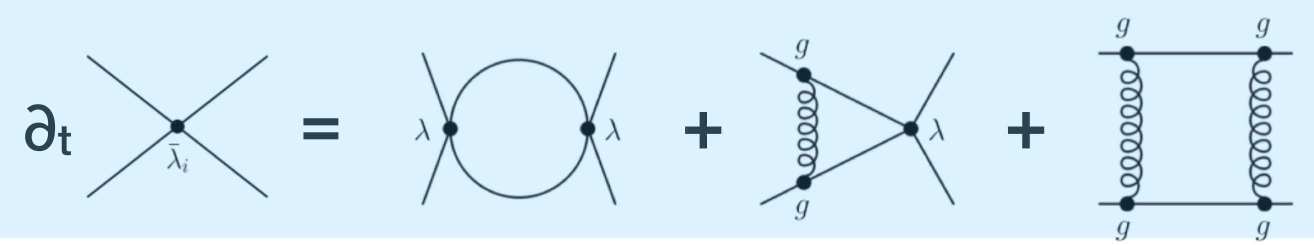
quarks, diquarks, mesons etc

k_{IR}

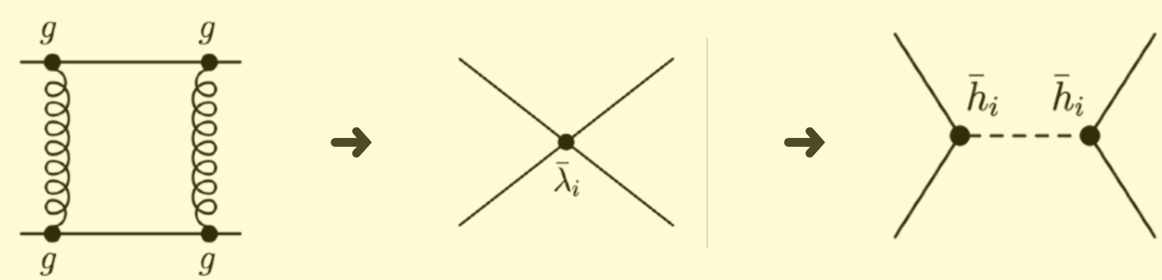
$$S = \int \mathrm{d}^4x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi} \left(\mathrm{i}\partial\!\!\!/ + \bar{g} A\!\!\!/ + \mathrm{i}\gamma_0 \mu \right) \psi \right\}$$

- quark-gluon vertex ➜ many quark self-interaction channels
- dynamical hadronisation:

4-quark correlators ➜ bound states /resonances



- general picture:



4-quark correlators

- **QCD procedure: start @O(100 GeV) (deep high-energy perturbative region)**

$\Lambda \sim O(10\,\mathrm{GeV})$ quarks, gluons

- symmetric regime

quarks, gluons ➜ mesons

k_Φ

- symmetry-broken regime

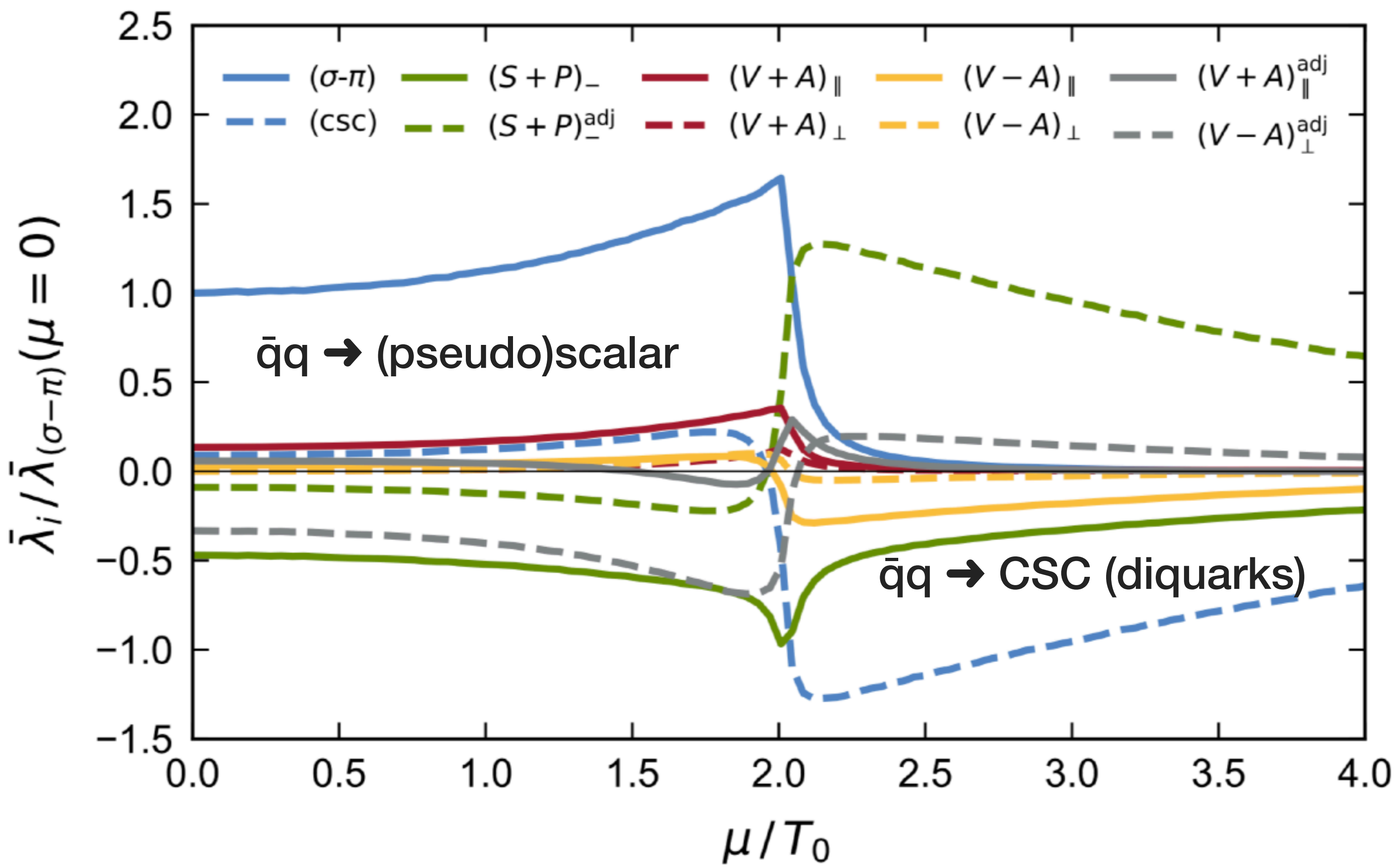
quarks, diquarks, mesons etc

k_{IR}

$$\text{pseudoscalar} \sim (\bar{q}q)^2 - (\bar{q}\gamma_5\vec{\tau}q)^2$$

$$\sigma \sim \bar{q}q \text{ and } \vec{\pi} \sim i\bar{q}\gamma_5\vec{\tau}q$$

[Braun et al. 2020]



$$\text{diquark} \sim (q^T \mathcal{C}\tau_2\lambda_A\gamma_5 q)(\bar{q}\gamma_5\tau_2\lambda_A\mathcal{C}\bar{q}^T)$$

$$\Delta_A^* \sim \bar{q}\gamma_5\tau_2\lambda_A\mathcal{C}\bar{q}^T$$

dynamical hadronization

- QCD procedure: start @O(100 GeV) (deep high-energy perturbative region)

[Braun et al. 2012++]

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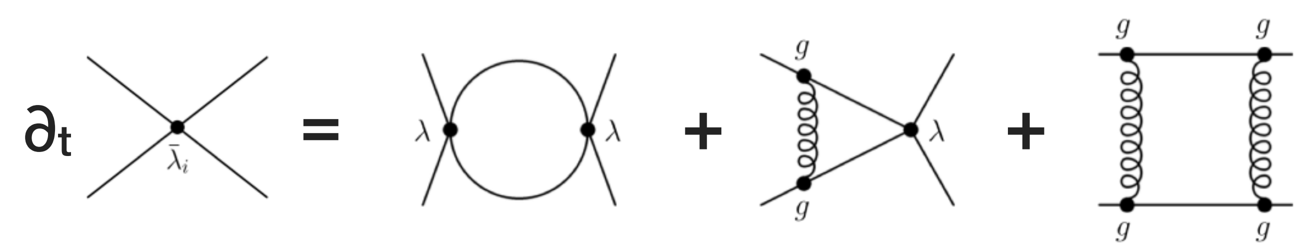
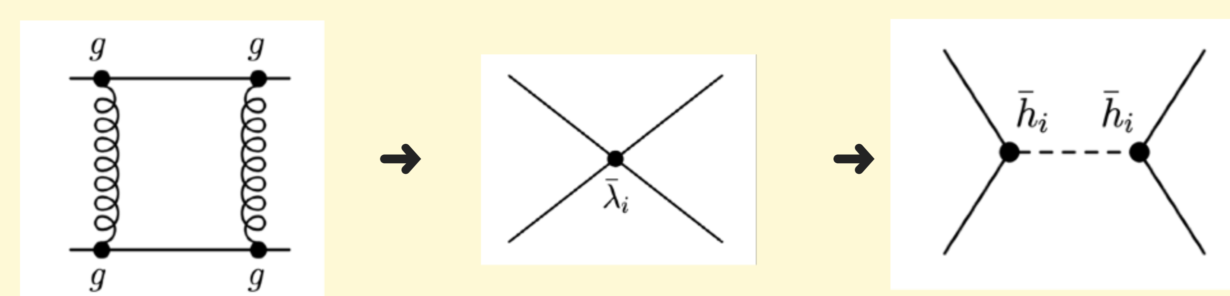
k_{IR}

$$S = \int \mathrm{d}^4x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar\psi \left(\mathrm{i}\partial\!\!\!/ + \bar g A\!\!\!/ + \mathrm{i}\gamma_0 \mu \right) \psi \right\}$$



- beyond pointlike approximation ➜ dynamical hadronization



- cast into quark-meson-diquark model truncation parametrizes low-energy regime with most important (pseudo)scalar and diquark channel

quark-meson-diquarks

- QCD procedure: start @O(100 GeV) (deep high-energy perturbative region)

[Mire, BJS to be published]

Λ ~ O(10GeV) quarks, gluons

- symmetric regime

quarks, gluons ➜ mesons

k_Φ

- symmetry-broken regime

quarks, diquarks, mesons etc

$k_{\rm IR}$

Quark-meson-diquark truncation at scale k_Φ

$$\mathcal{L}_{\rm QMD} = \bar q \left(\not\partial - \hat\mu\gamma_0 + g_\phi\left(\sigma + i\gamma_5\vec\pi\,\vec\tau\right)\right) q$$

$$+\frac{g_\Delta}{2}\left(\Delta^*_{\rm A}\bar q_C\gamma_5\tau_2\lambda_{\rm A} q - \Delta_{\rm A}\bar q\gamma_5\tau_2\lambda_{\rm A} q_C\right)$$

$$+\left(\left(\partial_\nu + \delta_{\nu 0}2\mu\right)\Delta^*_{\rm A}\right)\left(\partial_\nu - \delta_{\nu 0}2\mu\right)\Delta_{\rm A}$$

$$+\frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec\pi)^2 + U(\rho, d) - c\sigma$$

with $\rho = \frac{1}{2}(\sigma^2 + \vec\pi^2)$ and $d \equiv |\Delta|^2 = \sum_A \Delta^*_A\Delta_A$

(pseudo)-scalar (anti)-diquarks

quark-meson-diquarks

[Mire, BJS to be published]

Quark-meson-diquark truncation at scale k_Φ

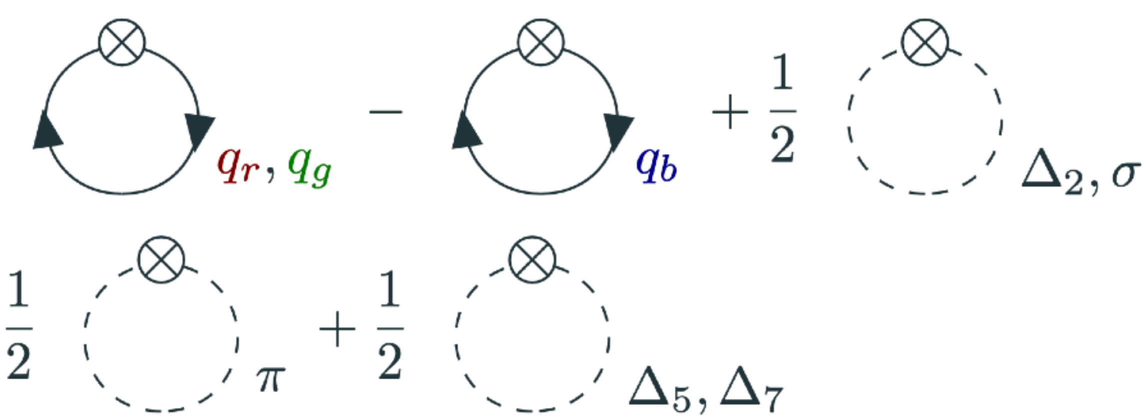
$$\mathcal{L}_{\mathrm{QMD}} = \bar{q}\left(\not{\partial} - \hat{\mu}\gamma_0 + g_\phi\left(\sigma + i\gamma_5 \vec{\pi}\,\vec{\tau}\right)\right) q$$

$$+\frac{g_\Delta}{2}\left(\Delta^*_{\mathrm{A}} \bar{q}_C \gamma_5 \tau_2 \lambda_{\mathrm{A}} q - \Delta_{\mathrm{A}} \bar{q} \gamma_5 \tau_2 \lambda_{\mathrm{A}} q_C\right)$$

$$+\left(\left(\partial_\nu + \delta_{\nu 0} 2\mu\right) \Delta^*_{\mathrm{A}}\right)\left(\partial_\nu - \delta_{\nu 0} 2\mu\right) \Delta_{\mathrm{A}}$$

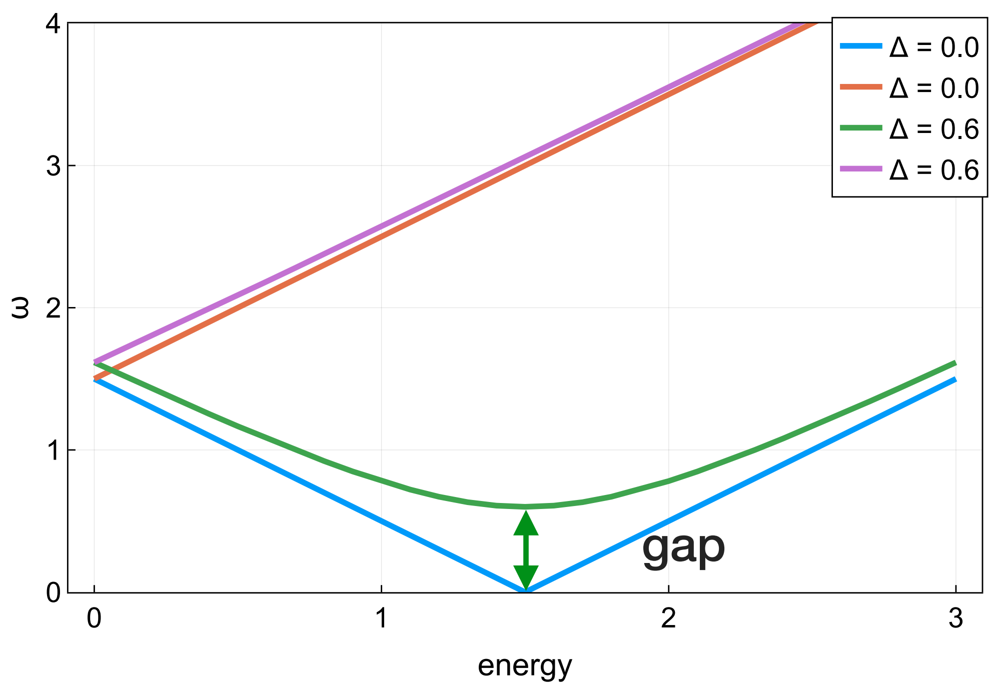
$$+\frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + U(\rho, d) - c\sigma$$

$\partial_t U_k(\sigma, \Delta) = -$



Fermi-surface

[Mire, BJS to be published]

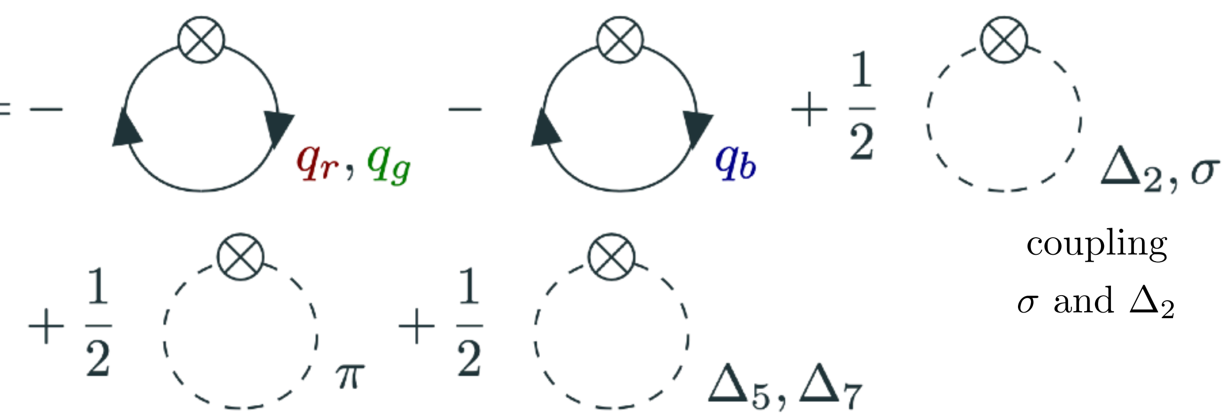


- dispersion relations

$$E_\Delta = \sqrt{(\epsilon_q \pm \mu)^2 + g_\Delta^2 d} \qquad \epsilon_{\{q,\pi,\sigma,\Delta\}} = \sqrt{k^2 + m^2_{\{q,\pi,\sigma,\Delta\}}}$$

- flow equation

$$\partial_t U_k(\sigma, \Delta) = -$$



$$\sim \frac{\epsilon_q - \mu}{\sqrt{(\epsilon_q - \mu)^2 + g_\Delta^2 d}} \qquad \sim \coth \frac{\epsilon_\Delta - 2\mu}{2T}$$

$$\longrightarrow \qquad \sim \frac{\epsilon_q - \mu}{((\epsilon_q - \mu)^2 + g_\Delta^2 d)^{3/2}}$$

diverges at Fermi-surface

flow around Fermi-surface?

no diquark loops

[Mire, BJS to be published]

Quark-meson-diquark truncation at scale k_Φ

$$\mathcal{L}_{\mathrm{QMD}} = \bar{q}\left(\not{\partial} - \hat{\mu}\gamma_0 + g_\phi\left(\sigma + i\gamma_5\vec{\pi}\,\vec{\tau}\right)\right) q$$

$$+\frac{g_\Delta}{2}\left(\Delta^*_{\mathrm{A}}\bar{q}_C\gamma_5\tau_2\lambda_{\mathrm{A}}q - \Delta_{\mathrm{A}}\bar{q}\gamma_5\tau_2\lambda_{\mathrm{A}}q_C\right)$$

$$+\left(\left(\partial_\nu + \delta_{\nu 0}2\mu\right)\Delta^*_{\mathrm{A}}\right)\left(\partial_\nu - \delta_{\nu 0}2\mu\right)\Delta_{\mathrm{A}}$$

no diquark loops

$$+\frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + U(\rho, d) - c\sigma$$

avoiding medium divergence

$$\sim \mu^2\Delta^2$$

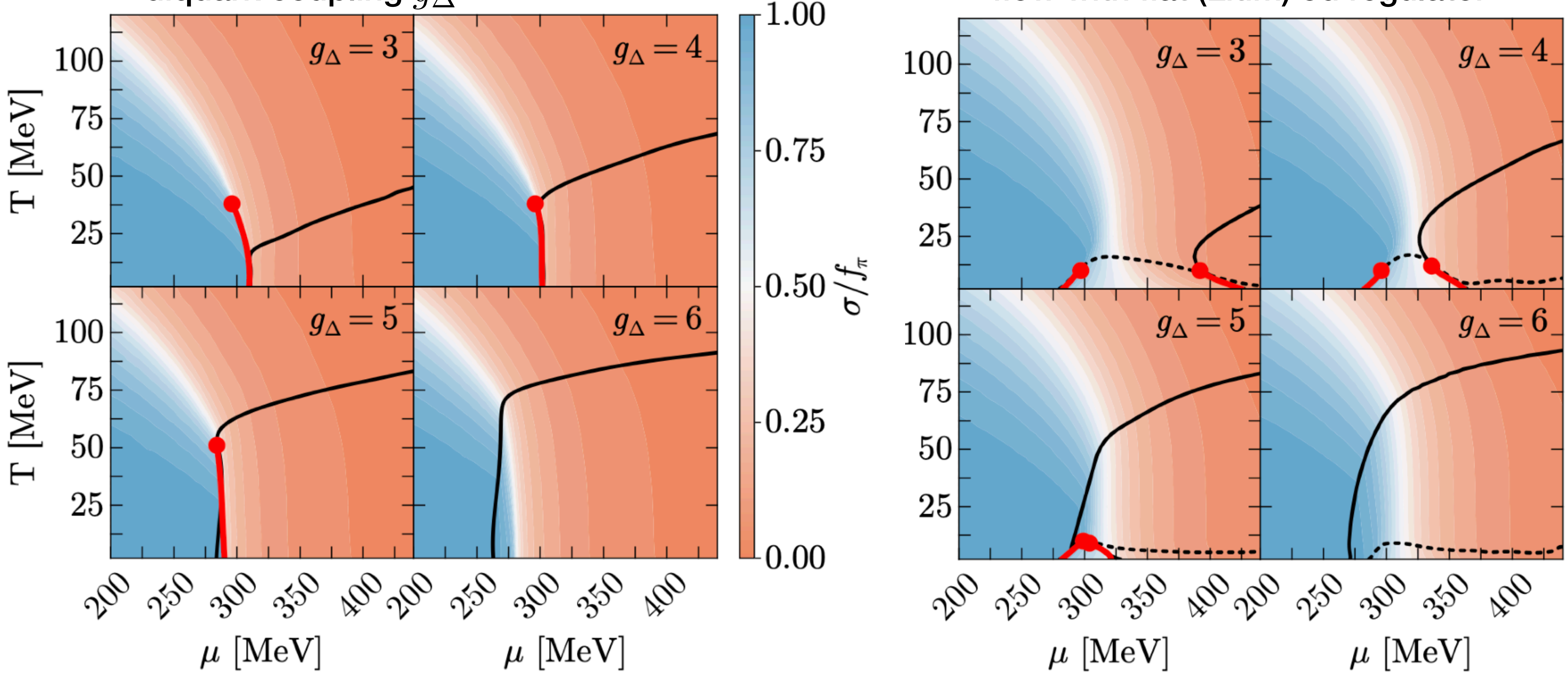
with $\rho = \frac{1}{2}(\sigma^2 + \vec{\pi}^2)$ and $d \equiv |\Delta|^2 = \sum_A \Delta^*_A\Delta_A$

(pseudo)-scalar (anti)-diquarks

Phase diagram: quark-meson-diquarks

[Mire, BJS to be published]

- diquark coupling g_Δ
- flow with flat (Litim) 3d regulator



(a) MFA

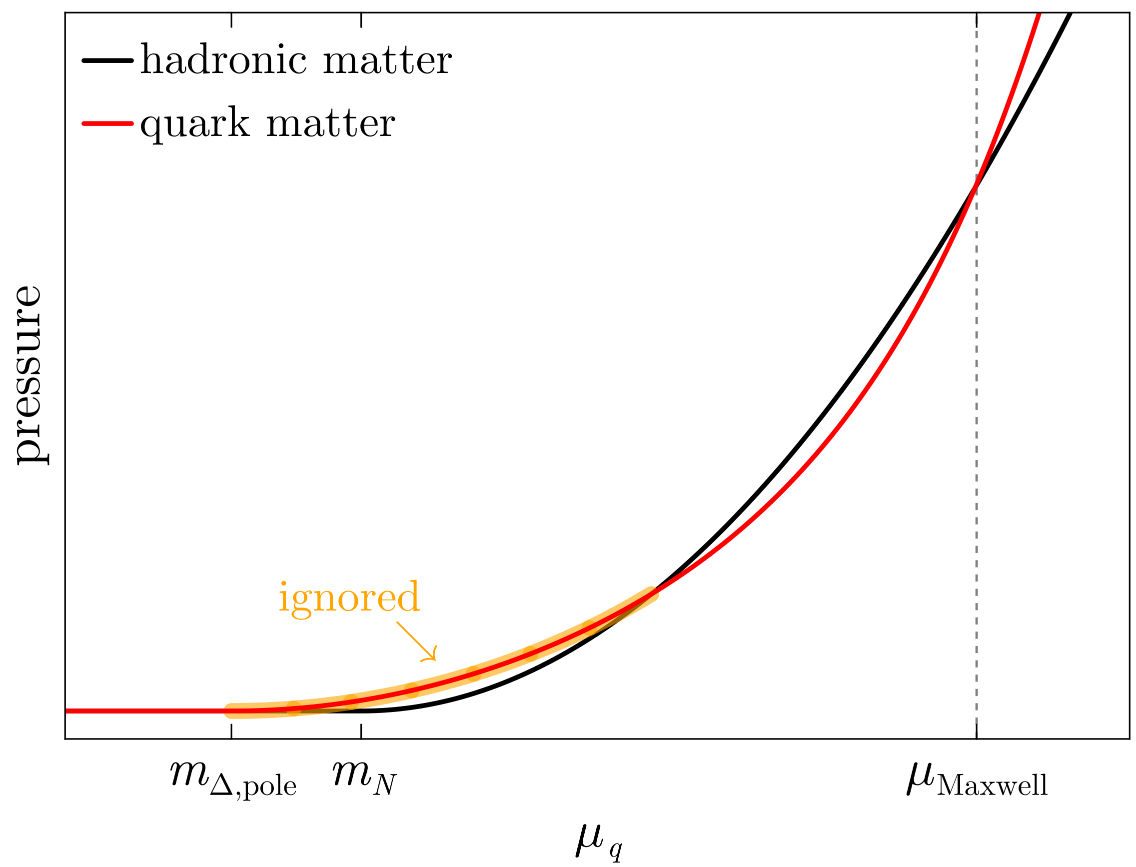
(b) FRG in LPA

EoS & mass-radius relation

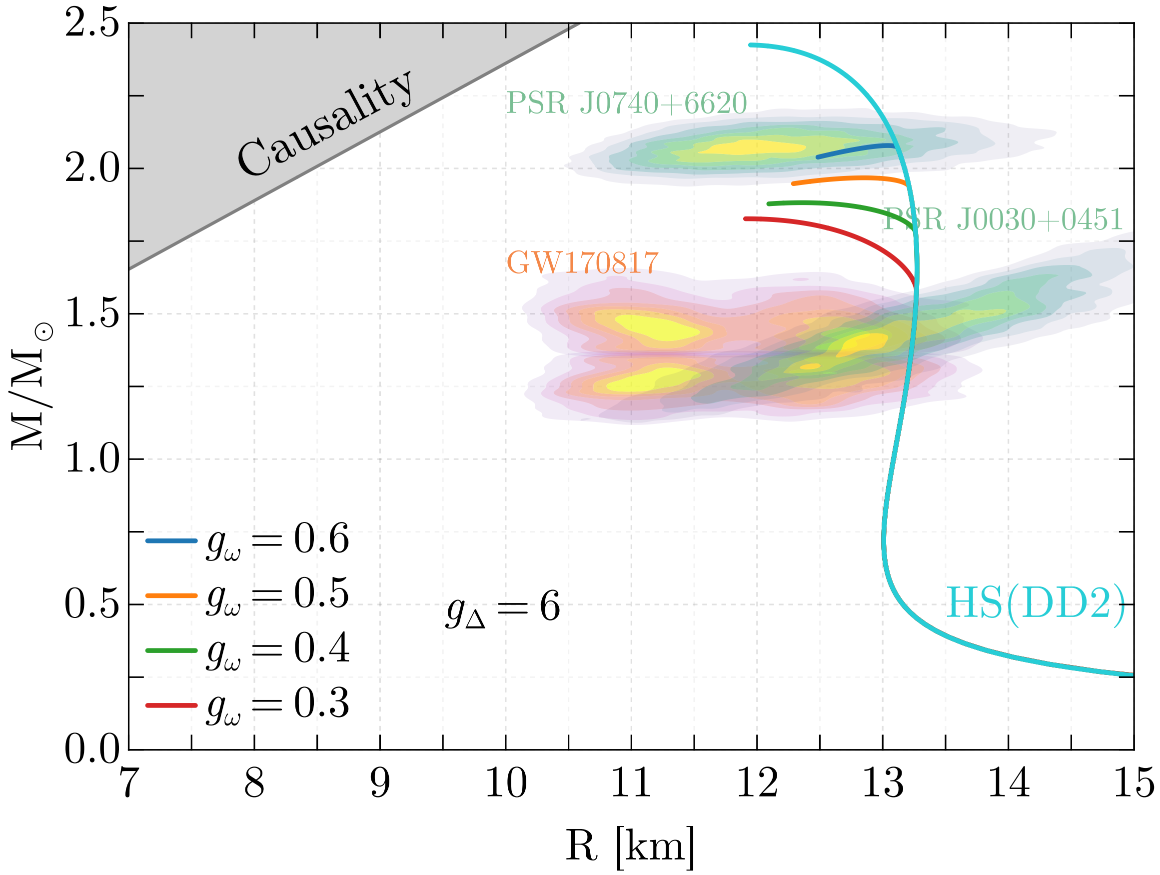
- **First Quark-meson-diquark FRG results**

[Mire, BJS to be published]

Maxwell construction: hadronic EoS (DD2)



M-R relation



- **onset quark matter EoS: diquark pole mass**

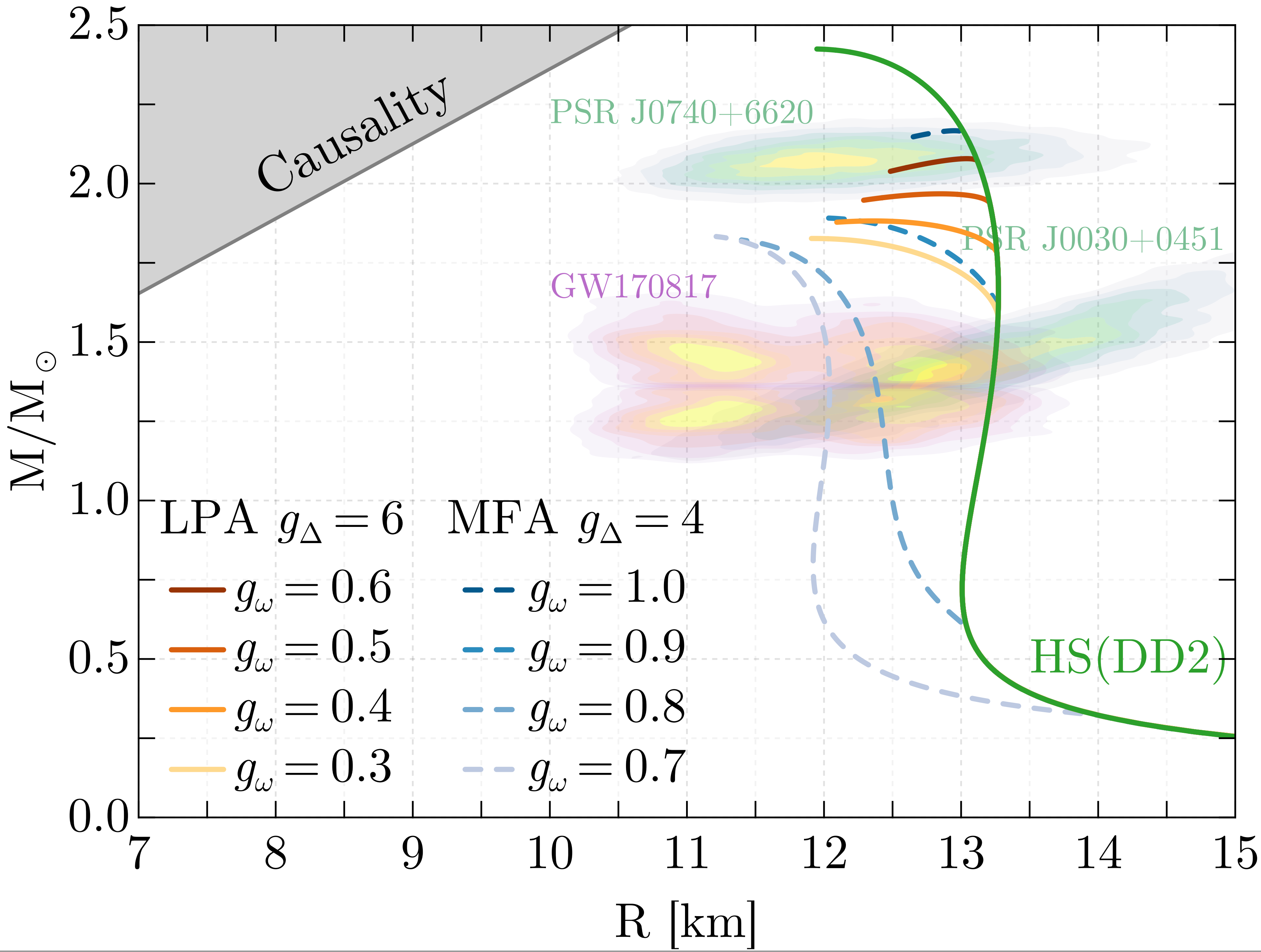
- **superconducting quark core**

already stable with present diquark parameters

mass-radius relation

- First Quark-meson-diquark FRG results vs. MFA

[Mire, BJS to be published]



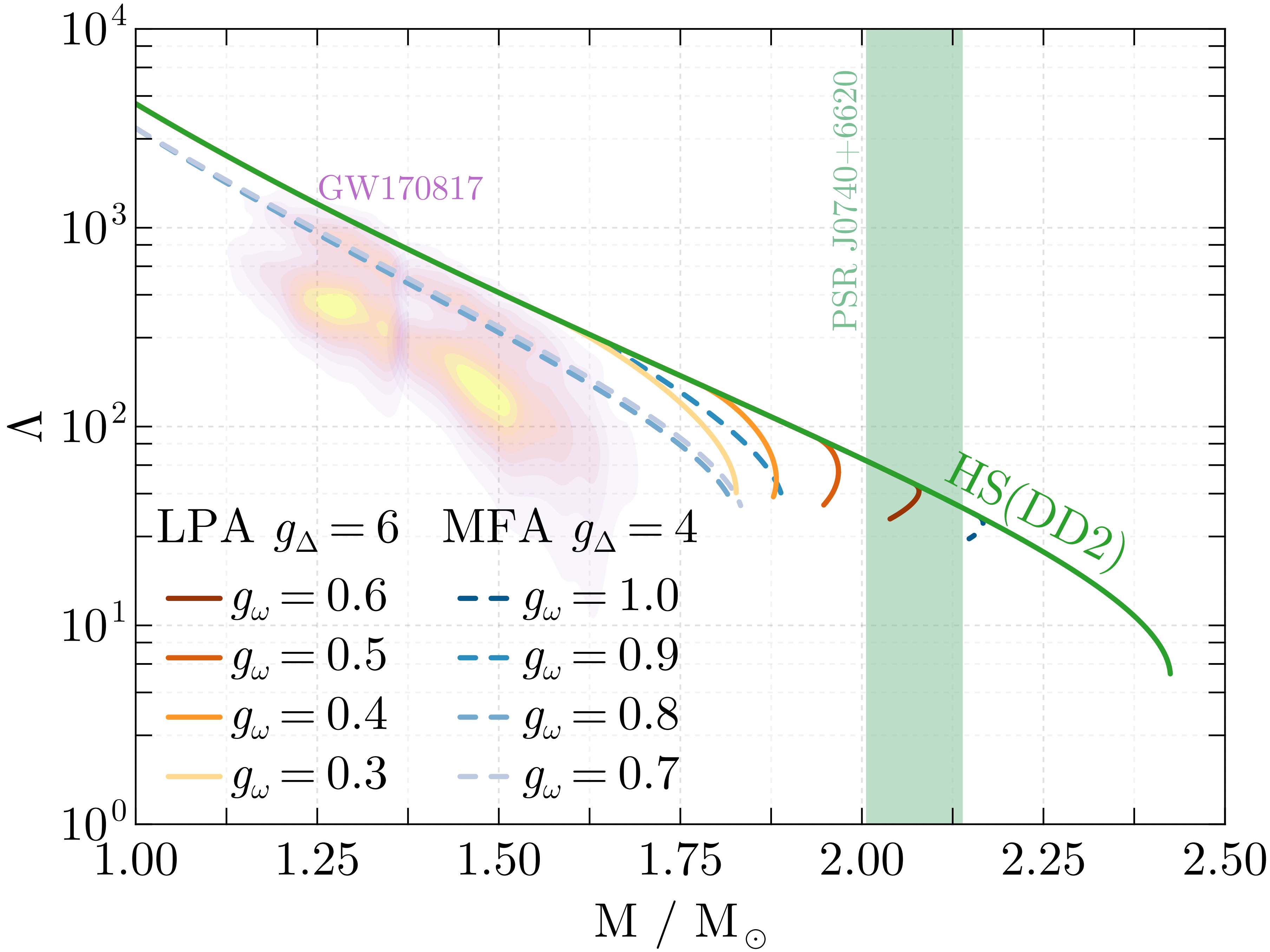
- note:

  different couplings!

tidal deformability

- First Quark-meson-diquark FRG results vs. MFA

[Mire, BJS to be published]



- note:

  different couplings!