A Phenomenological Estimate of the Binding Energy of the Tbc tetraquark

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ABSTRACT

The discovery of the dimeson T_{cc}+(1+) at CERN 2021 at the predicted energy supported the successful application of the quark model beyond 2-body and 3-body hadronic systems.

Now we are eager to get more support by studying experimentally and theoretically heavier double-heavy tetraquarks such as T_{bc}^{0} whose properties are expected to be between the dimeson T_{cc}^{+} and compact tetraquark T_{bb}^{-} .

Different estimates are designed to guide (or mislead!) future experiments. We assume that the wave functions of the two light antiquarks around the diquark bc in the tetraquark are very similar to those around the heavy quark in Λ_b and that the 1/m corrections are neglected.

We predict that T_{bc}^{0} (1+) is bound and T_{bc}^{0}(0+) is not.

WE SHALL CONCENTRATE ON THE QUESTION:

Is $T_{bc} = (b\bar{u})(c\bar{d})$ molecular (dimeson) ?

like $T_{cc}^{+} = DD^{*} = (c\bar{u})(c\bar{d})$

Or is $T_{bc} = \bar{u}(bc)\bar{d}$ atomic (compact) ?

like $T_{bb}^{-} = \bar{B}\bar{B}^{*} = \bar{u}(bb)\bar{d}$

IT IS A CHALLENGE!

T_{bc} lies inbetween T_{cc}^{+} and T_{bb}^{-}

and is a delicate test of popular quark models.

Is it like the H_2 molecule (covalent bond) or like the He atom

(He nucleus → bc diquark, electrons→light quarks) ?

The present study is qualitative and preliminary.
The emphasis is on the relative importance
of the atomic versus molecular configuration.

$\mathbf{T_{bb}} \rightarrow \mathbf{T_{bc}} \rightarrow \mathbf{T_{cc}}$

It could help us in theory (to choose the relevant parts of Hilbert space in detailed calculations in different quark models);

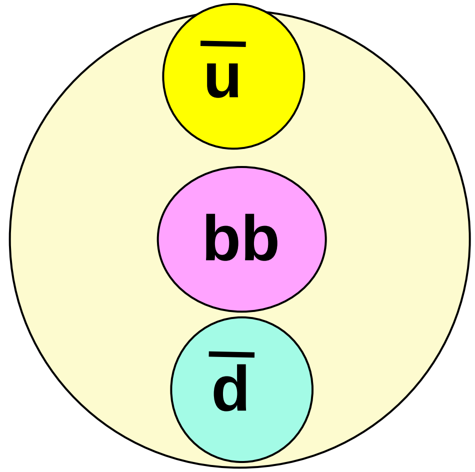
and in experiment (to look for relevant decay channels).

PHENOMENOLOGICAL ESTIMATE OF BINDING

Quark → diquark

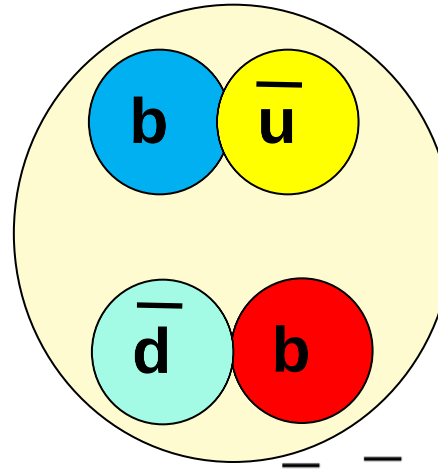
$\bar{b}$ ⇨ bb

5259 → 10115



like $\bar{\Lambda}_b$ antibaryon
(5620 ⇨ 10476)MeV

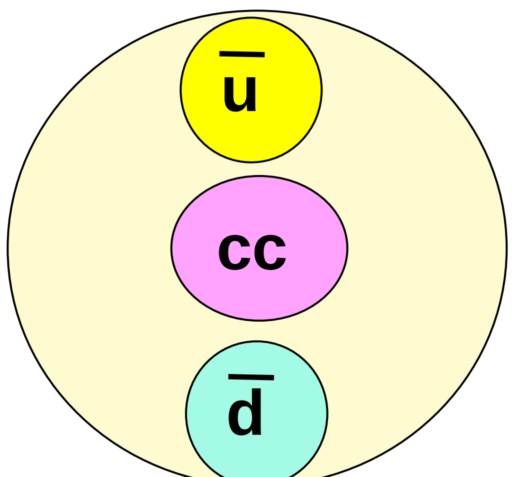
⇨
- 128
MeV



Threshold $\bar{B}+\bar{B}^*$
(5279+5325=10604)MeV

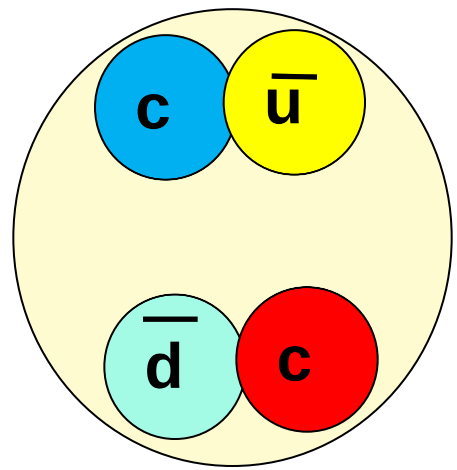
$\bar{c}$ ⇨ cc

1870 → 3556



like $\bar{\Lambda}_c$ antibaryon
(2286 ⇨ 3972)MeV

⇨
+92
MeV



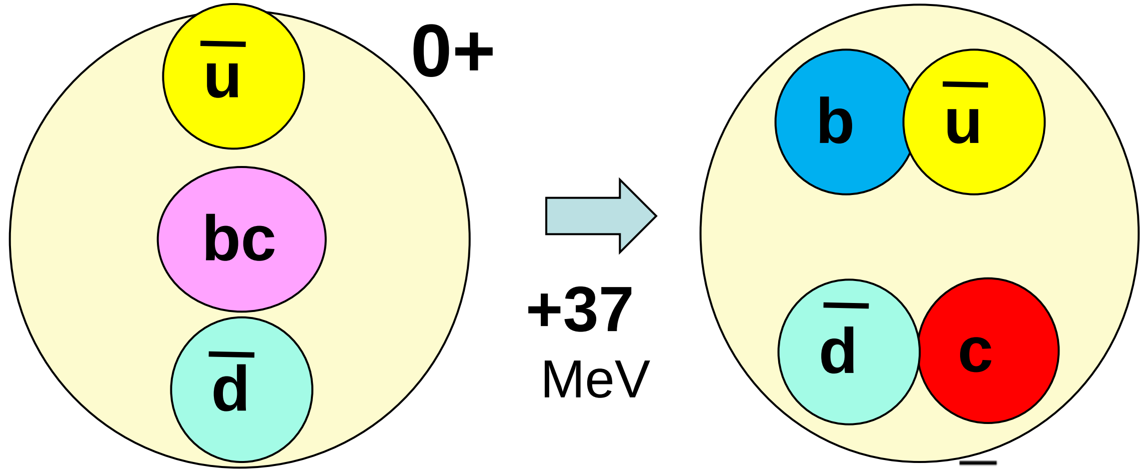
Threshold D+D*
(1867+2008=3875)MeV

PHENOMENOLOGICAL ESTIMATE OF BINDING

Quark → diquark

$\bar{b}$ ⇨ bc

5259 → 6820

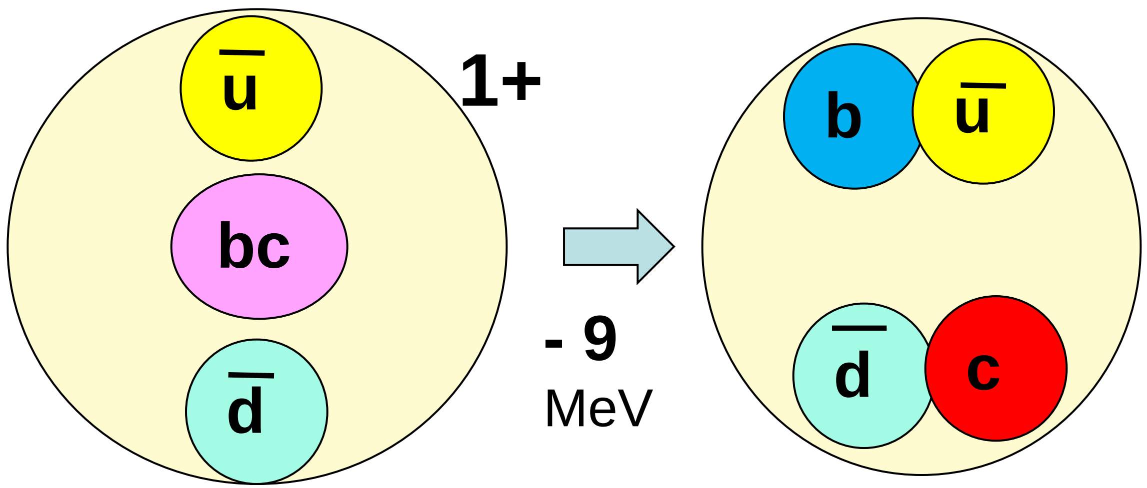


like $\bar{\Lambda}_b$ antibaryon

(5620 ⇨ 7181)MeV

Threshold $\bar{B}$+D

(5279+1865=7144)MeV



Threshold $\bar{B}$*+D

(5325+1865=7190)MeV

THE SUCCESSFUL CERN EXPERIMENT 2021

(THE BINDING ENERGY OF THE DD* „MOLECULE“)

The tetraquark (mesonic molecule) $\mathbf{T_{cc}^{+}}$ was produced in the proton-proton collision at the Large Hadron Collider and was detected as a narrow peak in the channel

$$\mathbf{T_{cc}^{+}} \to D^0 D^{*+} \to D^0 D^0 \pi^+ \to (K^- \pi^+)\,(K^- \pi^+)\,\pi^+$$

The peak position is **273 keV below the D^0D^{*+} threshold** (1683 keV below the D^+D^{*0} threshold)

The deduced **Breit-Wigner width is 410 keV.**

A more advanced model using a unitarised Breit-Wigner profile gives the peak at 360 keV below the D^0D^{*+} threshold and a width ~48 keV

(Effective range of T^+cc and other parameters [LHCb, arXiv:2109.01056], Mikhail Mikhasenko).

Observation of T_{cc}^+ state

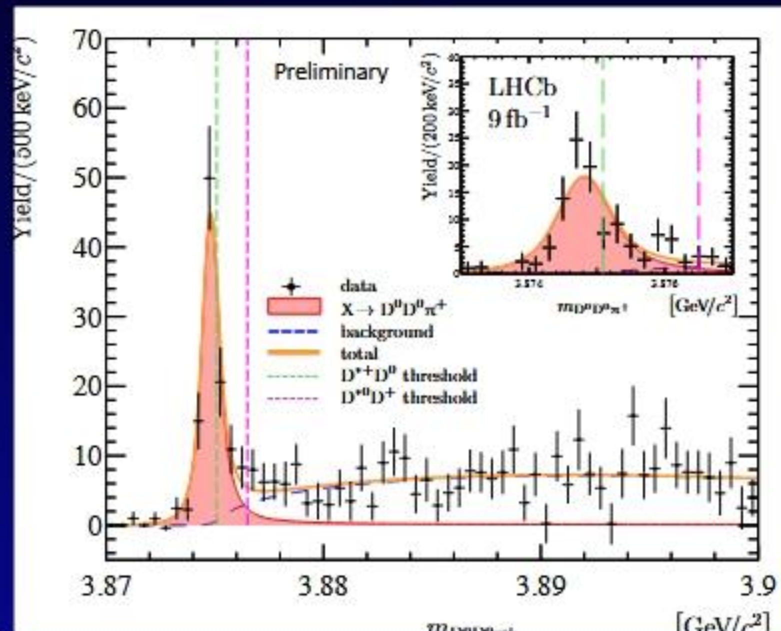
NEW

LHCb-PAPER-2021-031

- **First observation of a same-sign doubly charmed tetraquark T_{cc}^+**
  - Very narrow state in $D^0 D^0 \pi^+$ mass spectrum
  - Consistent with $cc\bar{u}\bar{d}$ tetraquark
  - Mass very close to $D^{*+}D^0$ mass thresholds
  - Manifestly exotic
- **Parameters of T_{cc}^+**
  - Fit structure with P-wave relativistic Breit-Wigner

$$\begin{aligned} \delta m_{\rm BW} &= -273 \pm 61 \pm 5\,^{+11}_{-14}\,\mathrm{keV}/c^2\,, \\ \Gamma_{\rm BW} &= 410 \pm 165 \pm 43\,^{+18}_{-38}\,\mathrm{keV}\,, \end{aligned}$$

  - Uncertainties stat, syst and due $J^P = 1^+$ assumption
  - Significance for signal > 10 σ
  - Significance for $\delta m_{BW} < 0$ 4.3 σ



BINDING ENERGY OF THE DD* „MOLECULE“

In 2004 Janc & Rosina predicted a weakly bound dimeson

Tcc+ – (D + D*) = - 0.6 MeV (Bhaduri potential)

- 2.7 MeV (Grenoble AL1 pot.)

It is a great satisfaction that it has been finally detected and confirmed 17 years later in CERN in July 2021.

The model parameters in the nonrelativistic calculation fitted all relevant mesons and baryons.
A rich 4-body model space was used including gaussians of Jacobi coordinats of different optimized widths and positions.

$$V_{ij}^B = -\frac{\lambda_i^C}{2}\cdot\frac{\lambda_j^C}{2}\left(U_0 + \frac{\alpha}{r_{ij}} + \beta r_{ij} + \alpha\frac{\hbar^2}{m_i m_j c^2}\frac{e^{-r_{ij}/r_0}}{r_0^2 r_{ij}}\sigma_i\cdot\sigma_j\right),$$

$$r_{ij} = |\boldsymbol{r}_i - \boldsymbol{r}_j|;$$

Few-Body Systems 35,175-196, 2004

The presented **phenomenological estimate** is based on the assumption that the wave functions of the two light quarks around the heavy quark in $\mathbf{\Lambda_c}$, $\mathbf{\Lambda_b}$ and around the antidiquark in the $\mathbf{\overline{cc}qq}$, $\mathbf{\overline{bc}qq}$ and $\mathbf{\bar{b}\bar{b}qq}$ are very similar.

This assumption implies that the heavy antidiquark in a colour triplet state acts just like a very heavy quark.

This assumption is supported by the experimental observation that many related masses differ by the same amount (essentially by the quark mass difference):

$\tilde{B} - \tilde{D} = 3341\text{MeV}$, $\tilde{B}_s - \tilde{D}_s = 3328\text{MeV}$, $\Lambda_b - \Lambda_c = 3340\text{MeV}$.

(The tilde means the hyperfine average between the scalar and vector meson.)

In order to get the diquark energy and mass we use

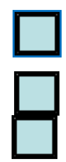
the „$V_{qq} = \frac{1}{2} V_{q\bar{q}}$ rule“

$$[p^2/2(b/2) + V_{bb}]\,\psi = \tfrac{1}{2}\,[p^2/2(b/4) + V_{b\bar{b}}]\,\psi = E_{bb}\psi = \tfrac{1}{2}\,F(b/4),$$

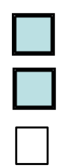
And similar for other diqarks

Justiication:

[A]: (color.color)=4/3 for $c\bar{c}$



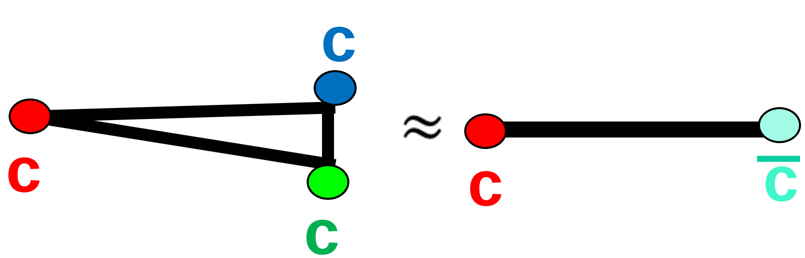
=2/3 for cc



Color singlet versus triplet

(SU(3) Casimir operator)

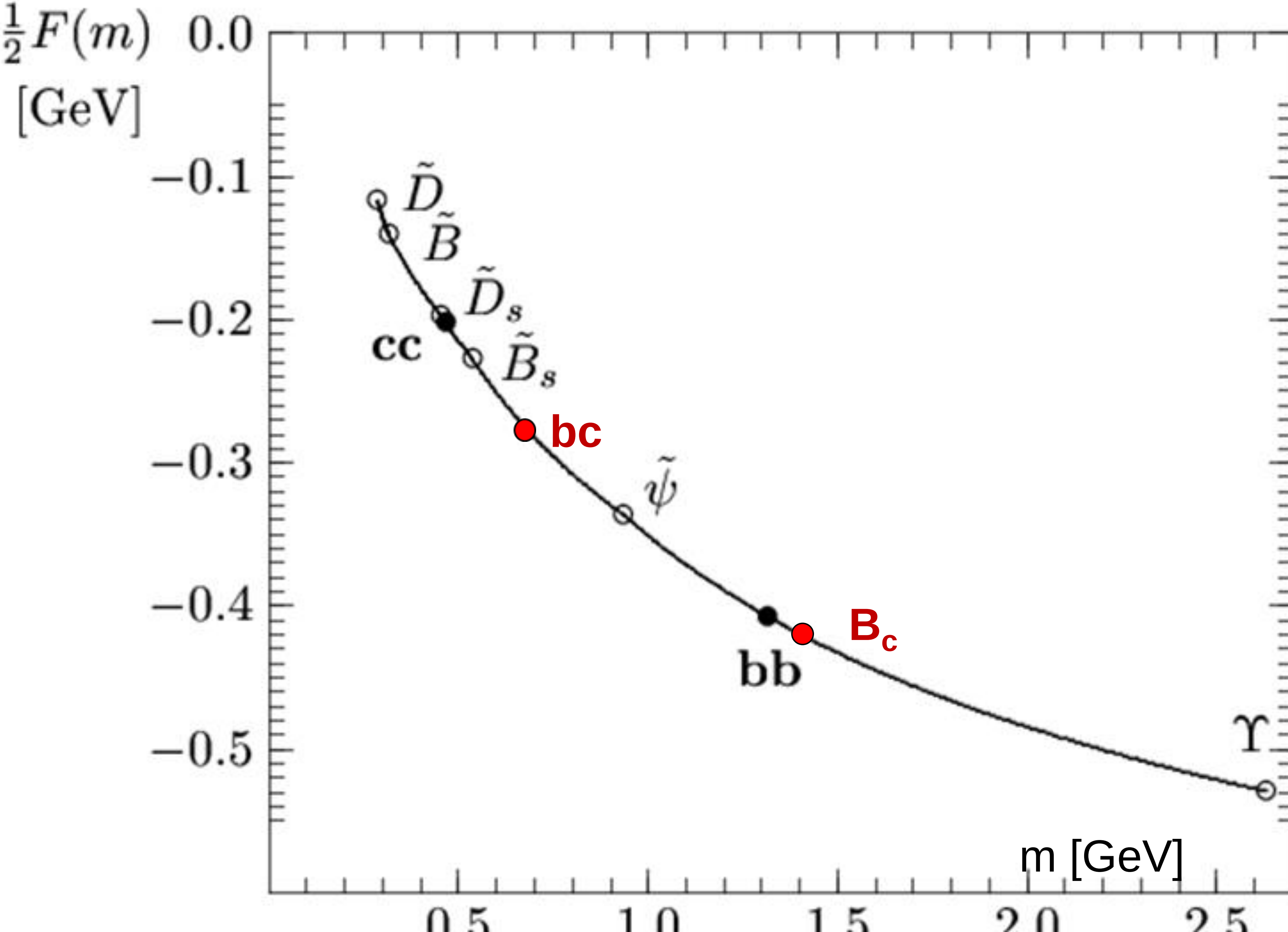
[B]: Flux tube model



At small angle is the

c – $\bar{c}$ flux tube similar to two

c – c flux tubes



$\frac{1}{2}F(m)$ [GeV]
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
$\tilde{D}$
$\tilde{B}$
$\tilde{D}_s$
cc
$\tilde{B}_s$
bc
$\tilde{\psi}$
bb
B_c
Υ
m [GeV]
0.5
1.0
1.5
2.0
2.5

$$T_{bc} = \Lambda_b + ½ (\tilde{\psi} - E_{c\bar{c}}) + E_{bc}$$

$$= 5619 + ½ (3012 + 672) - 280 = \mathbf{7181\ MeV}$$

$$\Delta T_{bc} = T_{bc} - B_0 - D_0 = T_{bc} - 7144 = \mathbf{37}\ \text{MeV} \quad (S=0)$$

$$\Delta T_{bc} = T_{bc} - B^*_0 - D_0 = T_{bc} - 7190 = \mathbf{-9}\ \text{MeV} \quad (S=1)$$

or

$$T_{bc} = \Lambda_b + c + E_{bc}$$

$$= 5619 + 1870 - 280 = \mathbf{7214\ MeV}$$

$$T_{bc} - B_0 - D_0 = T_{bc} - 7144 = \mathbf{+70}\ \text{MeV}$$

$$T_{bc} - B^*_0 - D_0 = T_{bc} - 7190 = \mathbf{+24}\ \text{MeV}$$

UNCERTAINTIES OF THE PHENOMENOLOGICAL ESTIMATES

1. Choice of quark masses:

Bhaduri: q=337, s=600, c=1870, b=5259 MeV

Bh/JR1* : q=337, s=600, c=1600, b=4941 MeV

KR: q=309, s=482, c=1656, b=4989 MeV

2. Interpolation

3. Averages of spin-spin (chromomagnetic) interaction
   (It does not scale the same way as the radial V)

COMPARISON OF DIFFERENT CHOICES

(INTERPOLATION)	ΔT_{cc}	ΔT_{bb}	ΔT_{bc} (J=0)	ΔT_{bc} (J=1)
Bhaduri masses:	+79	-141	+70	+24
--"-- ,different averages:	**+92**	**-128**	**+37**	**- 9**
Karliner, Rosner masses:	+31	-181	+62	+16
Janc, Rosina 2001:	+97	-128		
--"--smaller masses:	+43	-167		
Karliner, Rosner(*junctions*):	+125	- 89	+111	+65
Janc, Rosina (accurate):	-0.6	- 82		

–

DETECTION OF T_{bc}^{0}

Unfortunately, the decay channels of T_{bc}^{0}

are not so nice as in the case of T_{cc}^{+}

with all charged final particles:

$T_{cc}^{+} \longrightarrow D^0 D^{*+} \longrightarrow D^0 D^0 \pi^+ \longrightarrow (K^- \pi^+)\ (K^- \pi^+)\ \pi^+$.

If $T_{bc}^{0} > D^0B^{*0}$ ➡ strong decay $T_{bc}^{0} \longrightarrow D^0 + B^{*0}$

If $T_{bc}^{0} > D^0B^0$ ➡ $T_{bc}^{0} \longrightarrow D^0B^{*0} \longrightarrow D^0B^0\,\gamma \longrightarrow (K^- \pi^+)\,(K^- \pi^+)\,\gamma$

If $T_{bc}^{0} < D^0B^0$ ➡ weak decay (long lived)

For good statistics, a combination

of many channels is needed!

(Wait a few years!!!)

Weakly-decaying T_{bc}. Branching fractions

T^0_{bc} decay	$\mathcal{B}$ estimation	analogous B-decays	$\mathcal{B}$, exp
	$b \to D + hadrons$ decay modes		
D^0D^0	$10^{-3} \times f_b$ or $10^{-3} \times f_W$	—	—
$D^0D^+\pi^-$	$(2.5-5.0) \times 10^{-3} \times f_b$	$B^- \to D^0\pi^-$	$(4.61 \pm 0.10) \times 10^{-3}$
		$\overline{B}^0 \to D^+\pi^-$	$(2.51 \pm 0.08) \times 10^{-3}$
		$B^0_s \to D^+_s\pi^-$	$(2.98 \pm 0.14) \times 10^{-3}$
		$\Lambda^0_b \to \Lambda^+_c\pi^-$	$(4.9 \pm 0.4) \times 10^{-3}$
$D^{*0}D^+\pi^-$	$(5.2 \pm 0.5) \times 10^{-3} \times f_b$	$B^- \to D^{*0}\pi^-$	$(5.17 \pm 0.15) \times 10^{-3}$
$D^{*+}D^0\pi^-$	$(2.7 \pm 0.3) \times 10^{-3} \times f_b$	$\overline{B}^0 \to D^{*+}\pi^-$	$(2.74 \pm 0.13) \times 10^{-3}$
$D^+D^+\pi^-\pi^-$	$(1-8) \times 10^{-3} \times f_b$	$B^- \to D^+\pi^-\pi^-$	$(1.07 \pm 0.05) \times 10^{-3}$
		$\overline{B}^0 \to D^+\rho^-_{[\pi^-\pi^0]}$	$(7.6 \pm 1.2) \times 10^{-3}$
		$B^0_s \to D^-_s\rho^+_{[\pi^+\pi^0]}$	$(6.8 \pm 1.4) \times 10^{-3}$
$D^0D^0\pi^+\pi^-{}_{NR}$	$(0.9 \pm 0.4) \times 10^{-3} \times f_b$	$\overline{B}^0 \to D^0\pi^+\pi^-$	$(0.88 \pm 0.05) \times 10^{-3}$
$D^0D^+\pi^-\pi^0{}_{NR}$	$(1.0 \pm 0.3)\% \times f_b$	$B^- \to D^0\rho^-_{\pi^-\pi^0}$	$(1.34 \pm 0.18) \times 10^{-2}$
		$\overline{B}^0 \to D^+\rho^-_{\pi^-\pi^0}$	$(0.76 \pm 0.12) \times 10^{-2}$
	$b \to D + \mu X$ decay modes		
$D^0D^+\mu^-\nu$	$(2.3 \pm 0.1)\% \times f_b$	$B^- \to D^0\mu^-\nu$	$(2.30 \pm 0.09)\%$
		$\overline{B}^0 \to D^+\mu^-\nu$	$(2.24 \pm 0.09)\%$
$D^{*0}D^+\mu^-\nu$	$(5.1 \pm 0.2)\% \times f_b$	$B^- \to D^{*0}\mu^-\nu$	$(5.30 \pm 0.09)\%$
$D^0D^{*+}\mu^-\nu$	$(5.1 \pm 0.2)\% \times f_b$	$\overline{B}^0 \to D^{*+}\mu^-\nu$	$(4.97 \pm 0.12)\%$
$D^0D^+\mu^-X$	$(2-8)\% \times f_b$	$B^{+/0}_{mix} \to D^+\mu^-X$	$(2.6 \pm 0.5)\%$
		$B^{+/0}_{mix} \to D^0\mu^-X$	$(7.3 \pm 1.5)\%$
		$B^+ \to D\mu^-X$	$(9.6 \pm 0.7)\%$
		$\overline{B}^0 \to D\mu^-X$	$(9.3 \pm 0.8)\%$
	$b \to J/\psi + hadrons$ decay modes		
$J/\psi D^+K^-$	$(1.0 \pm 0.1) \times 10^{-3} \times f_b$	$B^- \to J/\psi K^-$	$(1.02 \pm 0.02) \times 10^{-3}$
$J/\psi D^0K^0_S$	$(0.45 \pm 0.05) \times 10^{-3} \times f_b$	$\overline{B}^0 \to J/\psi K^0_S$	$(0.89 \pm 0.02)/2 \times 10^{-}$
$J/\psi D^0K^{*-}$	$(1.3 \pm 0.2) \times 10^{-3} \times f_b$	$B^- \to J/\psi K^{*-}$	$(1.43 \pm 0.08) \times 10^{-3}$
		$\overline{B}^0 \to J/\psi K^{*0}$	$(1.27 \pm 0.05) \times 10^{-3}$
		$B^0_s \to J/\psi\phi$	$(1.04 \pm 0.04) \times 10^{-3}$
		$\Lambda^0_b \to J/\psi\Lambda(1520)$	$(0.32 \pm 0.06) \times (0.19 \pm$
$J/\psi D^0K^-\pi^+{}_{NR}$	$(0.3-1.2) \times 10^{-3} \times f_b$	$\overline{B}^0 \to J/\psi K^-\pi^+{}_{NR}$	$(1.15 \pm 0.05) \times 10^{-3}$
		$B^0_s \to J/\psi K^+K^-{}_{NR}$	$(0.79 \pm 0.07) \times 10^{-3}$
		$\Lambda^0_b \to J/\psi pK^-{}_{sum}$	$(0.32 \pm 0.06) \times 10^{-3}$

$f_b = 0.22 \pm 0.04$, $f_c = 0.72 \pm 0.04$, $f_W = 0.06 \pm 0.04$,

T^0_{bc} decay	$\mathcal{B}$ estimation	analogous B/D-decays	$\mathcal{B}$, exp
	$c \to s$ transition		
$\overline{B}^0K^-\pi^+$	$(4 \pm 1)\% \times f_c$	$D^0 \to K^-\pi^+$	$(3.95 \pm 0.03)\%$
$B^-K^-\pi^+\pi^+$	$(7.5 \pm 2.5)\% \times f_c$	$D^+ \to K^-\pi^+\pi^+$	$(9.38 \pm 0.16)\%$
		$D^+_s \to K^+K^-\pi^+$	$(5.39 \pm 0.15)\%$
		$\Lambda^+_c \to pK^-\pi^+$	$(6.28 \pm 0.32)\%$
$\overline{B}^0K^-\pi^+\pi^+\pi^-$	$(8 \pm 2)\% \times f_c$	$D^0 \to K^-\pi^+\pi^+\pi^-$	$(8.22 \pm 0.14)\%$
$\overline{B}^0K^-\mu^+\nu$	$(3 \pm 1)\% \times f_c$	$D^0 \to K^-\mu^+\nu$	$(3.41 \pm 0.04)\%$
$\overline{B}^0K^-\mu^+X$	$(2 \pm 1)\% \times f_c$	$D^0 \to K^{*-}\mu^+\nu$	$(1.89 \pm 0.24)\%$
		$D^0 \to K^-\pi^0e^+\nu$	$(1.6^{+1.3}_{-0.5})\%$
$B^-K^-\pi^+\mu^+\nu$	$(3.5 \pm 1.5)\% \times f_c$	$D^+ \to K^-\pi^+\mu^+\nu_{sum}$	$(3.65 \pm 0.34)\%$
	W exchange in $bc \to cs$		
$D^0K^-\pi^+$	$(5 \pm 4) \times 10^{-4} \times f_W$	—	—
D^+K^-	$(5 \pm 4) \times 10^{-4} \times f_W$	—	—
$D^0K^0_S$	$(5 \pm 4) \times 10^{-4} \times f_W$	—	—
	$b \to u$ transition		
$D^0\pi^+\pi^+$	$(3 \pm 2) \times 10^{-5} \times f_b$	$B^+ \to \pi^+\pi^0$	5.5×10^{-6}
$D^+\pi^-$	$(3 \pm 2) \times 10^{-5} \times f_b$	$B^+ \to \pi^+\pi^+\pi^-$	1.5×10^{-5}
$D^+\pi^+\pi^-\pi^-$	$(3 \pm 2) \times 10^{-5} \times f_b$	$B^0 \to \rho^+\pi^-$	2.3×10^{-5}

Channel	My estimation	Steve's number
DD+h	(0.02-0.2)%	1%
J/ψD+h	(0.01-0.03)%	1%
B+h	(3-6)%	1%
Dh	~10^{-5}	

Question to theory:

Can you provide us with better estimates?

Expected yields:

σ = 20 nb

L = 20 fb^{-1} (Run3)

yield ≈ 1 – 30 per channel

Thank you

for your attention!

The „$V_{qq} = \frac{1}{2} V_{q\bar{q}}$ rule“

$\frac{1}{2} E_{b\bar{b}} = \frac{1}{2}\, \tilde{Y} - b \equiv \frac{1}{2} F(\frac{1}{2}\, b) \quad = -529\text{MeV}$

$\frac{1}{2} E_{\bar{b}c} = \frac{1}{2}\, (\tilde{B}_c - b - c) \equiv \frac{1}{2} F((b^{-1} + c^{-1})^{-1})$
$= -428\text{ MeV}$

$E_{bc} = \frac{1}{2} F(\frac{1}{2}\, (b^{-1} + c^{-1})^{-1}) \quad = -276\text{ MeV}$

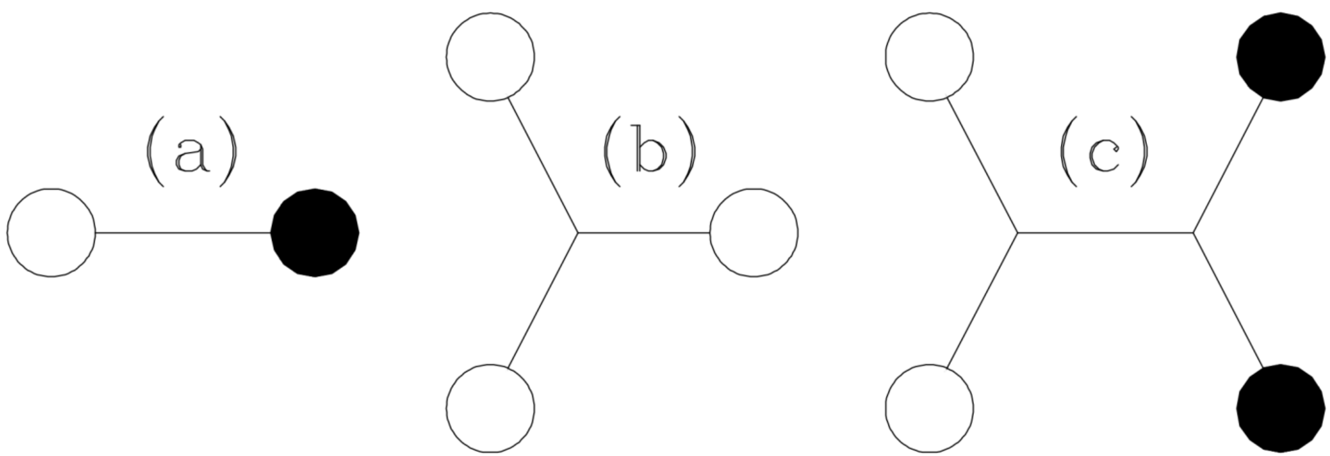
$\frac{1}{2} E_{c\bar{c}} = \frac{1}{2}\, \tilde{\psi} - c \equiv \frac{1}{2} F(\frac{1}{2}\, c) = -336\text{MeV}$

$\frac{1}{2} E_{\bar{c}s} = \frac{1}{2}\, (\tilde{D}_s - c - s) \equiv \frac{1}{2} F((c^{-1} + s^{-1})^{-1})$
$= -197\text{ MeV}$

KR = Karliner, Rosner, Phys. Rev. D 102, 094016, 2020

They assign S = 165 MeV to each string junction

and take into account the spin-spin („chromomagnetic“) term



meson (0×S)   baryon (1×S)   tetraquark (2×S)

REFERENCES:

JR1 = Janc, Rosina, Few-Body Systems 31,1-11,2001

JR2 = Janc, Rosina, Few-Body Systems 35,175-196, 2004

KR = Karliner, Rosner, Phys. Rev. D 102, 094016, 2020

Bh = Bhaduri, Cohler, Nogami, Nuovo Cim. A65, 376, 1981

Grenoble = Silvestre-Brac, , Few-Body Systems 20, 1 1996