A Unified Framework for Gluon GTMDs in the Small-x Regime

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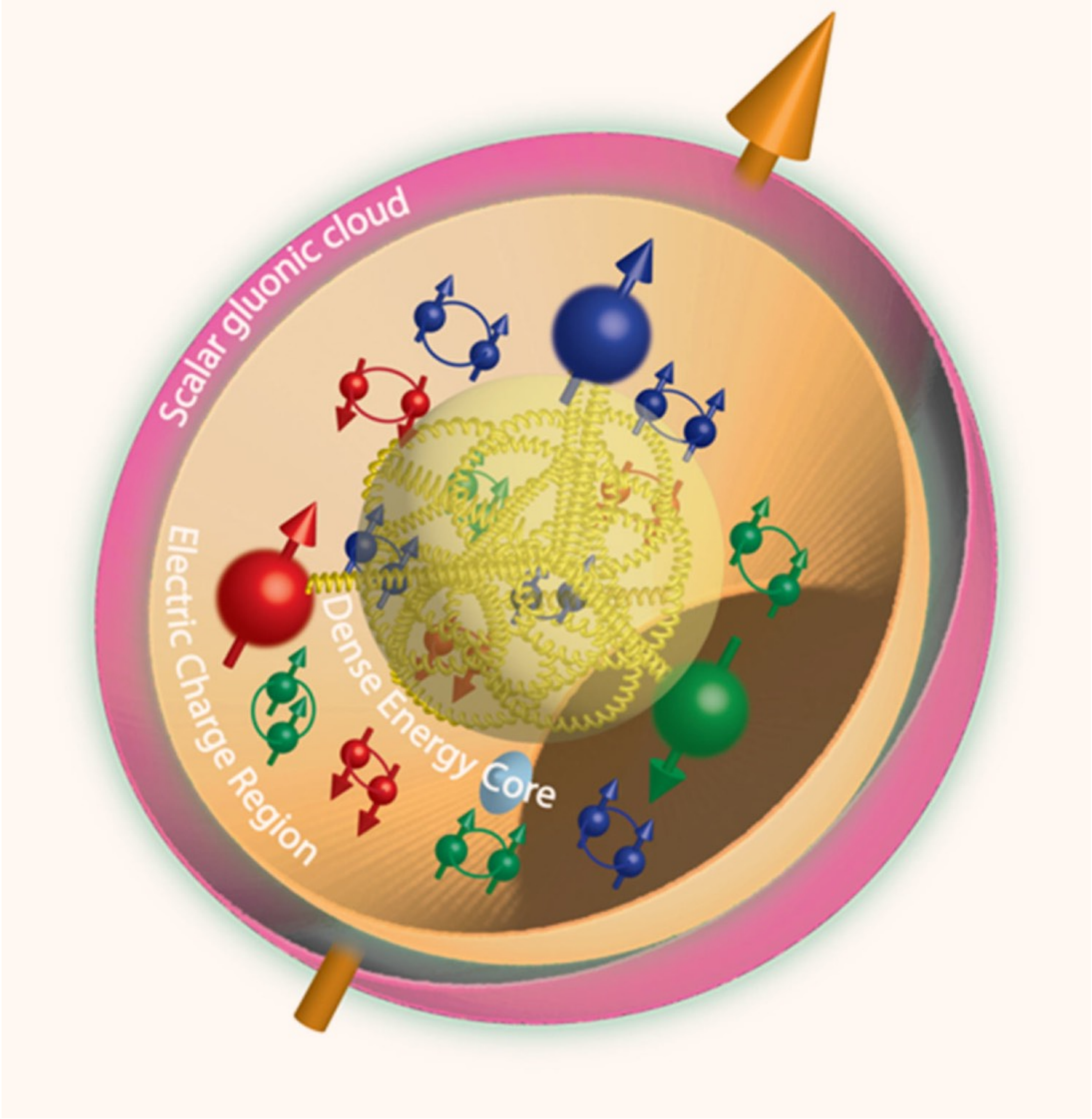
Contents

Understanding the proton(hadrons)

- Protons contribute to nearly 90% of the visible mass in the universe, making up the bulk of the mass of atoms.
  → proton mass

- The proton is a composite particle, made up of quarks and gluons.
  → proton spin, mechanical properties of proton

- No color-charged objects have been observed in isolation
  → color confinement mechanism



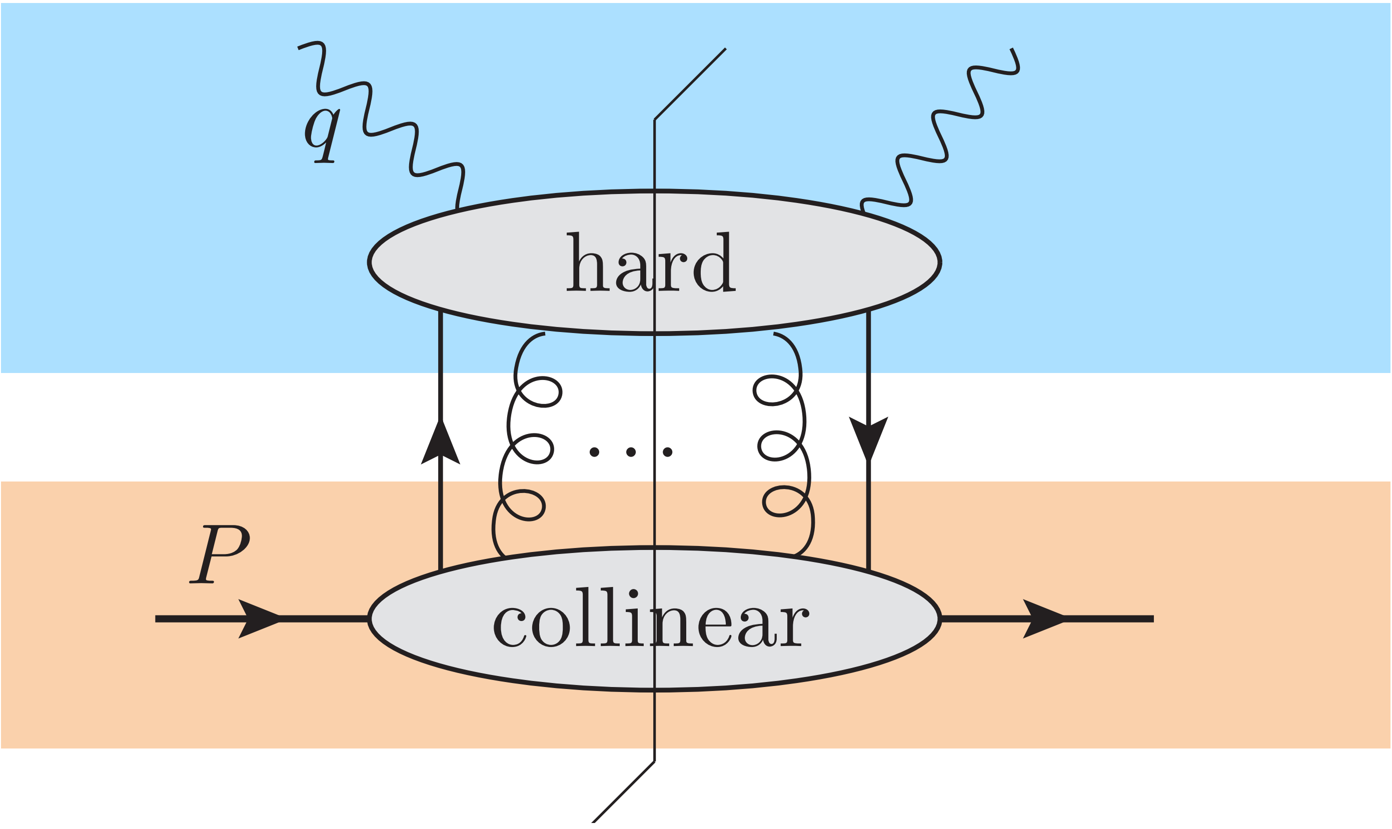
Nature volume 615, pages 813–816 (2023)

Parton distribution functions

Factorization theorem

QCD factorization theorem separate non perturbative and perturbative parts

J. Collins 2011



$$W^{\mu\nu} = \sum_j \int_{x-}^{1+} \frac{d\xi}{\xi} \operatorname{Tr} C_j^{\mu\nu}(q, \xi P; \alpha_s, \mu)\, \rho_j(\xi; \mu)\, f_j(\xi; \mu) + \text{p.s.c.}$$

Non perturbative part is universal and called Parton distribution functions

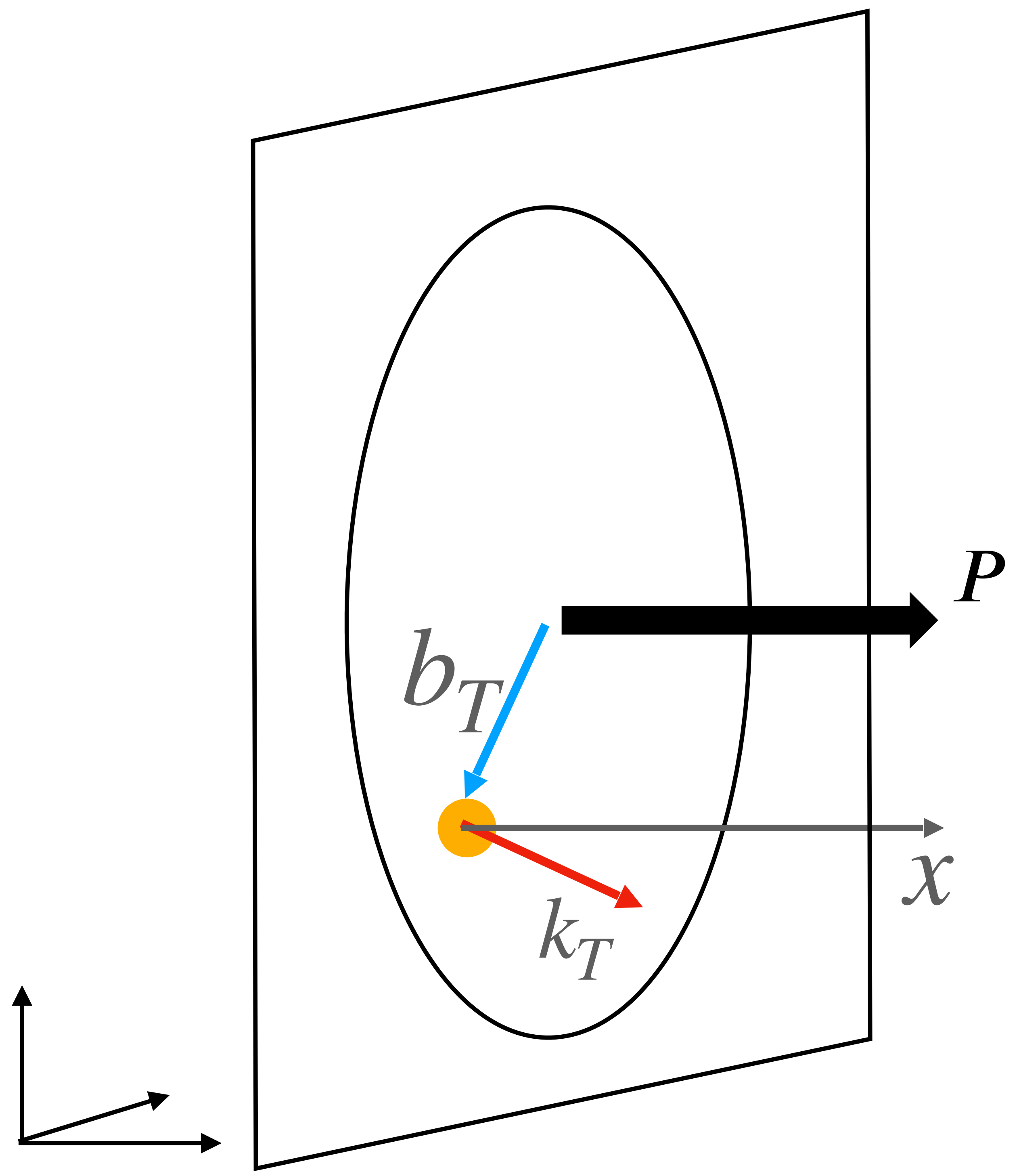
$$f_{(0)\, j/h}(\xi) = \int \frac{dw^-}{2\pi} e^{-i\xi P^+ w^-} \left\langle P \left| \overline{\psi}_j^{(0)}(0, w^-, \mathbf{0}_\mathrm{T}) W(w^-, 0) \frac{\gamma^+}{2} \psi_j^{(0)}(0) \right| P \right\rangle_\mathrm{c}$$

$$W(w^-, 0) = P\left\{ e^{-ig_0 \int_0^{w^-} dy^-\, A^+_{(0)\alpha}(0, y^-, \mathbf{0}_\mathrm{T}) t_\alpha} \right\}$$

Generalization of PDFs

More detailed structure of one parton

C. Lorcé, B. Pasquini 2013



Generalized Parton Distribution Functions

Kornelija's talk
Nikola's talk

$$\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-}\, \langle p', \Lambda' | \overline{\psi}(-\tfrac{z^-}{2}) \Gamma \mathcal{W} \psi(\tfrac{z^-}{2}) | p, \Lambda \rangle$$

$p' - p = \Delta_T$ $\xleftrightarrow{\text{F.T.}}$ b_T : Impact parameter

Transverse Momentum Dependent PDFs

Eduardo's talk

$$\frac{1}{2}\int \frac{\mathrm{d}z^-\,\mathrm{d}^2 z_T}{(2\pi)^3}\, e^{ixP^+z^- - i\vec{k}_T\cdot\vec{z}_T}\, \langle P, \Lambda' | \overline{\psi}(-\tfrac{z}{2}) \Gamma \mathcal{W} \psi(\tfrac{z}{2}) | P, \Lambda \rangle \Big|_{z^+=0}$$

k_T : Transverse momentum

GTMDs and other PDFs

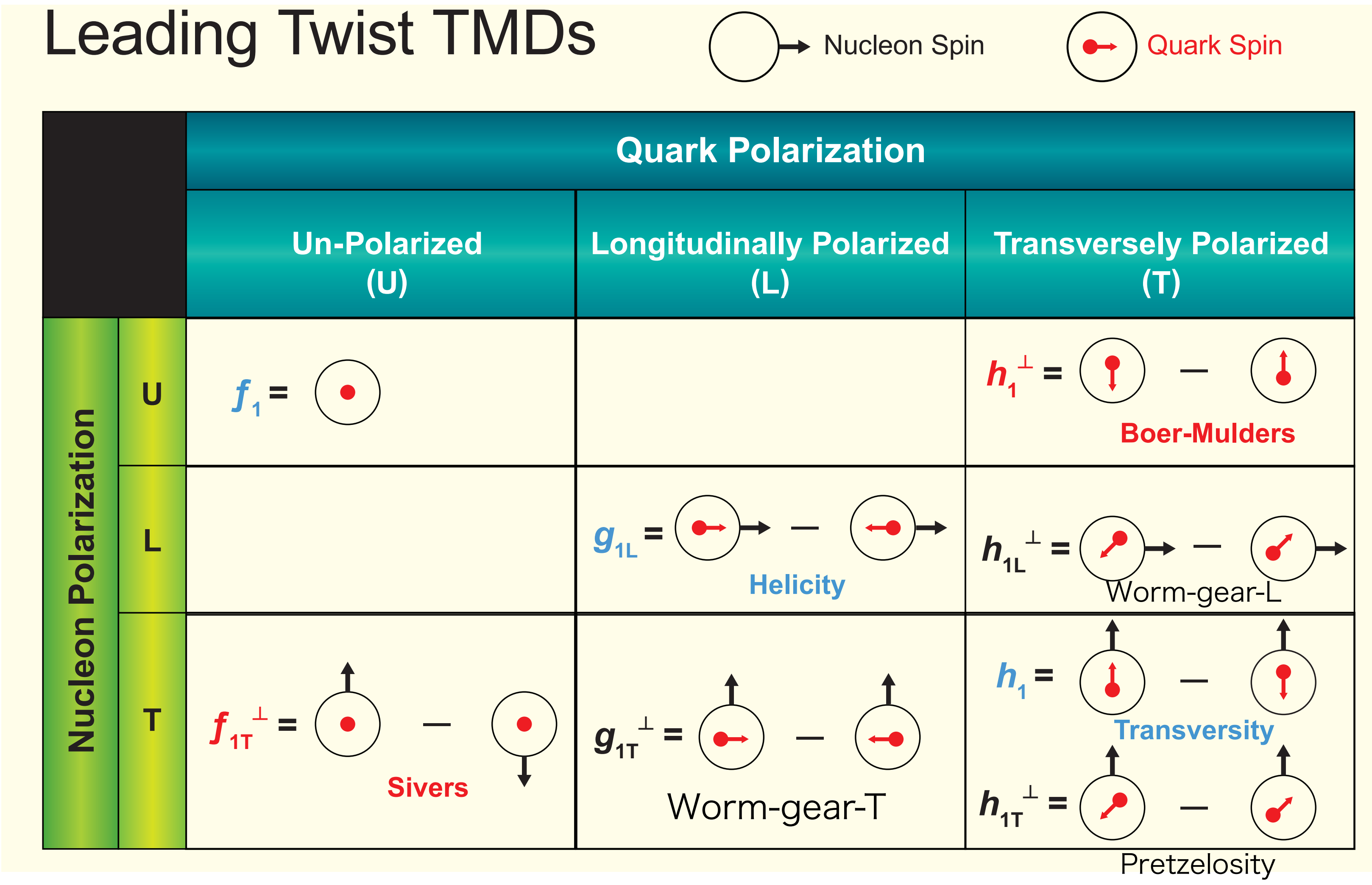
Relations for parton distribution functions

C. Lorcé, B. Pasquini 2013

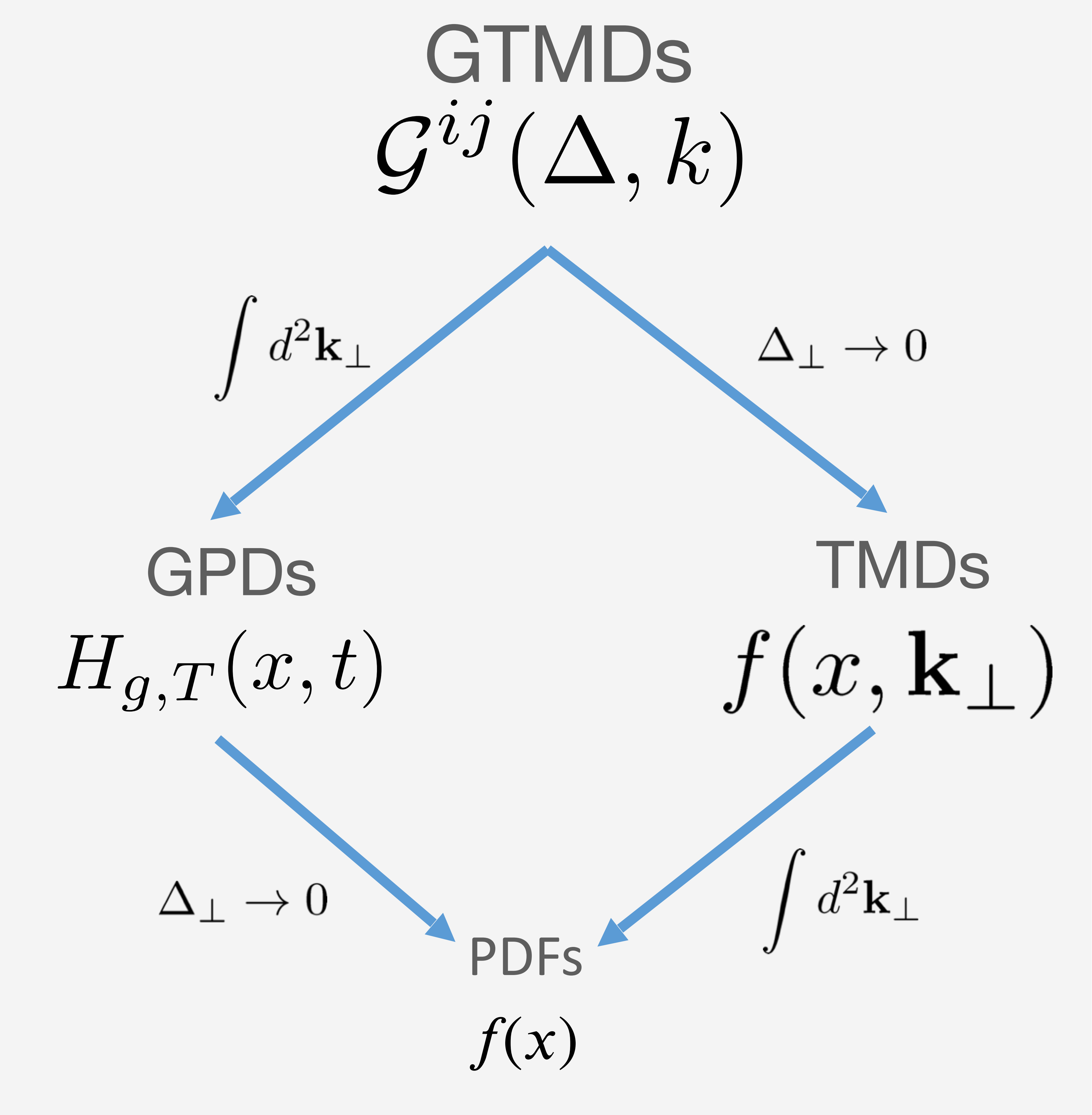
Generalized TMDs

$$\frac{1}{2}\int \frac{\mathrm{d}z^-\,\mathrm{d}^2z_T}{(2\pi)^3}\, e^{ixP^+z^- - i\vec{k}_T\cdot\vec{z}_T}\,\langle p',\Lambda'|\overline{\psi}(-\tfrac{z}{2})\Gamma\,\mathcal{W}\,\psi(\tfrac{z}{2})|p,\Lambda\rangle\Big|_{z^+=0}$$

Polarization dependences

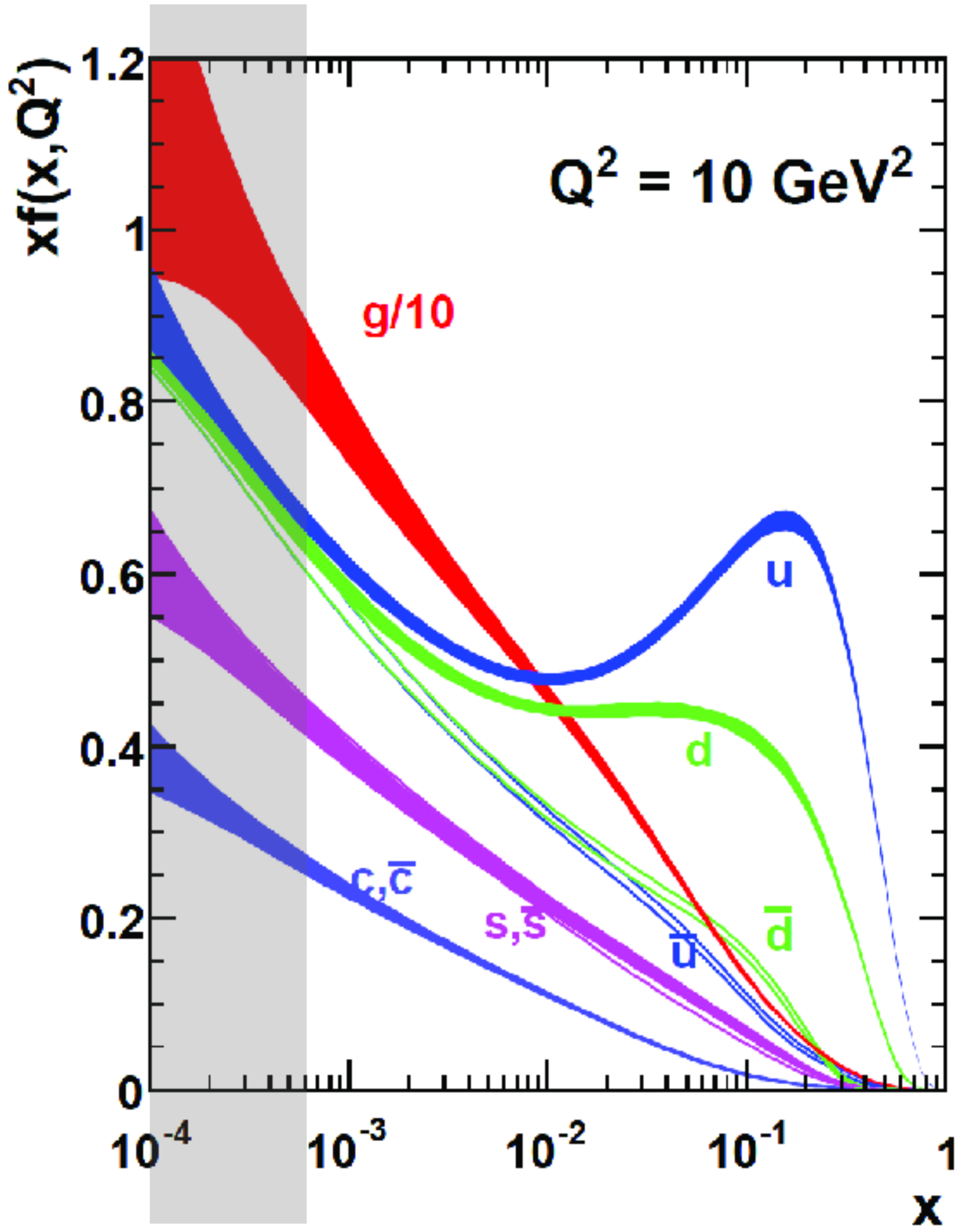


EIC white paper



Color Glass Condensate(gluon saturation)

Gluon dominance

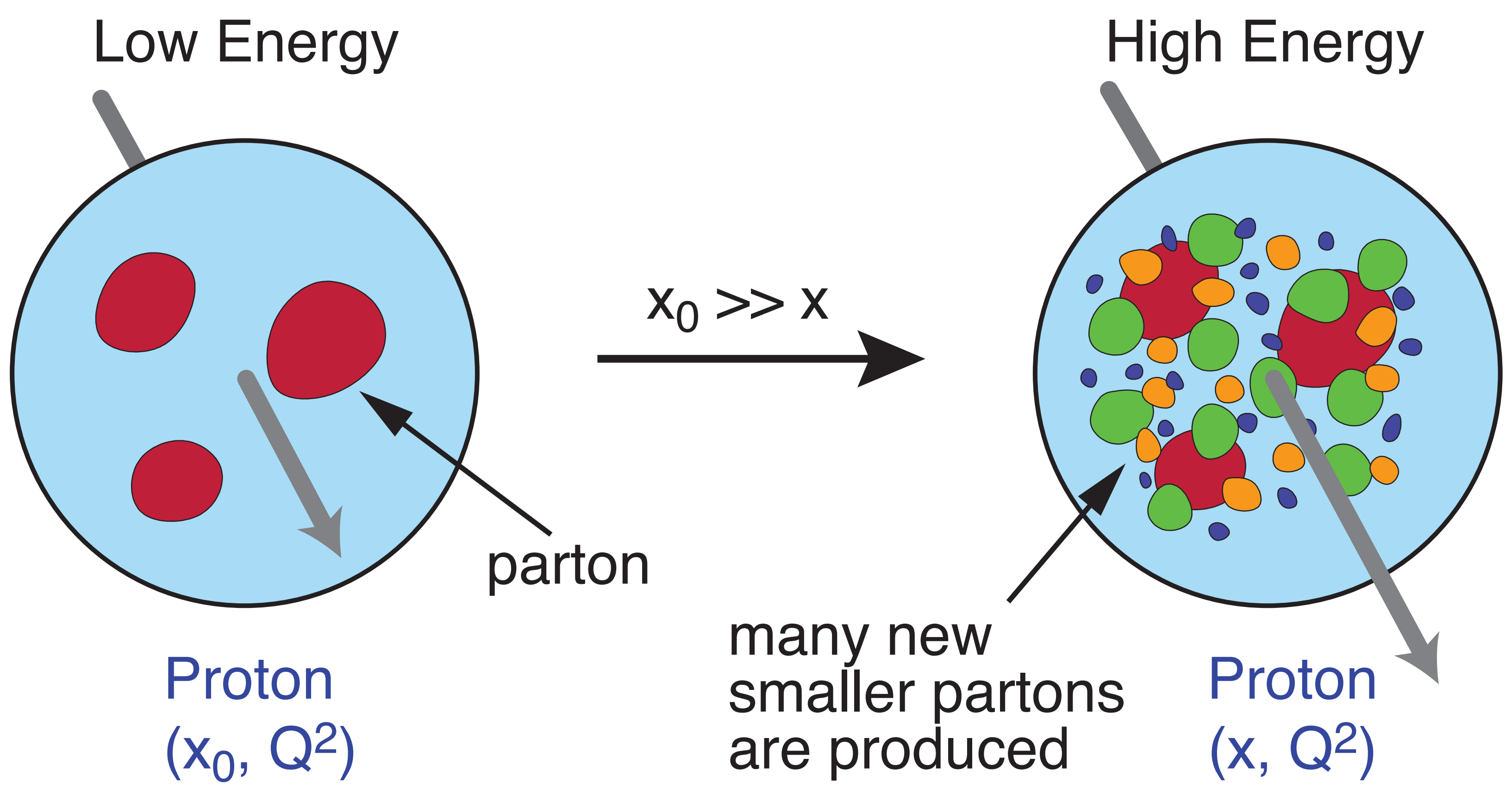


Martin, A.D. et al. Eur.Phys.J. C63 (2009) 189-285

Increase the CM energy (x becomes small)

Number of partons increase

The number of partons become saturate because of the gluon recombination process



"Color Glass Condensate"

EIC white paper

Color Glass Condensate

Meanings

E. Iancu 2005

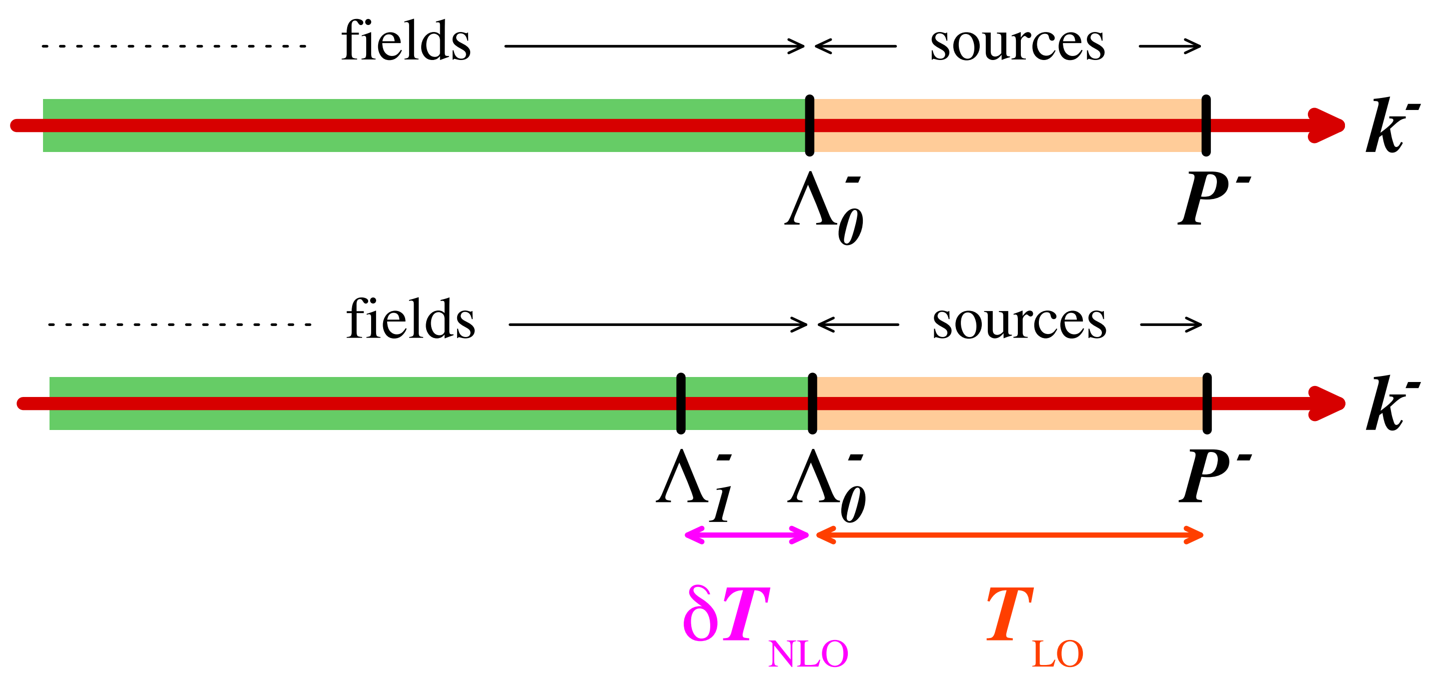
- Color: since the gluons carry the SU(3) ‘color’ charge of QCD
- Glass: since the associated color fields evolve very slowly relative to natural time scales, and are disordered
- Condensate: since the occupation numbers at saturation are of order $1/\alpha$, which is the largest value permitted by the gluon repulsive interactions. So, the saturated state is in fact a Bose condensate

Color Glass Condensate

Classical sources and fields

F.Gelis, E.Iancu, J.Jalilian-Marian and R.Venugopalan 2010

Color Glass Condensate separate soft and hard scale



Evolution of the amplitudes

$$\delta \boldsymbol{T}_{\mathrm{NLO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) = \ln\left(\frac{\Lambda_0^-}{\Lambda_1^-}\right) \mathcal{H}\, \boldsymbol{T}_{\mathrm{LO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)$$

Solving classical Yang Mills eq. → soft mode gluon field through hard mode charge

Lorenz gauge $\partial^\mu \mathcal{A}^\mu = 0$

$$D_\mu\ \mathcal{F}^{\mu\nu} = J^\nu$$

$$J_a^\mu = \delta^{\mu +}\delta(x^-)\rho_a(x^-, x_\perp)$$

$$\mathcal{A}^{a\mu} = \delta^{\mu +}\alpha_a(x^-, \boldsymbol{x})$$

$$-\nabla_\perp^2 \alpha_a(\vec{x}) = \rho^a(x)$$

Color Glass Condensate

F.Gelis, E.Iancu, J.Jalilian-Marian and R.Venugopalan 2010

Consider the physical expectation values as averaging the charge distributions(hard) inside hadrons with gluon field(soft)

$$\langle \mathcal{O} \rangle_{\Lambda^+} \equiv \int [D\rho]\, W_{\Lambda^+}[\rho]\, \mathcal{O}[\rho]$$

$$\int [D\rho]\, W_{\Lambda_0^-}[\rho]\ (\boldsymbol{T}_{\text{LO}} + \delta \boldsymbol{T}_{\text{NLO}}) = \int [D\rho]\, W_{\Lambda_1^-}[\rho]\, \boldsymbol{T}_{\text{LO}}$$

$$W_{\Lambda_1^-} \equiv (1 + \ln(\Lambda_0^-/\Lambda_1^-)\,\mathcal{H})\, W_{\Lambda_0^-}$$

Quantum renormalization evolution of the charge distribution

B-JIMWLK equation

$$\frac{\partial W_Y[\rho]}{\partial Y} = H\, W_Y[\rho]$$

$$H = \frac{1}{2} \int_{x_\perp y_\perp} \frac{\delta}{\delta \rho^a(x_\perp)}\, \chi^{ab}(x_\perp, y_\perp)[\rho]\, \frac{\delta}{\delta \rho^b(y_\perp)}$$

$$\chi^{ab}(\boldsymbol{x}, \boldsymbol{y}) = \frac{1}{\pi} \int \frac{d^2\boldsymbol{z}}{(2\pi)^2} \mathcal{K}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \left(1 + \widetilde{V}_{\boldsymbol{x}}^\dagger \widetilde{V}_{\boldsymbol{y}} - \widetilde{V}_{\boldsymbol{x}}^\dagger \widetilde{V}_{\boldsymbol{z}} - \widetilde{V}_{\boldsymbol{z}}^\dagger \widetilde{V}_{\boldsymbol{y}}\right)^{ab}$$

$$\mathcal{K}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \equiv \frac{(x^i - z^i)(y^i - z^i)}{(\boldsymbol{x} - \boldsymbol{z})^2 (\boldsymbol{z} - \boldsymbol{y})^2} \qquad \widetilde{V}^\dagger(\boldsymbol{x}) = \mathrm{P} \exp\left(ig \int dx^- \alpha^a(x^-, \boldsymbol{x}) T^a\right)$$

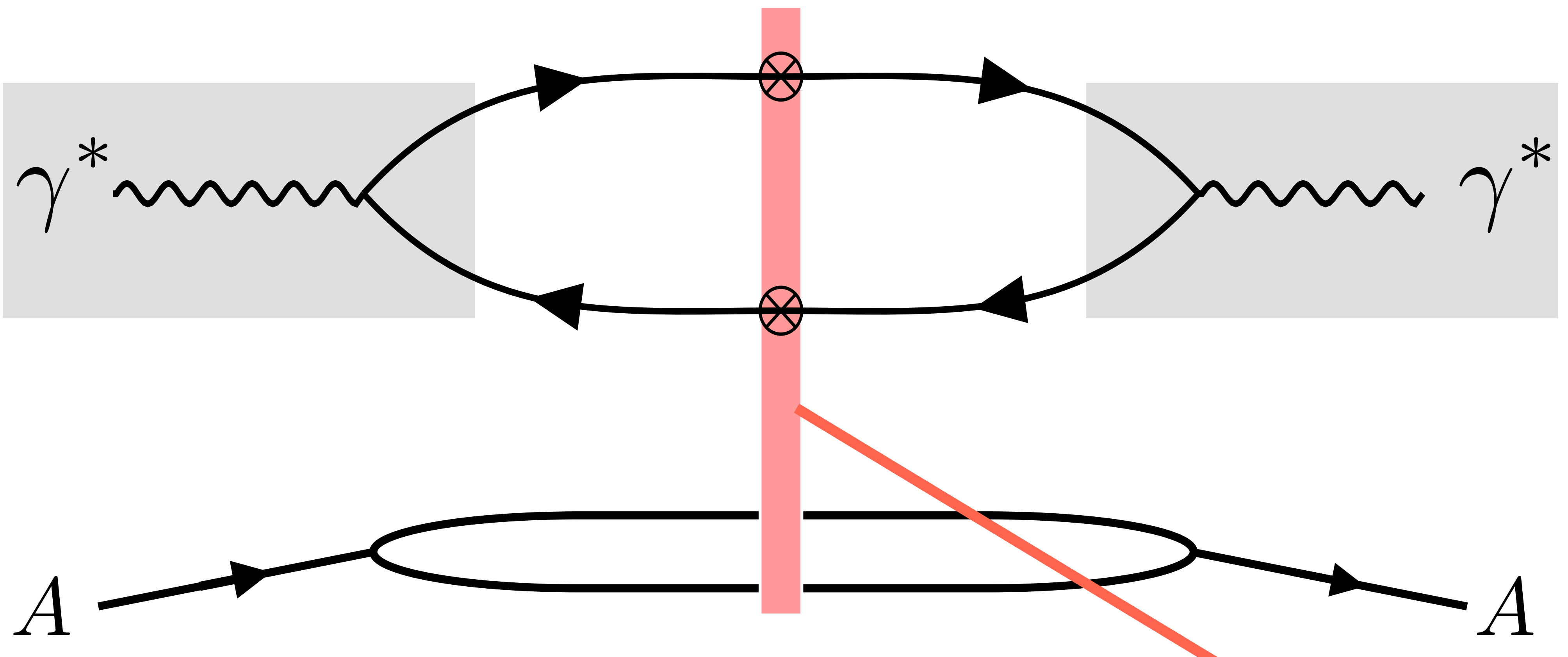
Initial distribution: McLerran Venugopalan model(Gaussian due to CLT)

$$W_{MV}[\rho] = \mathcal{C} \exp\left[-\int dx^- d^2x_\perp \frac{\rho^a(x^-, x_\perp)\rho^a(x^-, x_\perp)}{2\mu^2(x^-)}\right]$$

Meanings of S matrix in gluon saturation

Deep Inelastic Scattering in Mueller's dipole picture

A. H. Mueller, 94
Y. V. Kovchegov and E. Levin, 2012
A. Morreale and F. Salazar, 2021



Cross section for this DIS process

$$\sigma_\lambda^{\gamma^* A}(x, Q^2) = 2\int \mathrm{d}^2\boldsymbol{r}_\perp \mathrm{d}^2\boldsymbol{b}_\perp \int_0^1 \mathrm{d}z \left|\Psi_\lambda^{\gamma^*}(\boldsymbol{r}_\perp, Q^2, z)\right|^2 \left[1 - S_x^{(2)}\left(\boldsymbol{b}_\perp + \frac{\boldsymbol{r}_\perp}{2}, \boldsymbol{b}_\perp - \frac{\boldsymbol{r}_\perp}{2}\right)\right]$$

Dipole scattering amplitude

$$S_x^{(2)}\left(\boldsymbol{b}_\perp + \frac{\boldsymbol{r}_\perp}{2}, \boldsymbol{b}_\perp - \frac{\boldsymbol{r}_\perp}{2}\right) = \frac{1}{N_c}\left\langle \mathrm{tr}\left[U_{b_\perp + r_\perp/2} U^\dagger_{b_\perp - r_\perp/2}\right]\right\rangle_x$$

$$U_{x_\perp} = \mathcal{P}\exp\left[ig\int dz^+ \mathcal{A}^-(x_\perp, z^+)\right]$$

Twist 2 gluon GTMDs

Leading twist operators

C. Lorcé, B. Pasquini 2013

$$W^{ij}_{\Lambda'\Lambda} = \frac{2}{xP^+}\int \frac{\mathrm{d}z^-\mathrm{d}^2\boldsymbol{z}_\perp}{(2\pi)^3}\mathrm{e}^{\mathrm{i}xP^+z^- - \mathrm{i}\boldsymbol{k}_\perp\cdot\boldsymbol{z}_\perp}\langle p'\Lambda'|\mathrm{tr}\left(F^{+i}(-z/2)\mathcal{W}F^{+j}(z/2)\mathcal{W}'\right)|p\Lambda\rangle$$

Twist 2 operators

$$U = \delta^{ij}W^{ij}\,,$$

$$L = -\mathrm{i}\epsilon^{ij}W^{ij}\,,$$

$$T_R = -W^{RR}\,,$$

$$T_L = -W^{LL}\,.$$

$$a_{R,L} = a^1 \pm ia^2$$

Connections to helicity amplitudes

$$H_{\Lambda'\lambda',\Lambda\lambda}(P,k,\Delta,N;\eta_i) = \langle p',\Lambda'|\mathcal{O}_{\lambda'\lambda}(k,N;\eta_i)|p,\Lambda\rangle$$

$$\Downarrow$$

$$H_{\Lambda'\lambda',\Lambda\lambda} = \begin{pmatrix} \frac{1}{2}(U+L)_{\Lambda'\Lambda} & \frac{1}{2}T_{R,\Lambda'\Lambda} \\ \frac{1}{2}T_{L,\Lambda'\Lambda} & \frac{1}{2}(U-L)_{\Lambda'\Lambda} \end{pmatrix}_{\lambda'\lambda}$$

Helicity decomposed amplitude and GTMDs

A parametrization of twist 2 gluon GTMDs

C. Lorcé, B. Pasquini 2013

Parametrization through S, P, D, F states

	Helicity non-flip	Helicity flip
Spin non-flip	$H_{+\frac{1}{2}+1,+\frac{1}{2}+1} = \frac{1}{2}\left[(S^{0,+}_{1,1a} + S^{0,-}_{1,1a}) + \mathrm{i}\frac{\boldsymbol{k}_\perp \times \boldsymbol{\Delta}_\perp}{M^2}(S^{0,+}_{1,1b} + S^{0,-}_{1,1b})\right],$	$H_{+\frac{1}{2}+1,+\frac{1}{2}-1} = -\frac{1}{2}\left[\frac{k_R^2}{M^2}(D^{2,+}_{1,1a} + D'^{2,+}_{1,1a}) + \frac{\Delta_R^2}{M^2}(D^{2,+}_{1,1b} + D'^{2,+}_{1,1b})\right],$
	$H_{-\frac{1}{2}+1,-\frac{1}{2}+1} = \frac{1}{2}\left[(S^{0,+}_{1,1a} - S^{0,-}_{1,1a}) - \mathrm{i}\frac{\boldsymbol{k}_\perp \times \boldsymbol{\Delta}_\perp}{M^2}(S^{0,+}_{1,1b} - S^{0,-}_{1,1b})\right],$	$H_{-\frac{1}{2}+1,-\frac{1}{2}-1} = -\frac{1}{2}\left[\frac{k_R^2}{M^2}(D^{2,+}_{1,1a} - D'^{2,+}_{1,1a}) + \frac{\Delta_R^2}{M^2}(D^{2,+}_{1,1b} - D'^{2,+}_{1,1b})\right],$
Spin flip	$H_{+\frac{1}{2}+1,-\frac{1}{2}+1} = \frac{1}{2}\left[-\frac{k_L}{M}(P^{0,+}_{1,1a} - P^{0,-}_{1,1a}) - \frac{\Delta_L}{M}(P^{0,+}_{1,1b} - P^{0,-}_{1,1b})\right],$	$H_{+\frac{1}{2}+1,-\frac{1}{2}-1} = -\frac{1}{2}\left[\frac{k_R}{M}P^{2,+}_{1,1a} + \frac{\Delta_R}{M}P^{2,+}_{1,1b}\right],$
	$H_{-\frac{1}{2}+1,\frac{1}{2}+1} = \frac{1}{2}\left[\frac{k_R}{M}(P^{0,+}_{1,1a} + P^{0,-}_{1,1a}) + \frac{\Delta_R}{M}(P^{0,+}_{1,1b} + P^{0,-}_{1,1b})\right],$	$H_{-\frac{1}{2}+1,+\frac{1}{2}-1} = -\frac{1}{2}\left[\frac{k_R^3}{M^3}F^{2,+}_{1,1a} + \frac{\Delta_R^3}{M^3}F^{2,+}_{1,1b}\right].$

Parametrization of GTMDs at small x

Dipole amplitude representations

Small x limit of dipole gluon GTMDs

Y. Hatta, B. W. Xiao and F. Yuan, 2016,
D. Boer, T. Van Daal, P. J. Mulders, and E. Petreska, 2018

$$W^{ij}_{\Lambda'\Lambda} \to \frac{2N_c}{x\alpha_S}\left(k^i_\perp + \frac{1}{2}\Delta^i_\perp\right)\left(k^j_\perp - \frac{1}{2}\Delta^j_\perp\right)\mathcal{N}_{\Lambda'\Lambda}$$

$$\mathcal{N}_{\Lambda'\Lambda} = \int \frac{\mathrm{d}^2\boldsymbol{x}_\perp}{(2\pi)^2}\int \frac{\mathrm{d}^2\boldsymbol{y}_\perp}{(2\pi)^2} \mathrm{e}^{-\mathrm{i}\boldsymbol{k}_\perp\cdot(\boldsymbol{x}_\perp-\boldsymbol{y}_\perp)+\mathrm{i}\boldsymbol{\Delta}_\perp\cdot\frac{\boldsymbol{x}_\perp+\boldsymbol{y}_\perp}{2}} \frac{1}{N_c}\frac{\langle p'\Lambda'|\mathrm{tr}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right]|p\Lambda\rangle}{\langle P\Lambda|P\Lambda\rangle}$$

Relations to twist 2 operators

$$U_{\Lambda'\Lambda} = \frac{2N_c}{x\alpha_S}\left(\boldsymbol{k}^2_\perp - \frac{\boldsymbol{\Delta}^2_\perp}{4}\right)\mathcal{N}_{\Lambda'\Lambda},$$

$$L_{\Lambda'\Lambda} = \frac{2\mathrm{i}N_c}{x\alpha_S}(\boldsymbol{k}_\perp \times \boldsymbol{\Delta}_\perp)\mathcal{N}_{\Lambda'\Lambda},$$

$$T_{R,\Lambda'\Lambda} = -\frac{2N_c}{x\alpha_S}\left(k^2_R - \frac{\Delta^2_R}{4}\right)\mathcal{N}_{\Lambda'\Lambda}.$$

Parametrization of dipole amplitude

$$\mathcal{N}_{\Lambda'\Lambda} = \delta_{\Lambda\Lambda'}\mathcal{N} + \delta_{\Lambda,-\Lambda'}\frac{1}{M}(\Lambda k^1_\perp + \mathrm{i}k^2_\perp)\mathcal{N}^\perp_{1T} + \delta_{\Lambda,-\Lambda'}\frac{1}{M}(\Lambda\Delta^1_\perp + \mathrm{i}\Delta^2_\perp)\mathcal{N}_T$$

Matching of two parameterizations

Helicity non-flip part

We confirm the known results for unpolarized TMD and $k_\perp$-moment of spin orbit correlation S.Bhattacharya, R.Boussarie and Y.Hatta, 2024

Spin non-flip

$$xS^{0,+}_{1,1a} = \frac{2N_c}{\alpha_S}\left(\boldsymbol{k}_\perp^2 - \frac{\boldsymbol{\Delta}_\perp^2}{4}\right)\mathcal{N}\,,$$

$$xS^{0-}_{1,1a} = 0\,,$$

$$xS^{0+}_{1,1b} = 0\,,$$

$$xS^{0,-}_{1,1b} = \frac{2N_c}{\alpha_S}M^2\mathcal{N}\,.$$

Spin flip

$$xP^{0,+}_{1,1a} = \frac{2N_c}{\alpha_S}\left(\boldsymbol{k}_\perp^2 - \frac{\boldsymbol{\Delta}_\perp^2}{4}\right)\mathcal{N}^\perp_{1T}\,,$$

$$xP^{0,-}_{1,1a} = -\frac{2N_c}{\alpha_S}\left[(\boldsymbol{k}_\perp\cdot\boldsymbol{\Delta}_\perp)\mathcal{N}^\perp_{1T} + \boldsymbol{\Delta}_\perp^2\mathcal{N}_T\right]\,,$$

$$xP^{0,+}_{1,1b} = \frac{2N_c}{\alpha_S}\left(\boldsymbol{k}_\perp^2 - \frac{\boldsymbol{\Delta}_\perp^2}{4}\right)\mathcal{N}_T\,,$$

$$xP^{0,-}_{1,1b} = \frac{2N_c}{\alpha_S}\left[\boldsymbol{k}_\perp^2\mathcal{N}^\perp_{1T} + (\boldsymbol{k}_\perp\cdot\boldsymbol{\Delta}_\perp)\mathcal{N}_T\right]\,.$$

Matching of two parameterizations

Gluon helicity flip part

the PT symmetry of the Wilson lines → some zero components

Spin non-flip

$$xD^{2,+}_{1,1a} = -\frac{2N_c}{\alpha_S} M^2 \mathcal{N},$$

$$xD'^{2,+}_{1,1a} = 0,$$

$$xD^{2,+}_{1,1b} = \frac{2N_c}{\alpha_S} M^2 \mathcal{N},$$

$$xD'^{2,+}_{1,1b} = 0.$$

Spin flip

$$xP^{2,+}_{1,1a} = \frac{2N_c}{\alpha_S} \left[\left(\boldsymbol{k}_\perp^2 + \frac{\boldsymbol{\Delta}_\perp^2}{4} \right) \mathcal{N}^\perp_{1T} + 2(\boldsymbol{k}_\perp \cdot \boldsymbol{\Delta}_\perp) \mathcal{N}_T \right]$$

$$xP^{2,+}_{1,1b} = \frac{2N_c}{\alpha_S} \left[\frac{1}{2} (\boldsymbol{k}_\perp \cdot \boldsymbol{\Delta}_\perp) \mathcal{N}^\perp_{1T} + \left(\boldsymbol{k}_\perp^2 + \frac{\boldsymbol{\Delta}_\perp^2}{4} \right) \mathcal{N}_T \right]$$

$$xF^{2,+}_{1,1a} = -\frac{2N_c}{\alpha_S} M^2 \left[\frac{4(\boldsymbol{k}_\perp \cdot \boldsymbol{\Delta}_\perp)^2 - \boldsymbol{k}_\perp^2 \boldsymbol{\Delta}_\perp^2 - \frac{\boldsymbol{\Delta}_\perp^4}{4}}{4(\boldsymbol{k}_\perp \cdot \boldsymbol{\Delta}_\perp)^2 - \boldsymbol{k}_\perp^2 \boldsymbol{\Delta}_\perp^2} \mathcal{N}^\perp_{1T} + \frac{2(\boldsymbol{k}_\perp \cdot \boldsymbol{\Delta}_\perp) \boldsymbol{\Delta}_\perp^2}{4(\boldsymbol{k}_\perp \cdot \boldsymbol{\Delta}_\perp)^2 - \boldsymbol{k}_\perp^2 \boldsymbol{\Delta}_\perp^2} \mathcal{N}_T \right],$$

$$xF^{2,+}_{1,1b} = \frac{N_c}{\alpha_S} M^2 \left[\frac{\boldsymbol{k}_\perp^2 (\boldsymbol{k}_\perp \cdot \boldsymbol{\Delta}_\perp)}{4(\boldsymbol{k}_\perp \cdot \boldsymbol{\Delta}_\perp)^2 - \boldsymbol{k}_\perp^2 \boldsymbol{\Delta}_\perp^2} \mathcal{N}^\perp_{1T} + \frac{1}{2} \frac{4(\boldsymbol{k}_\perp \cdot \boldsymbol{\Delta}_\perp)^2 - \boldsymbol{k}_\perp^2 \boldsymbol{\Delta}_\perp^2 - 4\boldsymbol{k}_\perp^4}{4(\boldsymbol{k}_\perp \cdot \boldsymbol{\Delta}_\perp)^2 - \boldsymbol{k}_\perp^2 \boldsymbol{\Delta}_\perp^2} \mathcal{N}_T \right].$$

TMD limit

Consistency check

Relations between GTMDs and TMDs

$$xf_1 = x\mathrm{Re}(S^{0,+}_{1,1a}) \to \frac{2N_c}{\alpha_S}\boldsymbol{k}_\perp^2 \mathcal{P}\,,$$

$$xg_{1L} = x\mathrm{Re}(S^{0,-}_{1,1a}) \to 0\,,$$

$$xh_1 = x\mathrm{Im}(P^{2,+}_{1,1a}) \to \frac{2N_c}{\alpha_S}\boldsymbol{k}_\perp^2 \mathcal{O}^\perp_{1T}\,,$$

$$xh^\perp_{1L} = 2x\mathrm{Im}(D'^{2,+}_{1,1a}) \to 0\,,$$

$$xf^\perp_{1T} = -x\mathrm{Im}(P^{0,+}_{1,1a}) \to \frac{2N_c}{\alpha_S}\boldsymbol{k}_\perp^2 \mathcal{O}^\perp_{1T}\,,$$

$$xg_{1T} = x\mathrm{Re}(P^{0,-}_{1,1a}) \to 0\,,$$

$$xh^\perp_1 = 2x\mathrm{Re}(D^{2,+}_{1,1a}) = -\frac{4N_c}{\alpha_S} M^2 \mathcal{P}\,,$$

$$xh^\perp_{1T} = 2x\mathrm{Im}(F^{2,+}_{1,1a}) = -\frac{4N_c}{\alpha_S} M^2 \mathcal{O}^\perp_{1T}\,.$$

Parametrization of dipole amplitudes

$$\mathcal{N} = \mathcal{P} + \mathrm{i}\frac{(\boldsymbol{k}_\perp\cdot\boldsymbol{\Delta}_\perp)}{M^2}\mathcal{O}\,,$$

$$\mathcal{N}^\perp_{1T} = \frac{(\boldsymbol{k}_\perp\cdot\boldsymbol{\Delta}_\perp)}{M^2}\mathcal{P}^\perp_{1T} + \mathrm{i}\mathcal{O}^\perp_{1T}\,,$$

$$\mathcal{N}_T = \mathcal{P}_T + \mathrm{i}\frac{(\boldsymbol{k}_\perp\cdot\boldsymbol{\Delta}_\perp)}{M^2}\mathcal{O}_T\,.$$

Obtained known relations

$$f_1 = -\frac{\boldsymbol{k}_\perp^2}{2M^2}h_1^\perp\,, \qquad f^\perp_{1T} = h_1 = -\frac{\boldsymbol{k}_\perp^2}{2M^2}h^\perp_{1T}\,,$$

D. Boer, T. Van Daal, P. J. Mulders, and E. Petreska, 2018

TMD limit

Consistency check

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$$xf_1 = x\mathrm{Re}(S^{0,+}_{1,1a}) \to \frac{2N_c}{\alpha_S}\boldsymbol{k}_\perp^2 \mathcal{P}\,,$$

$$xg_{1L} = x\mathrm{Re}(S^{0,-}_{1,1a}) \to 0\,,$$

$$xh_1 = x\mathrm{Im}(P^{2,+}_{1,1a}) \to \frac{2N_c}{\alpha_S}\boldsymbol{k}_\perp^2 \mathcal{O}^\perp_{1T}\,,$$

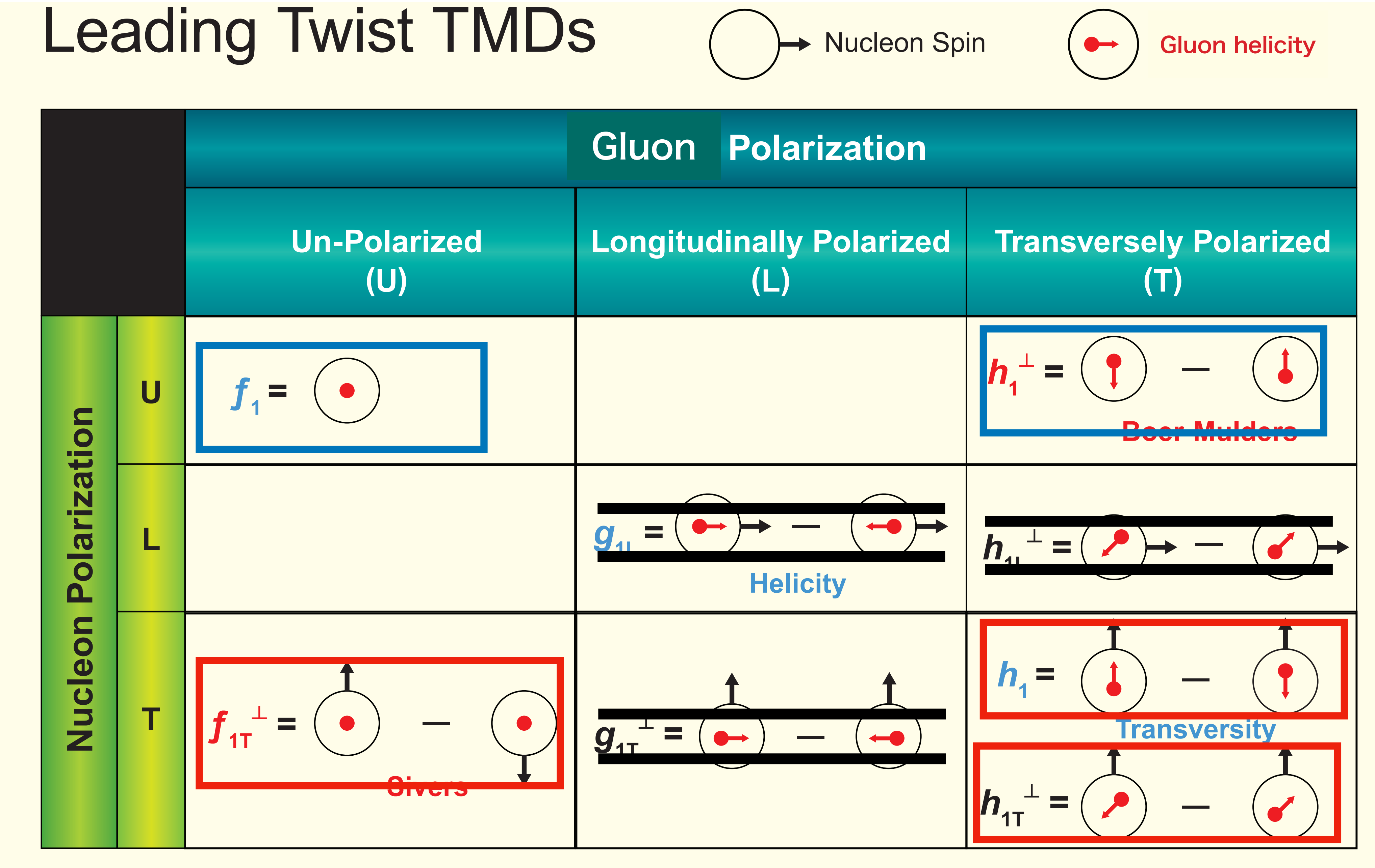
$$xh^\perp_{1L} = 2x\mathrm{Im}(D'^{2,+}_{1,1a}) \to 0\,,$$

$$xf^\perp_{1T} = -x\mathrm{Im}(P^{0,+}_{1,1a}) \to \frac{2N_c}{\alpha_S}\boldsymbol{k}_\perp^2 \mathcal{O}^\perp_{1T}\,,$$

$$xg_{1T} = x\mathrm{Re}(P^{0,-}_{1,1a}) \to 0\,,$$

$$xh^\perp_1 = 2x\mathrm{Re}(D^{2,+}_{1,1a}) = -\frac{4N_c}{\alpha_S}M^2\mathcal{P}\,,$$

$$xh^\perp_{1T} = 2x\mathrm{Im}(F^{2,+}_{1,1a}) = -\frac{4N_c}{\alpha_S}M^2\mathcal{O}^\perp_{1T}\,.$$



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$$f_1 = -\frac{\boldsymbol{k}_\perp^2}{2M^2}h_1^\perp\,, \qquad f^\perp_{1T} = h_1 = -\frac{\boldsymbol{k}_\perp^2}{2M^2}h^\perp_{1T}\,,$$

D. Boer, T. Van Daal, P. J. Mulders, and E. Petreska, 2018

GPD limit

Azimuthal expansion

$$\mathcal{P}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) = \mathcal{P}_0(k_\perp, \Delta_\perp) + 2\cos(2\phi_{k\Delta})\mathcal{P}_\epsilon(k_\perp, \Delta_\perp) + \dots ,$$

$$\mathcal{P}_{1T}^\perp(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) = \mathcal{P}_{1T,0}^\perp(k_\perp, \Delta_\perp) + 2\cos(2\phi_{k\Delta})\mathcal{P}_{1T,\epsilon}^\perp(k_\perp, \Delta_\perp) + \dots ,$$

$$\mathcal{P}_T(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) = \mathcal{P}_{T,0}(k_\perp, \Delta_\perp) + 2\cos(2\phi_{k\Delta})\mathcal{P}_{T,\epsilon}(k_\perp, \Delta_\perp) + \dots ,$$

Relation to GPDs

$$H = \frac{1}{\sqrt{1-\xi^2}}[\mathcal{S}_{1,1}^{0,+} + 2\xi^2\mathcal{P}_{1,1}^{0,+}] \to \mathcal{S}_{1,1}^{0,+} ,$$

$$E = 2\sqrt{1-\xi^2}\mathcal{P}_{1,1}^{0,+} \to 2\mathcal{P}_{1,1}^{0,+} ,$$

$$\tilde{H} = \frac{1}{\sqrt{1-\xi^2}}[\mathcal{S}_{1,1}^{0,-} + 2\xi\mathcal{P}_{1,1}^{0,-}] \to \mathcal{S}_{1,1}^{0,-} ,$$

$$\xi\tilde{E} = 2\sqrt{1-\xi^2}\mathcal{P}_{1,1}^{0,-} \to 2\mathcal{P}_{1,1}^{0,-} ,$$

$$H_T = -\frac{1}{\sqrt{1-\xi^2}}\left[\mathcal{P}_{1,1}^{2,+} - 4\xi\mathcal{D}_{1,1}^{\prime 2,+} + \frac{\boldsymbol{\Delta}_\perp^2}{M^2}\mathcal{F}_{1,1}^{2,+}\right] \to -\mathcal{P}_{1,1}^{2,+} - \frac{\boldsymbol{\Delta}_\perp^2}{M^2}\mathcal{F}_{1,1}^{2,+} ,$$

$$E_T = -\frac{4}{\sqrt{1-\xi^2}}\left[\mathcal{D}_{1,1}^{2,+} + \xi\mathcal{D}_{1,1}^{\prime 2,+} + 2\mathcal{F}_{1,1}^{2,+}\right] \to -4\left[\mathcal{D}_{1,1}^{2,+} + 2\mathcal{F}_{1,1}^{2,+}\right] ,$$

$$\tilde{H}_T = 4\sqrt{1-\xi^2}\mathcal{F}_{1,1}^{2,+} \to 4\mathcal{F}_{1,1}^{2,+} ,$$

$$\tilde{E}_T = -\frac{4}{\sqrt{1-\xi^2}}\left[\xi\mathcal{D}_{1,1}^{2,+} + \mathcal{D}_{1,1}^{\prime 2,+} + 2\xi\mathcal{F}_{1,1}^{2,+}\right] \to -4\mathcal{D}_{1,1}^{\prime 2,+} ,$$

Y.Hatta, B.W.Xiao and F.Yuan, 2017
For xH and xEt

Obtained relations

$$xH = \frac{2N_c}{\alpha_S}\int d^2\boldsymbol{k}_\perp \boldsymbol{k}_\perp^2 \mathcal{P}_0 ,$$

$$xE = \frac{8N_c}{\alpha_S}\int d^2\boldsymbol{k}_\perp \boldsymbol{k}_\perp^2 \left[\frac{\boldsymbol{k}_\perp^2}{M^2}\mathcal{P}_{1T,0}^\perp + 2\mathcal{P}_{T,0}\right] ,$$

$$x\tilde{H} = 0 ,$$

$$\xi\tilde{E} = 0 ,$$

$$xH_T = -\frac{4N_c}{\alpha_S}\int d^2\boldsymbol{k}_\perp \boldsymbol{k}_\perp^2 \left[\frac{\boldsymbol{k}_\perp^2}{M^2}\mathcal{P}_{1T,0}^\perp + 4\mathcal{P}_{T,0} - \frac{\boldsymbol{k}_\perp^2}{4M^2}\mathcal{P}_{1T,\epsilon}^\perp\right]$$

$$xE_T = \frac{8N_c}{\alpha_S}\frac{M^2}{\boldsymbol{\Delta}_\perp^2}\int d^2\boldsymbol{k}_\perp \boldsymbol{k}_\perp^2 \left[\mathcal{P}_\epsilon + \frac{\boldsymbol{k}_\perp^2}{M^2}\mathcal{P}_{1T,\epsilon}^\perp\right] ,$$

$$x\tilde{H}_T = -\frac{4N_c}{\alpha_S}\frac{1}{\boldsymbol{\Delta}_\perp^2}\int d^2\boldsymbol{k}_\perp \boldsymbol{k}_\perp^4 \mathcal{P}_{1T,\epsilon}^\perp ,$$

$$x\tilde{E}_T = 0 .$$

Conclusions

- The factorization theorem enables the separation of cross sections into perturbative and non-perturbative parts. The non-perturbative component is universal and encoded in the PDFs

- Within the CGC formalism, these PDFs can be expressed in terms of expectation values of Wilson lines

- By parameterizing PDFs through twist expansion and using the CGC framework, we establish connections among GTMDs. This provides a unified description of PDFs directly via the dipole scattering amplitude

Thank you!