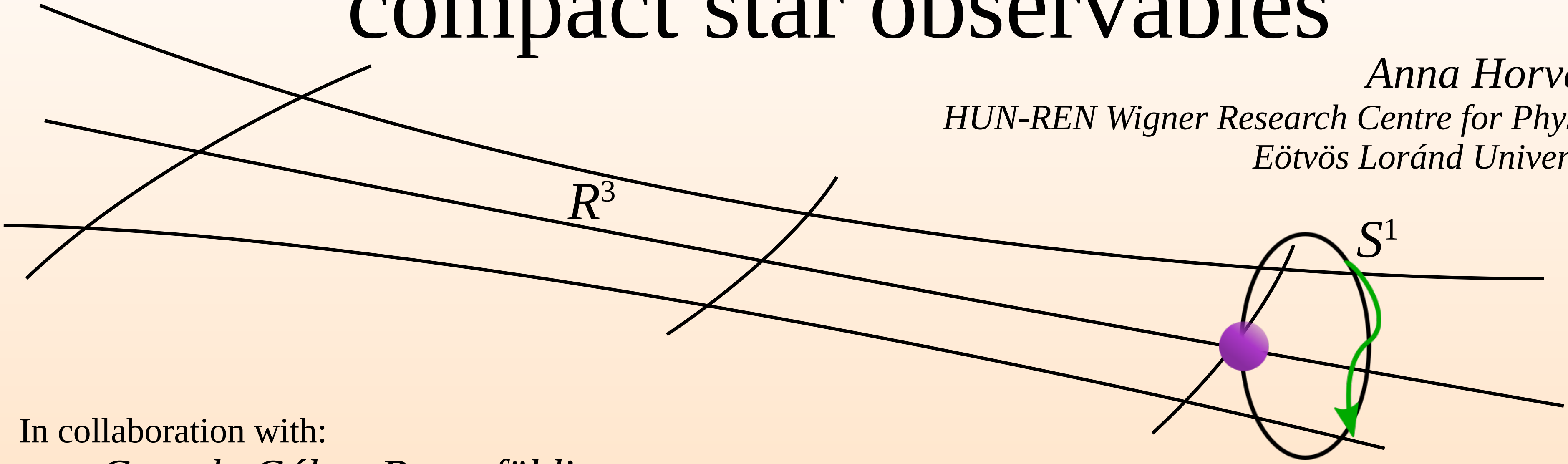
The effect of extra dimensions on compact star observables

Anna Horváth
HUN-REN Wigner Research Centre for Physics
Eötvös Loránd University



In collaboration with:
Gergely Gábor Barnaföldi
HUN-REN Wigner Research Centre for Physics
Emese Forgács-Dajka
Eötvös Loránd University

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ELTE DKÖP
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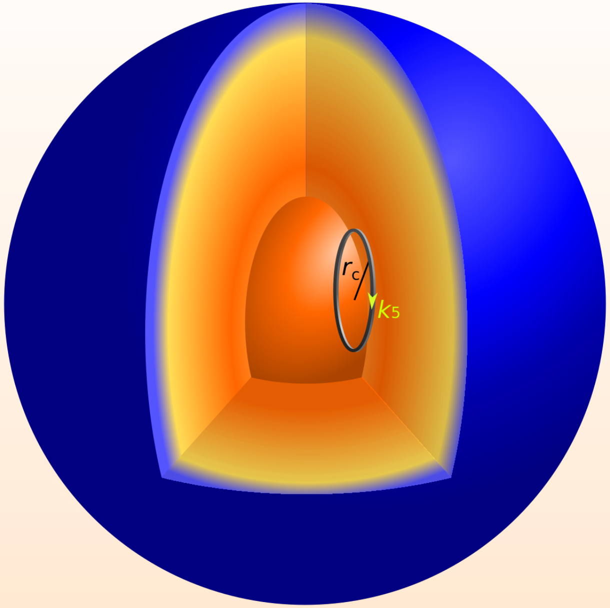
KMP-2023/101, KMP-2024/31

Outline of the talk



- Motivation
- Kaluza–Klein theory
- Testing possibilities
- Motivation
- Model
- Microscopic properties and thermodynamics
- Modeling neutron stars
- Results

[1] A. Horváth, E. Forgács-Dajka, G.G. Barnaföldi: "Application of Kaluza-Klein Theory in Modeling Compact Stars: Exploring Extra Dimensions", MNRAS (2024)
[2] A. Horváth, E. Forgács-Dajka, G.G. Barnaföldi: "The effect of multiple extra dimensions on the maximal mass of compact stars in Kaluza-Klein space-time", Accepted by IJMPA (2025)
[3] A. Horváth, E. Forgács-Dajka, G.G. Barnaföldi: “Speed of sound in Kaluza-Klein Fermi gas”, Sent to APP-B (2025)

Motivation I.

- GR and QM are not compatible
- Hierarchy problem
- Dark matter, dark energy?

↓

Search for new physics

- Many theories exist
- Phenomenological predictions often overlap

↓

Study a simple model in order to make testable predictions

Kaluza–Klein theory

1921 Theodor Kaluza

- Unify gravity with electromagnetism in a 5D spacetime
- Cylinder condition – no metric component depends on the fifth dimension

1926 Oskar Klein

- The fifth dimension is a microscopic one, curled up into a circle

↓

QM interpretation

Base of: scalar-tensor (Brans-Dicke) and string theories

Theodor Kaluza (1885-1954)

Oskar Klein (1894-1977)

Metric tensor

15 independent components: 10 + 4 + 1 extra

Usual 4D metric describing gravity

$$g_{AB} = \begin{bmatrix} g_{\alpha\beta} + \kappa^2\Phi^2 A_\alpha A_\beta & \kappa\Phi^2 A_\alpha \\ \kappa\Phi^2 A_\beta & \Phi^2 \end{bmatrix}$$

Scalar field

5D metric

Vector potential of electromagnetism

Overduin, J. M., & Wesson, P. S. 1997, Phys. Rept., 283, 303, doi: arXiv:gr-qc/9805018

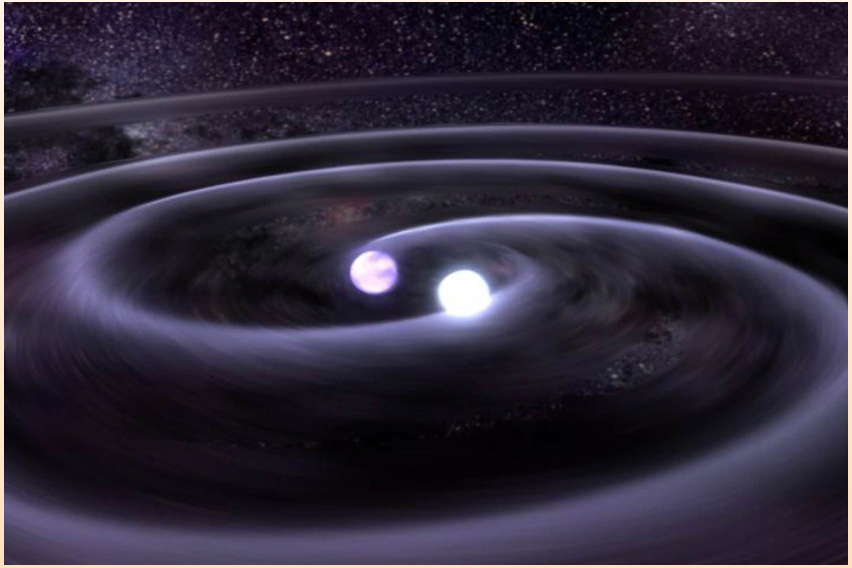
Testing possibilities

In general:

- Microscopic extra dimension ⟶ high energy needed
- Theory of gravity ⟶ strong gravitational field

In particular:

- Collider experiments
- Tabletop experiments
- Astrophysical observations
  - Neutron stars, black holes
  - Multi-messenger astronomy



https://www.quantamagazine.org/for-astronomers-a-neutron-star-merger-could-eclipse-the-eclipse-20170825/

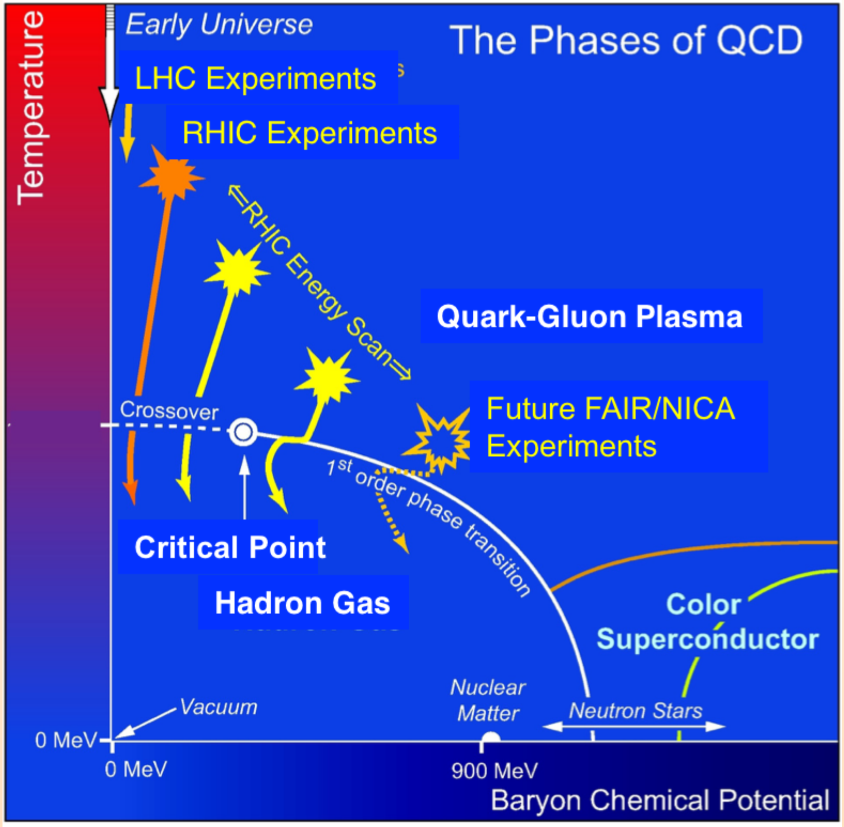
Motivation II.

The equation of state of cold dense nuclear matter is

- not well described by perturbative QCD or nuclear physics
- cannot be produced in laboratories

In particular:

- Collider experiments
- Tabletop experiments
- Astrophysical observations
  - Neutron stars, black holes
  - Multi-messenger astronomy

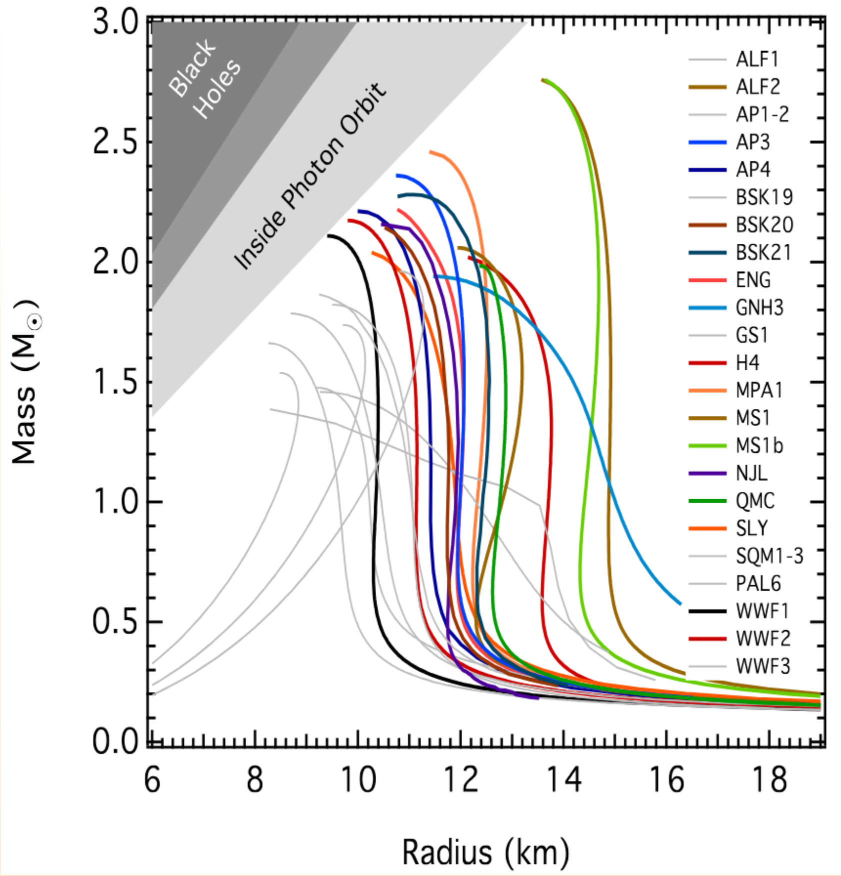


Tapan K. Nayak 2020 J. Phys.: Conf. Ser. 1602 012003

Model

Useful quantities for testing the EoS:

- Mass
- Radius
- Tidal deformability (not considered)



F. Özel and P. Freire, "Masses, Radii, and the Equation of State of Neutron Stars," Ann. Rev. Astron. Astrophys. **54** (2016), 401-440 doi:10.1146/annurev-astro-081915-023322 [arXiv:1603.02698 [astro-ph.HE]].

Model

Useful quantities for testing the EoS:

- Mass

Green Bank
West Virginia

Arecibo
Puerto Rico

CHIME
Canada

MeerKAT (SKA)
South Africa

Keck
Hawaii

LIGO
USA

Model

Useful quantities for testing the EoS:

- Mass

Constellation	Name	Mass, $[M_\odot]$	Detection device	Reference
Scorpius	PSR J1614−2230	$1.908^{+0.0016}_{-0.0016}$	Green Bank	Arzoumanian et al. (2018)
Scorpius	PSR J1614−2230	$1.97^{+0.04}_{-0.04}$	Green Bank	Demorest et al. (2010)
Taurus	PSR J0348+0432	$2.01^{+0.04}_{-0.04}$	Arecibo, Effelsberg, Green Bank	Antoniadis et al. (2013)
Camelopardalis	PSR J0740+6620	$2.08^{+0.07}_{-0.07}$	CHIME, Green Bank	Fonseca et al. (2021)
Hercules	PSR J1810+1744	$2.13^{+0.04}_{-0.04}$	Keck I	Romani et al. (2021)
Lacerta	PSR J2215+5135	$2.27^{+0.17}_{-0.15}$	IAC-80, William Herschel	Linares et al. (2018)
Columba	PSR J0514−4002E	$2.35^{+0.20}_{-0.18}$	MeerKAT	Barr et al. (2024)
Sextans	PSR J0952−0607	$2.35^{+0.17}_{-0.17}$	Keck I	Romani et al. (2022)
Sculptor	GW190814	$2.59^{+0.08}_{-0.09}$	LIGO/VIRGO	Abbott et al. (2020)

Model

Useful quantities for testing the EoS:

- Mass
- Radius
- Tidal deformability (not considered)

Assumptions:

- Static, spherically symmetric spacetime
- Neglect electromagnetism
- Isotropic, relativistic ideal fluid
- g_{55}:
  - set to 1
  - Allowed to vary (scalar field)

$$g_{\mu\nu} = \begin{bmatrix} g_{00} & \cancel{g_{01}} & 0 & 0 & \cancel{g_{05}} \\ \cancel{g_{01}} & g_{11} & 0 & 0 & \cancel{g_{15}} \\ 0 & 0 & g_{22} & 0 & 0 \\ 0 & 0 & 0 & g_{22}\sin^2\vartheta & 0 \\ \cancel{g_{05}} & \cancel{g_{15}} & 0 & 0 & \boxed{g_{55}} \end{bmatrix}$$

Microscopic to macroscopic

Microscopic properties of spacetime and nuclear matter are connected to the mass-radius relation via

- the Tolman–Oppenheimer–Volkoff (TOV) equation

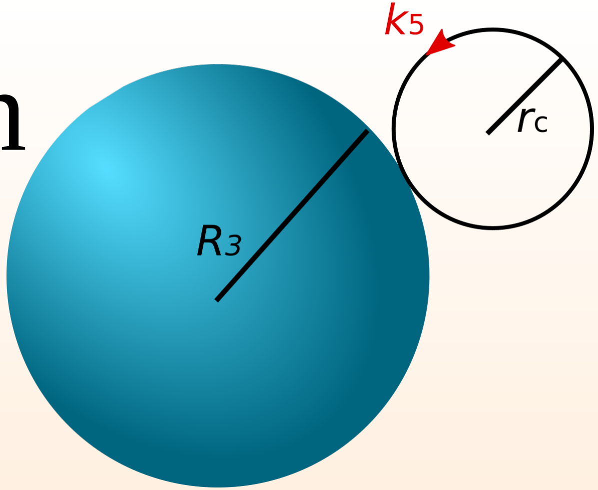
$$\frac{\mathrm{d}p(r)}{\mathrm{d}r} = -\frac{GM(r)\varepsilon(r)}{r^2} \times \left[1 + \frac{p(r)}{\varepsilon(r)}\right]\left[1 + \frac{4\pi r^3 p(r)}{M(r)}\right]\left[1 - \frac{GM(r)}{r}\right]^{-1}$$

$$M(r) = \int_0^r \mathrm{d}r' 4\pi r'^2 \varepsilon(r')$$

  Boundary conditions: central energy density ε_c, surface pressure $p(R)$

- and the equation of state (EoS)
  - Connects pressure (p) and energy density (ε)
  - Encodes microscopic properties of the theory

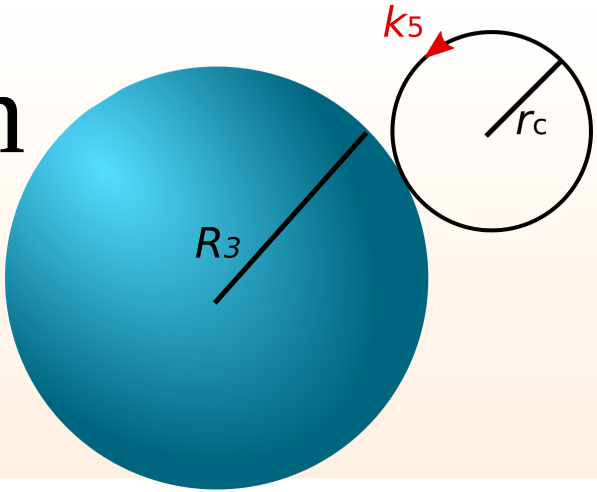
Modified dispersion relation



Particles moving in the extra dimension possess a modified effective mass from a 3-dimensional point of view.

$$E = \sqrt{\mathbf{k}^2 + k_5^2 + m^2} = \sqrt{\mathbf{k}^2 + \left(\frac{N_{\text{exc}}}{r_c}\right)^2 + m^2}$$

Modified dispersion relation



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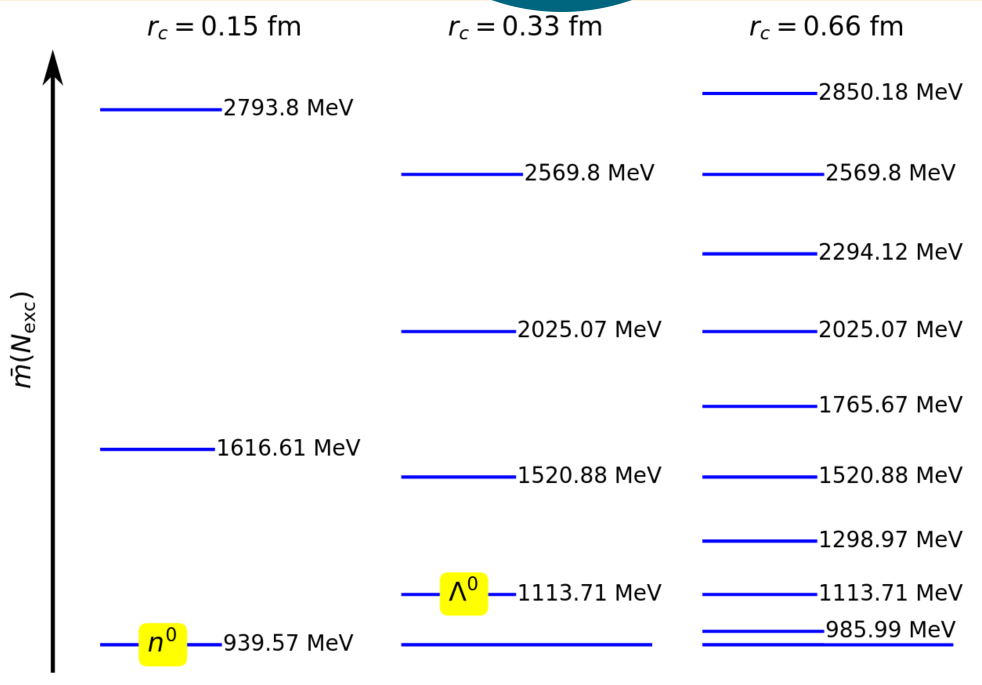
$$E = \sqrt{\mathbf{k}^2 + k_5^2 + m^2} = \sqrt{\mathbf{k}^2 + \left(\frac{N_{\text{exc}}}{r_c}\right)^2 + m^2}$$

$$\bar{m}^2(N_{\text{exc}}) = m^2 + k_5^2 \qquad k_5 = \frac{N_{\text{exc}}}{r_c}$$



Here the ground state of the Kaluza–
–Klein ladder is the neutron.

Equation of state

Thermodynamic potential of a Fermi gas:

$$\Omega = -V_{(d)} \sum_{i=0}^{N_{\text{exc}}} \frac{g_i}{\beta} \int \frac{\mathrm{d}^d \mathbf{k}}{(2\pi)^d} \times$$
$$\times \left[\ln\left(1 + \mathrm{e}^{-\beta(E_i-\mu)}\right) + \ln\left(1 + \mathrm{e}^{-\beta(E_i+\mu)}\right) \right]$$

Zero temperature approximation:

$$T \ln\left(1 + \mathrm{e}^{-(E-\mu)/T}\right)\Big|_{T=0} = \begin{cases} \mu - E, & \text{if } E < \mu \\ 0, & \text{if } E \geq \mu \end{cases}$$

Repulsive potential:

$$U(n) = \xi n$$

J. Zimanyi, B. Lukacs, P. Levai, J.P. Bondorf: „An Interpretable Family ofEquation of State for Dense Hadronic Matter", Nucl.Phys. A484 (1988) 647

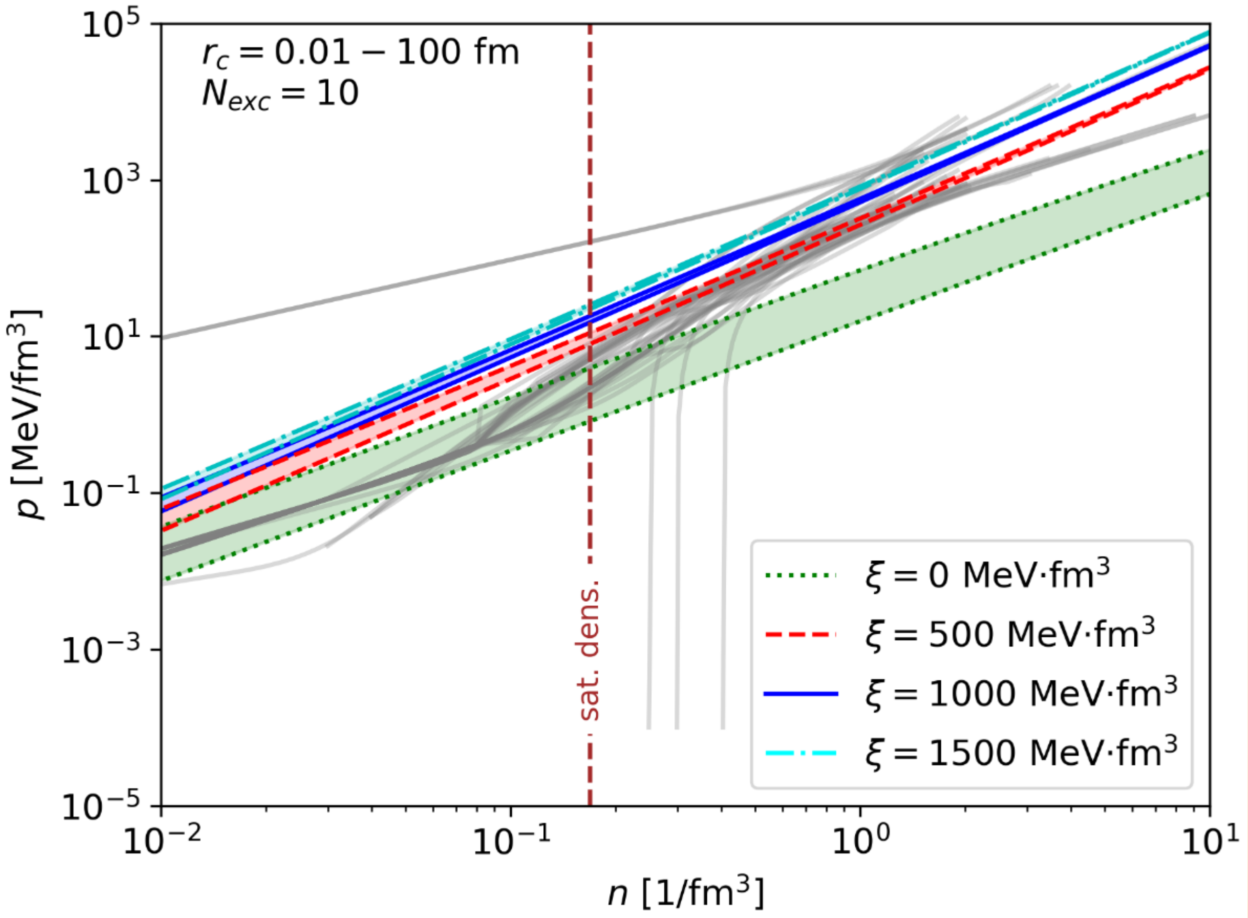
State variables:

$$p(\mu) = p_0(\bar{\mu}) + p_{\text{int}}$$
$$\varepsilon(\mu) = \varepsilon_0(\bar{\mu}) + \varepsilon_{\text{int}}$$

$$\bar{\mu} = \mu - U(n)$$

$$p_{int} = \varepsilon_{int} = \int U(n) dn = \frac{1}{2}\xi n^2$$

Equation of state

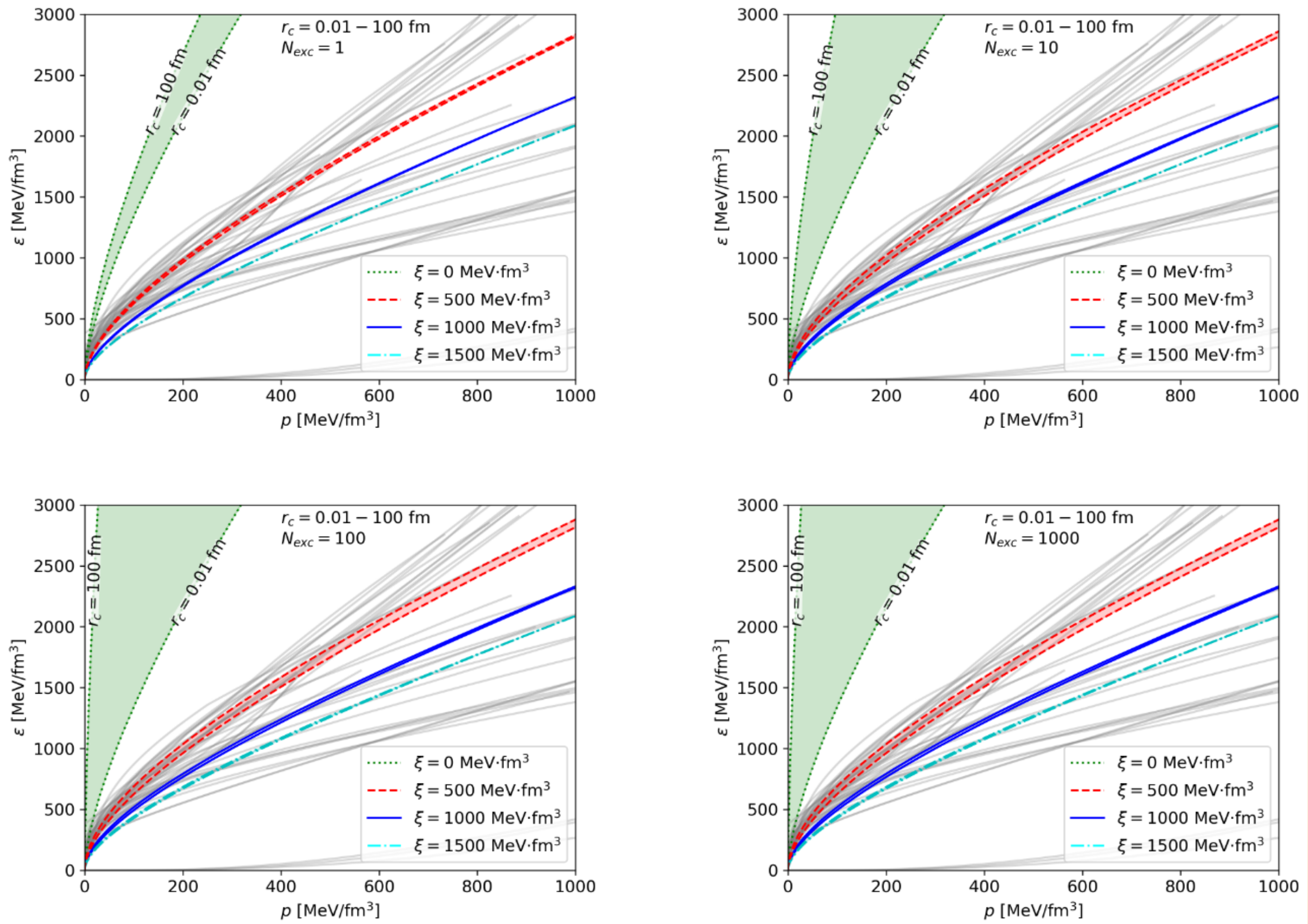


Pressure as a function of baryon number density.

F. Özel and P. Freire, "Masses, Radii, and the Equation of State of Neutron Stars," Ann. Rev. Astron. Astrophys. **54** (2016), 401-440 doi:10.1146/annurev-astro-081915-023322 [arXiv:1603.02698 [astro-ph.HE]].

https://compose.obspm.fr/

Equation of state

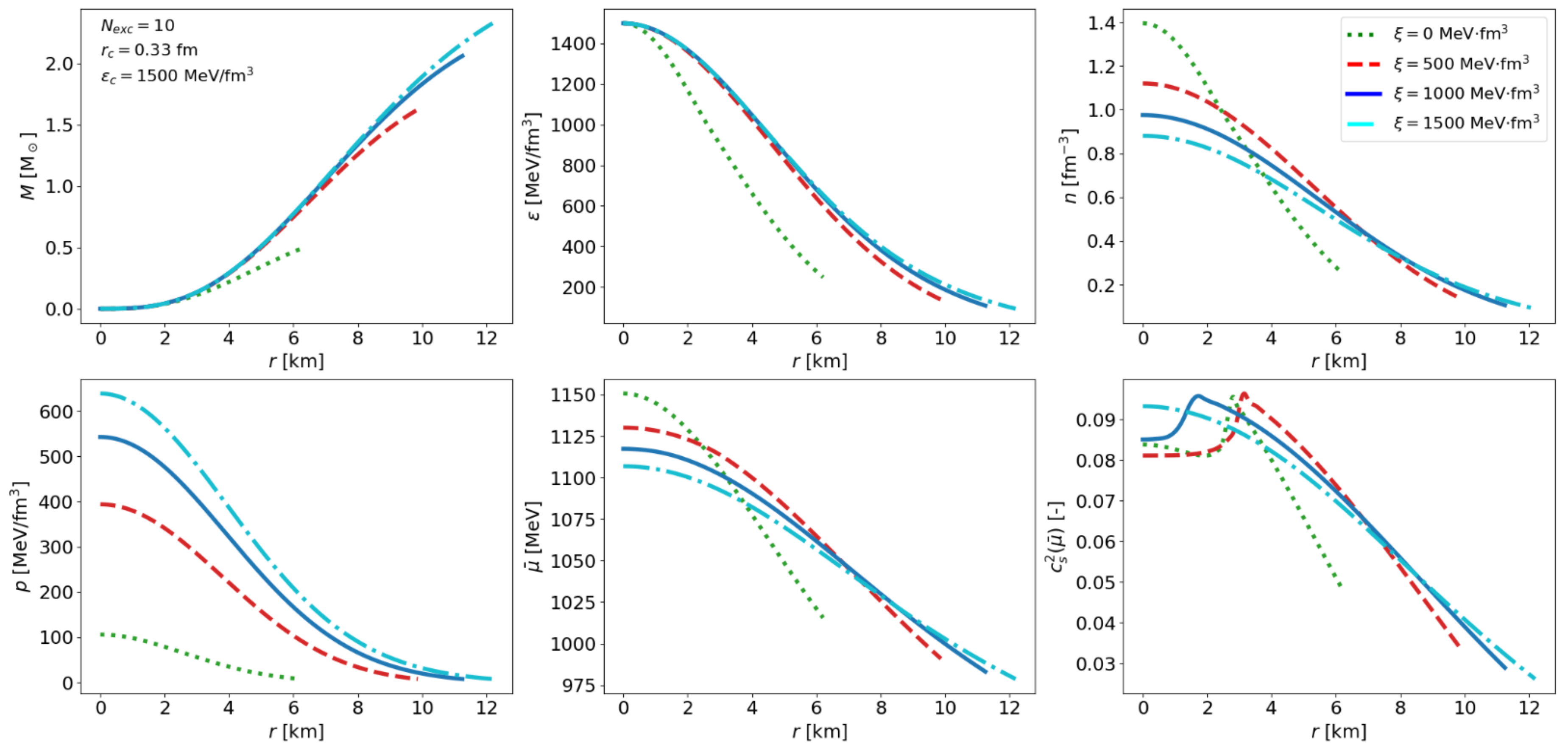


Energy density as a function of pressure.

F. Özel and P. Freire, "Masses, Radii, and the Equation of State of Neutron Stars," Ann. Rev. Astron. Astrophys. **54** (2016), 401-440 doi:10.1146/annurev-astro-081915-023322 [arXiv:1603.02698 [astro-ph.HE]].

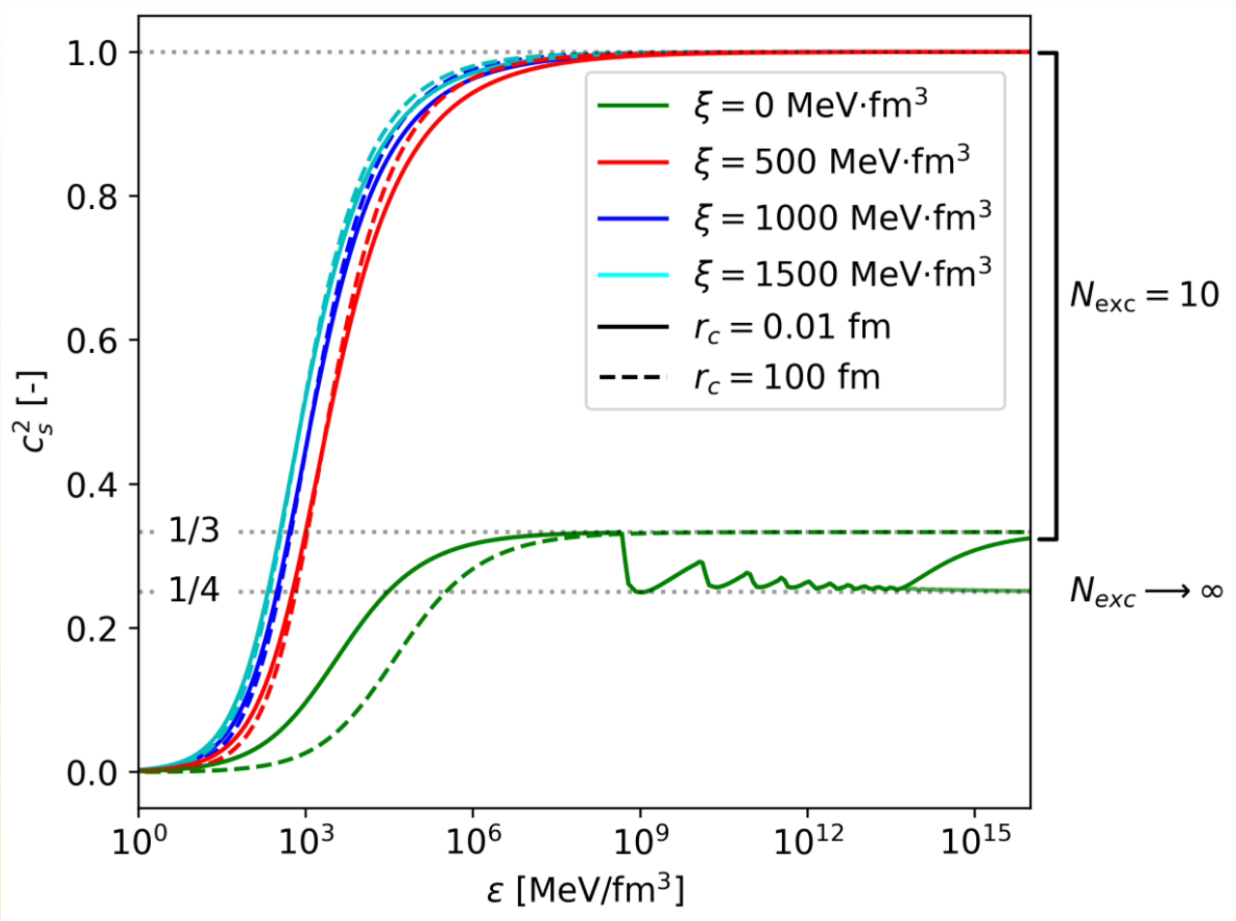
https://compose.obspm.fr/

Solving the TOV equation – stars

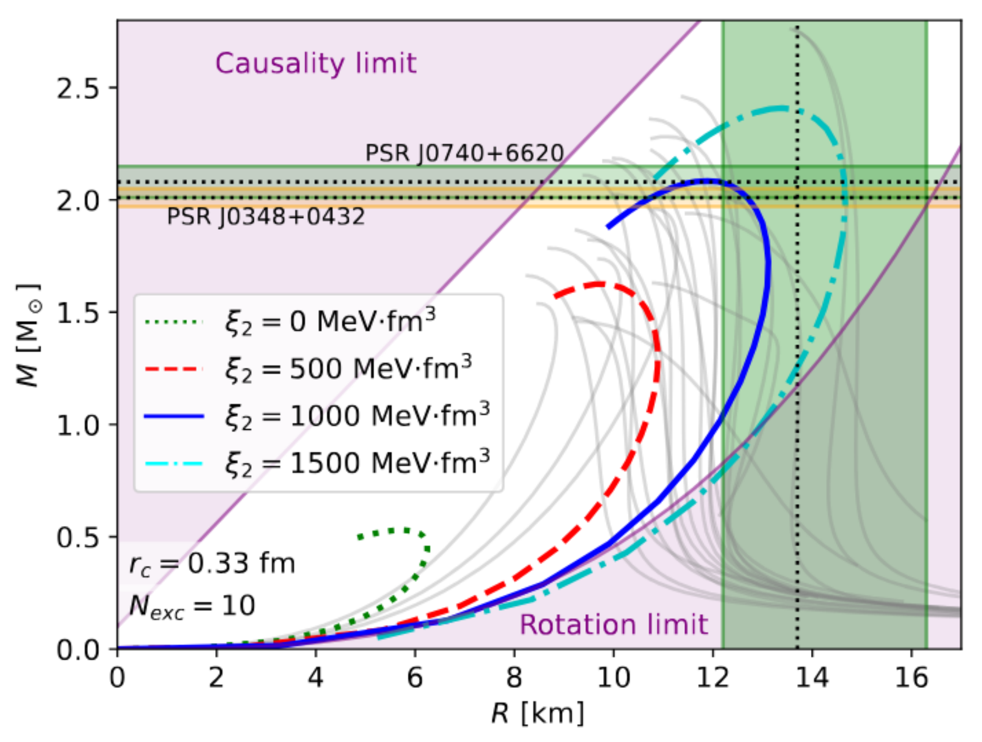


r: radius of star

Speed of sound

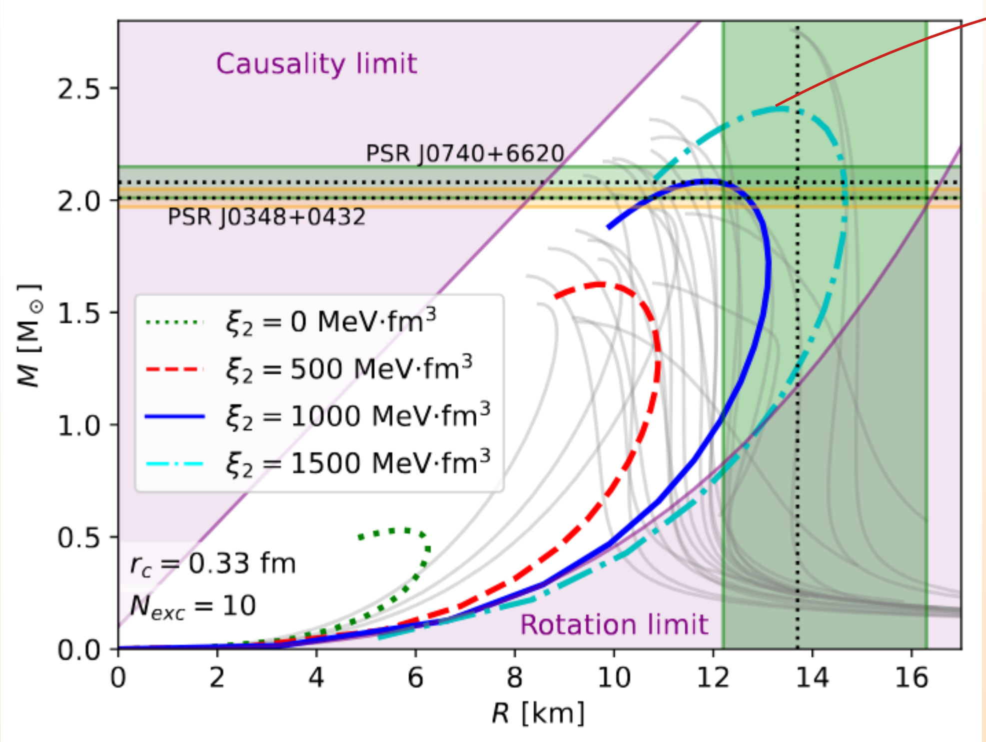


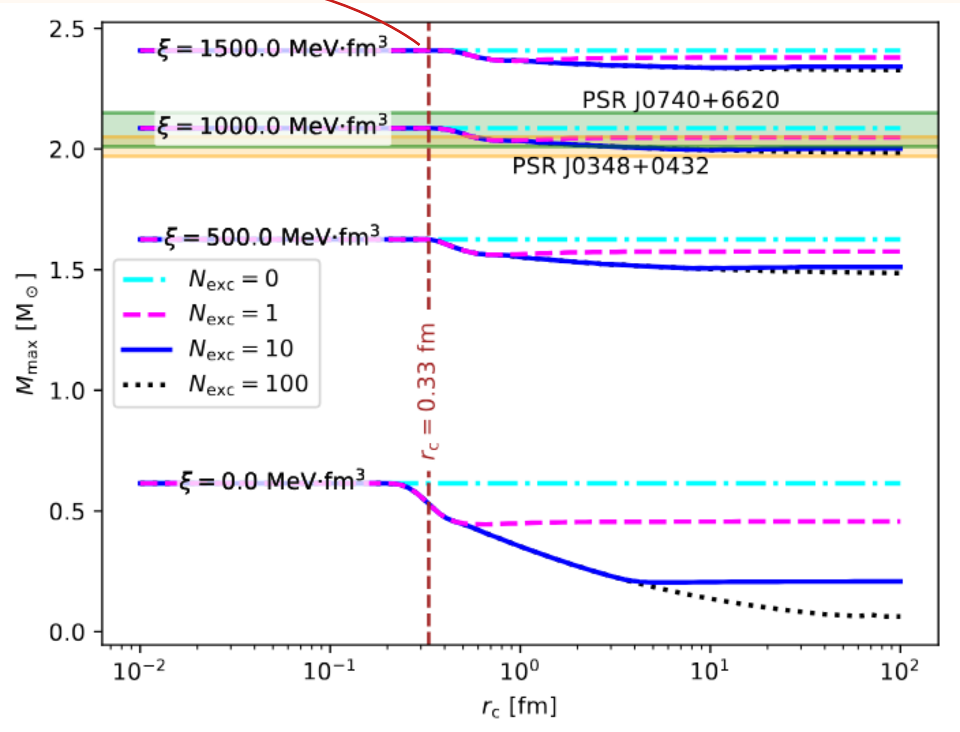
Comparison to observation



Fonseca E., et al., 2021, The Astrophysical Journal Letters, 915, L12
Miller M. C., et al., 2021, Astrophys. J. Lett., 918, L28
Antoniadis J., et al., 2013, Science, 340, 6131

Comparison to observation





Fonseca E., et al., 2021, The Astrophysical Journal Letters, 915, L12
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Extra-dimensional theories

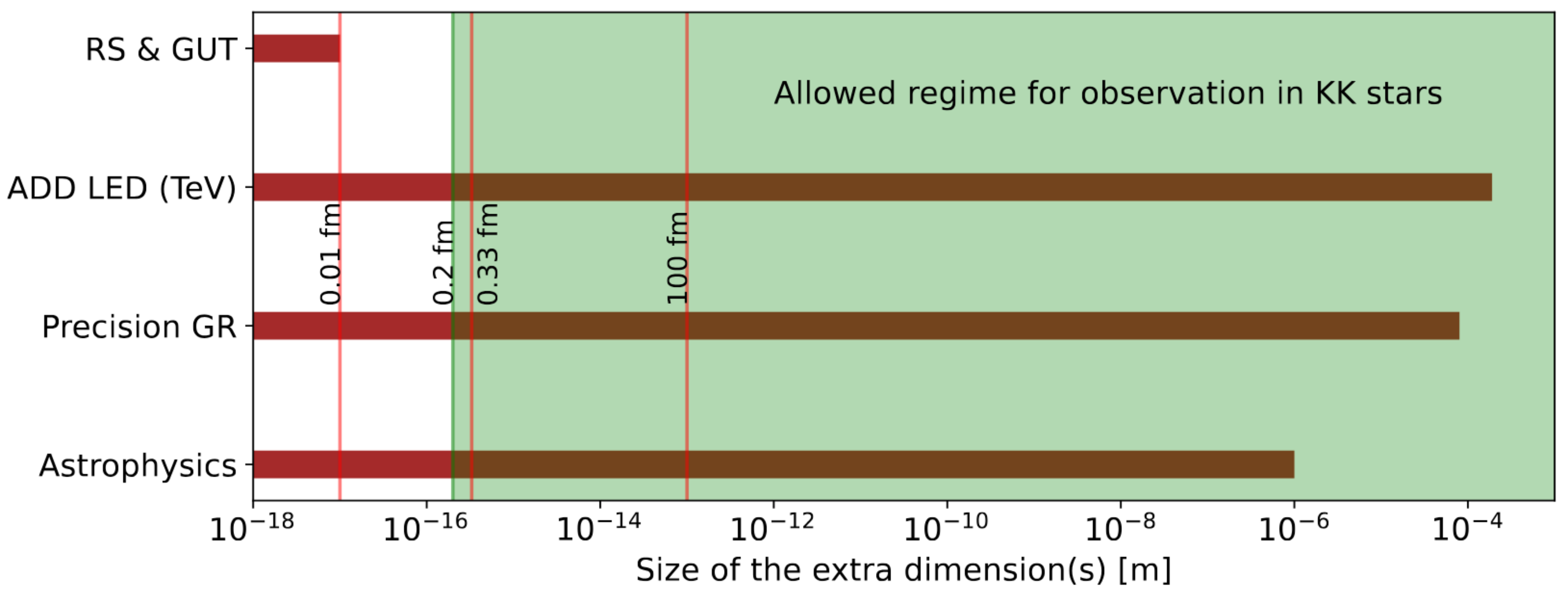
Observations regarding neutron stars can be relevant for constraining multiple beyond standard model extra-dimensional theories.

• Randall–Sundrum, GUT	$r_c < 10^{-17}$ m
• ADD large extra dimensions (TeV scale)	$r_c < 1.9\times10^{-4}$ m
• Precision measurements (tabletop)	$r_c < 8.0\times10^{-5}$ m
• Astrophysics – gravitational waves	$r_c < 10^{-6}$ m

Randall L., Sundrum R., 1999, Phys. Rev. Lett., 83, 3370
Cheung K., Landsberg G., 2002, Physical Review D, 65
Bernardi G., 2003. https://api.semanticscholar.org/CorpusID: 121772649
Abdallah J., et al., 2009, Eur. Phys. J. C, 60, 17
Adelberger E. G., Gundlach J. H., Heckel B. R., Hoedl S., Schlamminger S., 2009, Prog. Part. Nucl. Phys., 62, 102
Eötvös R. V., Pekár D., Fekete E., 1922, Annalen Phys., 68, 11
Péter G., Deák L., Gróf G., Kiss B., Szondy G., Tóth G., Ván P., Völgyesi L., 2022, Repeating the Eötvös-Pekár-Fekete equivalence principle measure-ments (arXiv:2205.14587), https://arxiv.org/abs/2205.14587
Murata J., Tanaka S., 2015, Class. Quant. Grav., 32, 033001
Abbott R., et al., 2021, Tests of General Relativity with GWTC-3 (arXiv:2112.06861)

Extra-dimensional theories

Observations regarding neutron stars can be relevant for constraining multiple beyond standard model extra-dimensional theories.



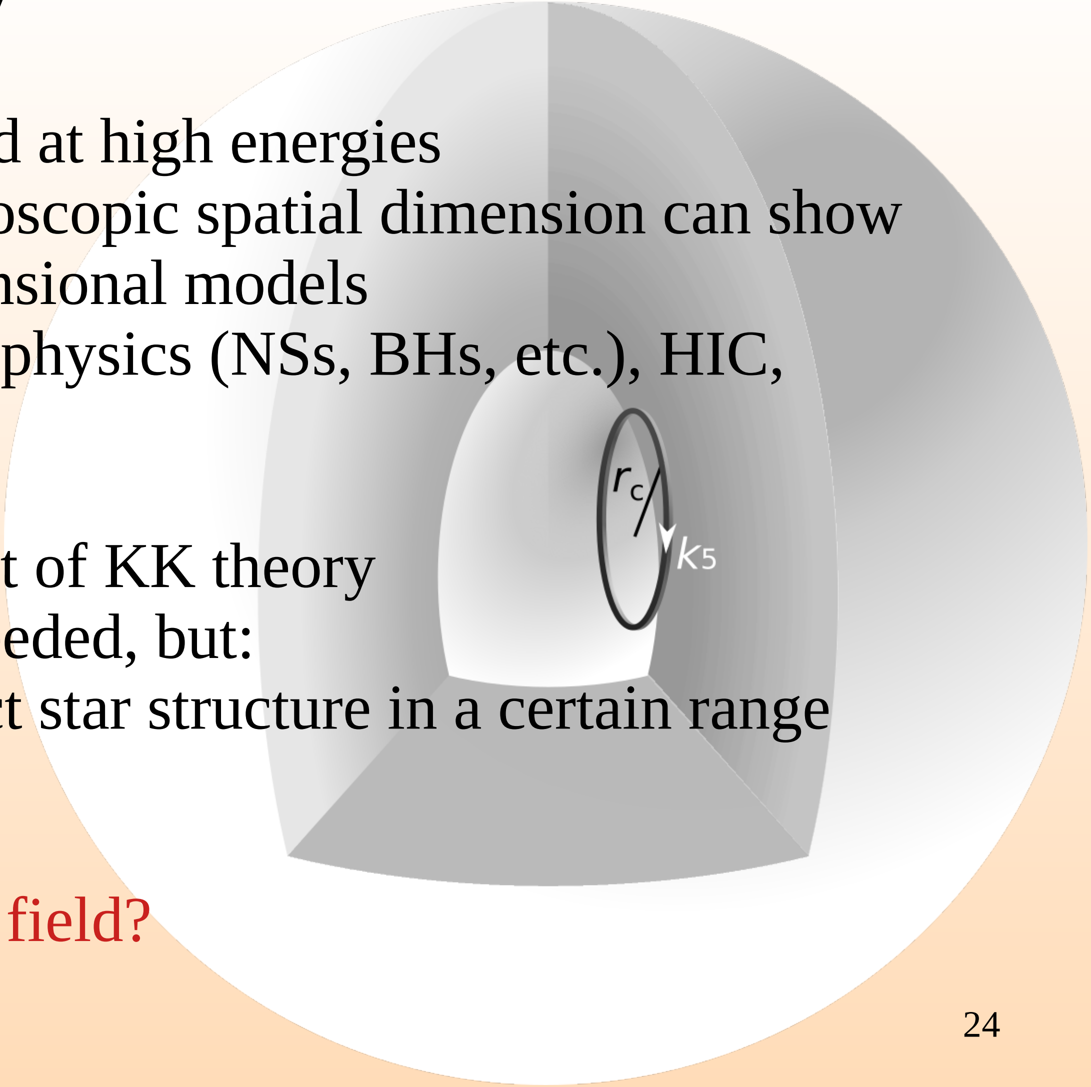
Summary

- New theories of physics are needed at high energies
- Kaluza–Klein with one extra microscopic spatial dimension can show the phenomenology of extra-dimensional models
- Observational possibilities in astrophysics (NSs, BHs, etc.), HIC, tabletop

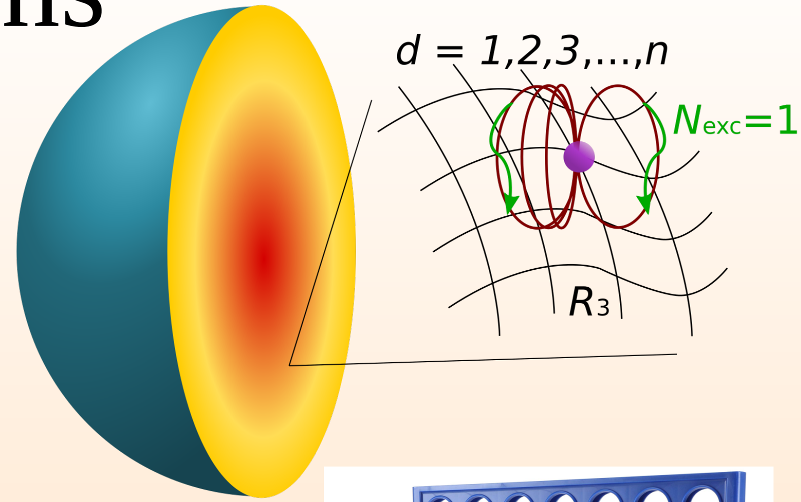
- Studied neutron stars in the context of KK theory
- More precise measurements are needed, but:
- extra dimensions do effect compact star structure in a certain range

$$r_c \gtrsim 0.2 \text{ fm}$$

- Multiple extra dimensions? Scalar field?



Multiple extra dimensions



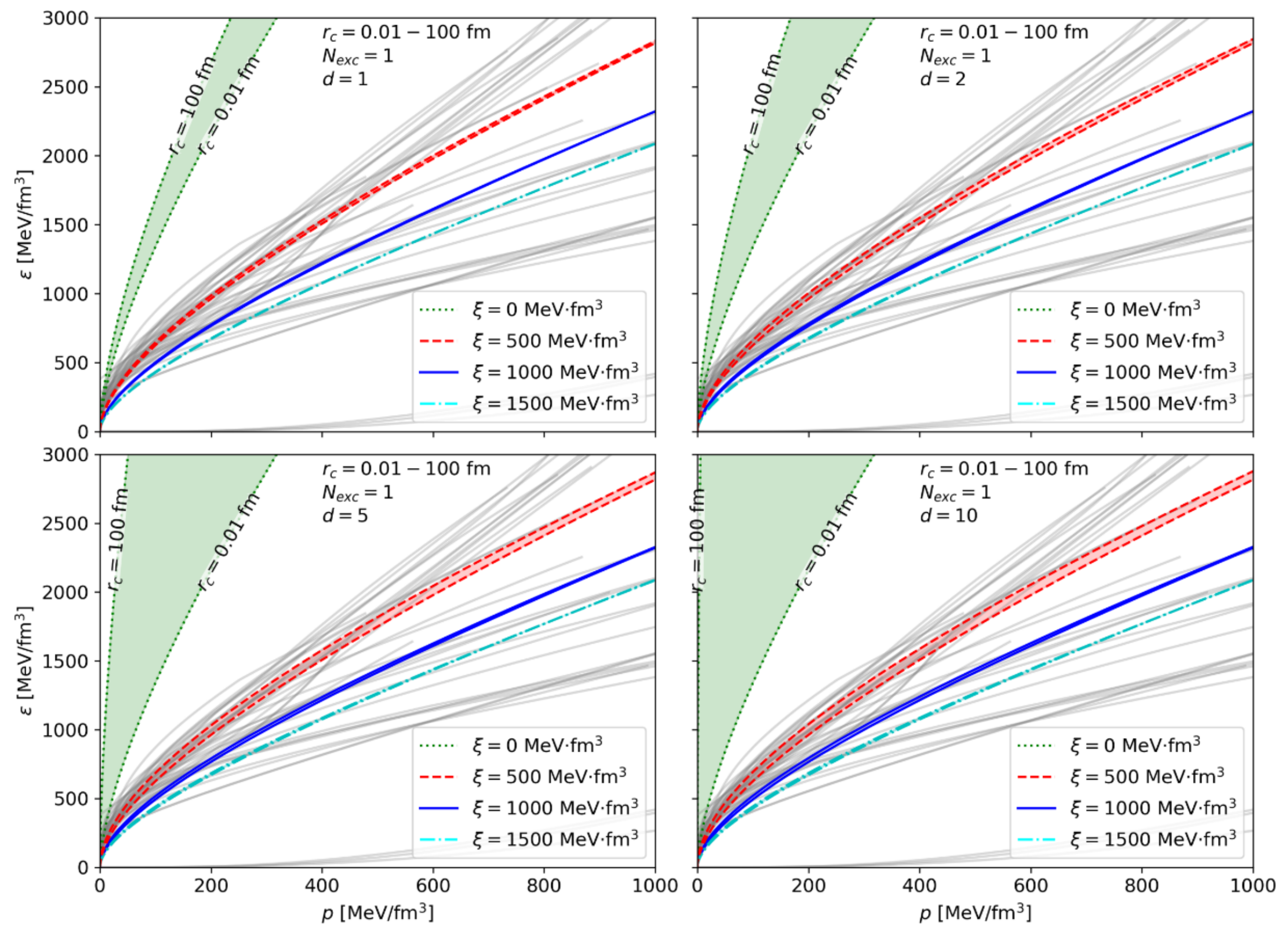
- All extra dimensions are the same size
- Particles are allowed to have excitations in all extra dimensions
- Effective mass:

$$\tilde{m}^2 = m^2 + \sum_{j=1}^{d} \tilde{k}_j^2$$

- Thermodynamic potential:

$$\Omega = -V_{(3+d)} \sum_i \underbrace{\sum_{j=0}^{N_{\mathrm{exc}}} \cdots \sum_{l=0}^{N_{\mathrm{exc}}}}_{d} \frac{g_i}{\beta} \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \times$$
$$\times \left[\ln\left(1 + \mathrm{e}^{-\beta(E_{ij\ldots l}-\mu)}\right) + \ln\left(1 + \mathrm{e}^{-\beta(E_{ij\ldots l}+\mu)}\right) \right]$$

Multiple extra dimensions



Modified phasespace

In collaboration with:
Aneta Wojnar

- Strong gravitational field could affect microscopic physics
- Relevance for the structure of NSs, BHs, WDs, planets?
- Modification to particle paths?
- Modified thermodynamics?

- Generalized uncertainty principle (GUP)

$$\Delta x \Delta p \geq \frac{\hbar}{2} + \beta \Delta p^2$$

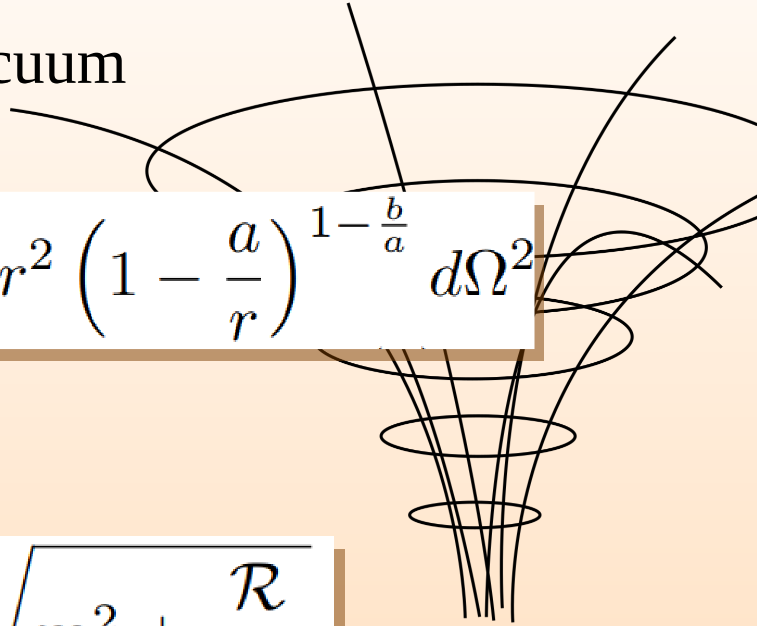
- In terms of the curvature

$$\sigma_p \rho \gtrsim \pi \hbar \left[1 - \frac{\rho^2 \mathcal{R}|_{p_0}}{12\pi^2} + \xi \frac{\rho^4}{\lambda_C^2} \nabla_j N_i \nabla^j N^i |_{p_0} \right]$$

L. Petruzziello and F. Wagner, Physical Review D 103, 104061 (2021).
M. P. Dabrowski and F. Wagner, The European Physical Journal C 80, 676 (2020).
Abdel Nasser Tawfik and Abdel Magied Diab 2015 Rep. Prog. Phys. 78 126001
Aleksander Kozak, Aneta Wojnar, “Earthquakes as probing tools for gravity theories”, 2023, arXiv:2308.01784

Schwarzschild-like solution

In collaboration with:
Aneta Wojnar



- g_{55} is allowed to vary ⟶ scalar field
- Non-zero energy-momentum tensor even for vacuum
- Schwarzschild-like metric

$$ds^2 = -\left(1-\frac{a}{r}\right)^{\frac{b}{a}} dt^2 + \left(1-\frac{a}{r}\right)^{-\frac{b}{a}} dr^2 + r^2\left(1-\frac{a}{r}\right)^{1-\frac{b}{a}} d\Omega^2$$

- Non-zero phase-space and spacetime curvatures
- Modified dispersion relation and effective mass

$$p^\mu p_\mu = -m^2c^2 - \frac{\mathcal{R}}{6}$$

$$m_{\text{eff}} = \sqrt{m^2 + \frac{\mathcal{R}}{6c^2}}$$

R. Coquereaux and G. Esposito-Farese, in Annales de l'IHP Physique th´eorique, Vol. 52 (1990) pp. 113–150
L. Petruzziello and F. Wagner, Physical Review D 103, 104061 (2021).
M. P. Dabrowski and F. Wagner, The European Physical Journal C 80, 676 (2020).

Speed of sound – modified potential