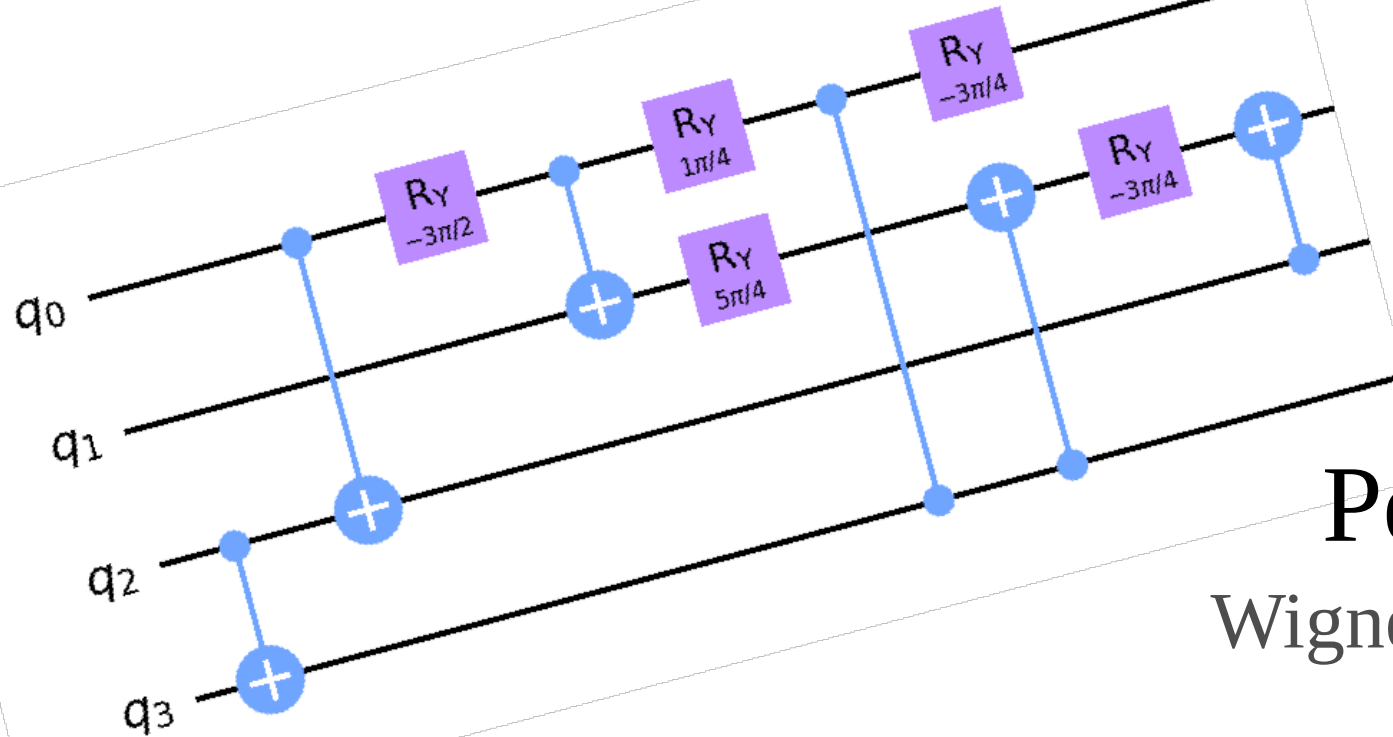


Batched Line Search Strategy for Navigating through Barren Plateaus in Quantum Circuit Training



Peter Rakyta
Wigner Research Centre
for Physics



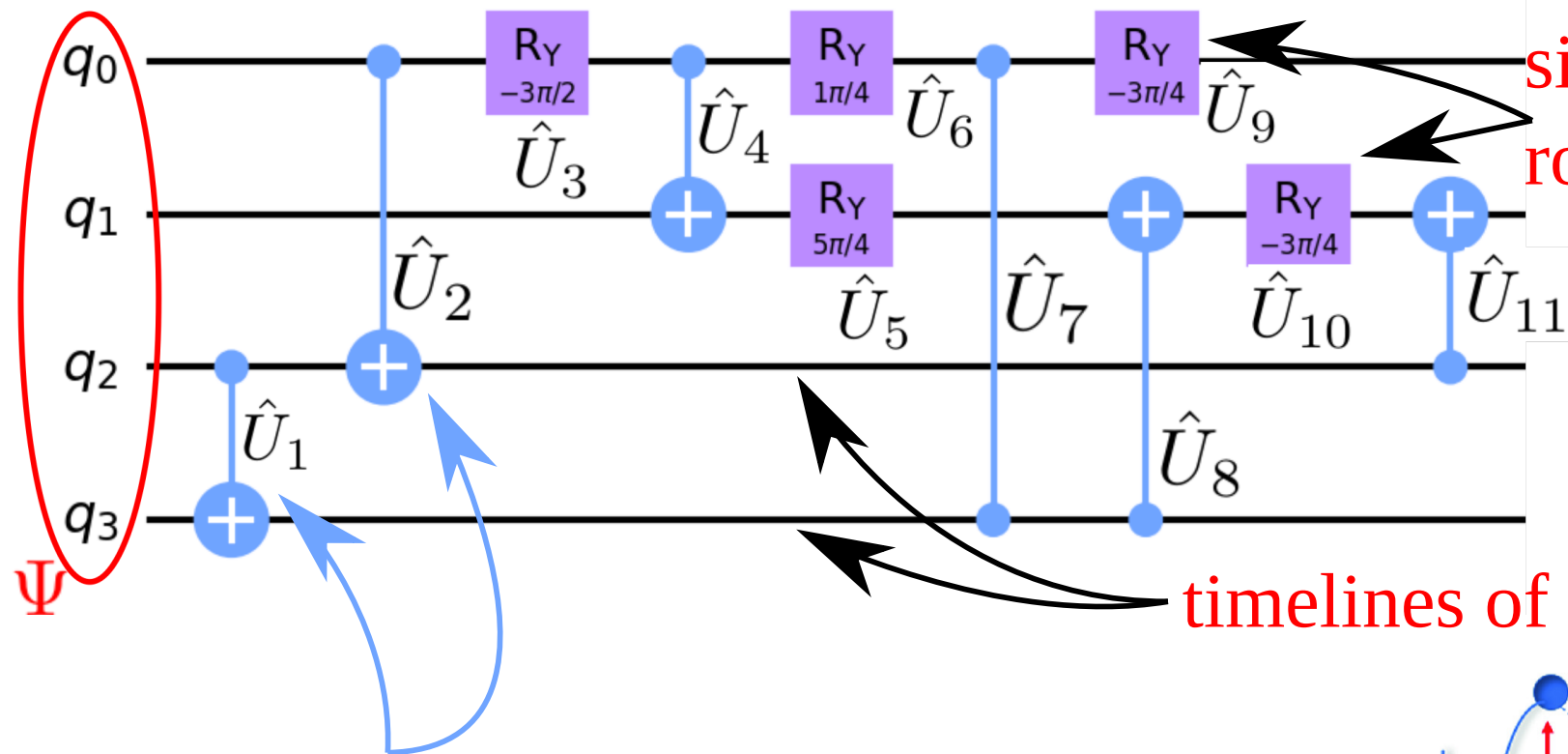
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Qubit based architecture

quantum program (unitary)

$$\hat{U} = \hat{U}_{11} \cdot \hat{U}_{10} \cdot \hat{U}_9 \cdot \hat{U}_8 \cdot \hat{U}_7 \cdot \hat{U}_6 \cdot \hat{U}_5 \cdot \hat{U}_4 \cdot \hat{U}_3 \cdot \hat{U}_2 \cdot \hat{U}_1$$



single qubit
rotations

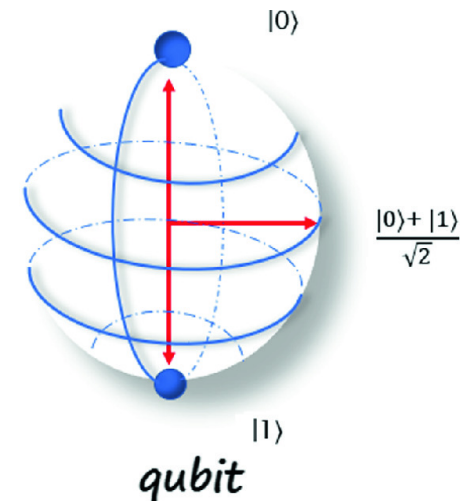
timelines of the qubits

controlled not gates

0

1

bit



qubit



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Software toolkits to train parametric quantum circuits

Gate decomposition utilities:

- Quantum Fast Approximate Synthesis Tool (QFAST)
- QSearch + LEAP

(Lawrence Berkeley National Laboratory)



QML utilities:



T|ket>: A Retargetable Compiler for NISQ Devices

(Cambridge Quantum Computing Ltd., University of Strathclyde)



Qulacs: a fast and versatile quantum circuit simulator for research purposes

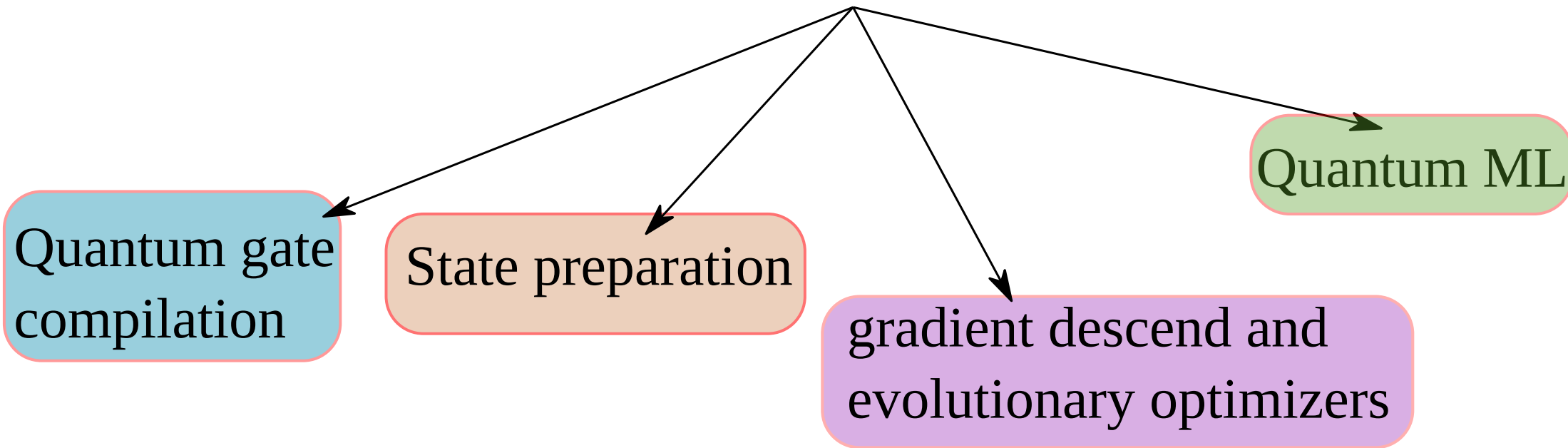
(QunaSys, Osaka University, NTT, and Fujitsu)



a framework for quantum simulation with hardware acceleration

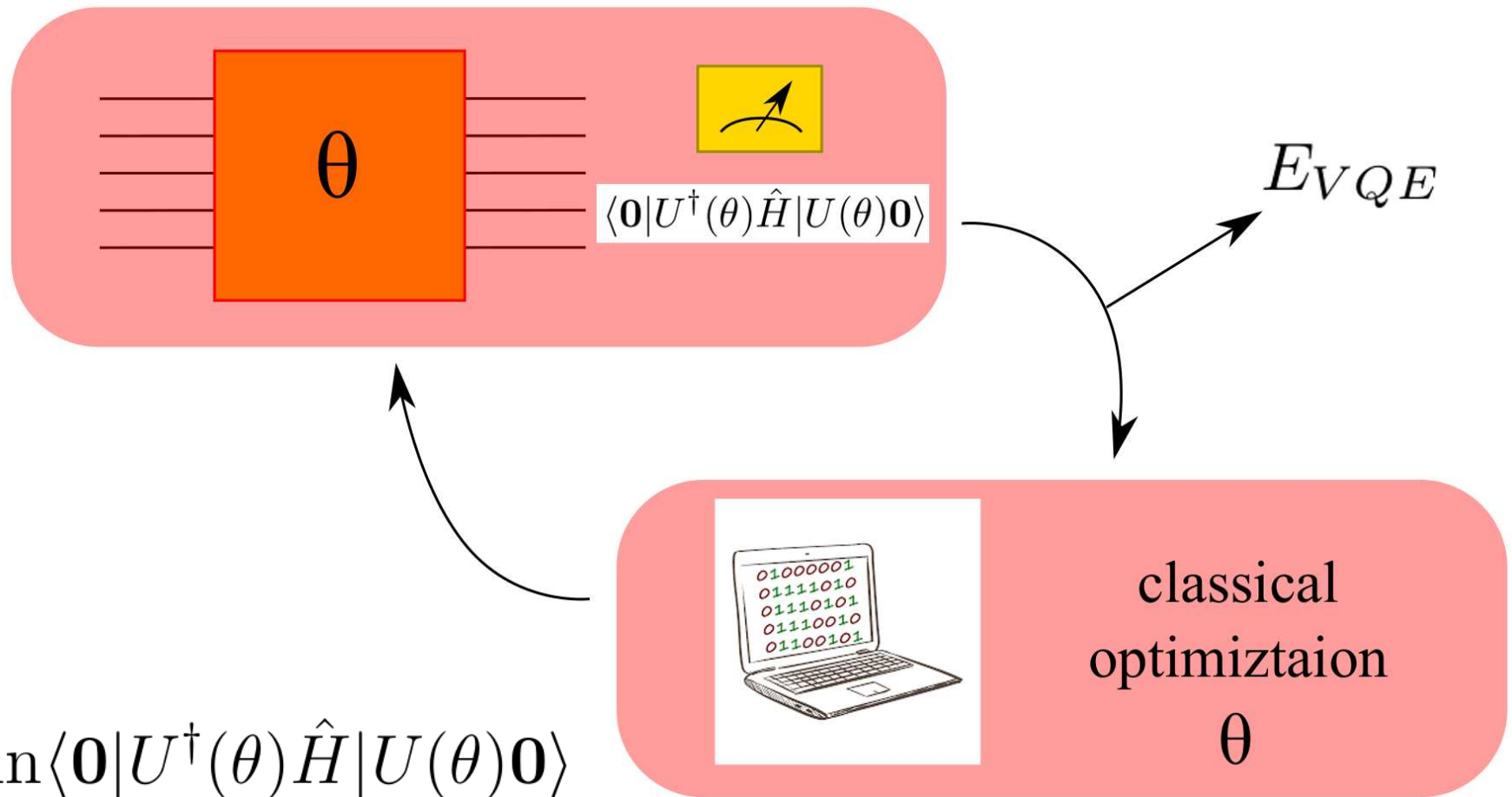
SQUANDER: toolkit to train quantum circuits

- SQUANDER: **S**equential **Q**uantum Gate **D**ecomposer



 **GitHub** <https://github.com/rakytap/sequential-quantum-gate-decomposer>

Variational quantum eigensolver



$$E_{VQE} = \min_{\theta} \langle \mathbf{0} | U^\dagger(\theta) \hat{H} | U(\theta) \mathbf{0} \rangle$$

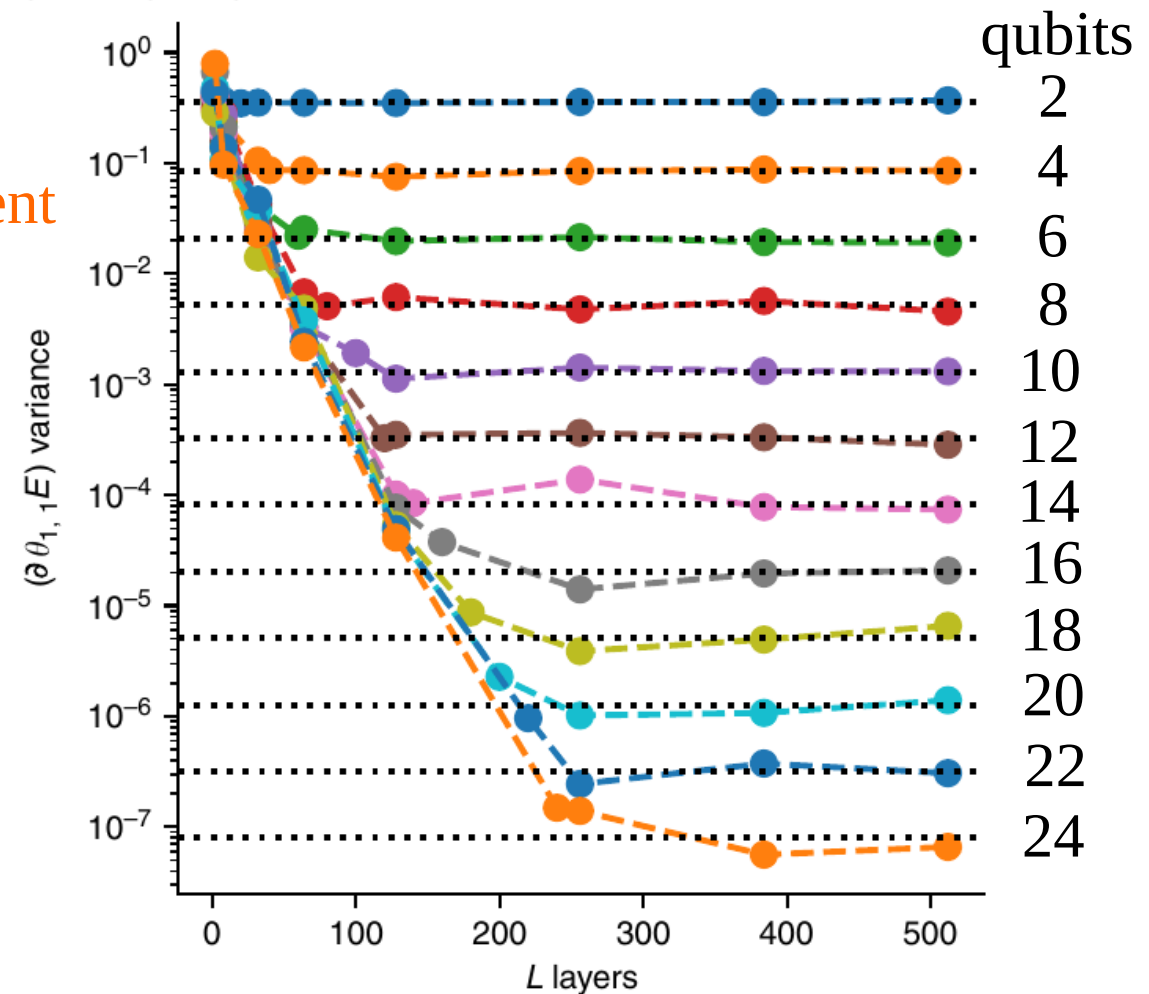
$$\hat{H} = \sum_{\alpha}^{\mathcal{P}} w_{\alpha} \hat{P}_{\alpha}$$

Barren plateaus in quantum neural network training landscapes

Jarrold R. McClean¹, Sergio Boixo¹, Vadim N. Smelyanskiy¹, Ryan Babbush¹ & Hartmut Neven¹

The variance of the cost function gradient
(and consequently its typical value)
vanishes exponentially
in the qubit number N

$$\text{Var}[\partial_{i,l} E(\boldsymbol{\theta})] \sim \mathcal{O}\left(\frac{1}{2^{2N}}\right)$$



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Barren plateau & entanglement entropy

controlling entanglement
to mitigate BP?

Second Rényi entropy

$$S_2 = -\ln \text{Tr} \rho_A^2$$

subsystem

Entanglement-Induced Barren Plateaus

Carlos Ortiz Marrero, Mária Kieferová,
and Nathan Wiebe

PRX Quantum 2, 040316

Entanglement devised barren plateau
mitigation

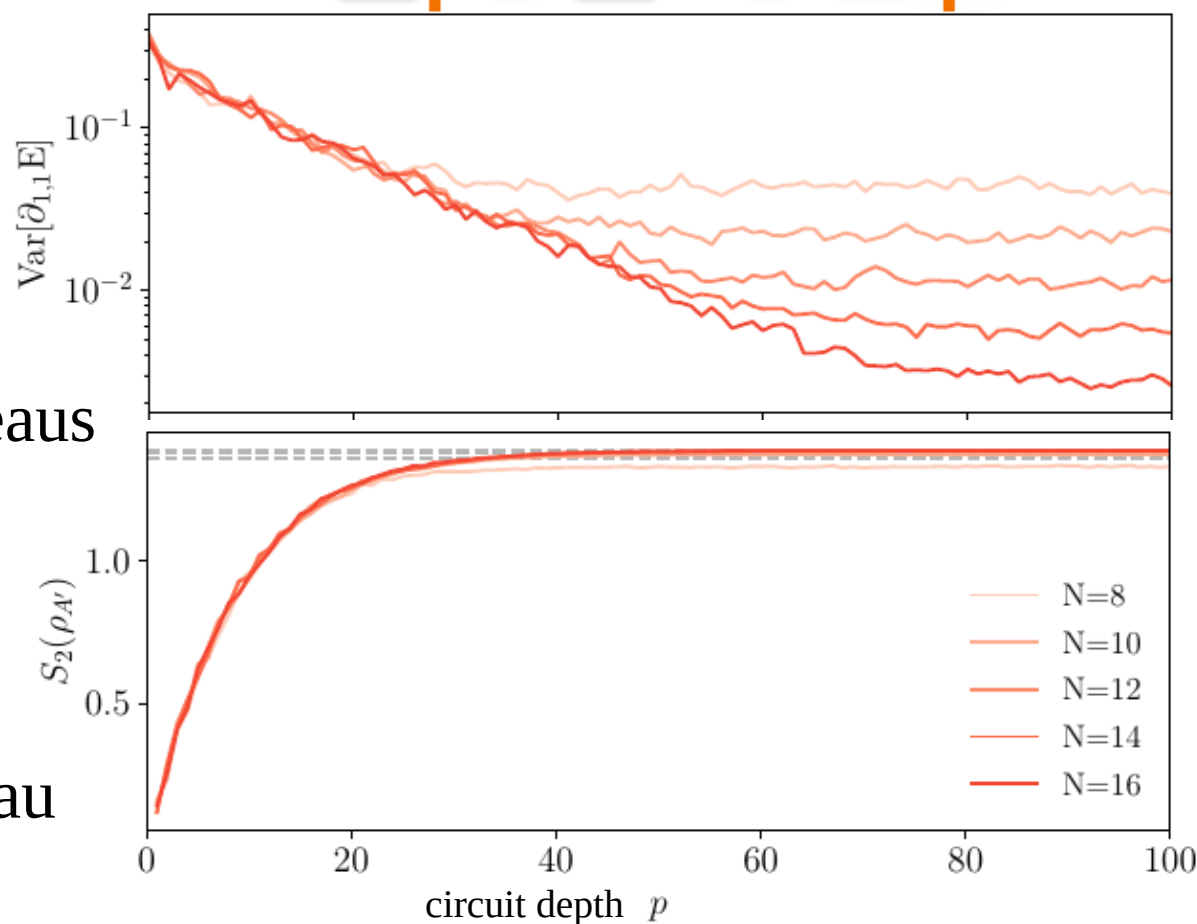
Taylor L. Patti, Khadijeh Najafi, Xun Gao, and Susanne F. Yelin
Phys. Rev. Research 3, 033090

Avoiding Barren Plateaus Using Classical Shadows

Stefan H. Sack, Raimel A. Medina, Alexios A. Michailidis,

Richard Kueng, and Maksym Serbyn

PRX QUANTUM 3, 020365 (2022)



Avoiding Barren Plateaus Using Classical Shadows

Stefan H. Sack, Raimel A. Medina, Alexios A. Michailidis,
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PRX QUANTUM 3, 020365 (2022)

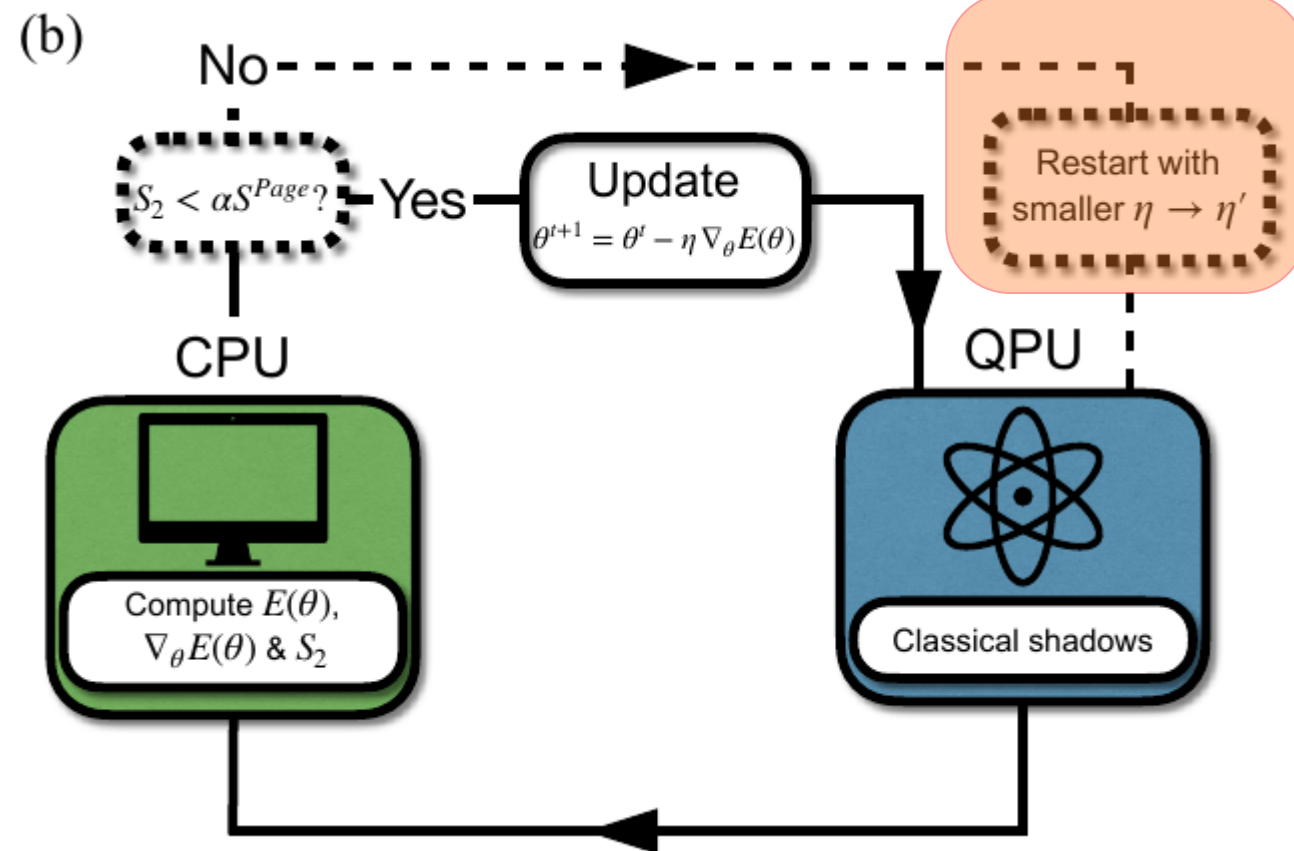
- monitor the entropy and control the learning rate

$$\theta^{t+1} = \theta^t - \eta \nabla_{\theta} E(\theta),$$

Decreasing the learning rate!!

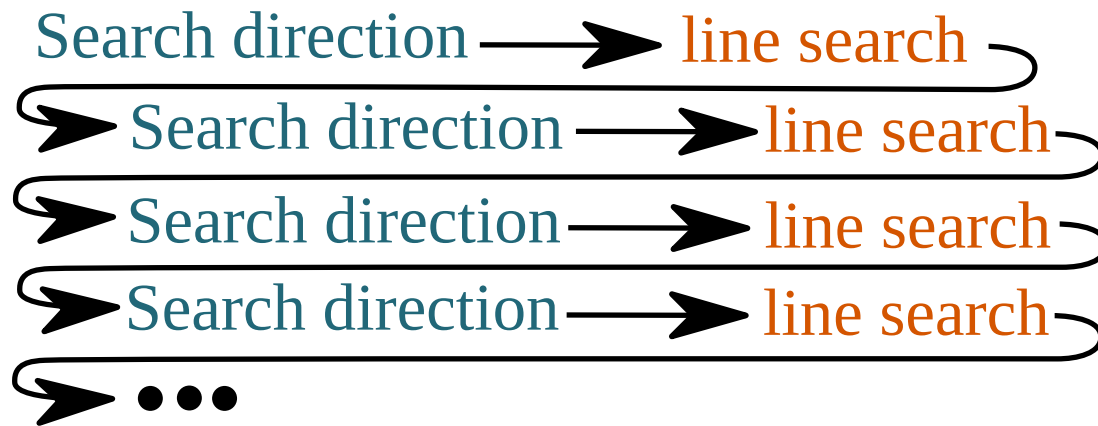
- parameter initialization to low entangling gates
Quantum 3, 214 (2019),
PRX Quantum 3, 010313 (2022)
- use local cost functions
Nature Communications 12, 1791 (2021)
- use matrix product states
PRX Quantum 3, 010313
- layer-by-layer optimization
Quantum Mach. Intell. 3, 5 (2021)

limiting the expressiveness
of the circuit

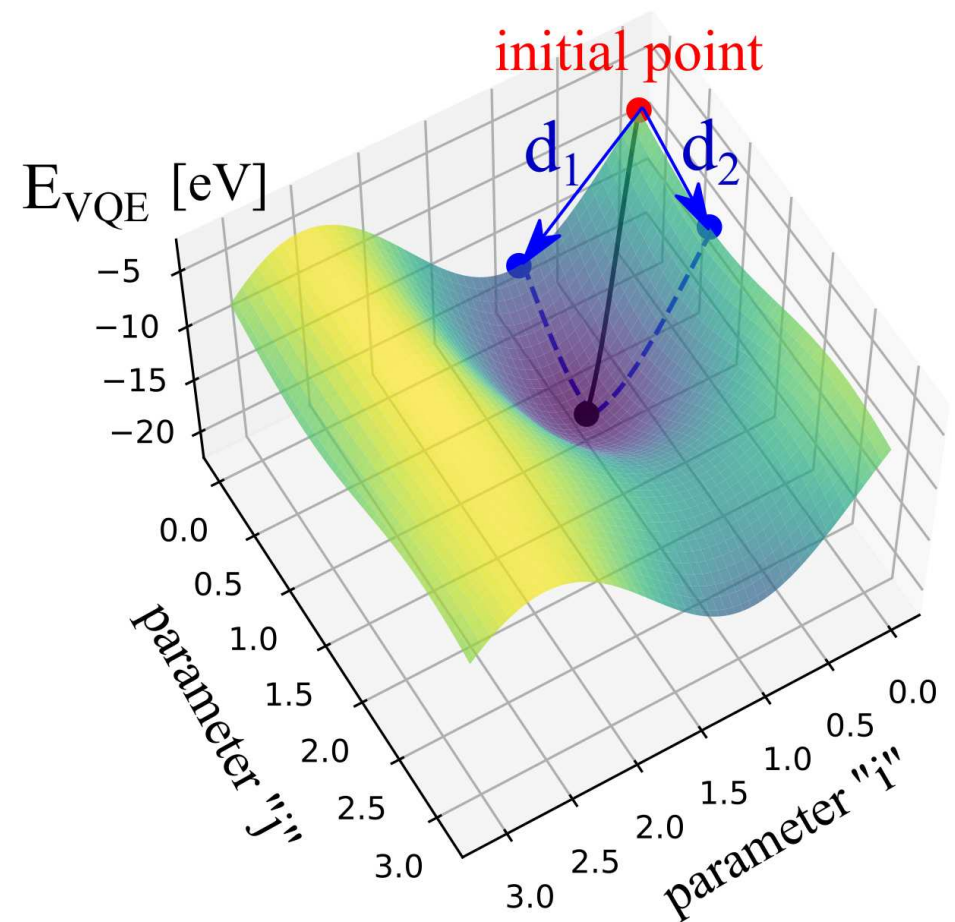


~~Decrease~~ the learning rate
Increase

- Perform line search along a well defined direction



- How to determine search direction?
- What is the range of the line search?
- layer-by-layer optimization



gate containing parameter p_i : U_p

$$|\Psi\rangle = U_1 \cdot U_2 \cdot U_3 \dots \overbrace{(A \cdot \cos(p_i) + B \cdot \sin(p_i) + C)}^{U_p} \dots U_{K-2} \cdot U_{K-1} \cdot U_K |0\rangle$$

matrices

$$|\Psi\rangle = \cos(p_i)|a\rangle + \sin(p_i)|b\rangle + |c\rangle$$

scalars

$$E = \langle \Psi | H | \Psi \rangle = \kappa \cdot \sin(2p_i + \xi) + \gamma \cdot \sin(p_i + \phi) + C$$

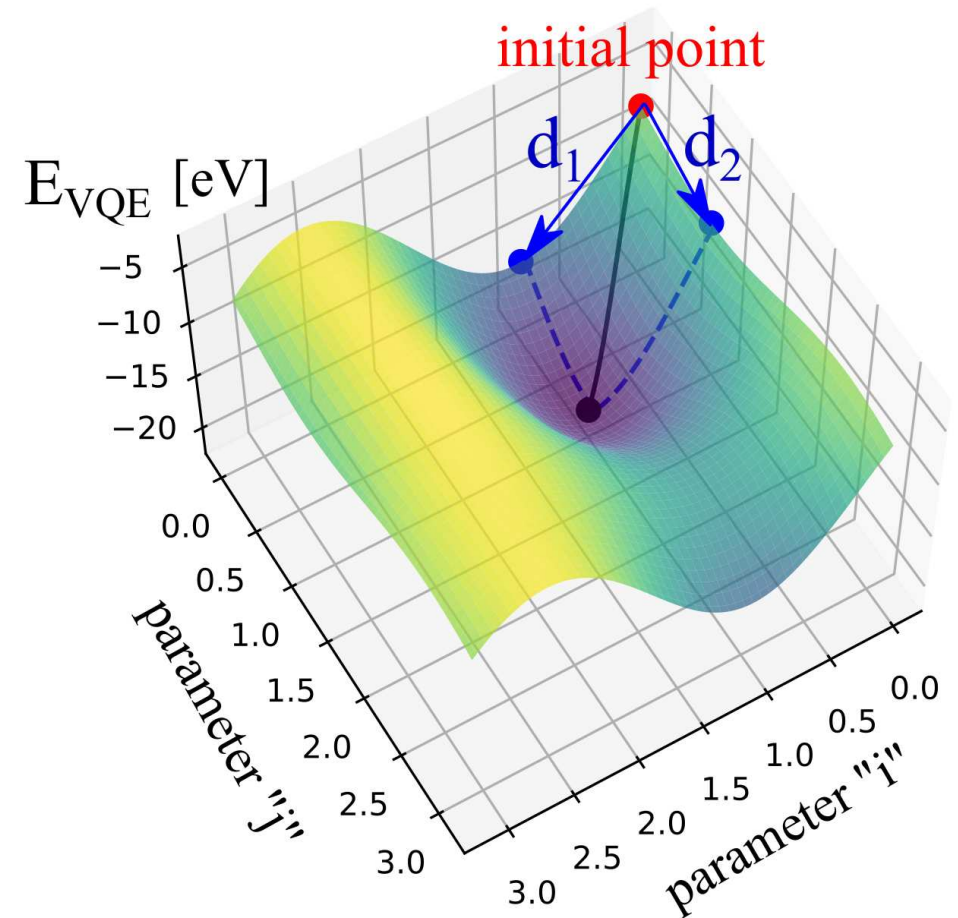
knowing the constants, the line search becomes efficient

$$E_{VQE} = \kappa \cdot \sin(2\theta_i + \xi) + C$$

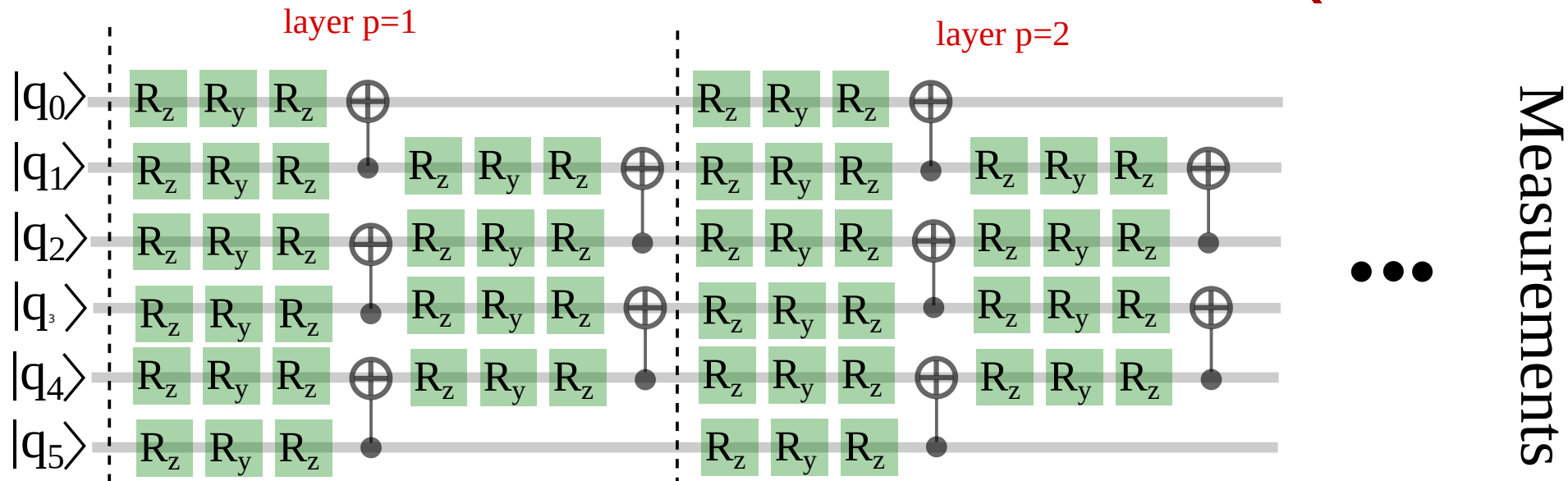
- randomly select a subset Λ of the parameters
- determine the parameter-wise minimum θ_i^*
- we define the search direction

$$d_i = \begin{cases} \theta_i^* - \theta_i & \text{if } i \in \Lambda \\ 0 & \text{otherwise.} \end{cases}$$

- decreasing values in E_{VQE} are **automatically** associated with moderate entanglement entropy.
- There are **no additional hyper-parameters** in the algorithm



The studied VQE problem



Heisenberg XXX model

$$\hat{H}_{XXX} = \sum_{i,j \in V_G} J (\hat{\sigma}_i^z \hat{\sigma}_j^z + \hat{\sigma}_i^y \hat{\sigma}_j^y + \hat{\sigma}_i^x \hat{\sigma}_j^x) + h_z \sum_N \hat{\sigma}_i^z$$

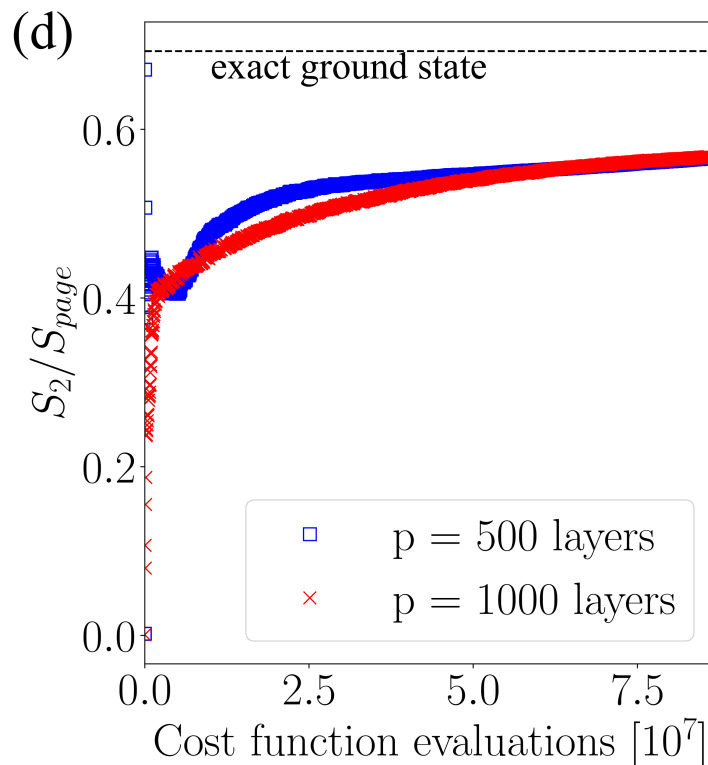
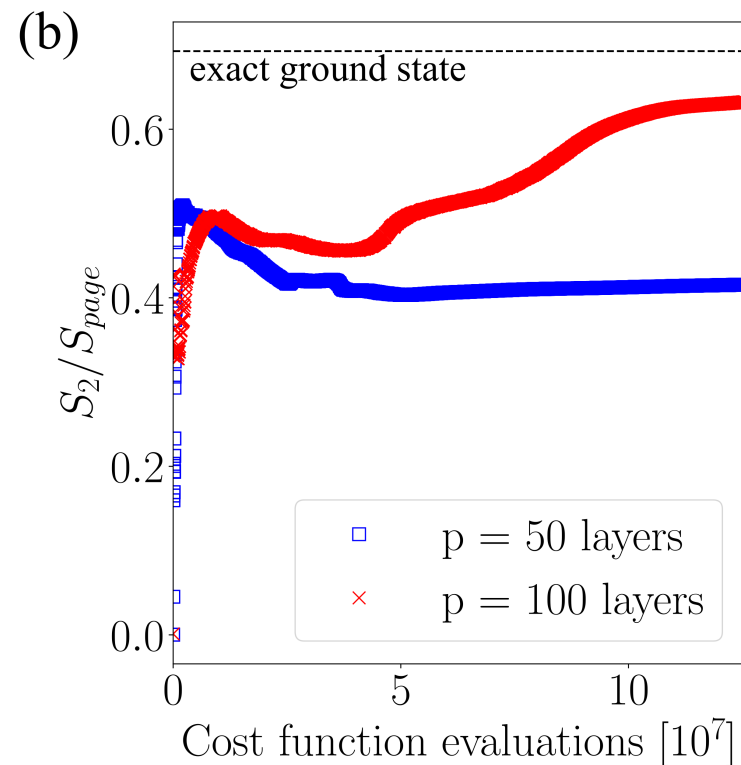
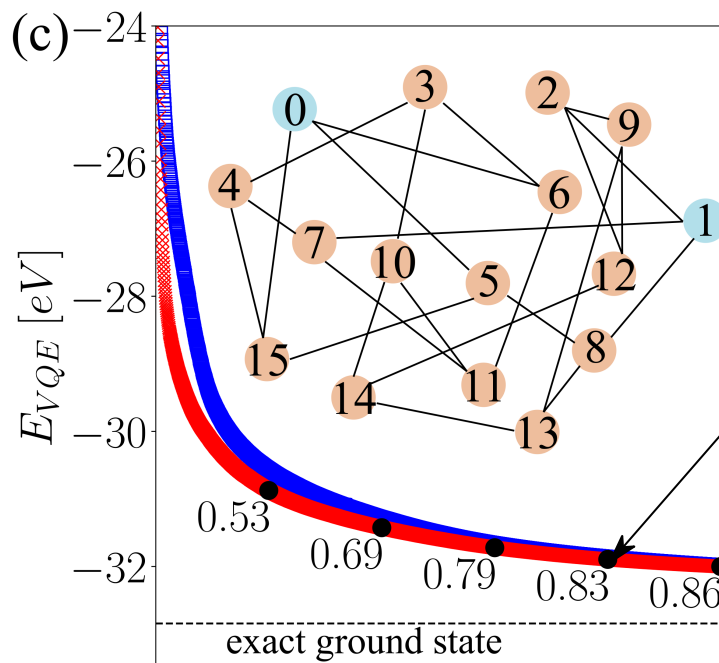
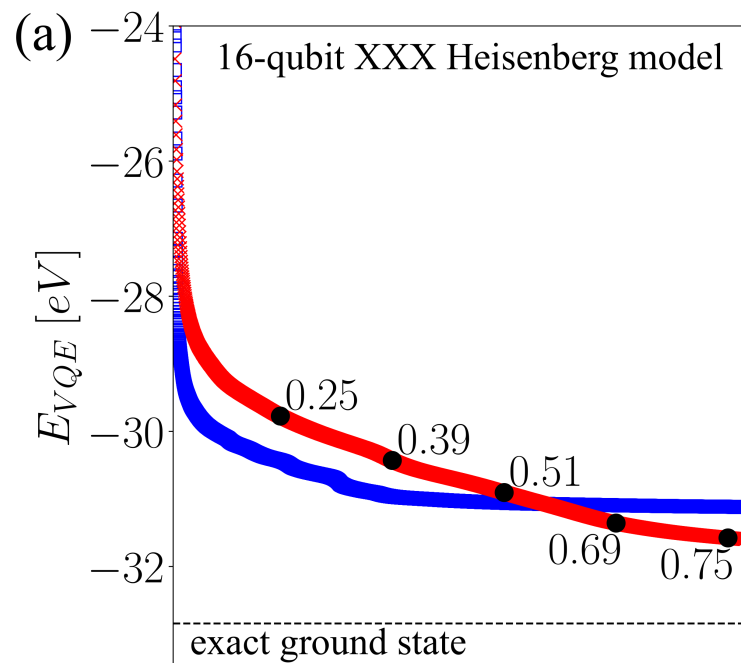
Sachdev-Ye-Kitaev (SYK) model describing Majorana fermions

$$\hat{H}_{SYK} = \sum_{1 \leq i < j < k < l \leq 2N} J_{i,j,k,l} \chi_i \chi_j \chi_k \chi_l$$

$$\{\chi_i, \chi_j\} = \delta_{i,j}$$

$J_{i,j,k,l}$ is taken from a Gaussian distribution

16-qubit system VQE



$$M = |\langle \Psi_0 | \Psi(\theta) \rangle|^2$$

16-qubit wave-function:
131070 free parameters

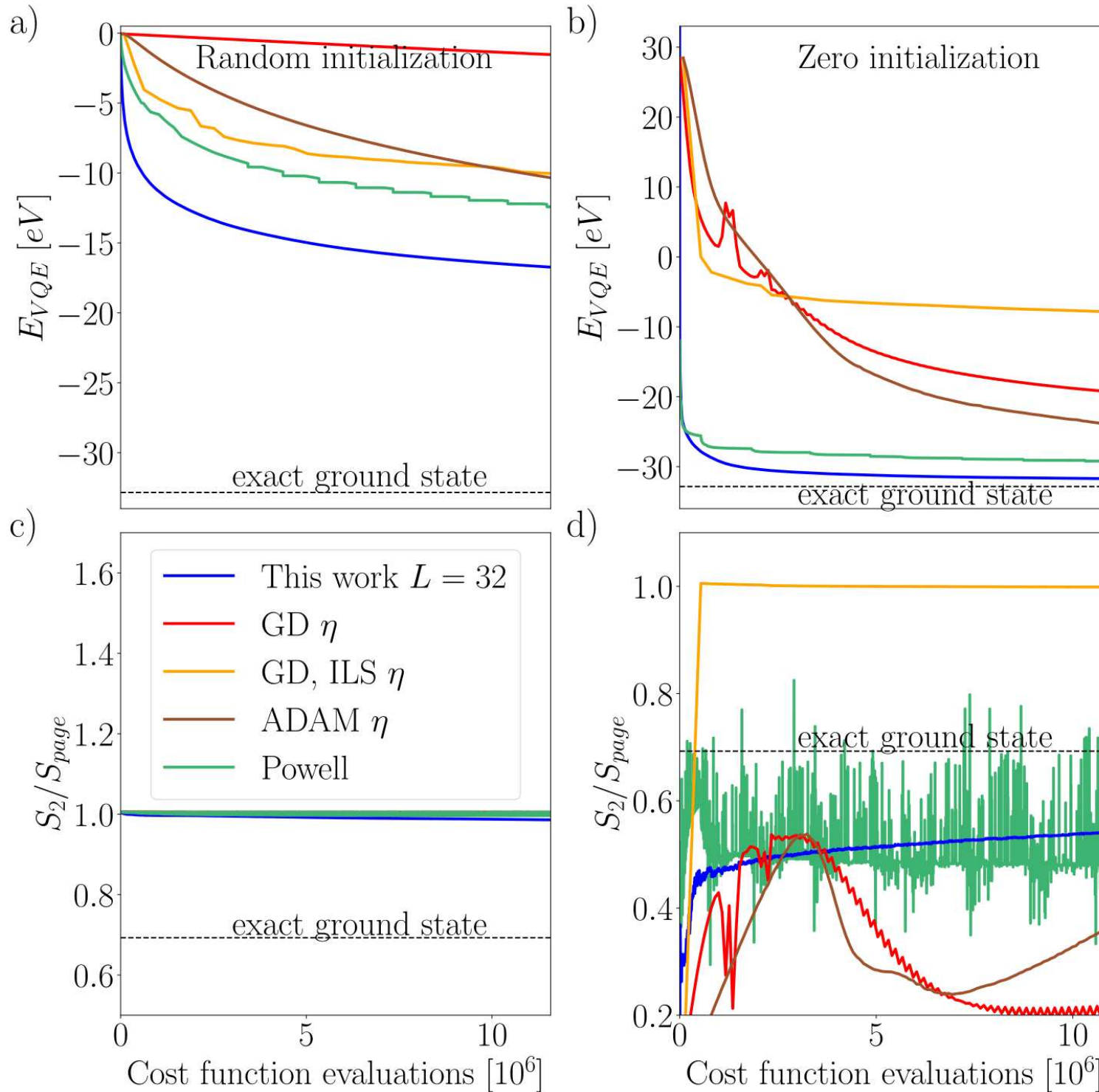
circuit with 100-layers:
9000 free parameters

circuit with 500-layers:
45000 free parameters

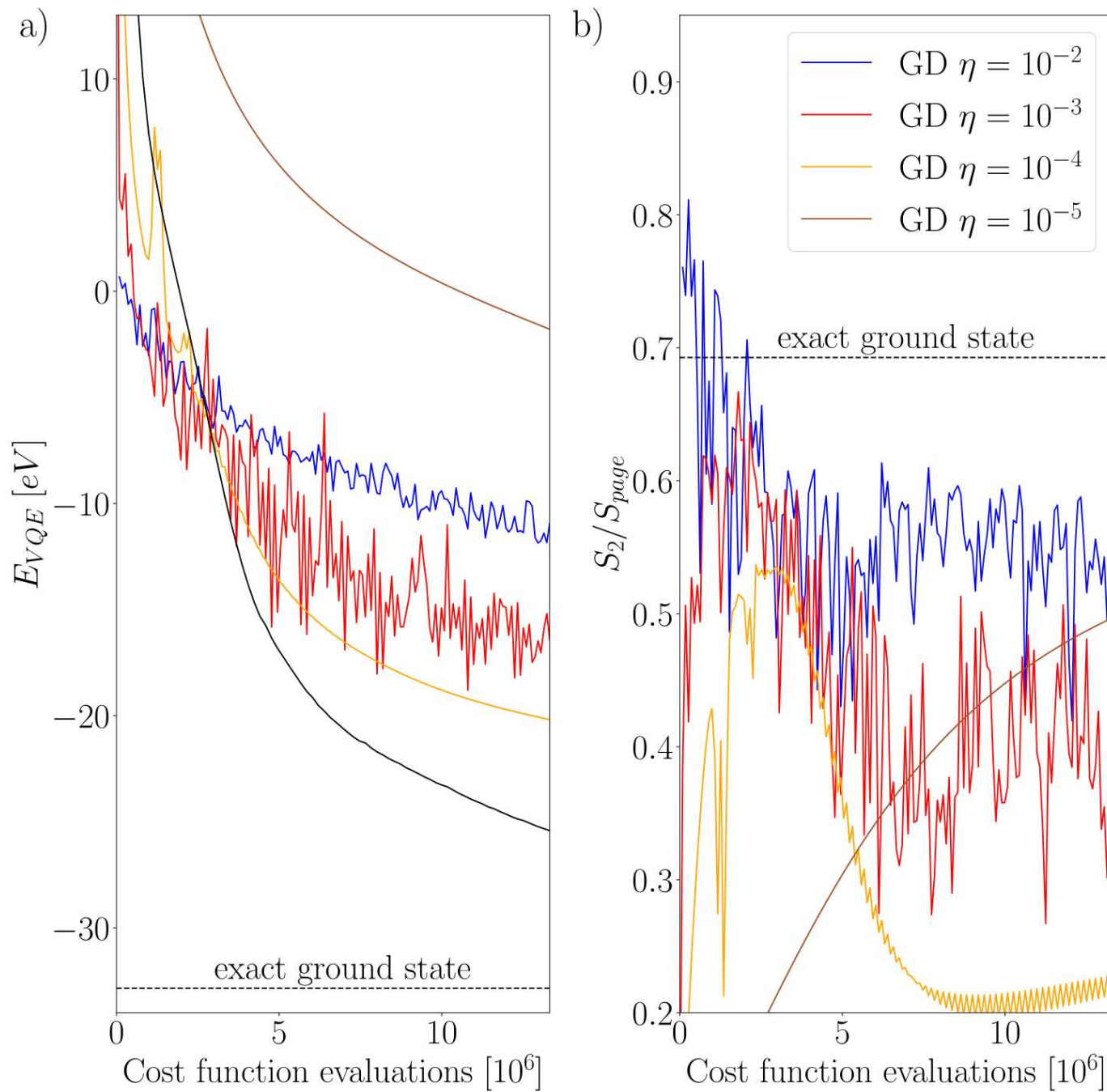
circuit with 1000-layers:
90000 free parameters



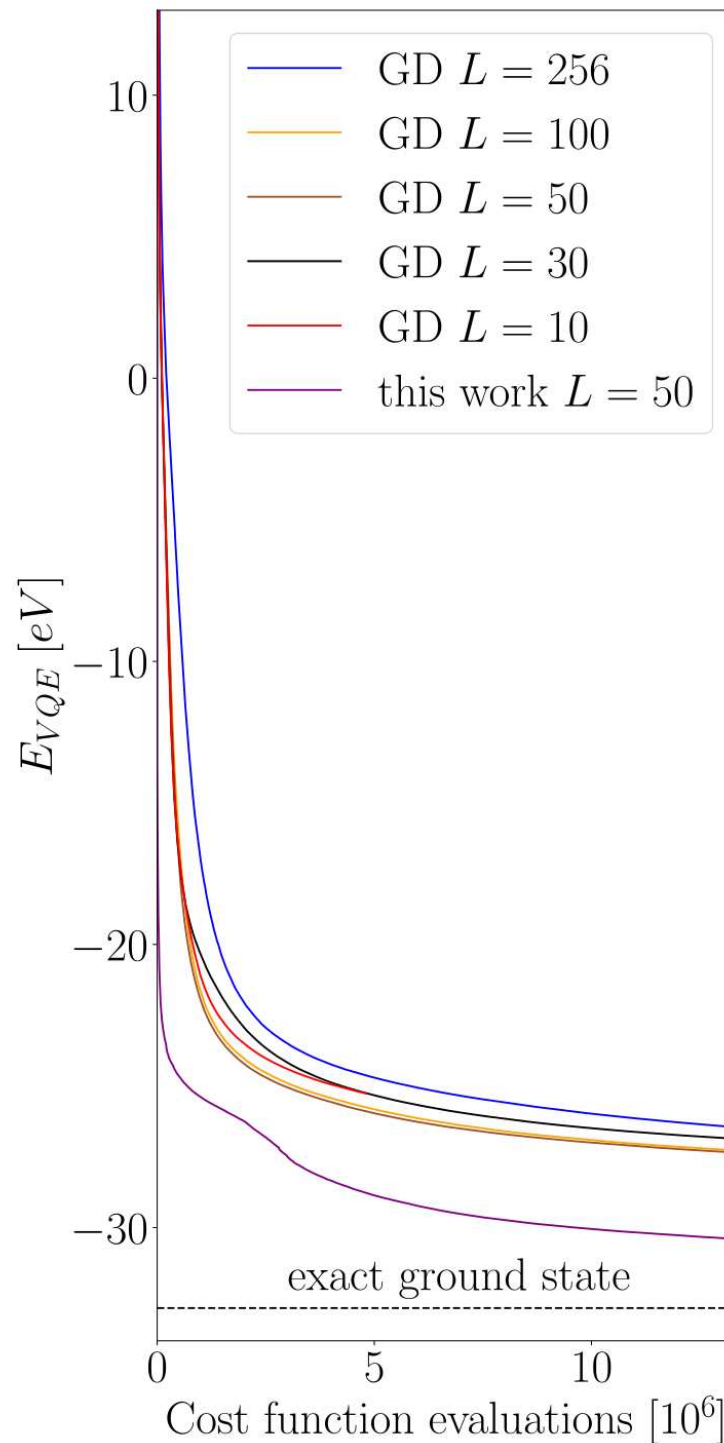
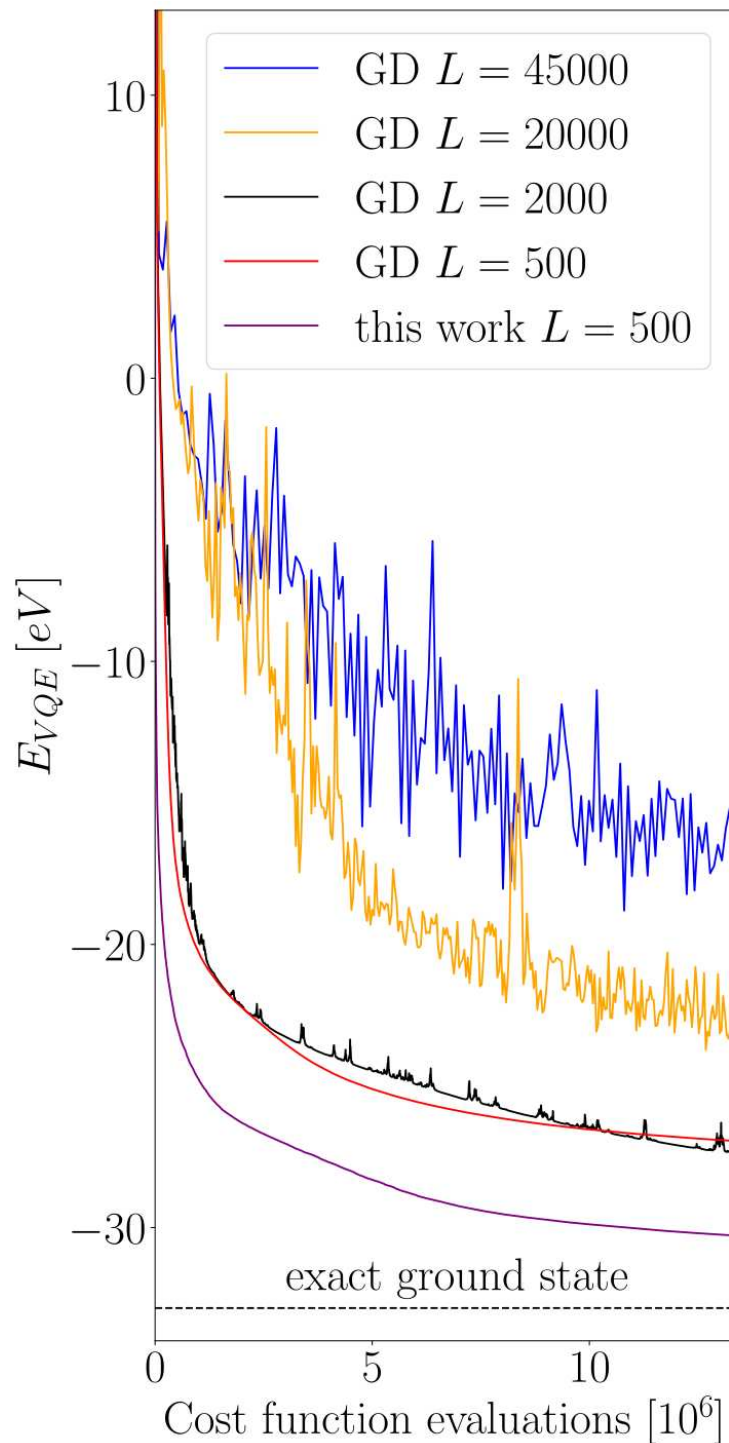
Compare optimization strategies



How much the learning rate influences the optimization?



Why is our method so efficient?



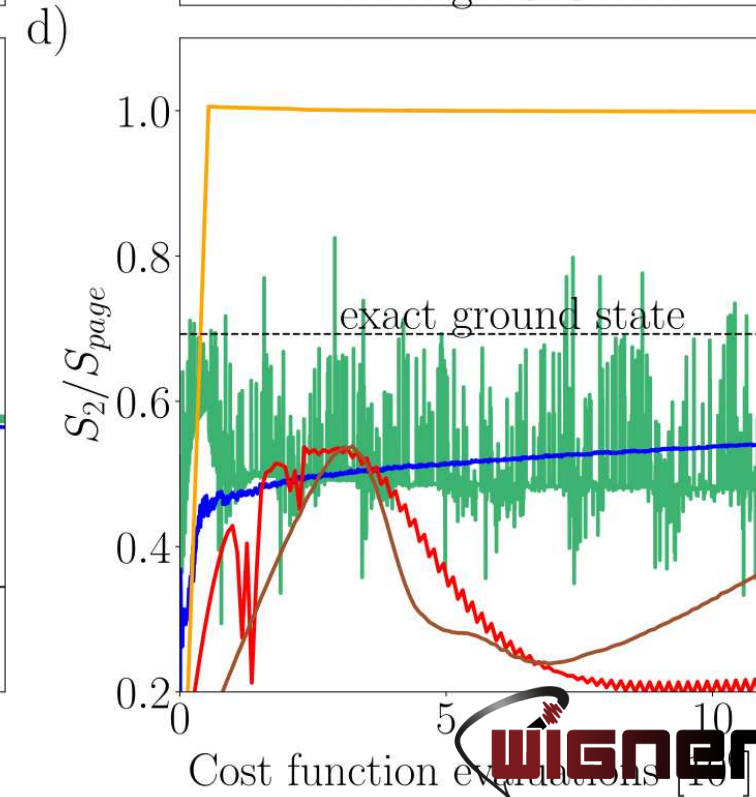
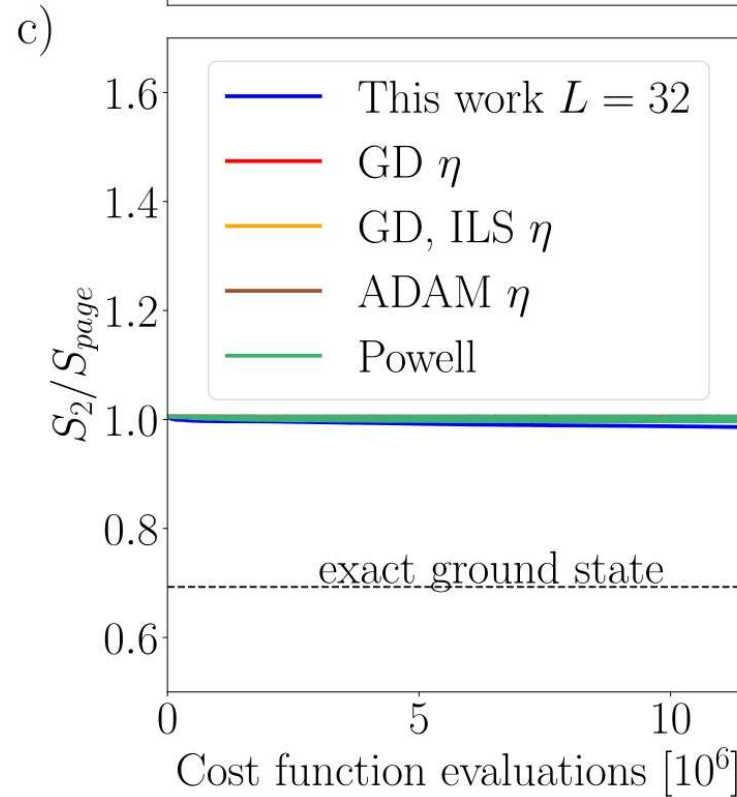
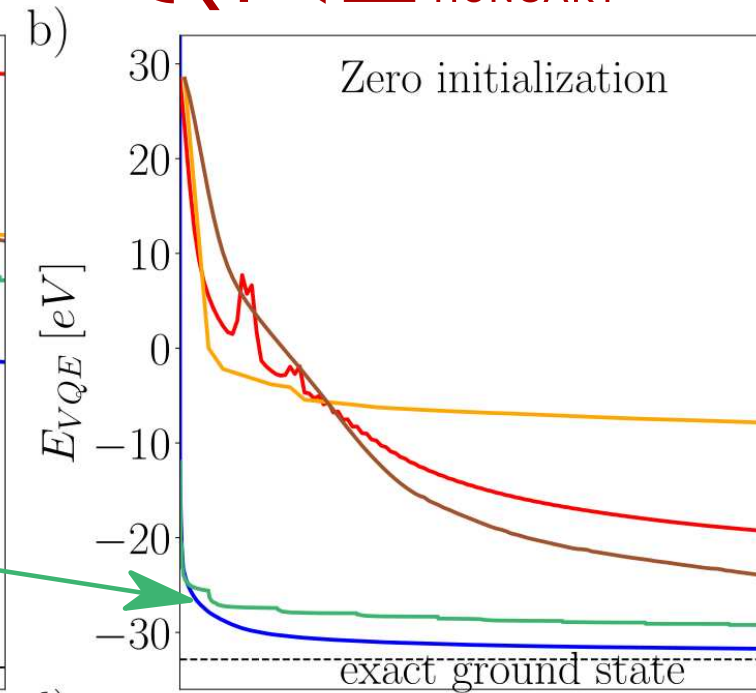
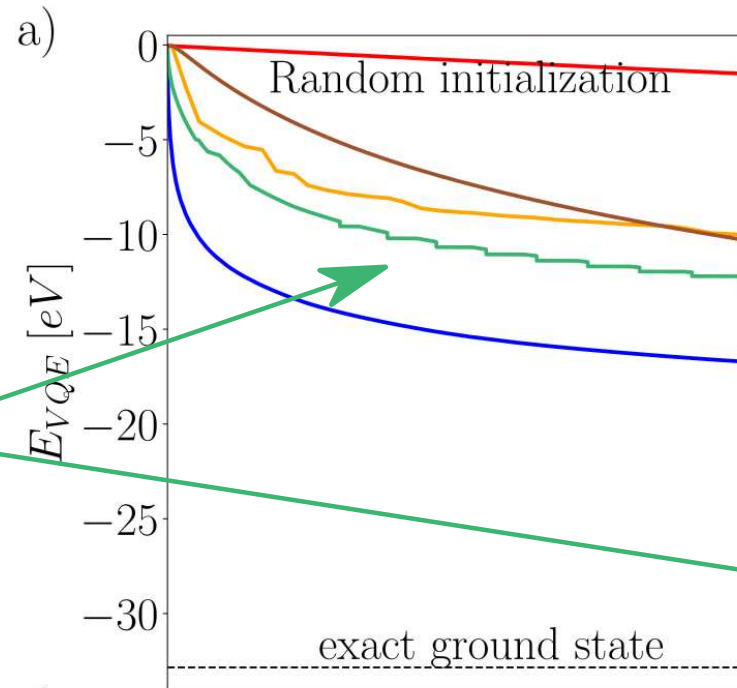
**L... the number of
parameters updated
in one iteration**

GD... Gradient descend



Compare optimization strategies

Steps in
the curve of
optimization



Further numerical improvements to scale up VQE experiments

- More efficient circuit ansatz adopted to
 - the coupling structure of the underlying model
 - the interaction types in the physical system
- Introducing noise into the model
- Using accelerators to evaluate the cost function



GPU

or



FPGA

or



AI data-flow chip



GPU QC simulation benchmark

qsim runtime comparison (noiseless random circuit)

● GPU: A100 ● CPU: C2-standard-60



Google Quantum AI

https://quantumai.google/qsim/choose_hw

Acknowledgement



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contact: Peter Rakyta, rakyta.peter@wigner.hu

