The effect of extra dimensions on compact star observables

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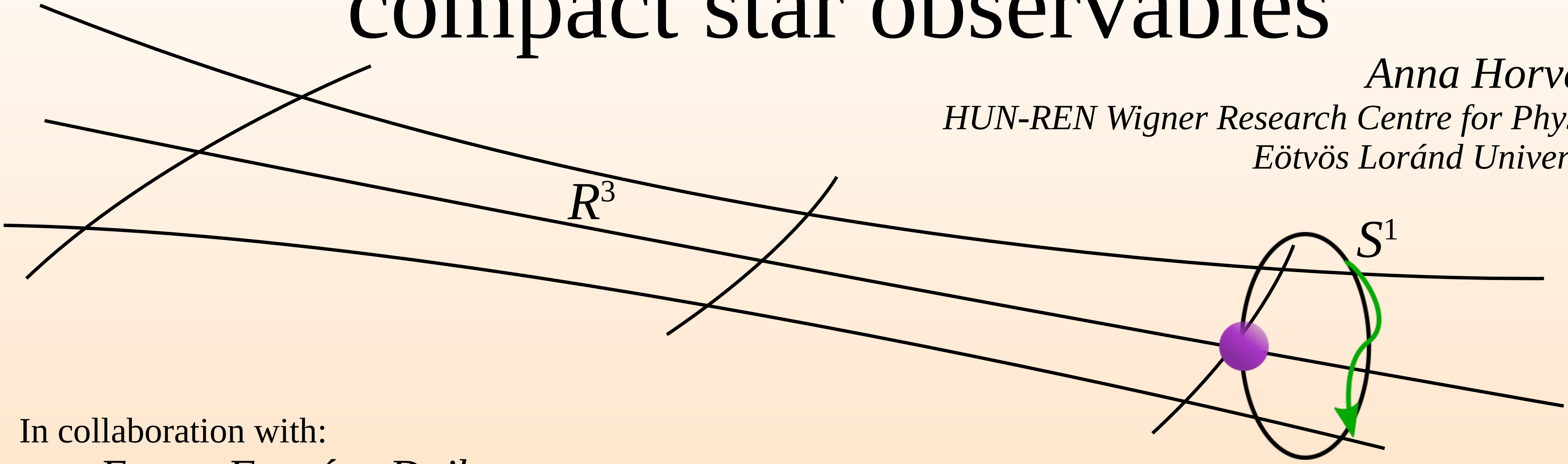

In collaboration with:
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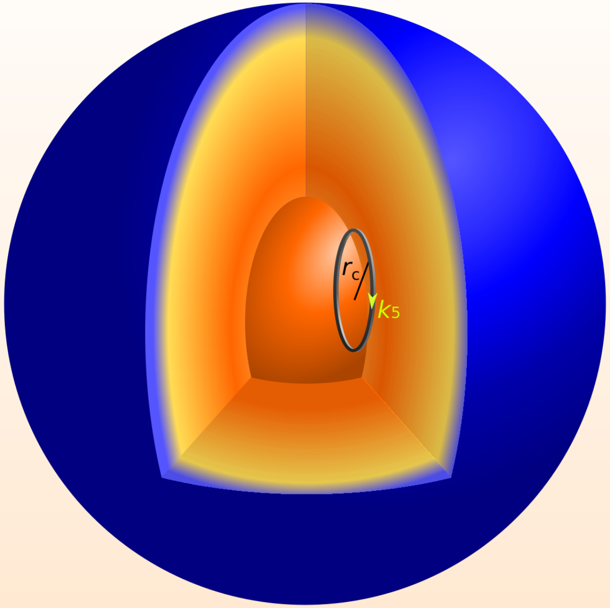
ELTE DKÖP
WSCLAB
KMP-2023/101, KMP-2024/31

Outline of the talk



- Motivation
- Kaluza–Klein theory
- Model
- Microscopic properties and thermodynamics
- Modeling neutron stars
- Results

[1] A. Horváth, E. Forgács-Dajka, G.G. Barnaföldi: "Application of Kaluza-Klein Theory in Modeling Compact Stars: Exploring Extra Dimensions", MNRAS (2024)
[2] A. Horváth, E. Forgács-Dajka, G.G. Barnaföldi: "The effect of multiple extra dimensions on the maximal mass of compact stars in Kaluza-Klein space-time", Accepted by IJMPA (2025)
[3] A. Horváth, E. Forgács-Dajka, G.G. Barnaföldi: “Speed of sound in Kaluza-Klein Fermi gas”, Sent to APP-B (2025)

Motivation

Questions of **fundamental physics**

- GR + QM = ?
- Hierarchy problem
- Dark matter, dark energy?

↓

Many theories with similar phenomenology

→ use a **simple model**

Motivation

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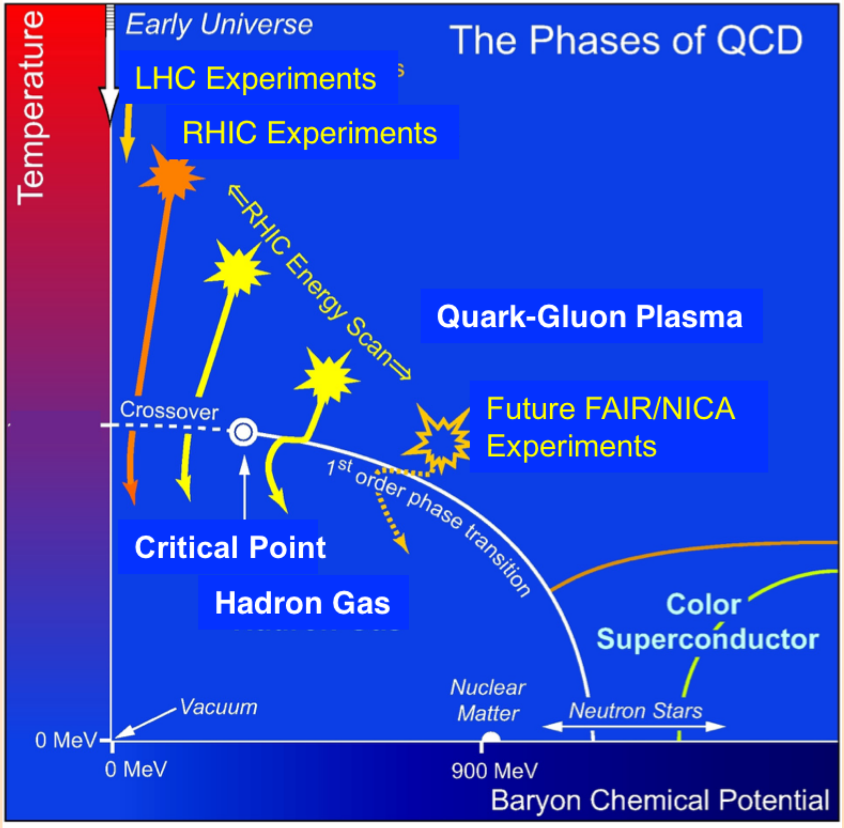
Many theories with similar phenomenology

→ use a **simple model**

Dense matter **equation of state**

- not well described by perturbative QCD or nuclear physics
- cannot be produced in laboratories

→ look for the **stars**



Tapan K. Nayak 2020 J. Phys.: Conf. Ser. 1602 012003

Kaluza–Klein theory

- Gravity + electromagnetism → 5D spacetime
- The 5th dimension is **microscopic**, curled up into a **circle**

  → QM interpretation


Theodor Kaluza (1885-1954)


Oskar Klein (1894-1977)

Base of: scalar-tensor (Brans-Dicke) and string theories

Kaluza–Klein theory

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Theodor Kaluza (1885-1954)

Metric tensor

10 + 4 + 1 independent components

4D metric of gravity

EM vector potential

$$g_{AB} = \begin{bmatrix} g_{\alpha\beta} + \kappa^2\Phi^2 A_\alpha A_\beta & \kappa\Phi^2 A_\alpha \\ \kappa\Phi^2 A_\beta & \Phi^2 \end{bmatrix}$$

5D metric

Scalar field

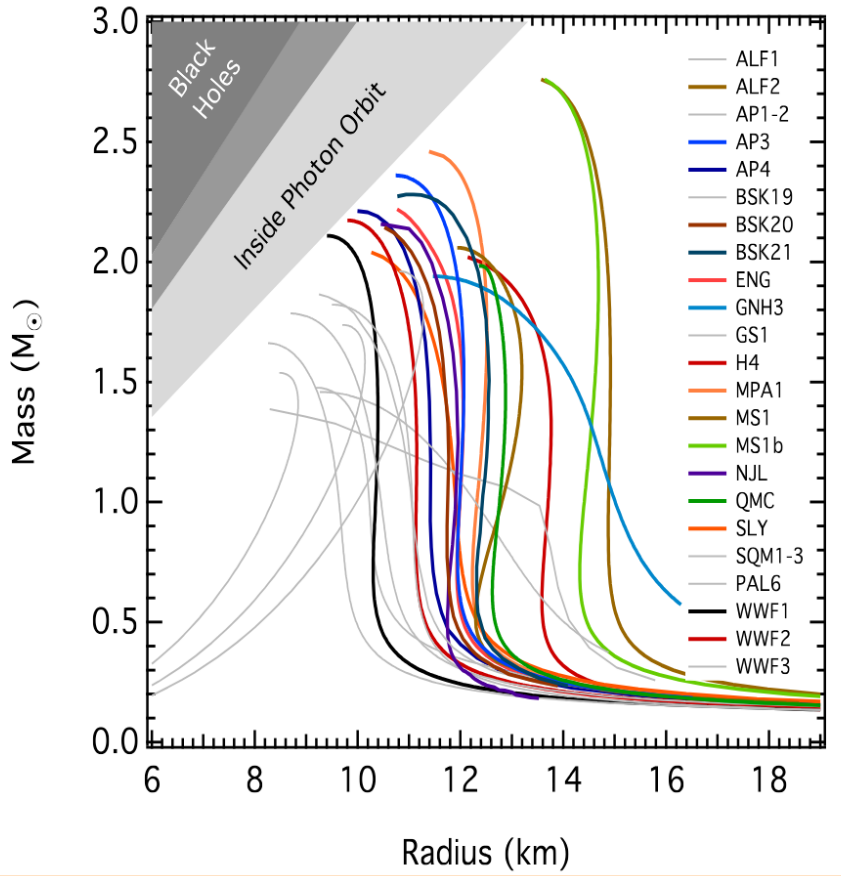
Oskar Klein (1894-1977)

Base of: scalar-tensor (Brans-Dicke) and string theories

Model

Useful quantities for testing the EoS:

- **Mass**
- Radius
- Tidal deformability (not considered)



F. Özel and P. Freire, "Masses, Radii, and the Equation of State of Neutron Stars," Ann. Rev. Astron. Astrophys. **54** (2016), 401-440 doi:10.1146/annurev-astro-081915-023322 [arXiv:1603.02698 [astro-ph.HE]].

Model

Useful quantities for testing the EoS:

- **Mass**
- Radius
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Assumptions:

- **Static, spherically symmetric** spacetime
- Neglect electromagnetism
- Isotropic, relativistic ideal fluid
- g_{55}:
    - **set to 1**
    - Allowed to vary (scalar field)

$$g_{\mu\nu} = \begin{bmatrix} g_{00} & \cancel{g_{01}} & 0 & 0 & \cancel{g_{05}} \\ \cancel{g_{01}} & g_{11} & 0 & 0 & \cancel{g_{15}} \\ 0 & 0 & g_{22} & 0 & 0 \\ 0 & 0 & 0 & g_{22}\sin^2\vartheta & 0 \\ \cancel{g_{05}} & \cancel{g_{15}} & 0 & 0 & \boxed{g_{55}} \end{bmatrix}$$

Microscopic to macroscopic

Microscopic properties of spacetime and nuclear matter are connected to the mass-radius relation via

- the **Tolman–Oppenheimer–Volkoff** (TOV) equation

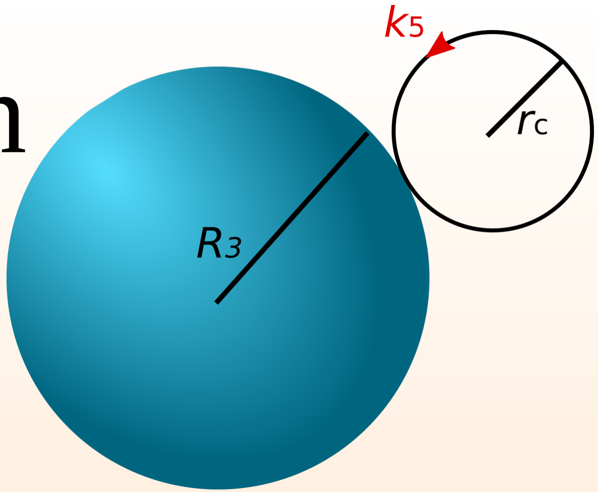
$$\frac{\mathrm{d}p(r)}{\mathrm{d}r} = -\frac{GM(r)\varepsilon(r)}{r^2} \times \left[1+\frac{p(r)}{\varepsilon(r)}\right]\left[1+\frac{4\pi r^3 p(r)}{M(r)}\right]\left[1-\frac{GM(r)}{r}\right]^{-1}$$

$$M(r) = \int_0^r \mathrm{d}r' 4\pi r'^2 \varepsilon(r')$$

  Boundary conditions: central energy density ε_c, surface pressure $p(R)$

- and the **equation of state** (EoS)
  - Connects pressure (p) and energy density (ε)
  - Encodes microscopic properties of the theory

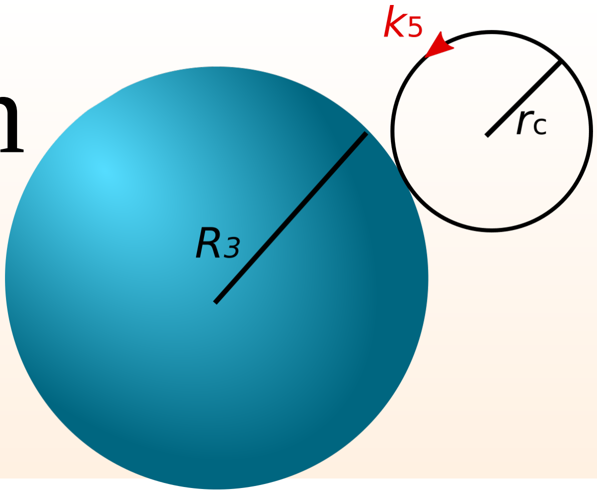
Modified dispersion relation



Particles **moving in** the **extra dimension** possess a modified **effective mass** from a 3-dimensional point of view.

$$E = \sqrt{\mathbf{k}^2 + k_5^2 + m^2} = \sqrt{\mathbf{k}^2 + \left(\frac{N_{\text{exc}}}{r_c}\right)^2 + m^2}$$

Modified dispersion relation



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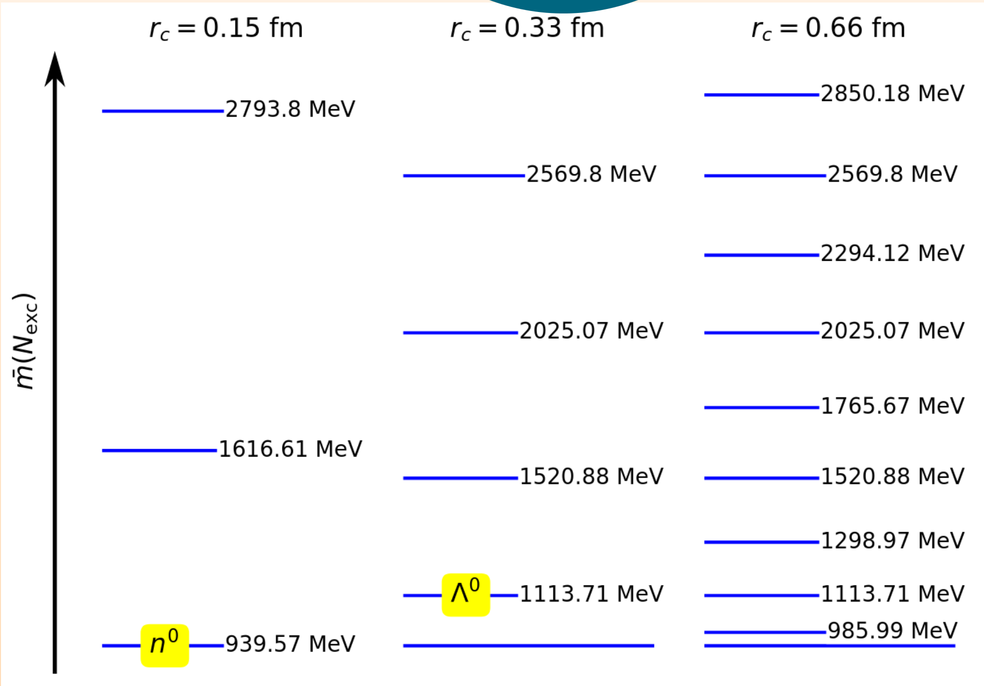
$$\bar{m}^2(N_{\mathrm{exc}}) = m^2 + k_5^2 \qquad k_5 = \frac{N_{\mathrm{exc}}}{r_{\mathrm{c}}}$$



Here the ground state of the Kaluza–
–Klein ladder is the **neutron**.

Equation of state

Thermodynamic potential of a **Fermi gas**:

$$\Omega = -V_{(d)} \sum_{i=0}^{N_{\rm exc}} \frac{g_i}{\beta} \int \frac{\mathrm{d}^d \mathbf{k}}{(2\pi)^d} \times \\ \times \left[\ln\left(1 + \mathrm{e}^{-\beta(E_i-\mu)}\right) + \ln\left(1 + \mathrm{e}^{-\beta(E_i+\mu)}\right)\right]$$

Zero temperature approximation:

$$T \ln\left(1 + \mathrm{e}^{-(E-\mu)/T}\right)\Big|_{T=0} = \begin{cases} \mu - E, & \text{if } E < \mu \\ 0, & \text{if } E \geq \mu \end{cases}$$

Repulsive **potential**:

$$U(n) = \xi n$$

J. Zimanyi, B. Lukacs, P. Levai, J.P. Bondorf: „An Interpretable Family ofEquation of State for Dense Hadronic Matter", Nucl.Phys. A484 (1988) 647

State variables:

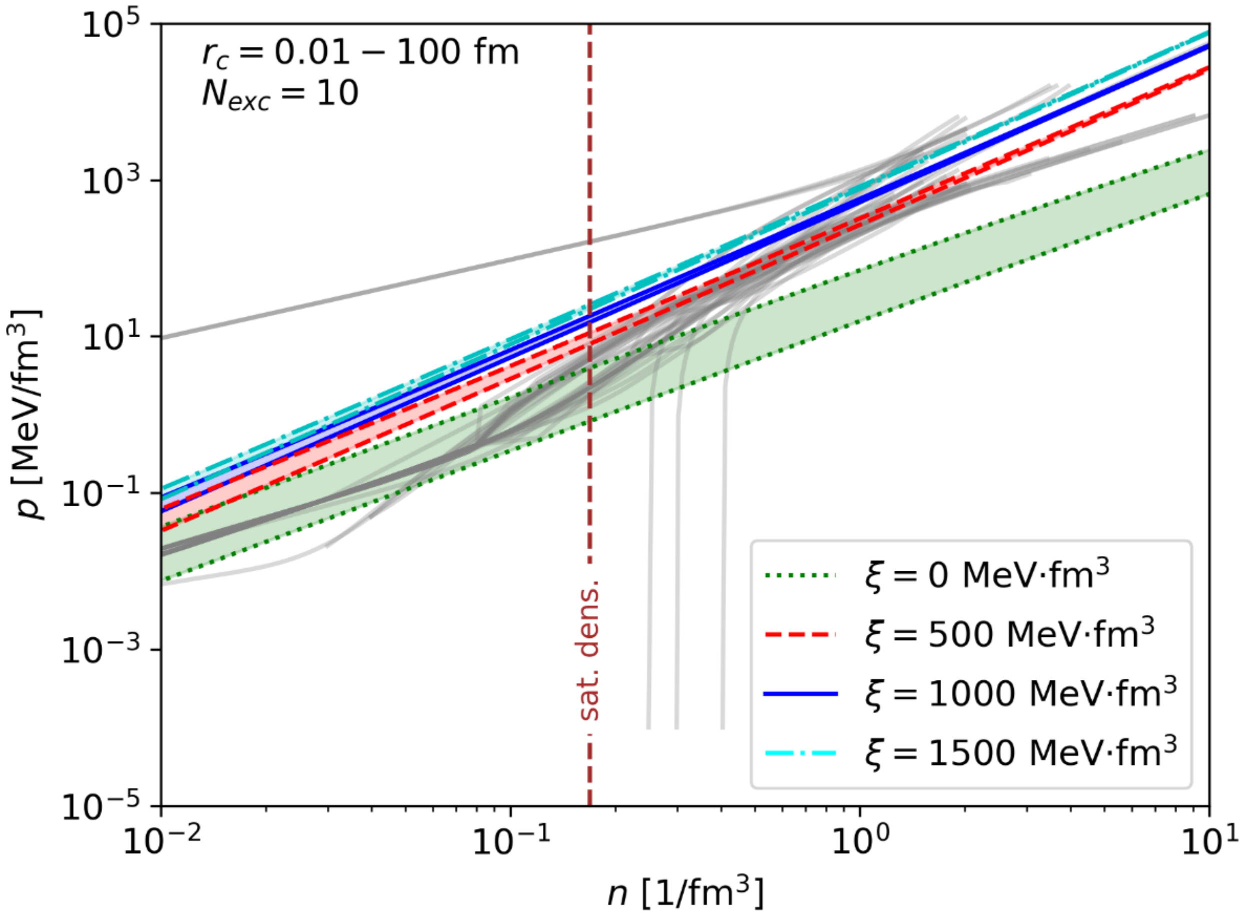
$$\bar{\mu} = \mu - U(n)$$

$$p(\mu) = p_0(\bar{\mu}) + p_{\rm int}$$

$$\varepsilon(\mu) = \varepsilon_0(\bar{\mu}) + \varepsilon_{\rm int}$$

$$p_{int} = \varepsilon_{int} = \int U(n) dn = \frac{1}{2}\xi n^2$$

Equation of state

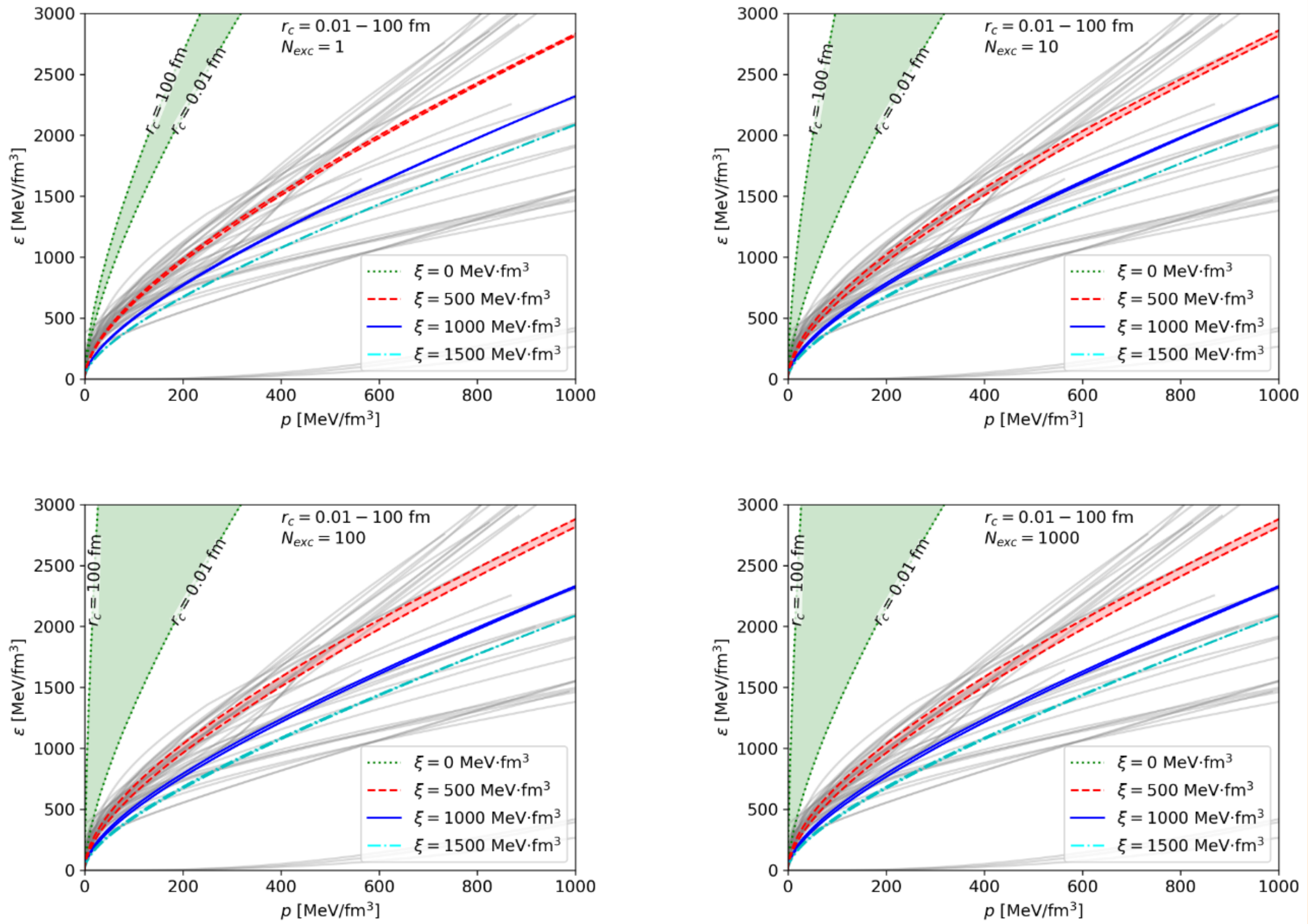


Pressure as a function of baryon number density.

F. Özel and P. Freire, "Masses, Radii, and the Equation of State of Neutron Stars," Ann. Rev. Astron. Astrophys. **54** (2016), 401-440 doi:10.1146/annurev-astro-081915-023322 [arXiv:1603.02698 [astro-ph.HE]].

https://compose.obspm.fr/

Equation of state



Energy density as a function of pressure.

F. Özel and P. Freire, "Masses, Radii, and the Equation of State of Neutron Stars," Ann. Rev. Astron. Astrophys. **54** (2016), 401-440 doi:10.1146/annurev-astro-081915-023322 [arXiv:1603.02698 [astro-ph.HE]].

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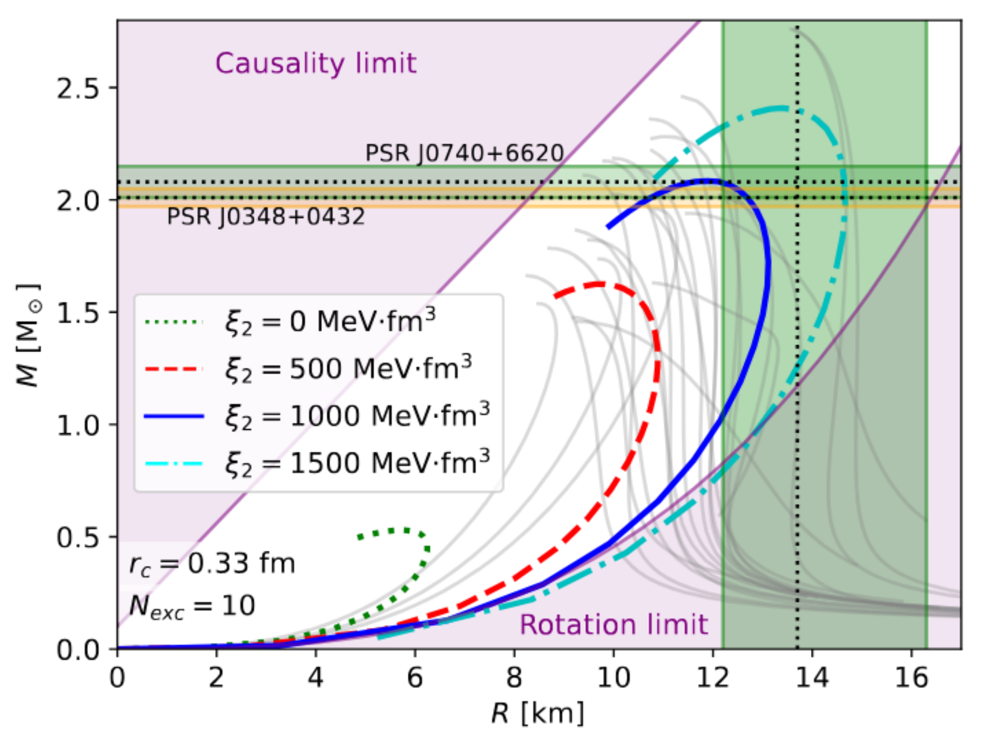
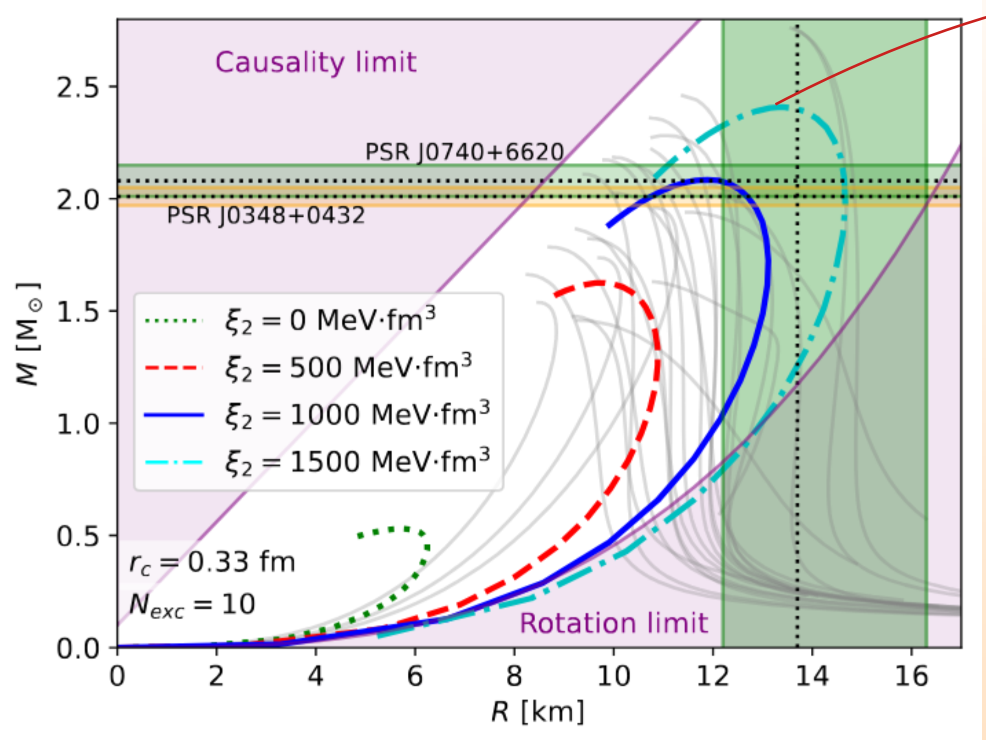
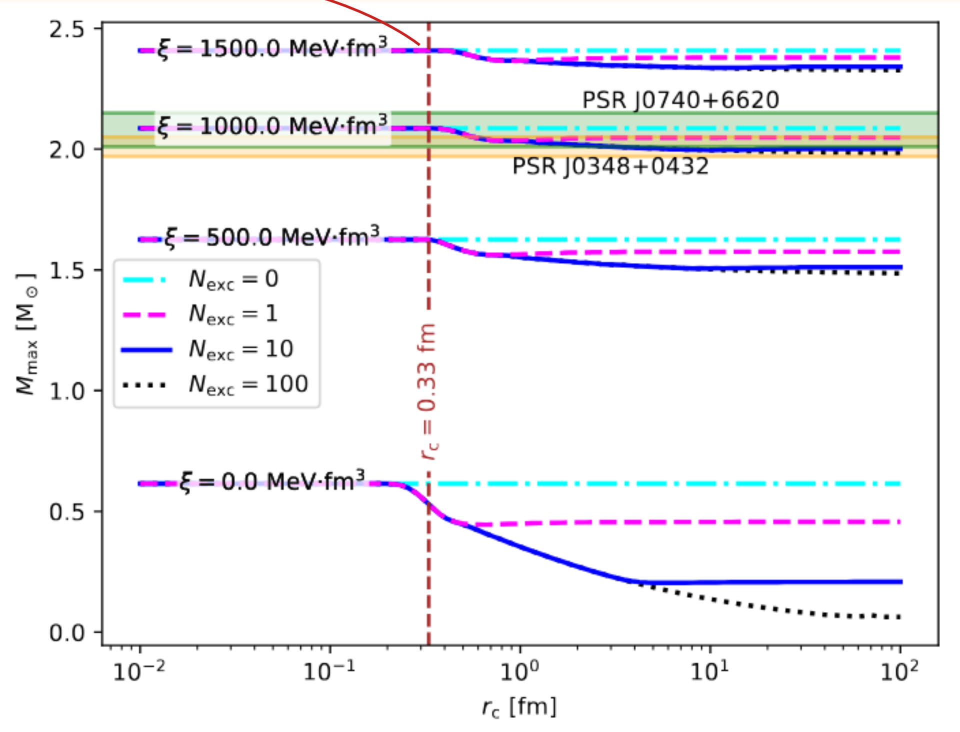
Comparison to observation



Fonseca E., et al., 2021, The Astrophysical Journal Letters, 915, L12
Miller M. C., et al., 2021, Astrophys. J. Lett., 918, L28
Antoniadis J., et al., 2013, Science, 340, 6131

Comparison to observation





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Extra-dimensional theories

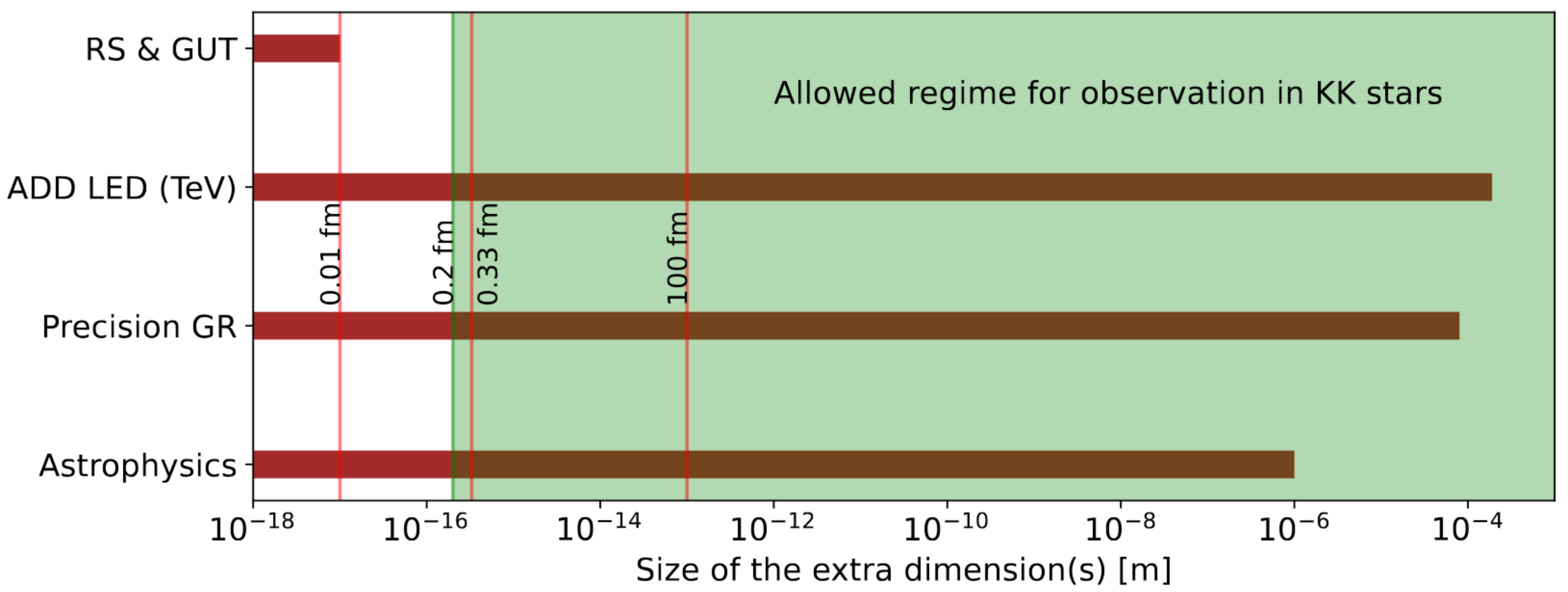
Observations regarding **neutron stars** can be **relevant for constraining** multiple beyond standard model extra-dimensional theories.

- Randall–Sundrum, GUT — $r_c < 10^{-17}$ m
- ADD large extra dimensions (TeV scale) — $r_c < 1.9\times10^{-4}$ m
- Precision measurements (tabletop) — $r_c < 8.0\times10^{-5}$ m
- Astrophysics – gravitational waves — $r_c < 10^{-6}$ m

Randall L., Sundrum R., 1999, Phys. Rev. Lett., 83, 3370
Cheung K., Landsberg G., 2002, Physical Review D, 65
Bernardi G., 2003. https://api.semanticscholar.org/CorpusID: 121772649
Abdallah J., et al., 2009, Eur. Phys. J. C, 60, 17
Adelberger E. G., Gundlach J. H., Heckel B. R., Hoedl S., Schlamminger S., 2009, Prog. Part. Nucl. Phys., 62, 102
Eötvös R. V., Pekár D., Fekete E., 1922, Annalen Phys., 68, 11
Péter G., Deák L., Gróf G., Kiss B., Szondy G., Tóth G., Ván P., Völgyesi L., 2022, Repeating the Eötvös-Pekár-Fekete equivalence principle measure-ments (arXiv:2205.14587), https://arxiv.org/abs/2205.14587
Murata J., Tanaka S., 2015, Class. Quant. Grav., 32, 033001
Abbott R., et al., 2021, Tests of General Relativity with GWTC-3 (arXiv:2112.06861)

Extra-dimensional theories

Observations regarding neutron stars can be relevant for constraining multiple beyond standard model extra-dimensional theories.



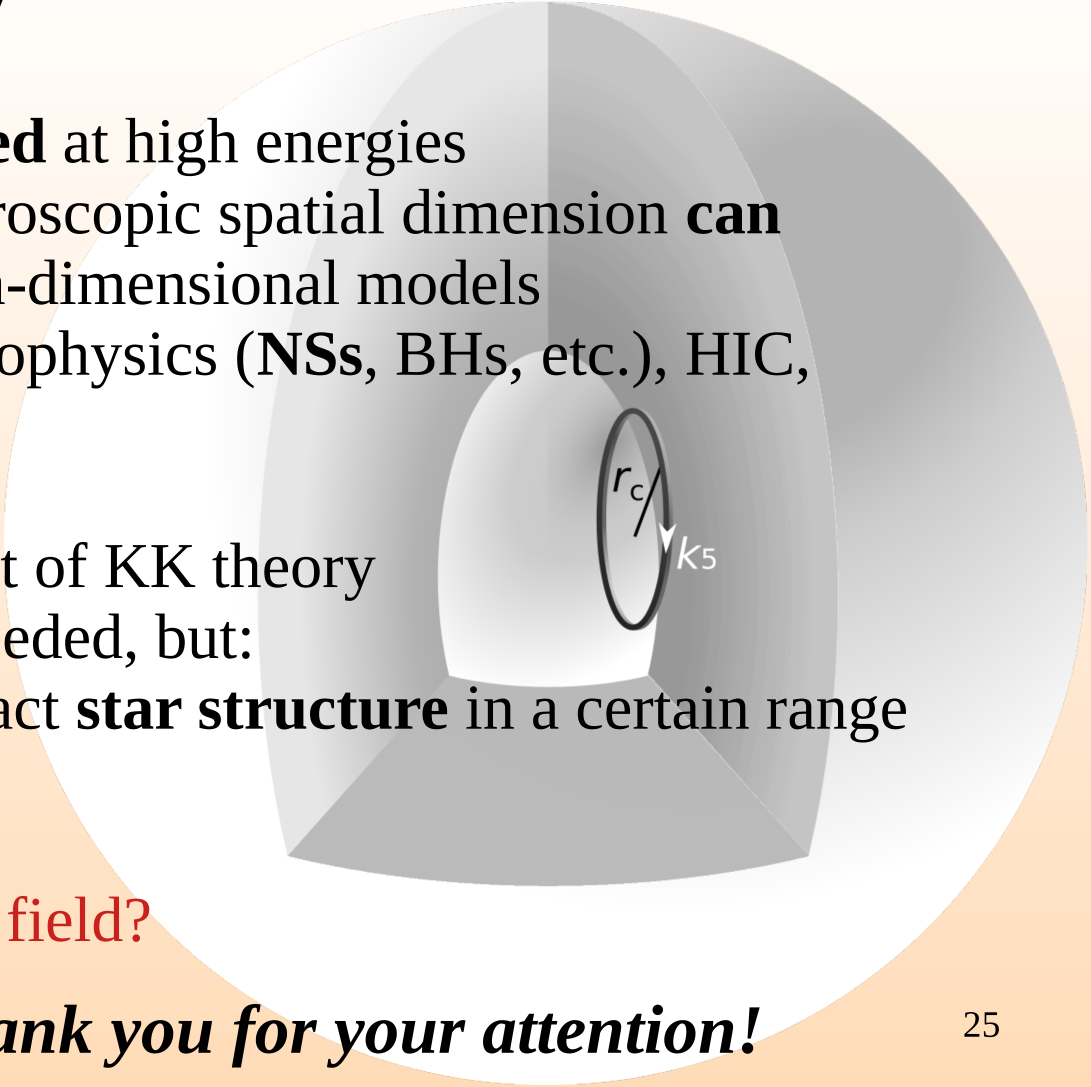
Summary

- **New theories** of physics are **needed** at high energies
- **Kaluza–Klein** with one extra microscopic spatial dimension **can show** the **phenomenology** of extra-dimensional models
- **Observational** possibilities in astrophysics (**NSs**, BHs, etc.), HIC, tabletop

- Studied neutron stars in the context of KK theory
- More precise measurements are needed, but:
- **extra dimensions** do **affect** compact **star structure** in a certain range

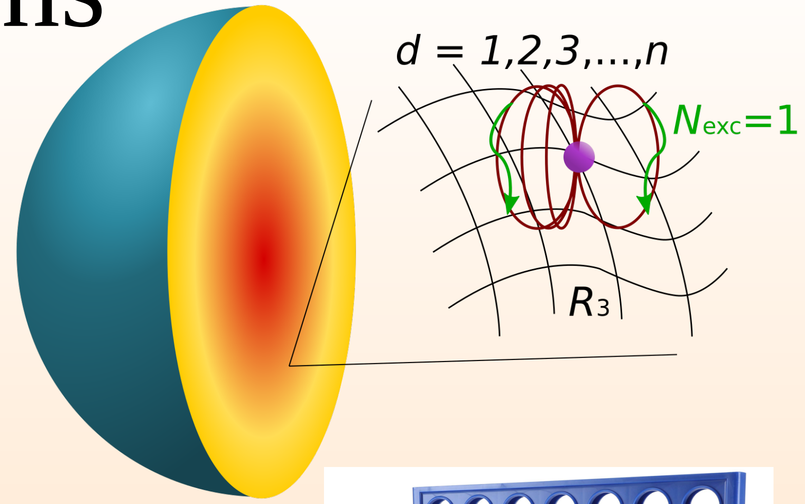
$$r_c \gtrsim 0.2\ \text{fm}$$

- Multiple extra dimensions? Scalar field?



Thank you for your attention!

Multiple extra dimensions



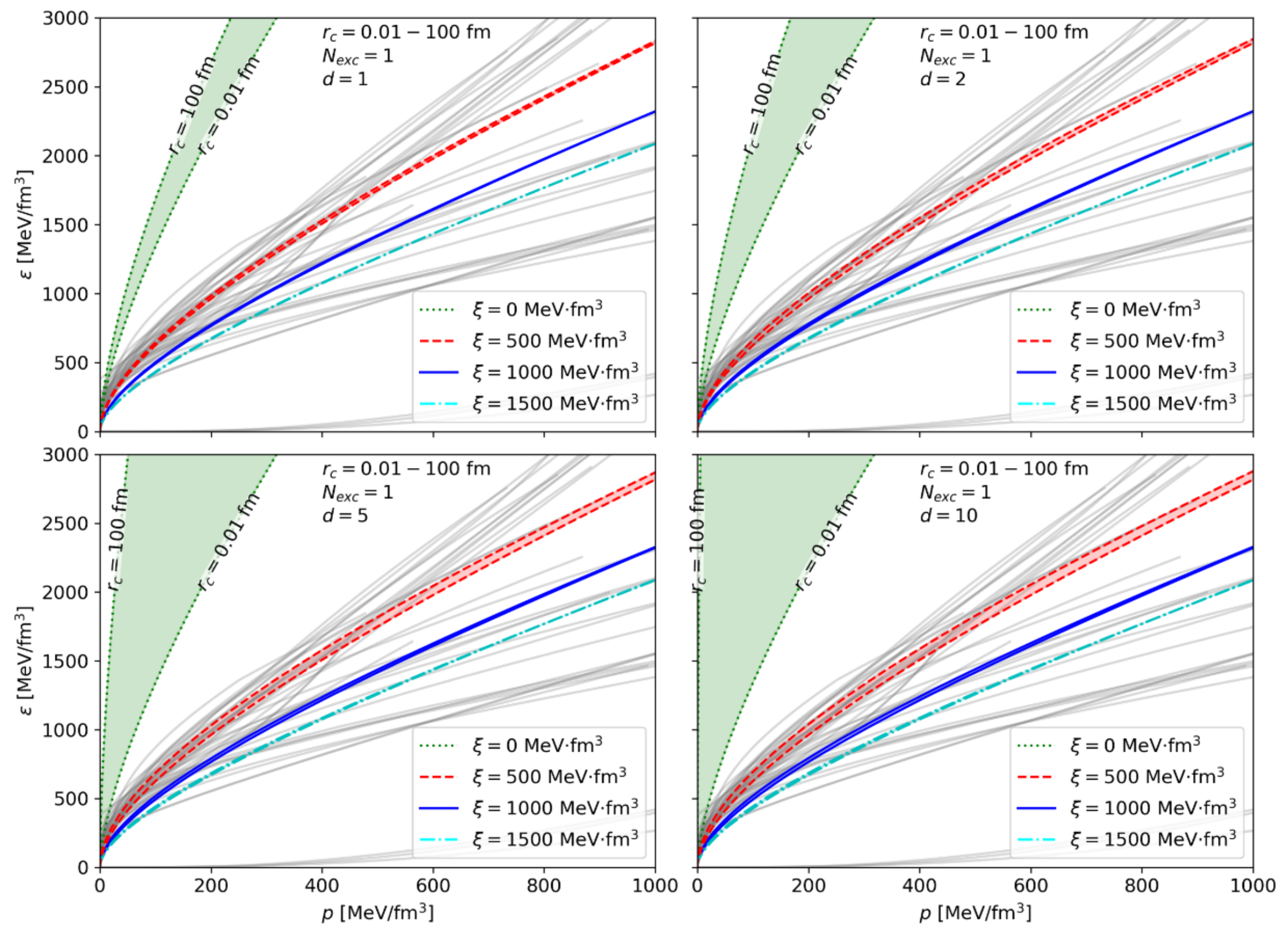
- All extra dimensions are the same size
- Particles are allowed to have excitations in all extra dimensions
- Effective mass:

$$\tilde{m}^2 = m^2 + \sum_{j=1}^{d} \tilde{k}_j^2$$

- Thermodynamic potential:

$$\Omega = -V_{(3+d)} \sum_i \underbrace{\sum_{j=0}^{N_{\mathrm{exc}}} \cdots \sum_{l=0}^{N_{\mathrm{exc}}}}_{d} \frac{g_i}{\beta} \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \times$$
$$\times \left[\ln\left(1 + \mathrm{e}^{-\beta(E_{ij\ldots l}-\mu)}\right) + \ln\left(1 + \mathrm{e}^{-\beta(E_{ij\ldots l}+\mu)}\right) \right]$$

Multiple extra dimensions



Modified phasespace

In collaboration with:
Aneta Wojnar

- Strong gravitational field could affect microscopic physics
- Relevance for the structure of NSs, BHs, WDs, planets?
- Modification to particle paths?
- Modified thermodynamics?

- Generalized uncertainty principle (GUP)

$$\Delta x \Delta p \geq \frac{\hbar}{2} + \beta \Delta p^2$$

L. Petruzziello and F. Wagner, Physical Review D 103, 104061 (2021).
M. P. Dabrowski and F. Wagner, The European Physical Journal C 80, 676 (2020).
Abdel Nasser Tawfik and Abdel Magied Diab 2015 Rep. Prog. Phys. 78 126001
Aleksander Kozak, Aneta Wojnar, “Earthquakes as probing tools for gravity theories”, 2023, arXiv:2308.01784

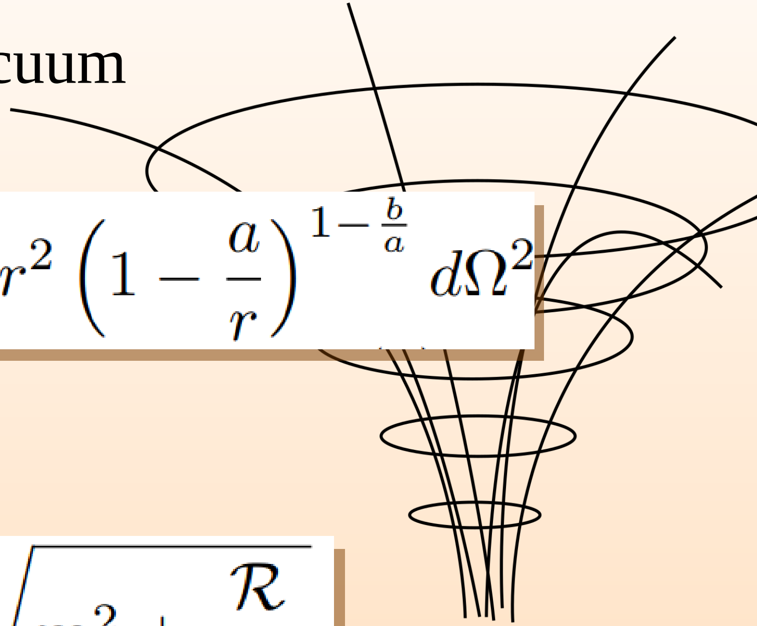
- In terms of the curvature

$$\sigma_p \rho \gtrsim \pi\hbar \left[1 - \frac{\rho^2 \mathcal{R}|_{p_0}}{12\pi^2} + \xi \frac{\rho^4}{\lambda_C^2} \nabla_j N_i \nabla^j N^i |_{p_0}\right]$$

Schwarzschild-like solution

In collaboration with:
Aneta Wojnar

- g_{55} is allowed to vary ⟶ scalar field
- Non-zero energy-momentum tensor even for vacuum
- Schwarzschild-like metric



$$ds^2 = -\left(1 - \frac{a}{r}\right)^{\frac{b}{a}} dt^2 + \left(1 - \frac{a}{r}\right)^{-\frac{b}{a}} dr^2 + r^2 \left(1 - \frac{a}{r}\right)^{1-\frac{b}{a}} d\Omega^2$$

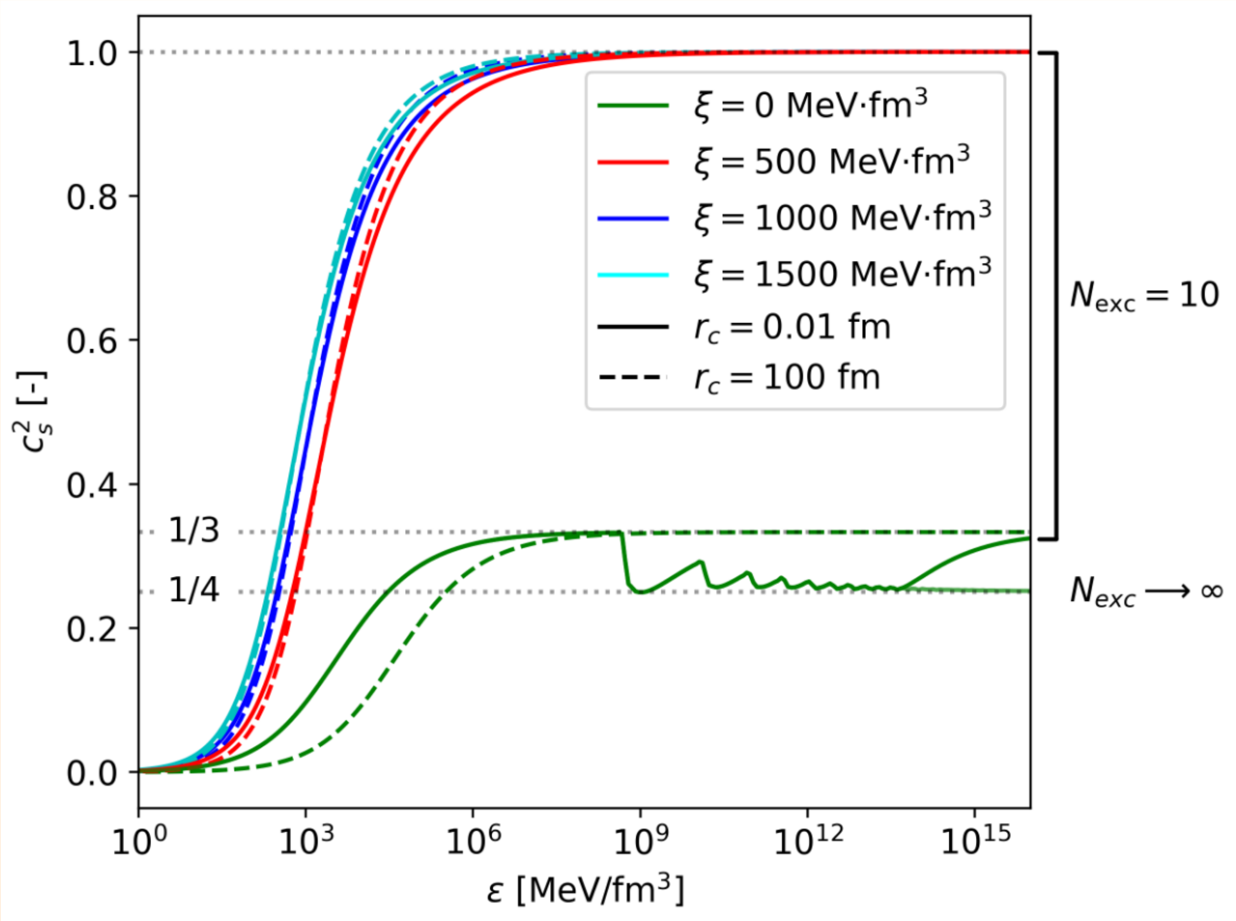
- Non-zero phase-space and spacetime curvatures
- Modified dispersion relation and effective mass

$$p^\mu p_\mu = -m^2 c^2 - \frac{\mathcal{R}}{6}$$

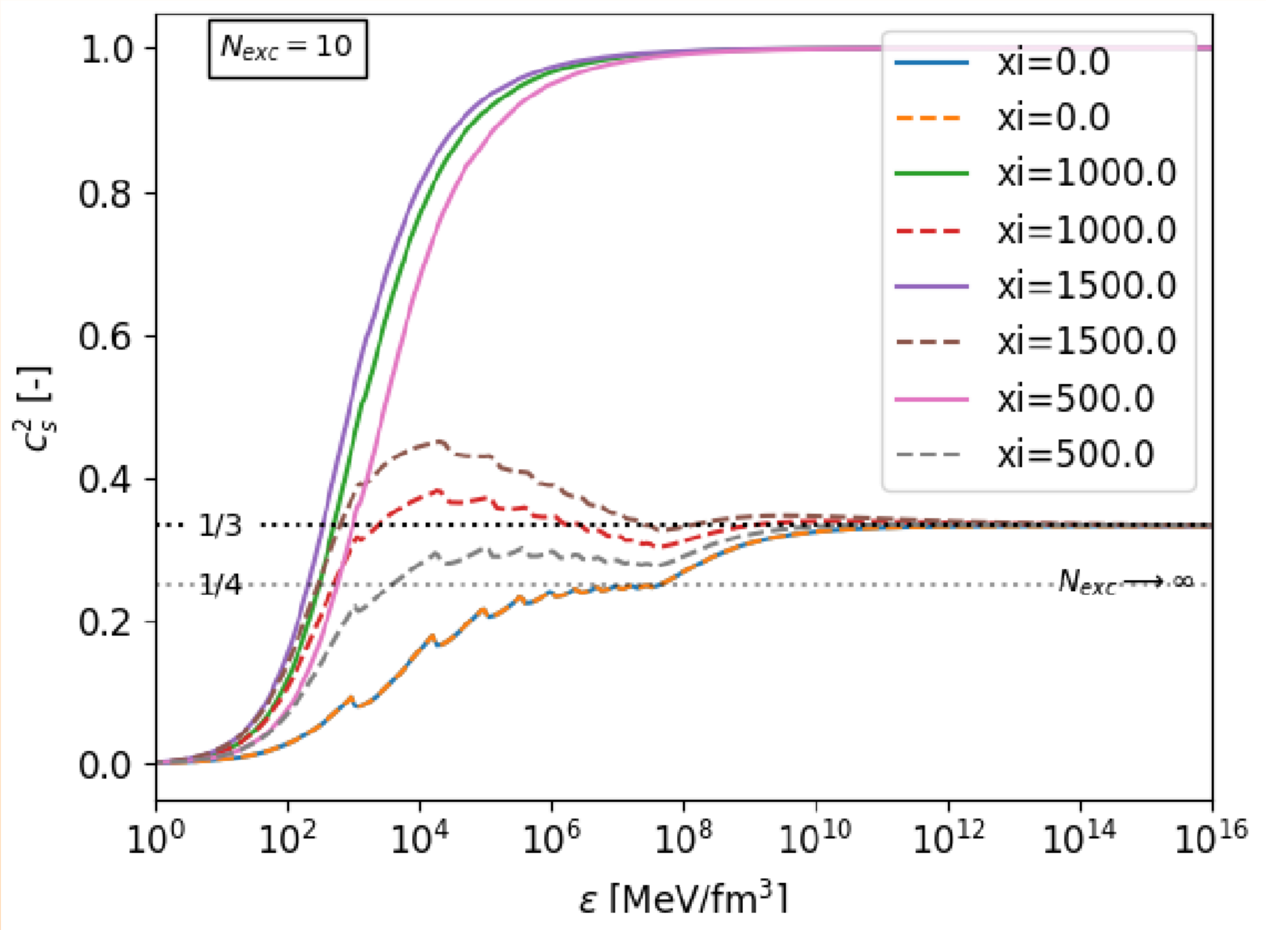
$$m_{\text{eff}} = \sqrt{m^2 + \frac{\mathcal{R}}{6c^2}}$$

R. Coquereaux and G. Esposito-Farese, in Annales de l'IHP Physique th´eorique, Vol. 52 (1990) pp. 113–150
L. Petruzziello and F. Wagner, Physical Review D 103, 104061 (2021).
M. P. Dabrowski and F. Wagner, The European Physical Journal C 80, 676 (2020).

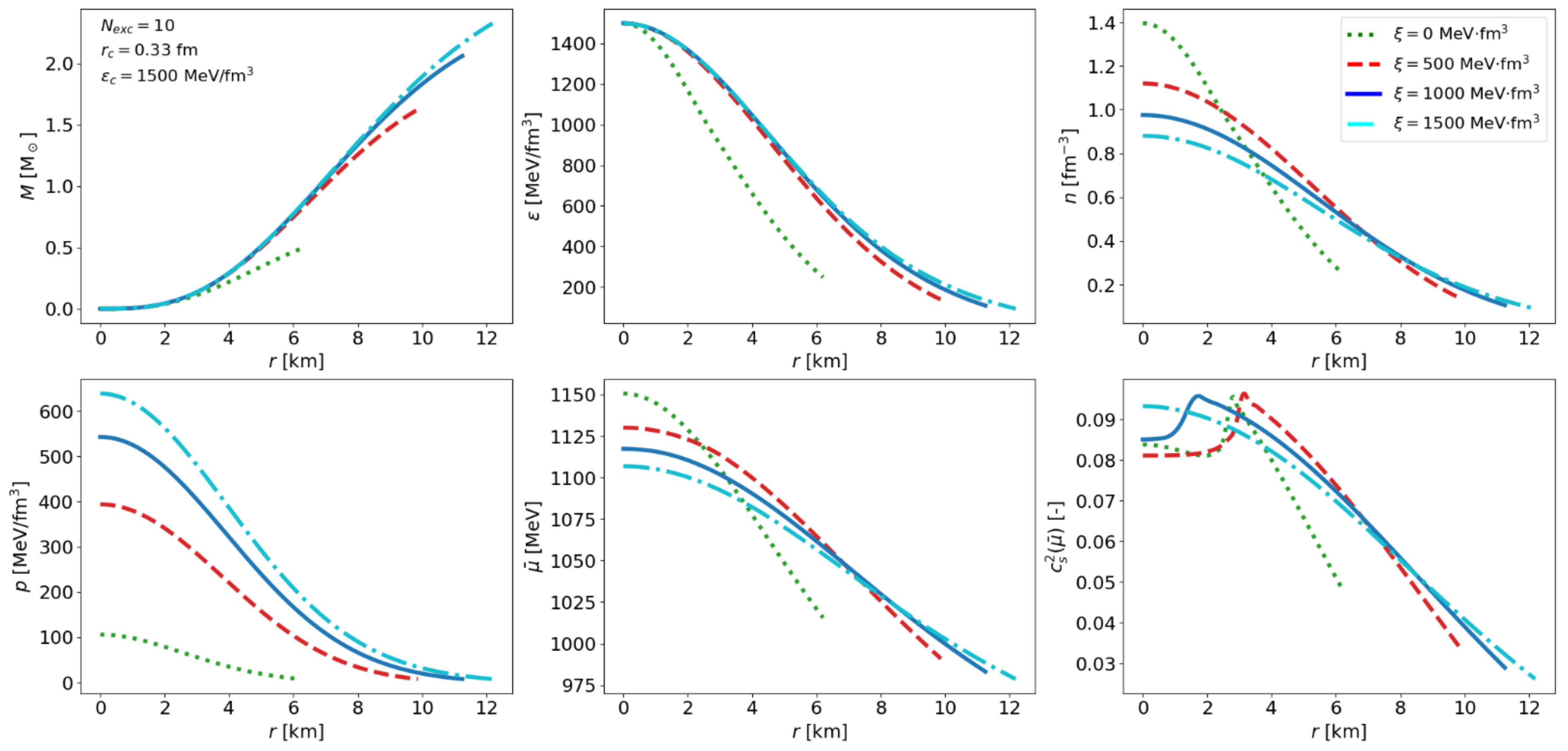
Speed of sound



Speed of sound – modified potential



Solving the TOV equation – stars



r: radius of star