Scattering of dark Pions

Yannick Dengler, Axel Maas, Fabian Zierler

ACHT 2025

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Outline

- Self-Interacting Dark Matter

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- Self-Interacting Dark Matter
- Minimal Realisation: $\mathrm{Sp}(4)_c$ with $N_f = 2$

Scattering of dark Pions

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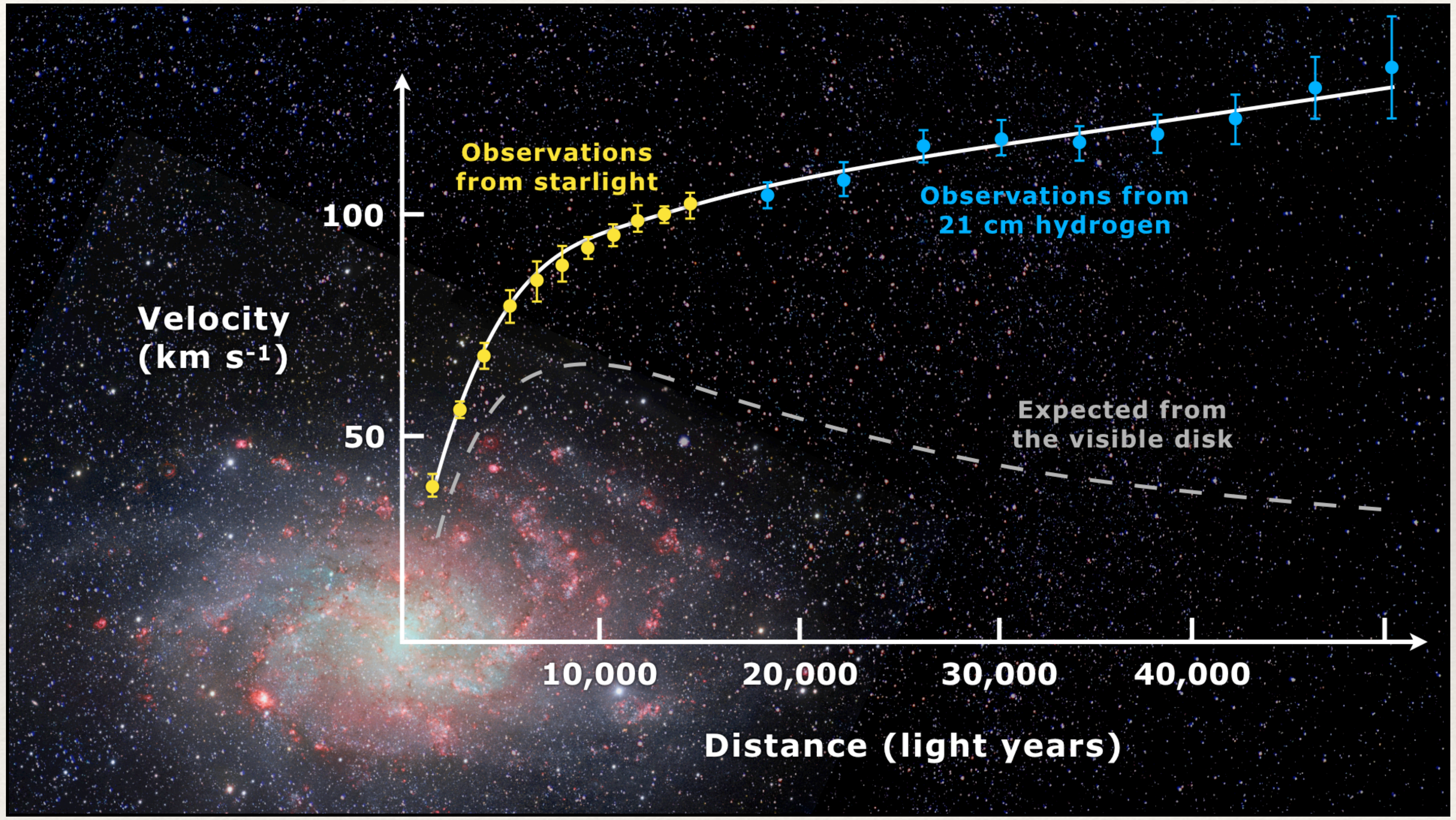
Outline

- Self-Interacting Dark Matter
- Minimal Realisation: $\mathrm{Sp}(4)_c$ with $N_f = 2$
- Lattice Results

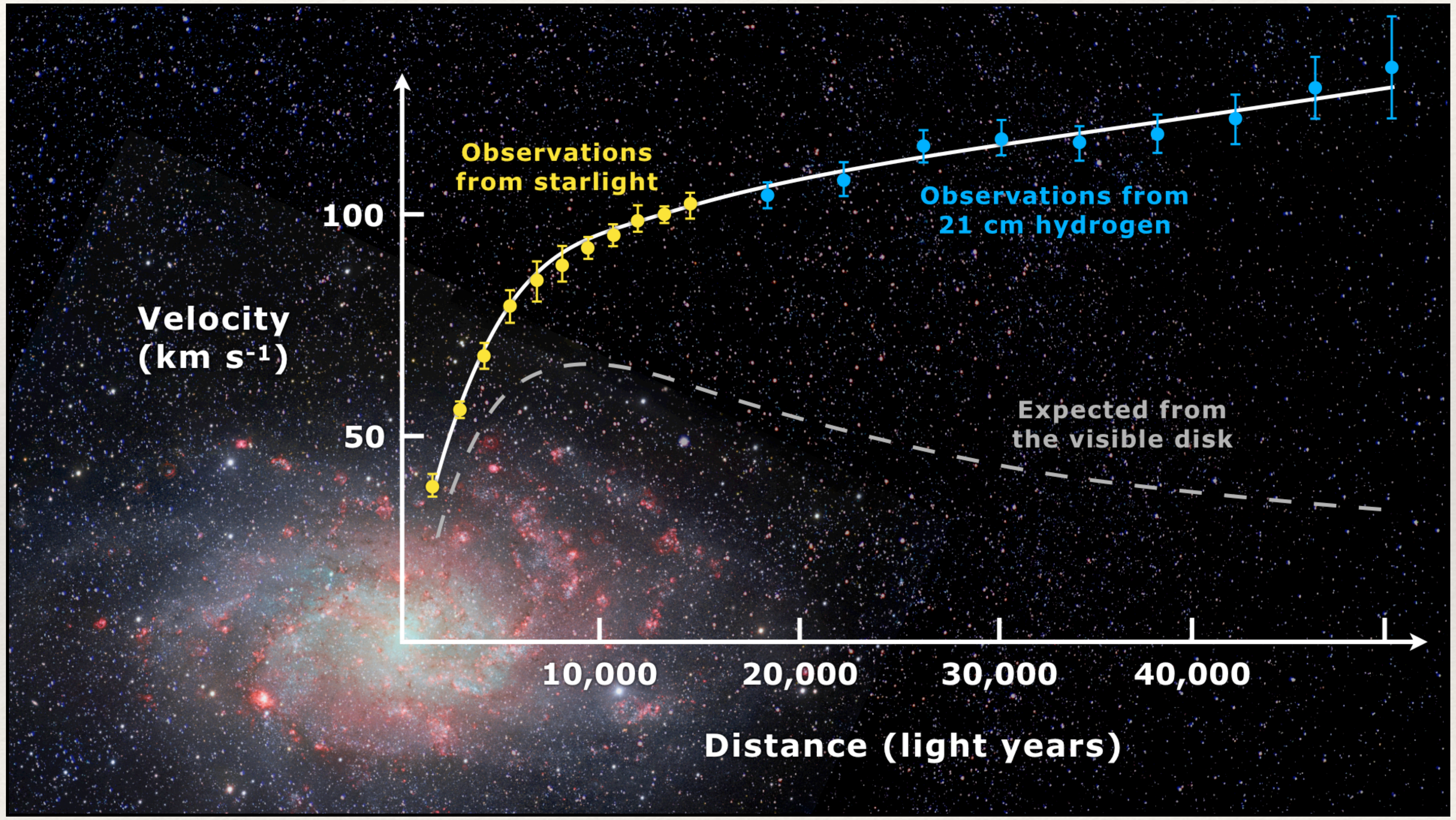
Dark Matter

- Collection of astronomical phenomena
  - Motion of objects, Large scale structure, gravitational lensing, …
  - In conflict with our current understanding



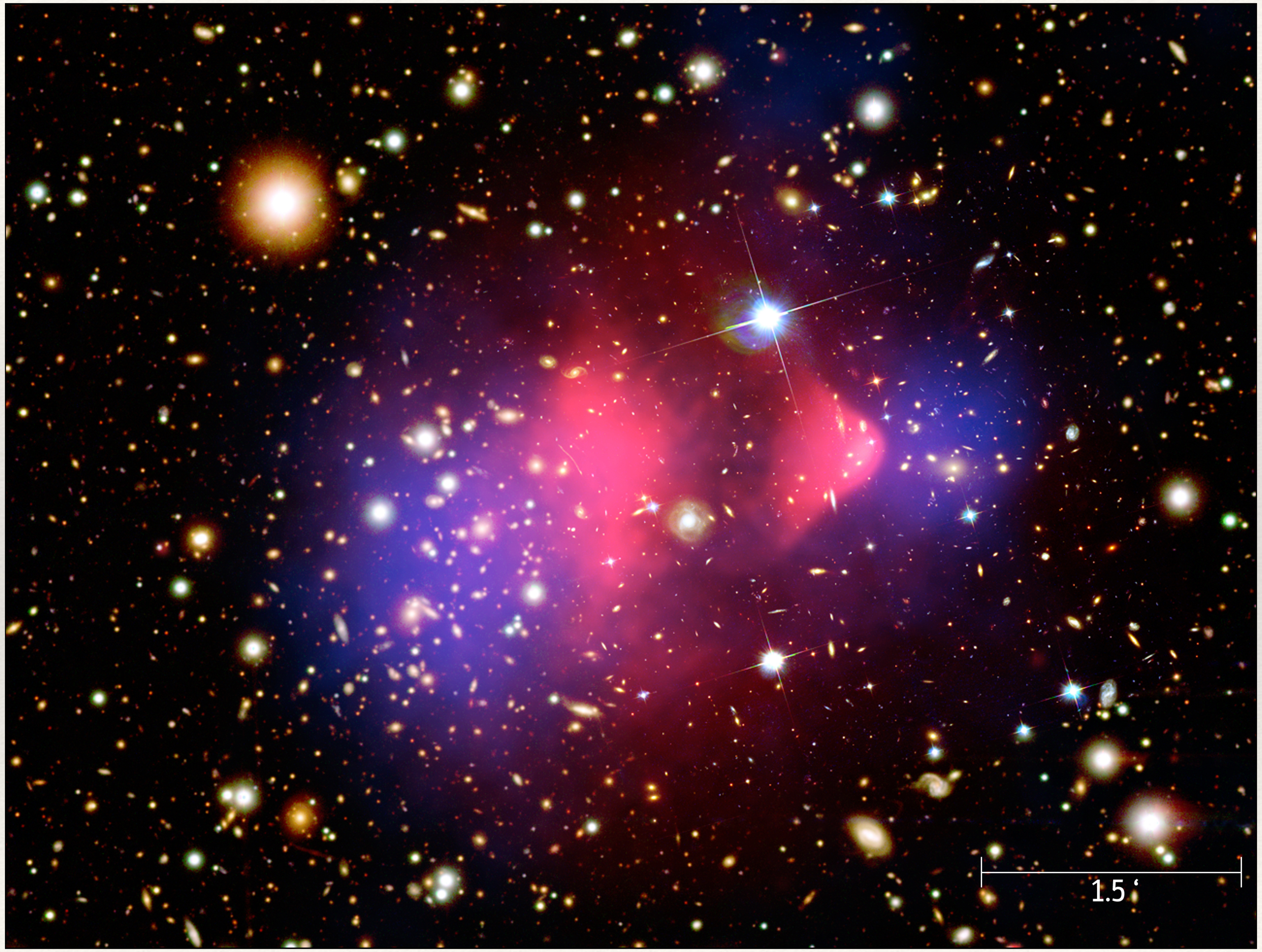
Dark Matter

- Collection of astronomical phenomena
  - Motion of objects, Large scale structure, gravitational lensing, …
  - In conflict with our current understanding
- Explanations:
  - Modified Gravity
  - Not observable sort of matter



Dark Matter Particle

- Evidence for particle DM:
  - i.e. "Bullet cluster"



Chandra X-ray Observatory

Dark Matter Particle

- Evidence for particle DM:
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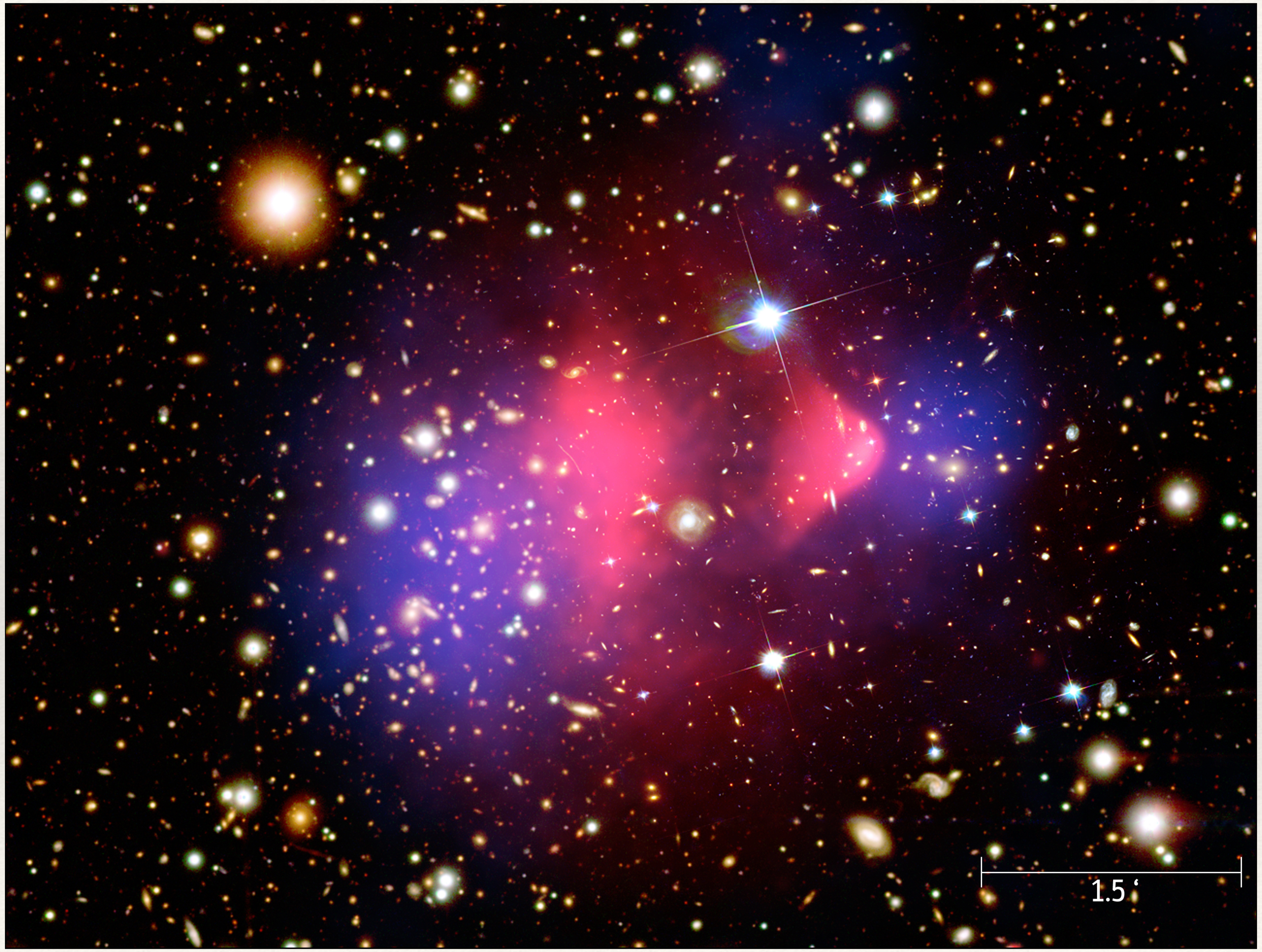
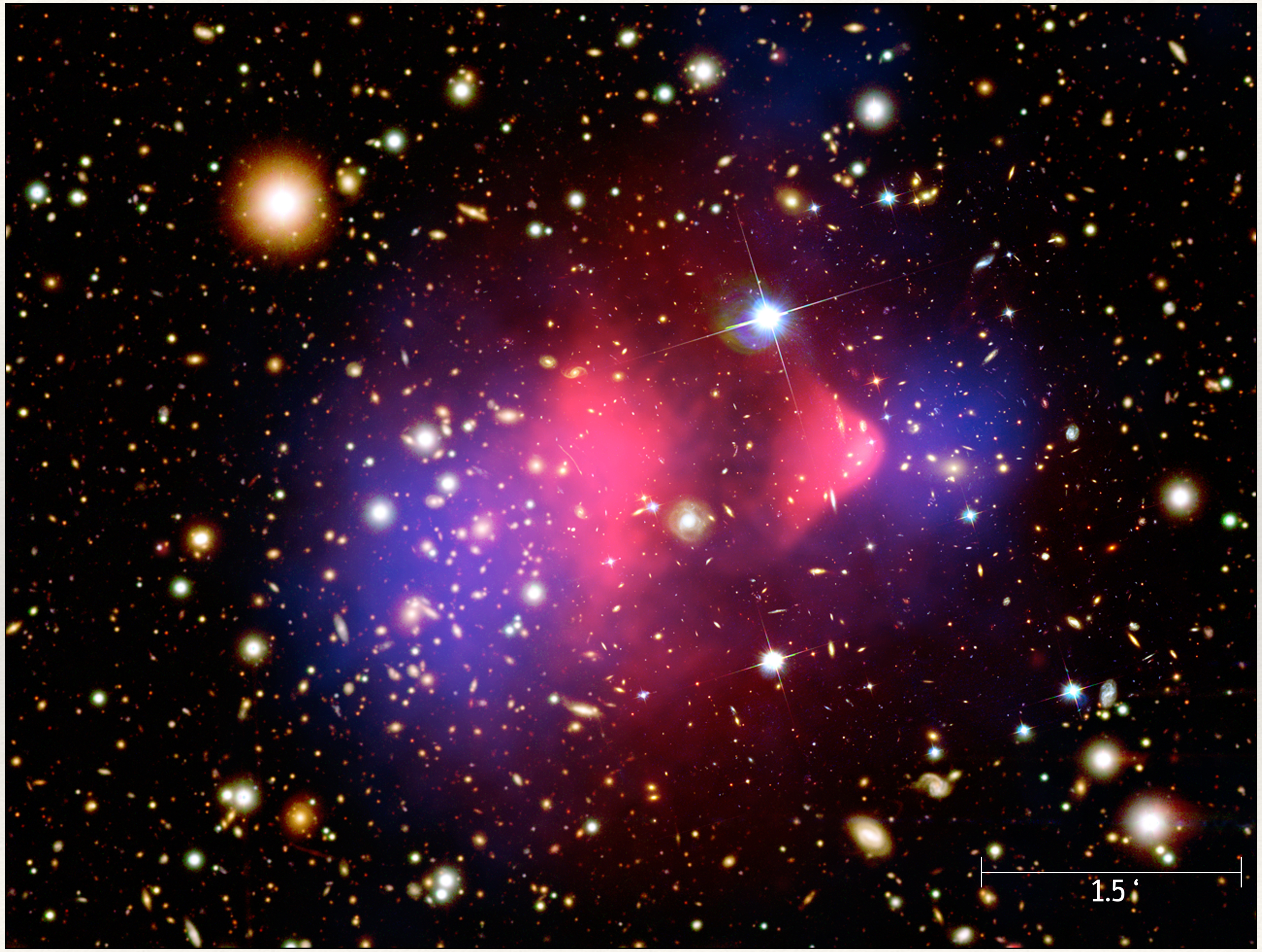
- Properties:
  - Massive, stable, "invisible"



Chandra X-ray Observatory

Dark Matter Particle

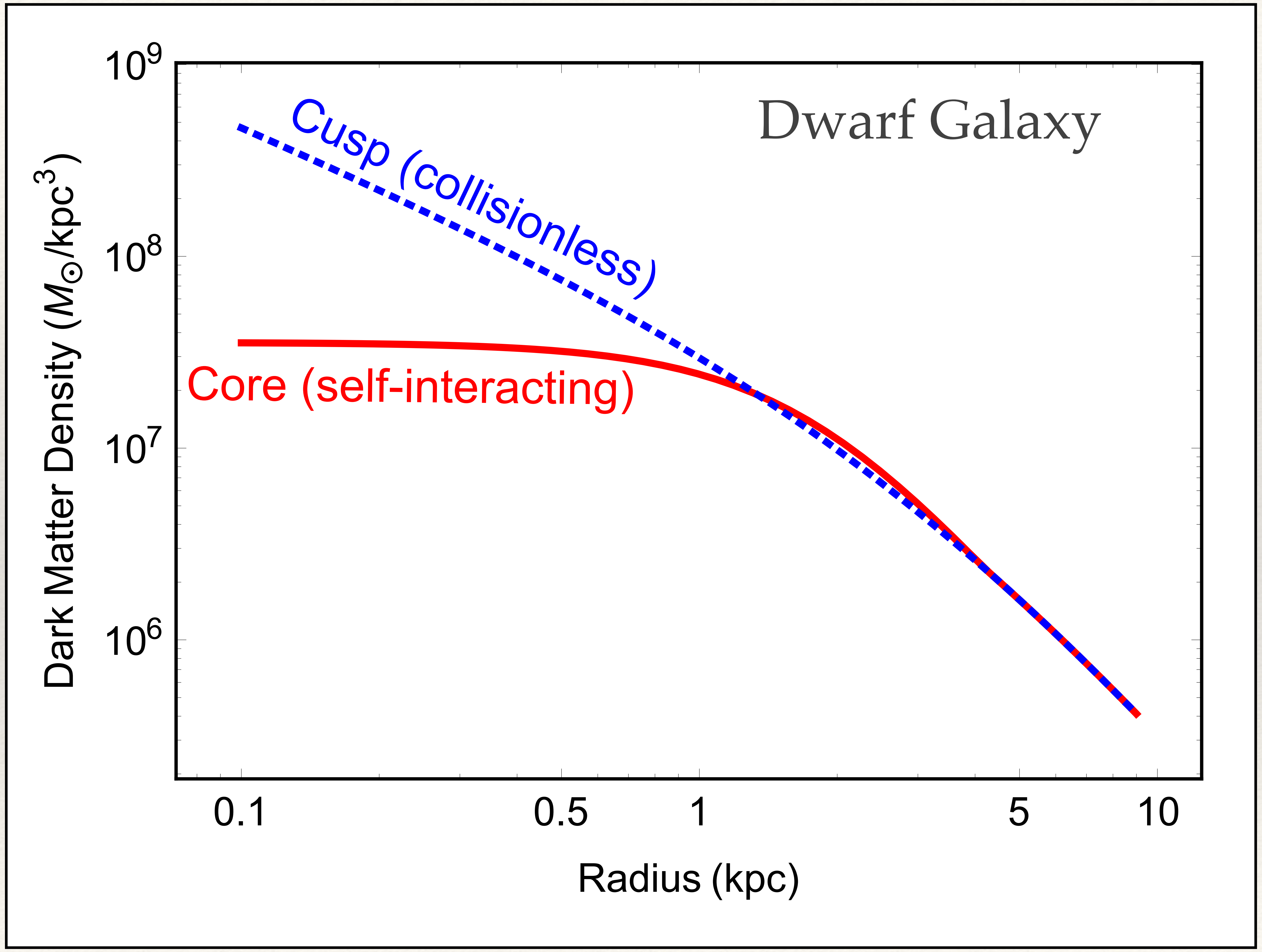
- Evidence for particle DM:
  - i.e. "Bullet cluster"
- Properties:
  - Massive, stable, "invisible"
- Interaction?
  - With SM: no (low)
  - Self: Maybe



Chandra X-ray Observatory

Self-interaction

- "Small structure problems"
  - *Diversity, too-big-to-fail, missing satellites, cusp vs. core*



Tulin, Yu: arXiv:1705.02358 (2017)

Self-interaction

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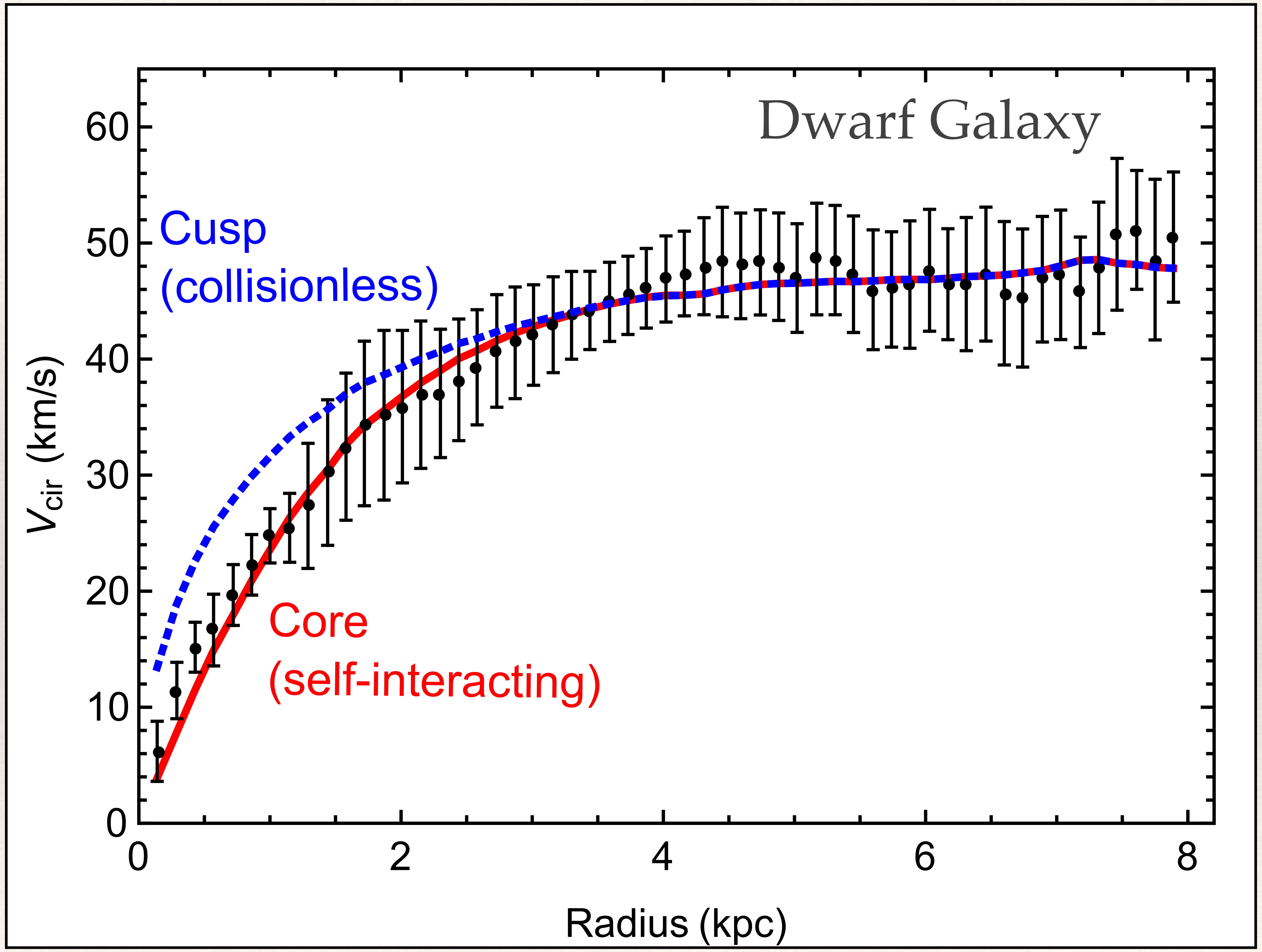
- Core-like shape preferred
  - Hints towards self-interaction



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Self-interaction

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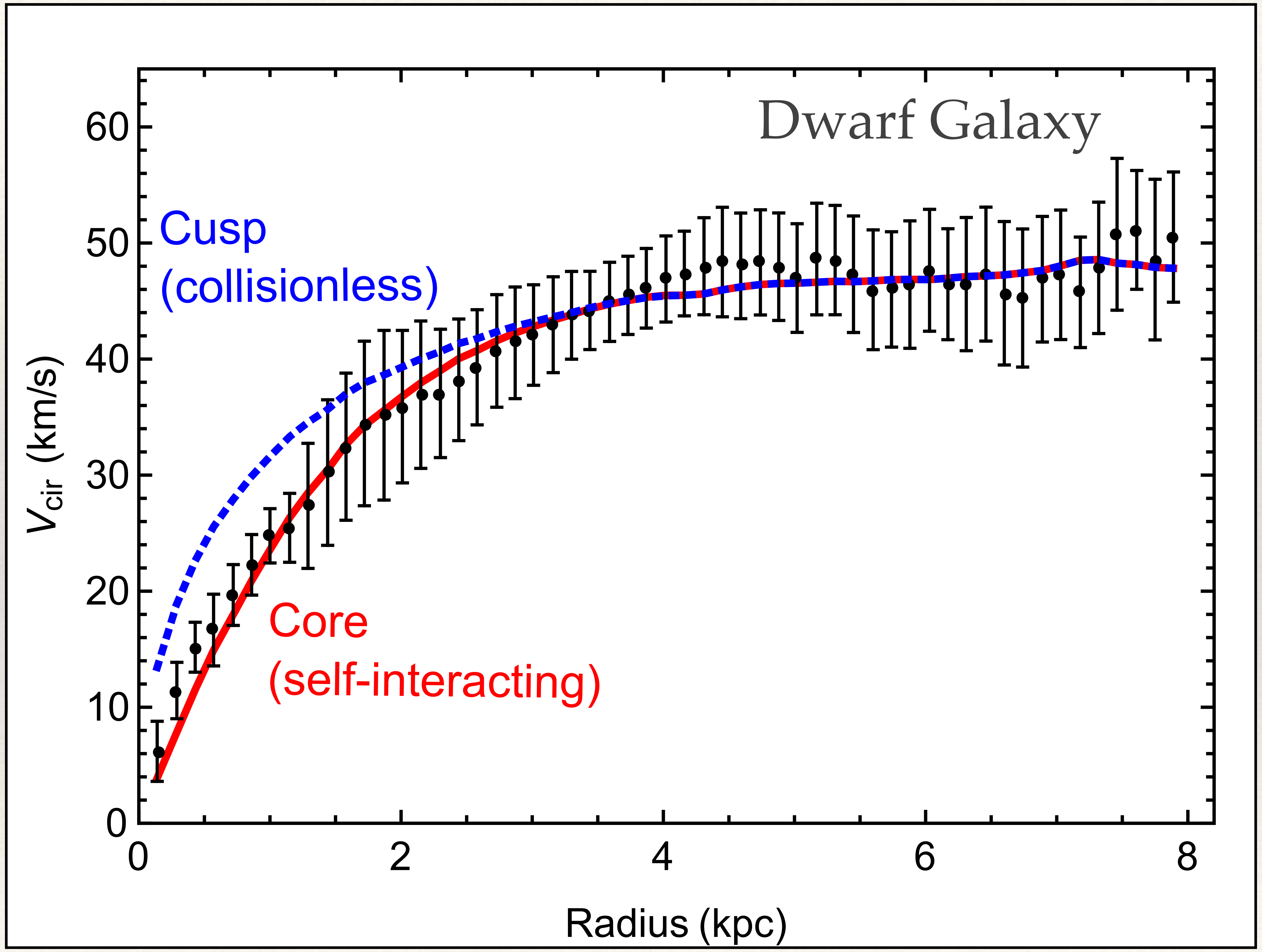
- Core-like shape preferred
  - Hints towards self-interaction
- Upper bounds on cross-section from the bullet cluster



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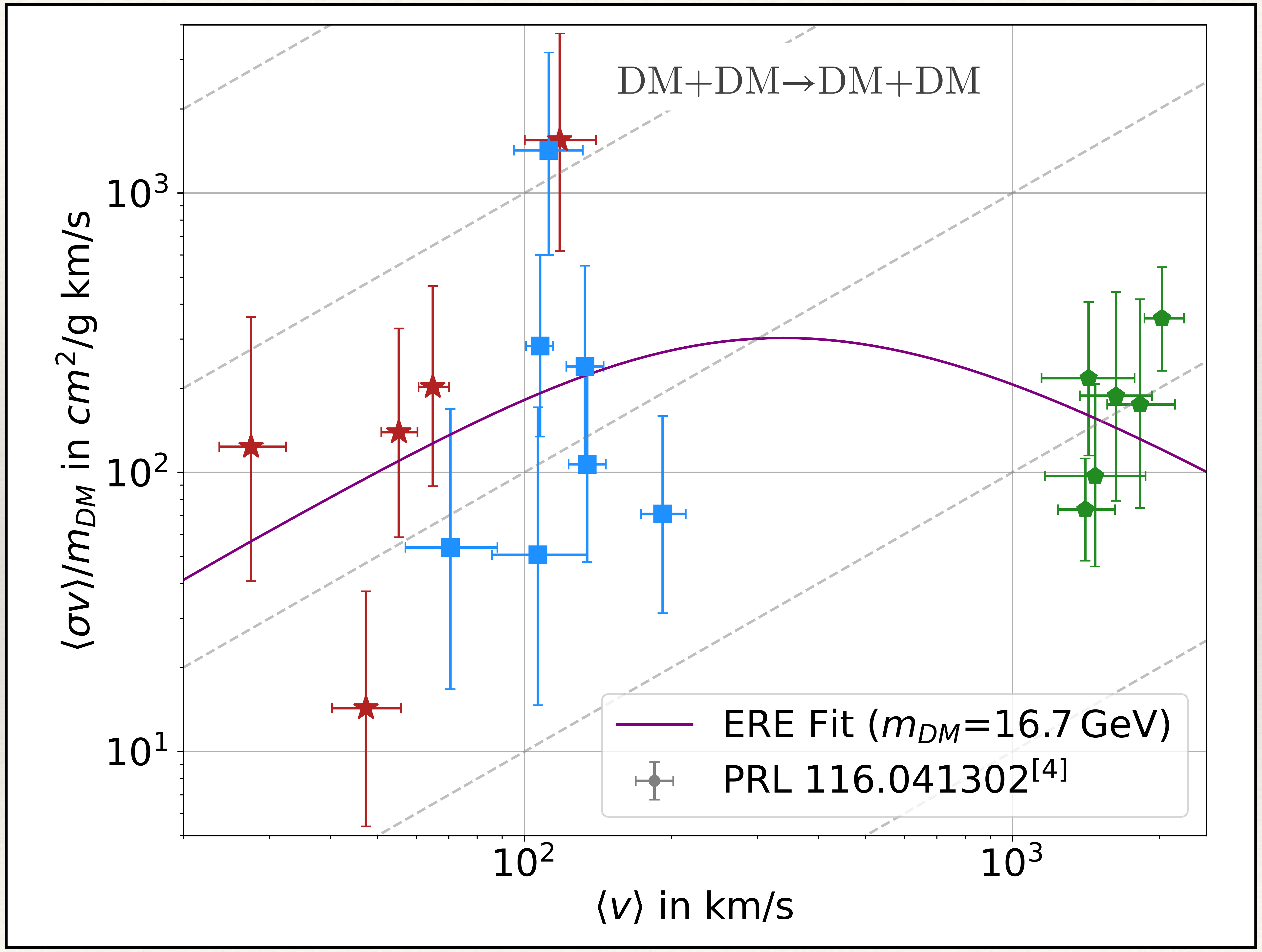
Velocity-dependent cross-section

- "Dark matter halos as particle colliders"
    - Larger object → larger $\langle v_i \rangle$ & $\langle v \rangle$



Kaplinghat et al: Phys. Rev. Lett. 116 (2016)

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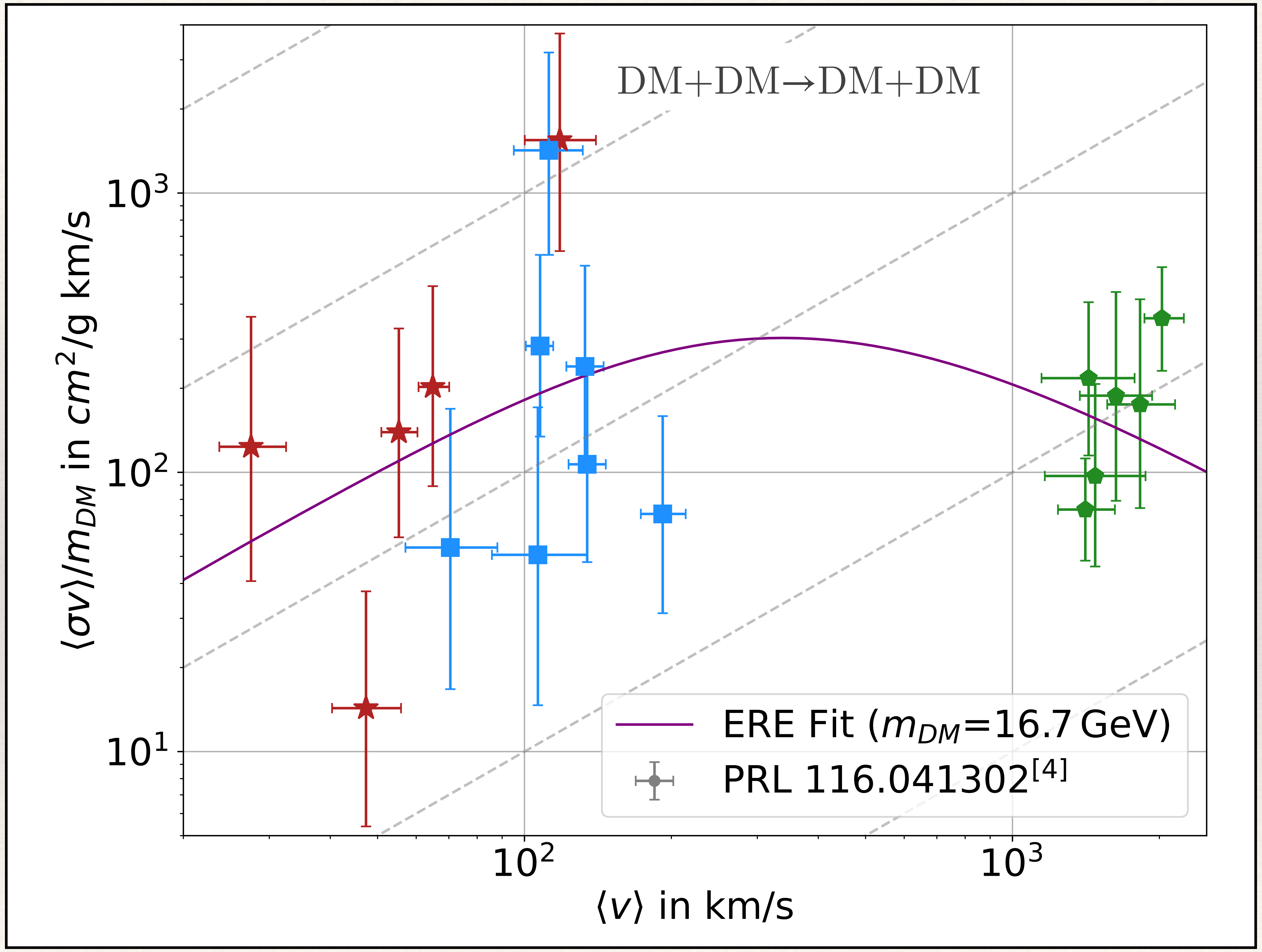

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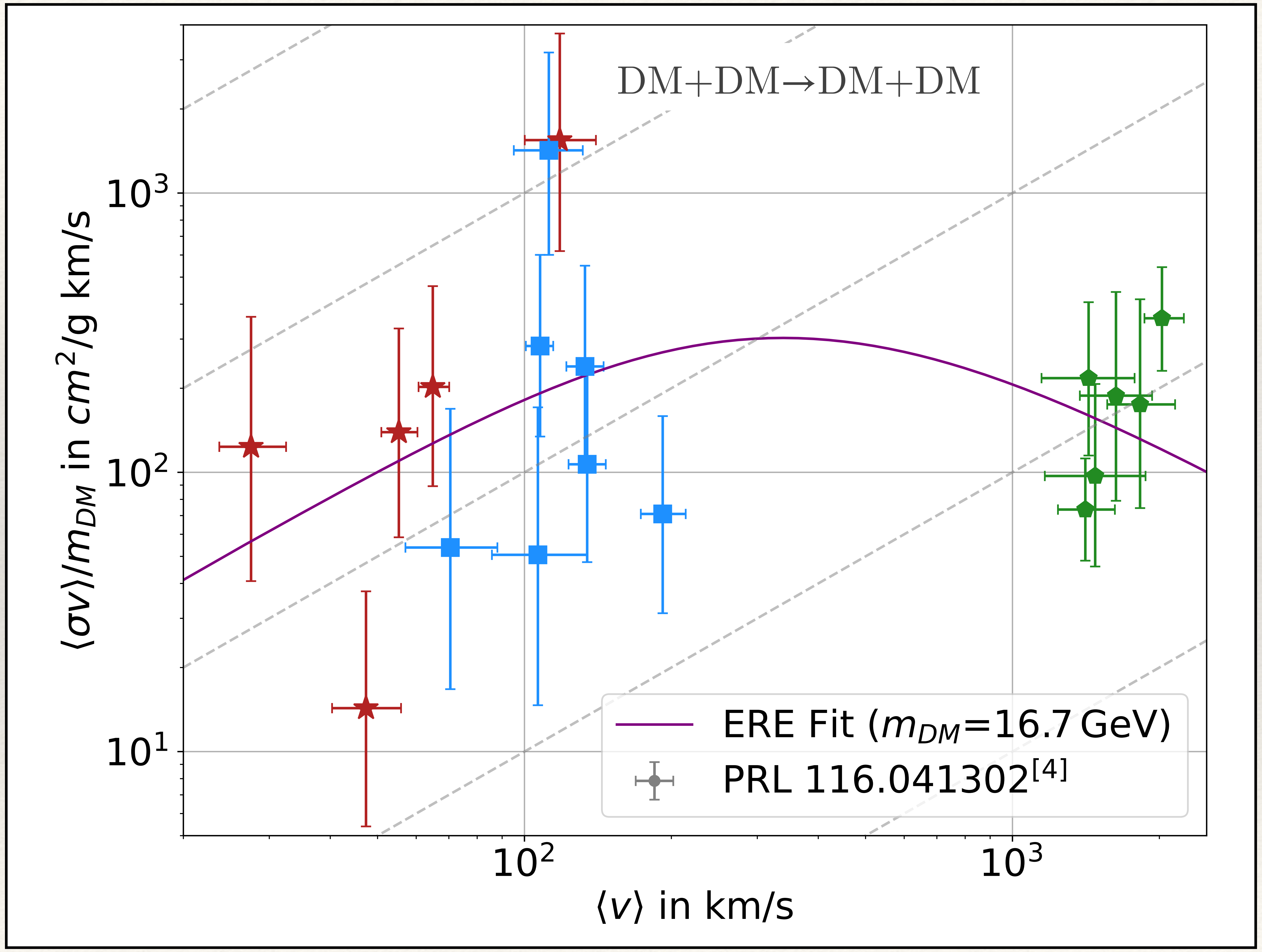

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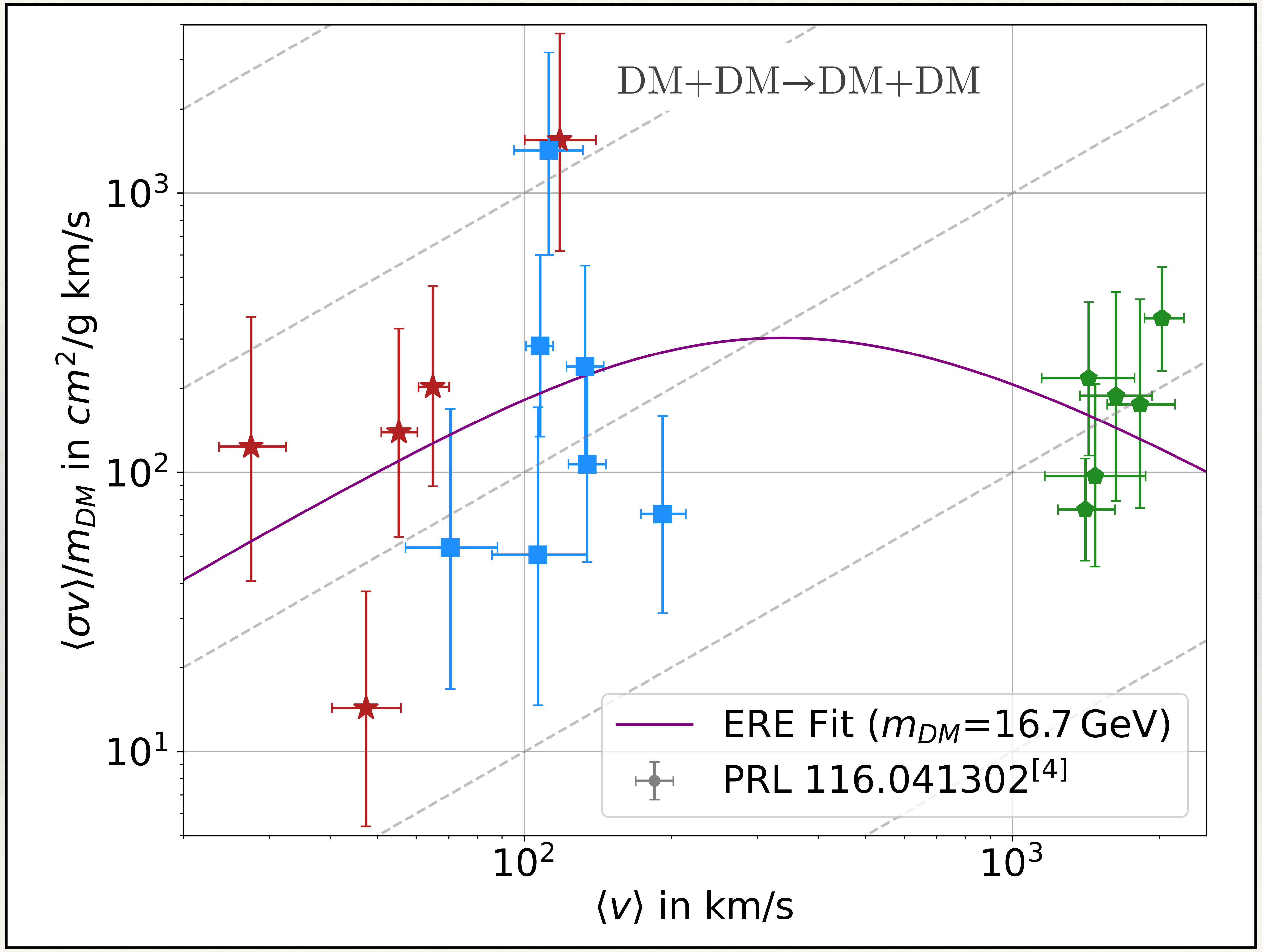
- Thermalized dark matter



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Velocity-dependent cross-section

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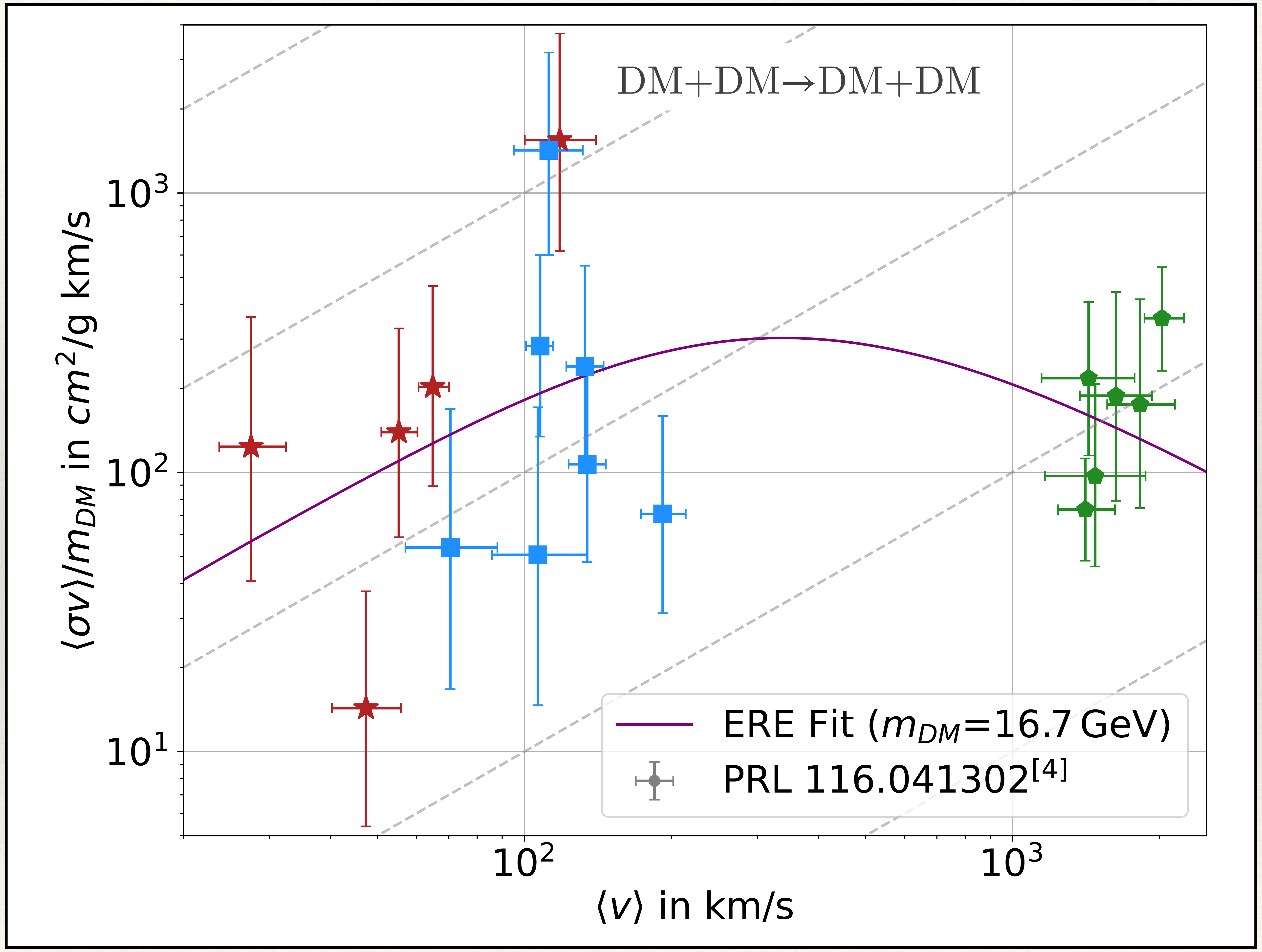
  - Larger object → larger $\langle v_i \rangle$ & $\langle v \rangle$
- Fit halo shape to simulations
- Mild velocity dependence @ non-relativistic velocities
- Thermalized dark matter
- Only $\sigma(s)$ needed

Can be done on the lattice



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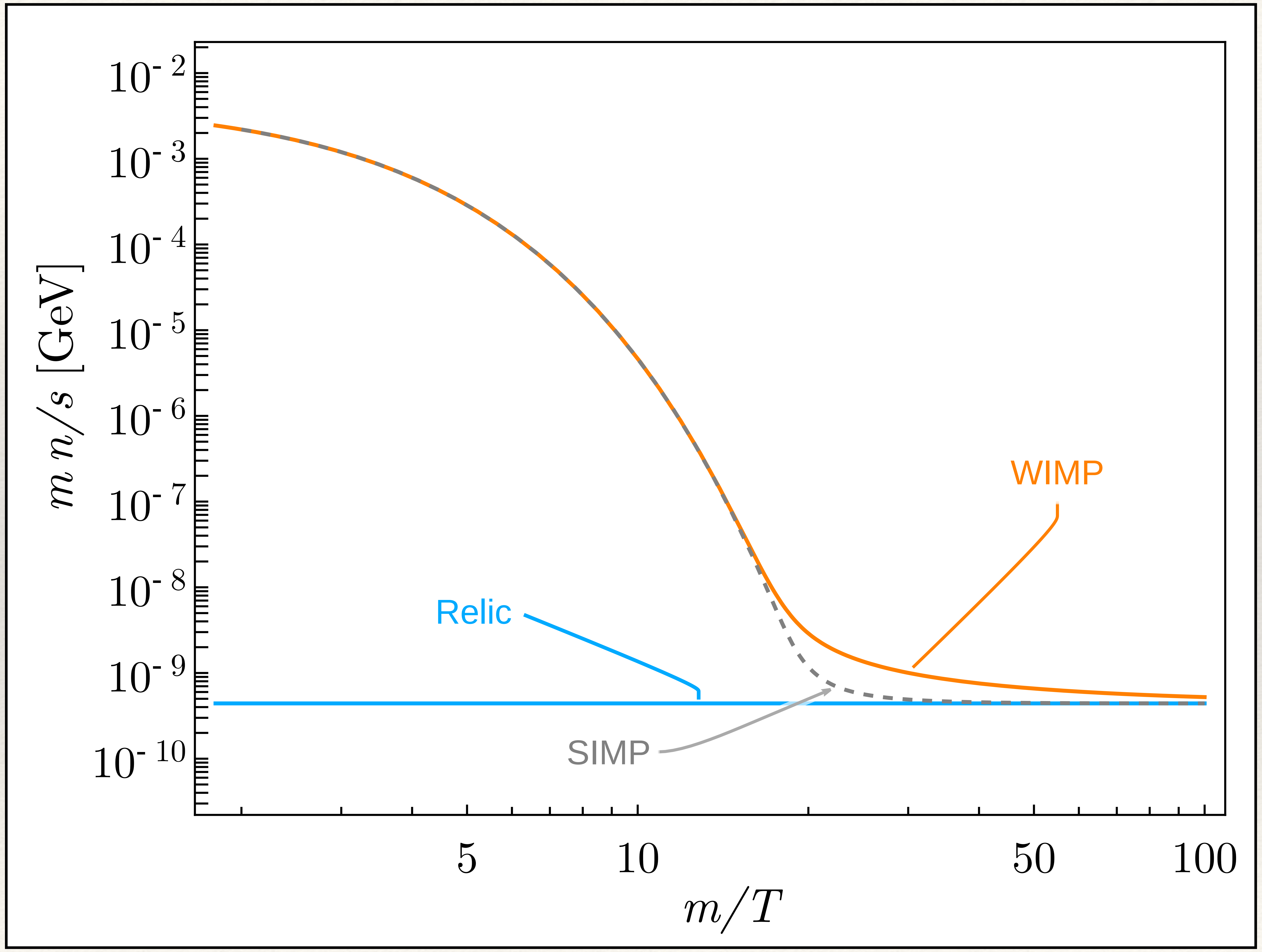
Relic density

- Possibility: Dark matter as as a thermal relic from the early universe



Frumkin et al: Phys.Rev.Lett. 130 (2023)

Relic density

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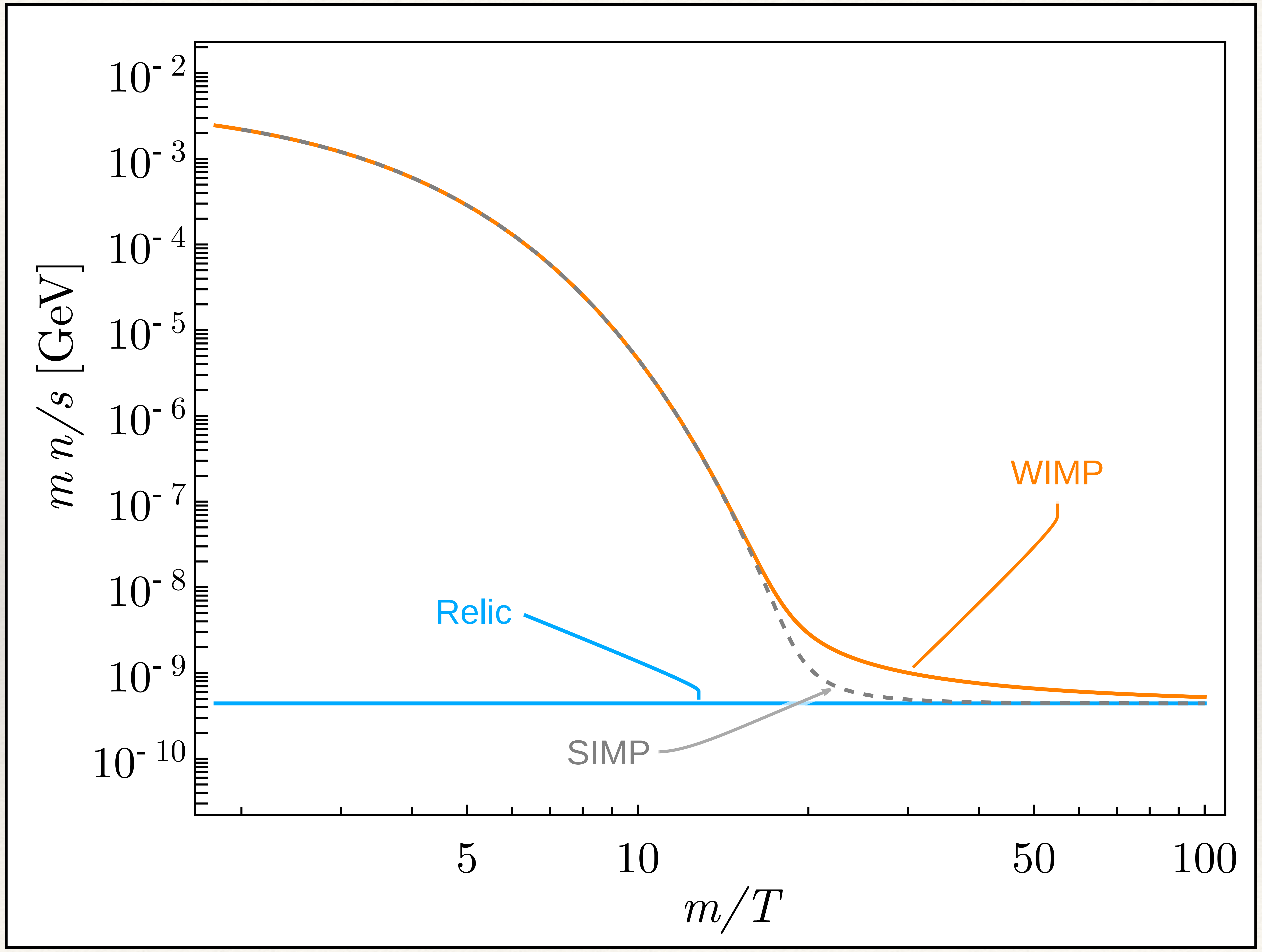

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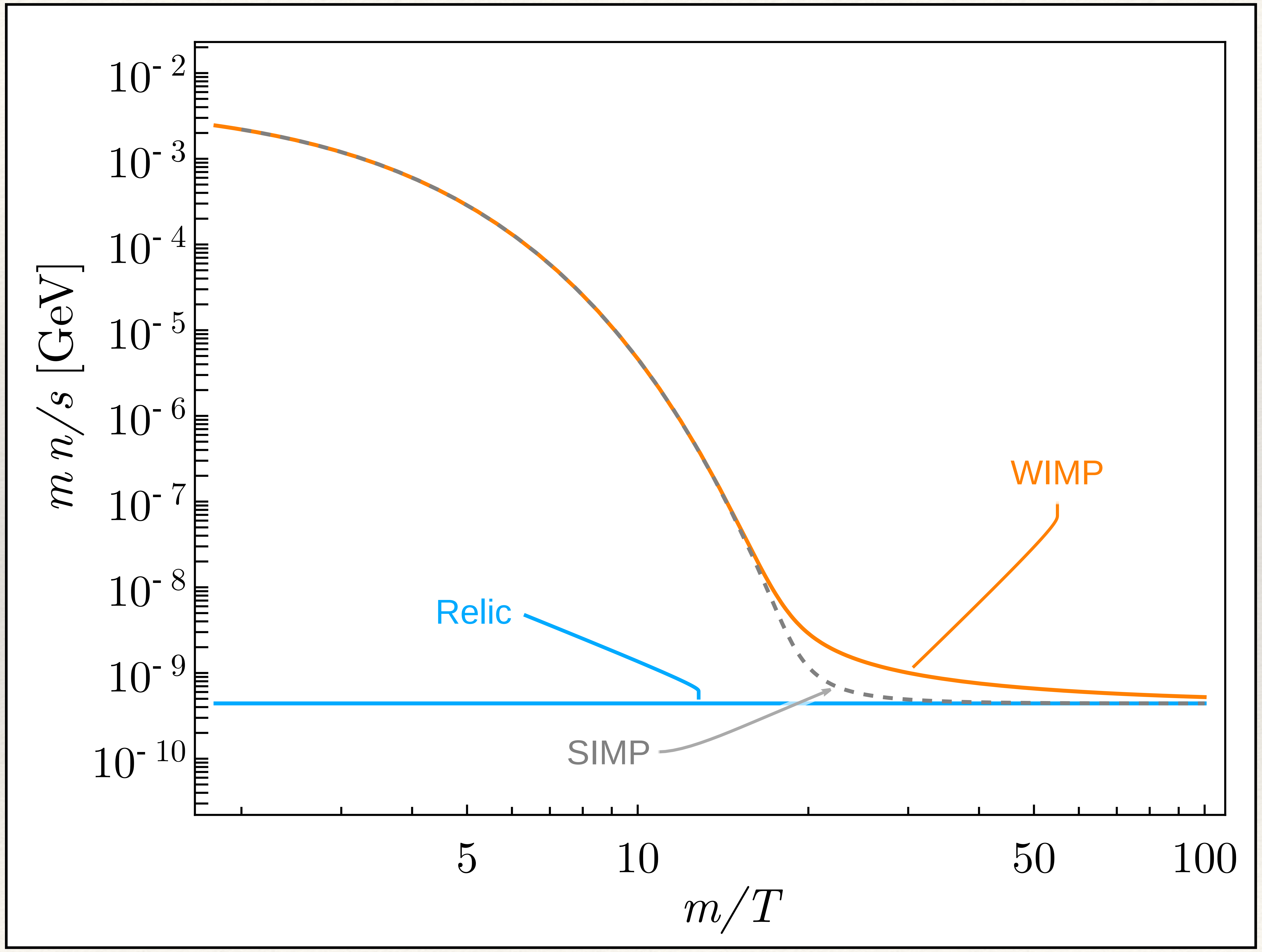
Relic density

- Possibility: Dark matter as as a thermal relic from the early universe
- Handle on the dark matter abundance
- Solve Boltzmann equations
  - Temperature decreases → interaction "freezes out"



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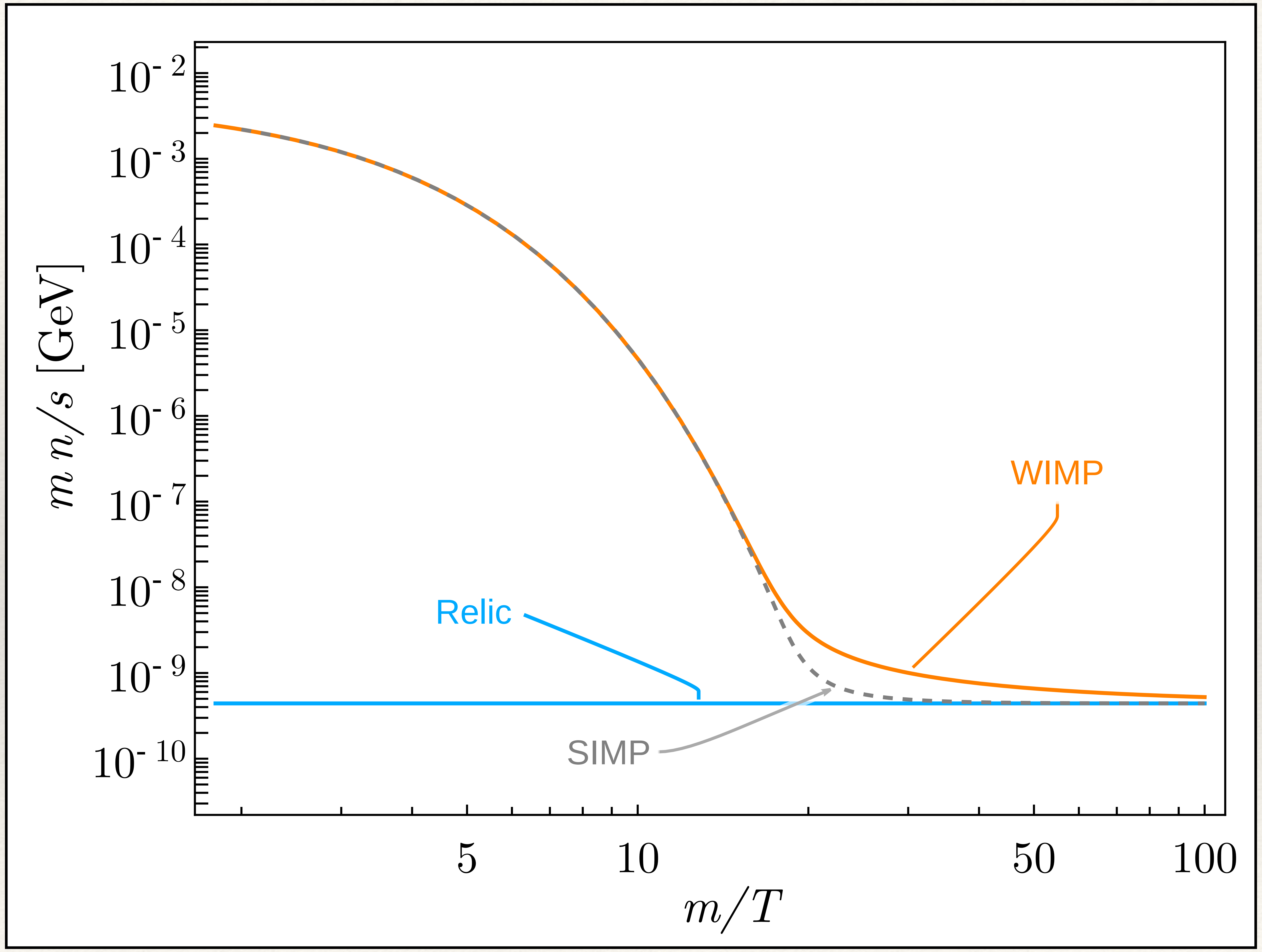
Relic density

- Possibility: Dark matter as as a thermal relic from the early universe
- Handle on the dark matter abundance
- Solve Boltzmann equations
  - Temperature decreases → interaction "freezes out"
- Example:
  - WIMP: DM + DM → SM + SM



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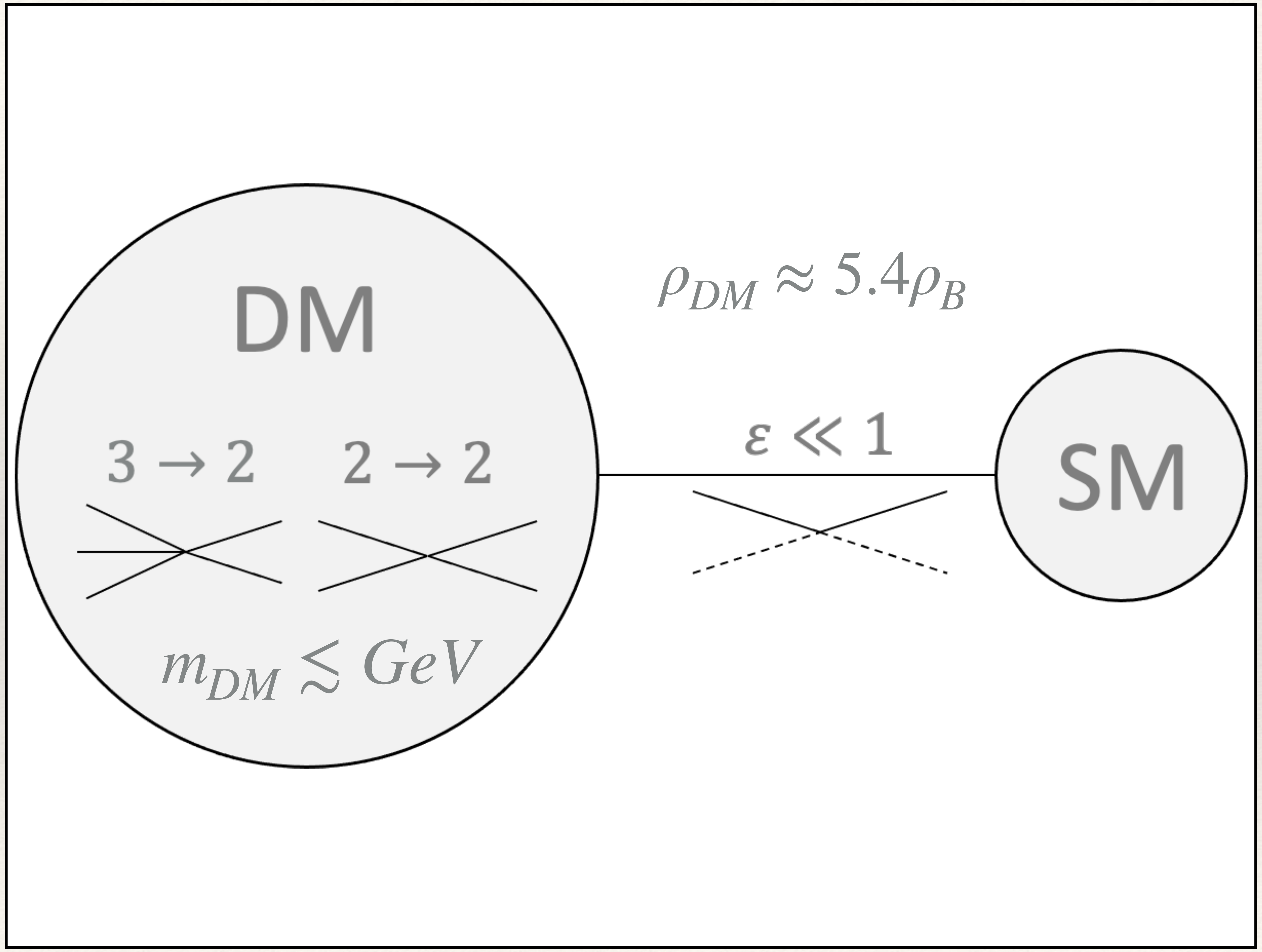
Strongly Interacting Massive Particles

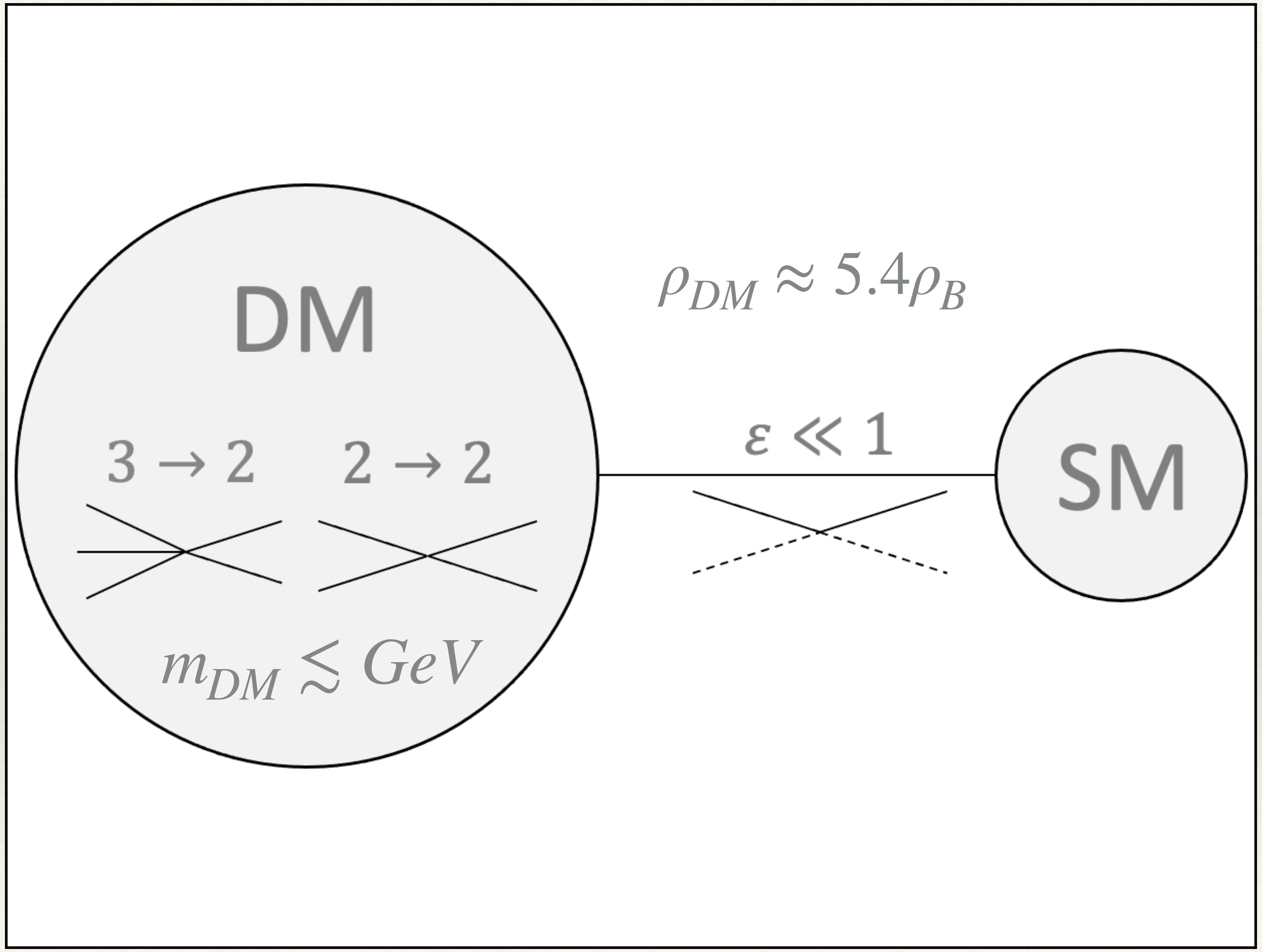
- Alternative freeze-out paradigm



Hochberg et al.: Phys. Rev. Lett. 113 (2014)

Strongly Interacting Massive Particles

- Alternative freeze-out paradigm
- Number lowering process in the dark sector
    - Addresses self-interaction



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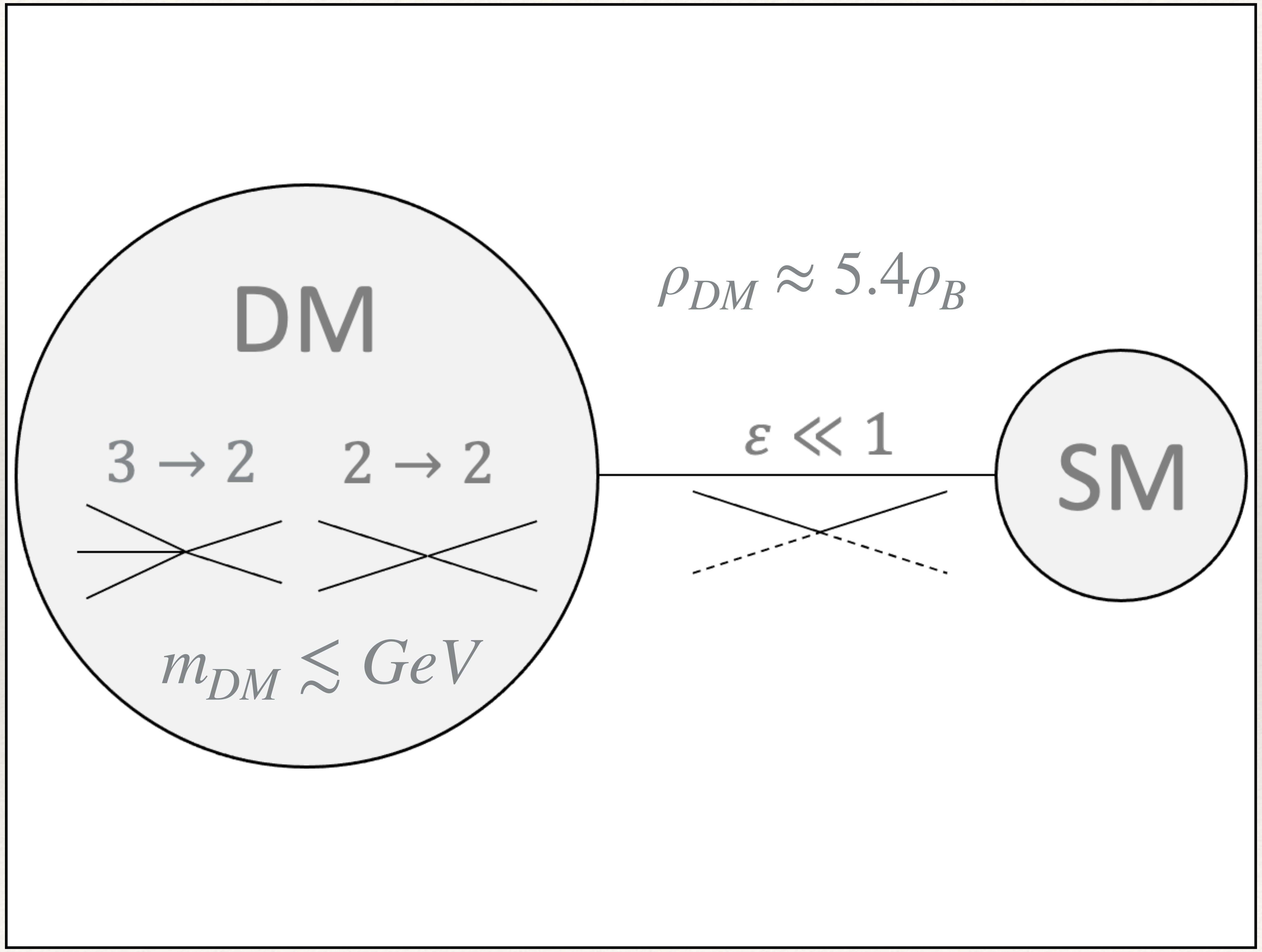
Strongly Interacting Massive Particles

- Alternative freeze-out paradigm
- Number lowering process in the dark sector
    - Addresses self-interaction
- Coupling to the SM sector needed to prevent heat-up
    - Mediator enables direct detection



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UV realisation

- Strong coupling arises *naturally* in confining gauge theories
  - $\mathscr{L} = -\frac{1}{2} F_{\mu\nu} F^{\mu\nu} + \bar{q}_i (i\gamma^\mu D_\mu - m_i) q_i$

Kulkarni et al.: SciPost Phys. 14 (2023)

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  - Fundamental, adjoint, antisymmetric, …

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Symmetry of the UV Lagrangian

Representation of gauge group	Flavour symmetry
Complex	U(2)xU(2)
Real	U(4)
Pseudoreal	U(4)

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- Symmetry depends on representation
  - Fundamental, adjoint, antisymmetric, …
- Also different breaking patterns

Symmetry of the UV Lagrangian

Representation of gauge group	Flavour symmetry
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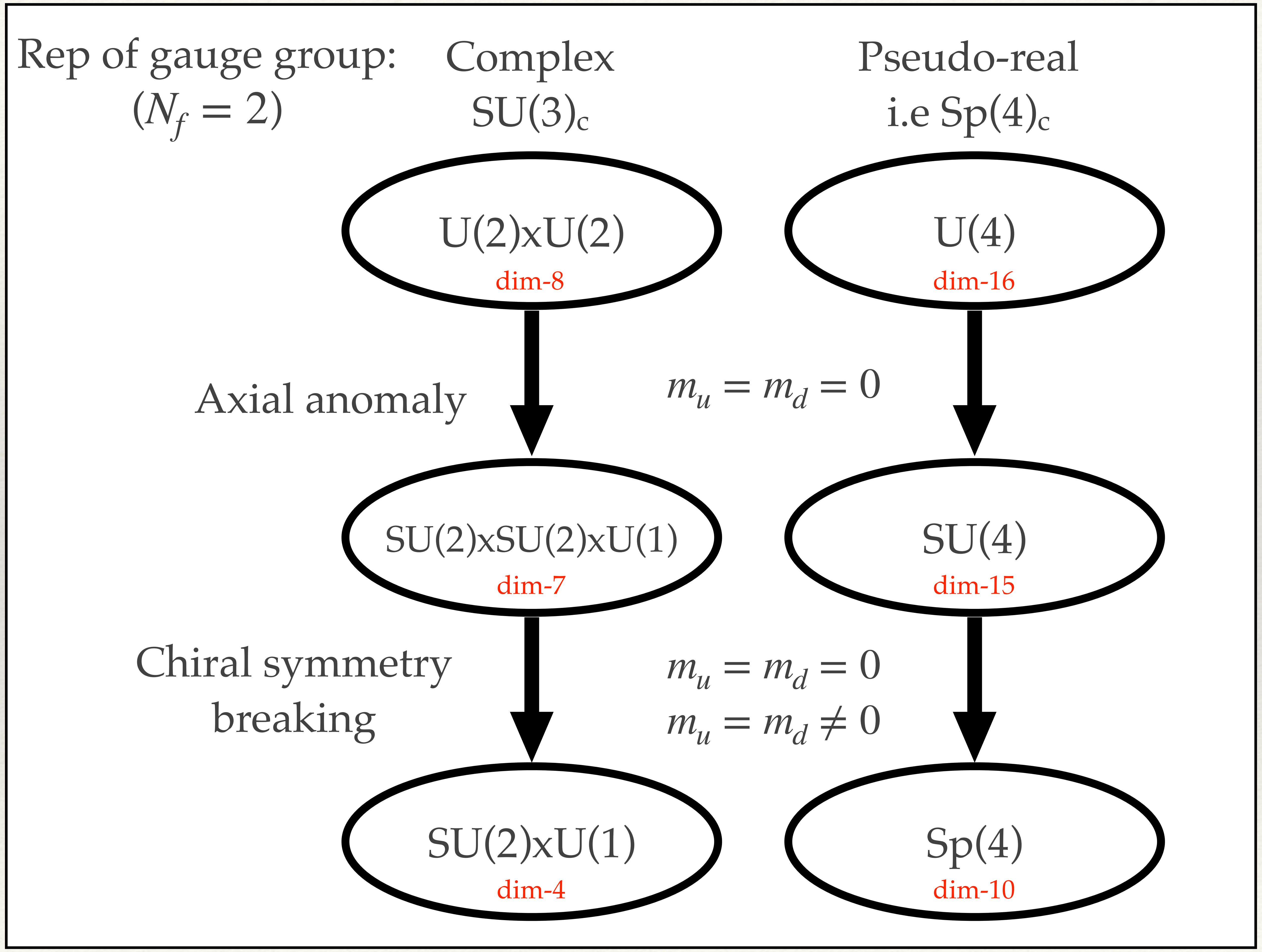
Minimal realisation

- Pseudo-real rep of gauge group with $N_f = 2$
  - Sp(4) flavour symmetry



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- Pseudo-real rep of gauge group with $N_f = 2$
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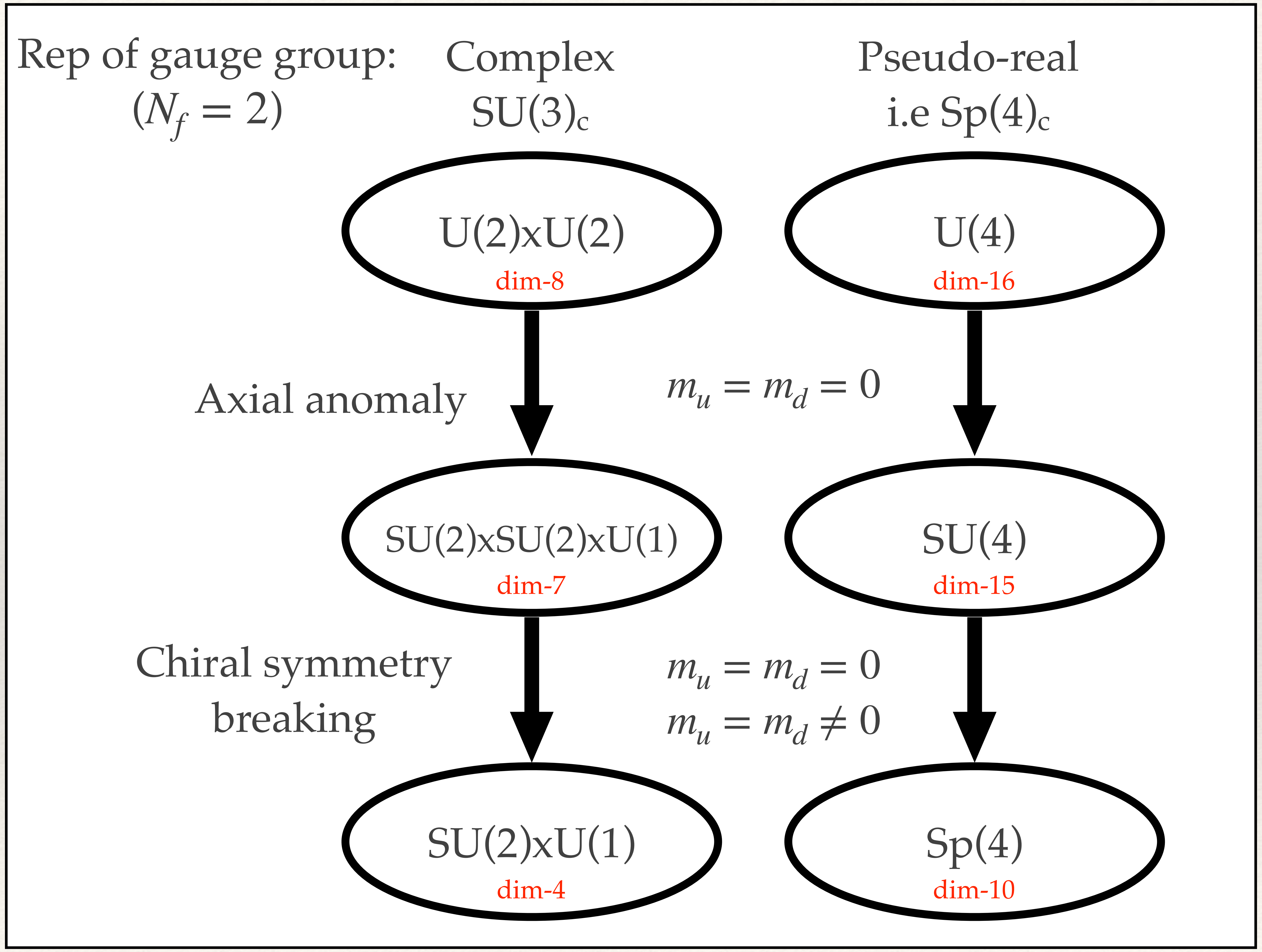
- Mixing of left- and right handed components (Weyl-fermions)
  - Symmetry is enlarged



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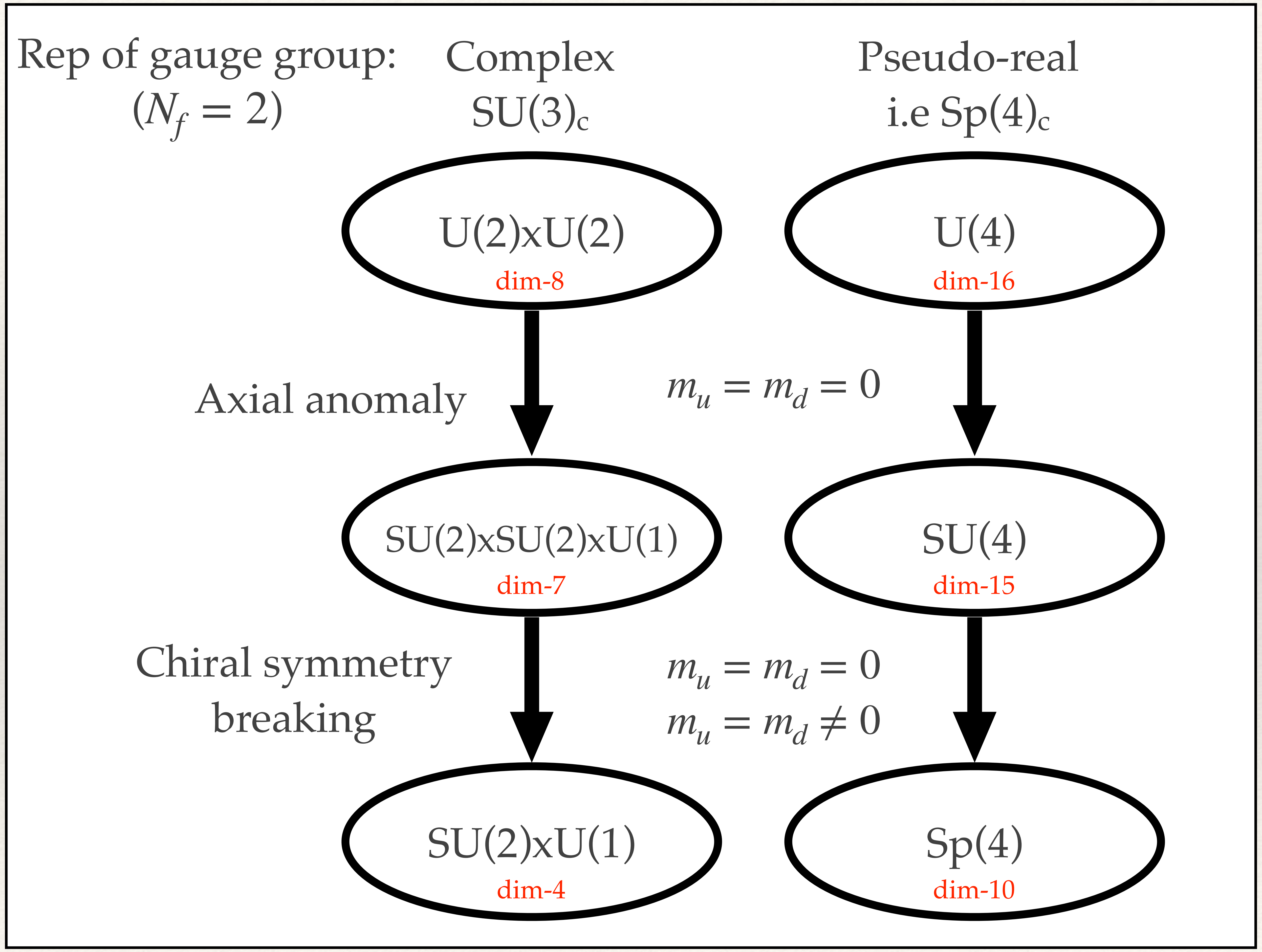
- Pseudo-real rep of gauge group with $N_f = 2$
  - Sp(4) flavour symmetry
- Mixing of left- and right handed components (Weyl-fermions)
  - Symmetry is enlarged
- Result: 5 pNGBs
  - $3 \to 2$ process possible
  - WZW description in ChPT



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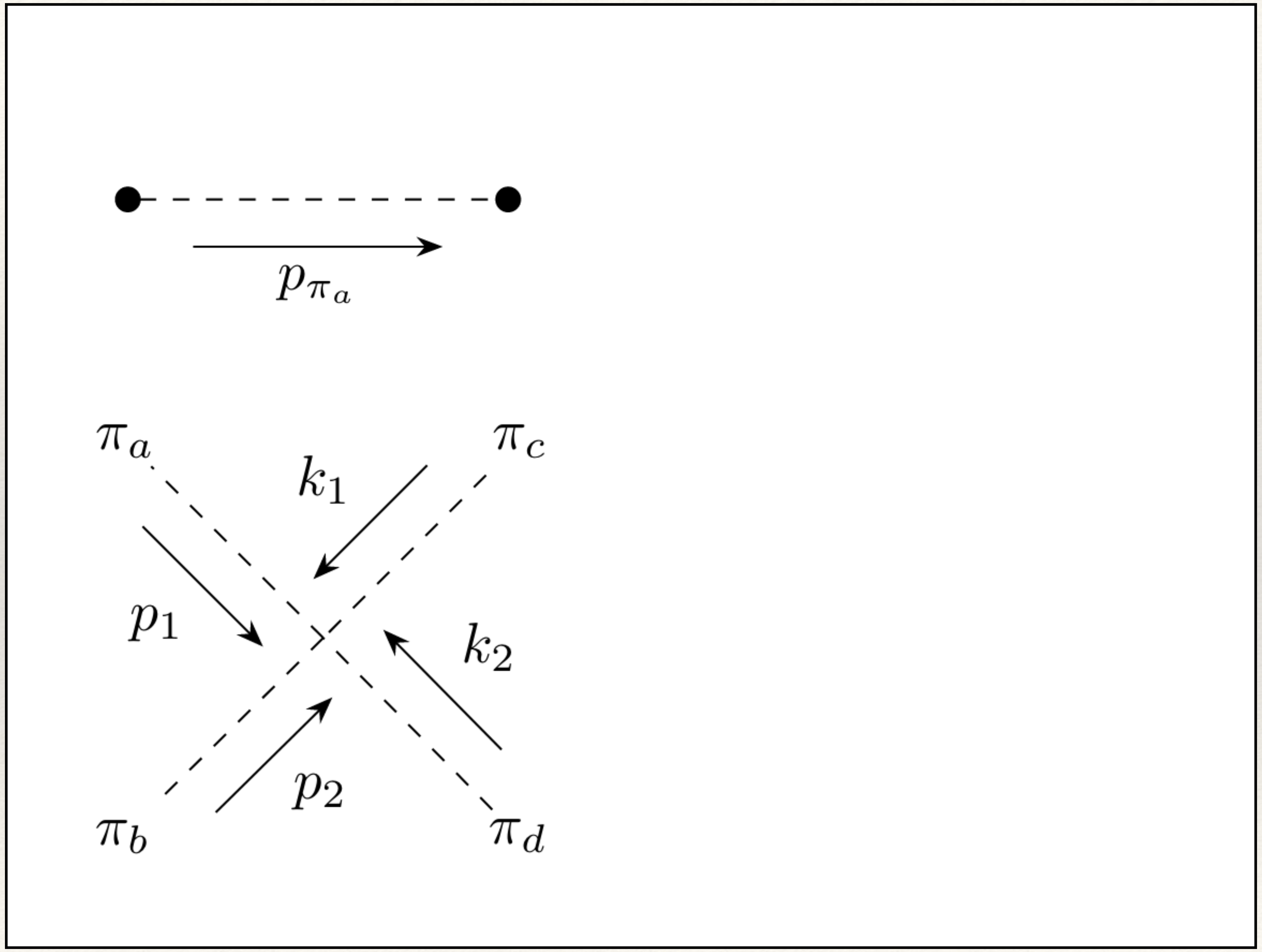
Sp(4) ChPT

- Effective description in terms of 5 *dark Pions*



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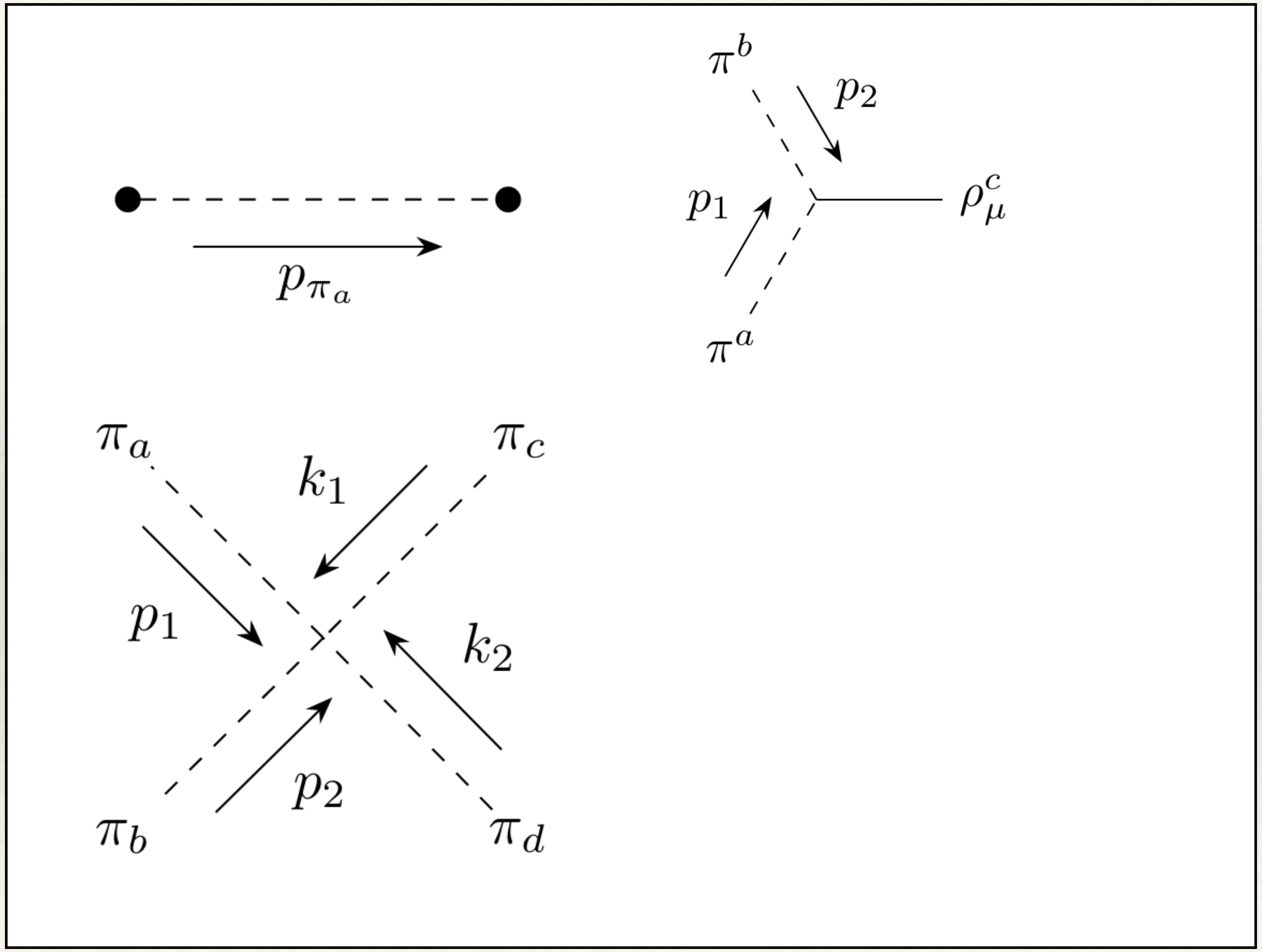
Sp(4) ChPT

- Effective description in terms of 5 *dark Pions*
- Include a vector particle and a mediator to the standard model



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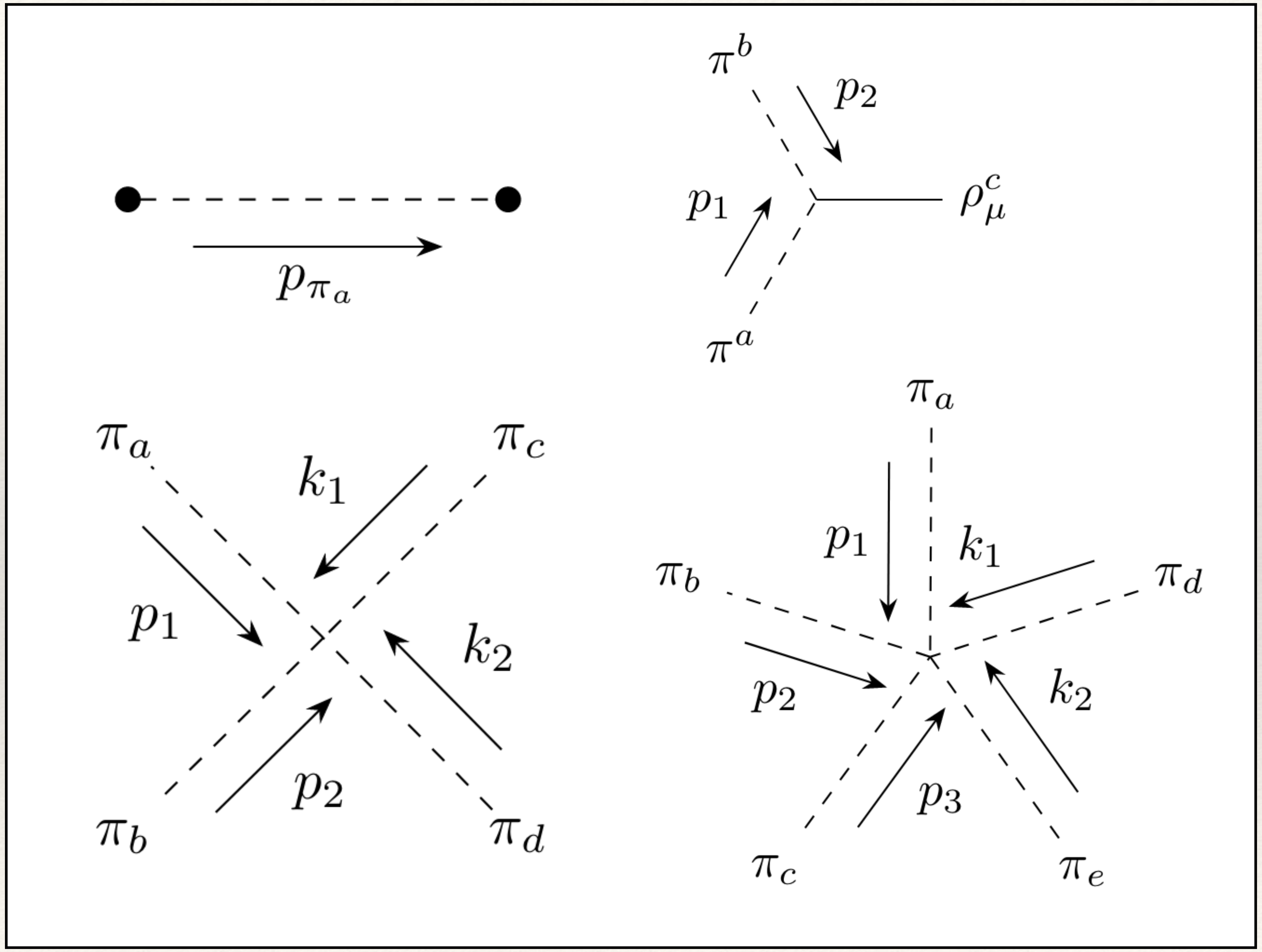
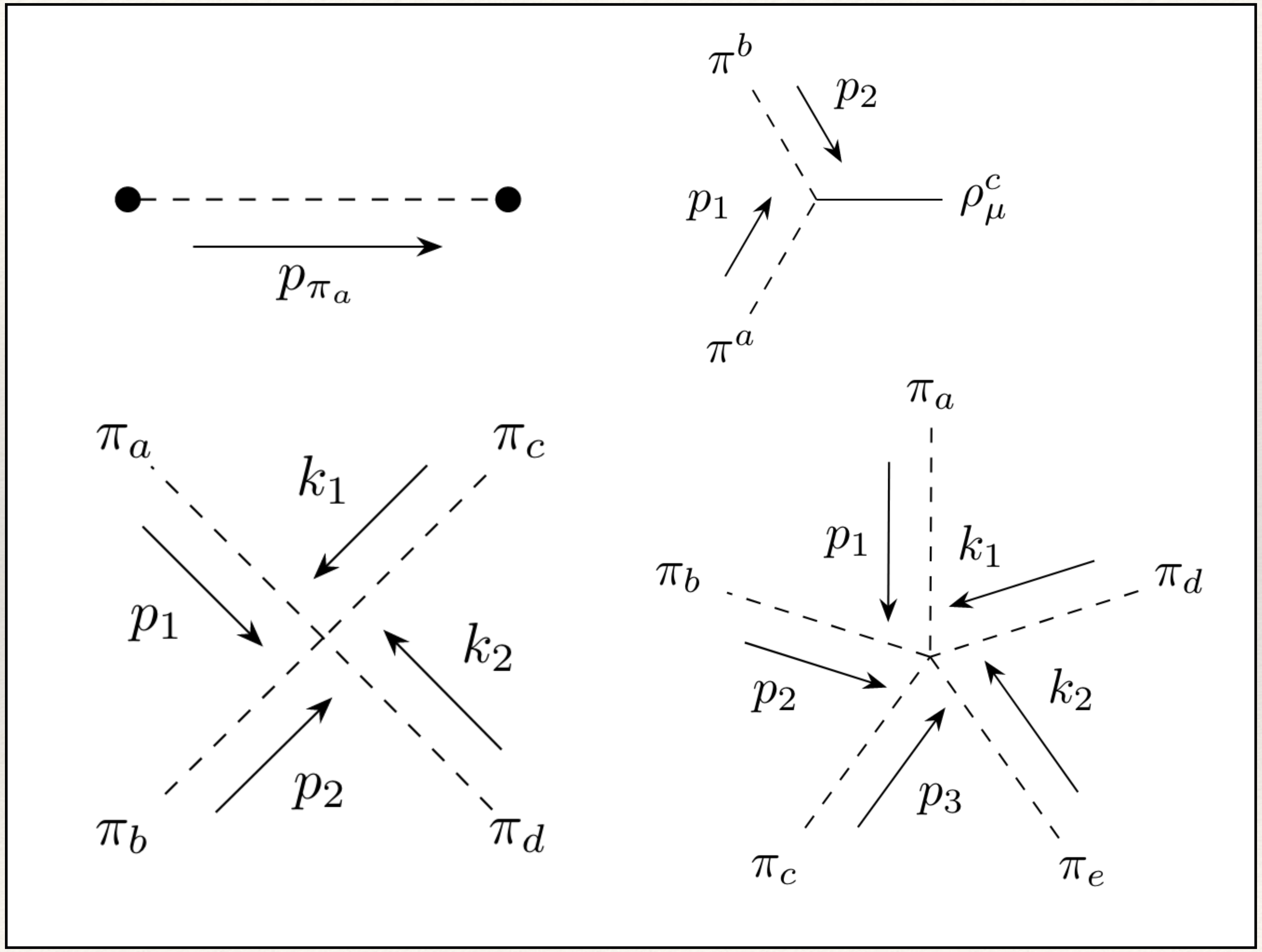


Kulkarni et al.: SciPost Phys. 14 (2023)

Sp(4) ChPT

- Effective description in terms of 5 *dark Pions*
- Include a vector particle and a mediator to the standard model
- Include 3 → 2 via Wess-Zumino-Witten term
- Relies on low energy constants
  - Masses, scattering length, …



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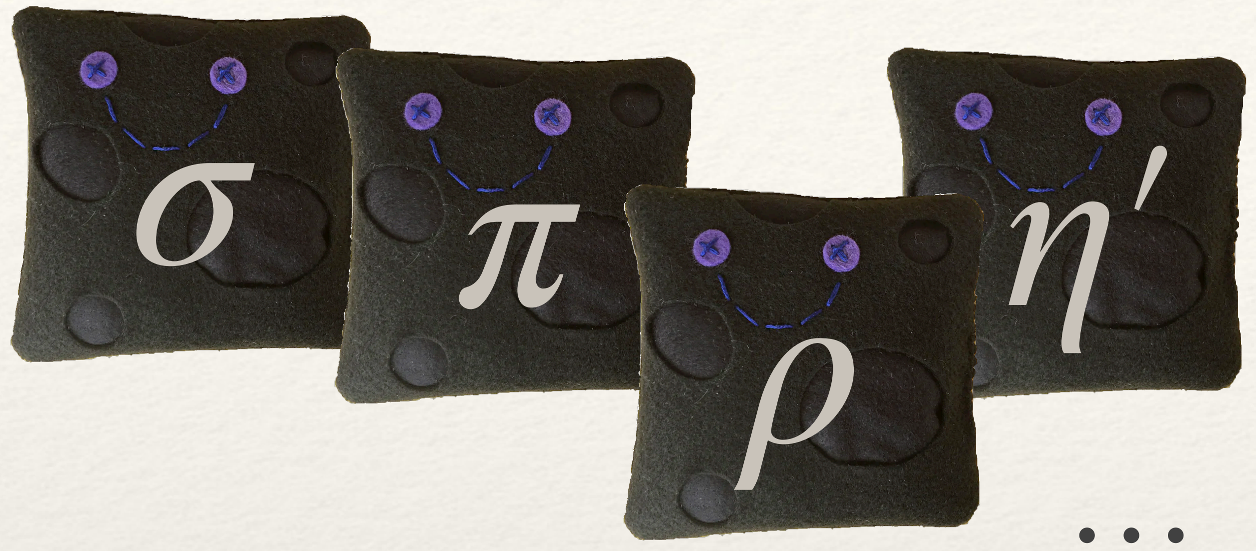
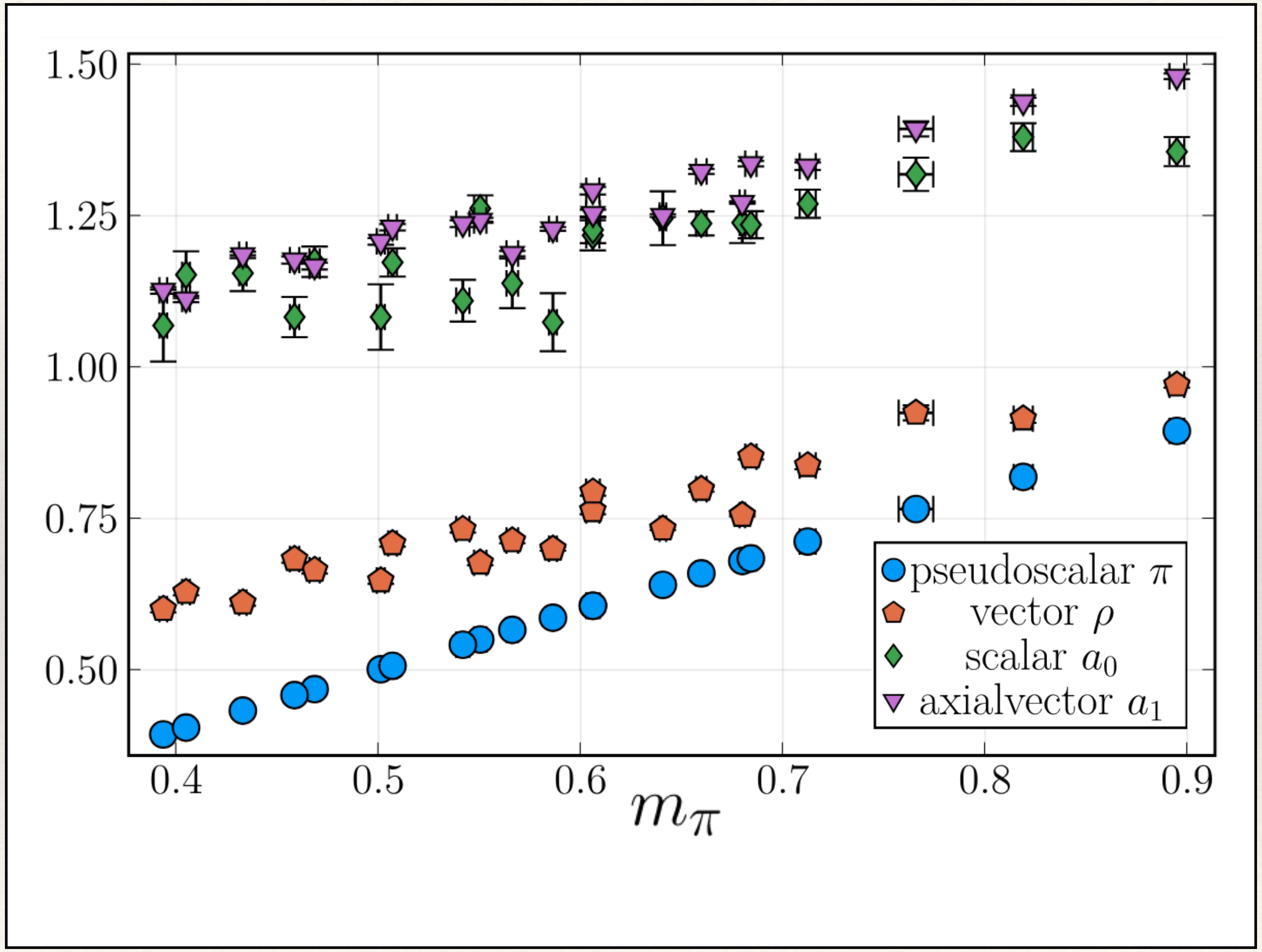
Particle phenomenology



- Zoo of dark hadrons



Bennett et al. - JHEP 12 (2019)
Bennett et al - Phys. Rev. D 109 (2024)

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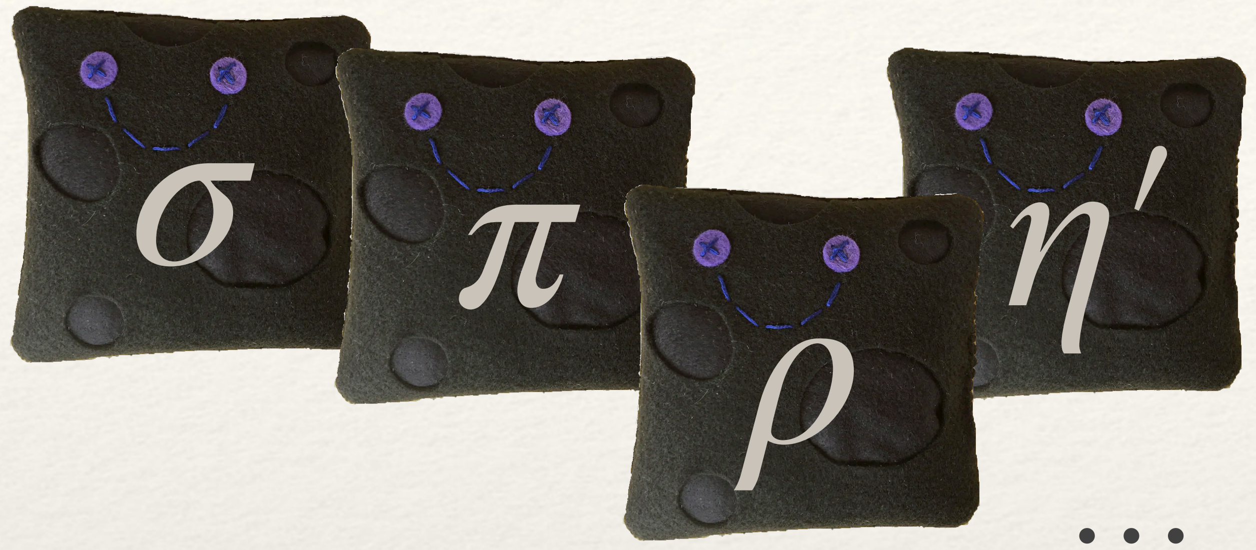
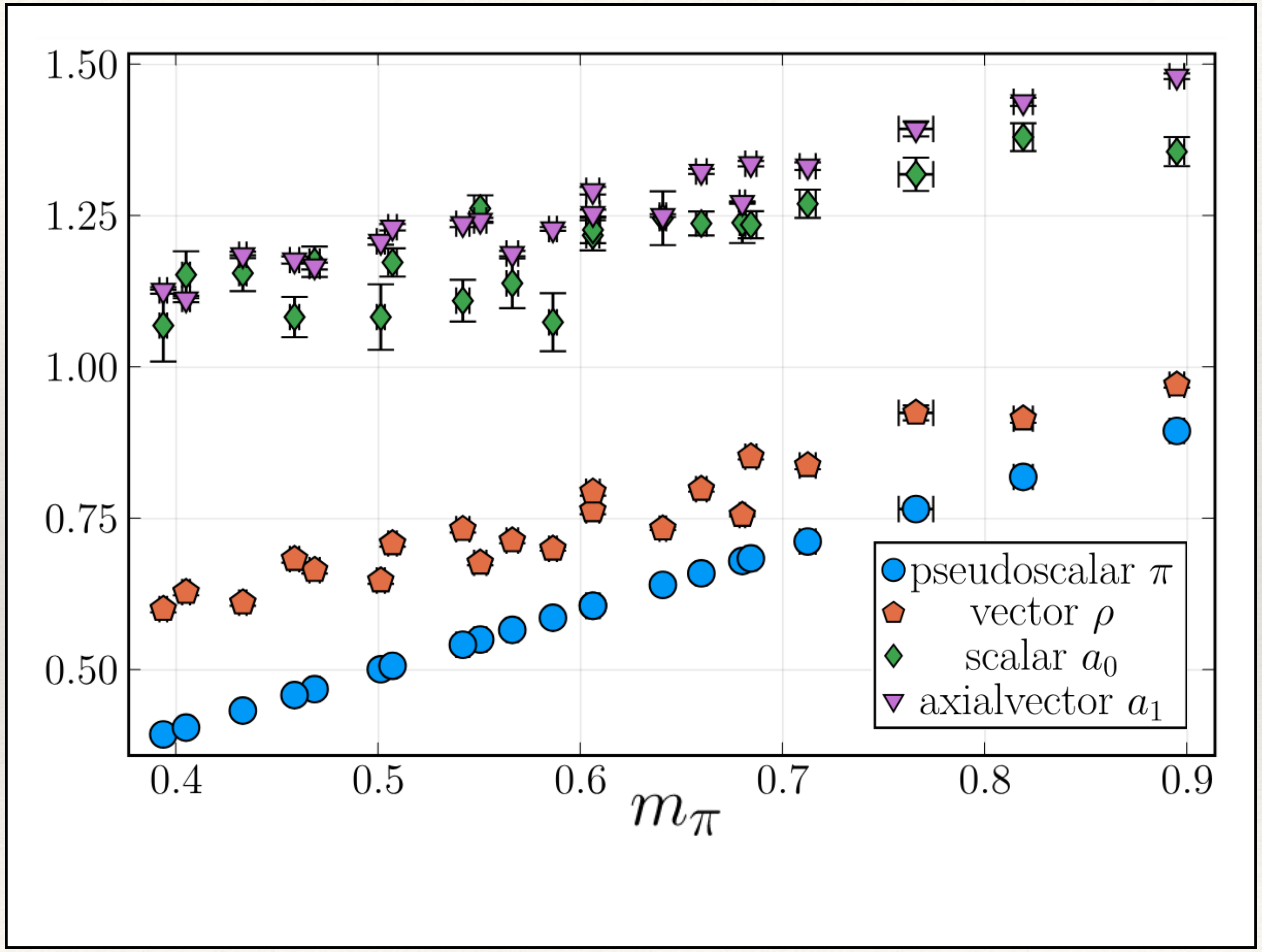


- Zoo of dark hadrons
- 5 Pions & 10 Rhos lightest non-singlets



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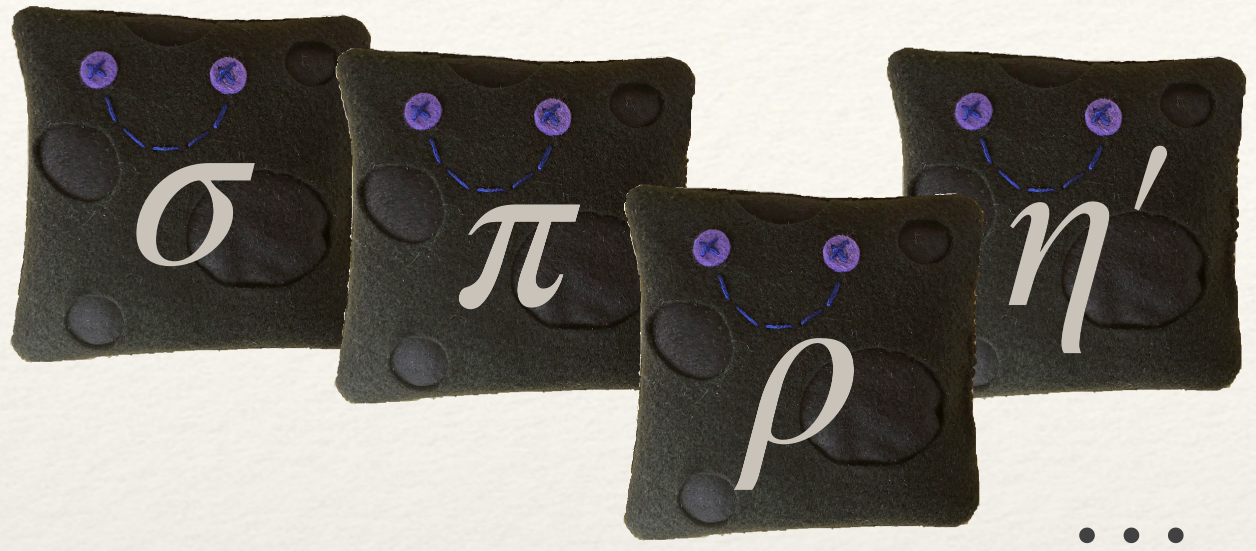
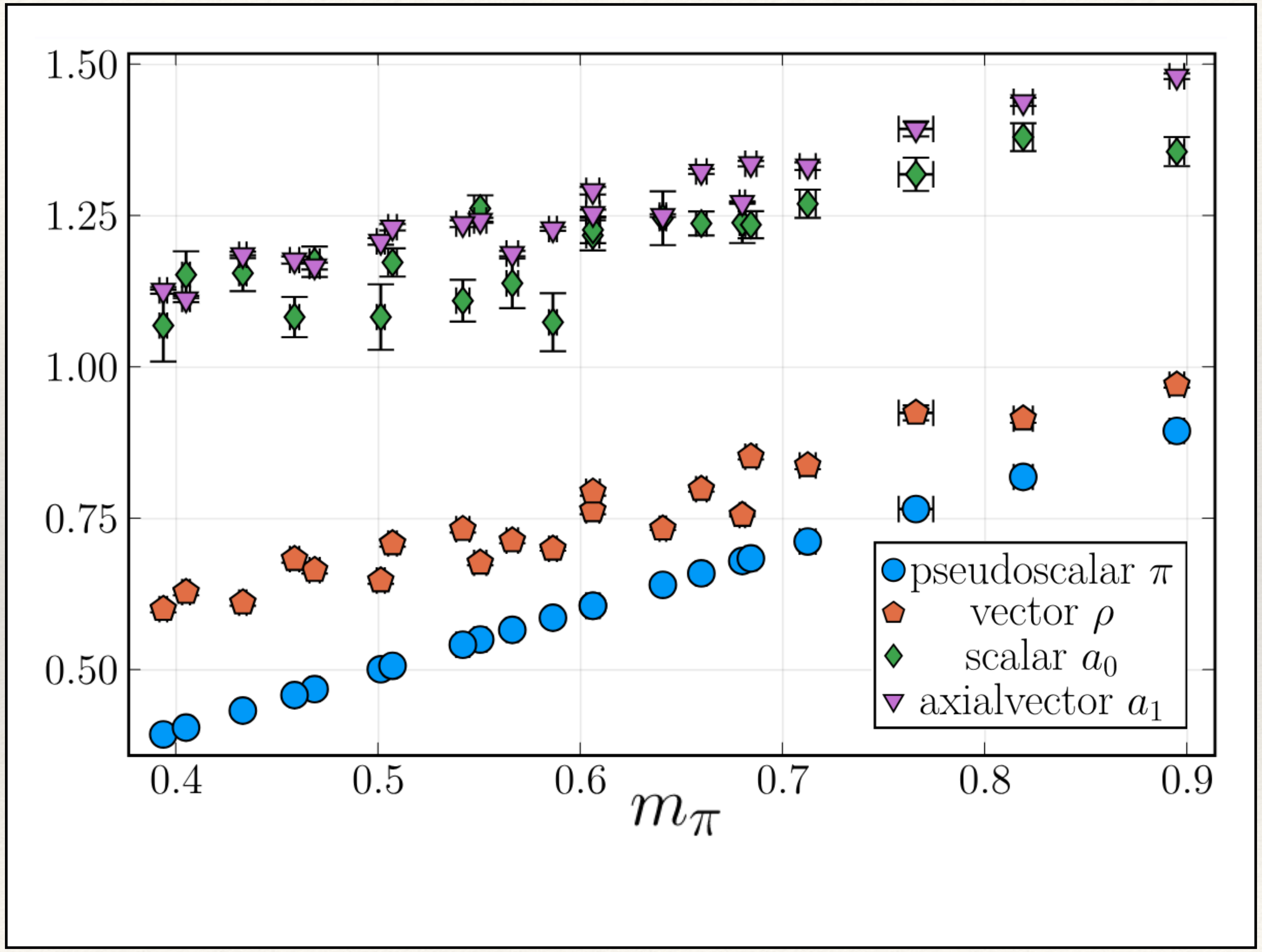


- Zoo of dark hadrons
- 5 Pions & 10 Rhos lightest non-singlets
- No fermionic bound states



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Bennett et al - Phys. Rev. D 109 (2024)

Scattering phenomenology

- **14**-dim:
  - (Probably) contributes most to $\pi\pi$ -scattering
  - 14 out of 25 possible combinations of Pions

$Sp(4)_f$

$$5 \otimes 5 = 1 \oplus 10 \oplus 14$$

$\pi\pi \rightarrow \pi\pi$ (I=0,1,2)

$\pi\pi \rightarrow \rho$ (I=1)

$\pi\pi \rightarrow \pi\pi\pi$ (I=1)

etc.

Feger et al.: Comp.Phys.Com 257 (2020)

Scattering phenomenology

- **1**-dim:
  - (Probably) no large contribution to $\pi\pi$ -scattering
  - Mixes in other scattering channel
  - Numerically challenging

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etc.

Feger et al.: Comp.Phys.Com 257 (2020)

Scattering phenomenology

- **10**-dim:
  - Mixing with the Rho
  - $\pi\pi\pi \to \pi\pi$
- Work in progress

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etc.

Feger et al.: Comp.Phys.Com 257 (2020)

Phenomenology of scattering channels

Done ✅

- **14-dim:**
  - Makes up most $\pi\pi$ scattering (14/25)
  - Easiest on the lattice
- **10-dim:**
  - Mixing with dark ρ
  - $\pi\pi\pi \to \pi\pi$
- **1-dim:**
  - Mixing with other states

Work in progress

$Sp(4)_f$

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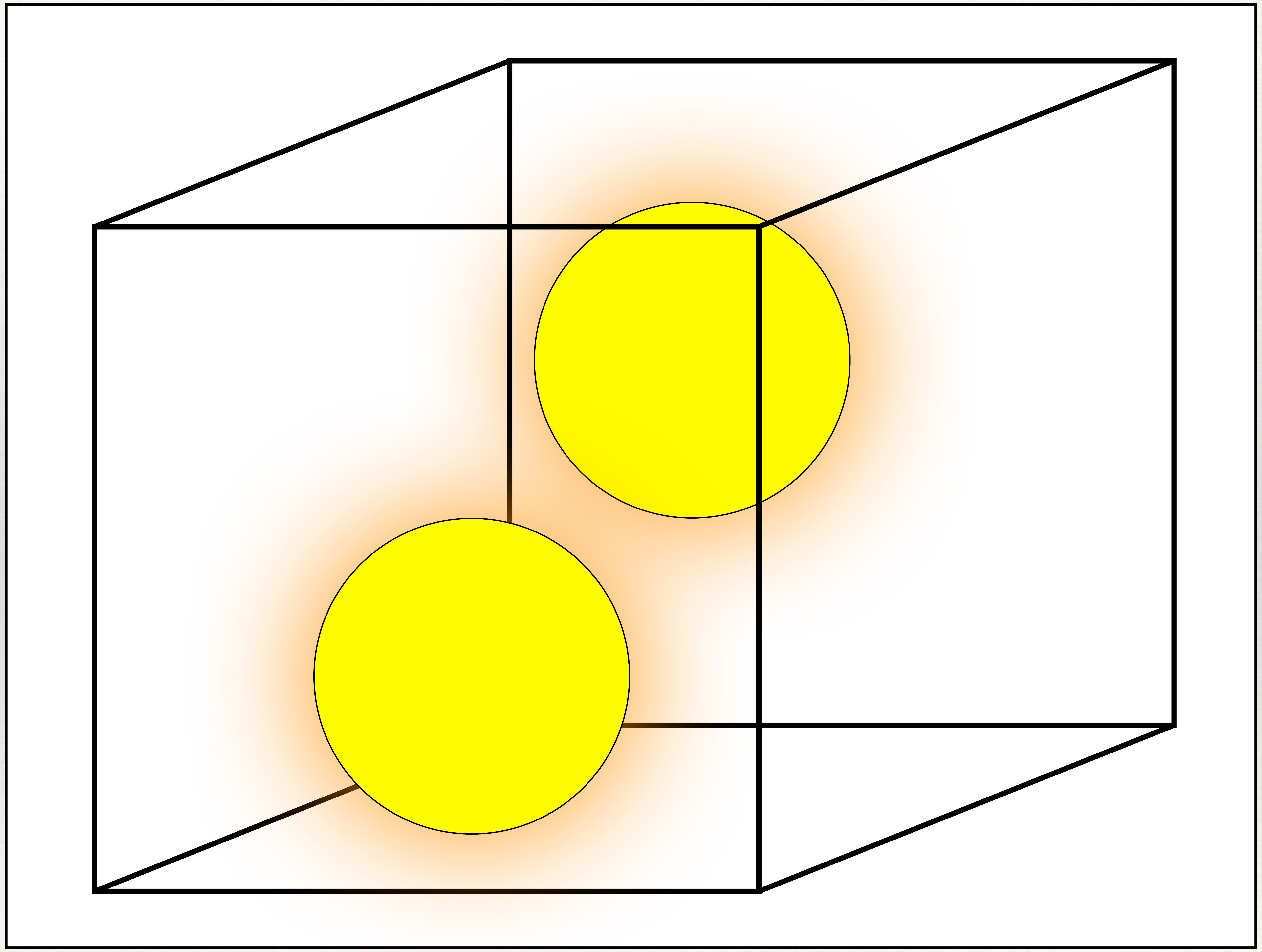
Scattering on the lattice

- Relate finite volume energy levels with infinite volume scattering properties
  - "Lüscher quantization condition"



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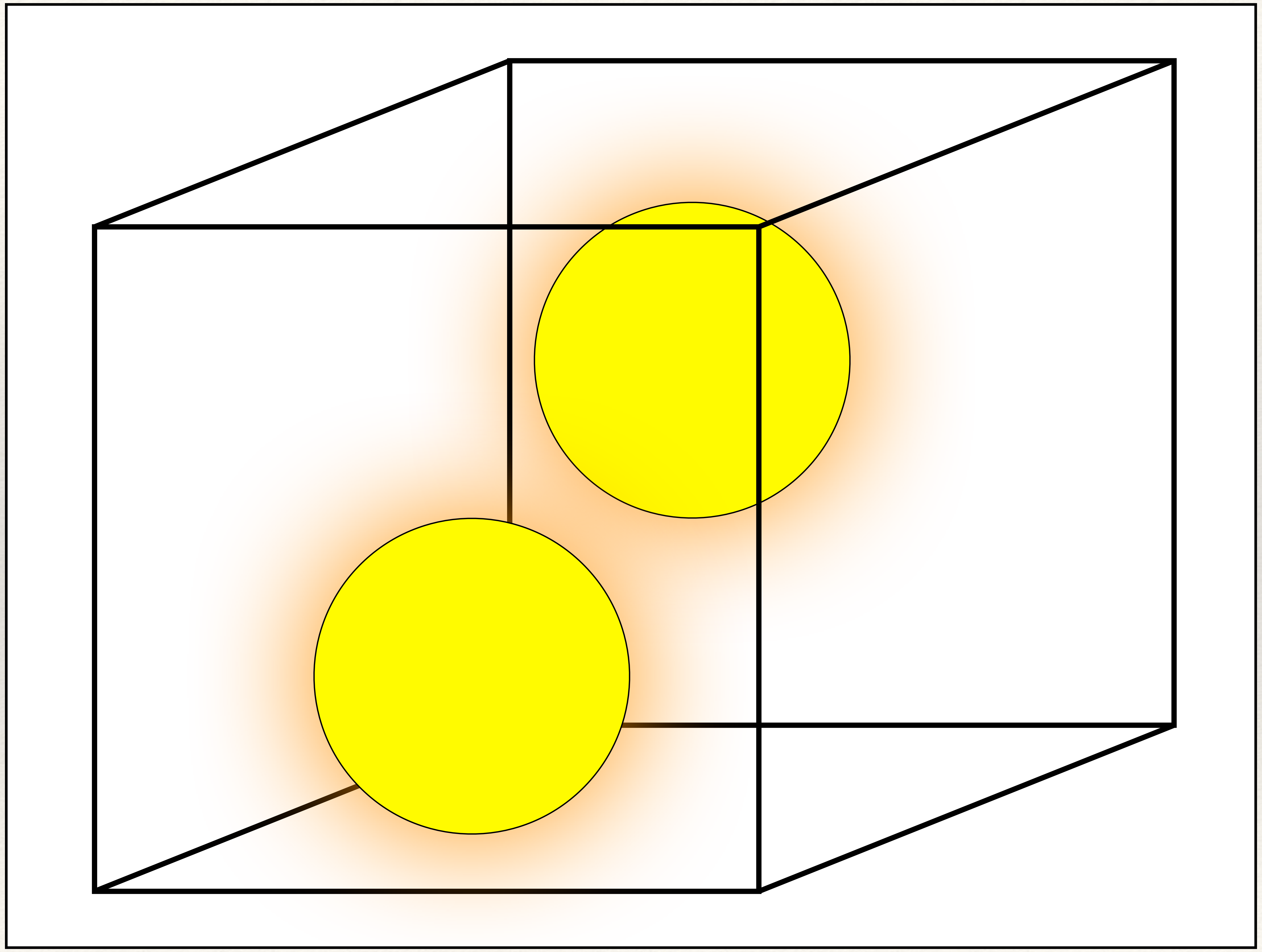

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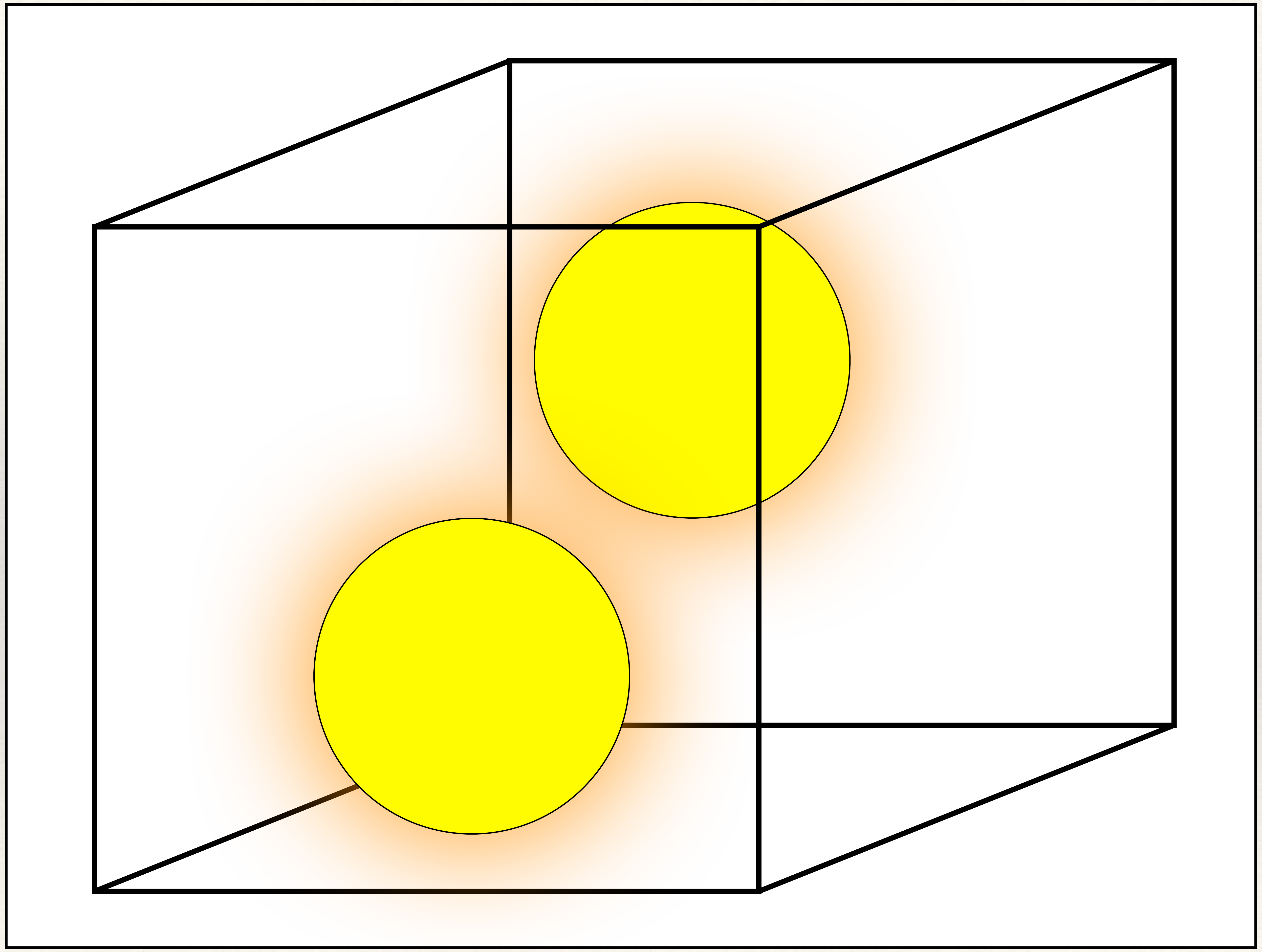
- Relate finite volume energy levels with infinite volume scattering properties
    - "Lüscher quantization condition"
- $\tan(\delta(\sqrt{s})) = f(E, \overrightarrow{P}, L)|_{E=E(L)}$
- Result: Energy-dependent phase-shift
    - Scattering length, cross-section, …



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- **14**-dim channel:
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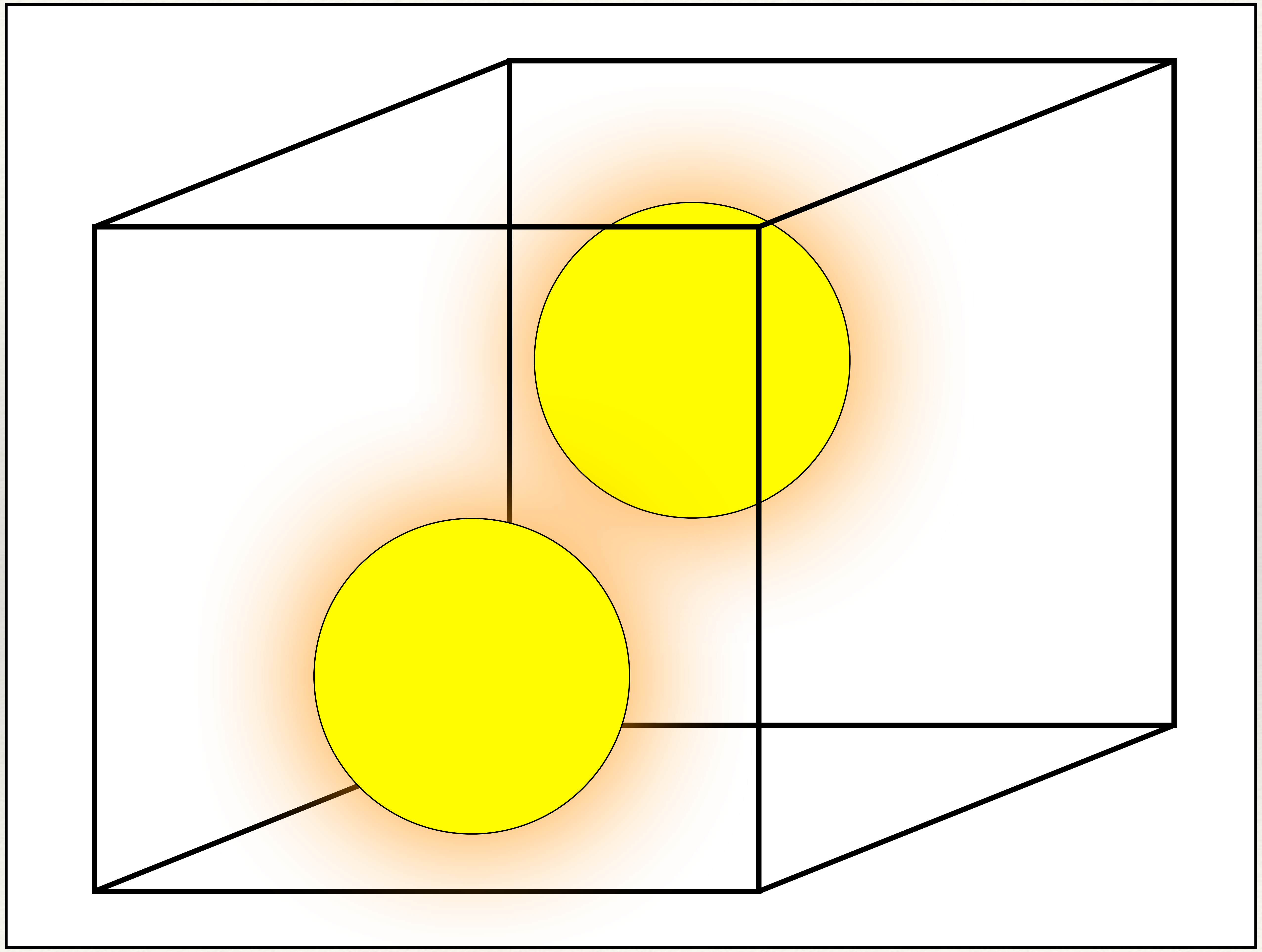
  - Comparison astro data



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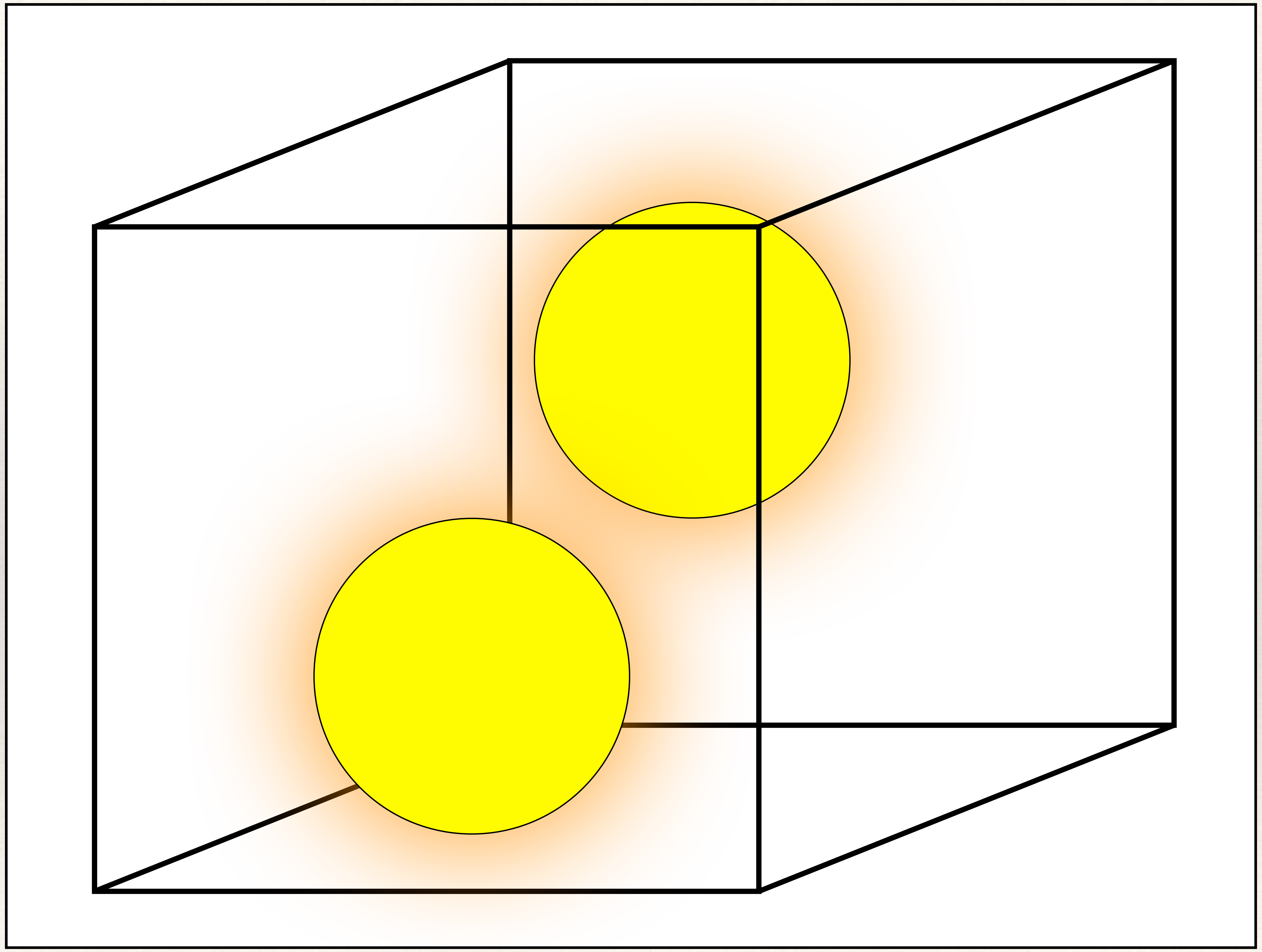
  - Comparison astro data
- **10**-dim channel:
  - Work in progress
  - Preliminary comparison to **14**



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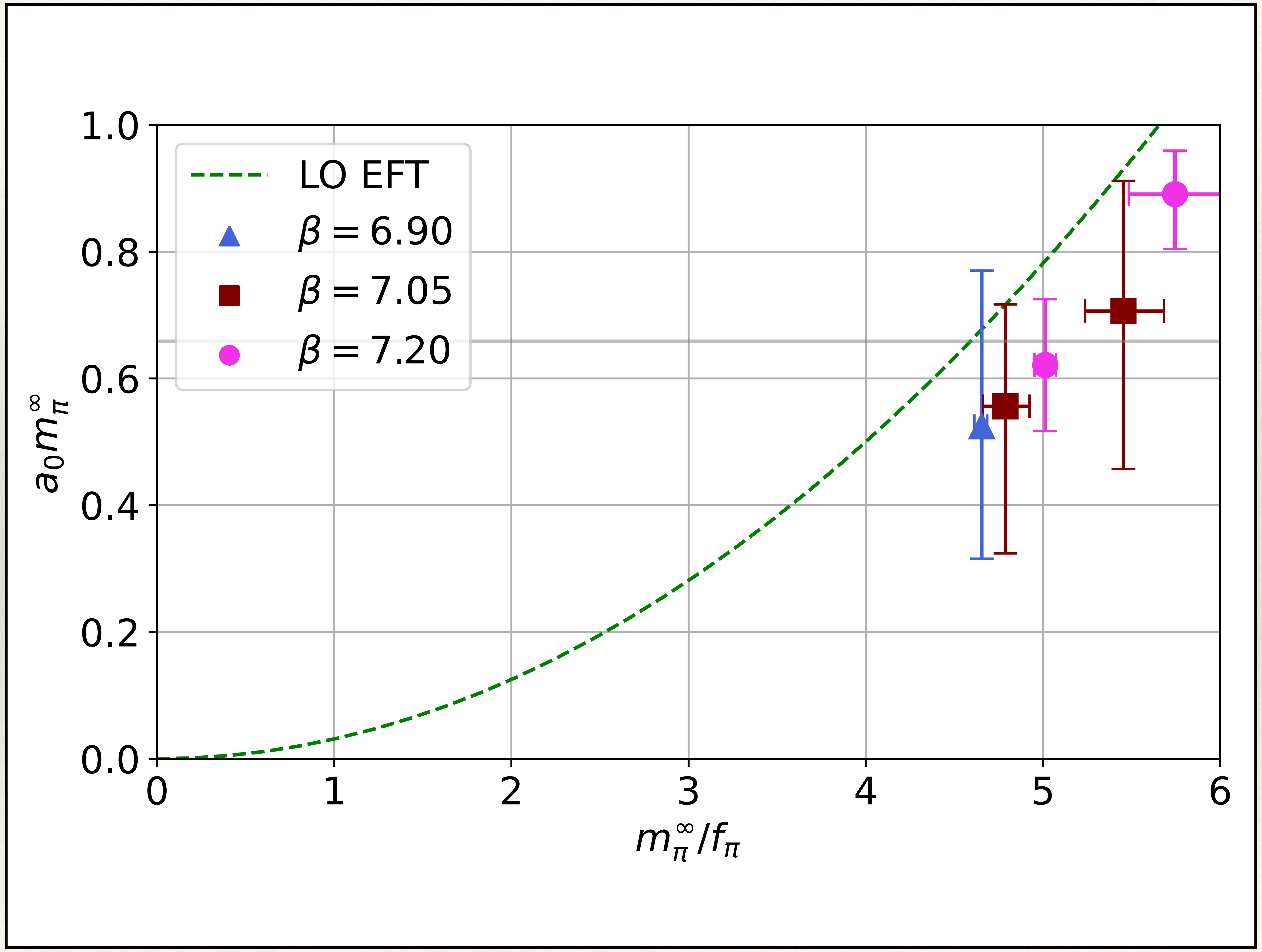
χ-pT comparison

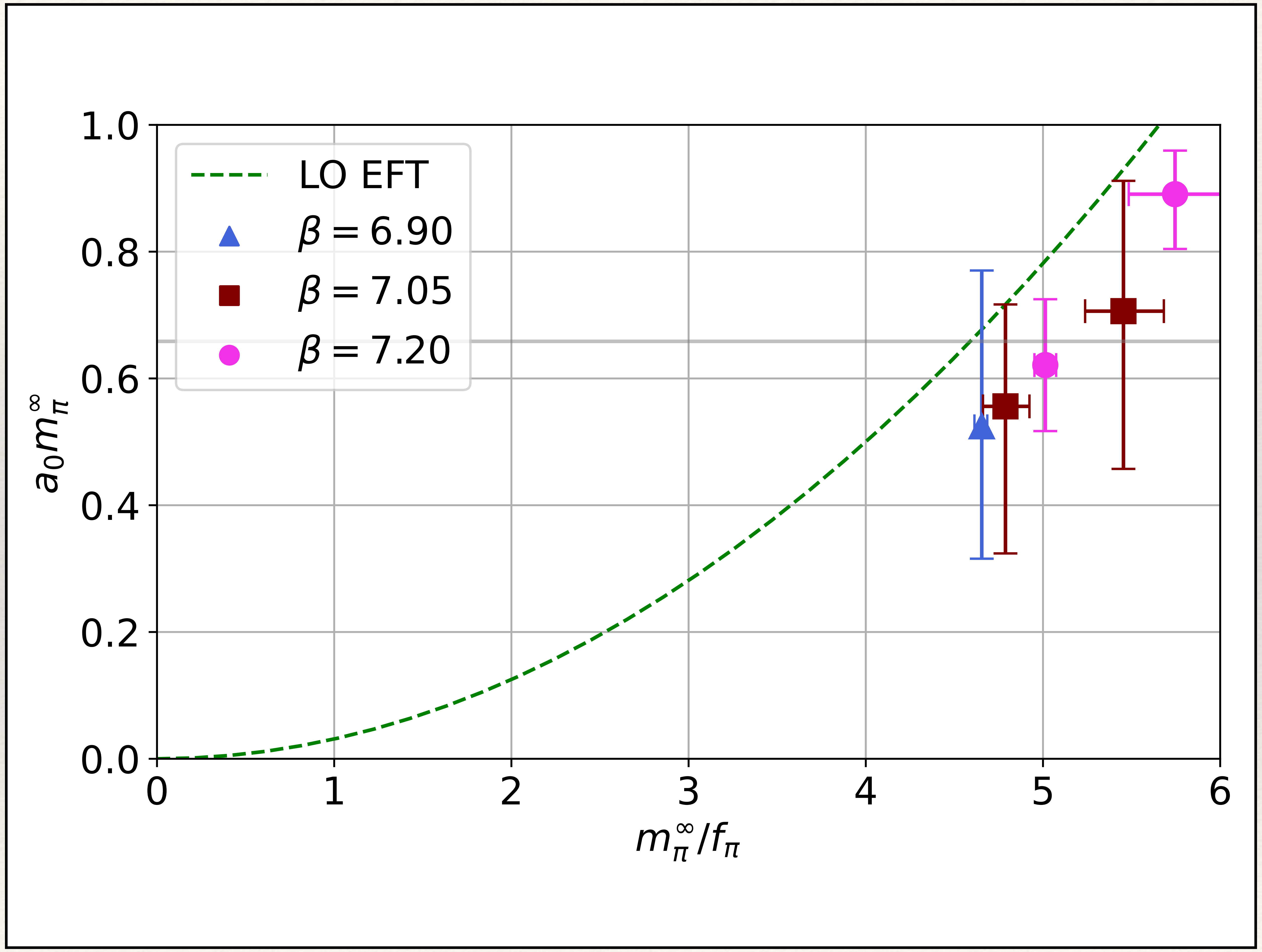
- Maximal channel



YD, F. Zierler and A. Maas: arXiv:2405.06506
Bijnens and Lu: JHEP, 03:028, (2011)

χ-pT comparison

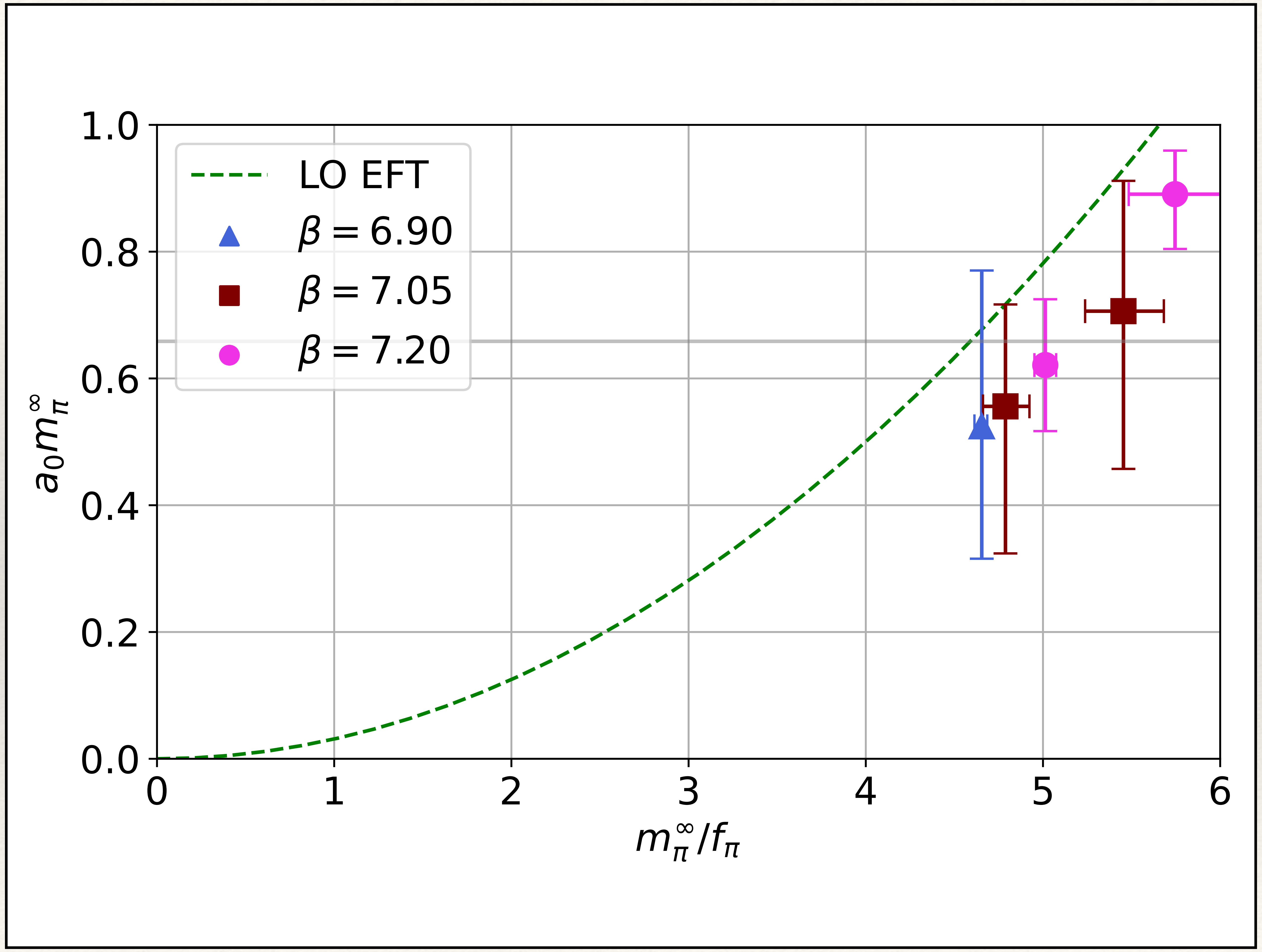
- Maximal channel
- Prediction: $a_0 m_\pi = \frac{1}{32}\left(\frac{m_\pi}{f_\pi}\right)^2$



YD, F. Zierler and A. Maas: arXiv:2405.06506
Bijnens and Lu: JHEP, 03:028, (2011)

χ-pT comparison

- Maximal channel
- Prediction: $a_0 m_\pi = \frac{1}{32}\left(\frac{m_\pi}{f_\pi}\right)^2$
- ChPT works well
  - NLO?



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$$\langle \sigma v \rangle = \int_0^{v_{esc}} dv\, \sigma(v)\, v\, f(v)$$

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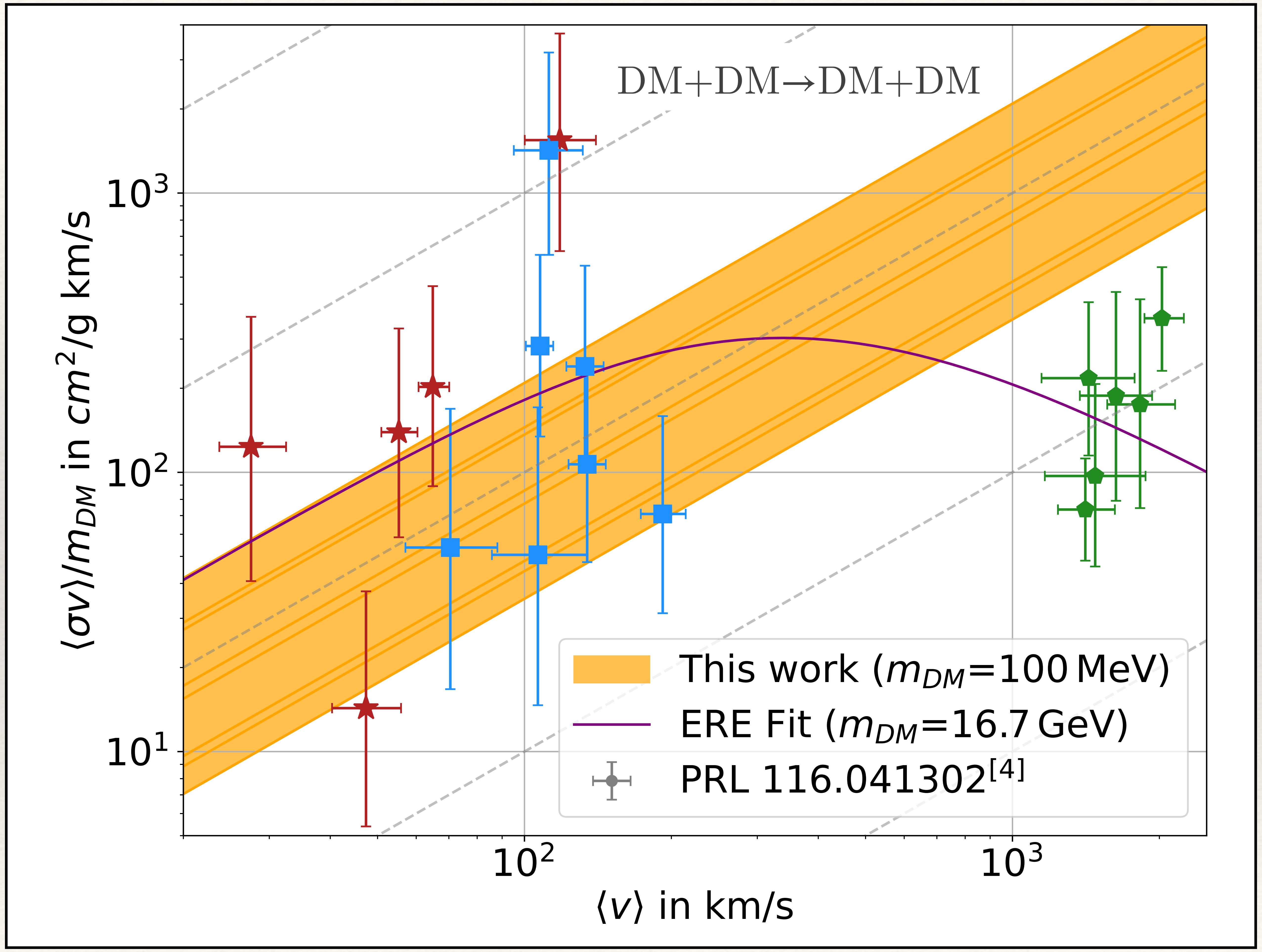

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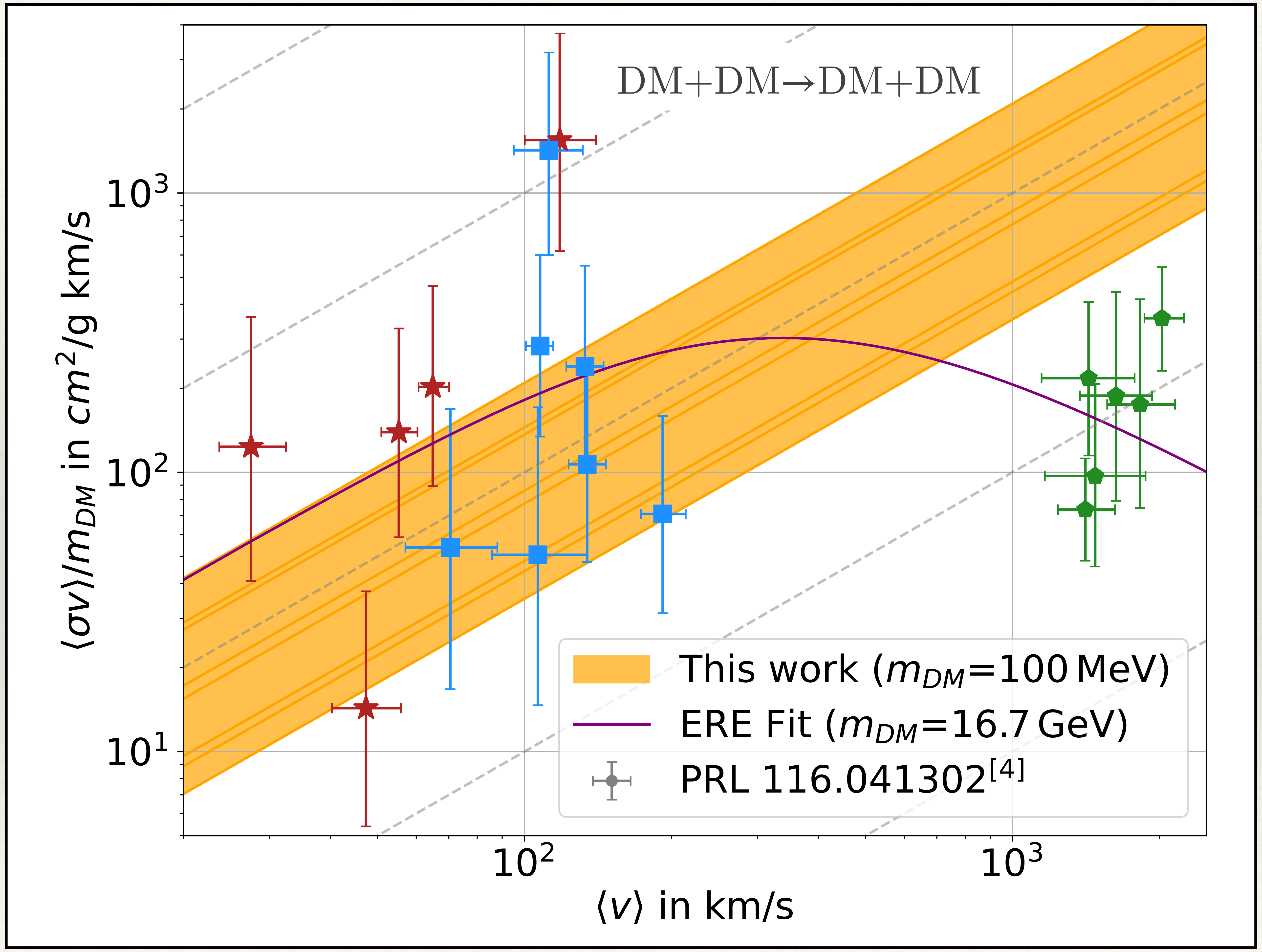

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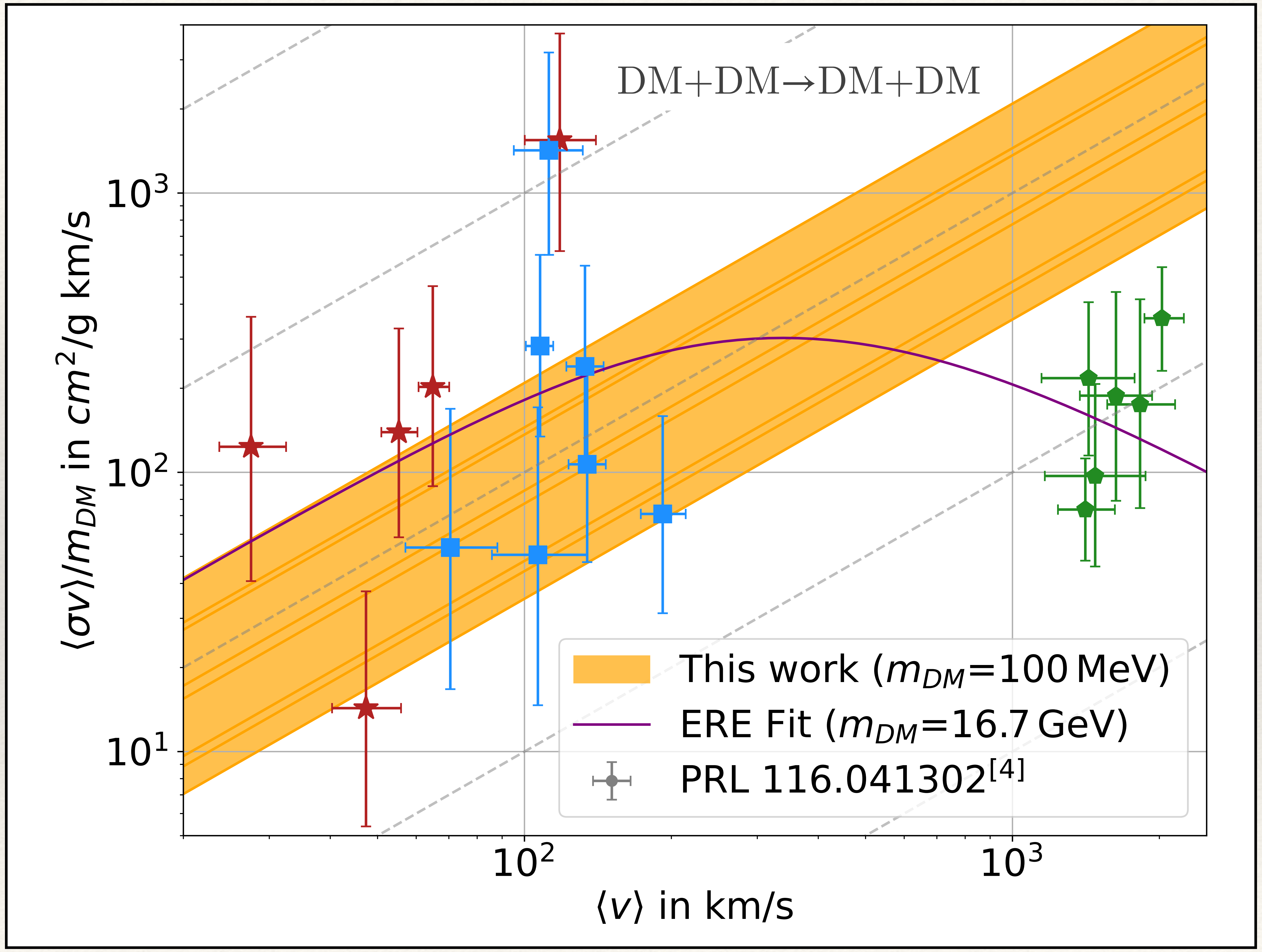

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Work in Progress

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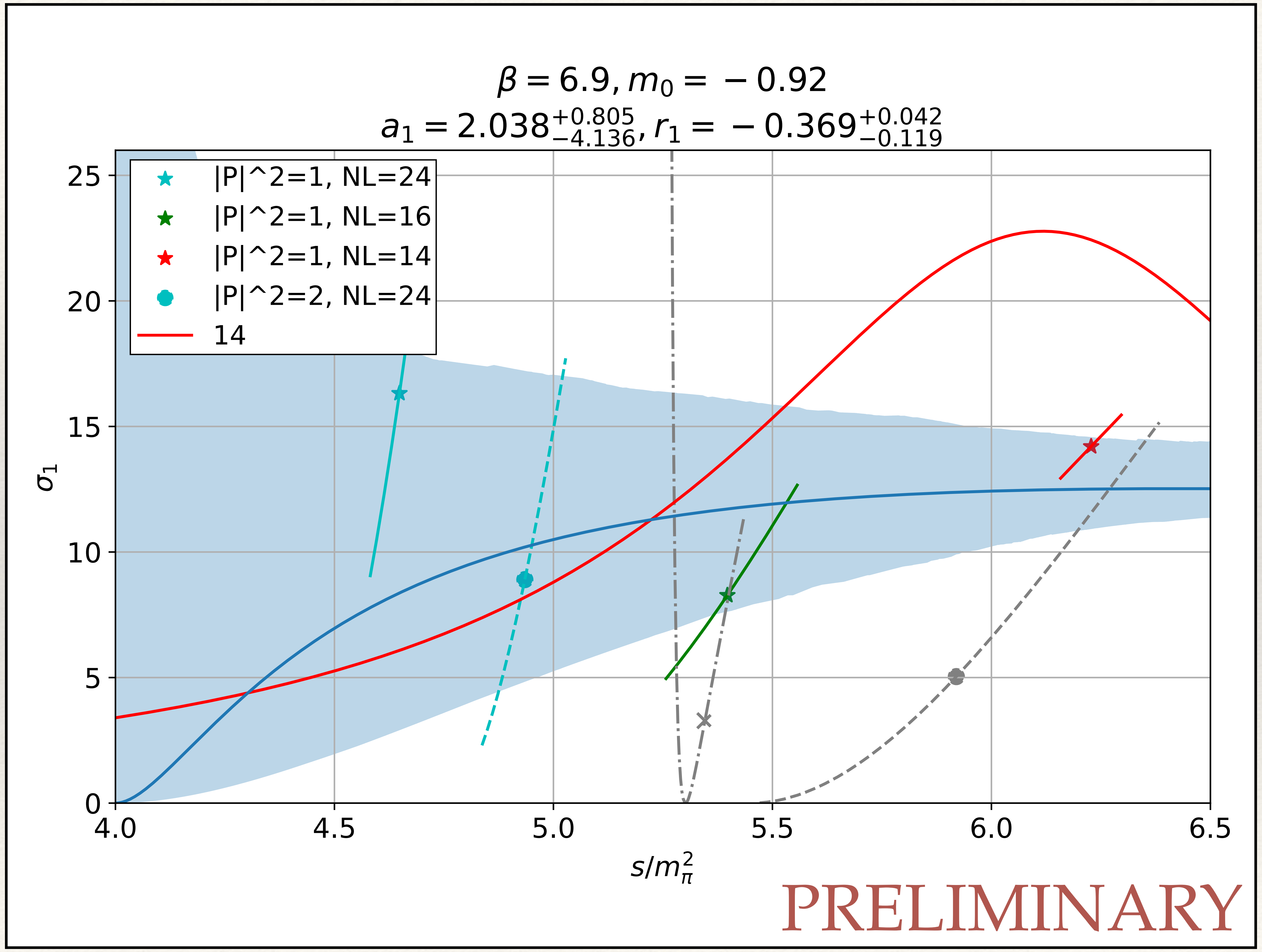
  - $\frac{m_\rho}{m_\pi} = \{1.6, 1.9, 2.3\}$



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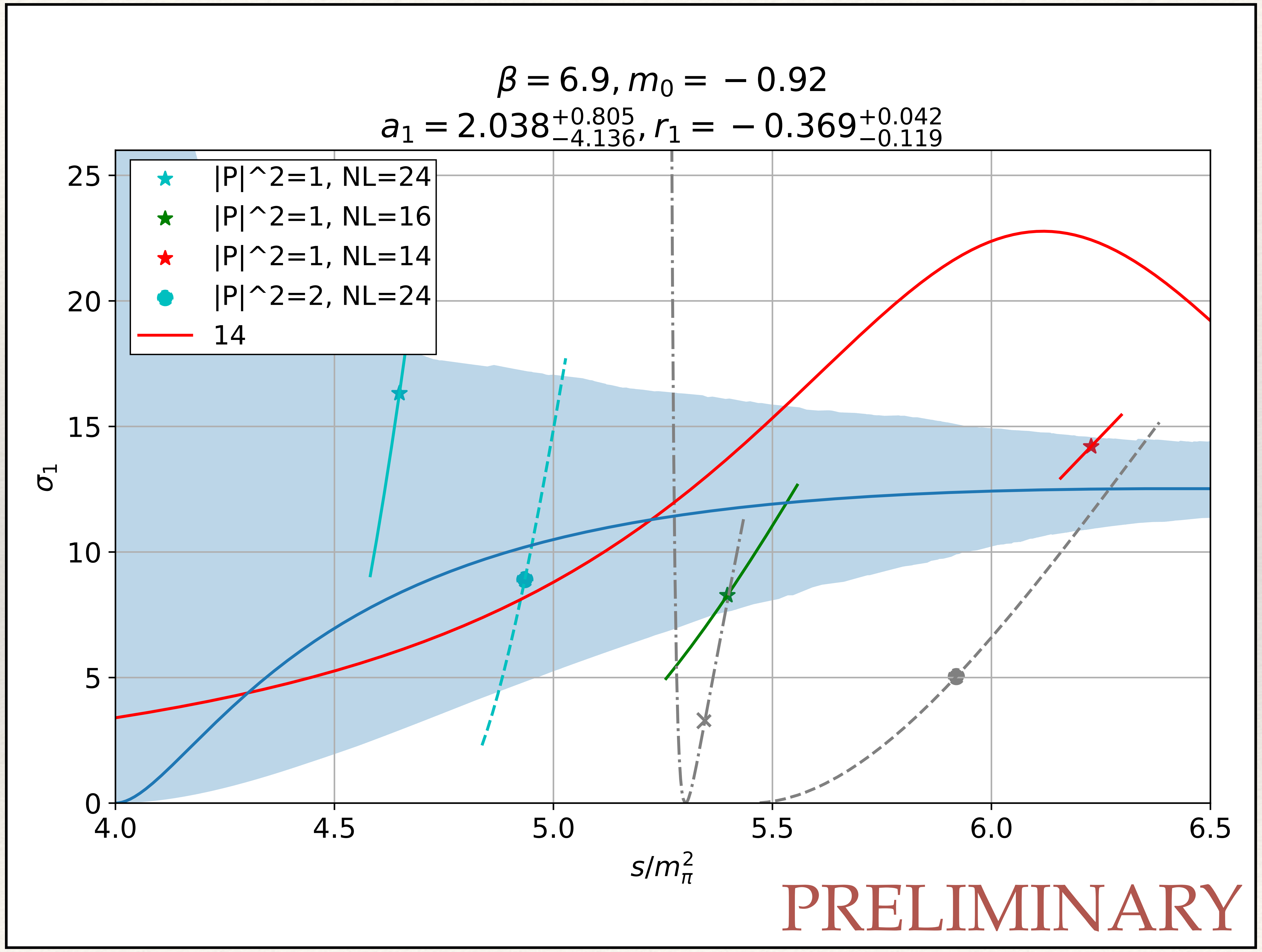

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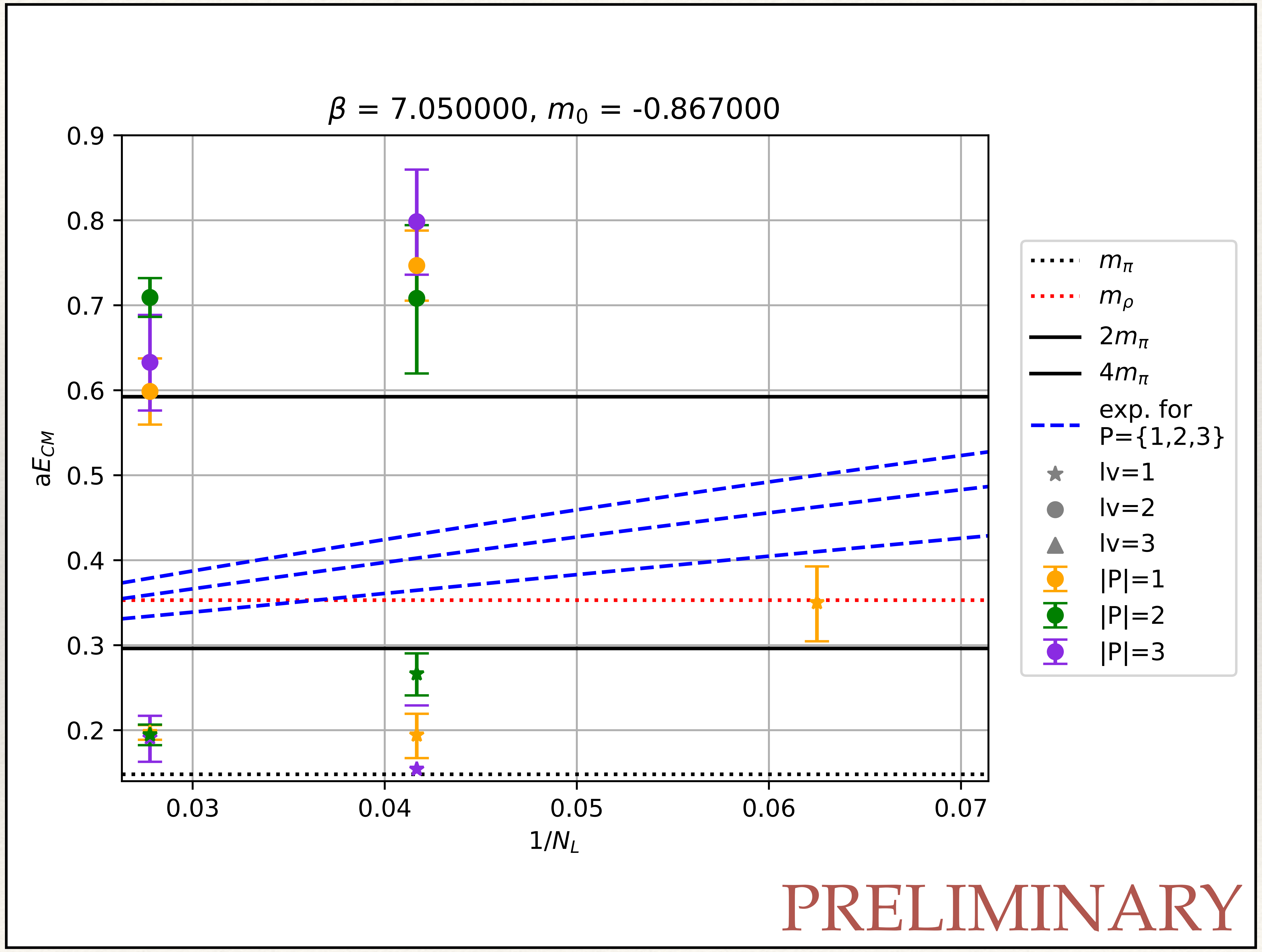

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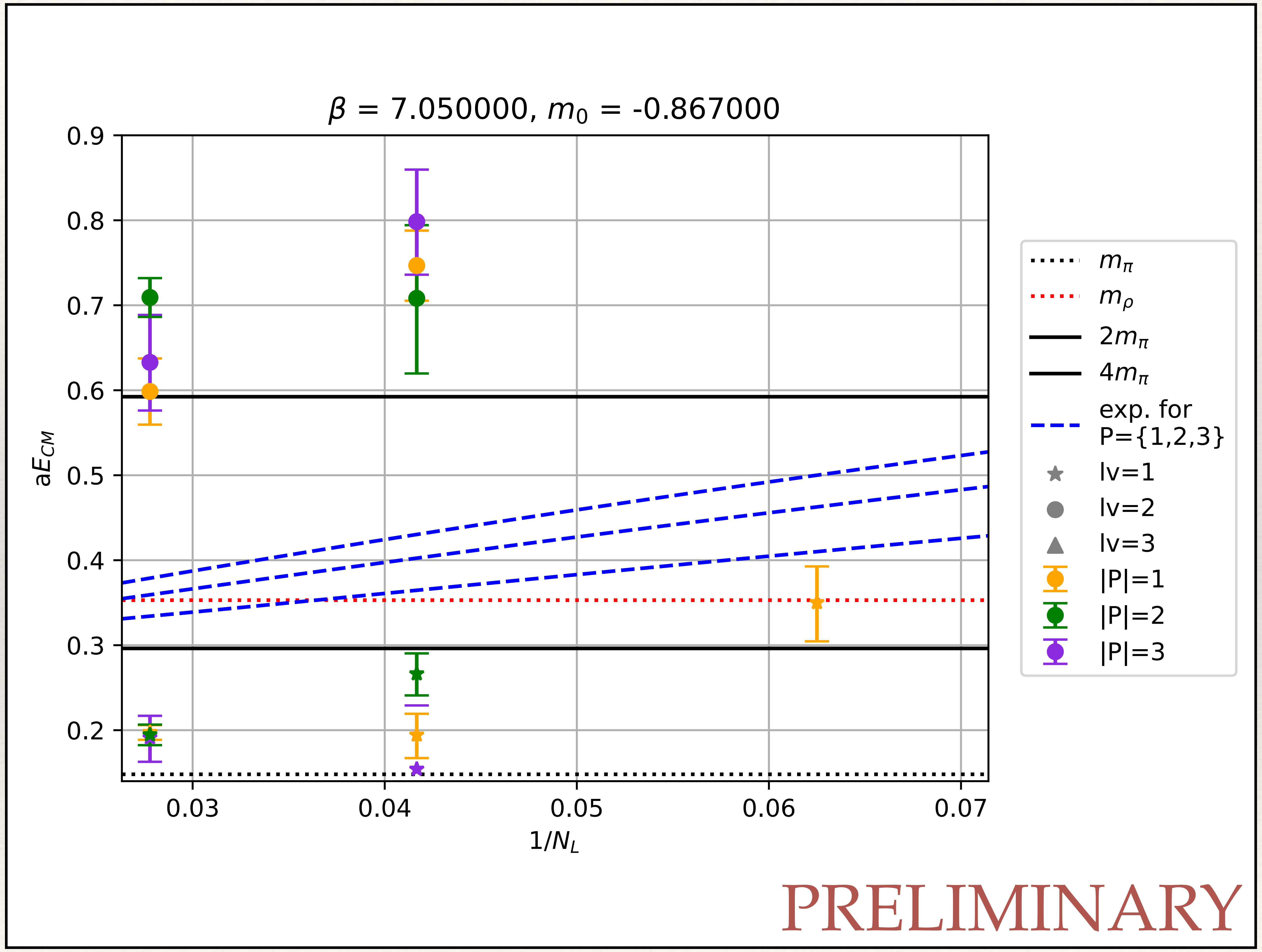
- Energy levels can be reused for $3\pi \to 2\pi$



Summary & Outlook

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- First principle verification for low energy constants

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Thank you!