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Lévy walk of pions in heavy-ion collisions

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https://www.nature.com/articles/s42005-025-01973-x

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ACHT 2025: Non-perturbative methods in strongly interacting quantum many-body systems

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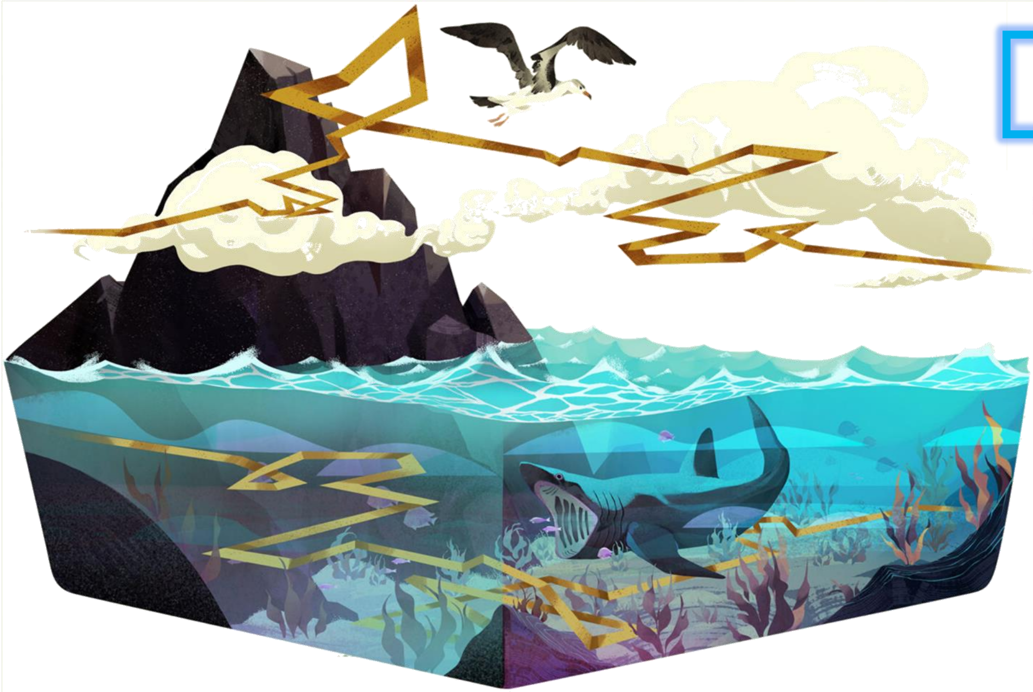
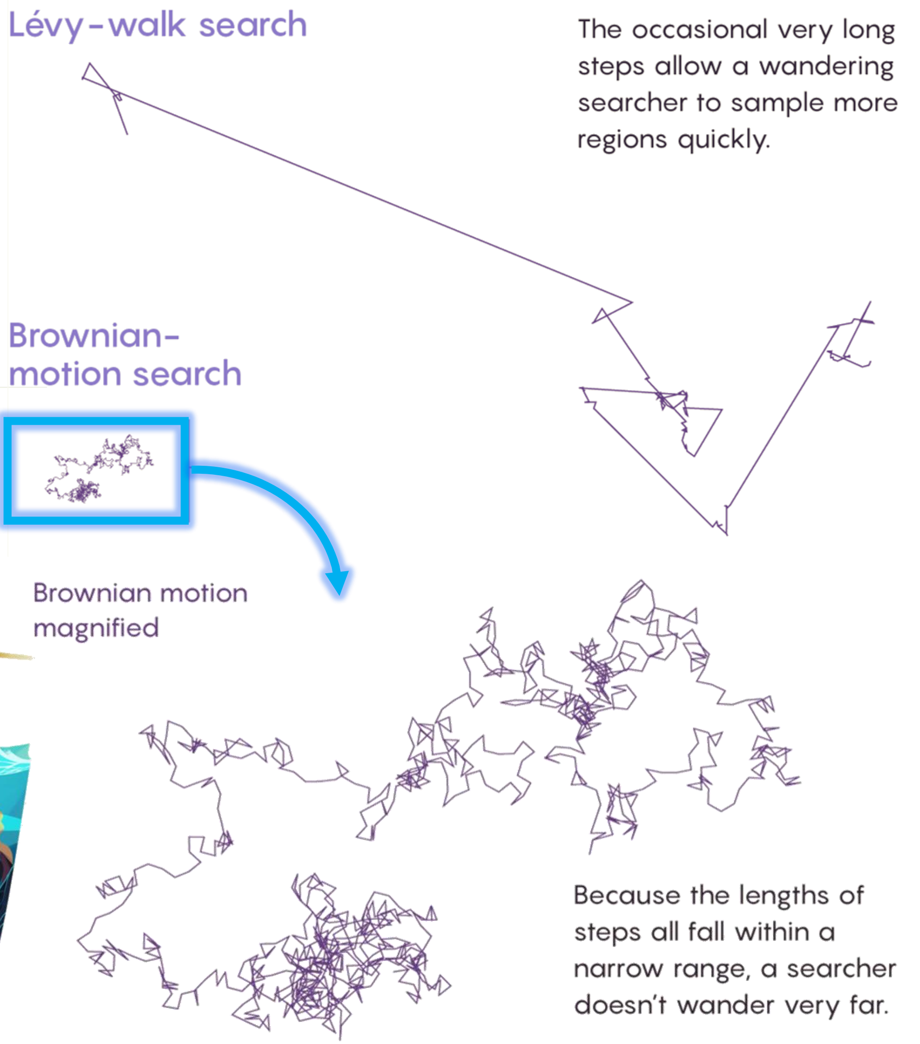
Nature 451(7182), 1098–1102 (2008)
Nature 449(7165), 1044–1048 (2007)
Nature 465(7301), 1066–1069 (2010)
Nature 486(7404), 545–548 (2012)
Nature 453(7194), 495–498 (2008)

Lévy-walk in Nature

- Wide range of phenomena (**foraging**, swarm dynamics, chemical, microbiological, physical processes)

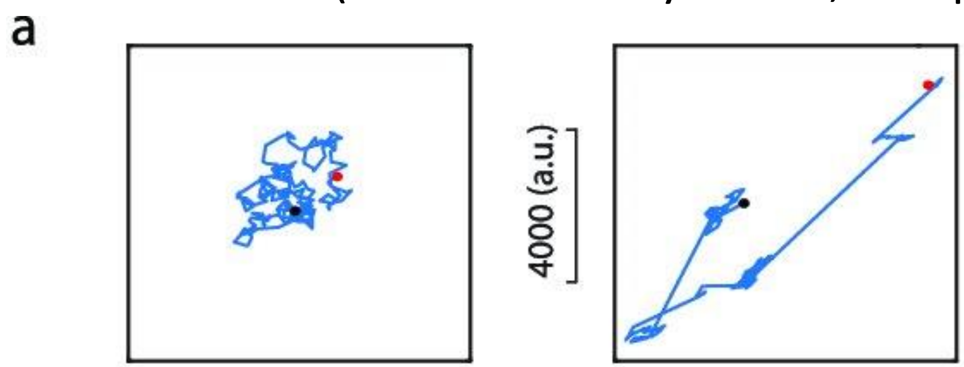




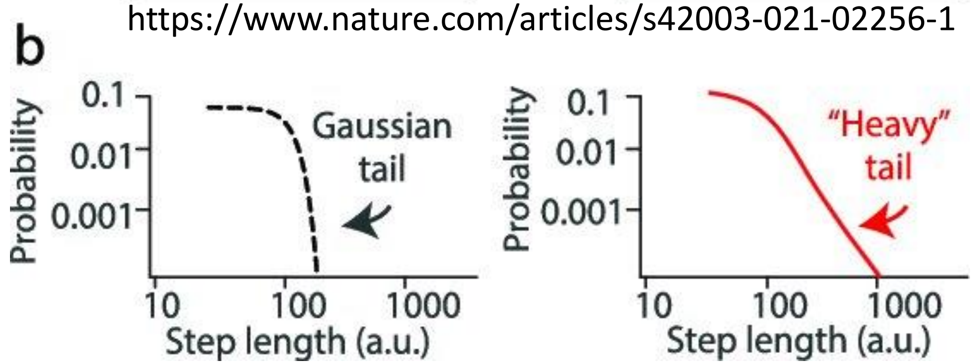
https://www.quantamagazine.org/random-search-wired-into-animals-may-help-them-hunt-20200611/

Lévy-walk in Nature

- Distribution of individual random variables → **no finite second moment**
- Generalized central limit theorem → sum of individual random variables follows a **stable distribution** (also called Lévy-stable, or alpha-stable)

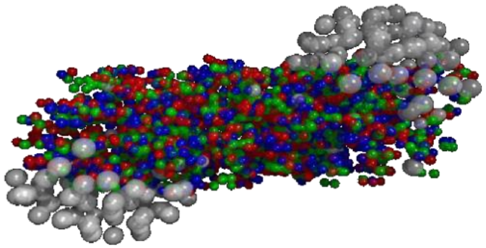


https://www.nature.com/articles/s42003-021-02256-1



Lévy-walk in hadronic scattering

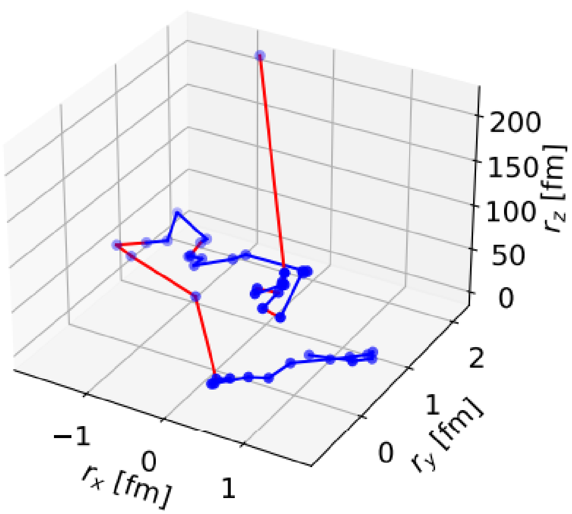
- Lévy-stable sources observed in heavy-ion experiments, many open questions
  $\rightarrow$ **see talks of M. Csanád, M. Molnár!**
- Idea: check pion movements in UrQMD history mode (where every individual scattering step is followed)

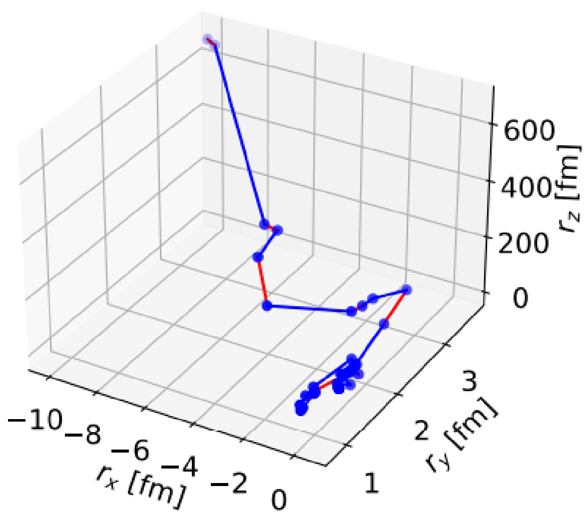


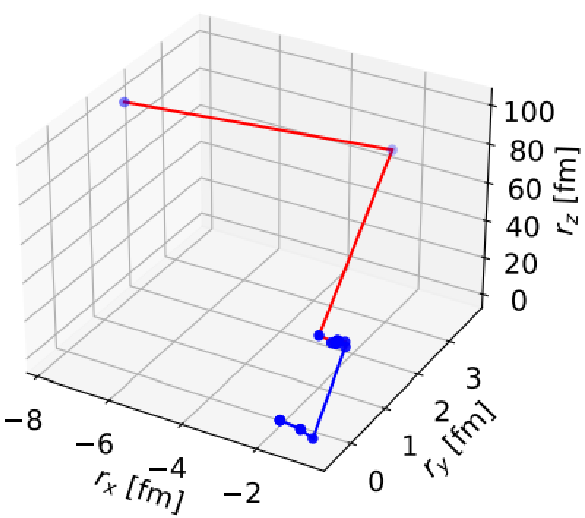
- **4 main types of interactions in UrQMD:**
  - **Scattering** (2 → 2 process, i.e. a 2-by-2 scattering, elastic or inelastic)
  - **Decay** (1 → N process with N > 1, i.e., 2 or 3 particles are created from one)
  - **Coalescence** (2 → 1 process; also called ’annihilation’ in UrQMD)
  - **String creation and subsequent fragmentation** (2 → N process, with N ≫ 2)
- Starting from the constituents of the colliding nuclei,
  a chain of interactions proceed until a large enough preset time

Lévy-walk in hadronic scattering

- Generated 100 UrQMD events, 0-10% Au+Au @200 GeV, full collision history
- **Selected pions at their last point of interaction,**
  and tracked their steps back to the constituents of the colliding nuclei
  (through scatterings, decays, and coalescence processes as well)
- A few example paths – resemble Lévy-walk!







Lévy-walk in hadronic scattering

- **Source function** investigated in femtoscopy:
  Distribution of points where pions start their straight flight toward the detectors

- Random variable representing the location of the freeze-out:
  vector-sum of the individual steps

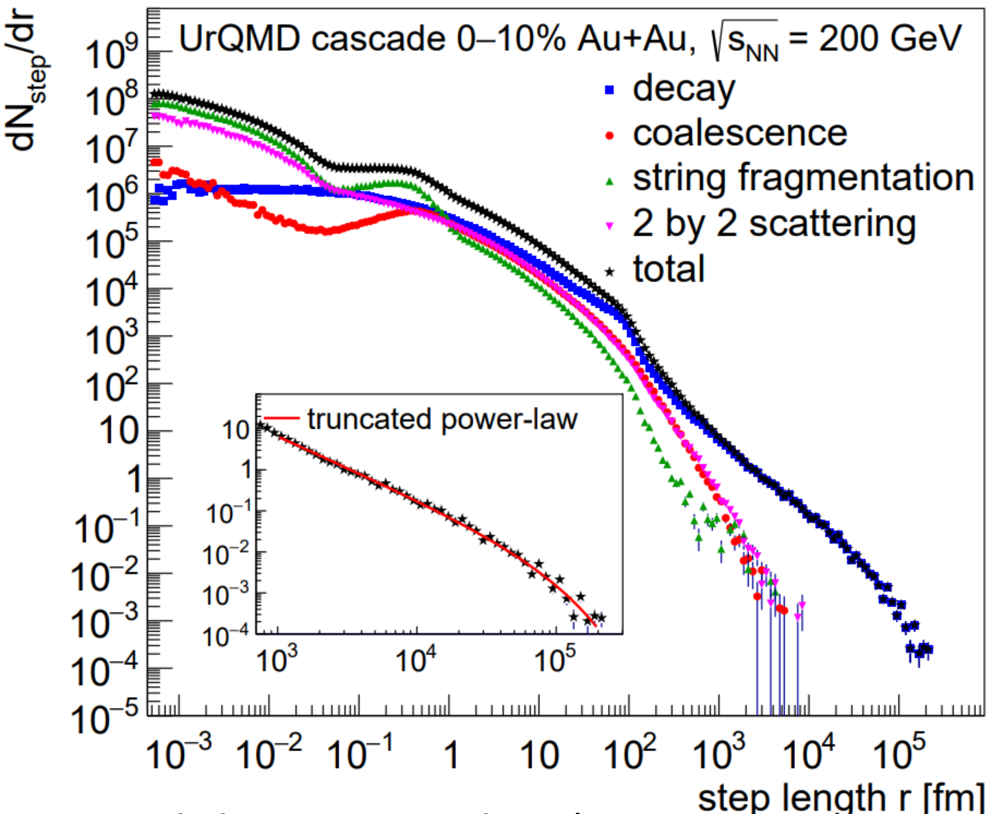
- **What is the second-moment of the step-length distribution?**
  - If finite → freeze-out coordinates follow a Gaussian distribution
  - If not finite → freeze-out coordinates follow a power-law-tailed stable distr.

- $\frac{dN_{step}}{dr} \sim r^{-1-\xi} \rightarrow \begin{cases} \xi > 2: & \text{finite mean \& variance} \\ 1 < \xi \leq 2: & \text{finite mean, infinite variance} \\ \xi \leq 1: & \text{infinite mean \& variance} \end{cases}$

  r: step-length

The pion step-length distribution

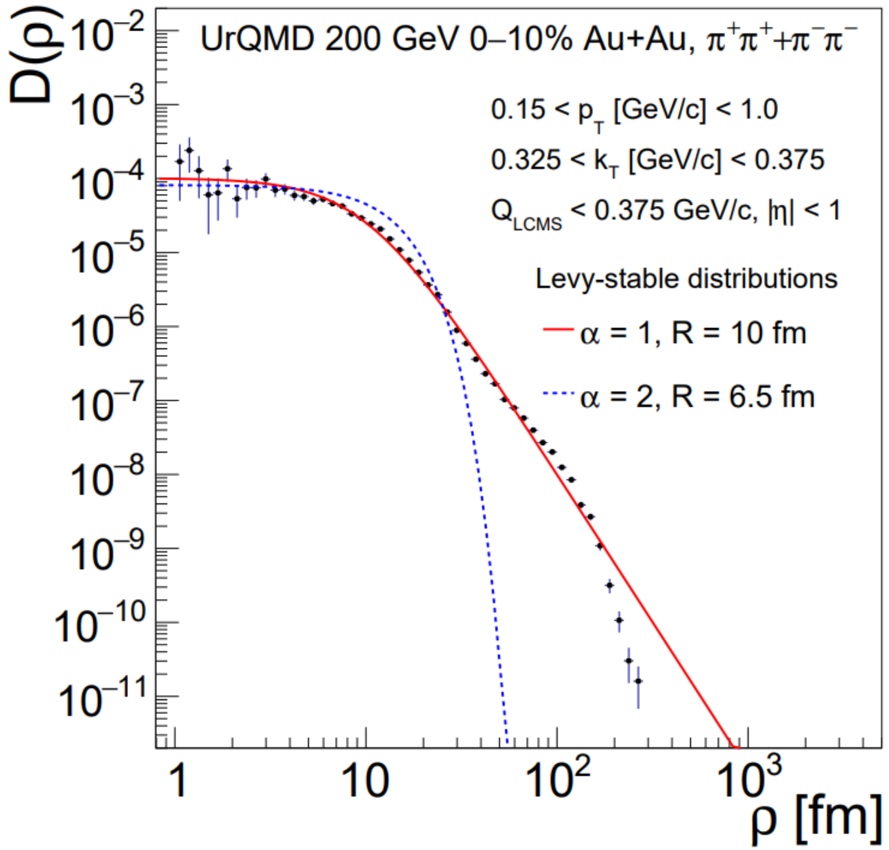
- Individual steps represent the distance covered **before** the actual process
- Related to the **mean free path** of the particle w.r.t the given process
- Expanding medium → decreasing density → m.f.p increasing → power-law tails
- All follow a power-law tail with varying exponents
- **Total distribution $\sim r^{-1.53}$, dominated by decays**
- **No second moment!**
- Finite upper-limit of phenomena → a truncation usually appears (but up until then, power-law!)



The pion pair-distance distribution at freeze-out

- Pair source function exhibits a power-law tail, close to a spherically symmetric Lévy-stable distribution (individual event shown as example)

$$\mathcal{L}(\alpha, R, \vec{r}) = \frac{1}{(2\pi)^3} \int d^3\vec{q} e^{i\vec{q}\vec{r}} e^{-\frac{1}{2}|\vec{q}R|^\alpha}$$



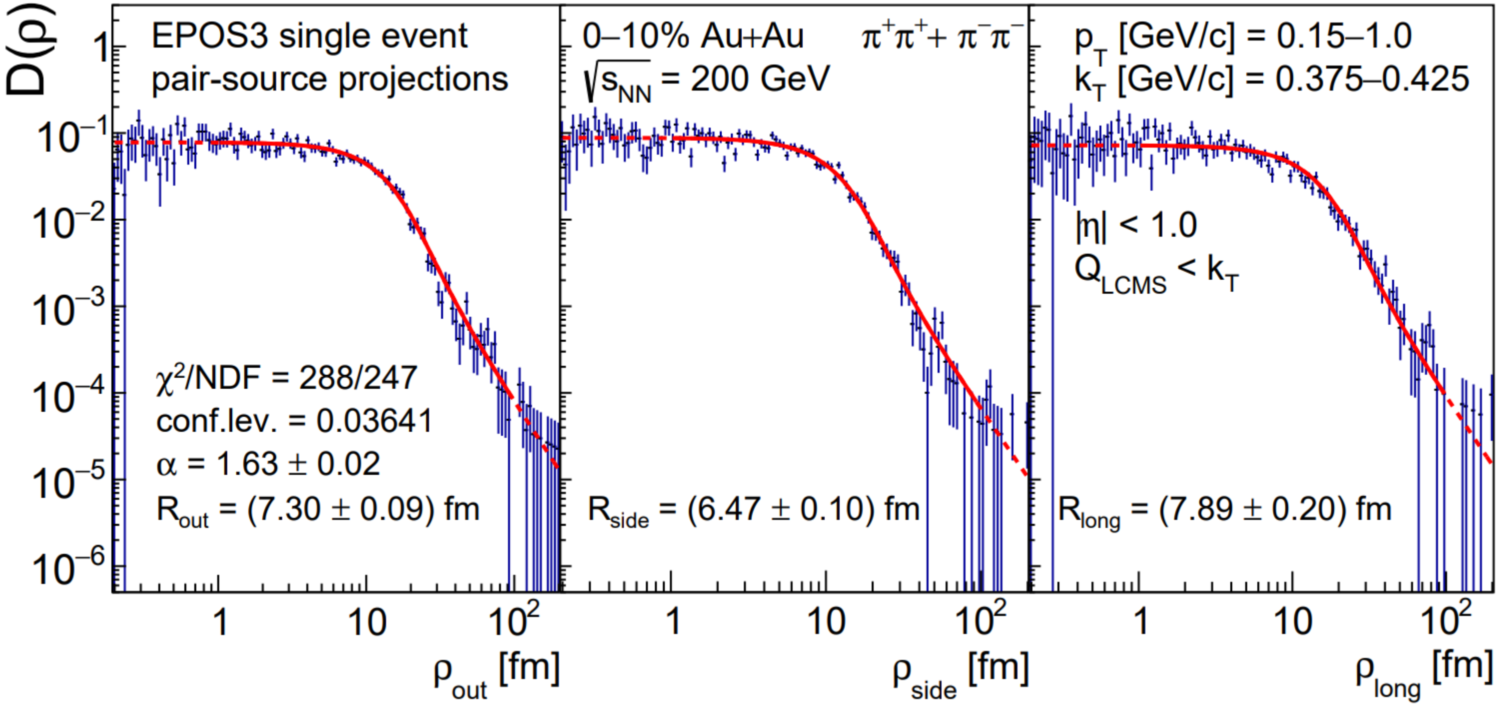
- Next steps:
  - Hybrid model, hydro+rescattering (EPOS3)
  - Multi-dimensional investigation of the source function

$$\mathcal{L}(\alpha, R^2, \vec{r}) = \frac{1}{(2\pi)^3} \int d^3\vec{q} e^{i\vec{q}\vec{r}} \, e^{-\frac{1}{2}\left|\vec{q}^T R^2 \vec{q}\right|^{\alpha/2}}$$

3D pion pair-source in EPOS

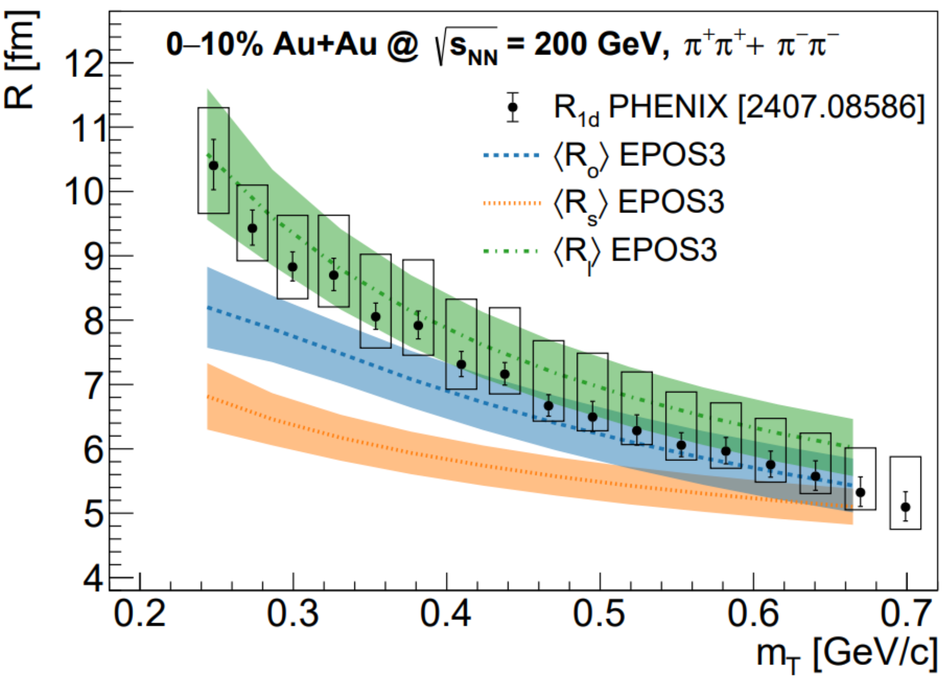
$$R^2 = \mathrm{diag}(R^2_{out}, R^2_{side}, R^2_{long})$$

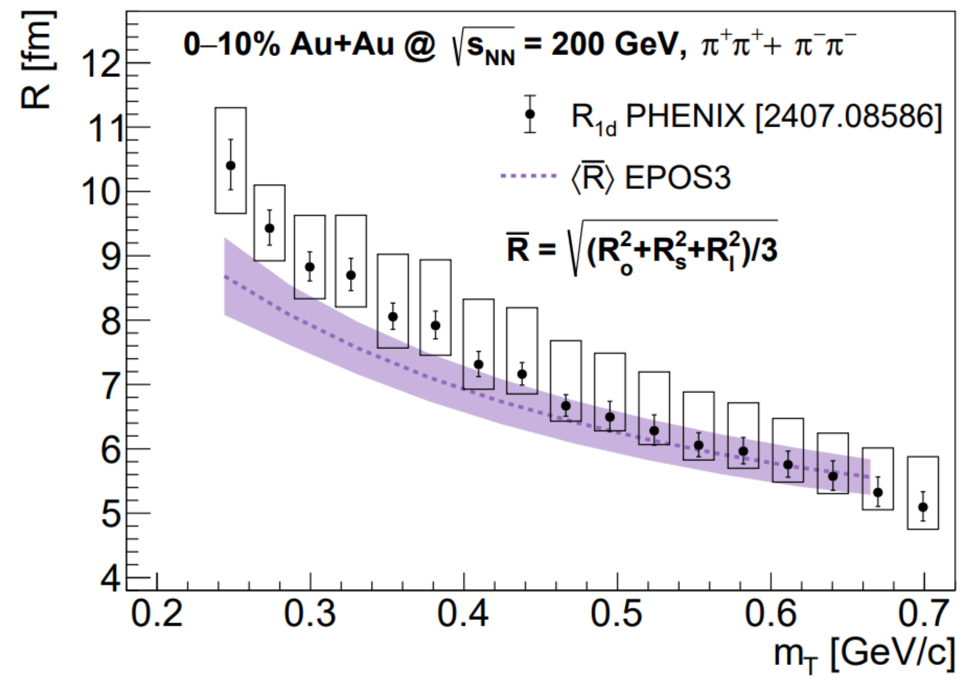
- 200 GeV Au+Au collisions, 0-10% centrality
- **Pair-distance distribution** of pions measured in **3D**, on an **event-by-event** basis, fitted with **elliptically contoured three-dimensional Lévy-stable distributions**:



3D pion pair-source parameters vs. PHENIX results

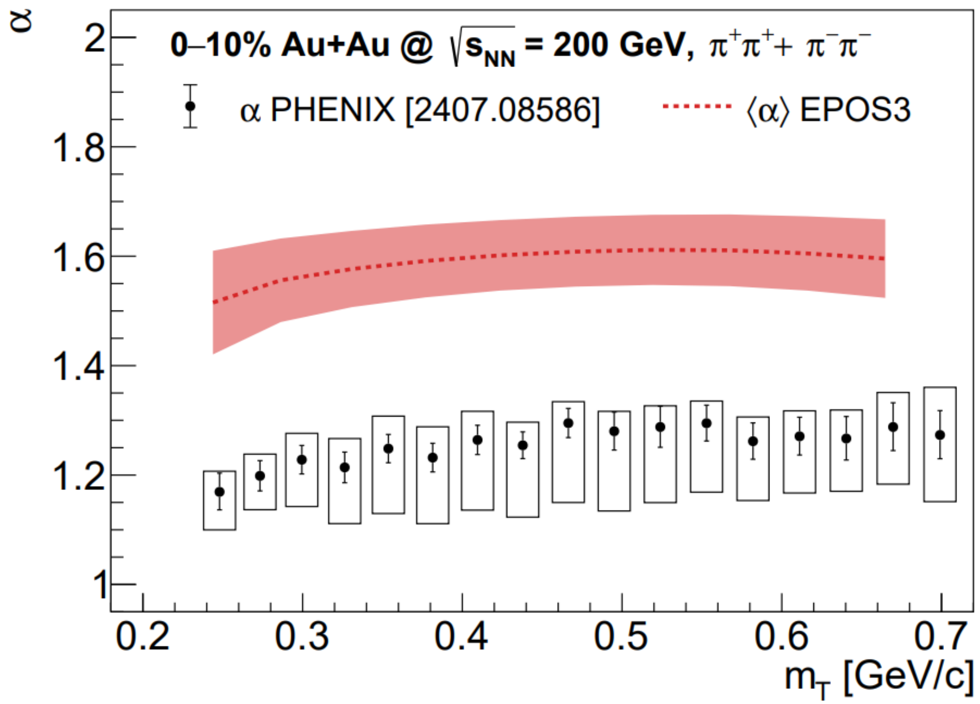
- Dashed line: mean of evt.by.evt fits, band: standard deviation of e.b.e fits
- Levy radii shows good agreement with recently published final experimental angle-averaged results (**see talk of Máté Csanád**!)





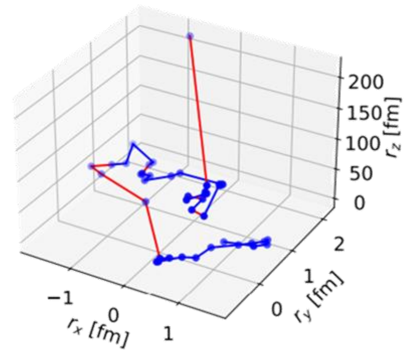
3D pion pair-source parameters vs. PHENIX results

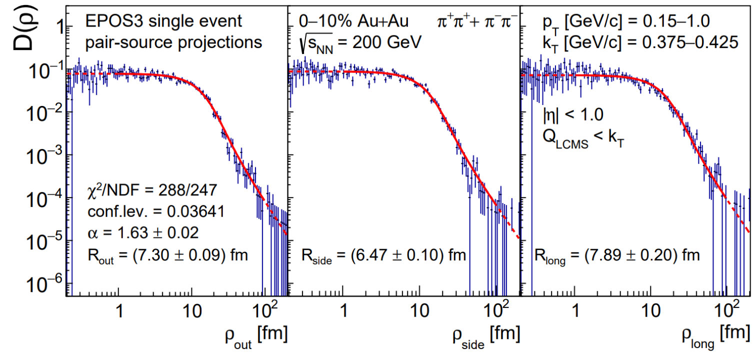
- Levy exponent is far from Gaussian, but not as low as the experimental result
- Besides Lévy walk, other phenomena may play a role?
  - Long-range Coulomb elastic scattering? (see talk by M. Csanád)
  - Maybe we can expect better agreement with data in peripheral events?
- Stronger effect in data than EPOS?
- What about centrality, coll. energy and particle species dependence?



Talks of M. Csanád, M. Molnár!

Summary





- Lévy-walk as a form of random movement appears in many areas of Nature
- Also present in hadronic scattering and decays as shown in UrQMD
- **In a hybrid model including hydro + hadronic scattering, three-dimensional elliptically contoured Lévy-stable sources appear!**
- **Next steps:**
  - Comparison with experiments
  - Centrality, collision energy, particle species dependence in EPOS
  - 3D correlation reconstruction with CRAB/CORAL, checking if results are consistent between the different methods

  → ongoing investigations, stay tuned!

Thank you!

Backup – kinematic variables

- Pair source $D(r, K)$:
  autocorrelation of $S(x, p)$ single particle phase-space density

$$D(r, K) = \int S\left(x + \frac{r}{2}, K\right) S\left(x - \frac{r}{2}, K\right) d^4x$$

$$K = (p_1 + p_2)/2$$
$$r = x_1 - x_2$$
$$q = p_1 - p_2$$

- Two-particle momentum correlation funcion $C_2(q, K)$:

$$C_2(q, K) = \int dr\, D(r, K) \left|\psi_q(r)\right|^2$$

- Four-momentum vectors: $q = (q_0, \vec{q}), K = \left(K_0, \vec{K}\right), r = (t, \vec{r})$
- For identical, on-shell particles $p_1^2 = p_2^2 = m^2$
- Thus, $q_0 = \vec{q}\vec{\beta}$, where $\vec{\beta} = \vec{K}/K_0$

- The proper spatial variable becomes $r \rightarrow \vec{\rho} \equiv \vec{r} - \vec{\beta}t$

Backup – frame choice

- Longitudinally Co-Moving System, Bertsch-Pratt coordinates:

$$K_{long} = K_{side} = 0$$
$$\vec{K} = (K_{out}, 0{,}0)$$
$$\vec{\beta} = (K_{out}/K_0, 0{,}0)$$
$$q_0 = q_{out}\beta_{out}$$

- $\vec{\rho} \equiv \vec{r} - \vec{\beta}t$

$$\rho_{out}^{LCMS} = r_x \cos\varphi + r_y \sin\varphi - \frac{K_T}{K_0^2 - K_z^2}(K_0 t - K_z r_z),$$

$$\rho_{side}^{LCMS} = -r_x \sin\varphi + r_y \cos\varphi,$$

$$\rho_{long}^{LCMS} = \frac{K_0 r_z - K_z t}{\sqrt{K_0^2 - K_z^2}}$$

$$\cos\varphi = K_x/K_T$$

$$\sin\varphi = K_x/K_T$$

$$K_T = \sqrt{K_x^2 + K_y^2}$$

Backup – Lévy scale vs. Gaussian scale

- Measure of widths:
  - R scale parameter of the distribution (in case of Gaussian, equal to RMS)
  - Half-width at half maximum (HWHM)
  - Half-width at half integral (HWHI)
- Relation of Gaussian widths (α = 2) to Lévy widths (α < 2):
  - 3D Gauss:
    HWHM ≈ 1.17·R_G, HWHI ≈ 1.54· R_G
  - Lévy α = 1.3:
    HWHM ≈ 0.61· R_L, HWHI ≈ 1.27· R_L
  - E.g., **α = 1.3 and R_L =7 fm** "means":
    - Same HWHM Gaussian: **R_G ≈ 3.6 fm**
    - Same HWHI Gaussian: **R_G ≈ 5.8 fm**