What neutron stars say about the properties of strong interaction

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ACHT 2025, Non-perturbative methods in strongly interacting quantum many-body systems, Budapest, 05-07.05.2025

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Phys. Rev. D105 (2022) 103014, D108 (2023) 043002.

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Overview

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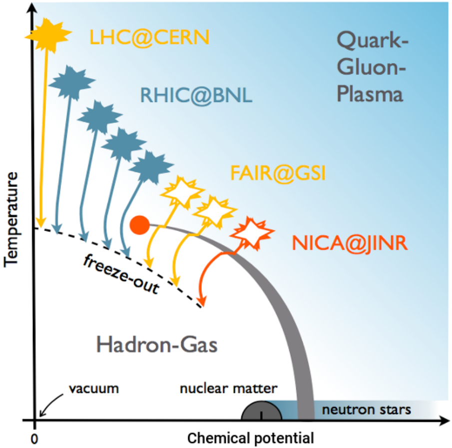
Dense strongly interacting matter



What is the phase diagram and EOS for dense strongly interacting matter?

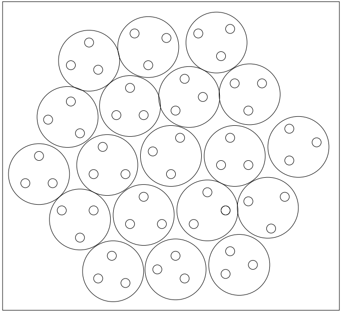
At $\mu = 0$: lattice and experiments (STAR/PHENIX and ALICE).

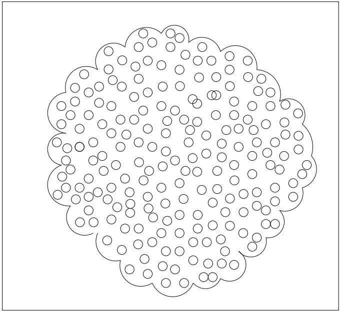
For $\mu >> 0$ no precise theory and no heavy ion experiment.

Dense matter at T=0

Are there different phases at T=0? If yes, at which densities?

heavy ion collisions: no sharp transition until 2-3 ρ_0





$V_{proton} = 2.22 fm^3$ (with r_{em}), densest packing with spheres: 74%

$\rightarrow \rho_{max} = 0.33 fm^{-3} \approx 1.8\rho_0$ by maximal packing

Model: nucleon = core + meson cloud

Reid hard core potential: $r_{hc} \approx 0.5 r_{em} \rightarrow \rho_{max} \approx 15\rho_0$ at hard core overlap

Form factors (scalar and vector em, axial vector and gluonic: μ absorption):

$r_{core} = \sqrt{< r^2 >} \approx 0.5 fm \rightarrow \rho_{max} \approx 8\rho_0$ at core overlap

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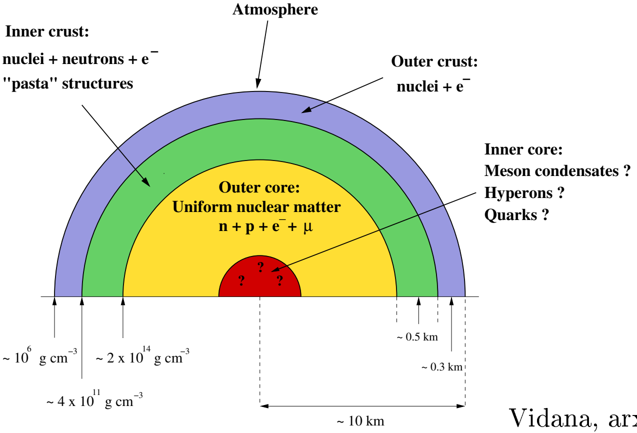
Neutron Stars a challenge and a possibility

Neutron stars are contain cold, dense matter (T≈0, $\rho > 3\rho_0$) not available in terrestial experiments (Laboratory for strong interaction)

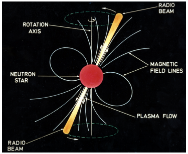
What is the structure of neutron stars (what are the constituents), hybrid stars? Superfluids?

YN, YNN interactions are important, three-body repulsion for Λ, Σ (Weise)

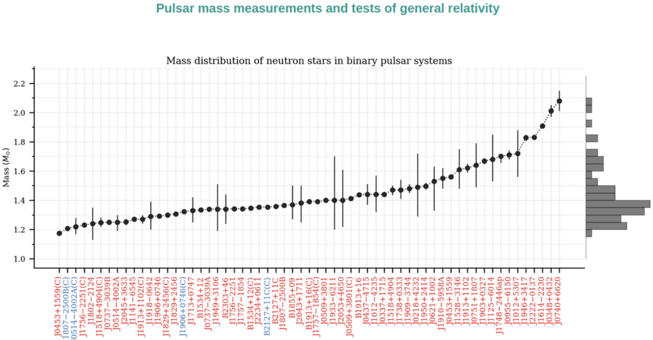


Vidana, arxiv:1805.00837

Pulsar mass distribution

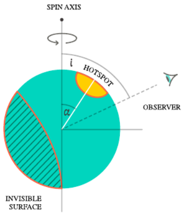


lighthouse effect, very precise frequency (1-700 Herz)



Neutron Star observations

- Discovering heavy neutron stars $M > 2M_\odot$ Demorest, et al., Nature. 467, 1081-1083 (2010). largest mass observed: $2.14 + 0.20 - 0.18 M_\odot$ (2019)
  (Shapiro-delay: pulsar+another star, at almost full covering the second member of the binary delays the radiation of the pulsar)
- Advanced gravitation wave detectors: Advanced Ligo and Virgo (soon Kagra):single neutron stars, multichannel astronomy
  neutron star collision: GW170817 (130 million lightyears)
- Modern telescopes: NICER X-ray telescope: precise (<5%) mass and radius measurements (2020) "for nearby" neutron stars,
  only 2 stars yet, but more to come
  radiation bent by strong gravity: hotspots observation: M, R



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Tolman-Oppenheimer-Volkoff (TOV) equation

Solving the Einstein's equation for spherically symmetric case and homogeneous matter $\longrightarrow$ TOV eqs.:

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{[p(r)+\varepsilon(r)]\,[M(r)+4\pi r^3 p(r)]}{r[r-2M(r)]} \qquad (1)$$

with

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2 \varepsilon(r)$$

These are integrated numerically for a specific $p(\varepsilon)$

- For a fixed ε_c central energy density Eq. (1) is integrated until $p = 0$
- Varying ε_c a series of compact stars is obtained (with given M and R)
- Once the maximal mass is reached, the stable series of compact stars ends

Observables for dense strongly interacting matter

1. Nuclear physics
   - $\rho = 0$
     nucleon-nucleon scattering, YN and YNN data from femtoscopy (ALICE), BB interaction and potential (lattice: HALQCD)
     femtoscopy data and HALQCD calculations are consistent
   - $\rho \approx \rho_0$
     masses of nuclei, isobaric analog states, hypernuclei, giant dipole and pigmy resonances, nuclear dipole polarizabilities, neutron skin thickness $\rightarrow$ normal nuclear density: ρ_0, binding energy, compressibility, symmetry energy (1st order in asymmetry expanded in density the 0th and 1st term)
2. Perturbative QCD: $\rho \approx 40\rho_0$
   N^3LO calculation, hard thermal loops: μ =2.6 GeV, p = 3.8 GeV/fm^3.
   T. Gorda, A. Kurkela, et al., Phys. Rev. Lett. 127 (2021) 162003, arXiv:2103.05658.
3. Heavy ion collisions: $\rho : 1 - 8\rho_0$
   not very conclusive; there are many competing effects, like momentum dependent interaction, nonequilibrium, nonzero temperature
4. Neutron stars: $\rho : 1 - 8\rho_0$
   M, R, Λ. Quite strong constraints even with not yet very precise data

EOS

1. $\rho \leq 2-4\rho_0$ ordinary nuclear potentials, CEFT, ...
2. $2-4\rho_0 \leq \rho \leq 6-8\rho_0$ quark matter model
3. $6-8\rho_0 \leq \rho$ extrapolation to the pQCD point

hadronic matter - soft: SFHo
(Steiner, A. W., Hempel, M., Fischer, T. Astrophys. J. 774 (2013) 17) and Hempel, M., Schaffner-Bielich, J. Nucl. Phys. A837 (2010) 210)
relativistic mean-field model (nucleons, σ, ω, ρ with quartic copulings), with K=245 MeV, L=47.1 MeV, m^*/m_n=0.76.

hadronic matter - stiff: DD2
(S. Typel, et al., Phys. Rev. C81 (2010) 015803
relativistic mean-field + light clusters, K = 243 MeV, L=58 MeV m^*/m_n=0.63

Quark matter: Quark-meson model - chiral $U(3)xU(3) \rightarrow SU(2)xU(1)$ model
degrees of freedom: 4 meson nonets, constituent quarks, Polyakov loops
condensates: 2 scalar (N,S), Polyakov loops (T>0), vector mesons ($\mu > 0$)
P. Kovács, Zs. Szép, Gy. Wolf, Phys. Rev. D93 (2016) 114014

Concatenation

It seems that a strong first order phase transition is ruled out by astrophysical constraints: J.-E. Christian and J. Schaffner-Bielich, Phys. Rev. D 103, 063042 (2021),
The allowed $p(\varepsilon)$ functions are in a rather narrow band, there can be no big jump

Hadron-quark crossover with polynomial interpolation ($\rho = \rho_B$):

$$\begin{aligned}
\varepsilon(\rho_B) &= \varepsilon_{hadronic}(\rho_B) \qquad \rho_B < \rho_{BL}, \\
\varepsilon(\rho_B) &= \sum_{k=0}^{5} C_k \rho_B^k \qquad \rho_{BL} \leq \rho_B \leq \rho_{BU} \\
\varepsilon(\rho_B) &= \varepsilon_{qm}(\rho_B) \qquad \rho_{BU} < \rho_B.
\end{aligned}$$

C_k is determined by the requirement that the energy density, ε and its first two derivatives with respect to ρ_B, pressure and sound velocity is continuous at the boundaries.
2 parameters: $\Gamma = 0.5 * (\rho_{BU} - \rho_{BL})$ and $\overline{\rho_B} = 0.5 * (\rho_{BL} + \rho_{BU})$

Features of our approach

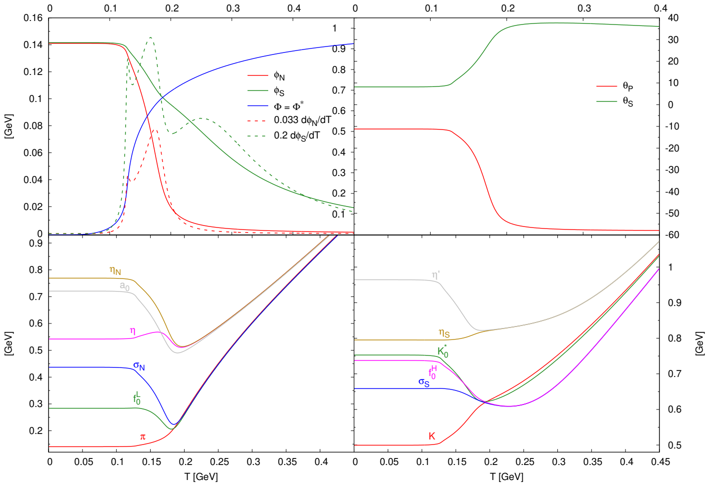
Effective field theory: the same symmetry pattern as in QCD

- D.O.F's: scalar, pseudoscalar, vector, axial vector nonets,
- Polyakov loop variables, $\Phi, \bar{\Phi}$ with U^{glue}
- u,d,s constituent quarks, $(m_u = m_d)$
- mesonic fluctuations included in the grand canonical potential:

$$\Omega(T, \mu_q) = -\frac{1}{\beta V} ln(Z)$$

- Fermion vacuum and thermal fluctuations
- Five order parameters $(\phi_N, \phi_S, \Phi, \bar{\Phi}, v_0) \rightarrow$ five T/μ-dependent equations

With low mass scalars, $m_{f_0^L} = 300$ MeV

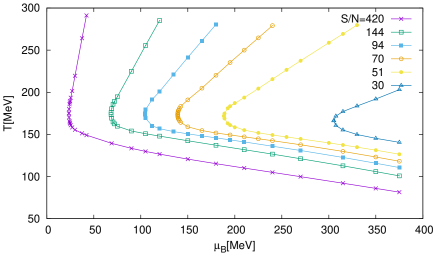


chiral symmetry is restored at high T as the chiral partners (π, f_0^L), (η, a_0) and $(K, K_0^\star)$, (η', f_0^H) become degenerate

$U(1)_A$ symmetry is not restored, as the axial partners (π, a_0) and (η, f_0^L) do not become degenerate

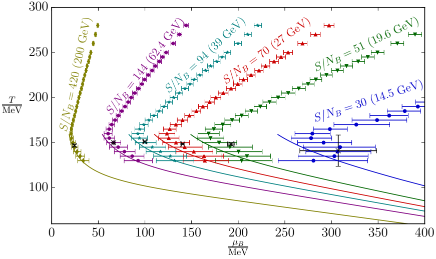
Isentropic trajectories in the $T - \mu_{\mathrm{B}}$ plane

our model, where $\mu_B^{\mathrm{CEP}} > 850\mathrm{MeV}$



lattice (analytic continuation)

Günther *et al.*, arXiv:1607.02493



same qualitative behavior of the isentropic trajectories for $\mu_B \leq 400$ MeV

$\Longrightarrow$ indication that in the lattice result there is no CEP in this region of μ_B

Bayesian inference

Unsetted parameters: $m_\sigma, g_v,\quad \overline{\rho_B} \equiv 0.5(\rho_{BL} + \rho_{BU}),\quad \Gamma \equiv 0.5(\rho_{BU} - \rho_{BL})$

290 MeV $\leq m_\sigma \leq$ 700 MeV
$0 \leq g_v \leq 10$
$2\rho_0 \leq \overline{\rho_B} \leq 5\rho_0$
$\rho_0 \leq \Gamma \leq 4\rho_0$ with the constraint: $\rho_{BL} = \overline{\rho_B} - \Gamma > \rho_0$

We created ∼ 18000 EOSs to be used in the Bayesian analysis
We believe that our model is quite generic and with running the parameters we cover the possible EOSs.

Bayes theorem:

θ is a parameter set, $p(\theta)$ is the prior probability for θ, $p(data|\theta)$ is the probability that for given θ, the data is measured. Then

$$p(\theta|data) = \frac{p(data|\theta)p(\theta)}{p(data)}$$

$p(data)$ is a normalization constant. We assume $p(\theta)$ is uniform in the allowed hypersurface. For independent observations:

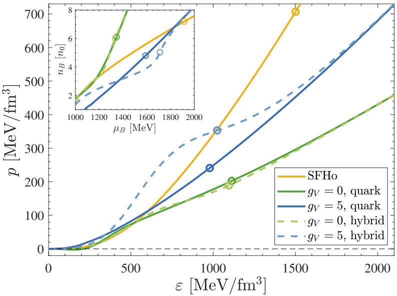
$$p(data|\theta) = p(M_{max}|\theta)p(NICER|\theta)p(\overline{\Lambda}|\theta)$$

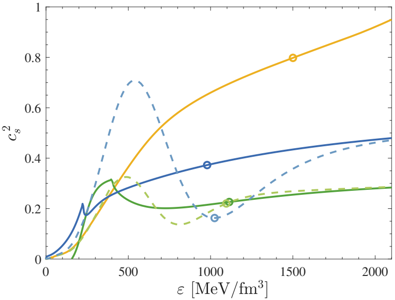
Phys. Rev. D105 (2022) 103014, Phys. Rev. D108 (2023) 043002.

Data

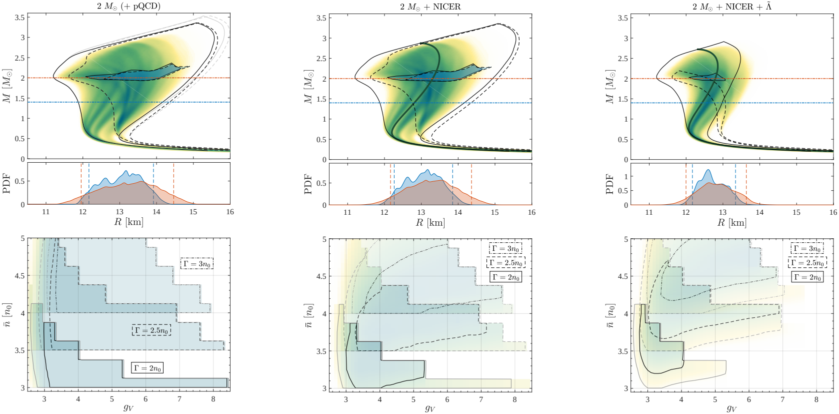
- $2M_\odot$: PSR J0348+0432 with a mass $2.01 \pm 0.04 M_\odot$,
  PSR J1614-2230 with a mass $1.908 \pm 0.016 M_\odot$
- perturbative QCD EOS should converge to it keeping $c_s < 1$,
  $\mu_{QCD} = 2.6GeV, n_{QCD} = 6.471/fm^3, p_{QCD} = 3823MeV/fm^3$
- NICER: (M,R) values for PSR J0030+0451, PSR J0740+6620 (Miller)
- tidal deformability GW170817: : $70 < \Lambda(1.4M_\odot) < 720$ Abbot (2019)
- hypermassive NS GW170817: no prompt black hole formation, (Rezzola):
  $2.01 \geq M_{TOV}/M_\odot \geq 2.16$
- massgap neutron star: $2.59 \pm 0.09 M_\odot$
- Hess J1731-347 neutron star: mass= $0.77 \pm 0.19 M_\odot$, $R = 10.4 \pm 0.8 km$

EOS and sound velocity



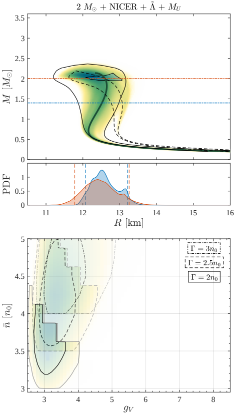


Bayesian analysis

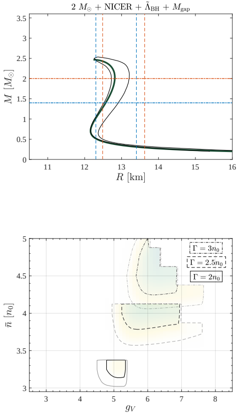


prior (M_{max} + p_{QCD}) | prior + NICER | prior + NICER + GW

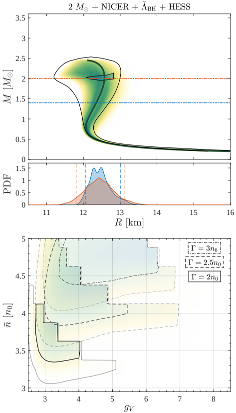
Bayesian analysis



prior + NICER + GW + HMNS

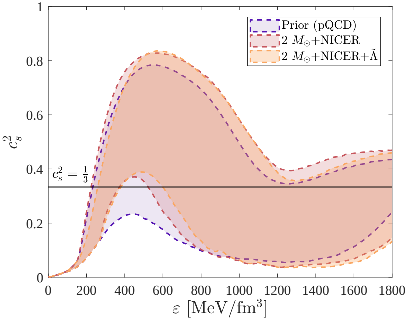


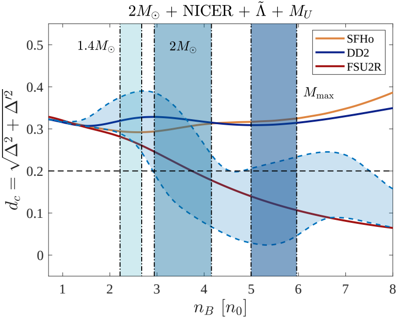
prior + NICER + GW + Mgap



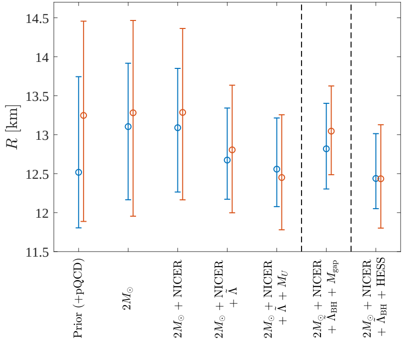
prior + NICER + GW + HESS

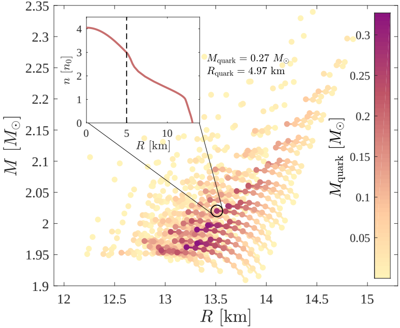
Sound velocity and Bayesian analysis





Bayesian analysis





Summary and Conclusions

- Our model can reproduce the lattice calculations at $\mu = 0$
- With our model we can fulfill the present astronomical constraints
- The central density do not go above $6\rho_0$.
- The radius of the neutron stars are 12.8 ± 0.8 km.
- hadronic and quark phase ought to be handled with the same model to drop ad-hoc parameters

Meson fields - pseudoscalar and scalar meson nonets

$$\Phi_{PS} = \sum_{i=0}^{8} \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_S = \sum_{i=0}^{8} \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} (\sim \bar{q}_i q_j)$$

Particle content:
Pseudoscalars: $\pi(138), K(495), \eta(548), \eta'(958)$
Scalars: $a_0(980$ or $1450), K_0^\star(800$ or $1430)$,
(σ_N, σ_S) : 2 of $f_0(500, 980, 1370, 1500, 1710)$

Included fields - vector meson nonets

$$V^\mu = \sum_{i=0}^{8} \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star+} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star -} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu$$

$$A^\mu = \sum_{i=0}^{8} b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

Particle content:

Vector mesons: $\rho(770), K^\star(894), \omega_N = \omega(782), \omega_S = \phi(1020)$
Axial vectors: $a_1(1230), K_1(1270), f_{1N}(1280), f_{1S}(1426)$

Spontaneous symmetry breaking

Interaction is approximately chiral symmetric, spectra not, so SSB:

$$\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S} \qquad \phi_{N/S} \equiv <\sigma_{N/S}>$$

For tree level masses we have to select all terms quadratic in the new fields. Some of the terms include mixings arising from terms like $\mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)]$:

$$\begin{aligned}
\pi_N - a^\mu_{1N} &: \quad -g_1\phi_N a^\mu_{1N}\partial_\mu\pi_N, \\
\pi - a^\mu_1 &: \quad -g_1\phi_N(a_1^{\mu+}\partial_\mu\pi^- + a_1^{\mu 0}\partial_\mu\pi^0) + \mathrm{h.c.}, \\
\pi_S - a^\mu_{1S} &: \quad -\sqrt{2}g_1\phi_S a^\mu_{1S}\partial_\mu\pi_S, \\
K_S - K^\star_\mu &: \quad \frac{ig_1}{2}(\sqrt{2}\phi_S - \phi_N)(\bar{K}^{\star 0}_\mu\partial^\mu K^0_S + K^{\star -}_\mu\partial^\mu K^+_S) + \mathrm{h.c.}, \\
K - K^\mu_1 &: \quad -\frac{g_1}{2}(\phi_N + \sqrt{2}\phi_S)(K_1^{\mu 0}\partial_\mu\bar{K}^0 + K_1^{\mu+}\partial_\mu K^-) + \mathrm{h.c.}.
\end{aligned}$$

Diagonalization→ Wave function renormalization

Thermodynamical Observables

We include mesonic thermal contribution to p for (π, K, f_0^l)

$$\Delta p(T) = -nT \int \frac{d^3q}{(2\pi)^3} ln(1 - e^{-\beta E(q)}), \quad E(q) = \sqrt{q^2 + m^2}$$

- pressure: $p(T, \mu_q) = \Omega_H(T = 0, \mu_q) - \Omega_H(T, \mu_q)$
- entropy density: $s = \frac{\partial p}{\partial T}$
- quark number density: $\rho_q = \frac{\partial p}{\partial \mu_q}$
- energy density: $\epsilon = -p + Ts + \mu_q \rho_q$
- scaled interaction measure: $\frac{\Delta}{T^4} = \frac{\epsilon - 3p}{T^4}$
- speed of sound at $\mu_q = 0$: $c_s^2 = \frac{\partial p}{\partial \epsilon}$

Inclusion of the vector meson- quark interaction

$$\mathcal{L}_{\mathrm{Vq}} = -g_V\sqrt{6}\bar{\Psi}\gamma_\mu V_0^\mu \Psi$$
$$V_0^\mu = \frac{1}{\sqrt{6}}\mathrm{diag}(v_0 + \frac{v_8}{\sqrt{2}}, v_0 + \frac{v_8}{\sqrt{2}}, v_0 - \sqrt{2}v_8) \qquad (2)$$

vector fields: like Walecka model, nonzero expectation values are built up at nonzero chemical potential. For simplicity

$$< v_0^\mu >= v_0\delta^{0\mu}, \quad < v_8^\mu >= 0$$

Modification of the grand canonical potential:

$$\Omega(T = 0, \mu_q, g_v) = \Omega(T = 0, \tilde{\mu}_q, gv = 0) - \frac{1}{2}m_v^2 v_0^2,$$

with $\tilde{\mu}_Q = \mu_q - g_v v_0$

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Lagrangian (2/1)

$$\mathcal{L}_{\mathrm{Tot}} = \mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2$$
$$-\frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \mathrm{Tr}\left[\left(\frac{m_1^2}{2}+\Delta\right)(L_\mu^2+R_\mu^2)\right] + \mathrm{Tr}[H(\Phi+\Phi^\dagger)]$$
$$+c_1(\det\Phi + \det\Phi^\dagger) + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\})$$
$$+\frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2\mathrm{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger).$$
$$+\bar{\Psi}i\partial\!\!\!/\Psi - g_F\bar{\Psi}(\Phi_S + i\gamma_5\Phi_{PS})\Psi + g_V\bar{\Psi}\gamma^\mu\left(V_\mu + \frac{g_A}{g_V}\gamma_5 A_\mu\right)\Psi$$

+Polyakov loops

P. Kovács, Zs. Szép, Gy. Wolf, Phys. Rev. D93 (2016) 114014

Lagrangian (2/2)

where

$$D^\mu\Phi = \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ieA_e^\mu[T_3,\Phi]$$

$$\Phi = \sum_{i=0}^{8}(\sigma_i + i\pi_i)T_i, \quad H = \sum_{i=0}^{8} h_i T_i \qquad T_i : U(3) \text{ generators}$$

$$R^\mu = \sum_{i=0}^{8}(\rho_i^\mu - b_i^\mu)T_i, \quad L^\mu = \sum_{i=0}^{8}(\rho_i^\mu + b_i^\mu)T_i$$

$$L^{\mu\nu} = \partial^\mu L^\nu - ieA_e^\mu[T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu[T_3, L^\mu]\}$$

$$R^{\mu\nu} = \partial^\mu R^\nu - ieA_e^\mu[T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu[T_3, R^\mu]\}$$

$$\bar\Psi = (\bar u, \bar d, \bar s)$$

Interaction is approximately chiral symmetric, spectra not, so SSB:

$$\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S} \qquad \phi_{N/S} \equiv <\sigma_{N/S}>$$

Determination of the parameters of the Lagrangian

16 unknown parameters ($m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S$, $\Phi_N, \Phi_S, g_F, g_V, g_A$) $\longrightarrow$ Determined by the min. of χ^2:

$$\chi^2(x_1, \dots, x_N) = \sum_{i=1}^{M} \left[\frac{Q_i(x_1, \dots, x_N) - Q_i^{\mathrm{exp}}}{\delta Q_i} \right]^2 ,$$

where $(x_1, \dots, x_N) = (m_0, \lambda_1, \lambda_2, \dots)$, $Q_i(x_1, \dots, x_N)$ calculated from the model, while Q_i^{exp} taken from the PDG

multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\rightarrow$ 2 physical quantities: f_π, f_K
- Tree-level masses $\rightarrow$ 15 physical quantities:
  $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\rightarrow$ 12 physical quantities:
  $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi}$,
  $\Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$
- $T_c = 155$ MeV from lattice

Polyakov loops in Polyakov gauge

Polyakov loop variables: $\Phi(\vec{x}) = \frac{\mathrm{Tr}_c L(\vec{x})}{N_c}$ and $\bar{\Phi}(\vec{x}) = \frac{\mathrm{Tr}_c \bar{L}(\vec{x})}{N_c}$ with

$$L(x) = \mathcal{P} \exp\left[i \int_0^\beta d\tau A_4(\vec{x}, \tau) \right]$$

$\longrightarrow$ signals center symmetry ($\mathbb{Z}_3$) breaking at the deconfinement

low T: confined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle = 0$

high T: deconfined phase, $\langle \Phi(\vec{x}) \rangle, \langle \bar{\Phi}(\vec{x}) \rangle \neq 0$

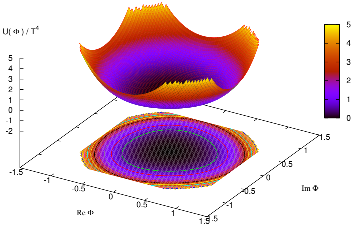
Polyakov gauge: the temporal component of the gauge field is time independent and can be gauge rotated to a diagonal form in the color space

Effects of the gauge fields:

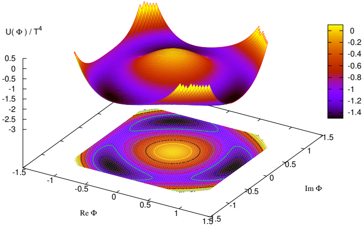
- In this gauge the effect of the gauge field on the quarks acts like an imaginary chemical potential
  $\longrightarrow$ modified quark distribution function.
- Polyakov potential: $\mathcal{U}\left(\Phi, \bar{\Phi}\right)$ models the free energy of a pure gauge theory, parameters are fitted to the pure gauge lattice data

Polyakov loop potential

"Color confinement"
$\langle\Phi\rangle = 0 \longrightarrow$ no breaking of $\mathbb{Z}_3$
one minimum



"Color deconfinement"
$\langle\Phi\rangle \neq 0 \longrightarrow$ spontaneous breaking of $\mathbb{Z}_3$
minima at $0, 2\pi/3, -2\pi/3$
one of them spontaneously selected



from H. Hansen et al., PRD75, 065004 (2007)

Effects of Polyakov loops on FD statistics

Inclusion of the Polyakov loop modifies the Fermi-Dirac distribution function

$$f(E_p - \mu_q) \longrightarrow f_\Phi^+(E_p) = \frac{\left(\bar{\Phi} + 2\Phi e^{-\beta(E_p-\mu_q)}\right) e^{-\beta(E_p-\mu_q)} + e^{-3\beta(E_p-\mu_q)}}{1 + 3\left(\bar{\Phi} + \Phi e^{-\beta(E_p-\mu_q)}\right) e^{-\beta(E_p-\mu_q)} + e^{-3\beta(E_p-\mu_q)}}$$

$$f(E_p + \mu_q) \longrightarrow f_\Phi^-(E_p) = \frac{\left(\Phi + 2\bar{\Phi} e^{-\beta(E_p+\mu_q)}\right) e^{-\beta(E_p+\mu_q)} + e^{-3\beta(E_p+\mu_q)}}{1 + 3\left(\Phi + \bar{\Phi} e^{-\beta(E_p+\mu_q)}\right) e^{-\beta(E_p+\mu_q)} + e^{-3\beta(E_p+\mu_q)}}$$

$\Phi, \bar{\Phi} \to 0 \Longrightarrow f_\Phi^\pm(E_p) \to f(3(E_p \pm \mu_q))$ $\Phi, \bar{\Phi} \to 1 \Longrightarrow f_\Phi^\pm(E_p) \to f(E_p \pm \mu_q)$

three-particle state appears: mimics confinement of quarks within baryons

at $T = 0$ there is no difference between models with and without Polyakov loop

Features of our approach

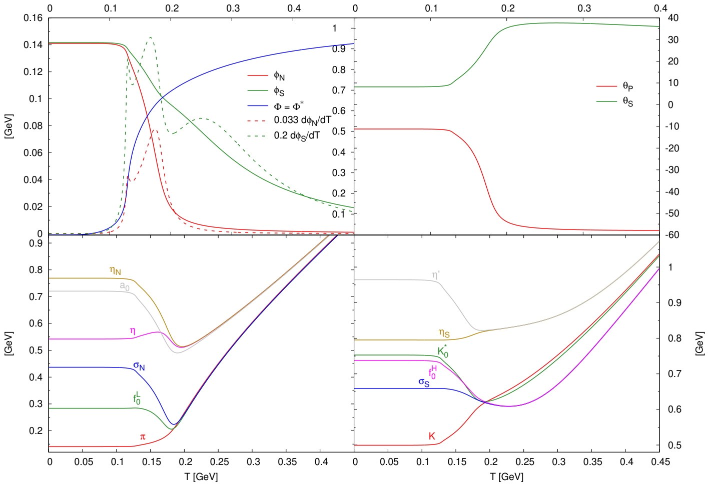
Effective field theory: the same symmetry pattern as in QCD

- D.O.F's: scalar, pseudoscalar, vector, axial vector nonets,
- Polyakov loop variables, $\Phi, \bar{\Phi}$ with U^{glue}
- u,d,s constituent quarks, $(m_u = m_d)$
- mesonic fluctuations included in the grand canonical potential:

$$\Omega(T, \mu_q) = -\frac{1}{\beta V} ln(Z)$$

- Fermion vacuum and thermal fluctuations
- Five order parameters $(\phi_N, \phi_S, \Phi, \bar{\Phi}, v_0) \rightarrow$ five T/μ-dependent equations

With low mass scalars, $m_{f_0^L} = 300$ MeV



chiral symmetry is restored at high T as the chiral partners (π, f_0^L), (η, a_0) and $(K, K_0^\star)$, (η', f_0^H) become degenerate

$U(1)_A$ symmetry is not restored, as the axial partners (π, a_0) and (η, f_0^L) do not become degenerate

Thank you for your attention!