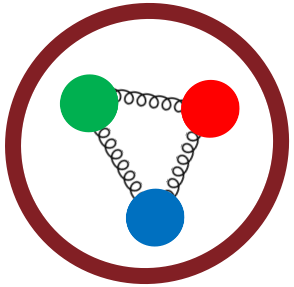
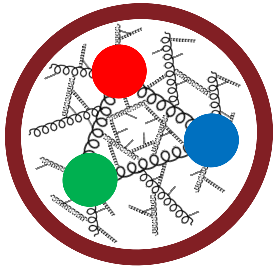
Single Spin Asymmetry from Pomeron-Odderon Interference



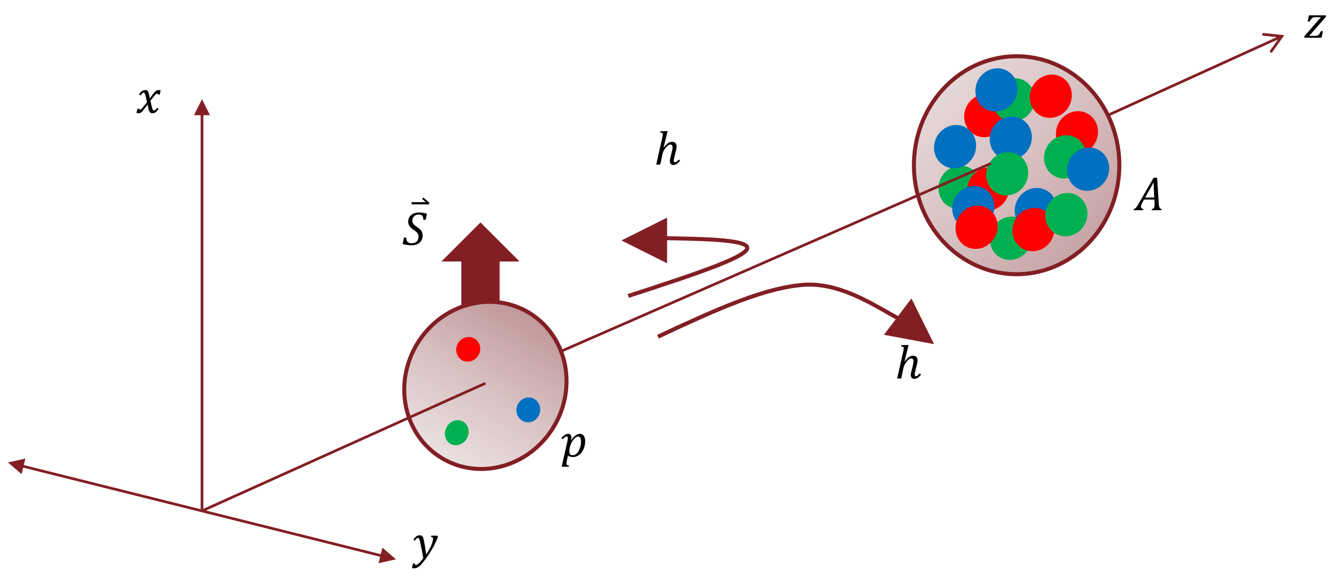
Eric Andreas Vivoda (University of Zagreb)
ACHT 2025, Budapest, 7.5.2025.



S. Benić, EAV, 2501.12847

Transverse Single Spin Asymmetry

- Left-right asymmetry of produced particles in collisions involving polarized hadrons



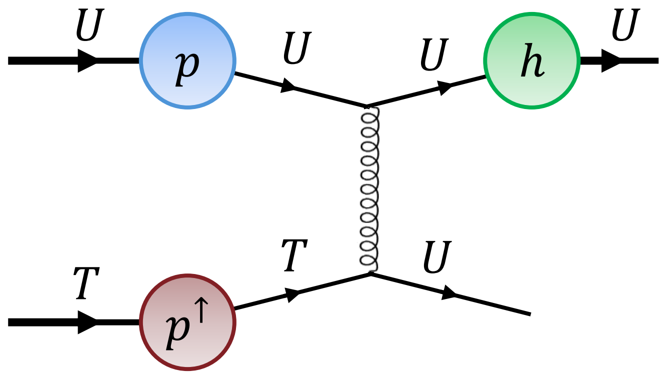
$$A_N \equiv \frac{N_L - N_R}{N_L + N_R} = \frac{\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}}{\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}} = \frac{\mathrm{d}\Delta\sigma}{2\mathrm{d}\sigma_{unp}}$$

Estimate in collinear QCD

Kane, Pumplin, Repko, PRL, 1978.

- The polarized cross section for $p^{\uparrow}p \to hX$ is written using QCD factorization:

$$\mathrm{d}\sigma^{p^{\uparrow}p\to hX} = f^{p} \otimes F^{p^{\uparrow}}_{\uparrow\uparrow} \otimes \mathrm{d}\hat{\sigma}^{\uparrow} \otimes D^{q\to h}$$
$$+ f^{p} \otimes F^{p^{\uparrow}}_{\uparrow\downarrow} \otimes \mathrm{d}\hat{\sigma}^{\downarrow} \otimes D^{q\to h}$$



1. $f^{p}(x)$: collinear PDF
2. $F^{p^{\uparrow}}_{\uparrow\uparrow(\downarrow)}(x)$: probability to find $q^{\uparrow(\downarrow)}$ in $p^{\uparrow}$
3. $d\sigma^{\uparrow(\downarrow)}$: parton level polarized cross sections
4. $D^{q\to h}$: fragmentation function
5. T: transverse polarization
6. U: unpolarized

- Rotational invariance of QCD:

$$\mathrm{d}\Delta\sigma = f^{p} \otimes \left(F^{p^{\uparrow}}_{\uparrow\uparrow} - F^{p^{\uparrow}}_{\uparrow\downarrow}\right) \otimes \left(\mathrm{d}\hat{\sigma}^{\uparrow} - \mathrm{d}\hat{\sigma}^{\downarrow}\right) \otimes D^{q\to h}$$

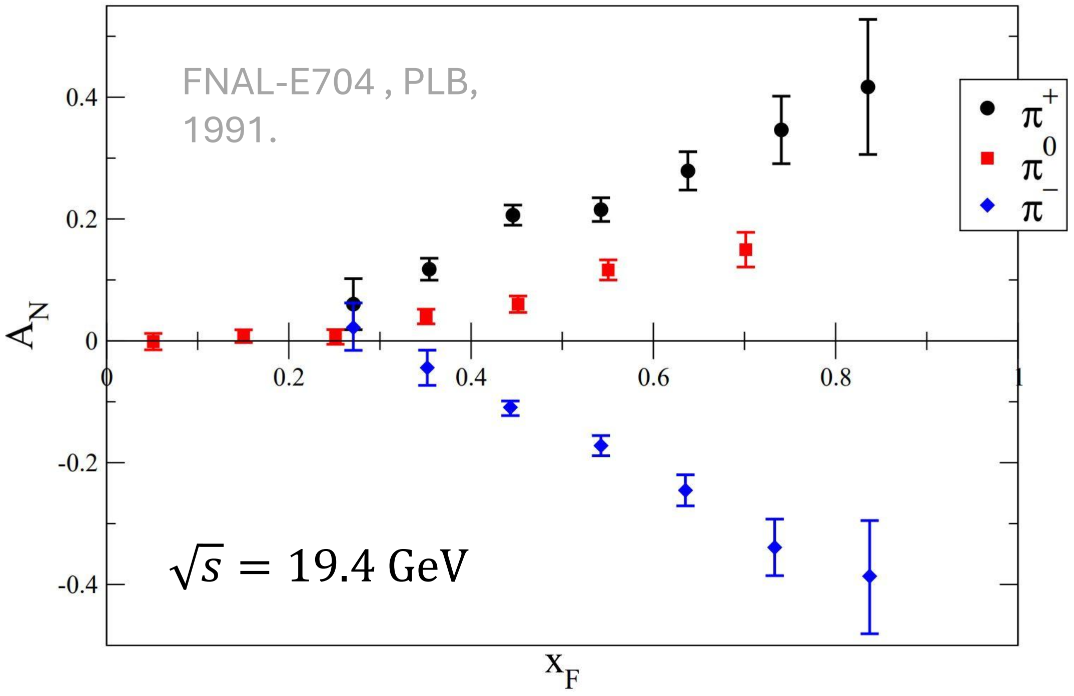
$$\mathrm{d}\hat{\sigma}^{\uparrow} - \mathrm{d}\hat{\sigma}^{\downarrow} \propto \mathrm{Re}(\mathcal{M}_{+}^{*}\mathcal{M}_{-})$$

Helicity flip! →

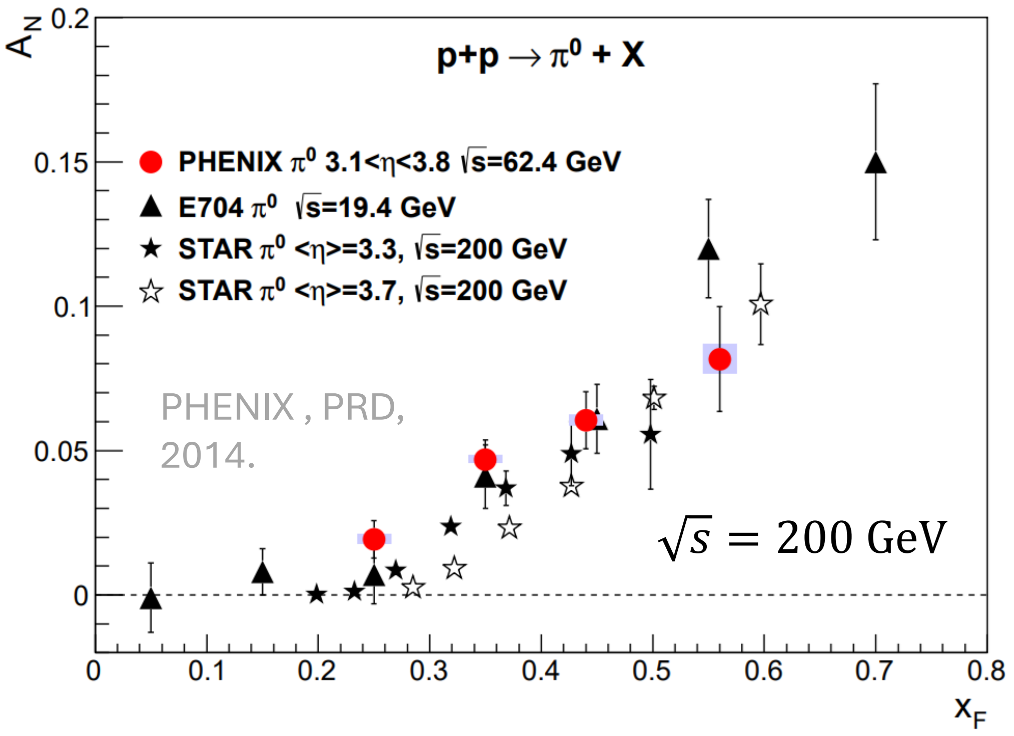
$$\boldsymbol{A_N \propto \alpha_s \frac{m_q}{P_{h\perp}}}$$

Vanish?

TSSA in $p^{\uparrow}p \to hX$ - experiments



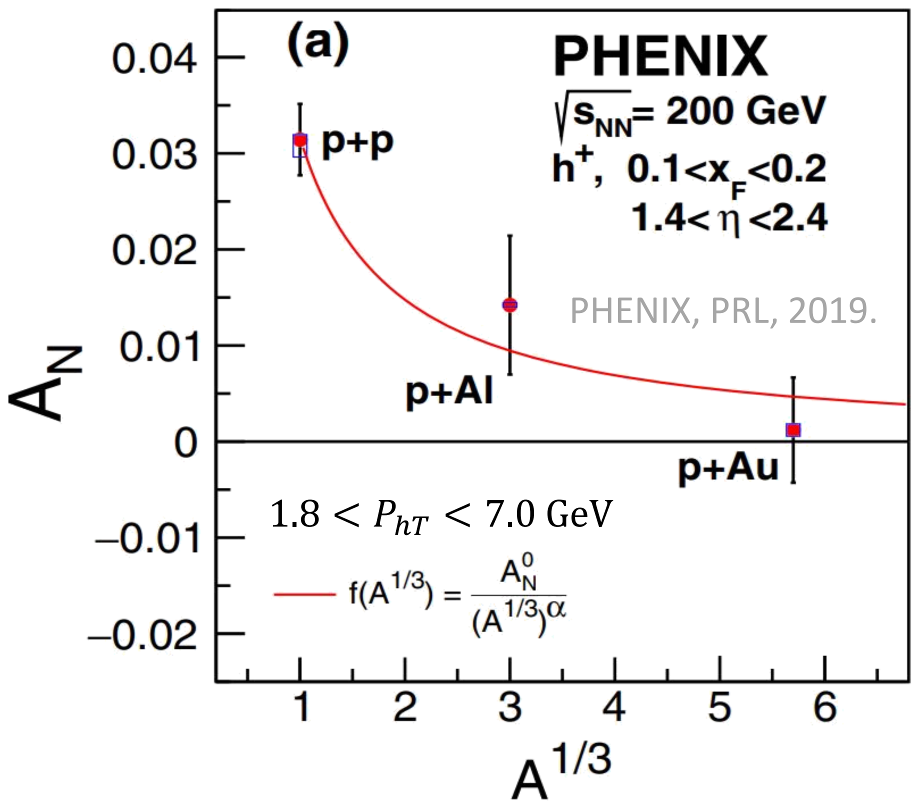
Larger energy

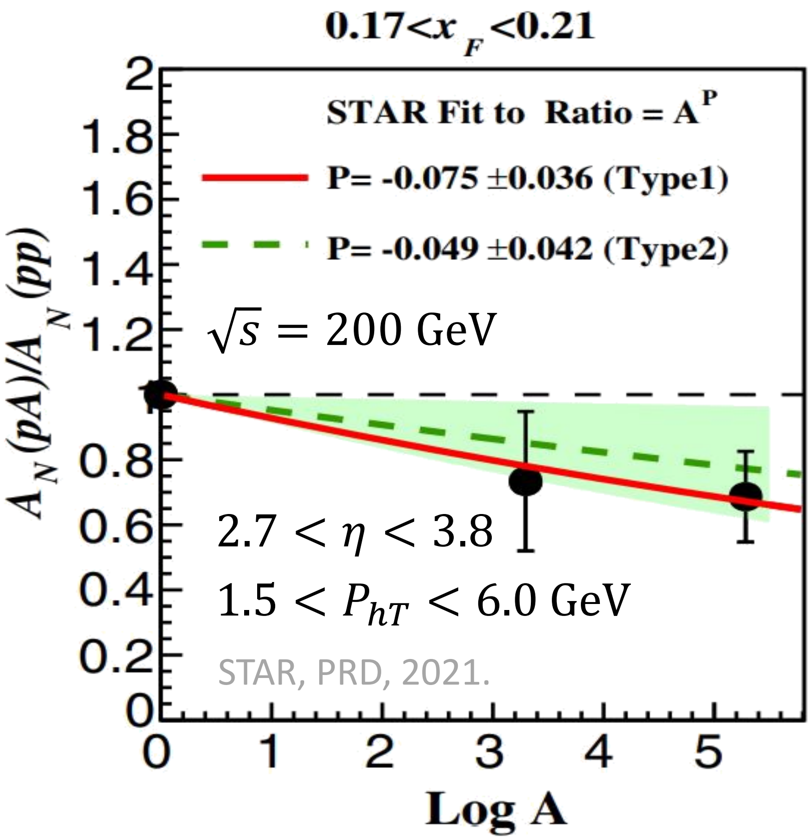


$$x_F = \frac{2P_h^z}{\sqrt{s}}$$

Asymmetry is largest in forward region of produced hadron!

TSSA in $p^{\uparrow}A \to hX$ - experiments





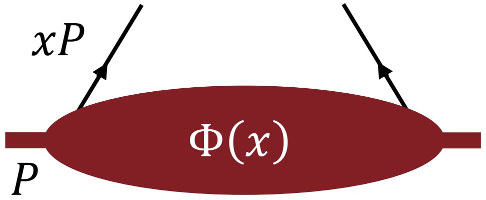
Significant A dependence!

$A_N \sim A^{(-1/3)}$

$A_N \sim A^{-0.027 \pm 0.005}$

I. Helicity flip should be **generated unperturbatively**!

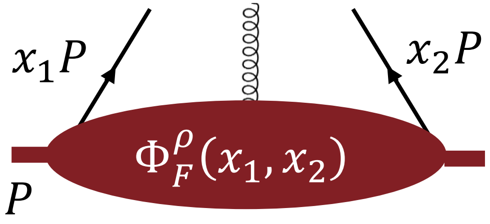
➢ **In pp and pA: Collinear twist-3 framework** → Considering full set of twist-3 distribution and fragmentation functions



Efremov and Teryaev, PLB, 1985.

→

Qiu and Sterman, PRL, 1991.



$$\langle P,S|\bar{\psi}\psi|P,S\rangle \propto \not{P} f(x) - \frac{i}{2}\gamma_5 P^+ S_{\perp i}\sigma^{-i} h_1(x) + \cdots$$

$$\langle P,S|\bar{\psi}F\psi|P,S\rangle \propto \frac{M_N}{4}\not{P}\epsilon^{\alpha P n S_\perp} G_F(x_1, x_2) + \cdots$$

ETQS function

II. Rise of TSSA with x_F → small-x (saturation) effects in target

Light-cone coordinates:

- $x^{\pm} = \frac{x^0 \pm x^3}{\sqrt{2}}$
- $\boldsymbol{x}_\perp = x_1 \hat{e}_1 + x_2 \hat{e}_2$

$\langle P,S|\bar{\psi}\psi|P,S\rangle \propto \not{P} f(x) - \frac{i}{2}\gamma_5 P^+ S_{\perp i}\sigma^{-i} h_1(x) + \cdots$

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ETQS function

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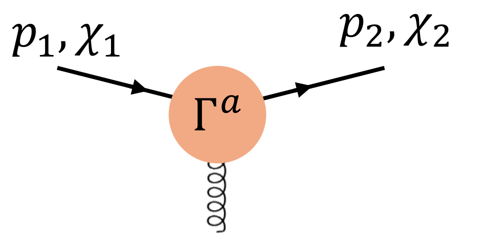
Structure of polarized cross section

- Consider a Dirac part of some Feynman amplitude:



$$T^a_{\chi_1\chi_2}(p_1,p_2) \equiv \bar{u}_{\chi_2}(p_2)\overbrace{\gamma^{\mu_{a_1}}\cdots\gamma^{\mu_{a_n}}}^{\Gamma^a}u_{\chi_1}(p_1) = (CPT) = \chi_1\chi_2 T_{-\chi_1-\chi_2}(p_1,p_2)^*$$

- Parametrization:

$$T^a_{\chi_1\chi_2}(p_1,p_2) = \delta_{\chi_1,\chi_2}[\overset{\text{Re}}{A^a(p_1,p_2)} + \chi_1 \overset{\text{Im}}{B^a(p_1,p_2)}] + \delta_{\chi_1,-\chi_2}[\overset{\text{Im}}{C^a(p_1,p_2)} + \chi_1 \overset{\text{Re}}{D^a(p_1,p_2)}]$$

- Cross section level: in general interference of two amplitudes and summation over χ_2

$$\sum_{\chi_2} T^a_{\chi_1\chi_2} {T^b_{\chi_1\chi_2}}^* = \overset{\text{Re}}{(A_a A_b^* + B_a B_b^* + C_a C_b^* + D_a D_b^*)} + \chi_1 \overset{\text{Im}}{(A_a B_b^* + B_a A_b^* + C_a D_b^* + D_a C_b^*)}$$

- If $a = b$ (the square of the amplitude), imaginary spin dependent part vanishes exactly

➢ **TSSA is consequence of amplitude interference!**

➢ **Cross section is real: we need another "i"**

Sievert,arxiv:1407.4047

Some known ways in $p^{\uparrow}p$ (on tree level)

- Generally: $\sigma \sim D \otimes F \otimes f \otimes H$

D	F	f	H
Fragmentation function	Distribution in projectile	Distribution in target	Hard factor
NONPERTURBATIVE	NONPERTURBATIVE	NONPERTURBATIVE	PERTURBATIVE

Pole-ETQS mechanism

Kouvaris et. al., PRD, 2006.

- Twist-3 collinear (ETQS) distribution from polarized proton
- Twist-2 collinear distribution in target
- Twist-2 fragmentation

- Phase comes from imaginary part of internal propagator (SGP)

$$\sigma \sim D_2 \otimes G_{F3} \otimes f_2 \otimes H_{pole}$$

Twist-3 fragmentation

Metz and Pitonyak, PLB, 2013.

- Twist-2 transversity distribution from polarized proton
- Twist-2 collinear distribution in target

- Imaginary part of twist-3 fragmentation function gives necessary phase

$$\sigma \sim \mathrm{Im}(\mathcal{D}_3) \otimes h_2 \otimes f_2 \otimes H$$

Twist-3 from the target

Kanazawa and Koike, PLB, 2000.

- Twist-2 fragmentation function
- Twist-2 transversity from polarized nucleon
- Twist-3 distribution for non-polarized target

$$\sigma \sim D_2 \otimes h_2 \otimes F_3 \otimes H_{pole}$$

- Phase from pole!

Some known ways in $p^{\uparrow}p$ (on tree level)

- Generally: $\sigma \sim D \otimes F \otimes f \otimes H$

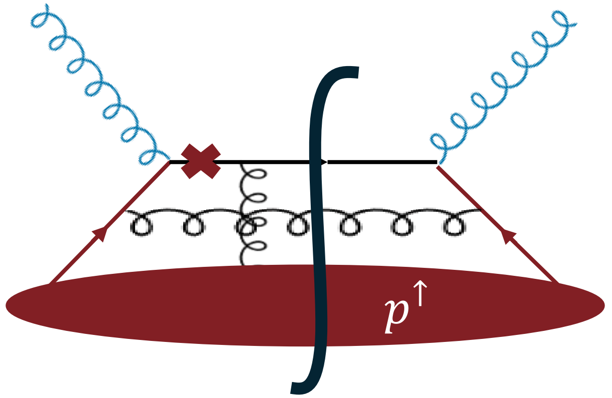
Fragmentation function — NONPERTURBATIVE

Distribution in projectile — NONPERTURBATIVE

Distribution in target — NONPERTURBATIVE

Hard factor — PERTURBATIVE

Pole-ETQS mechanism



- ETQS functions are real!

$$\frac{1}{p^2 + i\epsilon} = P\left(\frac{1}{p^2}\right) - i\pi\delta(p^2)$$

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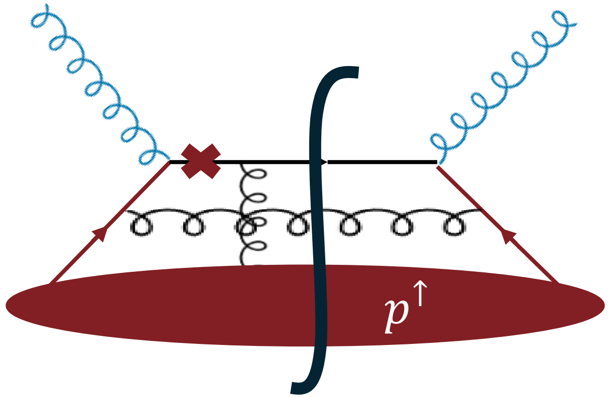

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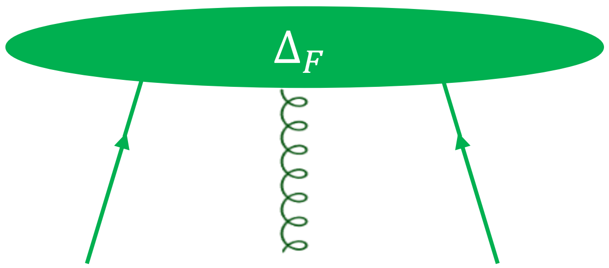

- Twist-3 FFs are complex function
- ➢ Consider only imaginary part!

Twist-3 from the target

Kanazawa and Koike, PLB, 2000.

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- Twist-2 transversity from polarized nucleon
- Twist-3 distribution for non-polarized target

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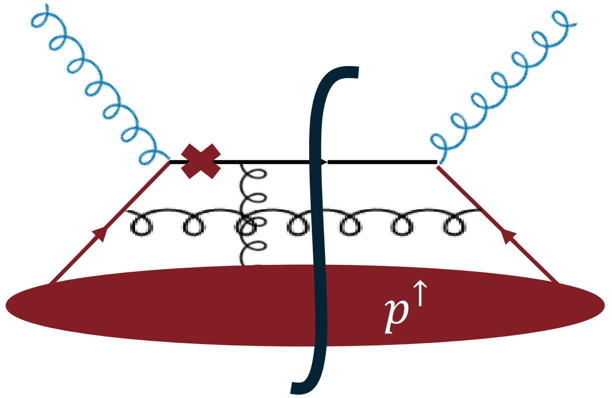
Pole-ETQS mechanism



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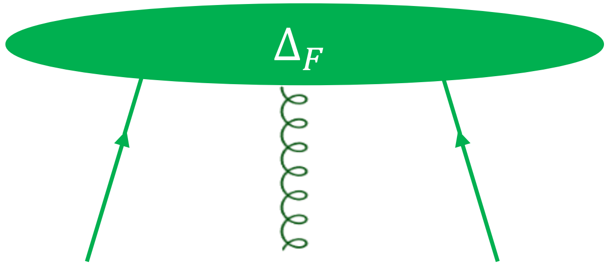


- Twist-3 FFs are complex function

➢ Consider only imaginary part!

Twist-3 from the target

➢ Suppressed mechanism in forward region

➢ This work: describe unpolarized target in saturation framework

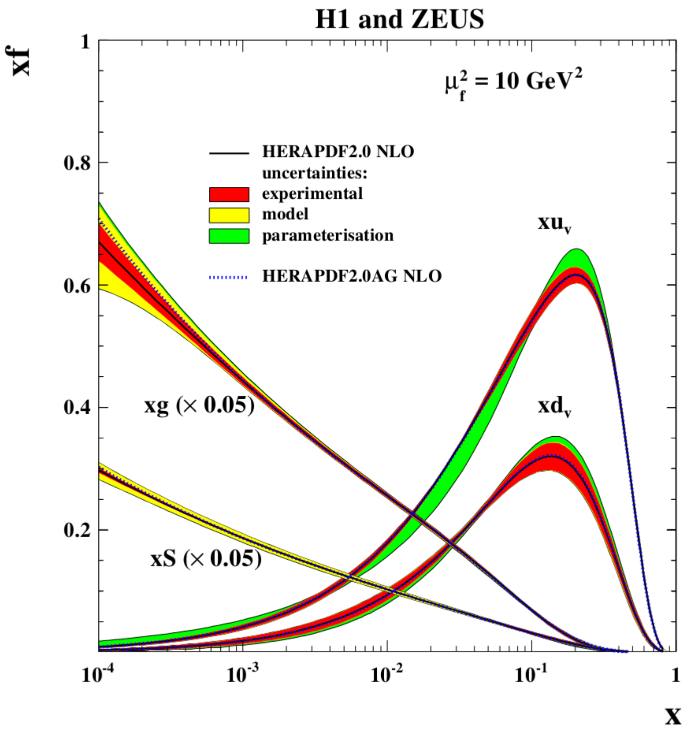
➢ Hybrid approach

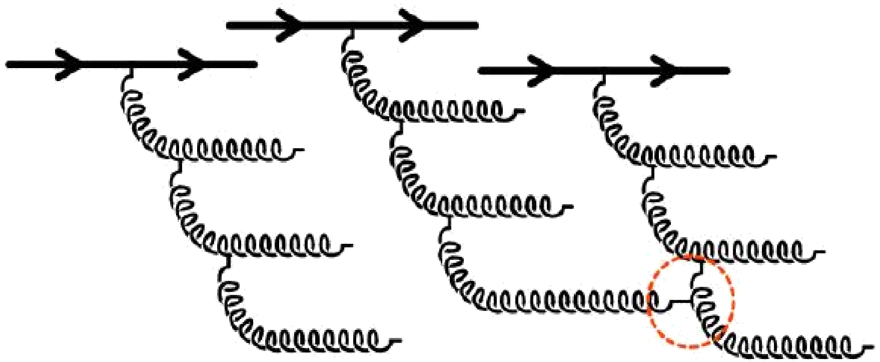
➢ Dilute-dense approximation

$p^{\uparrow}$ $\qquad$ p/A

Color Glass Condensate

- At high energies, small-x gluons dominate inside the target
- At some point, gluons wave functions start to overlap





F. Gelis, 1211.3327

$$\Gamma_{gg\to g} = \frac{\alpha_S N_C}{(N_C^2-1)Q^2}\frac{xf_g(x,Q^2)}{\pi R^2} \approx 1$$

Q_S- **Saturation scale**

$$(Q_S^A)^2 = A^{\frac{1}{3}}(Q_S^p)^2$$

Nuclear dependence!

- At Q_S the gluon occupation number is maximal
- Can be treated as classical field
- How to obtain this field?

Color Glass Condensate

Y. Hagiwara talk

- If the saturated target is moving in $-z$ direction all fast partons ($x > x_0$) form a color charge current:

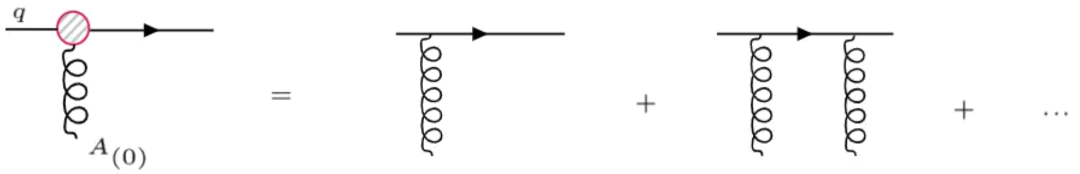
$$J^{\nu}_{x_0} = g\delta(x^+)\delta^{\nu -}\rho_{p/A}(\boldsymbol{x}_\perp, x_0)$$ → Color charge density is unknown stochastic function

- Classical gluon field consists of slow ($x < x_0$) gluons:

Blaizot, Gelis and Venugopalan, NPA,2004.

  - Classical YM equation: $[D_\mu, F^{\mu\nu}] = J^\nu$ → $A^{\mu}_{x_0} = -g\delta^{\mu -}\delta(x^+)\dfrac{1}{\boldsymbol{\nabla}^2_\perp}\rho_A(\boldsymbol{x}_\perp, x_0)$

- Interaction with CGC:



$\propto (V(\boldsymbol{z}_\perp) - 1)$

- Wilson line:

$$V(\boldsymbol{z}_\perp, x_0) = \mathcal{P}\exp\left[ig\int_{-\infty}^{\infty} \mathrm{d}z^+ A^-(z^+, \boldsymbol{z}_\perp, x_0)\right]$$

Cross section level

- Correlators of Wilson lines like the dipole distribution

$$Y = \ln\frac{1}{x}$$

$$\langle O\rangle_Y = \int \mathcal{D}\rho W(x_0,\rho)O[\rho]$$

$$W(x_0,\rho) \propto e^{-\rho^2}$$

McLerran, Venugopalan, PRD, 1994.

CGC average over all color configurations

- Independence on x_0 guaranteed by JIMWLK RGE

$$\mathcal{S}(\boldsymbol{x}_\perp,\boldsymbol{x'}_\perp) \equiv \frac{1}{N_C}\mathrm{tr}\langle V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{x'}_\perp)\rangle_{Y_0}$$

Jalilian-Marian, Kovner, Leonidov, Weigert, NPB, 1997.

Iancu, Leonidov, McLerran, PLB, 2001.

- C-parity of Wilson lines:

$$\boldsymbol{CVC^{-1} = V^*}$$

Real part of dipole distribution:

$$\mathcal{P}(\boldsymbol{x}_\perp,\boldsymbol{x'}_\perp) \equiv \frac{1}{2N_C}\mathrm{tr}\langle V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{x'}_\perp) + V(\boldsymbol{x}'_\perp)V^\dagger(\boldsymbol{x}_\perp)\rangle$$

$C\mathcal{P}C^{-1} = \mathcal{P}$ → Pomeron!

Imaginary part of dipole distribution:

$$\mathcal{O}(\boldsymbol{x}_\perp,\boldsymbol{x'}_\perp) \equiv \frac{1}{2iN_C}\mathrm{tr}\langle V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{x'}_\perp) - V(\boldsymbol{x}'_\perp)V^\dagger(\boldsymbol{x}_\perp)\rangle$$

$C\mathcal{O}C^{-1} = -\mathcal{O}$ → Odderon!

Hatta et. al, NPA, 2005.

Connection to dilute regime

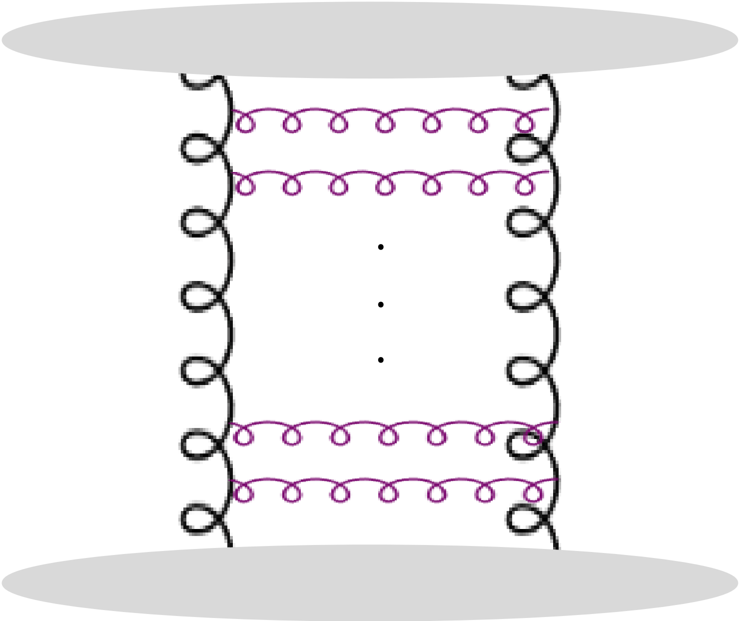
- Expansion of Wilson lines!

$\alpha^a_{\boldsymbol{x}_\perp} \equiv \int \mathrm{d}x^+ A^{+a}(x^+, \boldsymbol{x}_\perp)$

POMERON

Leading Pomeron contribution: 2 gluons bounded into a C-even singlet configuration

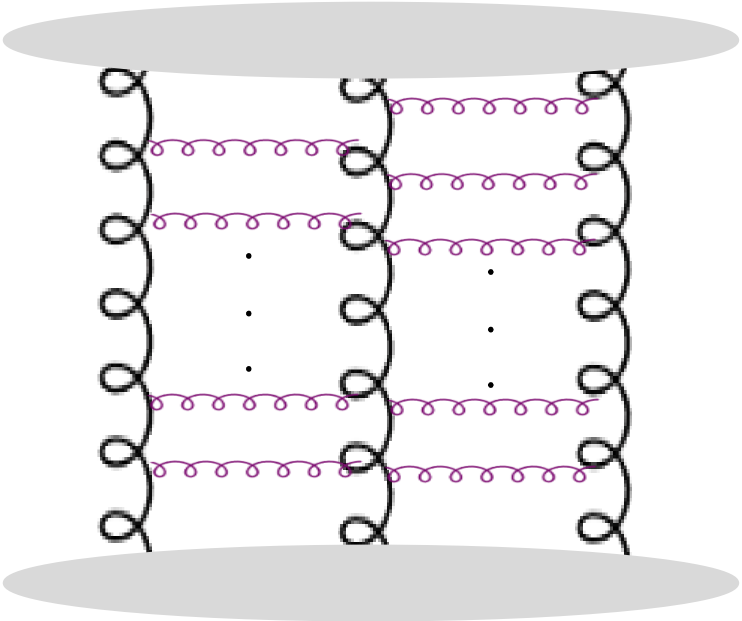
$$\mathcal{P}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) \approx 1 - \frac{g^2}{4N_C}\left\langle\left(\alpha^a_{\boldsymbol{x}_\perp} - \alpha^a_{\boldsymbol{y}_\perp}\right)^2\right\rangle$$



ODDERON

Leading Odderon contribution: 3 gluons bounded into a symmetric C-odd singlet configuration

$$\mathcal{O}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) \approx \frac{g^3}{24} d^{abc}\left\langle\left(\alpha^a_{\boldsymbol{x}_\perp} - \alpha^a_{\boldsymbol{y}_\perp}\right)\left(\alpha^b_{\boldsymbol{x}_\perp} - \alpha^b_{\boldsymbol{y}_\perp}\right)\left(\alpha^c_{\boldsymbol{x}_\perp} - \alpha^c_{\boldsymbol{y}_\perp}\right)\right\rangle$$



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- Dipolar form of the small-x evolution:

$$\frac{\partial \mathcal{P}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp, Y)}{\partial Y} = \frac{\alpha_S N_C}{2\pi^2} \int_{\boldsymbol{r}_{1\perp}} \frac{\boldsymbol{r}_\perp^2}{\boldsymbol{r}_{1\perp}^2 \boldsymbol{r}_{2\perp}^2} [\mathcal{P}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_{1\perp}, Y) + \mathcal{P}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_{2\perp}, Y) - \mathcal{P}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp, Y) - 1]$$

Mueller, NPB, 1994.

- Equivalent to the BFKL equation!

Hatta et. all, NPA, 2005.

Kuraev, Lipatov, Fadin, SPJETP, 1977.

Balitsky, Lipatov, SJNP, 1978.

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Kovchegov, Szymanowski, Wallon, PLB, 2004.

- Equivalent to the BKP equation!

Hatta et. all, NPA, 2005.

Bartels, NPB, 1980.

Kwiecinski, Praszalowicz, PLB, 1980.

Connection to dilute regime

$$\boldsymbol{r}_\perp = \boldsymbol{x}_\perp - \boldsymbol{y}_\perp \qquad \boldsymbol{r}_{1\perp} = \boldsymbol{x}_\perp - \boldsymbol{z}_\perp \qquad \boldsymbol{r}_{2\perp} = \boldsymbol{z}_\perp - \boldsymbol{y}_\perp$$

$$\boldsymbol{b}_\perp = \frac{1}{2}(\boldsymbol{x}_\perp + \boldsymbol{y}_\perp) \qquad \boldsymbol{b}_{1\perp} = \boldsymbol{b}_\perp + \frac{(\boldsymbol{r}_\perp - \boldsymbol{r}_{1\perp})}{2} \qquad \boldsymbol{b}_{2\perp} = \boldsymbol{b}_\perp + \frac{(\boldsymbol{r}_\perp - \boldsymbol{r}_{2\perp})}{2}$$

$$\frac{\partial \mathcal{P}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp, Y)}{\partial Y} = \frac{\alpha_S N_C}{2\pi^2} \int_{\boldsymbol{r}_{1\perp}} \frac{\boldsymbol{r}_\perp^2}{\boldsymbol{r}_{1\perp}^2 \boldsymbol{r}_{2\perp}^2} [\mathcal{P}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_{1\perp}, Y) + \mathcal{P}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_{2\perp}, Y) - \mathcal{P}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp, Y) - 1]$$

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BK evolution equation

- If we know dipole at some initial x_0, how to obtain it at lower values of x?

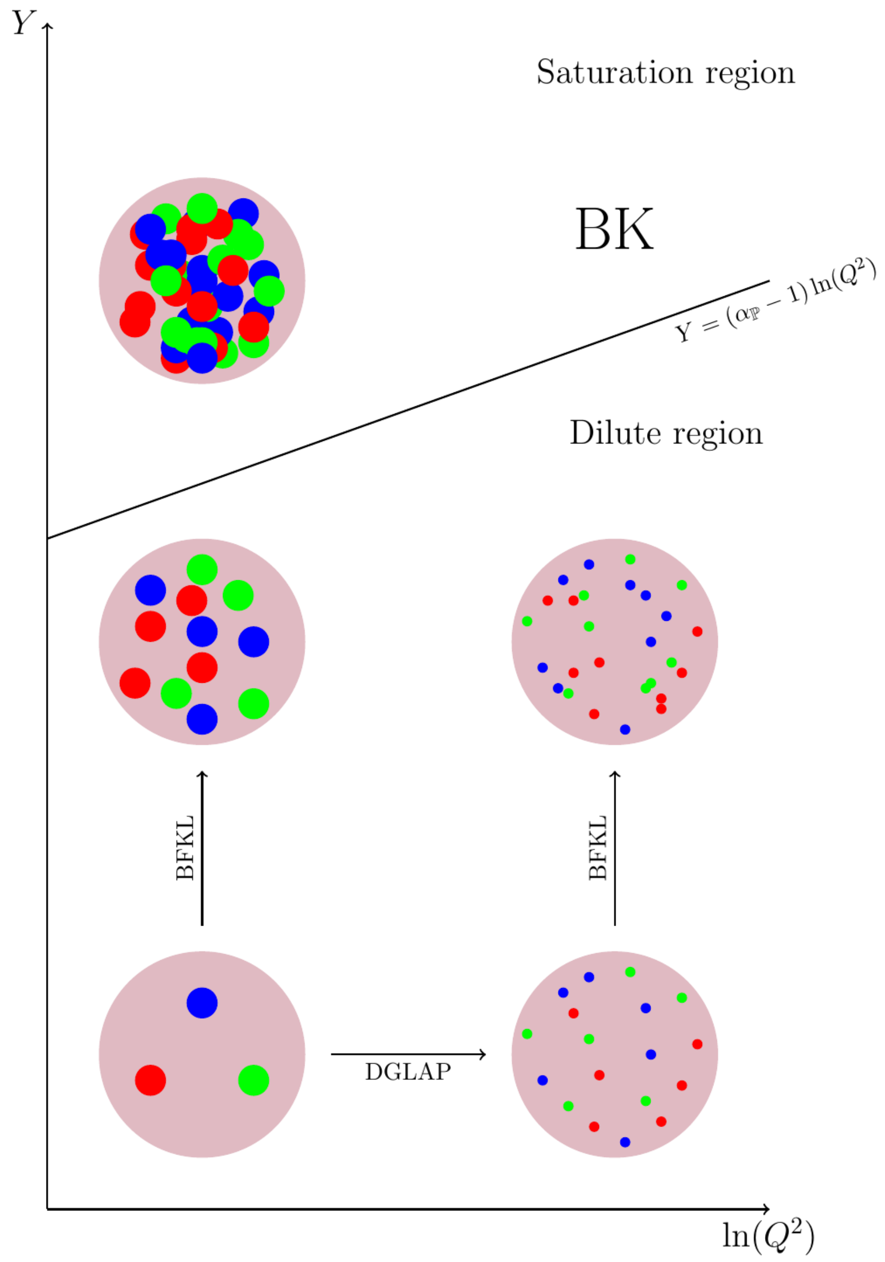
- Balitsky-Kovchegov evolution equation

Kovchegov, PRD, 1999.
Balitsky, NPB, 1996.

$$\frac{\partial \mathcal{S}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp, Y)}{\partial Y} = \frac{\alpha_S N_C}{2\pi^2} \int_{\boldsymbol{r}_{1\perp}} \frac{\boldsymbol{r}_\perp^2}{\boldsymbol{r}_{1\perp}^2 \boldsymbol{r}_{2\perp}^2} \times [\mathcal{S}(\boldsymbol{r}_{1\perp}, \boldsymbol{b}_{1\perp}, Y)\mathcal{S}(\boldsymbol{r}_{2\perp}, \boldsymbol{b}_{2\perp}, Y) - \mathcal{S}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp, Y)]$$

- Derivation idea:
  - Increasing the energy increases the rapidity interval between dipole and target
  - Larger phase space for the gluon radiation from $q\bar{q}$
  - Gluon could also come from target
  - Observables do not depend on that choice

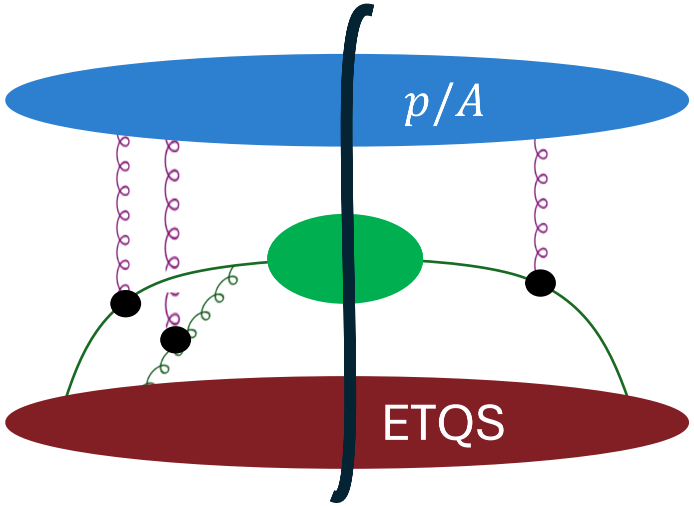
Nonlinearities are consequence of gluon recombination!



Hybrid approach in forward $p^{\uparrow}A$

Pole-ETQS in hybrid approach

Hatta et. all, PRD, 2016.

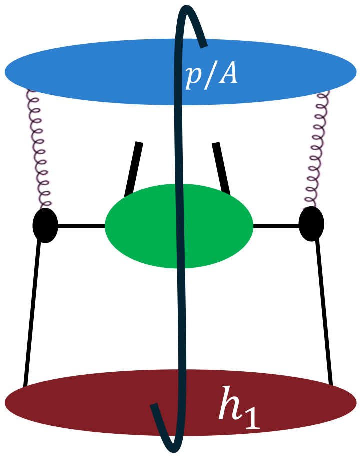


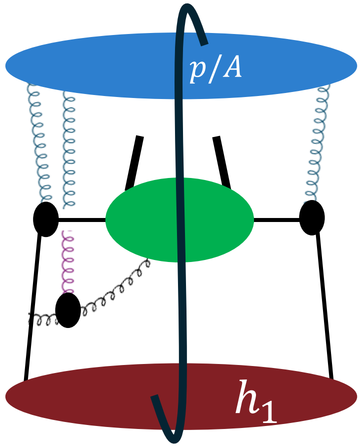
$$\frac{d\Delta\sigma^{SGP}}{dy_h d^2 P_{h\perp}} \approx \frac{\pi M x_F}{2(N_C^2-1)}\epsilon^{\alpha\beta}S_{\perp\beta}\int_{x_F}^{1}\frac{dz}{z^3}D(z)$$

$$\left\{\frac{2P_{h\alpha}/z}{(P_{h\perp}/z)^2}F(x_g, P_{h\perp}/z)\,x\frac{d}{dx}G_F(x,x)\right\}$$

Twist-3 FF in hybrid approach

Hatta et. all, PRD, 2017.





$$\frac{d\Delta\sigma}{dy_h d^2\boldsymbol{P}_{h\perp}} = \frac{M}{2}S_{\perp i}\epsilon^{ij}\int\frac{dz_2}{z_2^2}x_q h_1(x_q)\left\{-\mathrm{Im}\,\tilde{e}(z_2)\frac{d}{dP_h^j/z_2}F\left(x_g,\frac{P_{h\perp}}{z_2}\right)\right.$$

$$+4\frac{P_{hj}}{P_{h\perp}^2}\int_z^\infty\frac{dz_1}{z_1^2}P\left(\frac{z_2}{1/z_2-1/z_1}\right)\frac{\mathrm{Im}\,\hat{E}_F(z_1,z_2)}{N_C^2-1}$$

$$\left.\times\left(\frac{2\pi N_C^2}{\pi R_A^2}\int_0^{P_{h\perp}/z_1}l_\perp dl_\perp F(x_g,l_\perp)+\frac{1}{z_1(1/z_2-1/z_1)}\right)F\left(x_g,\frac{P_{h\perp}}{z_2}\right)\right\}$$

$$F(x_g,\kappa_\perp) = \frac{\pi R_A^2}{(2\pi)^2}\int_{\boldsymbol{r}_\perp} e^{i\boldsymbol{\kappa}_\perp\cdot\boldsymbol{x}_\perp}\,\mathcal{P}(x_g,\boldsymbol{x}_\perp)$$

Hybrid approach in forward $p^{\uparrow}A$

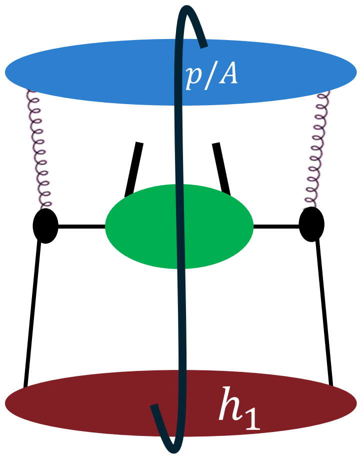
Pole-ETQS in hybrid approach

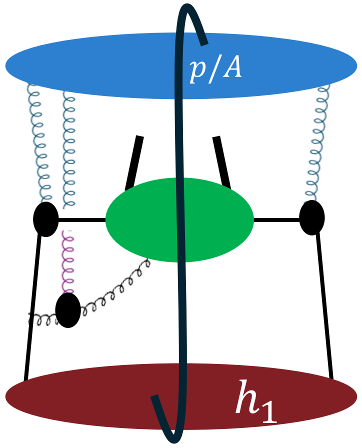
Hatta et. all, PRD, 2016.

$$\frac{A_N^{pA}}{A_N^{pp}} \approx 1$$

Twist-3 FF in hybrid approach

Hatta et. all, PRD, 2017.





$$\frac{d\Delta\sigma}{dy_h d^2\boldsymbol{P}_{h\perp}} = \frac{M}{2} S_{\perp i}\epsilon^{ij} \int \frac{dz_2}{z_2^2} x_q h_1(x_q) \left\{ -\mathrm{Im}\, \tilde{e}(z_2) \frac{d}{dP_h^j/z_2} F\left(x_g, \frac{P_{h\perp}}{z_2}\right) \right.$$

$$+4\frac{P_{hj}}{P_{h\perp}^2} \int_z^\infty \frac{dz_1}{z_1^2} P\left(\frac{z_2}{1/z_2 - 1/z_1}\right) \frac{\mathrm{Im}\, \hat{E}_F(z_1, z_2)}{N_C^2 - 1}$$

$$\left. \times \left(\frac{2\pi N_C^2}{\pi R_A^2} \int_0^{P_{h\perp}/z_1} l_\perp dl_\perp F(x_g, l_\perp) + \frac{1}{z_1(1/z_2 - 1/z_1)} \right) F\left(x_g, \frac{P_{h\perp}}{z_2}\right) \right\}$$

$$F(x_g, \kappa_\perp) = \frac{\pi R_A^2}{(2\pi)^2} \int_{\boldsymbol{r}_\perp} e^{i\boldsymbol{\kappa}_\perp \cdot \boldsymbol{x}_\perp} \mathcal{P}(x_g, \boldsymbol{x}_\perp)$$

Hybrid approach in forward $p^{\uparrow}A$

Pole-ETQS in hybrid approach

Hatta et. all, PRD, 2016.

$$\frac{A_N^{pA}}{A_N^{pp}} \approx 1$$

Twist-3 FF in hybrid approach

Hatta et. all, PRD, 2017.

$$\frac{A_N^{pA}}{A_N^{pp}} \approx A^{-1/3}$$

- Gets washed away by high energy evolution

Benić and Hatta, PRD, 2019.

$$F(x_g, \kappa_\perp) = \frac{\pi R_A^2}{(2\pi)^2} \int_{r_\perp} e^{i\boldsymbol{\kappa}_\perp \cdot \boldsymbol{x}_\perp} \mathcal{P}(x_g, \boldsymbol{x}_\perp)$$

Hybrid approach in forward $p^{\uparrow}A$

What about the Odderon???

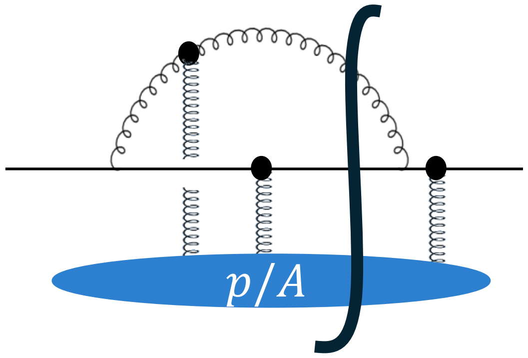
CGC-odderon mechanism for TSSA

- Odderon in CGC = imaginary part of dipole distribution:

$$\mathcal{S}(\boldsymbol{x}_\perp, \boldsymbol{x}'_\perp) \equiv \mathcal{P}(\boldsymbol{x}_\perp, \boldsymbol{x}'_\perp) + i\mathcal{O}(\boldsymbol{x}_\perp, \boldsymbol{x}'_\perp)$$

- Phase from Odderon? Kovchegov and Sievert, PRD 2012.

- Asymmetry calculated at parton level (up to NLO), i.e. $q^\uparrow A$ collisions



- No LO contribution; no interference

- Polarized cross section:

$$E_q \frac{\mathrm{d}\Delta\sigma}{\mathrm{d}^3 q} \propto i\alpha_S \int_{\boldsymbol{k}_\perp \boldsymbol{k}_{2\perp}} \int_{\boldsymbol{r}_\perp \boldsymbol{b}_\perp \boldsymbol{r}'_\perp} \mathcal{H}(\boldsymbol{r}_\perp, \boldsymbol{r}'_\perp, \boldsymbol{S}_\perp) \times [\mathcal{P}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)\mathcal{O}(\boldsymbol{r}'_\perp, \boldsymbol{b}'_\perp) - \mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)\mathcal{P}(\boldsymbol{r}'_\perp, \boldsymbol{b}'_\perp)]$$

- For $P_{h\perp} \approx Q_S$ the TSSA has significant nuclear suppression:

$$A_N \propto A^{-7/6}$$

CGC-odderon mechanism for TSSA

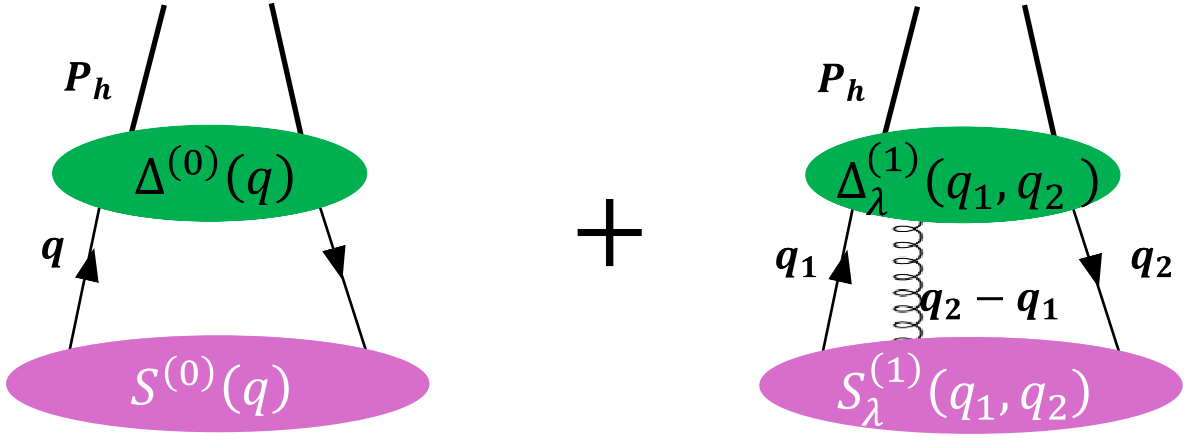
In this work we embed this idea at the level of $p^{\uparrow}A \to hX$ by considering twist-3 functions from the fragmentation side of cross section!

Twist-3 expansion of cross section

Kanazawa and Koike, PRD, 2013.

- Starting formula for cross section: $E_h \frac{d\Delta\sigma^{p^\uparrow A \to hX}}{d^3 P_h} = \frac{1}{2(2\pi)^3} \sum_{i=q,g} W_i(z)$

- Up to twist-3, two diagrams contribute to hadronic tensor $W(z)$

$W(z) =$  $+ \cdots$

$S^{(0,1)}$ contains hard factor and distributions from projectile and target

$$W(z) = \int_q \mathrm{Tr}\left(\Delta^{(0)}(q) S^{(0)}(q)\right) + \int_{q_1,q_2} \mathrm{Tr}\left(\Delta_\lambda^{(1)}(q_1, q_2) S^{(1)\lambda}(q_1, q_2)\right)$$

- Strategy: perform colinear expansion of $S^{(0)}$ and $S^{(1)}$ around $q^\mu = \frac{P_h^\mu}{z}$ and use QCD WT identities

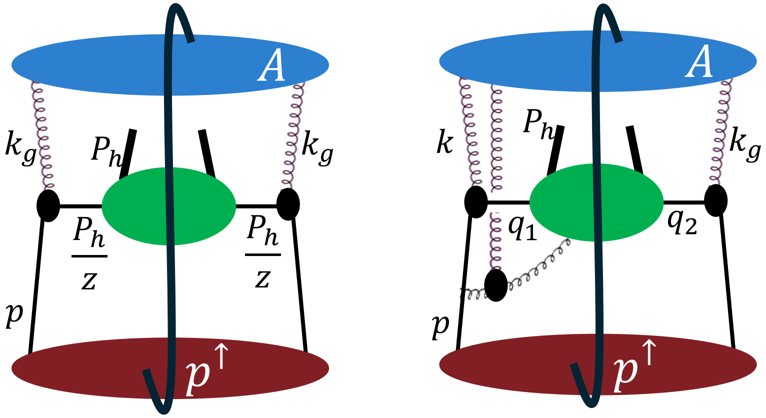
Kanazawa and Koike, PRD, 2013.

Hatta et. all, PRD, 2017.

- Gauge invariant twist-3 polarized cross section:

LO contributions!

$$\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}y_h \mathrm{d}^2 \boldsymbol{P}_{h\perp}} = \frac{1}{2(2\pi)^3}\left(\int \frac{\mathrm{d}z}{z^2}\mathrm{Tr}[\Delta(z)S^{(0)}(z)] + \int \frac{\mathrm{d}z}{z^2}\mathrm{Im\,Tr}\left[\Delta_\partial^\alpha(z)\frac{\partial S^{(0)}(K)}{\partial K^\alpha}\right]_{K=\frac{P_h}{z}} - \int \frac{\mathrm{d}z_1 \mathrm{d}z_2}{z_1^2 z_2^2}\mathrm{P}\left(\frac{1}{z_2}-\frac{1}{z_1}\right)^{-1} \mathrm{Im\,Tr}[\Delta_F^\alpha(z_1,z_2)S_\alpha^{1L}(z_1,z_2) + \bar{\Delta}_F^\alpha(z_2,z_1)S_\alpha^{1R}(z_1,z_2)]\right)$$



INTRINSIC $\langle 0|\psi|h,X\rangle\langle h,X|\bar{\psi}|0\rangle \propto \Delta(z) = \frac{M_N}{z}\hat{e}_1(z) + \frac{M_N}{2z}\sigma_{\lambda\alpha}i\gamma_5\epsilon^{\lambda\alpha w P_h}\hat{e}_{\bar{1}}(z) + \cdots$

KINEMATICAL $\langle 0|\partial_\perp\psi|h,X\rangle\langle h,X|\bar{\psi}|0\rangle \propto \Delta_\partial^\alpha = \frac{M_N}{2}\gamma_5\frac{\not{P}_h}{z}\gamma_\lambda\epsilon^{\lambda\alpha w P_h}\tilde{e}(z) + \cdots$

DYNAMICAL $\langle 0|\psi|h,X\rangle\langle h,X|\bar{\psi}F|0\rangle \propto \Delta_F^\alpha(z_1,z_2) = \frac{M_N}{2}\gamma_5\frac{\not{P}_h}{z_2}\gamma_\lambda\epsilon^{\lambda\alpha w P_h}\hat{E}_F(z_1,z_2) + \cdots$

Metz and Pitonyak, 2013.

Ji, PRD 1994.

$$w^2 = 0$$
$$P_h \cdot w = 1$$

Distribution in projectile: transversity

$$\langle P|\bar{\psi}\psi|P\rangle = \Phi(x_q) = -\frac{P^+ S_{\perp i}}{2}h_1(x_q)i\gamma_5\sigma^{-i}$$

Contribution from real part of $E_F(z_1, z_2)$

- Only dynamical twist-3 FF may contribute (they have real and imaginary part)
    - Imaginary part led to Pomeron dependent cross section   Hatta et. all, PRD, 2017.

- Contribution from real part: Benić and EAV, 2501.12847

$$\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}y_h \mathrm{d}^2\boldsymbol{P}_{h\perp}} = -\frac{1}{2(2\pi)^3}\frac{M_N}{2}\epsilon^{\lambda\alpha w P_h}\int \frac{\mathrm{d}z_1 \mathrm{d}z_2}{z_1^2 z_2^2} P\left(\frac{1}{z_2}-\frac{1}{z_1}\right)^{-1} \times \mathrm{Re}\,\hat{E}_F(z_1,z_2) \times \mathrm{Im\,Tr}\left(\gamma_5 \frac{\not{P}_h}{z_2}\gamma_\lambda S_\alpha^{(1)L}(z_1,z_2) + \gamma_\lambda \frac{\not{P}_h}{z_2}\gamma_5 \bar{S}_\alpha^{(1)L}(z_1,z_2)\right)$$

- Projectile and target distributions are contained in $S^{(1)}$

$$S_\alpha^{(1)L}(z_1,z_2) = \frac{1}{2P^+}\frac{2}{(N_C^2-1)}\int \mathrm{d}x_q (2\pi)\delta\left(\frac{P_h^+}{z_2} - x_q P^+\right)\langle \mathrm{tr_c}(\mathcal{M}_\alpha^a \Phi(x_q)\bar{\mathcal{M}} t^a)\rangle$$

Contribution from real part of $E_F(z_1, z_2)$

$$\mathcal{M}^a_\alpha(z_1,z_2) = i\int_{\boldsymbol{k}_\perp,\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} \mathrm{e}^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\mathrm{e}^{i\left(\frac{\boldsymbol{P}_{h\perp}}{z_2}-\boldsymbol{k}_\perp\right)\cdot\boldsymbol{y}_\perp} T_{qg\alpha}(\boldsymbol{k}_\perp,z_1,z_2)V(\boldsymbol{x}_\perp)t^b U^{ab}(\boldsymbol{y}_\perp)$$

Adjoint Wilson line:

$$U^{ab}(\boldsymbol{y}_\perp) \equiv \mathrm{tr_c}\left(t^a V(\boldsymbol{y}_\perp)t^b V^\dagger(\boldsymbol{y}_\perp)\right)$$

$$T_{qg\alpha}(\boldsymbol{k}_\perp,z_1,z_2) = i\int_{-\infty}^{+\infty}\frac{\mathrm{d}k^-}{(2\pi)}\gamma^+\frac{\frac{\not{P}_h}{z_1}-\not{k}}{\left(\frac{P_h}{z_1}-k\right)^2+i\epsilon}\gamma^\beta\frac{V_{\alpha\beta}}{\left(x_qP+k-\frac{P_h}{z_1}\right)^2+i\epsilon}$$

$$\mathcal{M} = -i\int_{\boldsymbol{k}_\perp,\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} \mathrm{e}^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\mathrm{e}^{i\left(\frac{\boldsymbol{P}_{h\perp}}{z_2}-\boldsymbol{k}_\perp\right)\cdot\boldsymbol{y}_\perp}\gamma^+V(\boldsymbol{x}_\perp)$$

$$S_\alpha^{(1)L}(z_1,z_2) = \frac{1}{2P^+}\frac{2}{(N_C^2-1)}\int \mathrm{d}x_q(2\pi)\delta\left(\frac{P_h^+}{z_2}-x_qP^+\right)\langle \mathrm{tr_c}(\mathcal{M}^a_\alpha\Phi(x_q)\bar{\mathcal{M}}t^a)\rangle$$

Final formula for the cross section

- After some algebra, the cross section is found to be:

$$\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}y_h \mathrm{d}^2\boldsymbol{P}_{h\perp}} = \frac{M_N}{2\pi^2}\frac{N_C^2}{N_C^2-1}\int\frac{\mathrm{d}z_1\mathrm{d}z_2}{z_1^2 z_2^2} P\left(\frac{1}{z_2}-\frac{1}{z_1}\right)^{-1} x_q h_1(x_q) \mathrm{Re}\,\hat{E}_F(z_1,z_2)$$

$$\int_{\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp} \mathcal{H}^{(1)}(\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp)\left[\mathcal{P}\left(\frac{\boldsymbol{P}_{h\perp}}{z_2},\boldsymbol{\Delta}_\perp\right)\mathcal{O}(\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp) - \mathcal{P}(\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp)\mathcal{O}\left(\frac{\boldsymbol{P}_{h\perp}}{z_2},\boldsymbol{\Delta}_\perp\right)\right]$$

$$\mathcal{H}(z_1,z_2,\boldsymbol{k}_\perp) = \frac{z_2}{z_1}\frac{\epsilon^{ij} S_{\perp i}\left(\frac{P_h}{z_1}-k\right)_j}{\left(\frac{\boldsymbol{P}_{h\perp}}{z_1}-\boldsymbol{k}_\perp\right)^2}$$

$$\mathcal{H}(\Delta_\perp=0) + \Delta_\perp\frac{\partial\mathcal{H}}{\partial\Delta_\perp}(\Delta_\perp=0) \equiv \mathcal{H}^{(0)} + \mathcal{H}^{(1)}$$

- For the Pomeron and Odderon we use separable form:

$$\mathcal{P}(\boldsymbol{r}_\perp,\boldsymbol{b}_\perp) = \mathcal{P}(r_\perp)T(b_\perp)$$

Lappi and Mäntysaari, PRD, 2013.

$$\mathcal{O}(\boldsymbol{r}_\perp,\boldsymbol{b}_\perp) = R_A\mathcal{O}(r_\perp)\frac{dT(b_\perp)}{db_\perp}\cos(\phi_{rb})$$

Jeon and Venugopalan, PRD, 2005.

- We applied the approximation:

$$z_1\int_0^{\frac{P_{h\perp}}{z_1}} \kappa_\perp F(\kappa_\perp)\mathrm{d}\kappa_\perp \approx z_2\int_0^{\frac{P_{h\perp}}{z_2}} \kappa_\perp F(\kappa_\perp)\mathrm{d}\kappa_\perp$$

Benić and Hatta, PRD, 2019.

- Approximation enables to use QCD EOM relation

$$\frac{\hat{e}_1(z_2)}{z_2} = -\int_{z_2}^{\infty} \frac{\mathrm{d}z_1}{z_1^2} P\left(\frac{1}{1/z_2 - 1/z_1}\right) \mathrm{Re}\hat{E}_F(z_1, z_2)$$

Koike, Pitonyak, Takagi, and Yoshida, PLB, 2016.

Kanazawa, Koike, Metz, Pitonyak, and Schlegel, PRD, 2016.

- Cross section used in numerical estimations is then:

$$\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}y_h \mathrm{d}^2\boldsymbol{P}_{h\perp}} = M_N \frac{N_C^2}{N_C^2 - 1} \frac{\epsilon^{ij} S_{\perp i} P_{hj}}{P_{h\perp}^3} \frac{1}{R_A^3} \int_{z_{min}}^{1} \frac{\mathrm{d}z}{z} \hat{e}_1(z, P_{h\perp}^2) x_q h_1(x_q, P_{h\perp}^2)\, G\left(\frac{P_{h\perp}}{z}\right) \int_0^{\frac{P_{h\perp}}{z}} \kappa_\perp F(\kappa_\perp) \mathrm{d}\kappa_\perp$$

$$F(\kappa_\perp) = \frac{\pi R_A^2}{(2\pi)^2} \int_{\boldsymbol{r}_\perp} e^{i\boldsymbol{\kappa}_\perp \cdot \boldsymbol{r}_\perp} \mathcal{P}(r_\perp)$$

$$i \cos(\phi_{\kappa\Delta}) G(\kappa_\perp) = \frac{\pi R_A^2}{(2\pi)^2} \int_{\boldsymbol{r}_\perp} e^{i\boldsymbol{\kappa}_\perp \cdot \boldsymbol{r}_\perp} \mathcal{O}(r_\perp) \cos(\phi_{r\Delta})$$

- For $\hat{e}_1(z)$ we use Chiral quark model

$$\hat{e}_1(z) = \frac{M_q}{M_N} \frac{z}{1-z} D_1(z) = \frac{1}{3} \frac{z}{1-z} D_1(z)$$

Ji, Zhu, 1993.

Yuan, PLB, 2004.

- Pomeron and Odderon obtained from BK solutions

Benić, Horvatić, Kaushik, and EAV, PRD, 2023.

➢ Other nonperturbative functions are from tables:

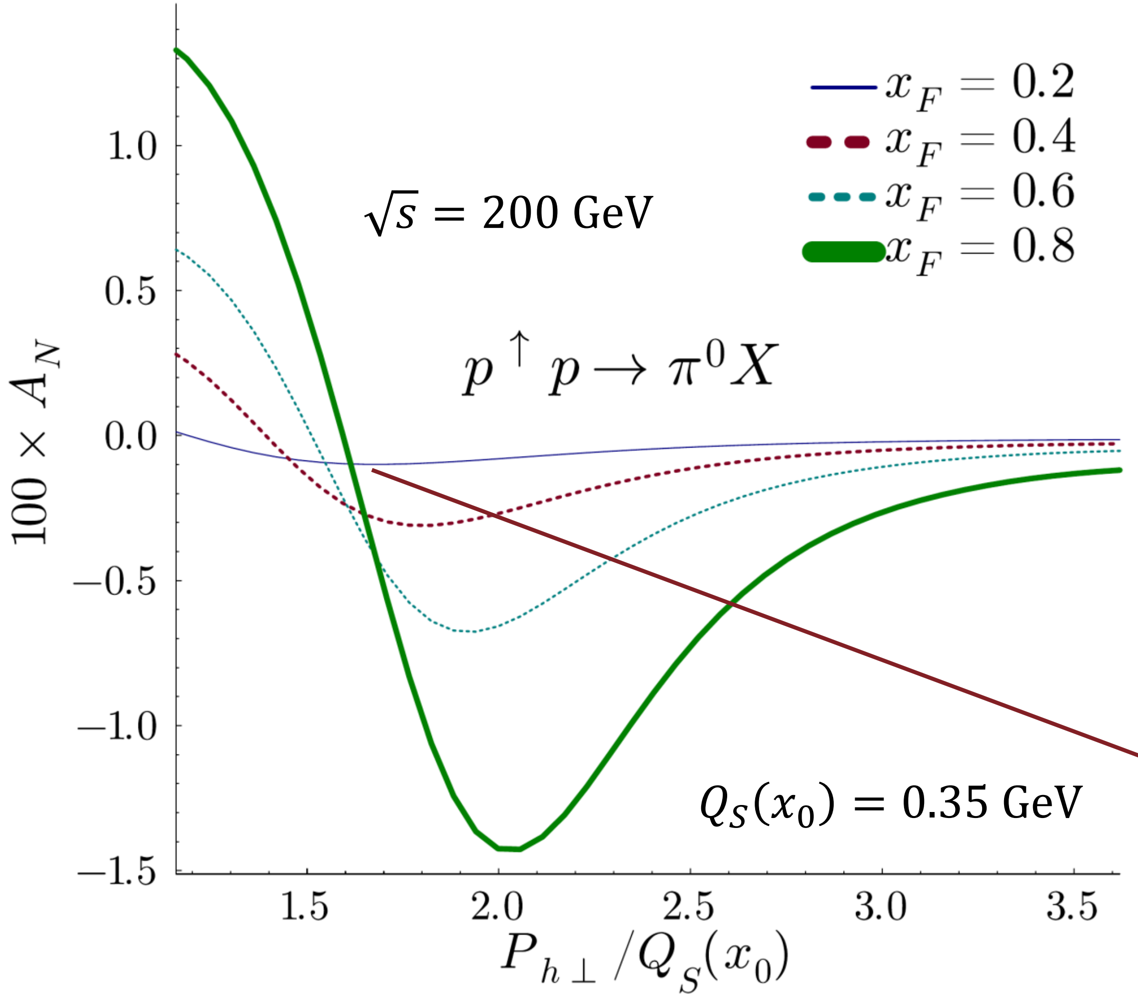
JAM,PRD, 2013. for twist-2 FF $D_1(z)$

Dulat et. al., PRD, 2016. for twist-2 PDF $f_1(x)$ in unpolarized CS

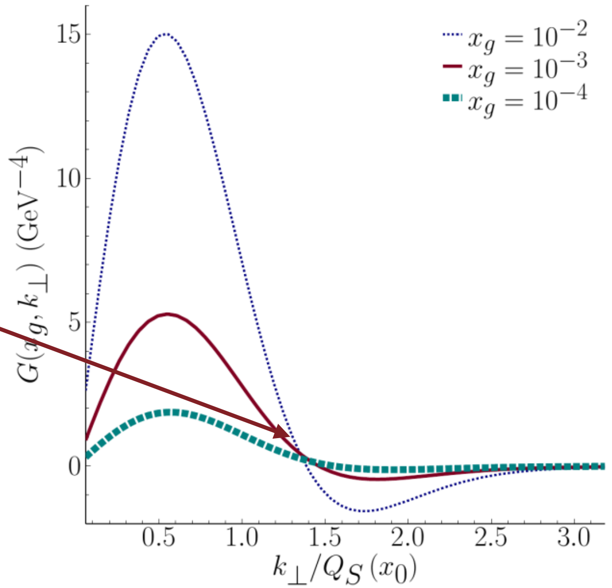
JAM, PRD, 2022. for transversity $h_1(x)$

Numerical solutions

Benić, Vivoda, 2501.12847



- TSSA grows as a function of Feynman-x
- This mechanism is significant for low transverse momenta of produced hadron
  - Smaller $P_{h\perp}$ than STAR and PHENIX
- TSSA obeys a sign change! Consequence of the node in the Odderon (small change of node position is consequence of $\kappa_\perp = P_{h\perp}/z$)



Nuclear effects

Benić, Vivoda, 2501.12847

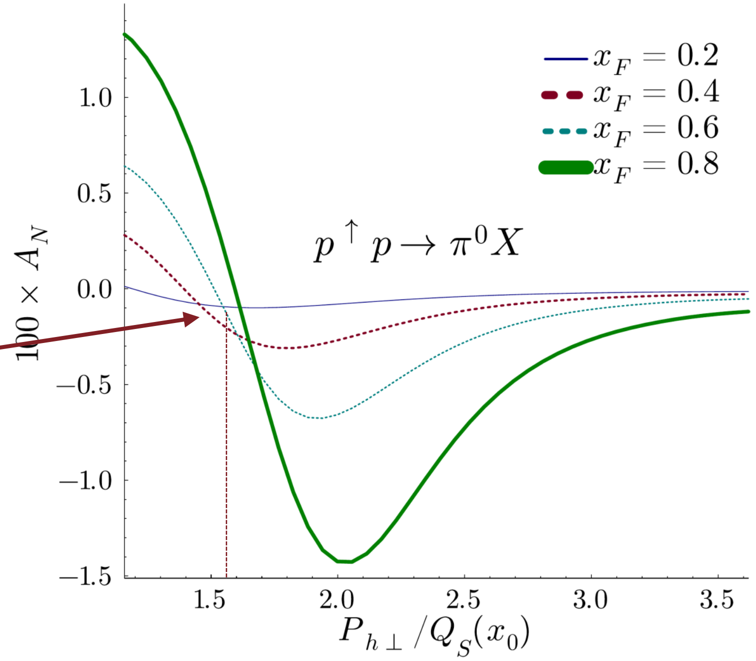
- The node position is shifting to higher values of $P_{h\perp}$

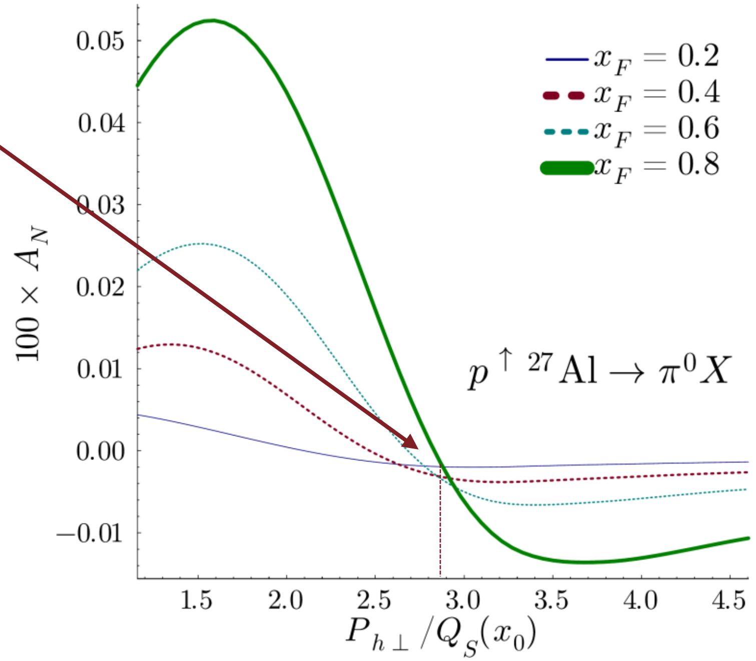
Scales as: $A^{1/6}$

- TSSA is decreasing with mass number

Numerical check: $A_N \approx A^{-7/6}$

In agreement with Kovchegov and Sievert (2012.)!





Summary:

1. Completely new contribution to TSSA coming from the combination of the real part of genuine twist-3 FF and the Odderon

2. TSSA changes sign as a function of $P_{h\perp}$

3. TSSA is largest for $P_{h\perp} \sim Q_S$ (saturation scale)

4. Numerically confirmed nuclear scaling

THANK YOU!

Details of calculation

- Object to calculate: color and Dirac traces

Benić and EAV, 2501.12847

$$\epsilon^{\lambda\alpha w P_h}\mathrm{Tr}\left(\gamma_5\frac{\not{P}_h}{z_2}\gamma_\lambda S_\alpha^{(1)L}(z_1,z_2)\right)=\frac{S_{\perp i}}{2(N_C^2-1)}\int \mathrm{d}x_q h_1(x_q)(2\pi)\delta\left(\frac{P_h^+}{z_2}-x_qP^+\right)\int_{\boldsymbol{k}_\perp,\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{k}'_\perp,\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp} e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}e^{i\left(\frac{\boldsymbol{P}_{h\perp}}{z_2}-\boldsymbol{k}_\perp\right)\cdot\boldsymbol{y}_\perp}e^{-i\boldsymbol{k}'_\perp\cdot\boldsymbol{x}'_\perp}e^{-i\left(\frac{\boldsymbol{P}_{h\perp}}{z_2}-\boldsymbol{k}'_\perp\right)\cdot\boldsymbol{y}'_\perp}$$

$$\epsilon^{\lambda\alpha w P_h}\mathrm{Tr}\left((i\gamma_5\sigma^{-i})\gamma^+\gamma_5\frac{\not{P}_h}{z_2}\gamma_\lambda T_{qg\alpha}(\boldsymbol{k}_\perp,z_1,z_2)\right)\langle\mathrm{tr}_c\left(V^\dagger(\boldsymbol{x}'_\perp)t^bV(\boldsymbol{x}_\perp)t^a\right)U^{ba}(\boldsymbol{y}_\perp)\rangle$$

- Color structure calculated with SU(N) Fierz identity; Large N:

$$\langle\mathrm{tr}_c\left(V^\dagger(\boldsymbol{x}'_\perp)t^bV(\boldsymbol{x}_\perp)t^a\right)U^{ba}(\boldsymbol{y}_\perp)\rangle=\frac{1}{2}\left(N_C^2\mathcal{S}(\boldsymbol{y}_\perp,\boldsymbol{x}'_\perp)\mathcal{S}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)-\mathcal{S}(\boldsymbol{x}_\perp,\boldsymbol{x}'_\perp)\right)$$

Imaginary part vanishes under transverse integration: NO ODDERON

- Dirac trace is calculated in forward limit; keeping leading power of P_h^+:

$$\epsilon^{\lambda\alpha w P_h}\mathrm{Tr}\left((i\gamma_5\sigma^{-i})\gamma^+\gamma_5\frac{\not{P}_h}{z_2}\gamma_\lambda T_{qg\alpha}(\boldsymbol{k}_\perp,z_1,z_2)\right)=-16\frac{P_h^+}{z_1}\epsilon^{ij}\frac{\left(\frac{P_h}{z_1}-k\right)_j}{\left(\frac{P_{h\perp}}{z_1}-k_\perp\right)^2}$$

The support properties of $\hat{E}_F$ $(0<z_2<1, z_2<z_1<\infty)$ enabled straightforward k^- integration

- After calculation of the mirror diagram (gluon on right side of the cut):

$$\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}y_h \mathrm{d}^2 \boldsymbol{P}_{h\perp}} = \frac{M_N}{2\pi^2}\frac{N_C^2}{N_C^2-1}\int \frac{\mathrm{d}z_1 \mathrm{d}z_2}{z_1^2 z_2^2} P\left(\frac{1}{z_2}-\frac{1}{z_1}\right)^{-1} x_q h_1(x_q) \mathrm{Re}\, \hat{E}_F(z_1,z_2)$$

$$\int_{\boldsymbol{k}_\perp,\boldsymbol{r}_\perp,\boldsymbol{b}_\perp,\boldsymbol{r}'_\perp} e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp} e^{i\frac{\boldsymbol{P}_{h\perp}}{z_2}\cdot\boldsymbol{r}'_\perp} \mathcal{H}(z_1,z_2,\boldsymbol{k}_\perp)[\mathcal{P}(\boldsymbol{r}_\perp,\boldsymbol{b}_\perp)\mathcal{O}(\boldsymbol{r}'_\perp,\boldsymbol{b}'_\perp)+\mathcal{P}(\boldsymbol{r}'_\perp,\boldsymbol{b}'_\perp)\mathcal{O}(\boldsymbol{r}_\perp,\boldsymbol{b}_\perp)]$$

$$\mathcal{H}(z_1,z_2,\boldsymbol{k}_\perp) = \frac{z_2}{z_1}\frac{\epsilon^{ij} S_{\perp i}\left(\frac{P_h}{z_1}-k\right)_j}{\left(\frac{\boldsymbol{P}_{h\perp}}{z_1}-\boldsymbol{k}_\perp\right)^2}$$

Benić, EAV, 2501.12847

- Fourier transforms of Pomeron and Odderon:

$$\mathcal{P}(\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp) = \int_{\boldsymbol{r}_\perp,\boldsymbol{b}_\perp} e^{i\boldsymbol{\kappa}_\perp\cdot\boldsymbol{r}_\perp} e^{i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\mathcal{P}(\boldsymbol{r}_\perp,\boldsymbol{b}_\perp)$$

$$\mathcal{O}(\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp) = \int_{\boldsymbol{r}_\perp,\boldsymbol{b}_\perp} e^{i\boldsymbol{\kappa}_\perp\cdot\boldsymbol{r}_\perp} e^{i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\mathcal{O}(\boldsymbol{r}_\perp,\boldsymbol{b}_\perp)$$

- As a function of $b_\perp$, Odderon peaks around R_A (small $\Delta_\perp$ approximation)

Benić, Horvatić, Kaushik, and EAV, PRD, 2023.

$$\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}y_h \mathrm{d}^2 \boldsymbol{P}_{h\perp}} = \frac{M_N}{2\pi^2}\frac{N_C^2}{N_C^2-1}\int \frac{\mathrm{d}z_1 \mathrm{d}z_2}{z_1^2 z_2^2} P\left(\frac{1}{z_2}-\frac{1}{z_1}\right)^{-1} x_q h_1(x_q) \mathrm{Re}\, \hat{E}_F(z_1,z_2)$$

$$\int_{\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp} \mathcal{H}^{(1)}(\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp)\left[\mathcal{P}\left(\frac{\boldsymbol{P}_{h\perp}}{z_2},\boldsymbol{\Delta}_\perp\right)\mathcal{O}(\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp)-\mathcal{P}(\boldsymbol{\kappa}_\perp,\boldsymbol{\Delta}_\perp)\mathcal{O}\left(\frac{\boldsymbol{P}_{h\perp}}{z_2},\boldsymbol{\Delta}_\perp\right)\right]$$

$$\mathcal{H} = \mathcal{H}(\Delta_\perp = 0) + \Delta_\perp \frac{\partial\mathcal{H}}{\partial\Delta_\perp}(\Delta_\perp = 0)$$

Vanishes under angular integral due to Odderon cosine modulation

$\mathcal{H}^{(1)}$

Separable Pomeron and Odderon

Benić, Vivoda, 2501.12847

- We assume separable form of Pomeron and Odderon:

$$\mathcal{P}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) = \mathcal{P}(r_\perp)T(b_\perp)$$

Lappi and Mäntysaari, PRD, 2013.

$$\mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp) = R_A \mathcal{O}(r_\perp)\frac{dT(b_\perp)}{db_\perp}\cos(\phi_{rb})$$

Jeon and Venugopalan, PRD, 2005.

$T(b_\perp)$ is a profile function normalized as: $\int_{\boldsymbol{b}_\perp} T(b_\perp) = \pi R_A^2$

- After performing angular integrations ϕ_κ and ϕ_Δ cross section becomes:

$$\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}y_h \mathrm{d}^2\boldsymbol{P}_{h\perp}} = -\frac{M_N}{2}\frac{N_C^2}{N_C^2-1}\frac{\epsilon^{ij}S_{\perp i}P_{hj}}{P_{h\perp}^3}\frac{1}{\pi^2 R_A^3}\int_0^\infty \Delta_\perp^3 T^2(\Delta_\perp)\mathrm{d}\Delta_\perp$$
$$\int_{z_{min}}^1 \frac{\mathrm{d}z_2}{z_2^2}\int_{z_2}^\infty \frac{\mathrm{d}z_1}{z_1^2} P\left(\frac{1}{z_2}-\frac{1}{z_1}\right)^{-1} \mathrm{Re}\,\hat{E}_F(z_1,z_2)x_q h_1(x_q) z_2 G\left(\frac{P_{h\perp}}{z_2}\right) z_1 \int_0^{\frac{P_{h\perp}}{z_1}} \kappa_\perp F(\kappa_\perp)\mathrm{d}\kappa_\perp$$

$$F(\kappa_\perp) = \frac{\pi R_A^2}{(2\pi)^2}\int_{\boldsymbol{r}_\perp} e^{i\boldsymbol{\kappa}_\perp\cdot\boldsymbol{r}_\perp}\mathcal{P}(r_\perp)$$

$$i\cos(\phi_{\kappa\Delta})G(\kappa_\perp) = \frac{\pi R_A^2}{(2\pi)^2}\int_{\boldsymbol{r}_\perp} e^{i\boldsymbol{\kappa}_\perp\cdot\boldsymbol{r}_\perp}\mathcal{O}(r_\perp)\cos(\phi_{r\Delta})$$

Setup for numerical calculations

- Due to support properties of twist-3 FFs we approximate:

Benić and Hatta, PRD, 2019.

Benić, Vivoda, 2501.12847

$$z_1 \int_0^{\frac{P_{h\perp}}{z_1}} \kappa_\perp F(\kappa_\perp) \mathrm{d}\kappa_\perp \approx z_2 \int_0^{\frac{P_{h\perp}}{z_2}} \kappa_\perp F(\kappa_\perp) \mathrm{d}\kappa_\perp$$

- Remaining z_1 dependence is of the form to use QCD EOM relation:

$$\frac{\hat{e}_1(z_2)}{z_2} = -\int_{z_2}^{\infty} \frac{\mathrm{d}z_1}{z_1^2} P\left(\frac{1}{1/z_2 - 1/z_1}\right) \mathrm{Re}\hat{E}_F(z_1, z_2)$$

Koike, Pitonyak, Takagi, and Yoshida, PLB, 2016.

Kanazawa, Koike, Metz, Pitonyak, and Schlegel, PRD, 2016.

- For the profile function we take the Gaussian form: $T(b_\perp) = e^{-b^2/R_A^2}$

$$\frac{\mathrm{d}\Delta\sigma}{\mathrm{d}y_h \mathrm{d}^2\boldsymbol{P}_{h\perp}} = M_N \frac{N_C^2}{N_C^2 - 1} \frac{\epsilon^{ij} S_{\perp i} P_{hj}}{P_{h\perp}^3} \frac{1}{R_A^3} \int_{z_{min}}^{1} \frac{\mathrm{d}z}{z} \hat{e}_1(z, P_{h\perp}^2) x_q h_1(x_q, P_{h\perp}^2)\, G\left(\frac{P_{h\perp}}{z}\right) \int_0^{\frac{P_{h\perp}}{z}} \kappa_\perp F(\kappa_\perp) \mathrm{d}\kappa_\perp$$

In numerical calculations: Chiral-quark model

$$\hat{e}_1(z) = \frac{M_q}{M_N} \frac{z}{1-z} D_1(z)$$

Ji, Zhu, 1993.

Yuan, PLB, 2004.

Pomeron and Odderon obtained from solutions of BK equation

Benić, Horvatić, Kaushik, and EAV, PRD, 2023.