Chiral magnetic effect: from perturbation theory to lattice QCD

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- Introduction and preliminaries
- Global CME
- Local CME
- Dynamical CME
- Summary

CRC-TR 211
Strong-interaction matter under extreme conditions

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Introduction

- **Non-dissipative**[1] transport effects in background **magnetic fields**,
- originating from the $U_A(1)$ **anomaly**.
- The most prominent is the Chiral Magnetic Effect[2] $\equiv$ **CME**,
- an **electric current** in the presence of **chiral imbalance** and a **magnetic field**,
- parallel to the magnetic field:

$$\langle J_3 \rangle_{\mu_5, B} = \left\langle \bar{\psi} \gamma_3 \psi \right\rangle_{\mu_5, B} = c_{\mathrm{CME}} \, \mu_5 \, q B_3 + \mathcal{O}(\mu_5^3, B^3) \, .$$

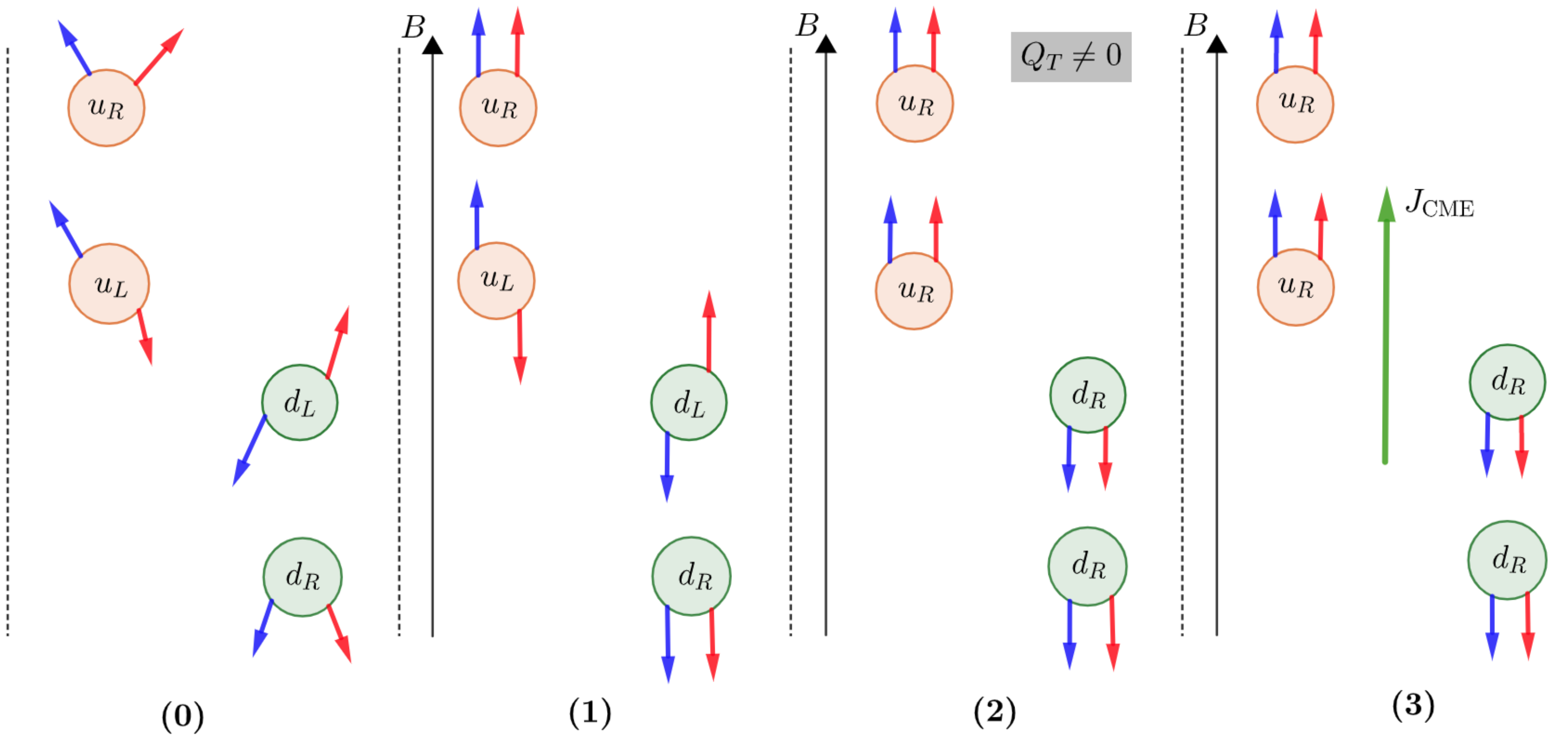
- The coefficient can be obtained as

$$\frac{\delta \langle J_3 \rangle}{\delta \mu_5} = \left\langle J_3 J_0^5 \right\rangle_{\mu_5 = 0, B} = c_{\mathrm{CME}} \, q B_3 \, .$$

[1] Chen et al., PRL 115 (2015) 2, 021601 [2] Fukushima, Kharzeev and Warringa, PRD 78 (2008) 074033

Introduction

CME

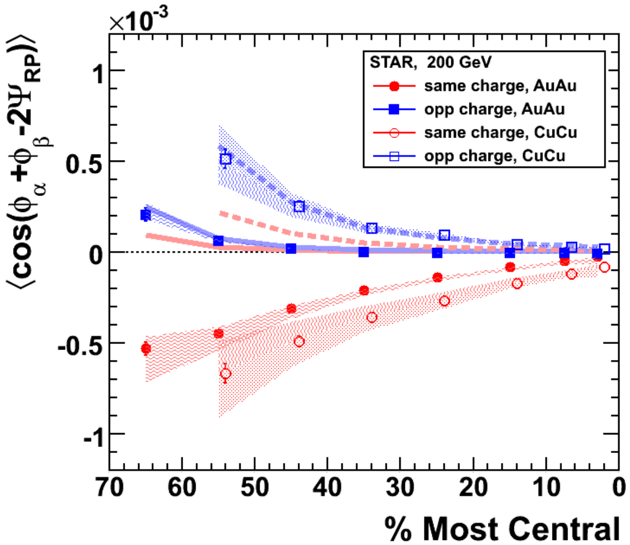


blue arrows≡spin, red arrows≡momentum

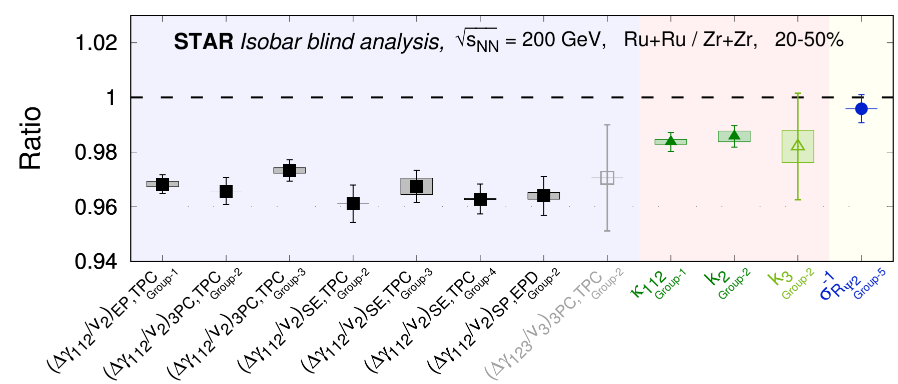
by Eduardo Garnacho Velasco

Experimental search for CME

- **Anomalous origin** $\rightarrow$ detection $\equiv$ detection of **topological nature** of QCD.
- Topological charge is zero on average $\rightarrow$ search for **local fluctuations** in HIC.
- Look for charge separation in angular correlations, big experimental effort.



STAR, PRL **103** (2009), 251601



STAR, PRC **105** (2022) 014901

- **No significant signal**, but the search continues.
- **Was found**[3] in low-dimensional condensed matter experiments, with **EM topology**!

[3] Li et al., Nature Phys. **12** (2016), 550-554

Background fields

- **Chiral "chemical potential"** $A_0^5 \equiv \mu_5$ is treated **homogeneous**, at most time dependent.

- Choose z-axis to point in the direction of $\vec{B}$.

Homogeneous magetic fields

- If B_3 is homogeneous $\to$ Dirac propagator is[4]

$$S_B(x,y) = \underbrace{\Phi(x,y)}_{\text{Schwinger phase}} \underbrace{\int_p \mathrm{e}^{-ip(x-y)} \widetilde{S}_B(p)}_{\text{translation invar.}},$$

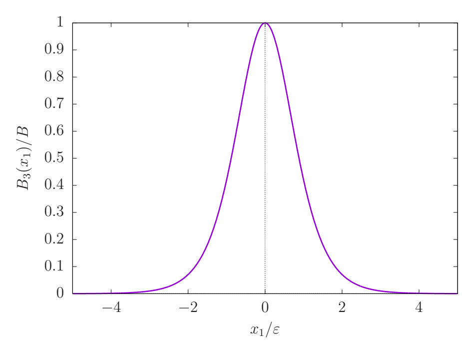
- $\widetilde{S}_B$ has a **Landau level sum** or a **Schwinger proper time** representation.

Inhomogeneous magetic fields

- Perturbatively could treat anything, but on the lattice we simplify and choose

$$B_3 = \frac{B}{\cosh^2\left(\frac{x_1}{\varepsilon}\right)}.$$

- Similar to a Gaussian (width=ε), motivated by simulated HIC magnetic field profiles[5].



[4] Shovkovy, Lect. Notes Phys. 871 (2013). [5] Deng and Huang, PRC 85 (2012) 044907.

Absence of global CME in equilibrium

- Bloch's theorem[6]
  Conserved **global** currents cannot flow in **equilibrium** ground state.

Some other approaches:

- Triangle diagram [7]
- Dirac eigenvalues + Lattice (free overlap) [8]
- Weyl-Wigner formalism [9,10]
- Vacuum polarization in background B + Lattice (full QCD staggered) [11]
- ...

End of story, why am I talking about this?

[6] Yamamoto, PRD 92, 085011 (2015)

[7] Hou, Liu, Ren, JHEP 05 (2011) 046. [8] Buividovich, NPA 925 (2014).

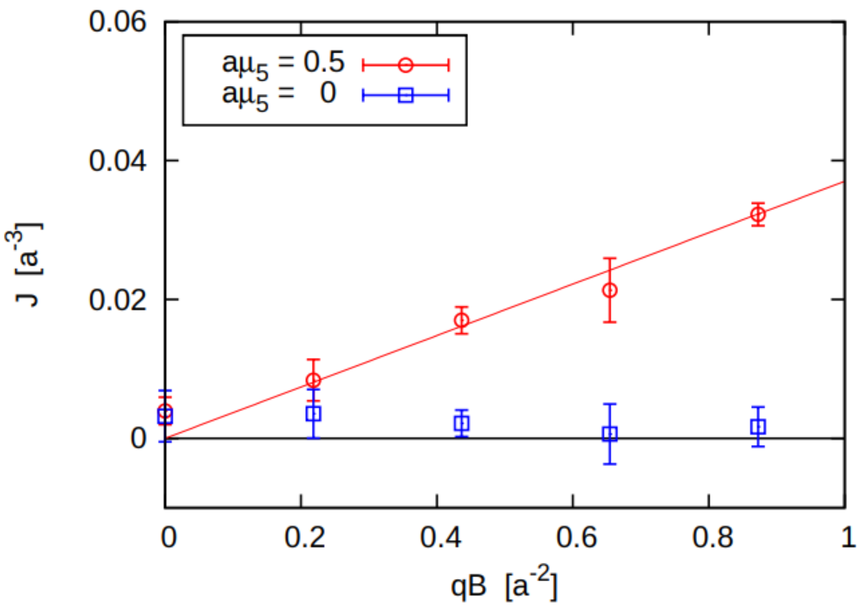
[9] Zubkov, PRD 93, 105036 (2016), [10] Banerjee et al., PLB 819, 136457 (2021).

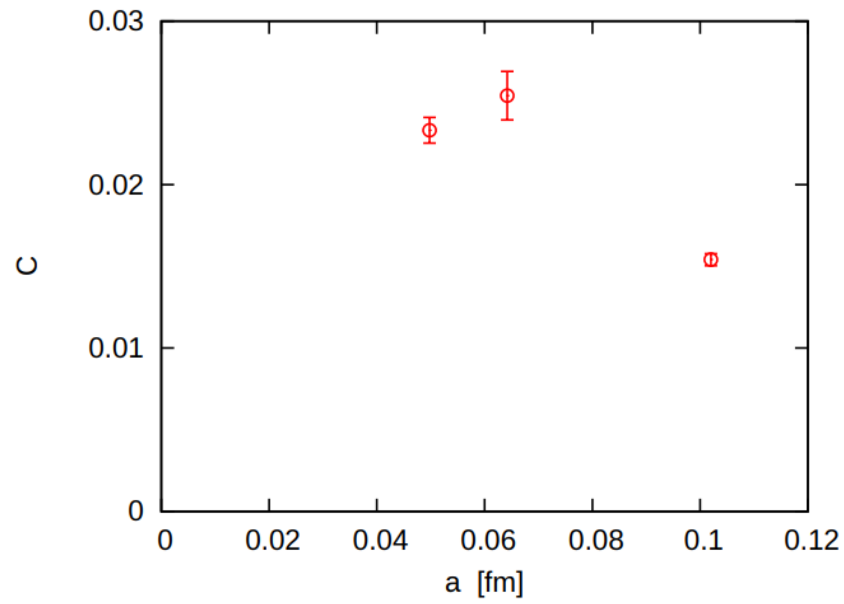
[11] Brandt, Endrődi, Garnacho, GM, JHEP 09 (2024) 092.

Lattice simulations

- Simulations at finite μ_5 are possible!
- Quenched and full QCD: Wilson[12]

$$C_{\mathrm{CME}} \approx 0.025 \neq 0? \qquad 1/(2\pi^2) \approx 0.05$$

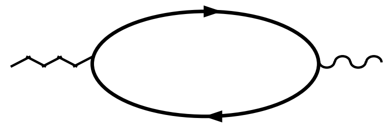




[12] Yamamoto, PRD 84 (2011) 114504, PRL 107 (2011) 031601

Absence of global CME in equilibrium

$$c_{\text{CME}} qB = \left\langle J_3 J_0^5 \right\rangle_B =$$



- Pauli-Villars: $\Gamma(m_{\text{phys}}) \to \sum_{s=0}^{3} c_s \Gamma(m_s)$, $c_s = \pm 1$, $m_{1,2,3} \to \infty$, $s = 0$ is physical.

$$c_{\text{CME}} qB = \frac{iT}{V} \sum_{s=0}^{3} c_s \int d^4x \int d^4y \, \text{Tr} \left[\gamma_3 S_B(x,y) \gamma_0 \gamma_5 S_B(y,x) \right] \, ,$$

- Using Schwinger proper time representation and some algebra

$$c_{\text{CME}} = \frac{1}{4\pi^3} \sum_{s=0}^{3} c_s \int_{-\infty}^{\infty} dp_3 \, dp_4 \frac{m_s^2 + p_4^2 - p_3^2}{(m_s^2 + p_4^2 + p_3^2)^2}$$

- The integral for a single PV species is ill-defined!

$$\frac{1}{4\pi^3} \int_{-\infty}^{\infty} dp_3 \, dp_4 \frac{m_s^2 + p_4^2 - p_3^2}{(m_s^2 + p_4^2 + p_3^2)^2} = \frac{1}{2\pi^2} \neq \frac{1}{4\pi^3} \int_{-\infty}^{\infty} dp_4 \, dp_3 \frac{m_s^2 + p_4^2 - p_3^2}{(m_s^2 + p_4^2 + p_3^2)^2} = 0 \, .$$

- But regularized by the PV sum

$$c_{\text{CME}} \propto \sum_{s=0}^{3} c_s = 0 \, .$$

Non-conserved vector current?

- lattice Ward identities for Wilson fermions [13]

- Local vector current $\to$ no lattice WI

$$j_\mu^{VL}(n) = \bar{\psi}(n)\gamma_\mu\psi(n)$$

- Point-split vector current $\to$ lattice WI

$$j_\mu^{VC}(n) = \frac{1}{2}\Big[\bar{\psi}(n)(\gamma_\mu - r)U_\mu(n)\psi(n+\hat{\mu}) + \bar{\psi}(n)(\gamma_\mu + r)U_\mu^\dagger(n-\hat{\mu})\psi(n-\hat{\mu})\Big]$$

- Breaking of lattice WI is a **UV effect**.

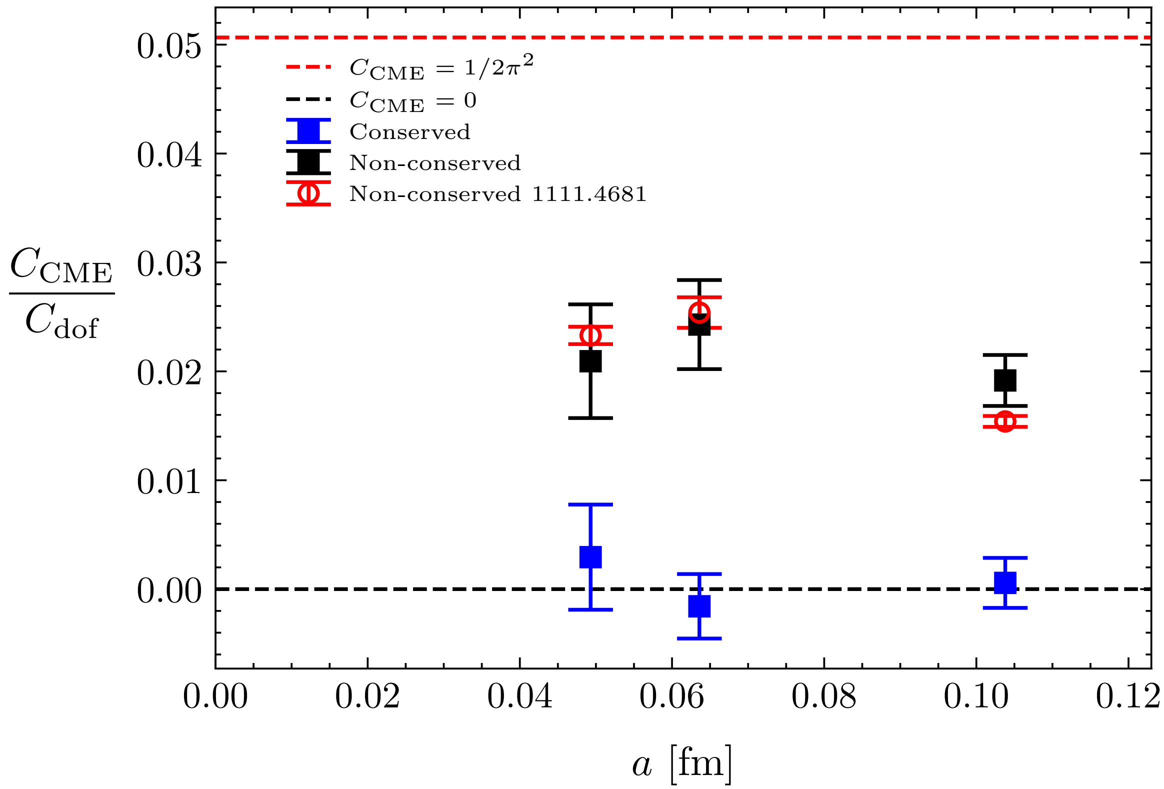
- Local version used in hadron spectroscopy, transport effects . . . and the 2011 lattice study.

- Does it matter for CME?

[13] Karsten, Smit, NPB 183 (1981) 103

Conserved currents & CME

- Quenched QCD (Wilson) [14]



- Using a non-conserved current “circumvents” Bloch’s theorem, allows non-vanishing result.

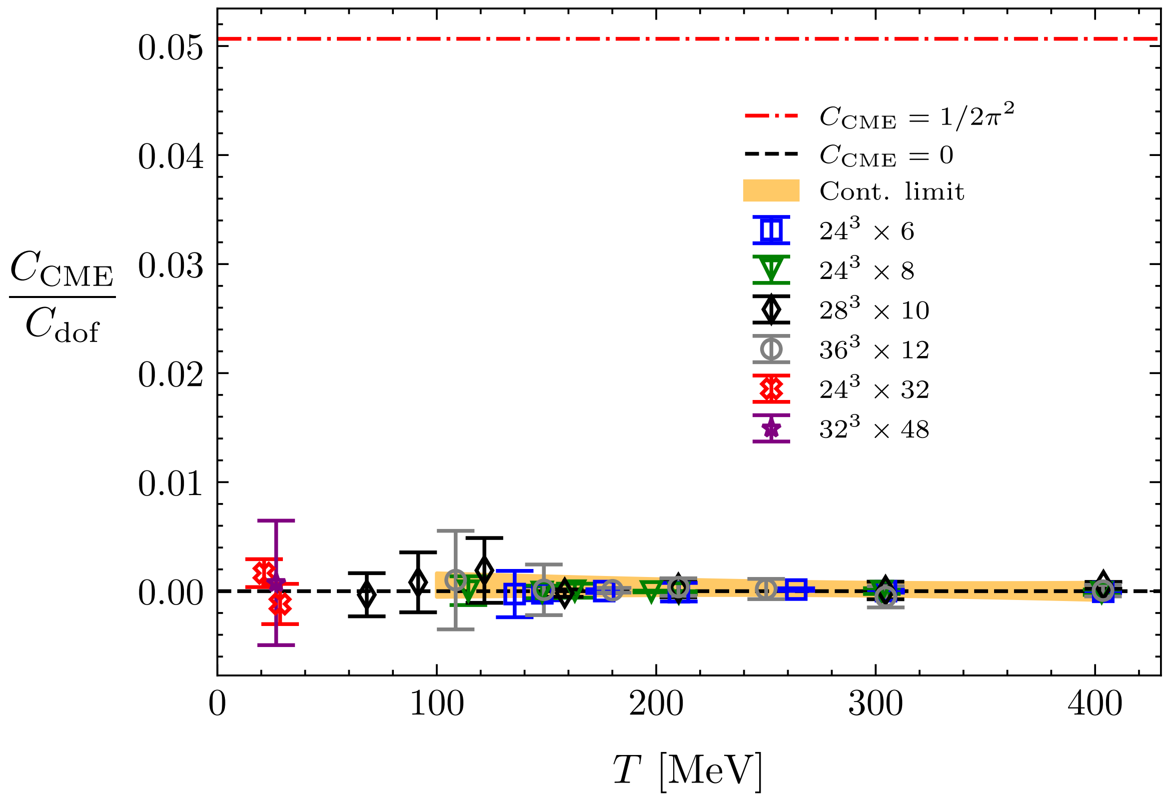
- Crucial to use a conserved vector current

[14] Brandt, Endrődi, Garnacho, GM, JHEP 09 (2024) 092.

Abscence of global CME in equilibrium QCD

- **global CME vanishes** in equilibrium, also in QCD[15]

(full QCD staggered)

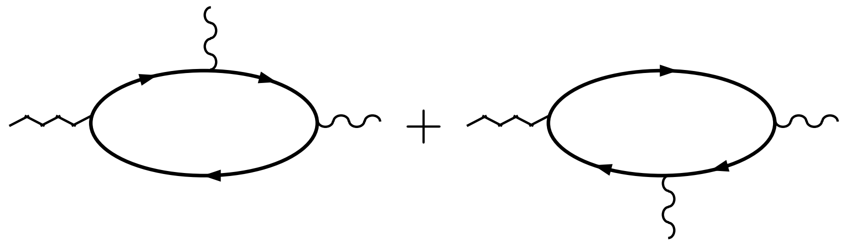


[15] Brandt, Endrődi, Garnacho, GM, JHEP 09 (2024) 092.

Local CME in equilibrium

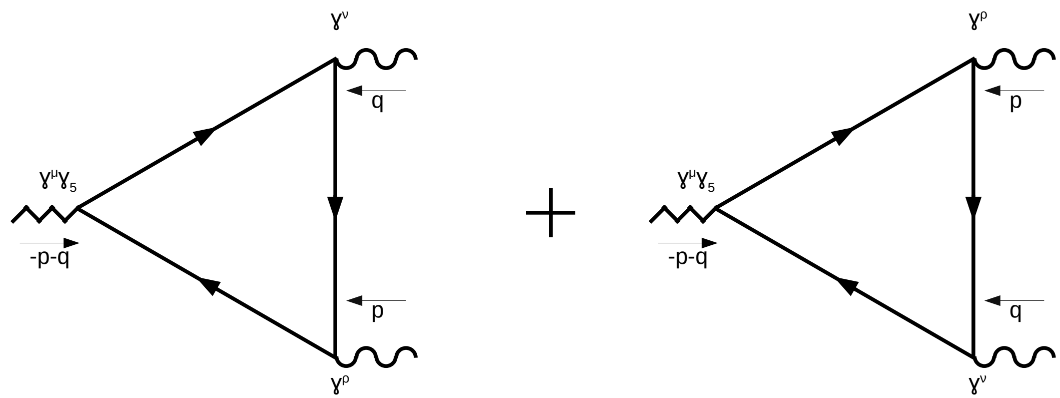
- **Local** CME current is **not forbidden** by Bloch's theorem
- **Inhomogeneous response** to an **inhomogeneous** B but homogeneous μ_5

$$c_{\rm CME}(q) = \frac{\delta \left\langle J_3 J_0^5 \right\rangle}{\delta B(q)} =$$



- We need the axial-vector–vector–vector 3-point function

$$\Gamma^{\rm AVV}_{\mu\nu\rho}(p+q,q,p) =$$



$$= -\, i \sum_{s=0}^{3} c_s \int_K \frac{{\rm Tr}\,\left[\gamma^\mu\gamma_5(\not{K}+m_s)\gamma^\nu(\not{K}+\not{q}+m_s)\gamma^\rho(\not{K}+\not{q}+\not{p}+m_s)\right]}{(K^2-m_s^2)((K+q)^2-m_s^2)((K+q+p)^2-m_s^2)}$$

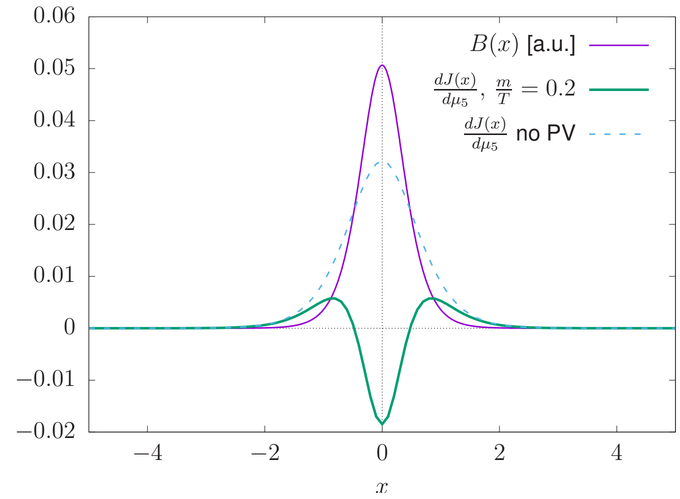
$$+\,(\{\nu,q\} \leftrightarrow \{\rho,p\})\,.$$

$$\langle J_3(x_1)\rangle = \mu_5 \int\ {\rm e}^{\,iq_1x_1}\frac{1}{q_1}\Gamma^{\rm AVV}_{023}(0,-q_1,q_1)B_3(q_1) \neq 0\,.$$

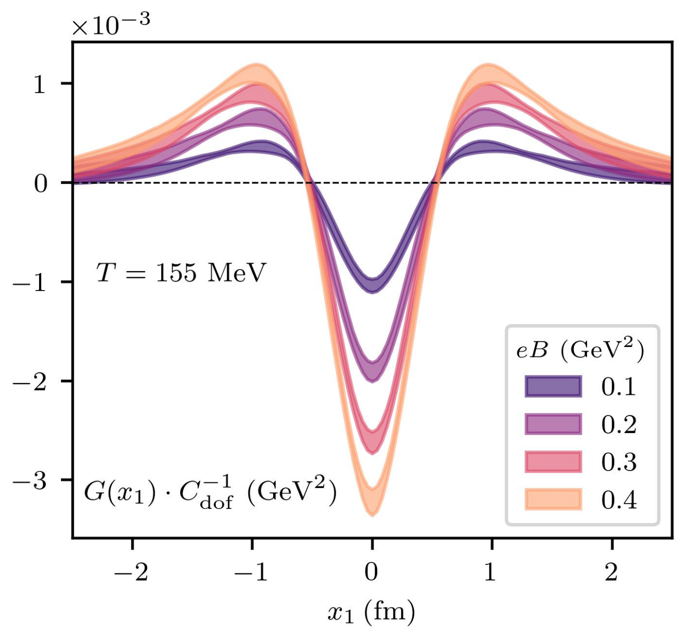
Local CME in equilibrium

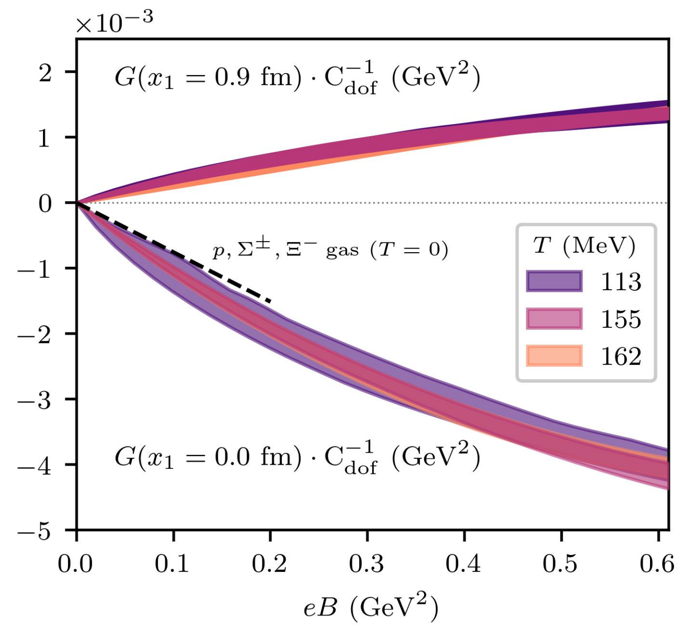
- For our specific magnetic field profile,

$$B_3 = \frac{B}{\cosh^2\left(\frac{x_1}{\varepsilon}\right)}$$



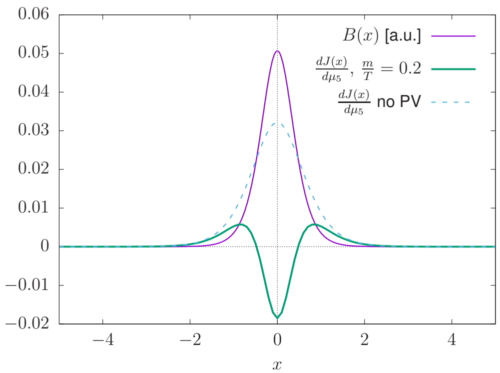
- the perturbative result
- full QCD lattice simulations[16]

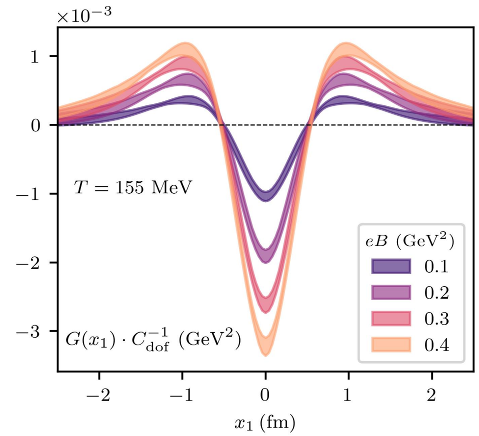


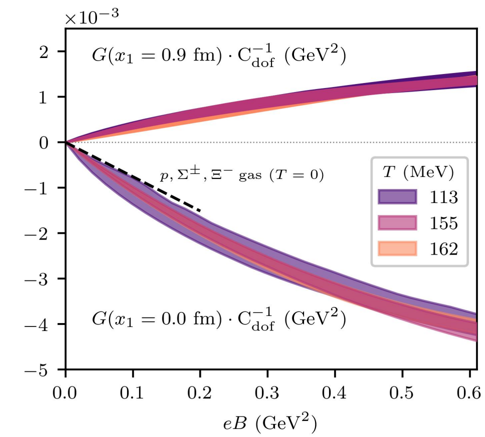


[16] Brandt, Endrődi, Garnacho, GM, Valois arXiv:2409.17616

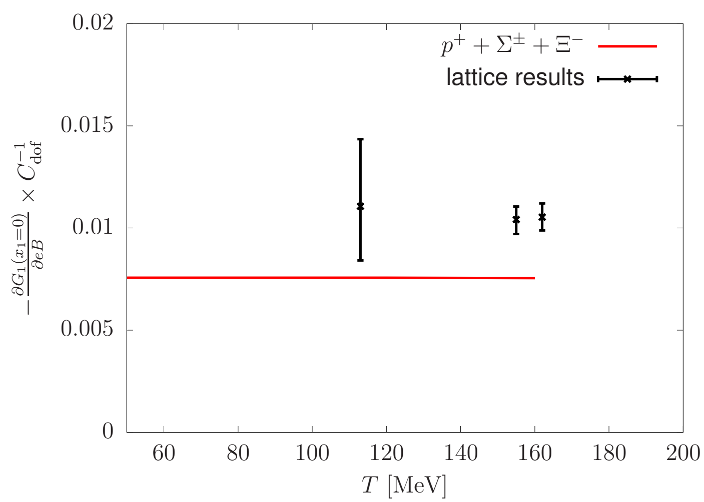
Local CME in equilibrium



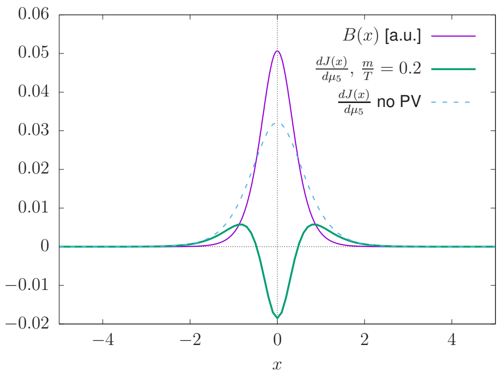


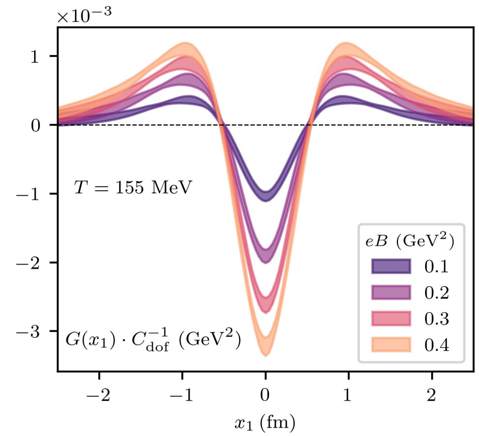


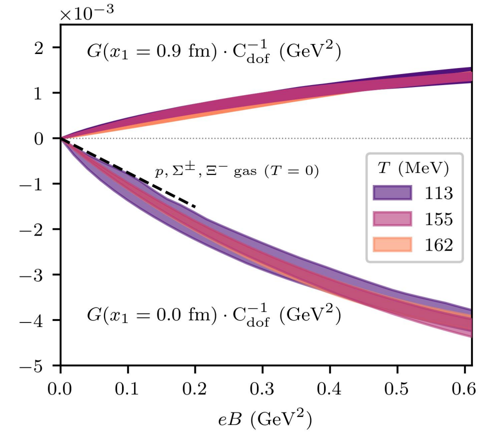
- Perturbative profile is qualitatively correct
- Profile is practically T independent. **Why?**
- Use free results to model: HRG ($p + \Sigma^\pm + \Xi^-$).



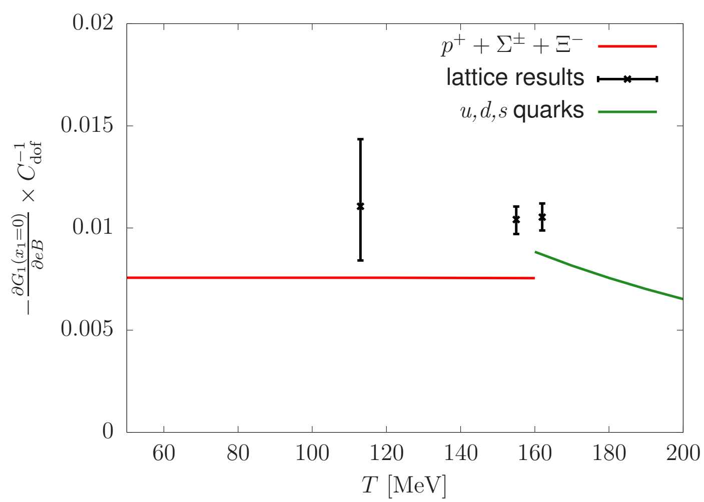
Local CME in equilibrium







- Perturbative profile is qualitatively correct
- Profile is practically T independent. **Why?**
- Use free results to model: HRG $(p + \Sigma^\pm + \Xi^-)$ vs free quarks $(u + d + s)$.



Dynamical CME

- To avoid Bloch's theorem global CME must be an **out-of-equilibrium** phenomenon.

- To calculate **linear response** (Kubo formulas), we avoid real-time Schwinger-Keldysh formalism[17].

- Instead use **imaginary time** and careful **analytical continuation** in frequency space, to recover the spectral function ϱ.

- We can also think in terms of spectral reconstruction (this is the lattice way)

$$G(\tau) = \int d\omega \varrho(\omega) K(\omega, \tau) \, .$$

- **Disclaimer**: **perturbation theory** is known to have **issues for transport peaks**[18], in principle resummations would be needed.

[17] Horváth et al., PRD 101 (2020), 076026 [18] Arnold, Moore and Yaffe, JHEP 11 (2000)

Dynamical CME, perturbative results

- Linear response functions to homogeneous but (Euclidean) **time dependent** $\boldsymbol{\mu_5}$[19]

$$G(\tau, \vec{x}, \tau', \vec{x}') = \Big\langle J_3(\tau, \vec{x}) J_0^5(\tau', \vec{x}') \Big\rangle_B \Longrightarrow G(\tau) = \int \frac{d^3x}{V} \Big\langle J_3(\tau, \vec{x}) J_0^5(0, 0) \Big\rangle_B \, .$$

- After careful analytic continuation one finds for the spectral function

$$\varrho(\omega) = \alpha \left(\frac{m}{T} \right) \omega \delta(\omega) + \Theta(\omega^2 - 4m^2) \frac{m^2}{\pi} \frac{\tanh \frac{|\omega|}{4T}}{\omega \sqrt{\omega^2 - 4m^2}} \, .$$

- The Kubo formula for the CME coefficient is

$$c_{\mathrm{CME}}^{\mathrm{neq}} = \frac{1}{qB\,T} \lim_{\omega \to 0} \frac{\varrho(\omega)}{\omega} \, .$$

- Shows perturbative **IR divergence** at $\omega = 0$, the **Kubo formula limit does not exist**.
- The transport peak is expected to widen through interactions, **resummations needed**.
- The same happens with the electric conductivity.[20]

[19] Buividovich, PRD 110 (2024), 094508 [20] Aarts, Martinez, NPB 726 (2005)

Dynamical CME on the lattice

- ϱ **not** directly **accessible** from lattice QCD

- Spectral representation of Euclidean correlators

$$G_E(\tau) = \int d\omega \, \frac{\rho(\omega)}{\omega} K(\tau,\omega)$$

Lattice — Want — Known

- On the lattice: $N_t \sim \mathcal{O}(10)$ **ill-posed inverse problem.**

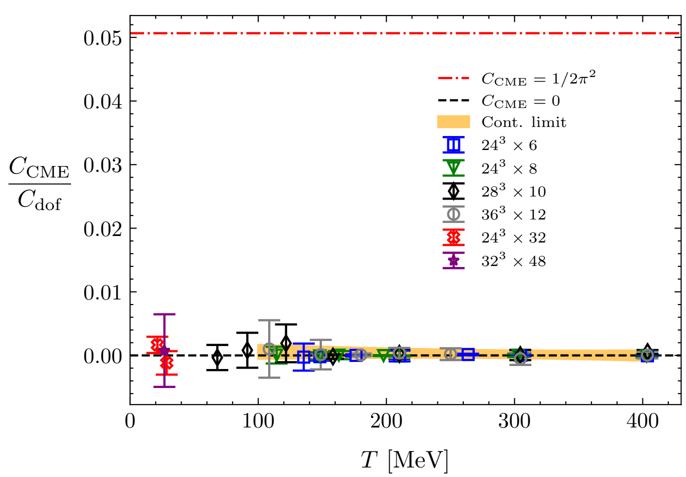
- Many methods on the market $\rightarrow$ applied to get other transport coefficients.

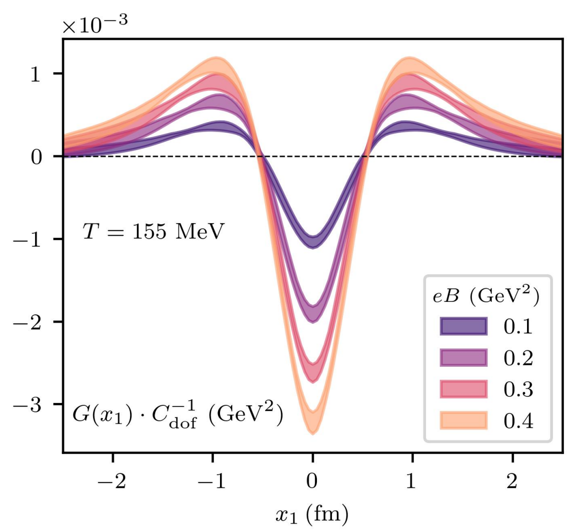
- We are combining several methods to get a robust answer.

- Stay tuned for full QCD results!

Summary

- We discussed the **chiral magnetic effect**
  - as a **global** current in **equilibrium**,
  - as a **local** current in **equilibrium**,
  - as a **global** current **out-of-equilibrium**.
- We learned from perturbation theory that **UV regulator effects** are important!
- It causes the global, equilibrium results to **vanish**, in accordance with **Bloch's theorem**.
- The **local, equilibrium** result is **non-vanishing**, and has a **weak temperature dependence**.
- Perturbation theory allows us to **understand** this in terms of the **HRG** model.
- **Out-of-equilibrium**, perturbation theory might be useful to test numerical approaches.
- In full QCD spectral functions are a tough nut to crack, **ask me about it before you go home**!





$$\varrho(\omega) = \alpha \left(\frac{m}{T}\right) \omega\delta(\omega) + \Theta(\omega^2 - 4m^2)\frac{m^2}{\pi}\frac{\tanh\frac{|\omega|}{4T}}{\omega\sqrt{\omega^2 - 4m^2}}$$