Recent developments of a multi-phase transport model for relativistic nuclear collisions

Zi-Wei Lin
East Carolina University (ECU)

HUN-REN Wigner Research Centre for Physics
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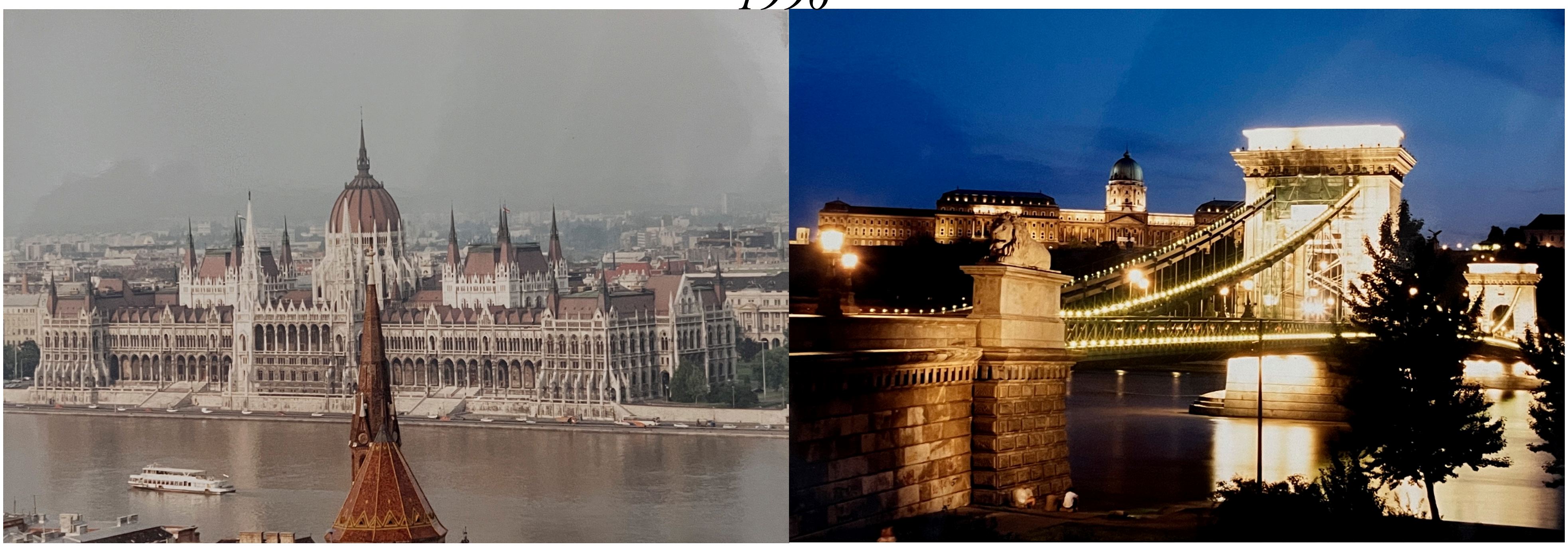
Outline

- Motivation
- Earlier status of the AMPT model (before ~2019)
- Recent improvements of AMPT
- Summary

1996



Motivation: for comprehensive simulations of high energy heavy ion collisions

What we need:	***What we can use:***
Initial particle/energy production	*Soft+hard model (+string melting), CGC, Glauber model, pQCD, Lund strings, ...*
Pre-equilibrium interactions: *equilibration, thermalization, pre-flow*	*Parton cascade (ZPC, MPC, BAMPS), free streaming, CGC, AdS/CFT, ...*
Space-time evolution of QGP	*Parton cascade (ZPC, MPC, BAMPS), (ideal or viscous) hydrodynamics, ...*
Hadronization /QCD phase transition	*Quark coalescence/parton recombination, fragmentation, Cooper-Frye, statistical hadronization, rate equations, ...*
Hadronic interactions	*Hadron cascade (ART, UrQMD, SMASH), ...*

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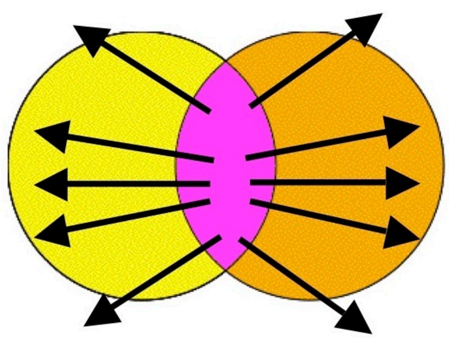
Red choices: a hybrid model (hydrodynamics+hadron cascade)

Green choices: a multi-phase transport (AMPT) model

Motivation: transport models are great for non-equilibrium studies

- For large systems at very high energies:
  transport models are similar to hydrodynamics
  (*for the same EoS, η, ζ*), but in different language:
  transport models (using microscopic particles & scatterings via σ)
  versus hydrodynamics-based models (using $T_{\mu\upsilon}$, EoS &
  transport coefficients η,ζ…).

- For finite/small systems at finite energies,
  or early time & hard probes of large systems at very high energies:
  non-equilibrium effects are expected to be important.
  One example is **the parton escape mechanism**:
  interaction-induced response from kinetic theory
  to the anisotropic spatial geometry.



L He et al. PLB 753 (2016);
ZWL et al. NPA 956 (2016);
HL Li et al. PRC 99 (2019)

Motivation: transport models are great for non-equilibrium studies

Small system data show tantalizing similar flow features as large systems:

are they real signals from collectivity?

is a parton matter formed in small systems?

is the matter dominated by gluons or quarks?

is the matter close to equilibrium or far off equilibrium (core-corona)?

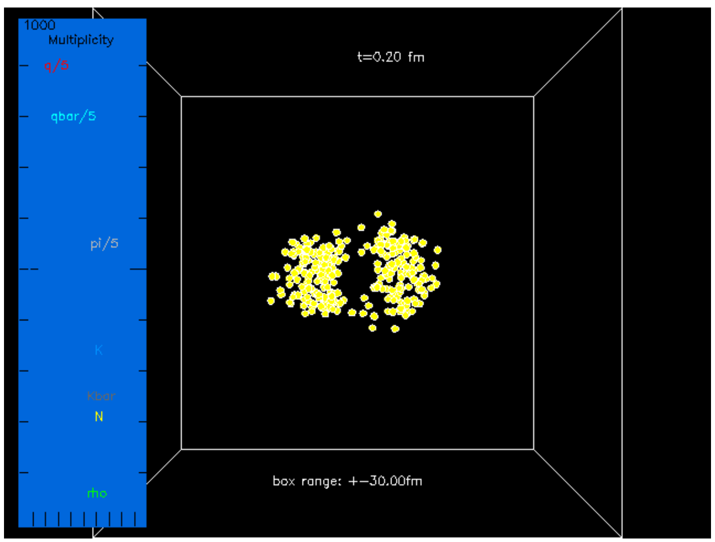
To answer these questions and study properties of parton matter/QGP, transport models / kinetic theory are crucial as they don't assume equilibrium & can address non-equilibrium dynamics.

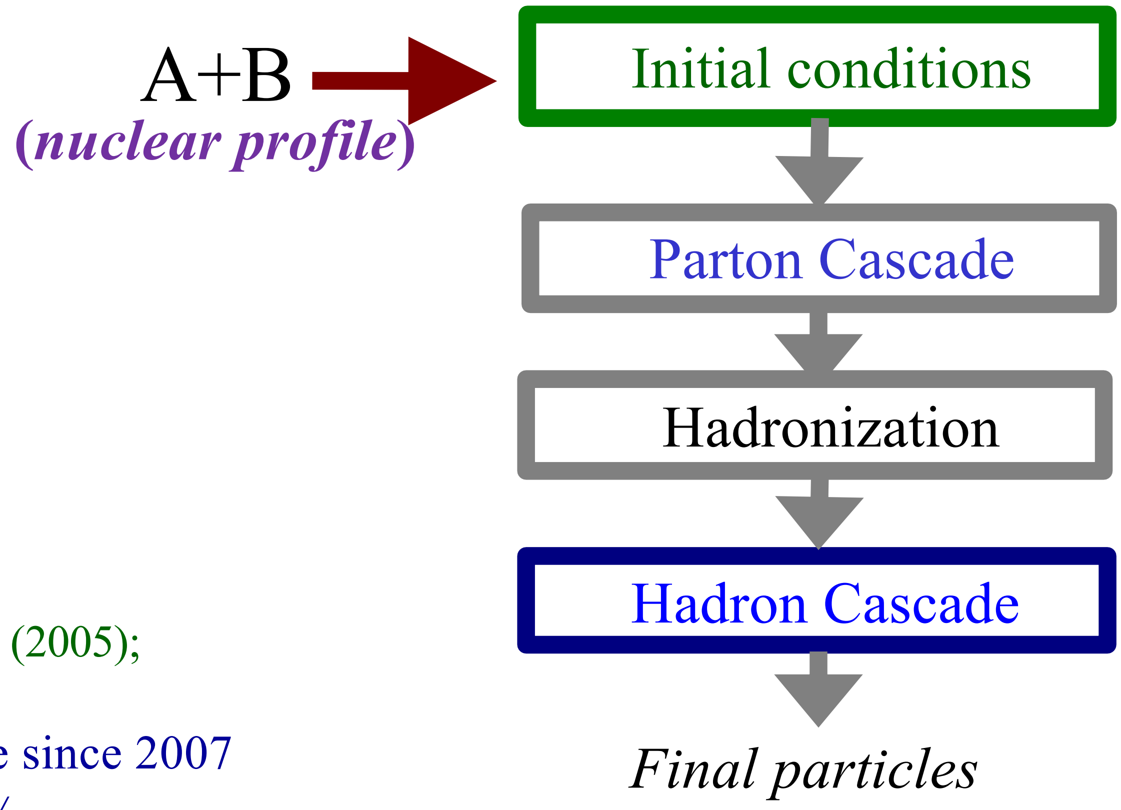
B Zhang, CPC 109 (1998); Heiselberg & Levy, PRC 59 (1999);
ZWL & Ko, PRC 65 (2002); Z Xu & Greiner, PRC 71 (2005);
Bzdak & Ma, PRL 113 (2014);
Kurkela et al. PLB 783 (2018) & EPJC 79 (2019);
Kurkela, Törnkvist & Zapp, EPJC 84 (2024); ...

Earlier status of the AMPT model

A multi-phase transport (AMPT) model
was constructed as a self-contained kinetic description of nuclear collisions:

- evolves the system from initial condition to final observables
- includes 3D productions of all flavours & conserved charges (B/Q/S/C...)
- includes fluctuating initial conditions & non-equilibrium dynamics/evolution

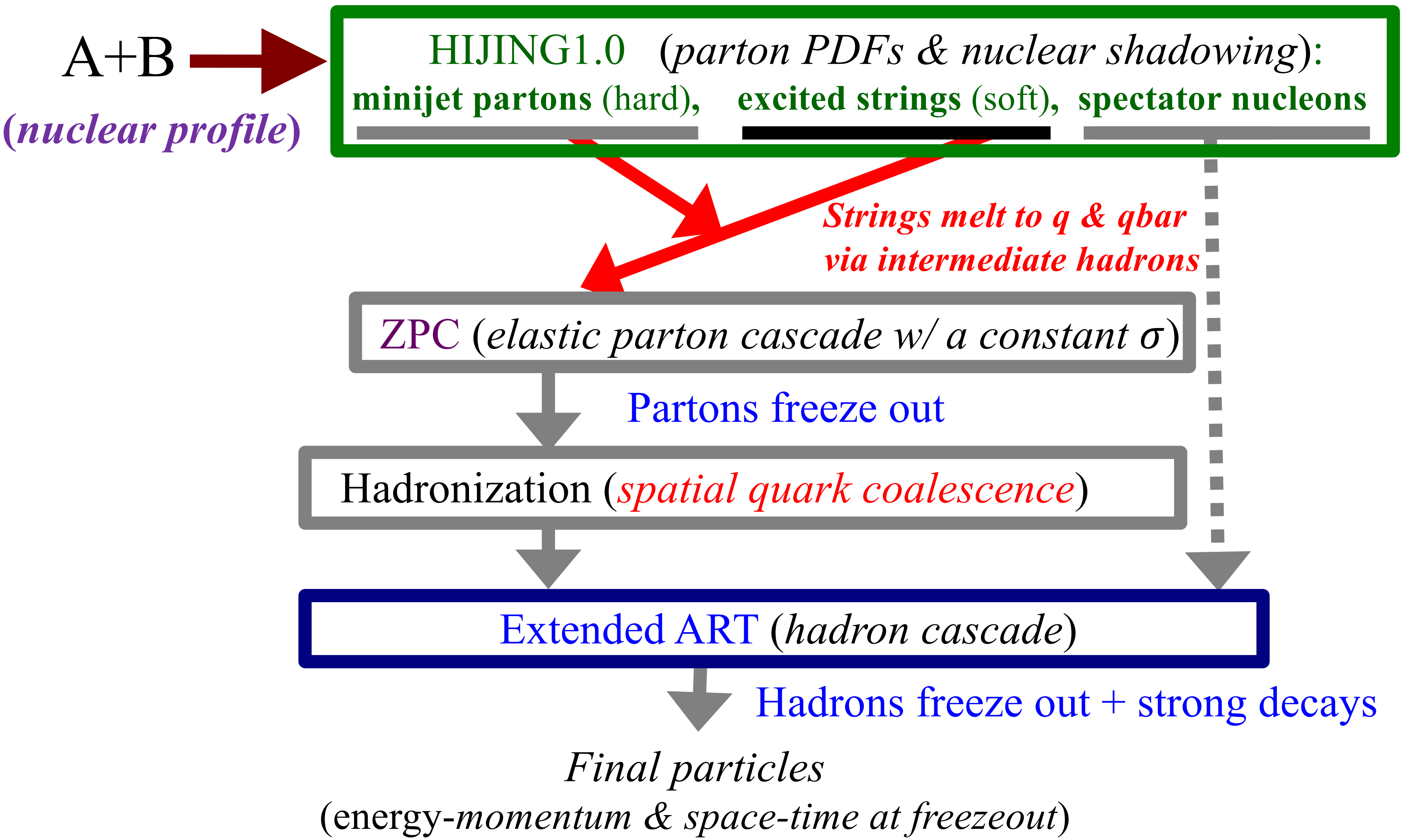




ZWL, Ko, Li, Zhang & Pal, PRC 72 (2005);
ZWL & Zheng, NST 32 (2021)
Source codes at the ECU website since 2007
https://myweb.ecu.edu/linz/ampt/

The string melting version of the AMPT model:

applicable when we expect the formation of a parton matter



Earlier status of the AMPT model

myweb.ecu.edu

AMPT source codes

(updated December 25, 2018):

A Multi-Phase Transport (AMPT) model is a Monte Carlo transport model for nuclear collisions at relativistic energies.
Each of the following versions contains:
the source codes, an example input file, a Makefile, a readme, a required subdirectory for storing output files, and a script to run the code.

1. ampt-v1.11-v2.11.tgz (11/2004)
2. ampt-v1.21-v2.21.tgz (10/2008)
3. *Other older versions inbetween*
4. ampt-v1.26t5-v2.26t5.zip (4/2015)
5. ampt-v1.26t7-v2.26t7.zip (10/2016)
6. ampt-v1.26t7b-v2.26t7b.zip (5/2018)
7. ampt-v1.26t9-v2.26t9.zip (9/2018)
8. ampt-v1.26t9b-v2.26t9b.zip (12/2018)

String Melting AMPT since 4/2015 can reasonably describe the bulk matter at high energies at RHIC and LHC.

This readme file lists the main changes up to version v1.26t9b-v2.26t9b ("t" means a version under test):

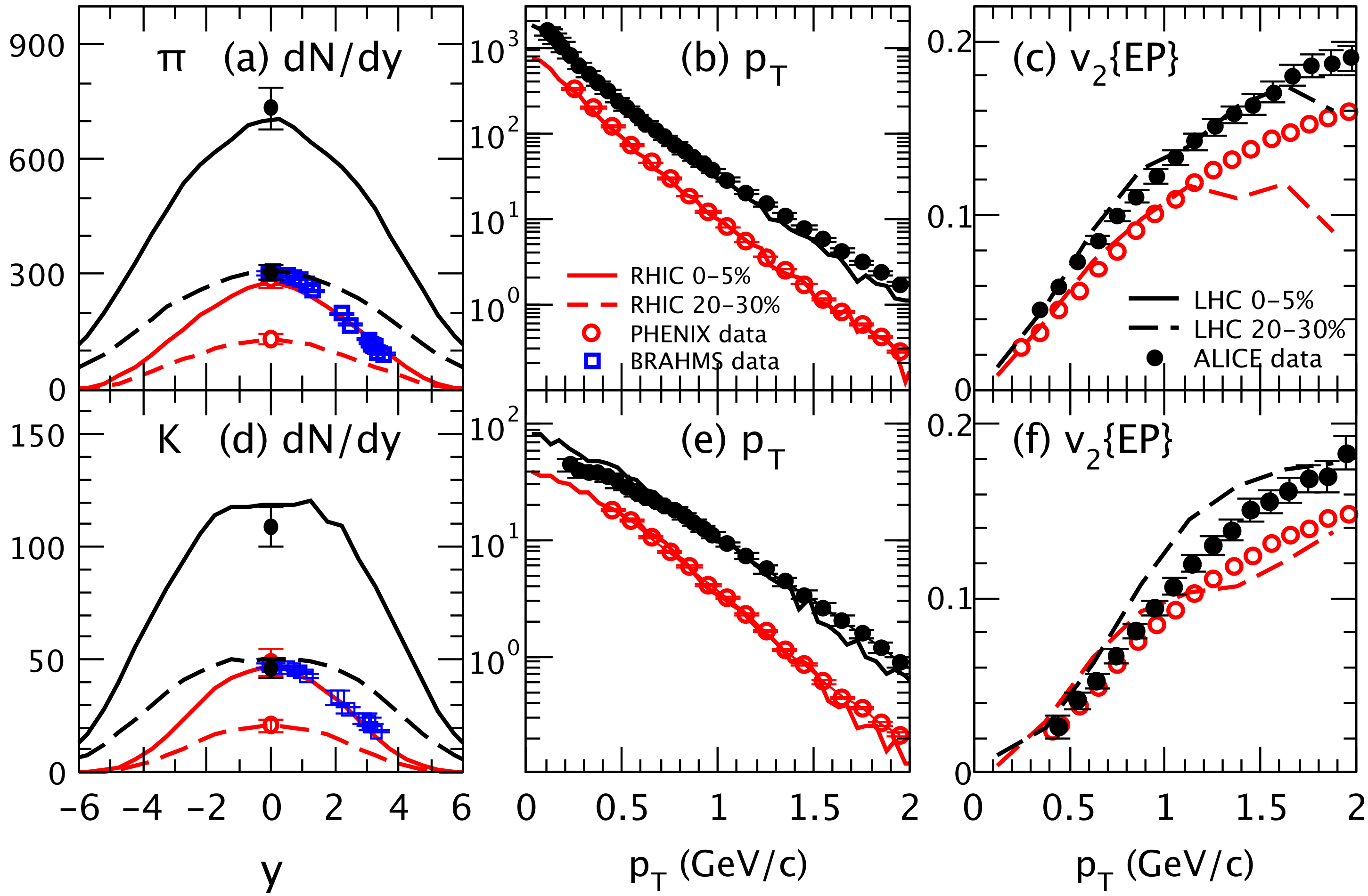
```
 AMPT Users' Guide

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
12/2018 test version v1.26t9b/v2.26t9b:
     * Fixed bugs that can cause segmentation fault (especially for default
       AMPT at high energies):
       exclude endpoints of 0. and 1. in random values from RANART()
         in amptsub.f in order to avoid crash in case of 0 branching ratio,
       resample random value from RLU(0) when TEVB in PYSSPA does not change
         (this may happen when RLU(0)>~0.9999 and thus
         ex==|LOG(RLU(0))*B0/(5.*WTSUM)|<~3e-8 and thus EXP(ex)=1.00000000,
         which may cause IS(2) to be undefined and be >10^8 (out of bound)).
    The above modification can be found by searching "clin-12/2018".
```

Earlier status of the AMPT model

- The string melting AMPT model is applicable
  when we expect the formation of a parton matter.
- It can reasonably describe the bulk matter observables
  at low p_T in high energy A+A collisions
  (after using a very small Lund parameter b_L=0.15/GeV^2):

ZWL, PRC (2014)



Recent improvements of AMPT

I will focus on some of the improvements since 2019:

1. Modern PDF & spatially-dependent nuclear shadowing — Zhang, Zheng, Liu, Shi & ZWL, PRC (2019)
2. Improvements of heavy flavours — Zheng, Zhang, Shi & ZWL, PRC (2020); Zhang, Zheng, Shi & ZWL, PLB (2023)
3. Local nuclear scaling for self-consistent size dependence — Zhang, Zheng, Shi & ZWL, PRC (2021)
4. Benchmark and improve ZPC parton transport — Zhao, Ma, Ma & ZWL, PRC (2020); Mendenhall & ZWL, arXiv:2507.23107

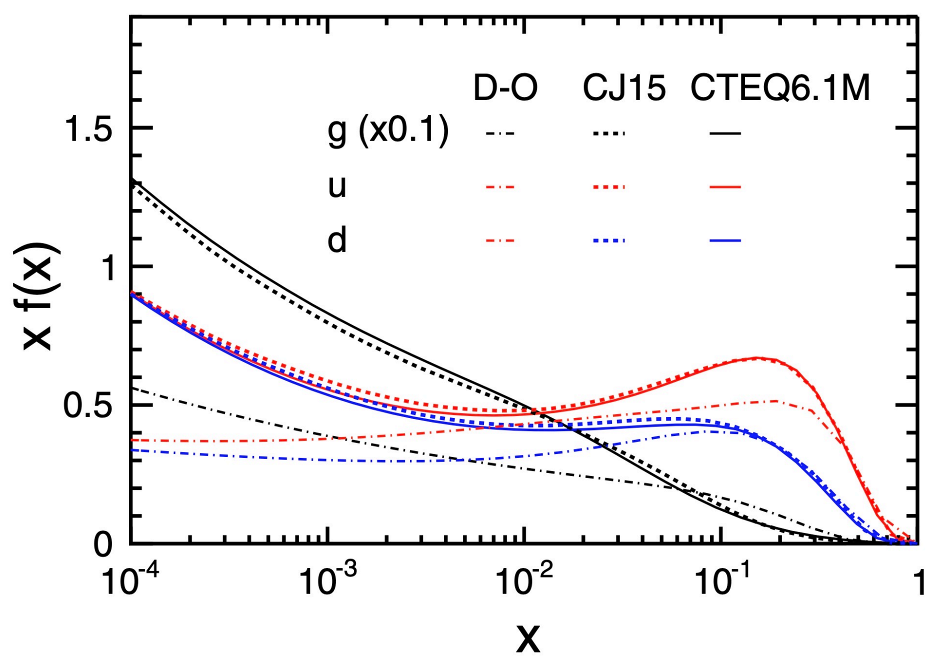
Improvement 1: modern PDF of nuclei

Modern nPDFs should improve AMPT on pQCD observables such as heavy flavor & high p_T:

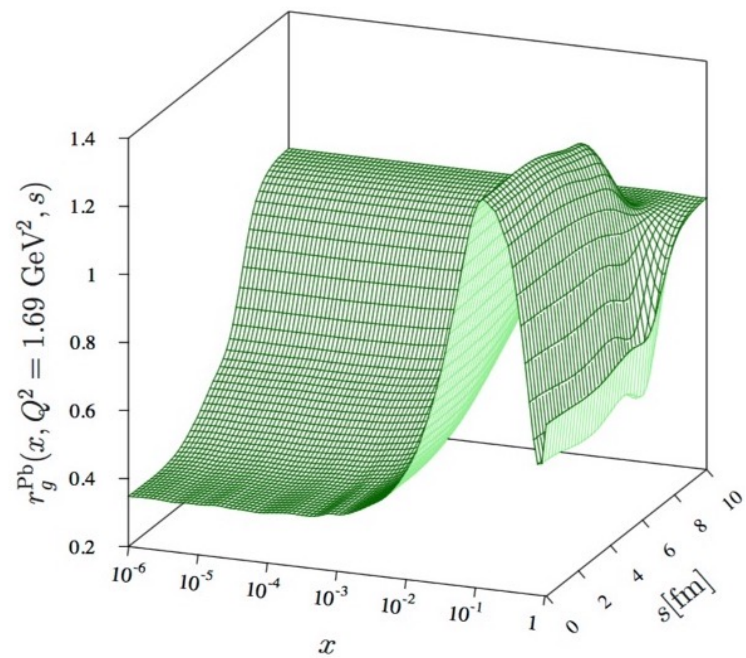
$$\frac{d\sigma^{Q\bar{Q}}}{dp_T^2 dy_1 dy_2} = K \sum_{a,b} x_1 f_a(x_1, \mu_F^2) x_2 f_b(x_2, \mu_F^2) \frac{d\sigma^{ab\to Q\bar{Q}}}{d\hat{t}}$$

$$f_i^{p/A}(x, Q^2) \equiv R_i^A(x, Q^2) f_i^p(x, Q^2) \qquad R_i^A(x, Q^2) \equiv \frac{1}{A}\int d^2\mathbf{s}\, T_A(\mathbf{s})\, r_i^A(x, Q^2, \mathbf{s})$$

We have incorporated CTEQ6.1M PDFs for the free nucleon & EPS09s nuclear shadowing.

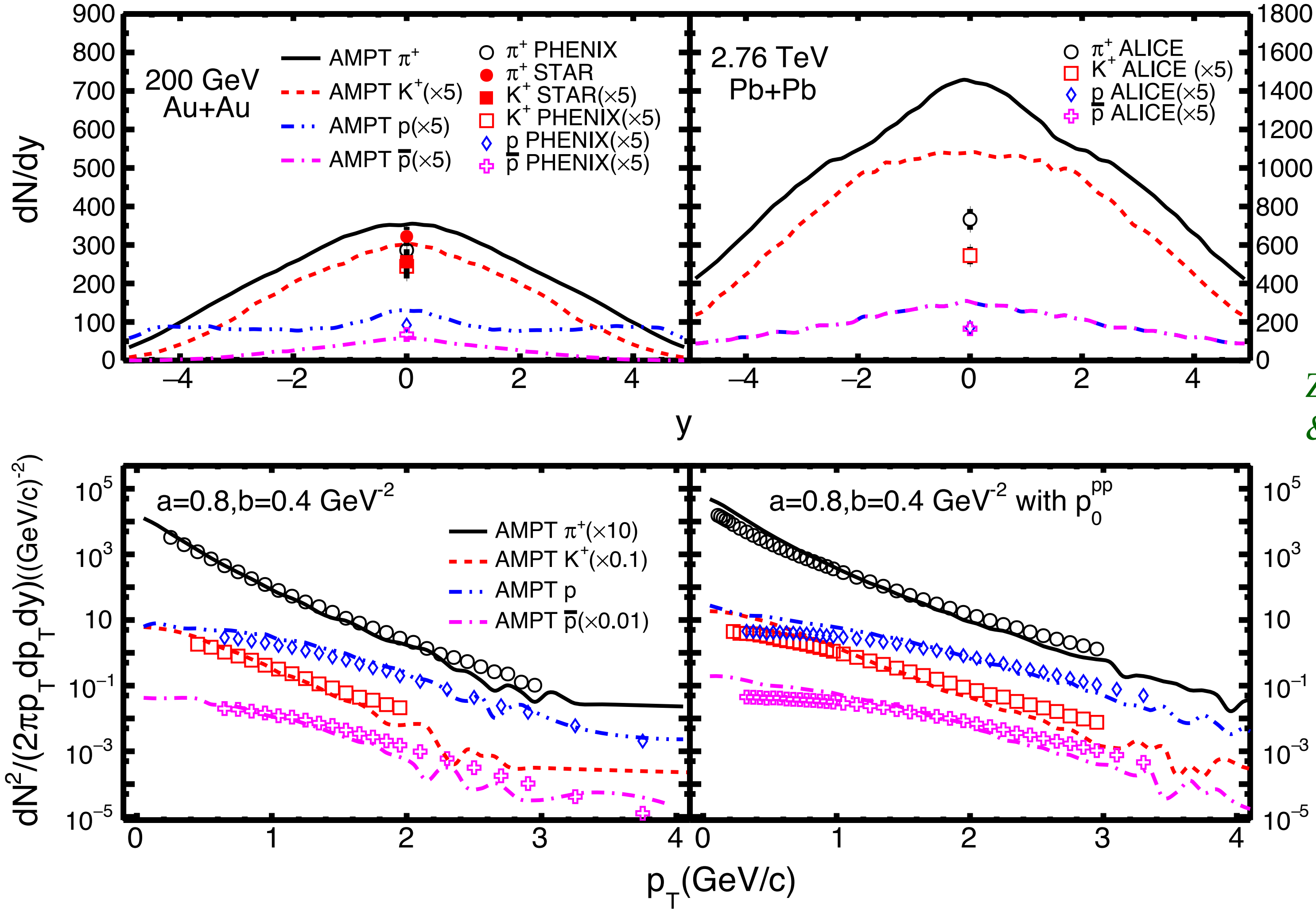


Free proton PDFs vs x from the old Duke-Owens set and newer sets



Shadowing function for Pb vs x and s (transverse position)

Improvement 1: modern PDF of nuclei



Zhang, Zheng, Liu, Shi & ZWL, PRC (2019)

We first fixed the model parameters ($p_0, \sigma_{\text{soft}}$, *Lund* a_L & b_L) with *pp* data,

then string melting AMPT **fails** to describe central AA data:

it overestimates most particle yields and also gives too-soft p_T spectra.

This issue was known/solved before: ZWL, PRC (2014)

Improvement 1: modern PDF of nuclei

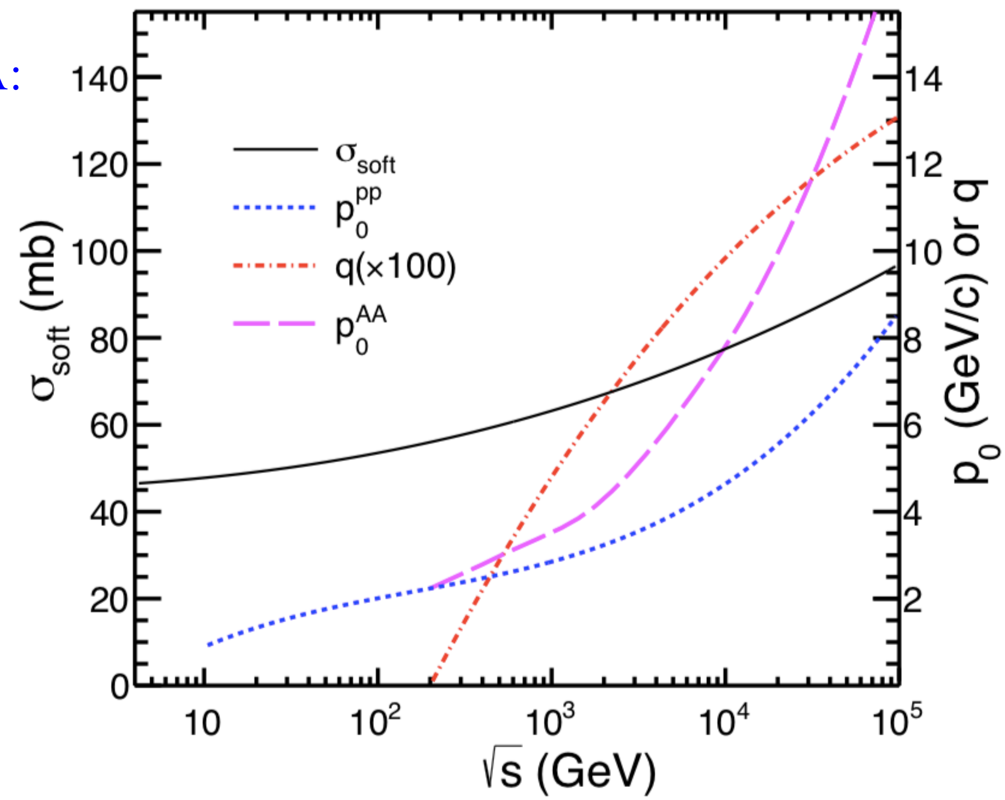
For AA collisions at high energies, we need to use a higher minijet cutoff p_0 to suppress σ_{jet} and the pQCD/hard contribution to particle yields:

$$\sigma_{\text{jet}} = \sum_{c,d} \frac{1}{1+\delta_{cd}} \int_{p_0^2}^{s/4} dp_T^2 dy_1 dy_2 \frac{d\sigma^{cd}}{dp_T^2 dy_1 dy_2}$$

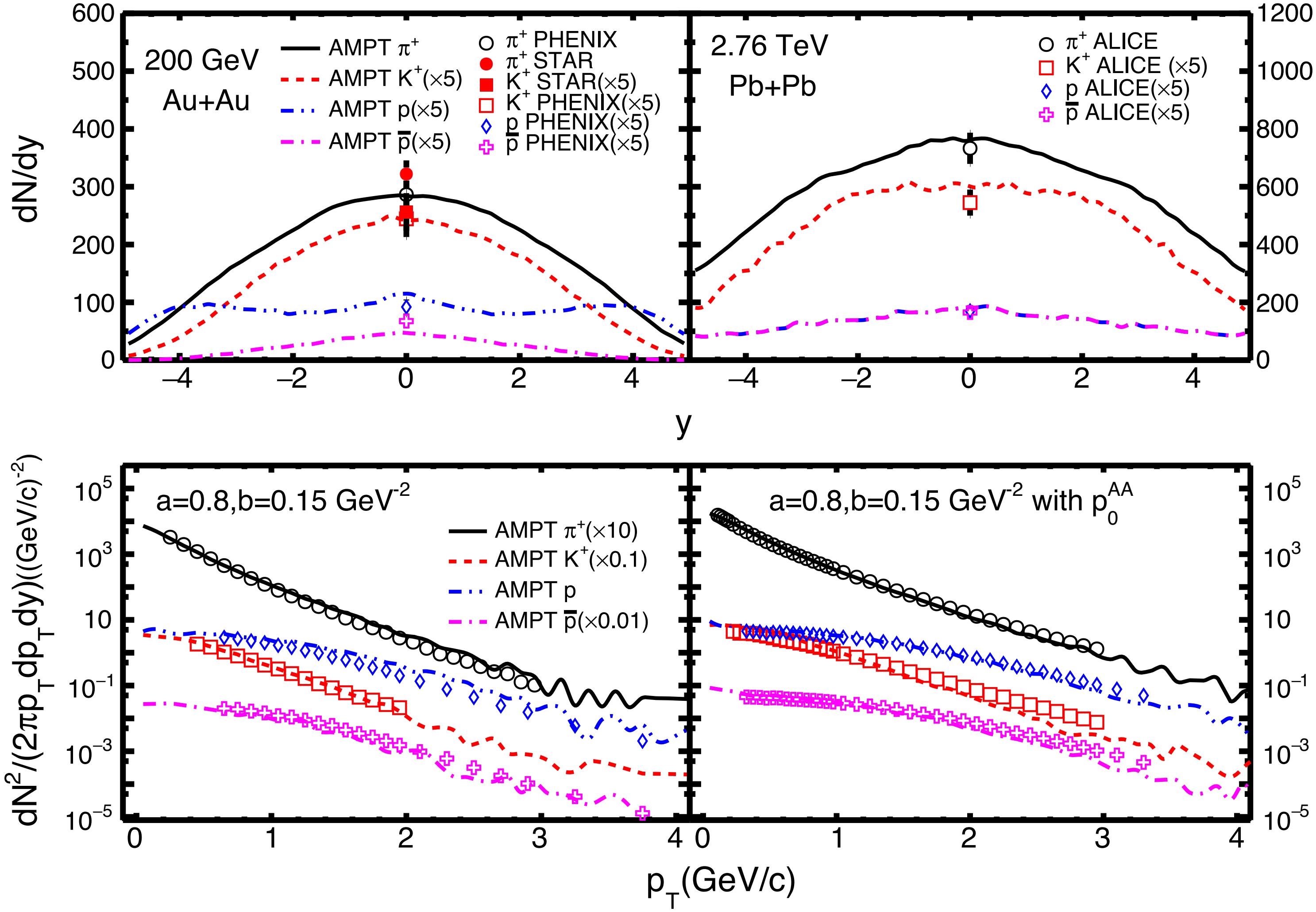
→ We introduce the A-scaling for central AA:

$$p_0^{AA} = p_0^{pp} A^{q(s)}$$

- the scaling is motivated by the CGC/saturation model, where $Q_s \propto A^{1/6}$ in the saturation regime.
- *q(s):* starts from 0 at 200A GeV, ~0.16 at ~10^7A GeV
- This increase of p_0 with energy is similar to HIJING2



Improvement 1: modern PDF of nuclei



After A-scaling of p_0 *(and decreasing Lund b_L to 0.15/GeV^2)*,
the string melting AMPT model can reasonably describe these central AA data.

Improvement 2: heavy flavor (HF)

$gg \rightarrow gg$ cross section in pQCD
is divergent for massless g,
so HIJING uses a minijet cutoff p_0 (for minijets of ALL flavours).

$$\frac{d\sigma}{dt} \sim \frac{9\pi\alpha_s^2}{2t^2}$$

But due to heavy quark mass, heavy flavor production
has a finite cross section and does not need a cutoff, (e.g. in FONLL):

$$g + g \rightarrow Q + \bar{Q}, \quad q + \bar{q} \rightarrow Q + \bar{Q}, \; \ldots$$

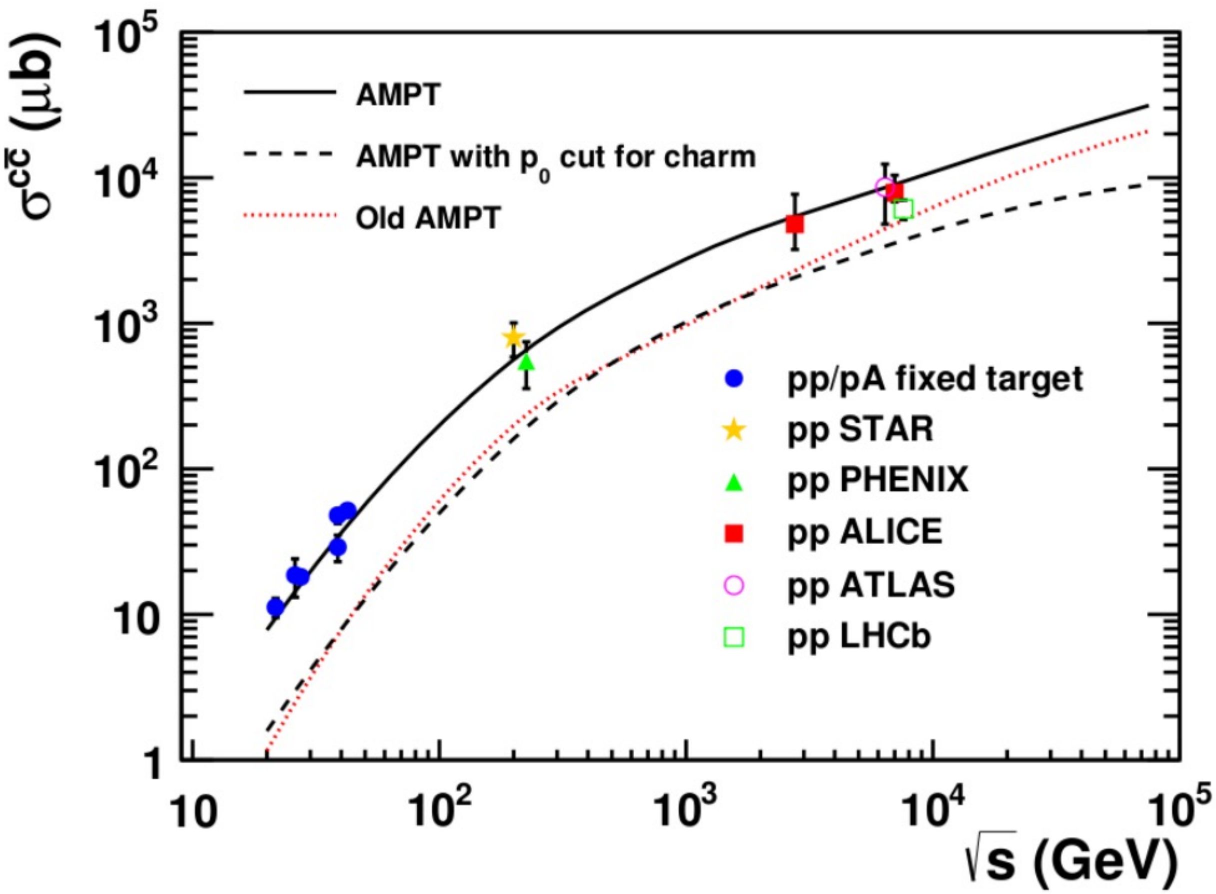
- So we removed the p_0 cut on HF productions
  in HIJING/AMPT.

Zheng, Zhang, Shi
& ZWL, PRC (2020)

- Unlike HIJING, we include HF in σ_{jet}: $\sigma_{jet} = \sigma_{jet}^{LF} + \sigma^{HF}$

- We also correct factor of ½ in certain σ_{jet} channels

Improvement 2: heavy flavor (HF)



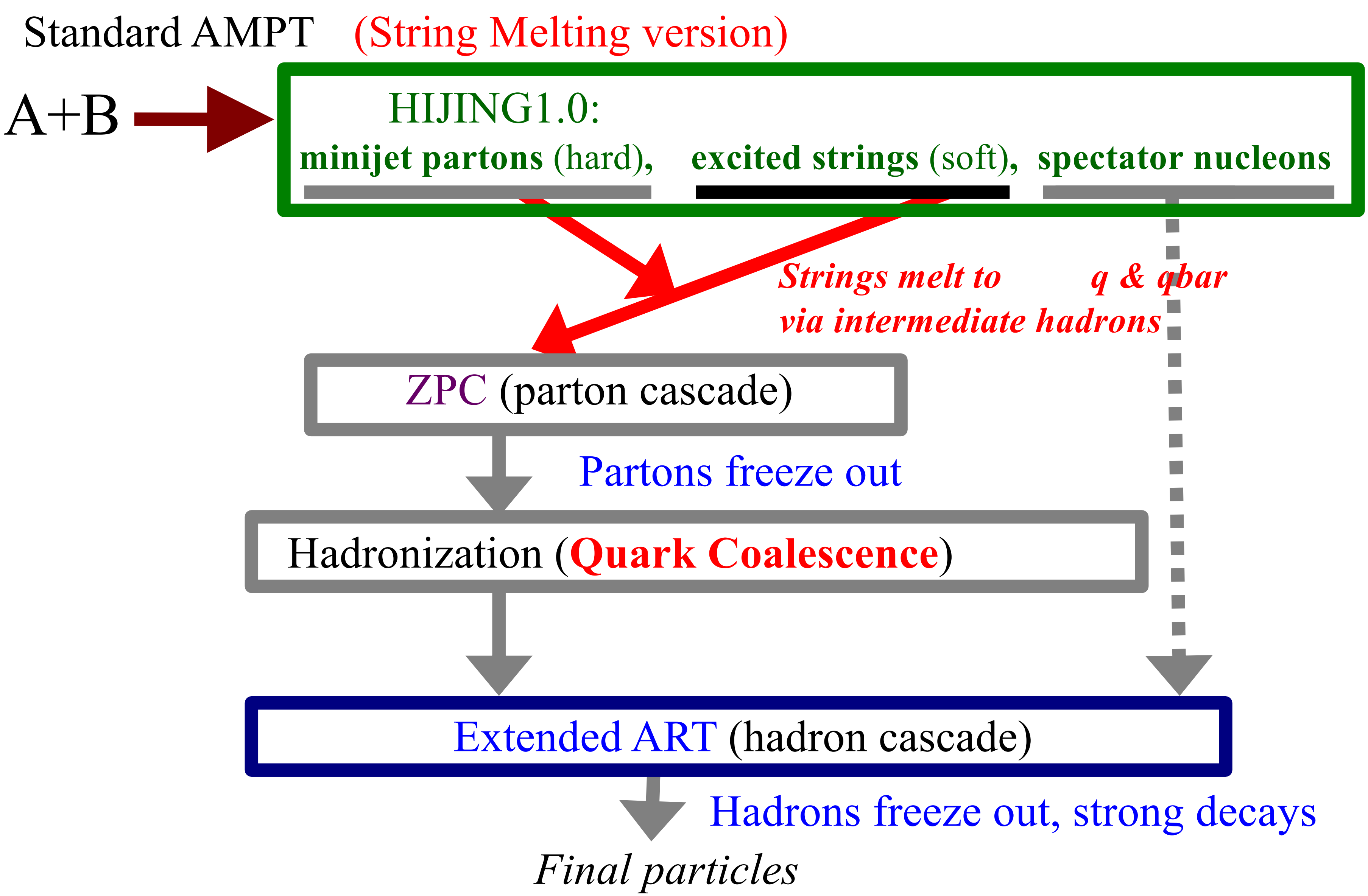
Zheng, Zhang, Shi & ZWL, PRC (2020)

- Old/public AMPT charm yield << data
- Removing p_0 in HF production greatly enhances charm yield
- AMPT now well describes world *pp* data on total $c\bar{c}$ cross section

Improvement 2: heavy flavor (HF)

We made further improvements recently:

Zhang, Zheng, Shi & ZWL, PLB (2023)



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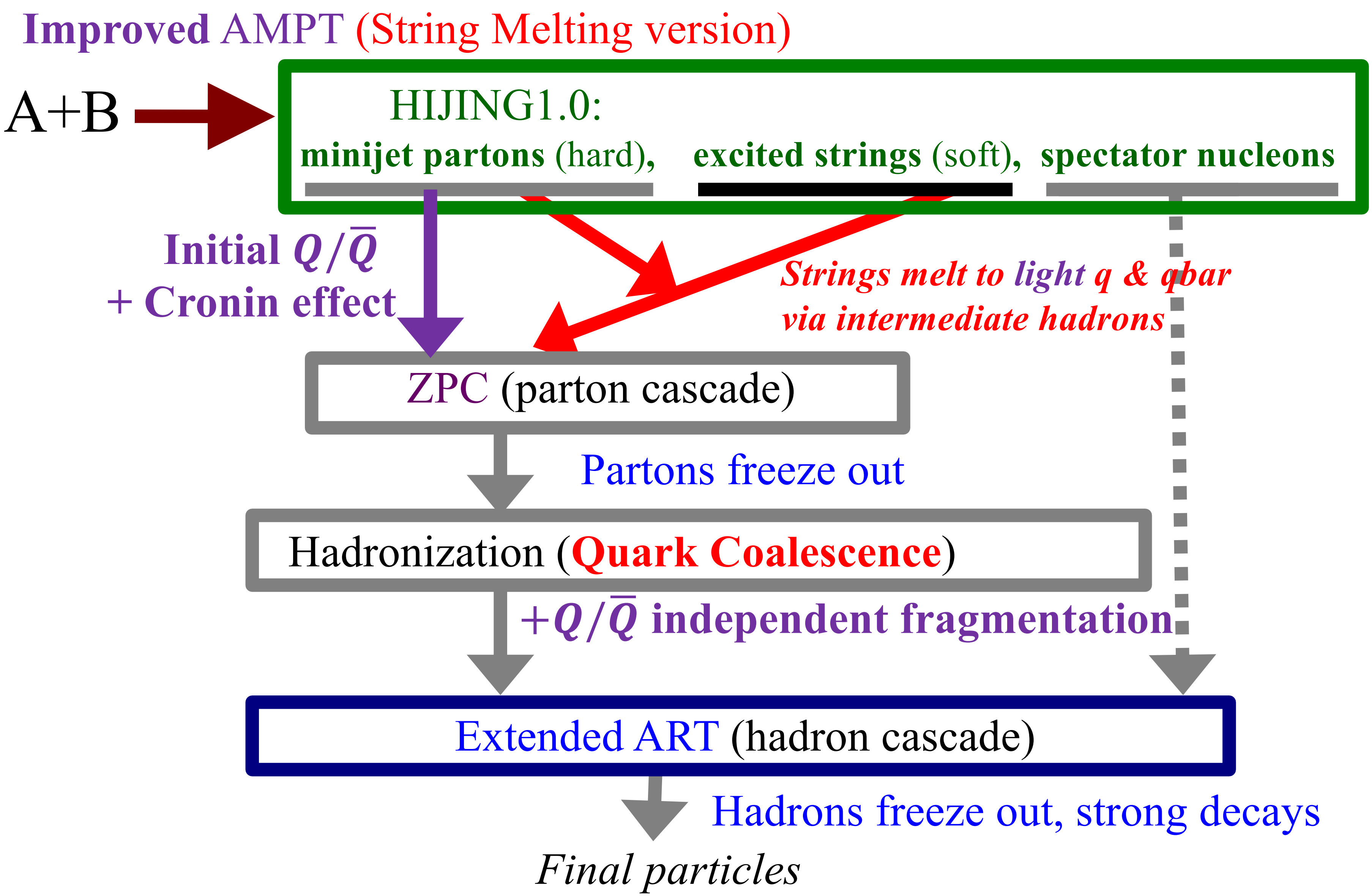
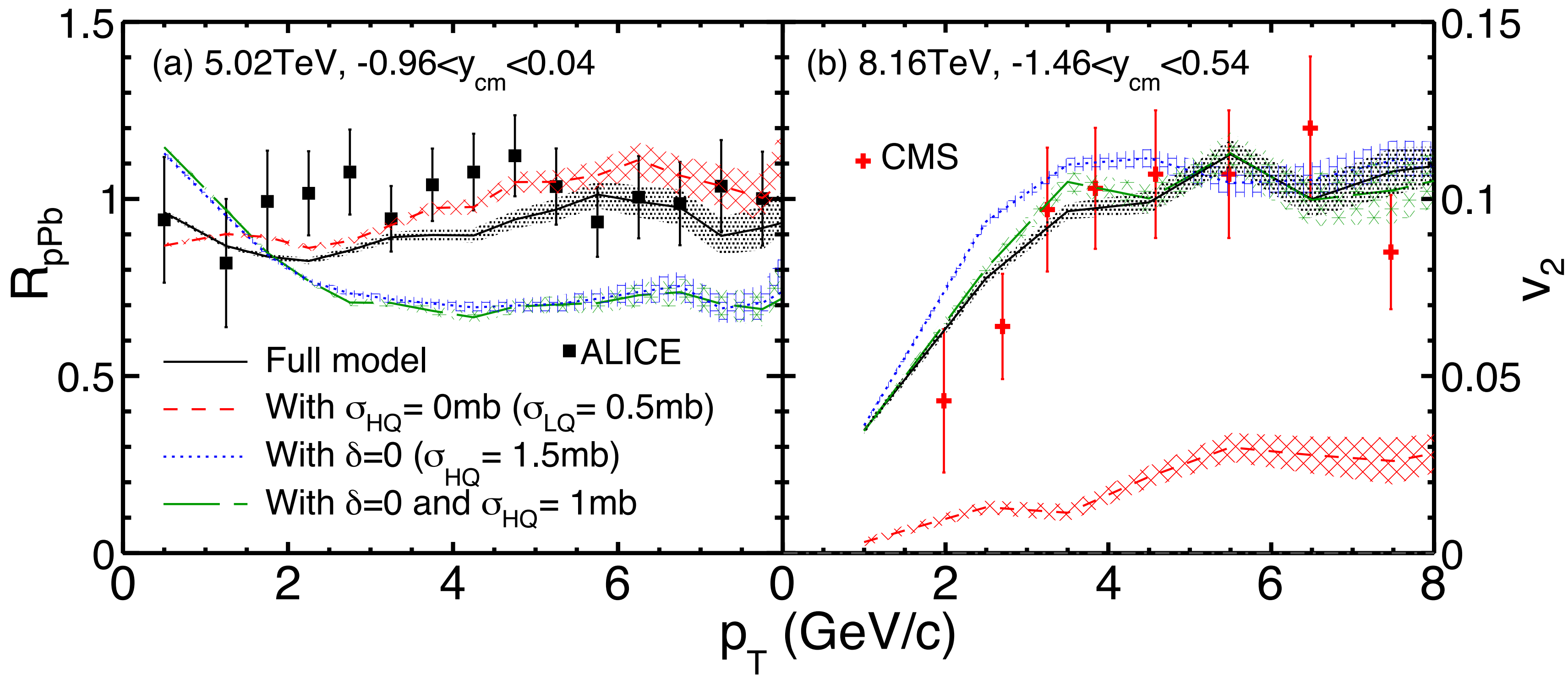


Improvement 2: heavy flavor (HF)

We have proposed the Cronin effect (k_T broadening) as a possible solution of the D^0 R_{pA}/v_2 puzzle.

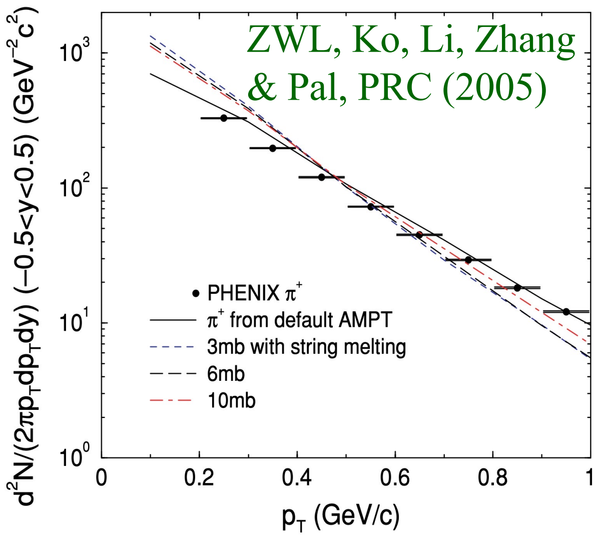
Zhang, Zheng, Shi & ZWL, PLB (2023)

Without the Cronin effect (δ=0): we can get sizable D^0 v_2, but R_{pA} is underestimated due to charm scatterings with medium (via σ_{HQ}).



The Cronin effect enhances charm R_{pA} at moderate/high p_T but has little effect on charm v_2 → resolves the R_{pA}/v_2 puzzle

Improvement 3: local nuclear scaling

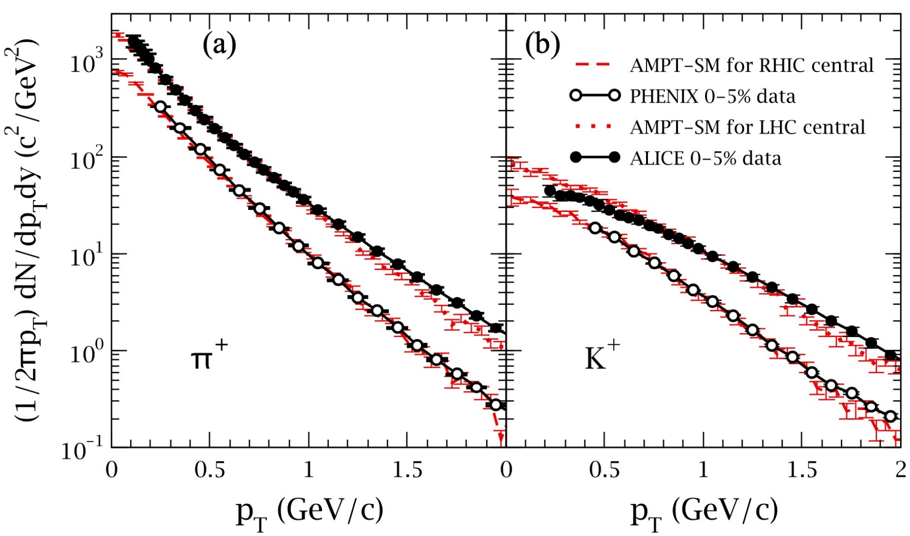


- String Melting AMPT describes flows and HBT but used to (*before 2014*) fail badly in hadron spectra.

Lund symmetric string fragmentation function:

$$f(z) \propto z^{-1}(1-z)^{a_L} e^{-b_L m_T^2/z}$$

b_L typical values (in 1/GeV^2):
0.5 (AMPT), ~0.58 (PYTHIA6.2), 0.9 (HIJING1)



ZWL, PRC (2014)

- We later realized that the model can simultaneously describe dN/dy, p_T –spectra & v_2 at low p_T in high energy central AA collisions if we use a very small Lund parameter b_L ~0.15,

  a small $b_L \rightarrow$ a higher string tension κ:

$$\kappa \propto \frac{1}{b_L(2+a_L)}$$

Improvement 3: local nuclear scaling

Different values of $\boldsymbol{b_L}$ are needed for pp and central AA,
same for the minijet cutoff scale $\boldsymbol{p_0}$ (*after using modern nPDFs*).
We have used A-scaling of p_0 for central AA as motivated by CGC.

Zhang, Zheng, Shi
& ZWL, PRC (2021)

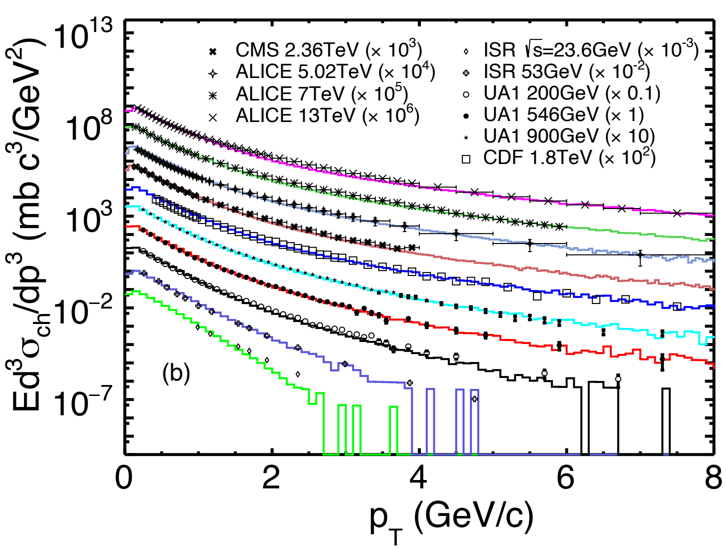
→ We propose a more general scaling by using local nuclear densities:

$$b_L(s_A, s_B, s) = \frac{b_L^{pp}}{[\sqrt{T_A(s_A)T_B(s_B)}/T_p]^{\beta(s)}}$$

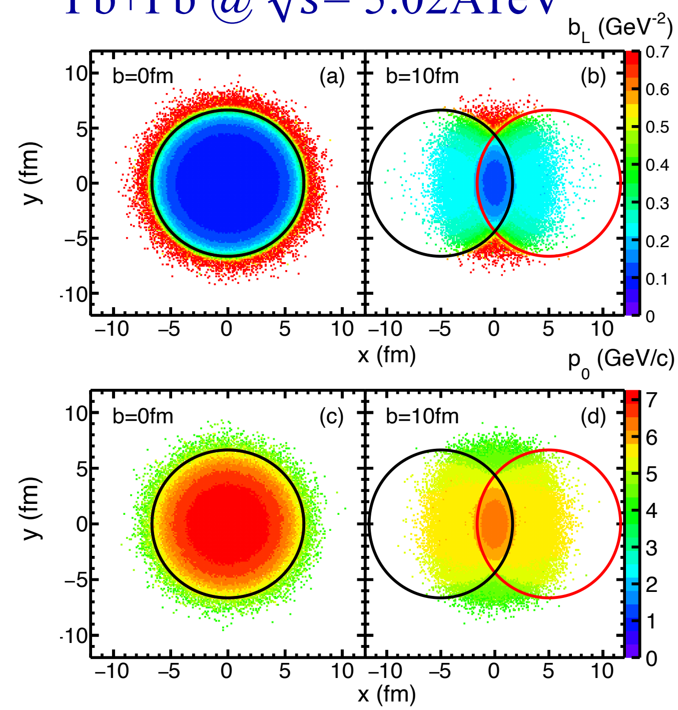
$$p_0(s_A, s_B, s) = p_0^{pp}(s)[\sqrt{T_A(s_A)T_B(s_B)}/T_p]^{\alpha(s)}.$$

We fit charged hadrons in *pp* to determine b_L^{pp} =0.7,
then used central Au+Au/Pb+Pb data to fit α(s), β(s)

pp
or $p\bar{p}$



Pb+Pb @ $\sqrt{s}$= 5.02ATeV

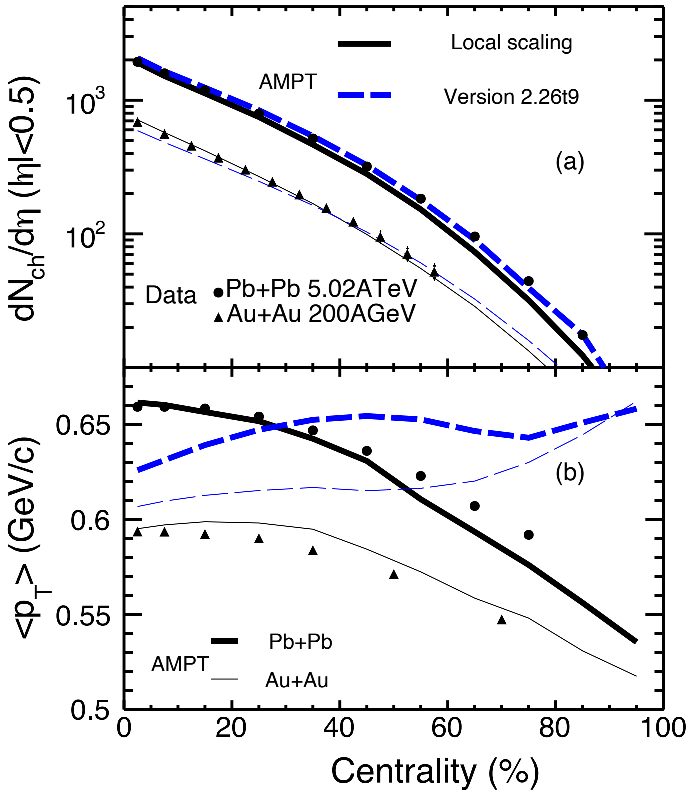


Improvement 3: local nuclear scaling

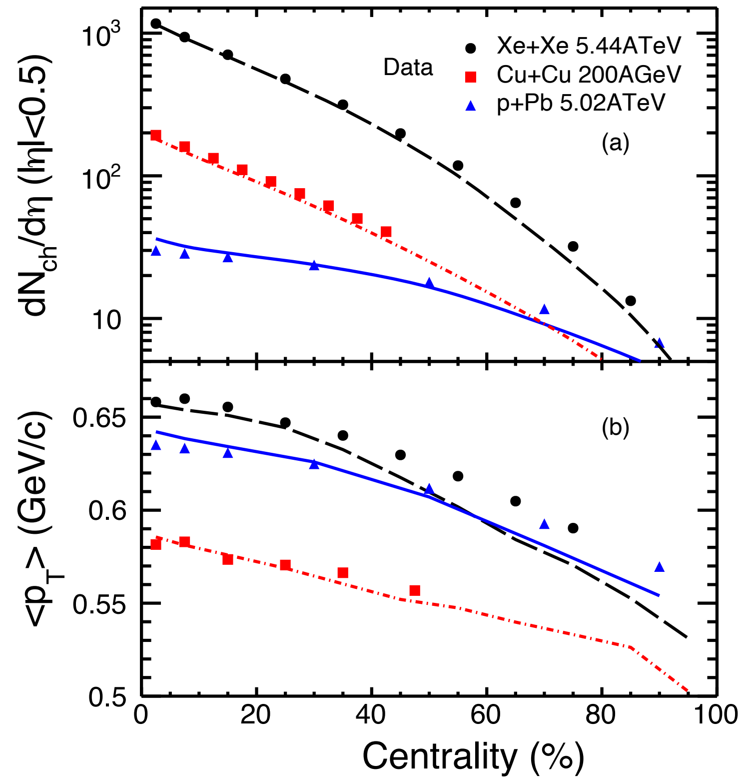
The scaling allows AMPT-SM to self-consistently describe the system size dependence, including centrality dependences of Au+Au & Pb+Pb and smaller systems.

Zhang, Zheng, Shi & ZWL, PRC (2021)

Centrality dependences of <p_T> are now reasonable, much better than public AMPT (v2.26t9)



Scaling also works for smaller systems:



Key input parameters of AMPT: a_L b_L p_0 σ (parton cross section)

No longer free parameters → can focus on QGP properties like σ & η

Improvement 4: parton transport

Flows like v_2 & v_3 at high energies mostly come from parton interactions:

$$\partial_t f + \frac{\partial \boldsymbol{x}}{\partial t} \cdot \nabla_x f = C[|M^2| f_1 f_2] \propto \sigma f_1 f_2$$

for 2-body scatterings

But ZPC/MPC cascade solution of the Boltzmann equation at large densities n and/or cross sections σ is well known to suffer from causality violation.

Zhang, Comp Phys Comm (1998);
Monlar & Gyulassy, PRC (2000);
Cheng et al. PRC (2002); …

Naively, the cascade solution using geometric cross sections is only accurate in the dilute limit when the opacity parameter χ is small:

$$\chi \equiv \frac{r}{\lambda} = \frac{\sigma^{3/2} n}{\sqrt{\pi}} < 1,$$

Zhang, Gyulassy & Pang, PRC (1998)

where the range of particle interaction $r <$ mean free path λ

$$r \equiv \sqrt{\frac{\sigma}{\pi}} \qquad \lambda = \frac{1}{\sigma\, n}$$

Improvement 4: parton transport

Pang, CU-TP-815 (1996)
Gyulassy, Zhang & Pang, PRC (1998)

Particle subdivision reduces causality violation:

$$\partial_t f + \frac{\partial \boldsymbol{x}}{\partial t} \cdot \nabla_x f = C[|M^2| f_1 f_2] \propto \sigma f_1 f_2$$

This is because the above Boltzmann equation is invariant under transformation:

$$f \longrightarrow f * l \qquad \text{and} \qquad \sigma \longrightarrow \frac{\sigma}{l}$$

which reduces the opacity χ:

$$\chi \equiv \frac{\sigma^{3/2} n}{\sqrt{\pi}} \longrightarrow \frac{\chi}{\sqrt{l}}$$

l: subdivision factor

However, subdivision method is very CPU-consuming;
more importantly, it changes event-by-event fluctuations & correlations.

→ Test then improve the accuracy of parton transport (*without using subdivision*)

Improvement 4: parton transport

We have tested ZPC for partons in a box:

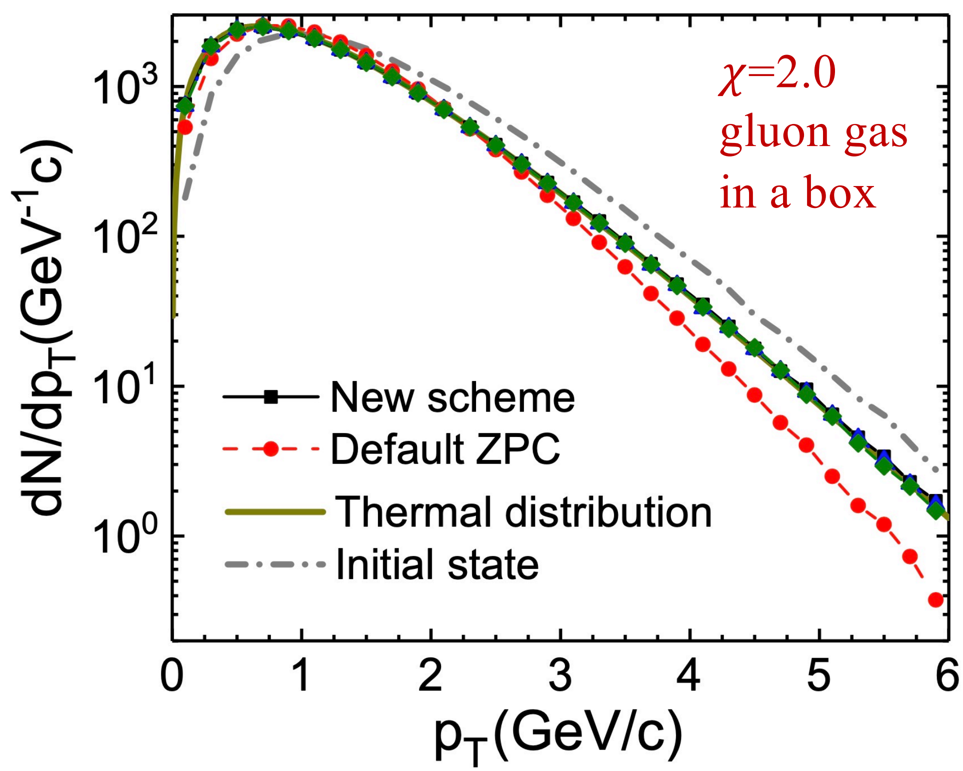
Zhao, Ma, Ma & ZWL, PRC (2020)

Ordering time \ Collision time	ct_1 & ct_2	$min(ct_1, ct_2)$	$(ct_1 + ct_2)/2$	$max(ct_1, ct_2)$
$min(ct_1, ct_2)$	A	B (new scheme)	C	D
$(ct_1 + ct_2)/2$	E	F	G (default ZPC scheme)	H
$max(ct_1, ct_2)$	I	J	K	L

- Parton cascade has freedom in choosing collision time (ct) and/or collision ordering time

- Default ZPC (t-avg scheme) fails to maintain thermal equilibrium at high opacities

- A new choice (t-min scheme) gives the expected thermal distribution



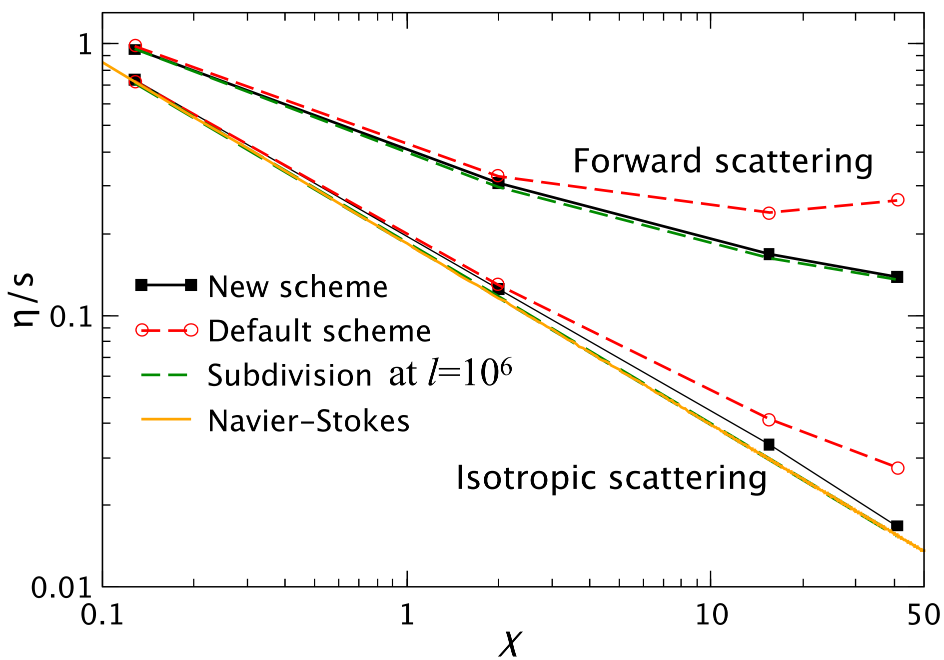
Improvement 4: parton transport

Shear viscosity η and η/s:
the new t-avg scheme agrees well with
Navier-Stokes result for isotropic scatterings
even at very high opacities up to χ~40 !

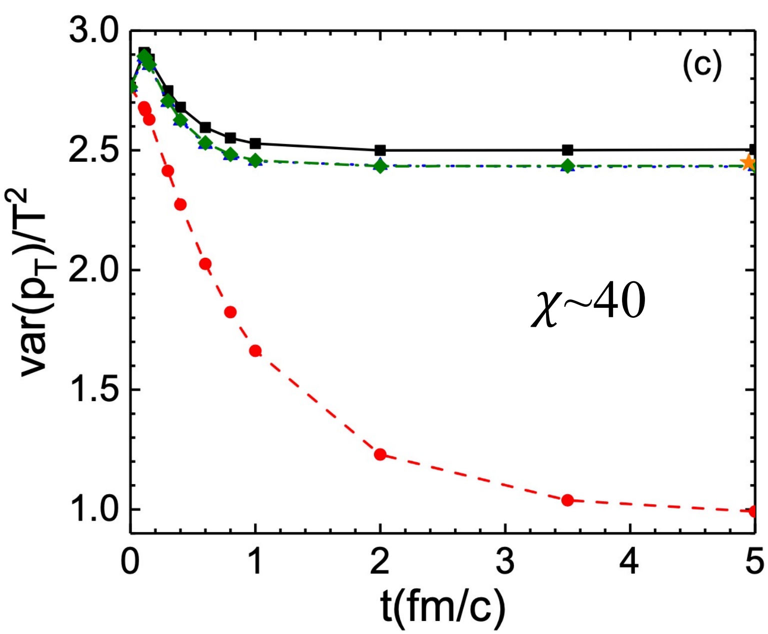
- Default ZPC scheme fails at high χ.

$$\eta^{NS}=1.265\,\frac{T}{\sigma}$$

De Groot, Van Leeuwen & Van Weert, Relativistic Kinetic Theory (1980);
Huovinen & Molnar, PRC (2009);
Plumari, Puglisi, Scardina & Greco, PRC (2012);
MacKay and ZWL, EPJC (2022)



Zhao, Ma, Ma & ZWL, PRC (2020);
ZWL & Zheng, NST (2021)



Time evolution of spectrum agrees well with subdivision results.

Improvement 4: parton transport

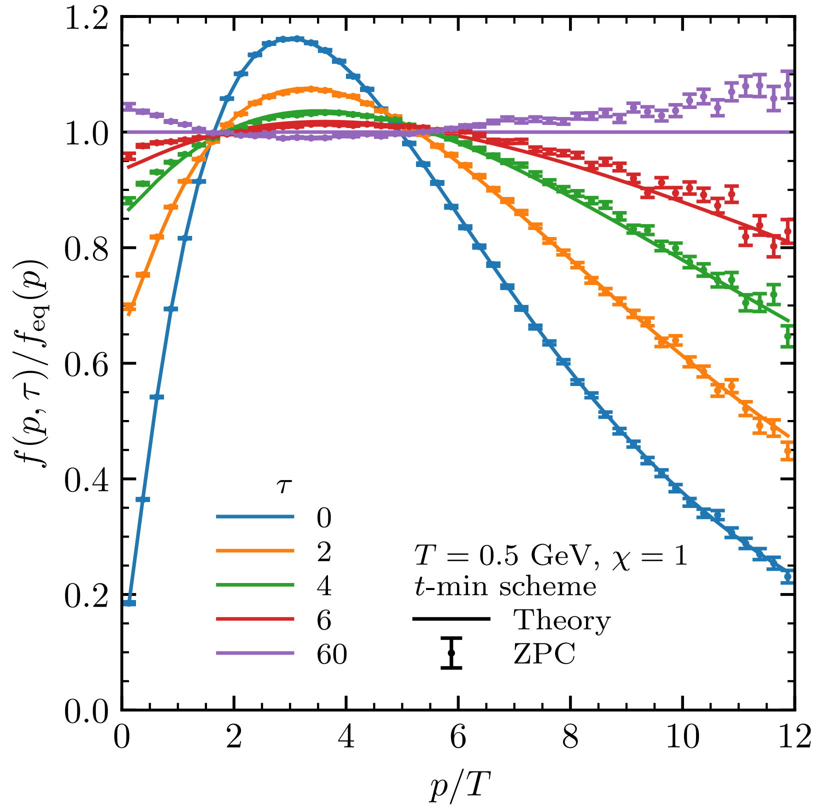
Recently, an exact solution of the relativistic Boltzmann equation has been found for a massless homogeneous gas under 2-body isotropic scatterings.

→ Test the full time evolution of the momentum spectra & improve parton transport.

Bazow, Denicol, Heinz, Martinez & Noronha, PRL (2016) & PRD (2016)

For non-expanding spacetime, the solution is

$$f_{\text{theory}}(p,\tau) = \exp\left(-\frac{p}{T\kappa(\tau)}\right)\left[\frac{4\kappa(\tau)-3}{\kappa^4(\tau)} + \frac{p}{T}\frac{1-\kappa(\tau)}{\kappa^5(\tau)}\right].$$



$\tau \propto t$ is a scaled time,

$$\kappa(\tau) = 1 - \exp(-\tau/6)/4$$

- Spectra evolves from highly off-equilibrium to a thermal distribution $f_{\text{eq}}(p)$
- ZPC with t-min scheme performs quite well.

Mendenhall & ZWL, arXiv:2507.23107

Improvement 4: parton transport

We then use a more general scheme for parton collision time:

$$ct = \min(ct_1, ct_2) + r\,|ct_1 - ct_2|$$

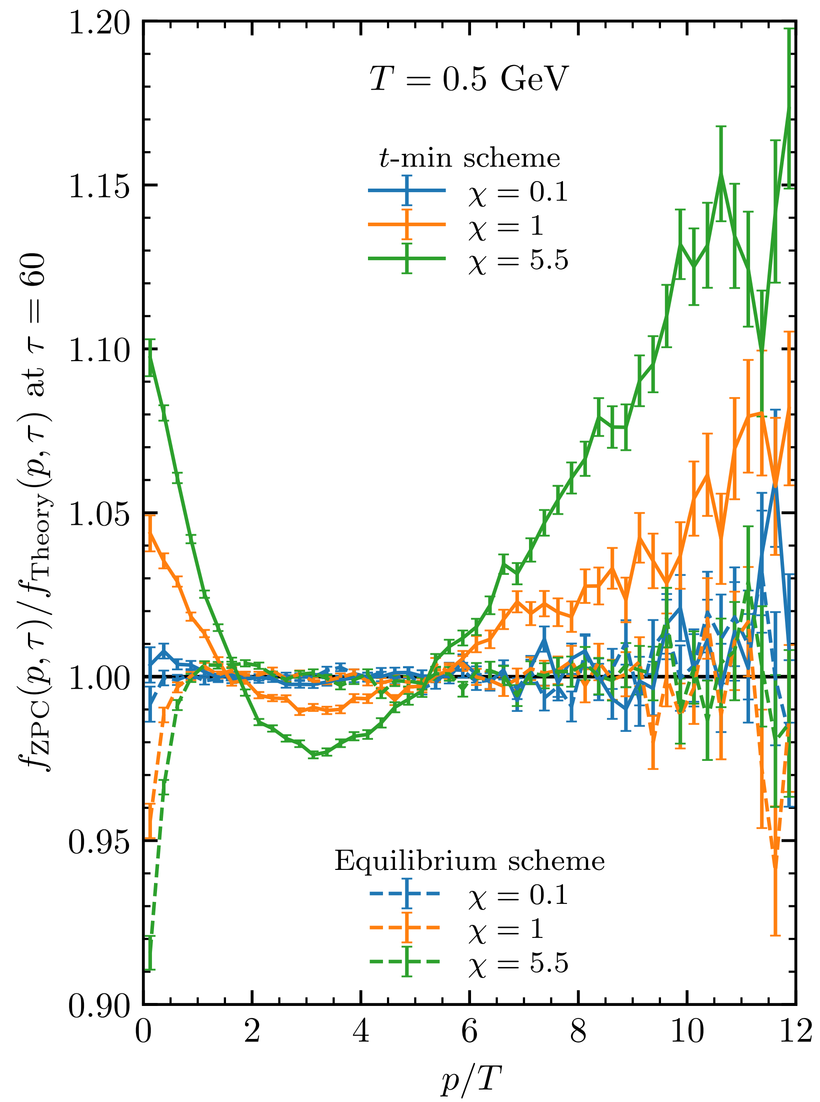
Default ZPC (t-avg scheme): $r = 1/2$

The t-min scheme: $r = 0$

General schemes: $r(\chi)$

fitted to minimize spectra difference;

gives even better spectra in equilibrium and during the time evolution even at very high opacities, with mean relative deviation <1%.



Mendenhall & ZWL, arXiv:2507.23107

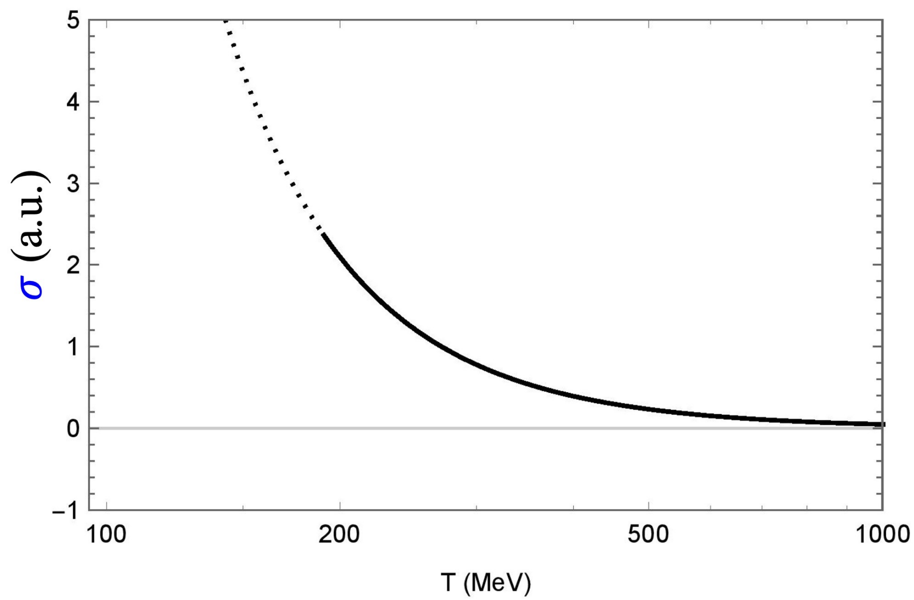
Improvement 4: parton transport

Causality violation in current AMPT is small due to small σ (<=3mb) — Molnar 1906.12313

But finite-temperature pQCD → $\mu \propto gT$
→ σ will be T-dependent
& much larger close to hadronization, reflecting the small QGP η/s

Arnold, Moore & Yaffe, JHEP (2003);
Csernai, Kapusta & McLerran, PRL (2006)

→ ZPC with improved collision scheme will lead to accurate results even at high opacities



So far, AMPT always uses constant σ & μ.

With $\mu \propto gT$
→ $\sigma \propto 1/\mu^2$ will be larger at lower T
→ $\eta \propto T/\sigma$, $\eta/s \propto \frac{1}{T^2\sigma}$
will have the expected T- & t-dependences

MacKay and ZWL, EPJC (2022)

→ improve ZPC/AMPT as a dynamical model to solve finite-T kinetic theory

Ohanaka, Ross & ZWL, in preparation

Summary

A multi-phase transport (AMPT) model
provides a self-contained kinetic description of heavy ion collisions
and is especially suitable for studies of non-equilibrium dynamics

Recent improvements make AMPT more versatile and accurate:

- Modern PDF & nuclear shadowing
  enable better studies of pQCD productions
  such as high-p_T and heavy flavor observables
- Local nuclear scaling of two key parameters
  significantly reduces uncertainty from free model parameters
  and enables us to focus on QGP properties like η/s
- Improved parton transport gives accurate results at very high opacities
  and lays the foundation to develop ZPC/AMPT into
  a dynamical model of finite-temperature QCD kinetic theory

Thank you!