Scales in Sauter-Schwinger pair production

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- Determination of time scales and spatial scales in pair production from fields
- **Why?** Understand the temporal and spatial resolution of non-equilibrium quantum processes
- **Problem:**
  - Pair densities in background at non-asymptotic times are unphysical.

Solution?

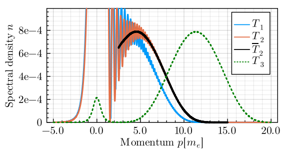
Overview

Sauter-Schwinger effect

- Dirac-Heisenberg-Wigner-formalism
- Sauter pulse $E(t) = \frac{1}{\cosh^2(\frac{t}{\tau})}$

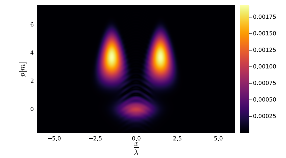
Time scales for spatially homogeneous field

- Time dependence of adiabatic particle number
- Time scales of single Sauter pulse
- Time evolution of double Sauter pulse



Scales for spatially inhomogeneous fields

- Time evolution in phase space
- Spatial dependence of time Scales
- Spatial separation



Sauter-Schwinger effect

- Spontaneous creation of electron-positron pairs from the vacuum in ultrastrong electromagnetic fields
- Sauter Sauter, F. Über das Verhalten eines Elektrons im homogenen elektrischen Feld nach der relativistischen Theorie Diracs. Z. Physik 69, 742–764 (1931). https://doi.org/10.1007/BF01339461
- "Tunneling" from Dirac sea

Vacuum decay rate:

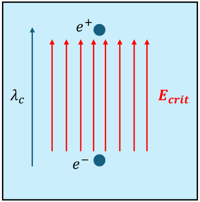
$$w = \frac{\alpha E^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} exp\left(-\frac{n\pi m_e^2}{|eE|}\right)$$

Pair production rate:

$$\Gamma = \frac{2(eE)^2}{(2\pi)^3} \exp\left(-\frac{\pi m^2}{eE}\right)$$

Schwinger limit:

$$E_{crit} = \frac{m_e^2 c^3}{e\hbar} \approx 1.3 \times 10^{18} \frac{V}{m}$$



Julian Schwinger, On Gauge Invariance and Vacuum Polarization Phys. Rev. 82, 664 (1951)

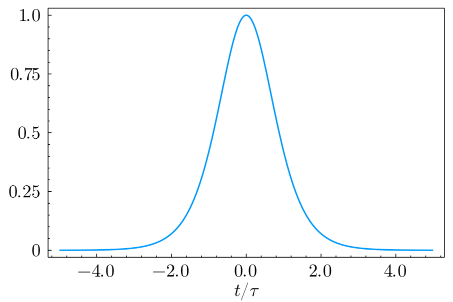
Dirac-Heisenberg-Wigner-formalism

$$w_{\alpha\beta} = -\frac{1}{2}\int d^3y \, \langle\Omega|\, e^{-i\boldsymbol{p}\cdot\boldsymbol{y}} U(A,x,y) \left[\psi_\alpha\left(\boldsymbol{x}-\frac{\boldsymbol{y}}{2},t\right), \overline{\psi}_\beta\left(\boldsymbol{x}+\frac{\boldsymbol{y}}{2},t\right)\right] |\Omega\rangle$$

- Equations of motion for $w_{\alpha\beta}$ via Dirac equation, where A treated in a mean-field approximation
- Suitable for studying pair production in space and time-dependent electromagnetic fields
- Wigner coefficients are related to observable quantities as particle number densities, charge densities...
- These quantities only then have a physical interpretation, when the background field is switched off
- Pseudo-differential operators need careful treatment

D. Vasak, M. Gyulassy, and H. T. Elze, Ann. Phys. (N.Y.) 173, 462 (1987), O. T. Serimaa, J. Javanainen, and S. Varró, Phys. Rev. A, 33, 2913 (1986) I. Bialynicki-Birula, P. Górnicki, and J. Rafelski, Phys. Rev. D 44, 1825 (1991)

Sauter-pulse



$$E(t) = \varepsilon \frac{1}{\cosh(t/\tau)^2}$$

- ε: field strength relative to E_{crit}
- τ: pulse duration

Keldysh parameter:

$$\gamma = \frac{1}{\varepsilon\tau}$$

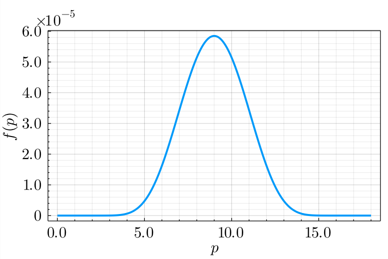
- $\gamma \gg 1$ perturbative, multiphoton
- $\gamma \approx 1$ crossover regime
- $\gamma \ll 1$ Schwinger regime

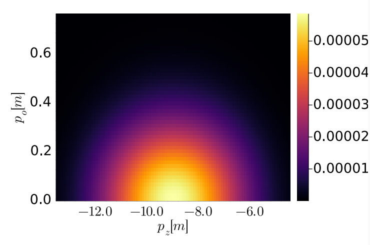
Inhomogeneous Sauter pulse:

$$E_{inh}(t,x) = E(t) exp\left(\frac{-x^2}{2\lambda^2}\right)$$

Asymptotic particle yield

- Spatially homogeneous Sauter-pulse $\longrightarrow$ analytical solution of DHW-equations
- Final yield: $N = \int \frac{dp}{2\pi} f(p)$
- Maximum of $f(p)$ is at $p = \varepsilon\tau$
- For short τ, f_{max} is proportional to the squared Fourier amplitude at $\omega = 2m_e$

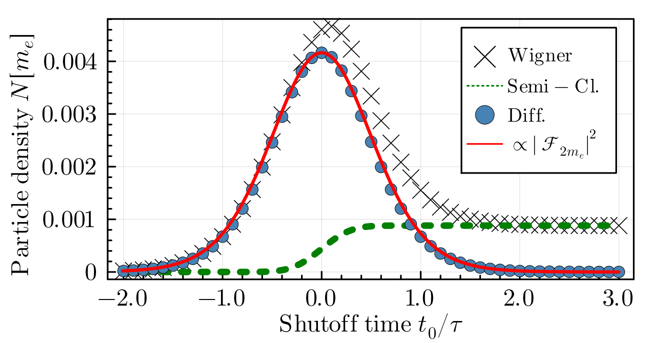




Parameters: $\varepsilon = 0.3, \quad \tau = \frac{30}{m}$

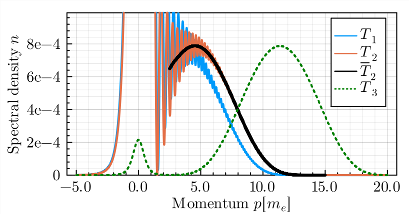
Time dependence of adiabatic particle number

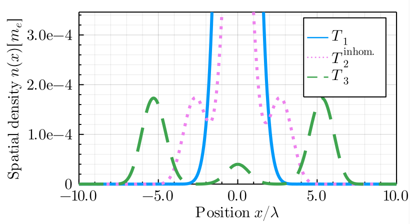
- Instantaneous switch-off of E causes the pair production rate $\dot{f}$ to immediately vanish
- The particle number at time t_0 is then the number of pairs produced if the pulse is shut-off exactly at t_0 A. Ilderton, Phys. Rev. D **105**, 016021 (2022)
- We assume that the excess is associated with multi-photon, perturbative production as it vanishes for $t_0 \to \infty$



Time scales

- We looked at the time evolution of particle spectra in momentum space
- We identified 3 time scales:
  1. $T_1 \approx 0.65\,\tau^{1/4}\,t_C^{3/4}/\varepsilon^2$, Appearance of a Side-peak
  2. $T_2 = T_1 + 0.06\tau^{3/4}t_C^{1/4}/\varepsilon^{3/2}$ side-peak follows a classical trajectory
  3. $T_3 = 1.8\tau$ at which quantum fluctuations fade out.





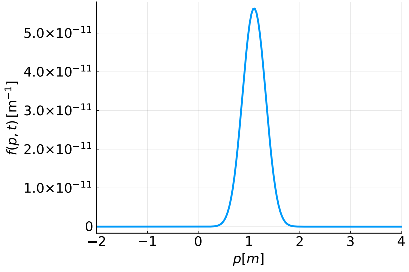
[MD, R Alkofer, C Kohlfürst, Identifying time scales in particle production from fields, Physics Letters B, Volume 844, 2023,]

Dynamically assisted Sauter-Schwinger effect

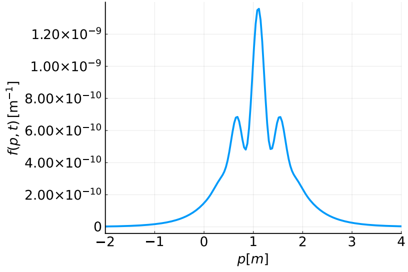
- Combination of a strong long pulse and a short weak pulse: [Ralf Schützhold, Holger Gies, and Gerald Dunne, Phys. Rev. Lett. 101, 130404 (2008).]

$$E(t) = E_{\text{crit}} \left[\varepsilon_1 \frac{1}{cosh^2(t/\tau_1)} + \varepsilon_2 \frac{1}{cosh^2(t/\tau_2)} \right]$$

- For the right choice of parameters, this can lead to a significant increase in particle production, well below the Schwinger limit.



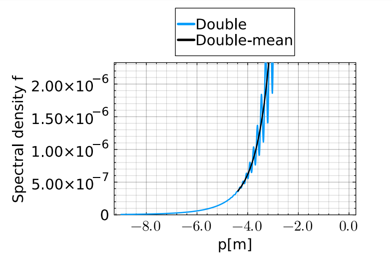
$\varepsilon_1 = 0.11,\ \tau_1 = 10m^{-1}$



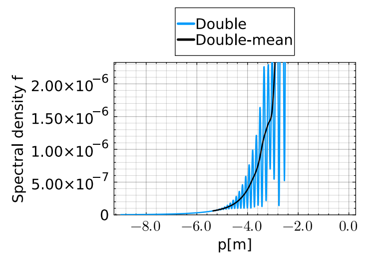
$\varepsilon_2 = \frac{1}{100}\varepsilon_1,\ \tau_2 = \frac{1}{50}\tau_1$

Time evolution of double Sauter pulse

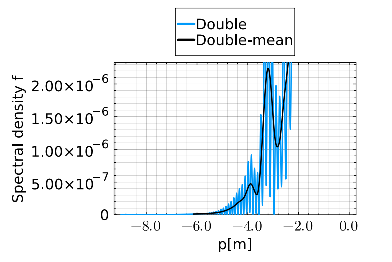
$t = 0.51 \cdot \tau$



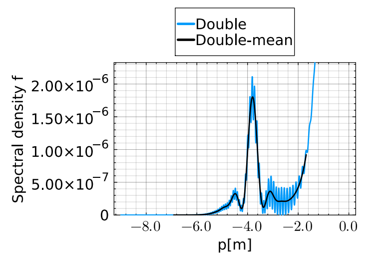
$t = 0.79\tau$



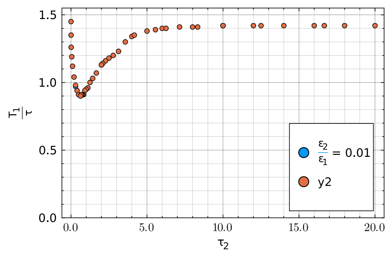
$t = 1.1 \cdot \tau$

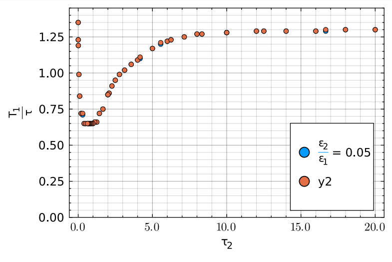


$t = 1.81\tau$

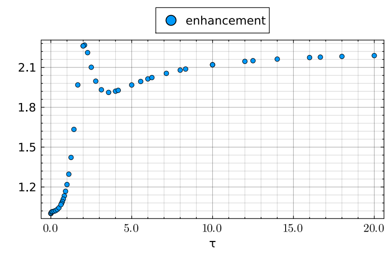


T_1 for double Sauter pulse



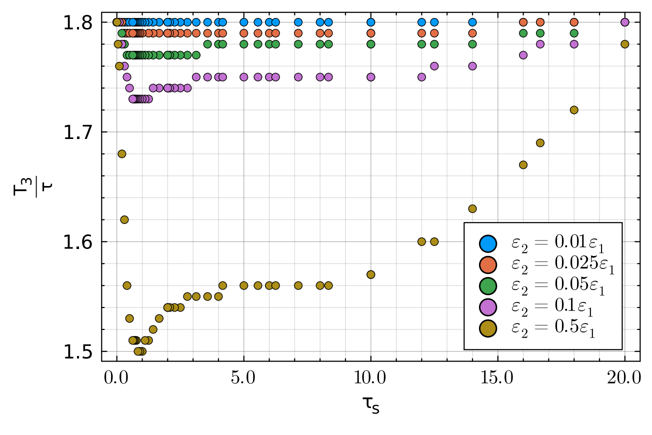


$\varepsilon_1 = 0.2$, $\tau_1 = \frac{20}{m}$



- Reduction of T_1 in DA
- Single pulse limit for:
  1. $\tau_2 \to \tau_1$
  2. $\tau_2 \to 0$
- At minimum of T_1 short pulse dominates pair production

T_3 for the double Sauter pulse



- Single pulse limit for $\tau_2 \to 0$ and $\tau_2 \to \tau_1$
- Maximum size of central peak scales with $(\varepsilon_1 + \varepsilon_2)^2$
- Asymptotic size of central peak scales for $\tau_s < \tau_1$ with ε_1^2

Inhomogeneous Fields

Why do we look at spatially inhomogeneous fields?

- Phase space information
- How do spatial inhomogeneities influence pair production:
    - Time scales
    - Spatial separation
    - Correlation between temporal and spatial structures

Electric field model:

$$E_{inh}(x,t) = \varepsilon E_{crit} \frac{1}{\cosh(\frac{t}{\tau})^2} \exp\left(-\frac{x^2}{2\lambda^2}\right)$$

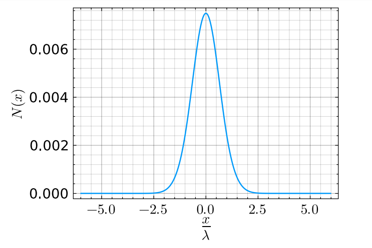
λ... pulse width

τ... pulse length

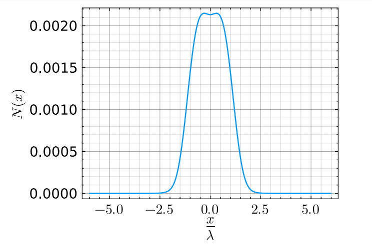
ε... field strength parameter

Reduced density $N(x,t) = \int \frac{dx}{2\pi} N(x,p,t)$

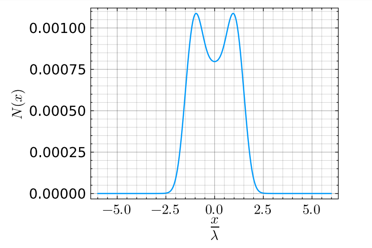
$t = 0 \cdot \tau$



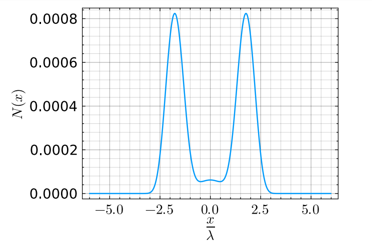
$t = 0.8 \cdot \tau$



$t = 1.1 \cdot \tau$

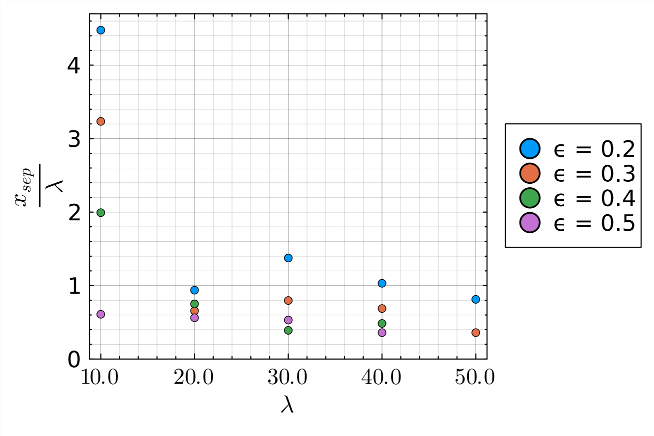


$t = 1.8\tau$



Spatial separation distance

x_{sep}...distance between two peaks in position space at their appearance



Conclusion & Outlook

1. The quantum distribution function contains sensible physical information even at finite, pre-asymptotic times
2. At least three time-scales identified:
   - Maximum appearance : $T_1 \approx 0.65\,\tau^{1/4}\,t_C^{3/4}/\varepsilon^2$
   - Classical trajectory: $T_2 \approx T_1 + 0.06\tau^{3/4}t_C^{1/4}/\varepsilon^{3/2}$
   - Fading out of quantum fluctuations : $T_3 \approx 1.8\tau$
3. Spatial scales identified

Currently under investigation:

- Study of combinations of Sauter pulses
- Spatial and temporal correlations of emerging scales
- Time evolution in phase space