NNLO corrections to Unpolarized and Polarized SIDIS

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: Phys. Rev. Lett. 133, 211905

HUN-REN Wigner Research Centre for Physics

Outline

- Introduction
- Theoretical Framework
- Relevant Proceses
- Calculation of Diagrams
- Calculation of Master Integrals
- UV and IR divergences
- Numerical Impact
- Taking Forward…
- Abelianization
- Conclusions and Future directions

Introduction

- Processes with identified final state hadrons play important roles in QCD. They provide crucial information on the splitting function and fragmentation function.

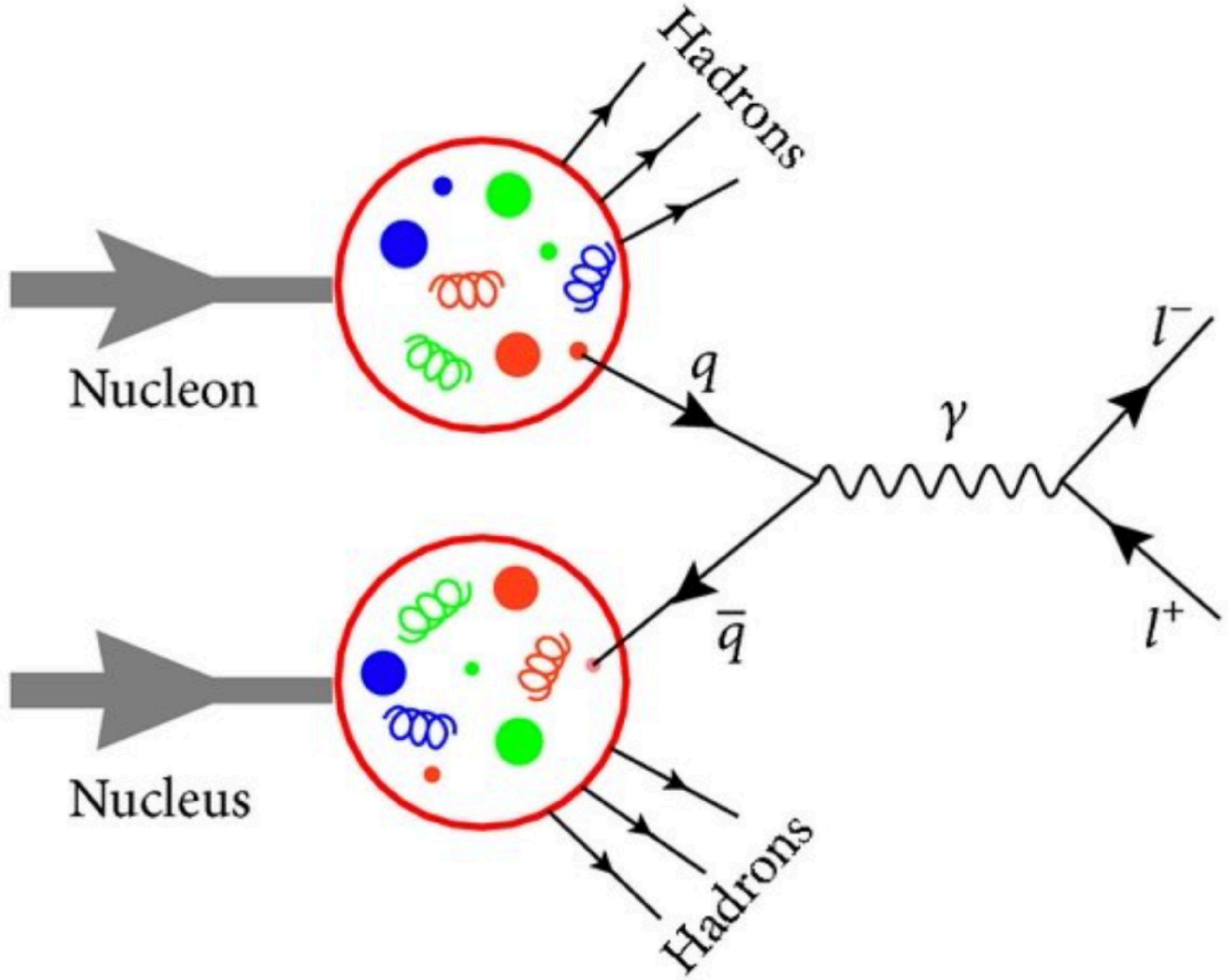
- Hadron production serves as a powerful probe of nucleon or nuclear structure.

- Hadron production data tests our key concepts in QCD at high energies such as factorization, universality of splitting functions, and perturbative calculations.

- Because electrons do not manifest any internal structure, they can be used as a precise probe of the more complicated nucleons and nuclei.

Drell-Yan Production at LHC

$$\sigma(q^2,\tau) = \sigma_0(\mu_R^2)\int \frac{dz}{z}\Phi_{ab}\left(\frac{\tau}{z},\mu_F^2\right)\Delta_{ab}(q^2,\mu_F^2,z)$$



Parton Distribution Function

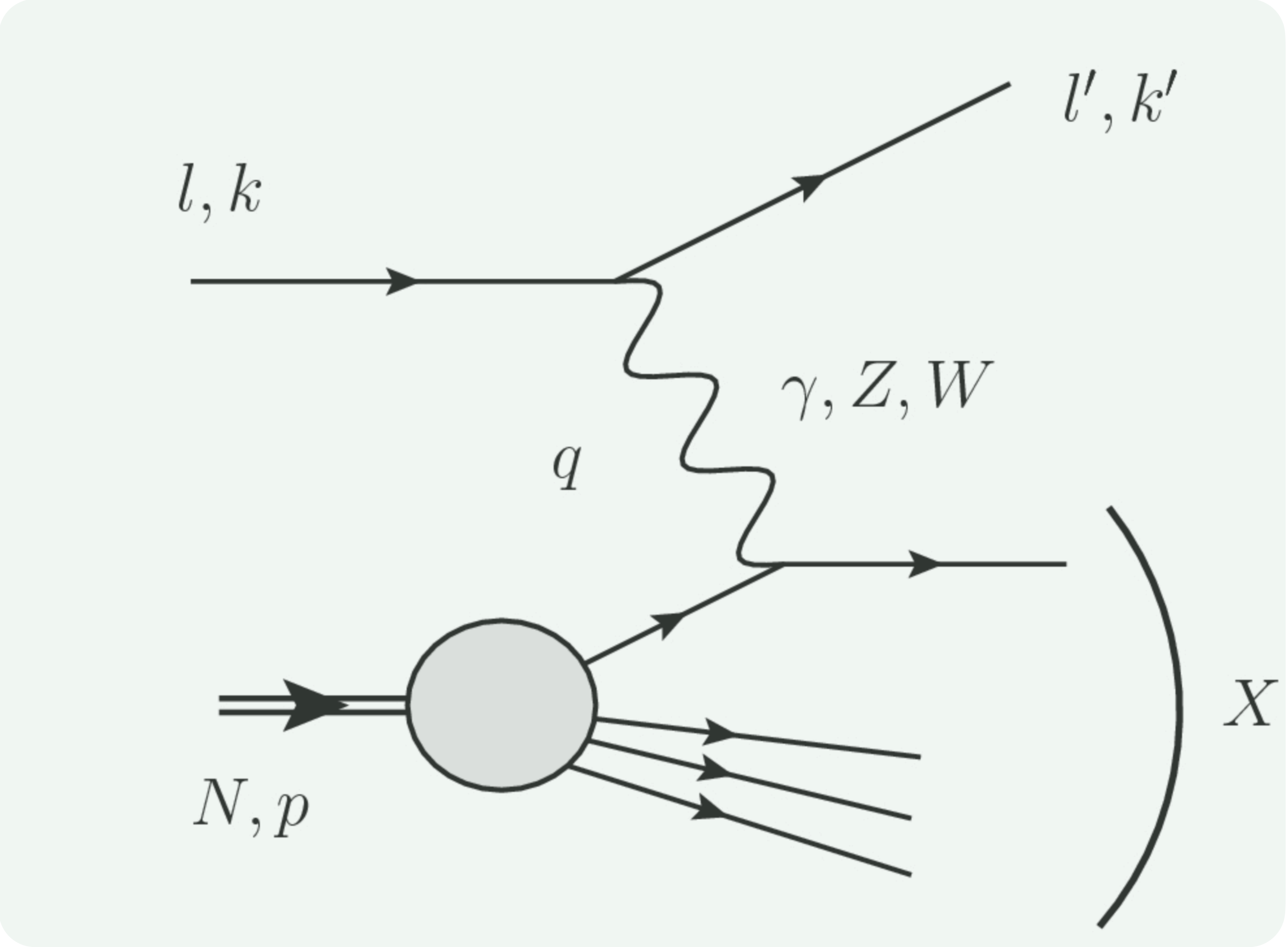
$$\Phi_{ab}(\mu_F^2,z) = \int \frac{dy}{y} f_a(y,\mu_F^2) f_b\left(\frac{z}{y},\mu_F^2\right)$$

Role of DIS

Inclusive DIS (Deep Inelastic Scattering),

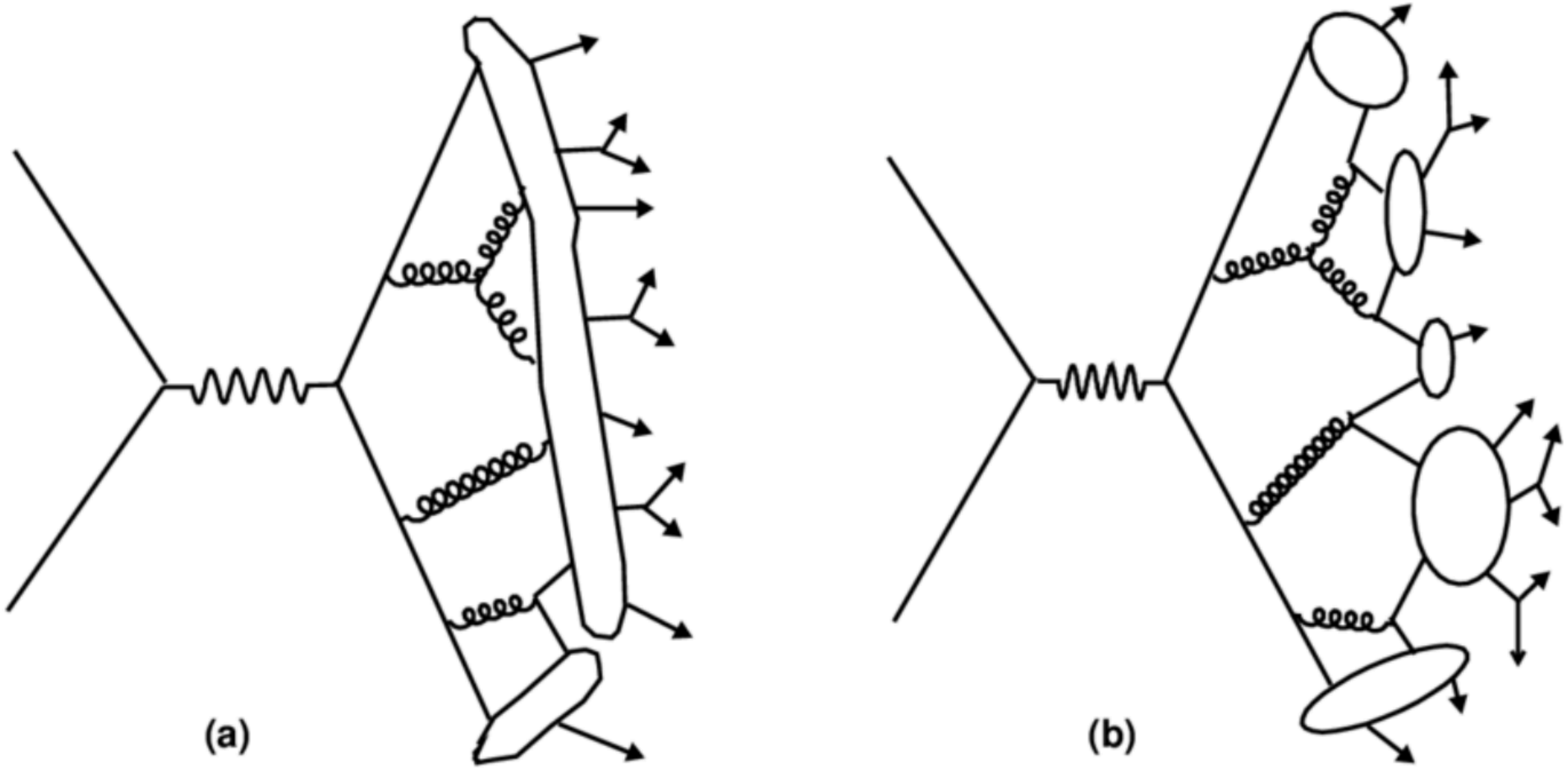
lepton + hadron → lepton + X

One sums up all the particles in the final state, except the scattered lepton



Depends on Parton Distribution Function (PDF) of Incoming hadron.

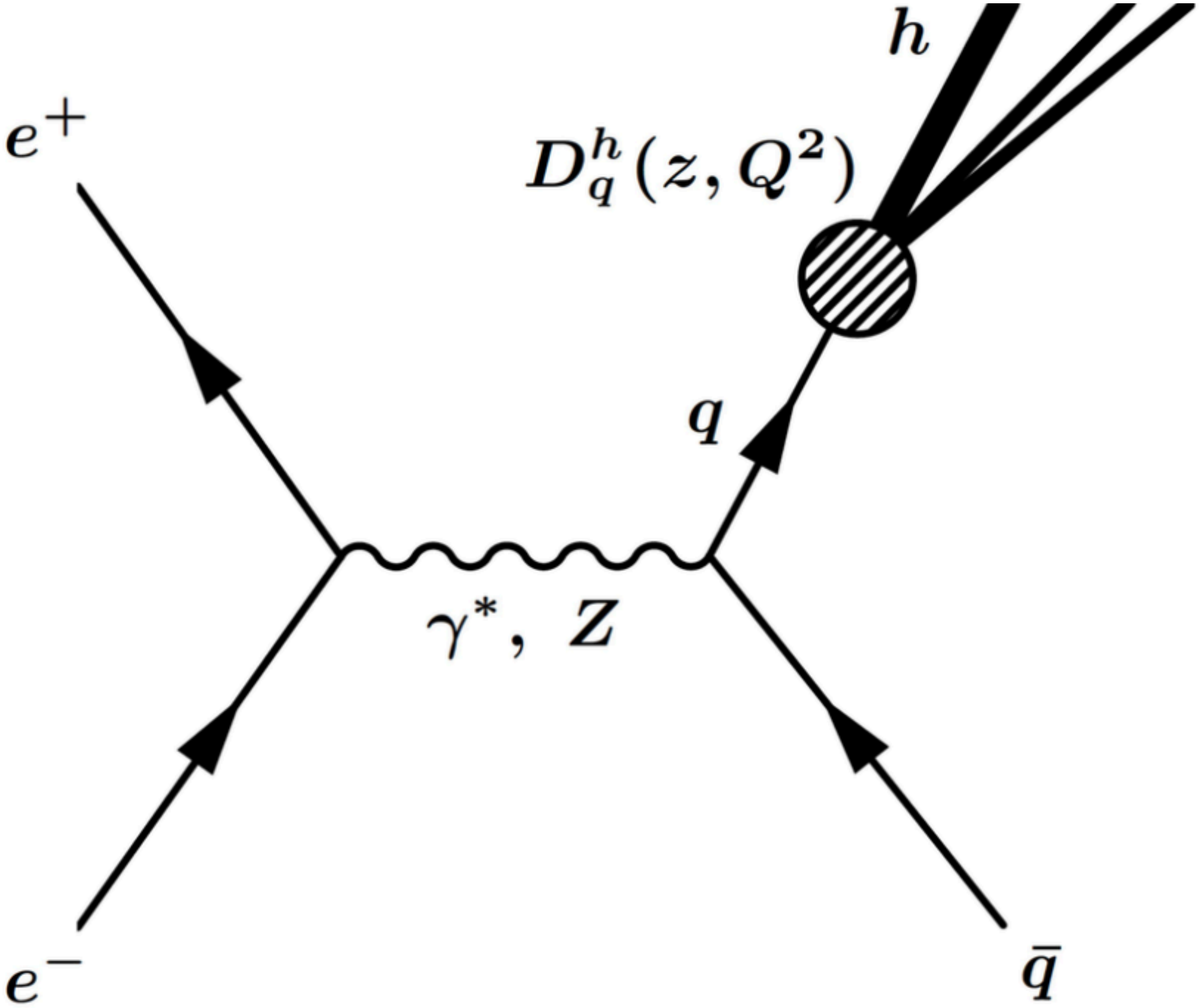
Hadronization



Fragmentation Function:

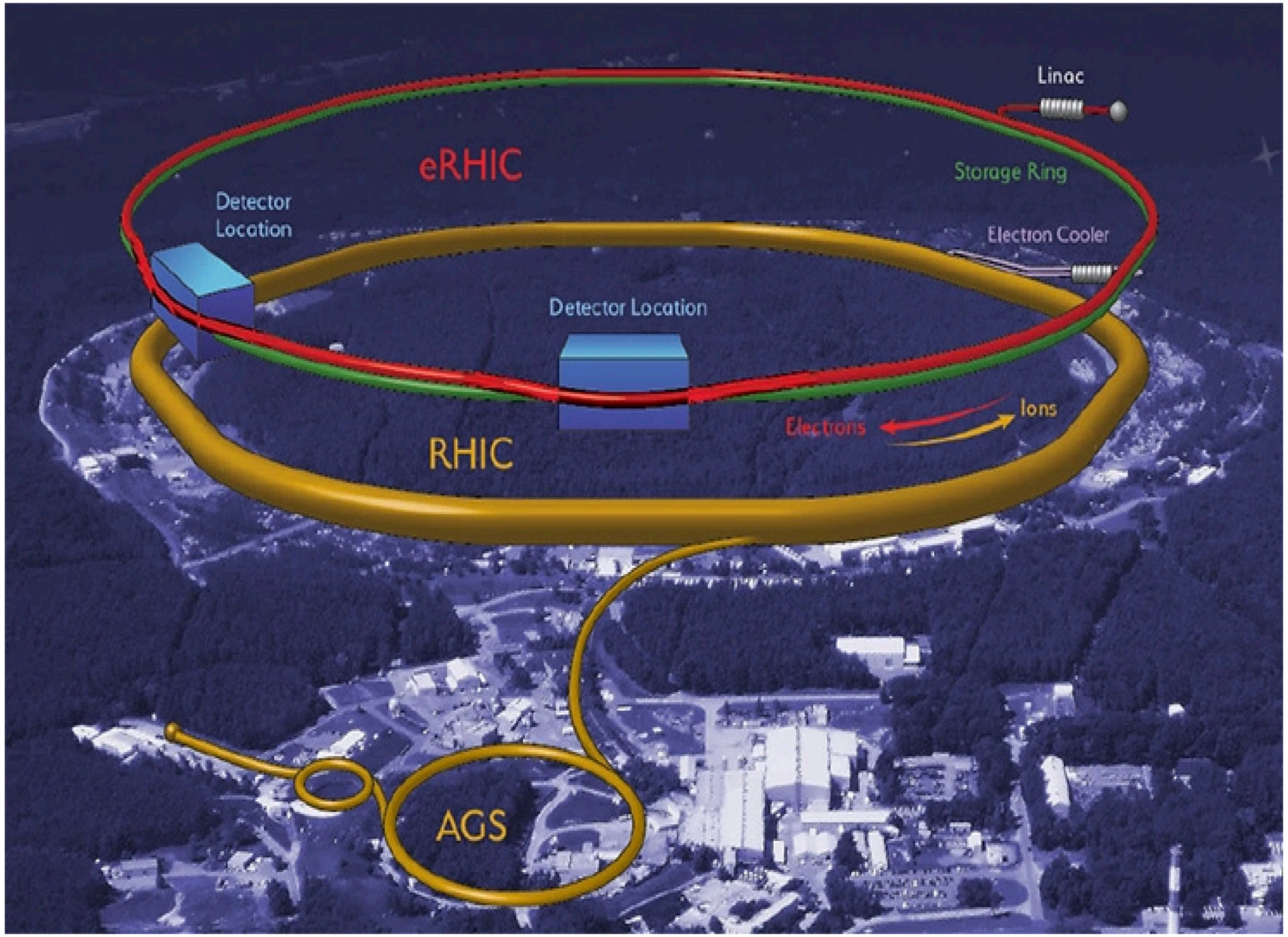
Probability of a Parton converting to Hadron

Fragmentation Function



EIC Goals

- Precision 3D imaging of hadrons.
- Solving the proton spin puzzle.
- Gluon saturation and Color glass condensate.
- Quark and gluon confinement.
- Mass problem of nucleons.

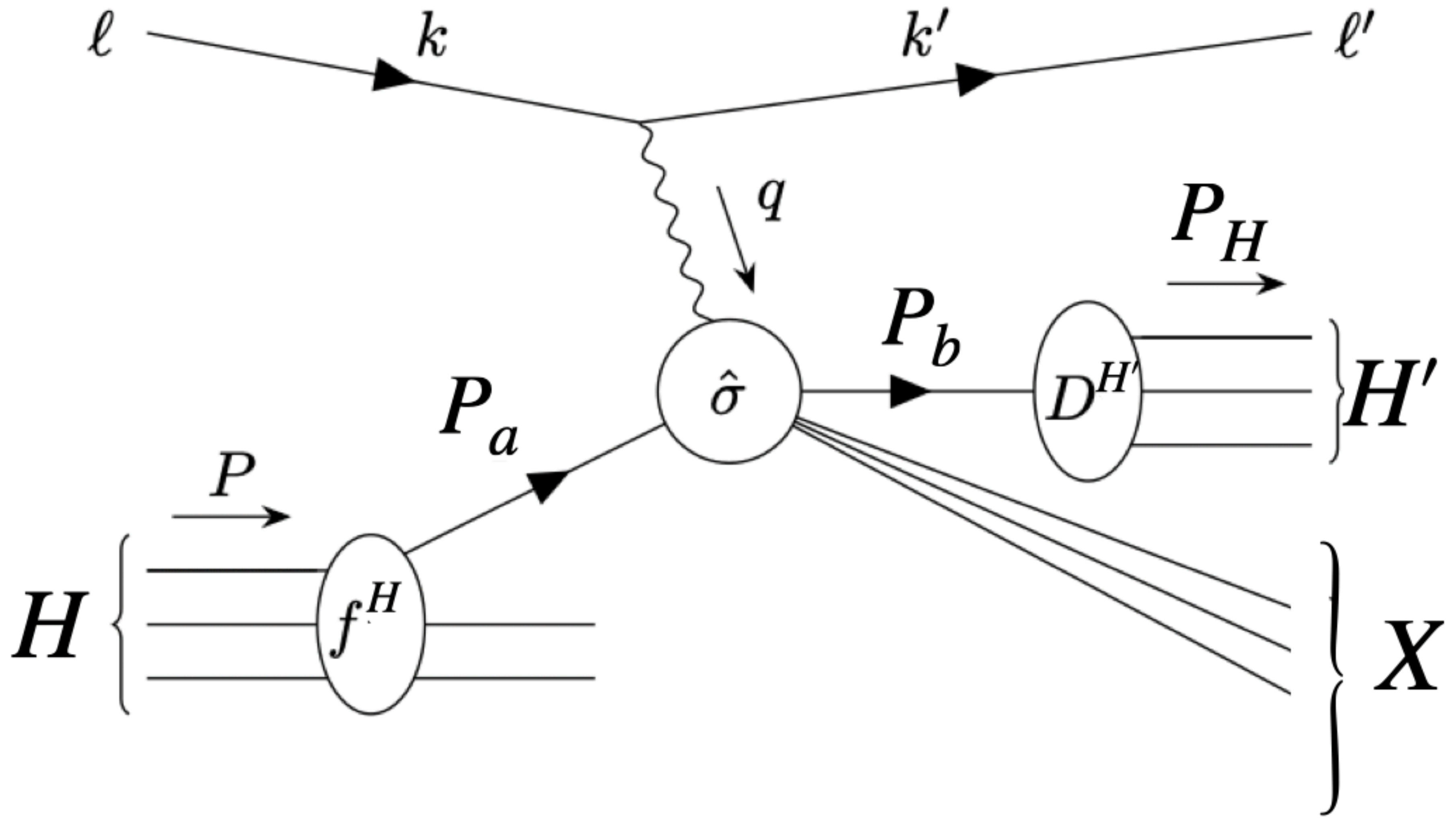


SIDIS ?

SIDIS (Semi-Inclusive Deep Inelastic Scattering),

lepton + hadron ⟶ lepton + hadron + X

In SIDIS, in addition to the scattered lepton, we tag on one of the outgoing hadron



Depends on Parton Distribution Function (PDF) of Incoming hadron and Fragmentation Function (FF) of Outgoing hadron.

NLO: Altarelli et al 1979

SV NNLO

Vogelsang et al 2022

Nuclear Physics B

Volume 160, Issue 2, 3 December 1979, Pages 301-329

Processes involving fragmentation functions beyond the leading order in QCD ☆

G. Altarelli, R.K. Ellis, G. Martinelli, So-Young Pi

Threshold resummation at N^3LL accuracy and approximate N^3LO corrections to semi-inclusive DIS

March 16, 2022

Maurizio Abele[a], Daniel de Florian[b], Werner Vogelsang[a]

Approximate NNLO QCD corrections to semi-inclusive DIS

March 16, 2022

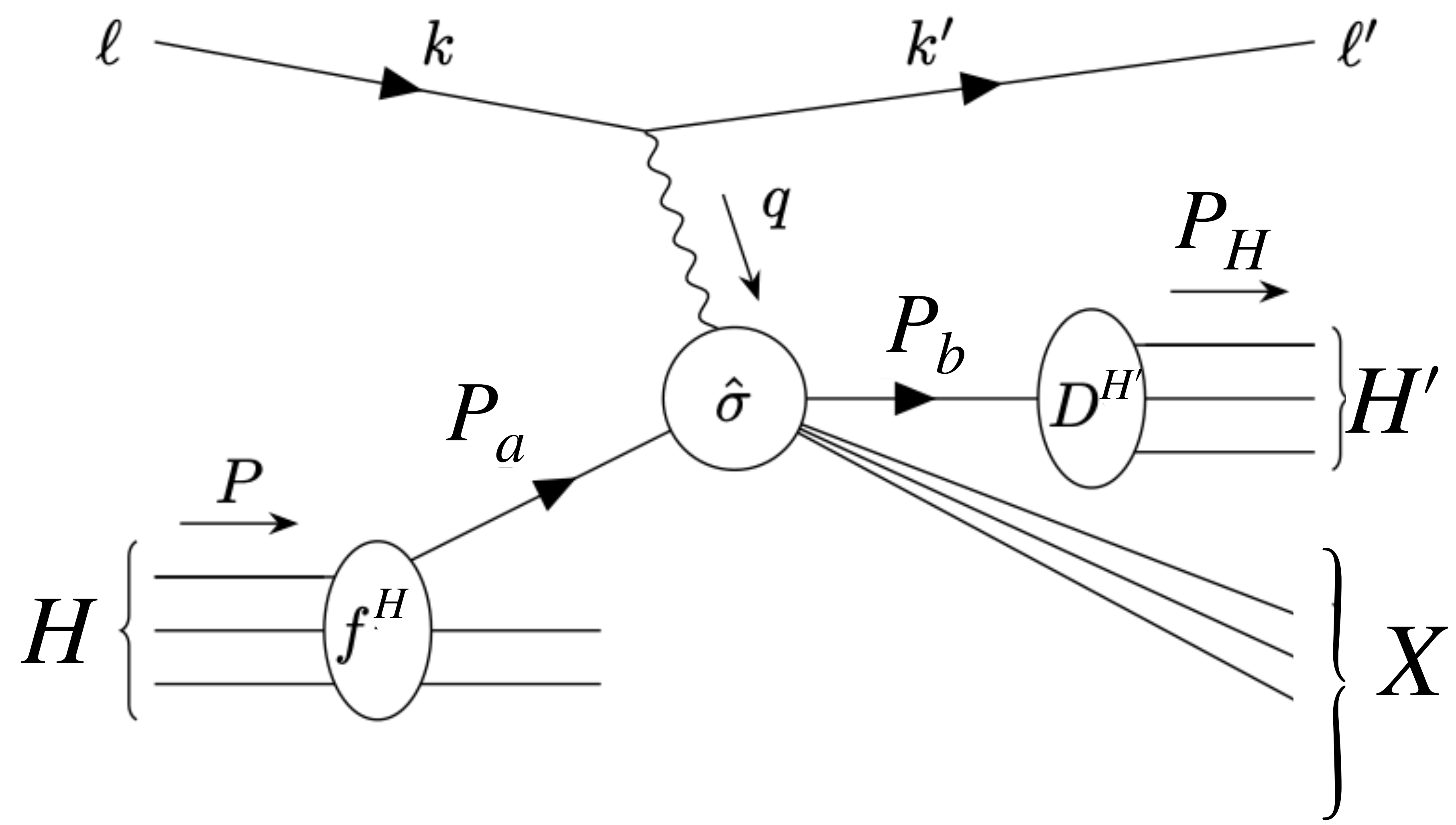
Theoretical Framework

- Semi-Inclusive Deep Inelastic process,

$$l(k_l) + H(P) \rightarrow l(k'_l) + H'(P_H) + X$$

- At Partonic level cross-section,

$$q/g/\gamma(P_a) + \gamma^*(q) \rightarrow q/g/\gamma(P_b) + \mathscr{X}$$



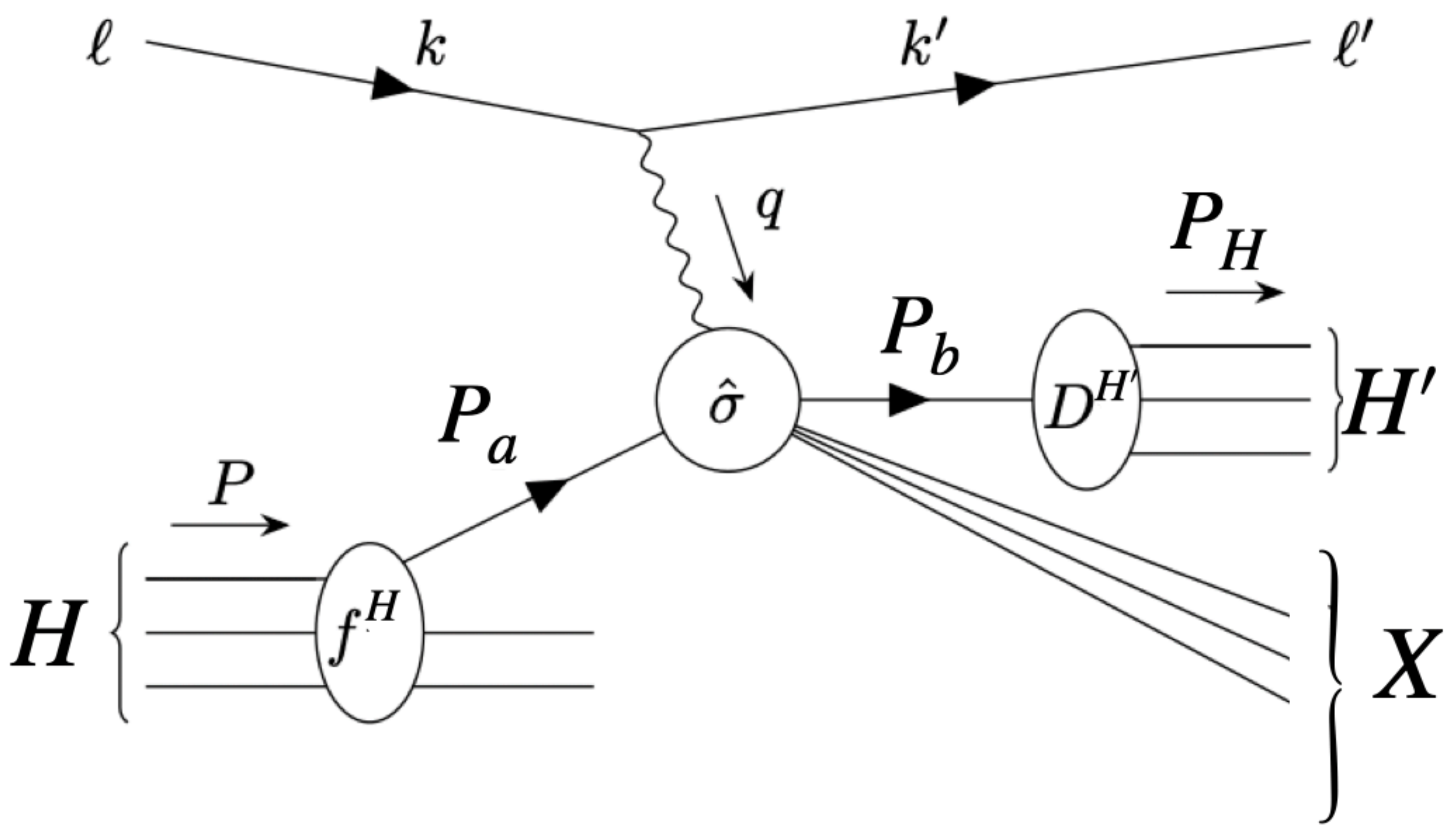
where $P_a^2 = P_b^2 = 0$ and $q^2 = -Q^2$. Our kinematic variables are

$$x = \frac{Q^2}{2P\,.\,q} \Rightarrow x' = \frac{Q^2}{2P_a\,.\,q}$$ and $$z = \frac{P\,.\,P_H}{P\,.\,q} \Rightarrow z' = \frac{P_a\,.\,P_b}{P_a\,.\,q}$$

Theoretical Framework

- The Differential Hadronic cross-section for

$$l(k_l) + H(P) \rightarrow l(k'_l) + H'(P_H) + X$$ process is,



$$\frac{d^3(\Delta)\sigma}{dxdydz} = \frac{2\pi y \alpha_e^2}{Q^4}(\Delta)L^{\mu\nu}(k_l, k_l', q)(\Delta)W_{\mu\nu}(P, P_H, q)$$

where Laptonic tensor is,

$$L^{\mu\nu} = 2k_l^{\mu}k_l^{'\nu} + 2k_l^{'\mu}k_l^{\nu} - Q^2 g^{\mu\nu} \quad \text{and} \quad \Delta L^{\mu\nu} = 2i\epsilon^{\mu\nu\sigma\lambda}q_\sigma s_{l,\lambda}$$

- Hadron compositeness makes hadronic tensor, $W_{\mu\nu}$ evaluation non-trivial.

Theoretical Framework

- The (polarized) unpolarized $(\Delta)W_{\mu\nu}$ can be parametrize as,

$$W_{\mu\nu} = \sum_{J=1,2} W_J(x, z, Q^2) T_{J,\mu\nu}(P, q)$$

$$\Delta W_{\mu\nu} = \sum_{J=1,2} g_J(x, z, Q^2) S_{J,\mu\nu}(P, q)$$

- The tensors $T_{J,\mu\nu}$ for the Unpolarized case,

$$T_{1,\mu\nu} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}$$

$$T_{2,\mu\nu} = \frac{1}{M^2}\left(P_\mu - \frac{P.q}{q^2} q_\mu\right)\left(P_\nu - \frac{P.q}{q^2} q_\nu\right)$$

- The tensors $S_{J,\mu\nu}$ for the Polarized case,

$$S_{1,\mu\nu} = \frac{i}{P.q} \epsilon_{\mu\nu\sigma\lambda}\, q^\sigma S^\lambda$$

$$S_{2,\mu\nu} = \frac{i}{P.q} \epsilon_{\mu\nu\sigma\lambda}\, q^\sigma \left(S^\lambda - \frac{S.q}{P.q} P^\lambda\right)$$

with $S^2 = 1$ and $S \cdot P = 0$.

Theoretical Framework

- Substituting $T_{J,\mu\nu}$ and $S_{J,\mu\nu}$ in the eqn. for $(\Delta)W_{\mu\nu}$ and expressing the phase space of the leptonic tensor in terms of x and y,

$$\frac{d^3\sigma}{dxdydz} = \frac{4\pi\alpha_e^2}{Q^2}\left[y\, F_1(x,z,Q^2) + \frac{(1-y)}{y}\, F_2(x,z,Q^2)\right]$$

F_J are related to W_J through $W_1 = F_1/M$ and $W_2 = F_2/(Ey)$, and E is the energy of the incoming lepton and y is inelasticity.

- Similarly, the spin-dependent cross-section is found to be

$$\frac{d^3\Delta\sigma}{dxdydz} = \frac{4\pi\alpha_e^2}{Q^2}(2-y)\; g_1(x,z,Q^2)$$

- Note that g_2 does not contribute since we restrict ourselves to longitudinally polarized hadron in the initial state.

Theoretical Framework

- SFs are not calculable due to the non-perturbative nature of the incoming and outgoing hadrons.

- Defining $(g_1(x,z,Q^2))F_J(x,z,Q^2) = (\Delta)\mathcal{S}(x,z,Q^2)$ we find,

$$(\Delta)\mathcal{S}_J(x,z,Q^2) = \sum_{a,b=q,\bar{q},g} \int_x^1 \frac{dx_1}{x_1}(\Delta)f_{a/P}\left(x_1,\mu_F^2\right) \int_z^1 \frac{dz_1}{z_1} D_{H/b}\left(z_1,\mu_F^2\right)(\Delta)\mathscr{C}_{J,ab}\left(\frac{x}{x_1},\frac{z}{z_1},Q^2,\mu_F^2\right)$$

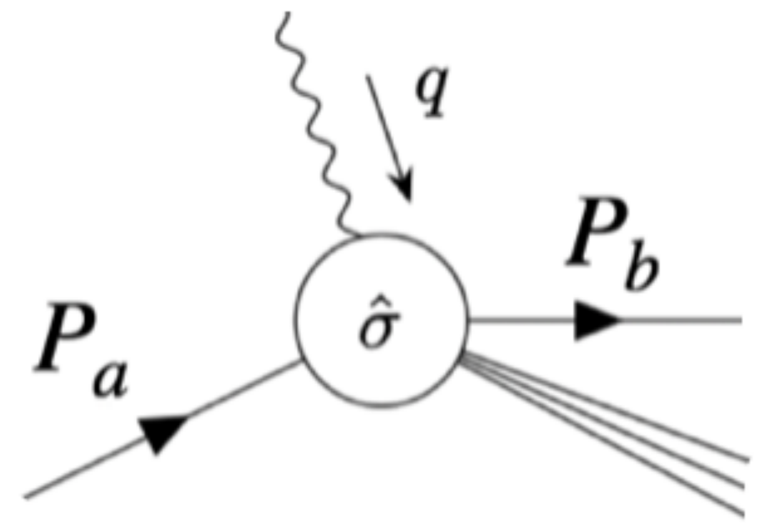
Here $f_{a/P}$ denotes the spin averaged PDF and $\Delta f_{a/P} = f_{a(\uparrow)/P(\uparrow)} - f_{a(\downarrow)/P(\uparrow)}$.

- Our goal is to calculate coefficient function $(\Delta)\mathscr{C}_{J,ab}$ at NNLO.

Theoretical Framework

- The partonic level coefficient function can be computed as

$$\frac{d^3(\Delta)\hat{\sigma}_{i,ab}}{dxdydz} = \frac{(\Delta)\mathscr{P}_i^{\mu\nu}}{4\pi}\int \mathrm{dPS}_{\mathcal{X}+b}\,\Sigma\,|(\Delta)M_{ab}|^2_{\mu\nu} \times \delta\left(\frac{z}{z_1} - \frac{p_a\cdot p_b}{p_a\cdot q}\right)$$



where $(\Delta)\mathscr{P}_i^{\mu\nu}$ is the projectors to project out the CFs and

$(\Delta)M_{ab} = M_{a(\uparrow)b} + (-)M_{a(\downarrow)b}$ is amplitude for the $\gamma^* + a(p_a, s_a) \to b(p_b) + \mathcal{X}$.

where the partonic projectors in d space-time dimensions

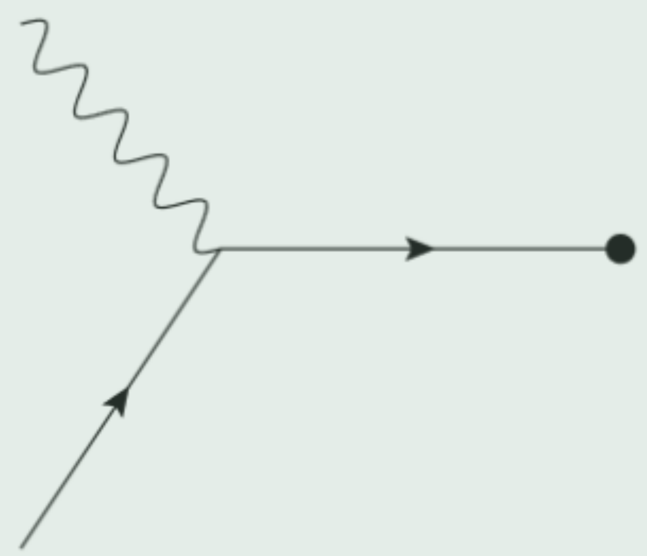
$$\mathscr{P}_1^{\mu\nu} = \frac{1}{(d-2)}\left(T_{1,\mu\nu} + 2xT_{2,\mu\nu}\right),\quad \mathscr{P}_2^{\mu\nu} = \frac{2x}{(d-2)x_1}\left(T_{1,\mu\nu} + 2x(d-1)T_{2,\mu\nu}\right)$$

$$\Delta\mathscr{P}_1^{\mu\nu} = \frac{i}{(d-2)(d-3)}\epsilon^{\mu\nu\sigma\lambda}\frac{q_\sigma p_{a,\lambda}}{p_a\cdot q}$$

Relevant Processes

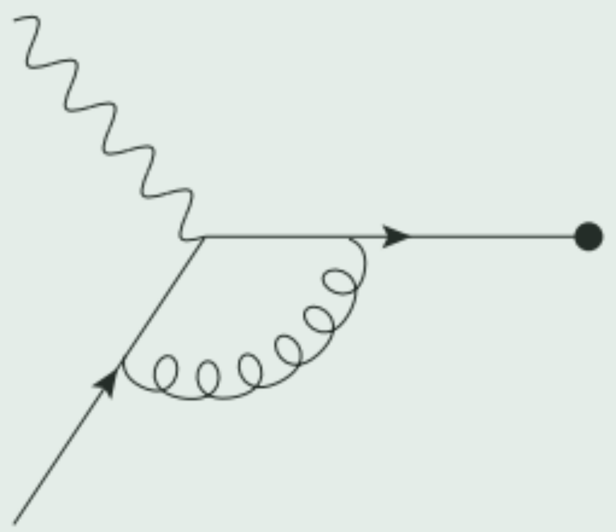
Leading Order (LO) :

$$q(\overline{q}) \; + \; \gamma^* \rightarrow q(\overline{q})$$

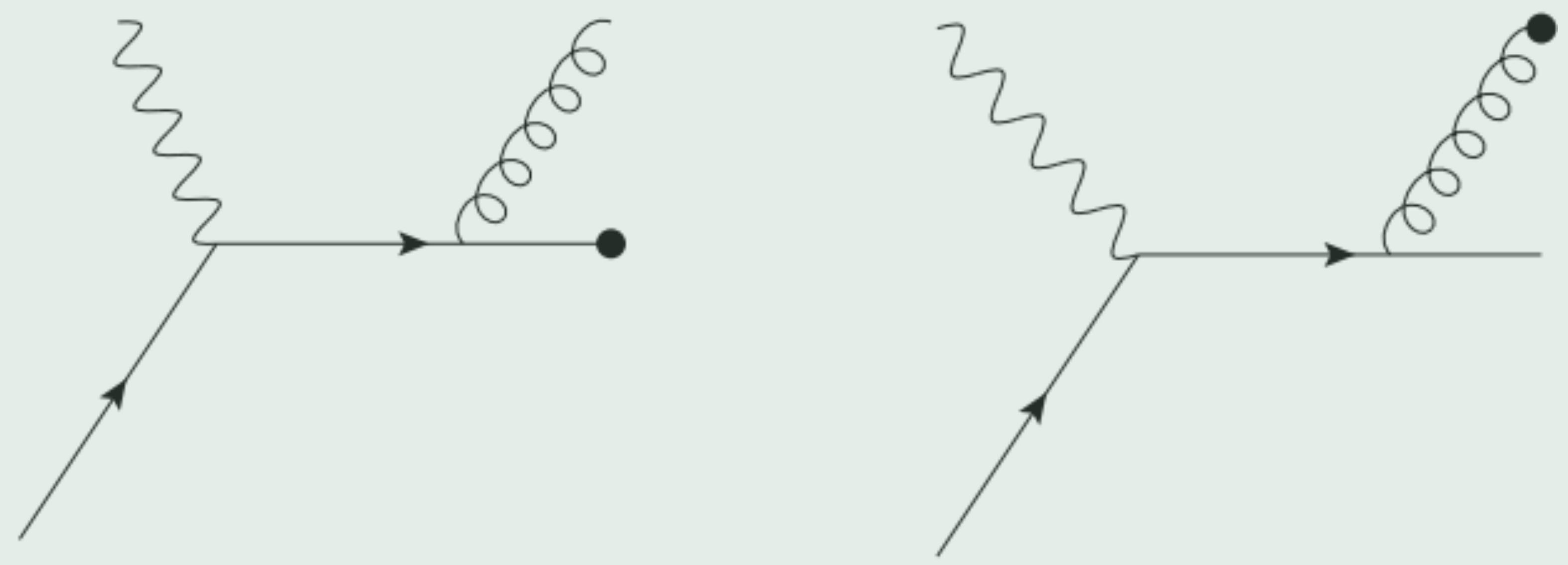


Next Leading Order (NLO) :

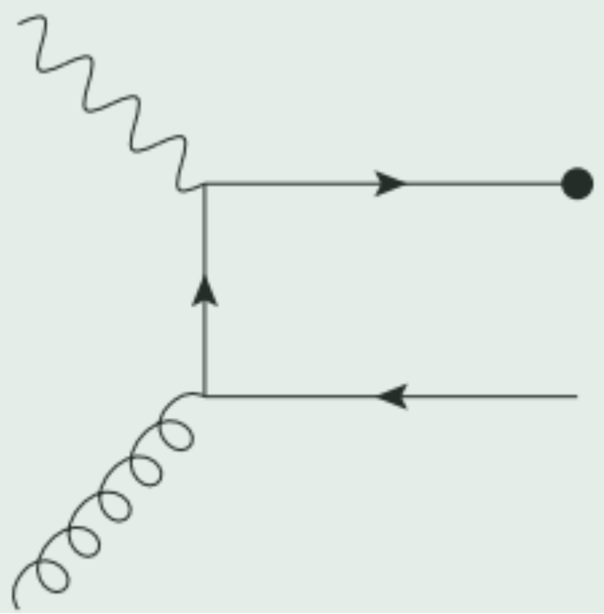
$$q(\overline{q}) \; + \; \gamma^* \rightarrow q(\overline{q}) \; + \; 1 \text{ loop}$$



$$q(\overline{q}) \; + \; \gamma^* \rightarrow q(\overline{q}) \; + \; g$$



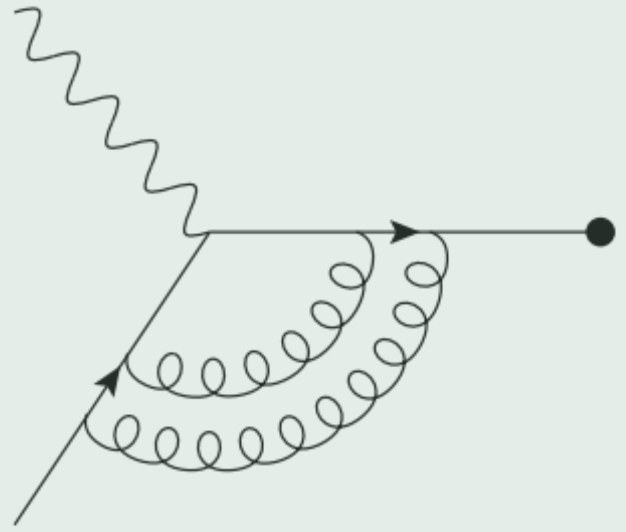
$$g \; + \; \gamma^* \rightarrow q \; + \; \overline{q}$$



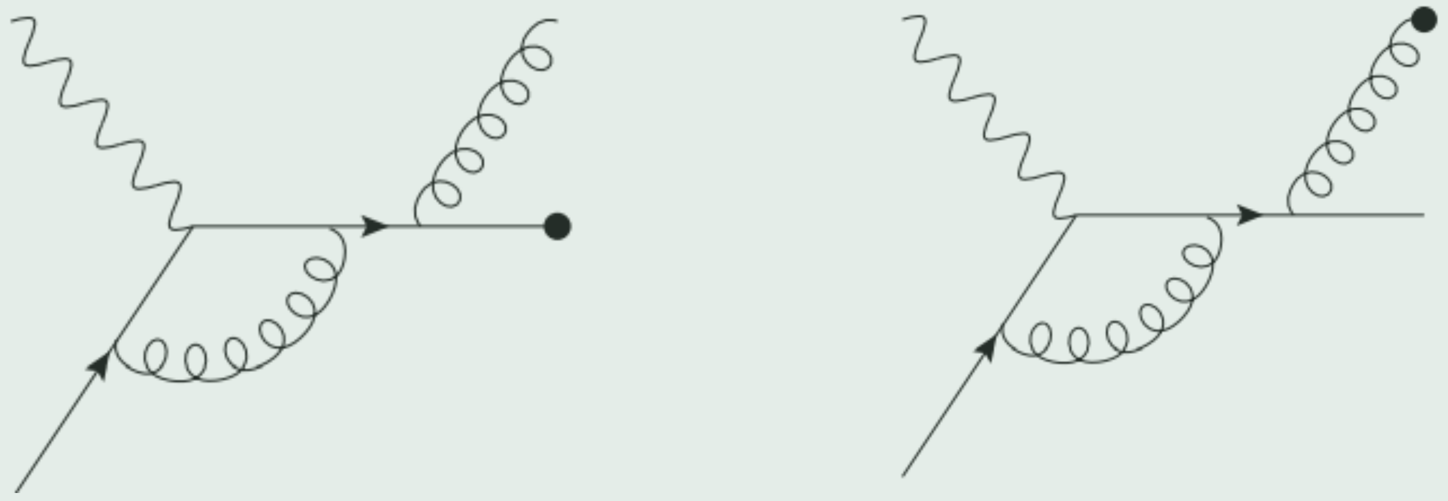
Relevant Processes

Next-to-Next Leading Order (NNLO) :

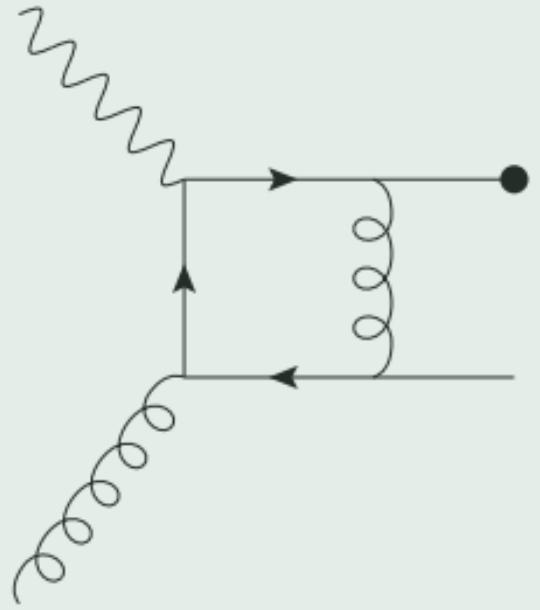
$$q(\overline{q}) \ + \ \gamma^* \rightarrow q(\overline{q}) \ + \text{2 loop}$$



$$q(\overline{q}) \ + \ \gamma^* \rightarrow q(\overline{q}) \ + \ g \ + \ \text{1 loop}$$



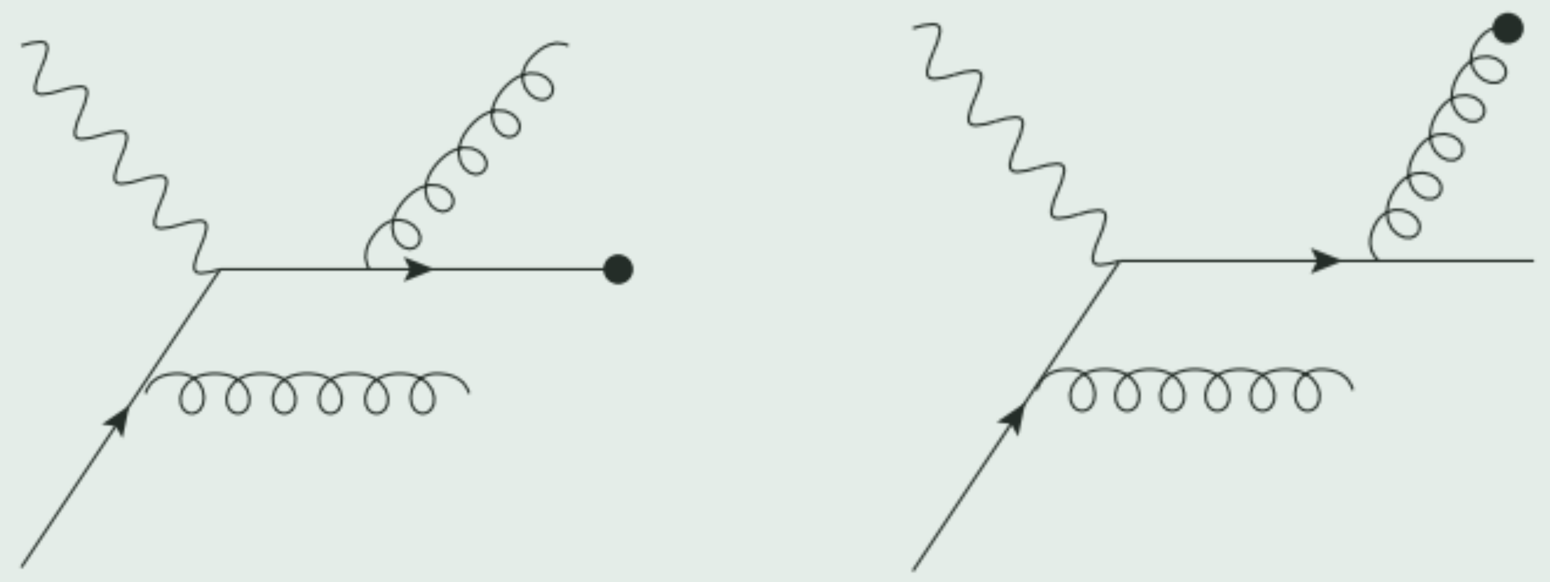
$$g \ + \ \gamma^* \rightarrow q \ + \ \overline{q} \ + \ \text{1 loop}$$



Relevant Processes

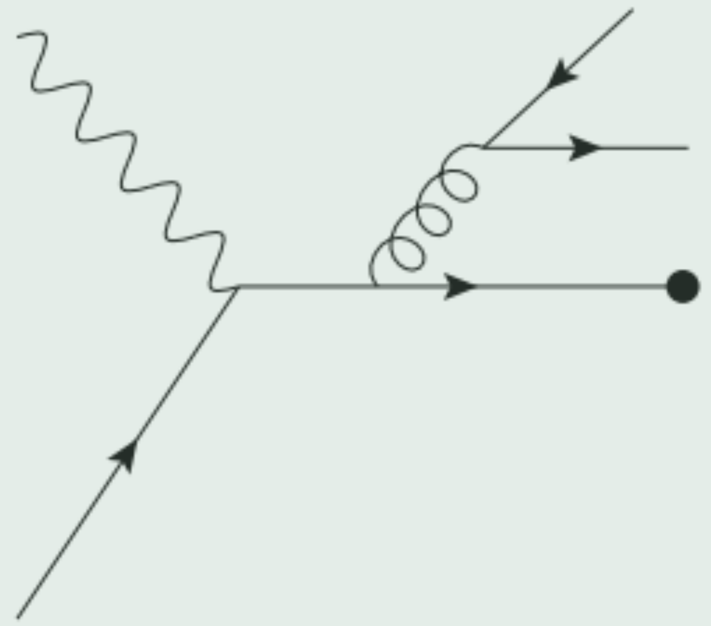
Next-to-Next Leading Order (NNLO) :

$$q(\overline{q}) \;+\; \gamma^* \rightarrow q(\overline{q}) \;+\; g \;+\; g$$



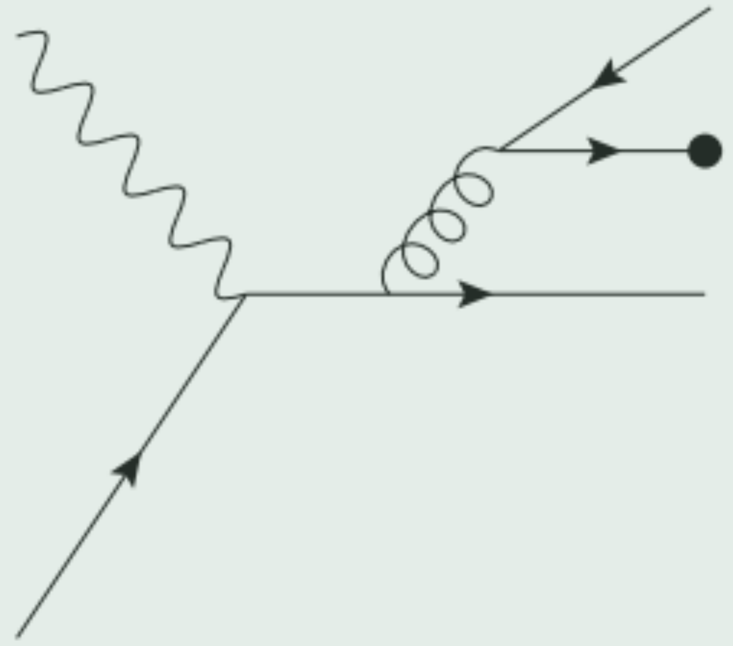
$$q(\overline{q}) \;+\; \gamma^* \rightarrow q(\overline{q}) \;+\; q \;+\; \overline{q}$$

$$q(\overline{q}) \;+\; \gamma^* \rightarrow q(\overline{q}) \;+\; q' \;+\; \overline{q}'$$



A-type

$$q(\overline{q}) \;+\; \gamma^* \rightarrow q(\overline{q}) \;+\; q \;+\; \overline{q}$$



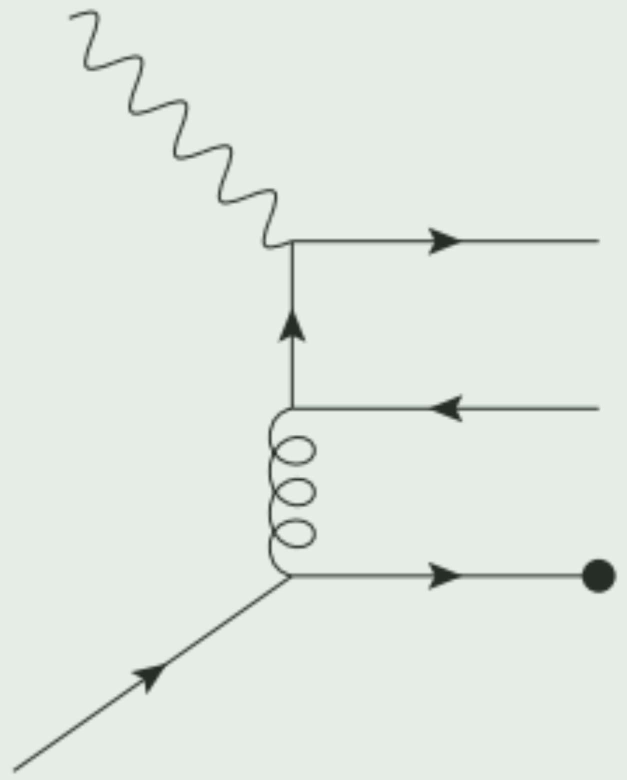
B-type

Relevant Processes

Next-to-Next Leading Order (NNLO) :

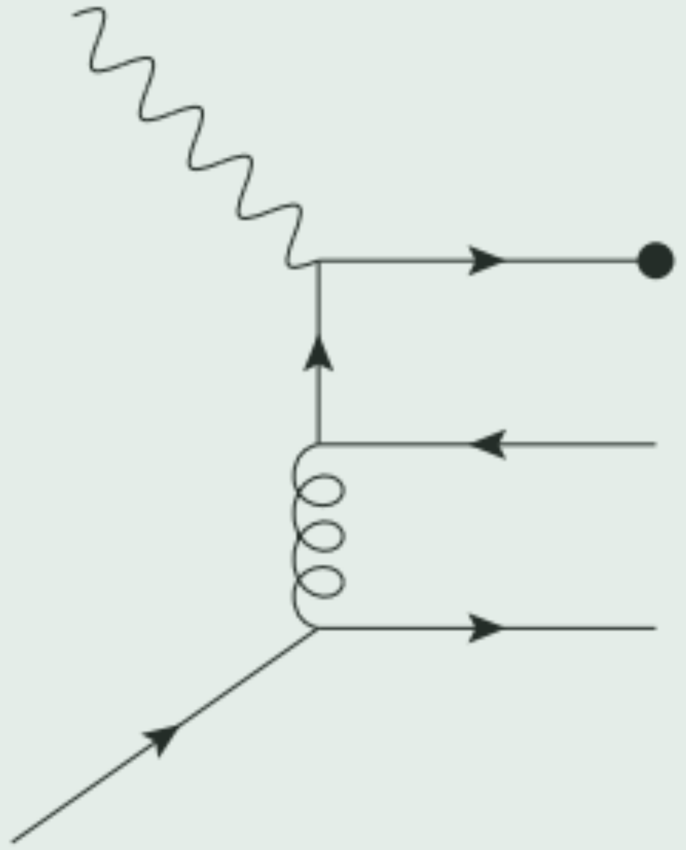
$$q(\overline{q}) \ + \ \gamma^* \rightarrow q(\overline{q}) \ + \ q \ + \ \overline{q}$$

$$q(\overline{q}) \ + \ \gamma^* \rightarrow q(\overline{q}) \ + \ q' \ + \ \overline{q}'$$



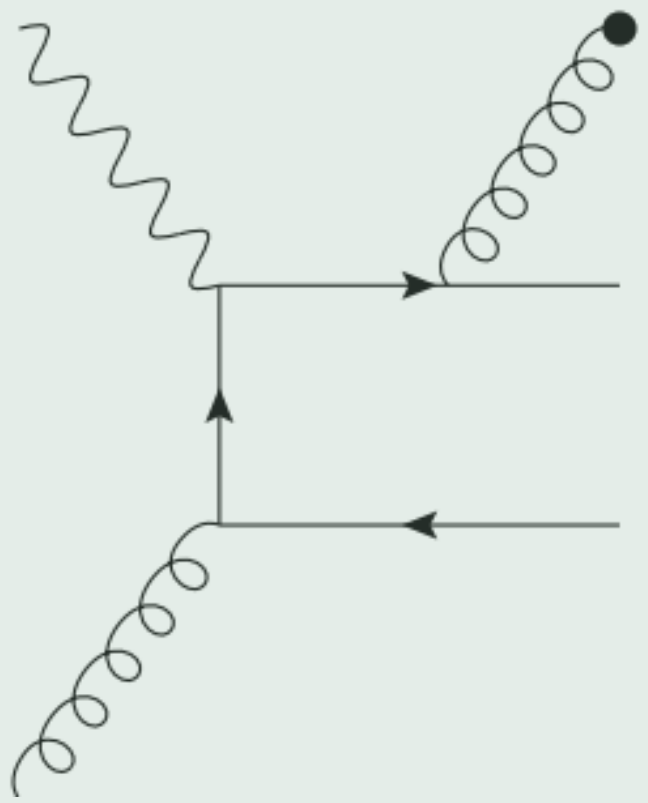
C-type

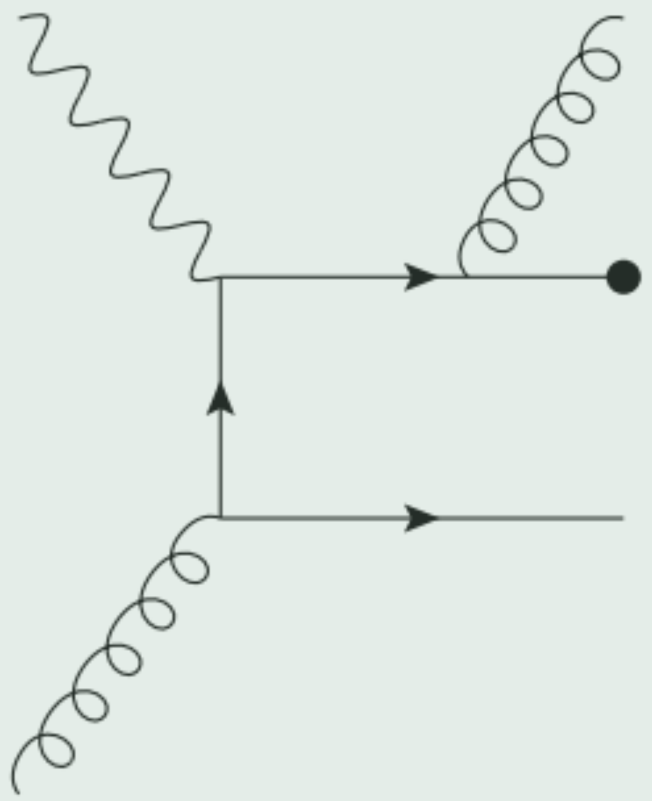
$$q(\overline{q}) \ + \ \gamma^* \rightarrow q(\overline{q}) \ + \ q \ + \ \overline{q}$$



D-type

$$g \ + \ \gamma^* \rightarrow q \ + \ \overline{q} \ + \ g$$





Calculation of Diagrams

Generate Feynman diagrams in symbolic form
QGRAF

↓

Feynman Rules,SU(N) color algebra, Lorentz and Spin algebra
FORM + Mathematica

↓

Classify loop integrals into families
FORM + Mathematica

↓

Reduce to minimal set of integrals
LiteRed

↓

Master Integrals obtained

Calculation of Diagrams

- In Spin-dependent partonic amplitude squared contain $|\Delta M_{ab}|^2$ contain Dirac matrix γ_5 or Levi-Civita tensors from spin dependent quarks (anti-quarks) wave functions or the polarization of gluons or photons.

- Since the γ_5 matrix and the Levi-Civita tensor are intrinsically four-dimensional objects, we need to choose a prescription to define them in $d = 4 + \varepsilon$ dimensions.

- In our work, we use Larin's scheme which is very straightforward to implement in FORM. To define γ_5 in $d = 4 + \varepsilon$ dimensions,

$$\not{p}_a \gamma_5 = -\frac{i}{6} \epsilon_{\mu\nu\sigma\lambda} p_a^\mu \gamma^\nu \gamma^\sigma \gamma^\lambda.$$

- In this scheme, we add finite counter terms to restore chiral ward identity.

Calculation of Diagrams

- 3-Particle Phase-space is

$$\int_{z'} [\text{dPS}]_3 = \prod_{i=1}^{2} \left(\int \frac{d^d k_i}{(2\pi)^{d-1}} \delta(k_i^2) \right) \int d^d p_b (2\pi) \delta(p_b^2) \times \delta^d(p_b + k_1 + k_2 - p_a - q) \delta\left(z' - \frac{p_a \cdot p_b}{p_a \cdot q} \right)$$

- Phase-space integrals via Reverse Unitarity: delta functions → propagators

$$\delta(p^2 - m^2) \to \frac{i}{p^2 - m^2 - i\epsilon} - \underset{leave}{c.c.}$$

- IBP identities are based on the property that integral of a total derivative evaluates to a surface term which can be shown to vanish

$$0 = \int d^d k \frac{\partial}{\partial k_i^\mu} \frac{[k^\mu, q_i^\mu]}{D_1^{k_1} D_2^{k_2} \dots D_n^{k_n}}$$

k^μ and q_i^μ are the loop momentum and external momentas respectively.

- The resulting expression contain Feynman loop integrals which are reduced to a minimal set of integrals called master integrals through IBP Identities.

Calculation of Master Integrals

- Linear differential equation system w.r.t both the kinematic variable in the form

$$\frac{\partial \vec{J}}{\partial x'} = M_{x'}(x', z', \varepsilon)\vec{J},$$
$$\frac{\partial \vec{J}}{\partial z'} = M_{z'}(x', z', \varepsilon)\vec{J}$$

where $M_{x'}$ and $M_{z'}$ are 20 X 20 lower triangular matrix.

- Completeness of Master Integrals basis → "Integrability Condition"

$$\frac{\partial M_{x'}(x', z', \varepsilon)}{\partial z'} - \frac{\partial M_{z'}(x', z', \varepsilon)}{\partial x'} + [M_{x'}, M_{z'}] = 0$$

Calculation of Master Integrals

- The differential equations can be expressed in the ε-factorized form or canonical form.

- A new MI basis is obtained via transformations defined by a matrix T which changes the MI basis from $\vec{J}$ to $\overrightarrow{\widetilde{J}}$ i,e $\vec{J} = T\overrightarrow{\widetilde{J}}$. Finally we obtain

$$\frac{d\overrightarrow{\widetilde{J}}}{dx'} = \varepsilon \widetilde{M}_{x'}(x', z')\overrightarrow{\widetilde{J}}$$

$$\frac{d\overrightarrow{\widetilde{J}}}{dz'} = \varepsilon \widetilde{M}_{z'}(x', z')\overrightarrow{\widetilde{J}}$$

into ε-form. where $\widetilde{M}_{x'} = T^{-1}\left(M_{x'}T - \frac{dT}{dx'}\right)$ and $\widetilde{M}_{z'} = T^{-1}\left(M_{z'}T - \frac{dT}{dz'}\right)$.

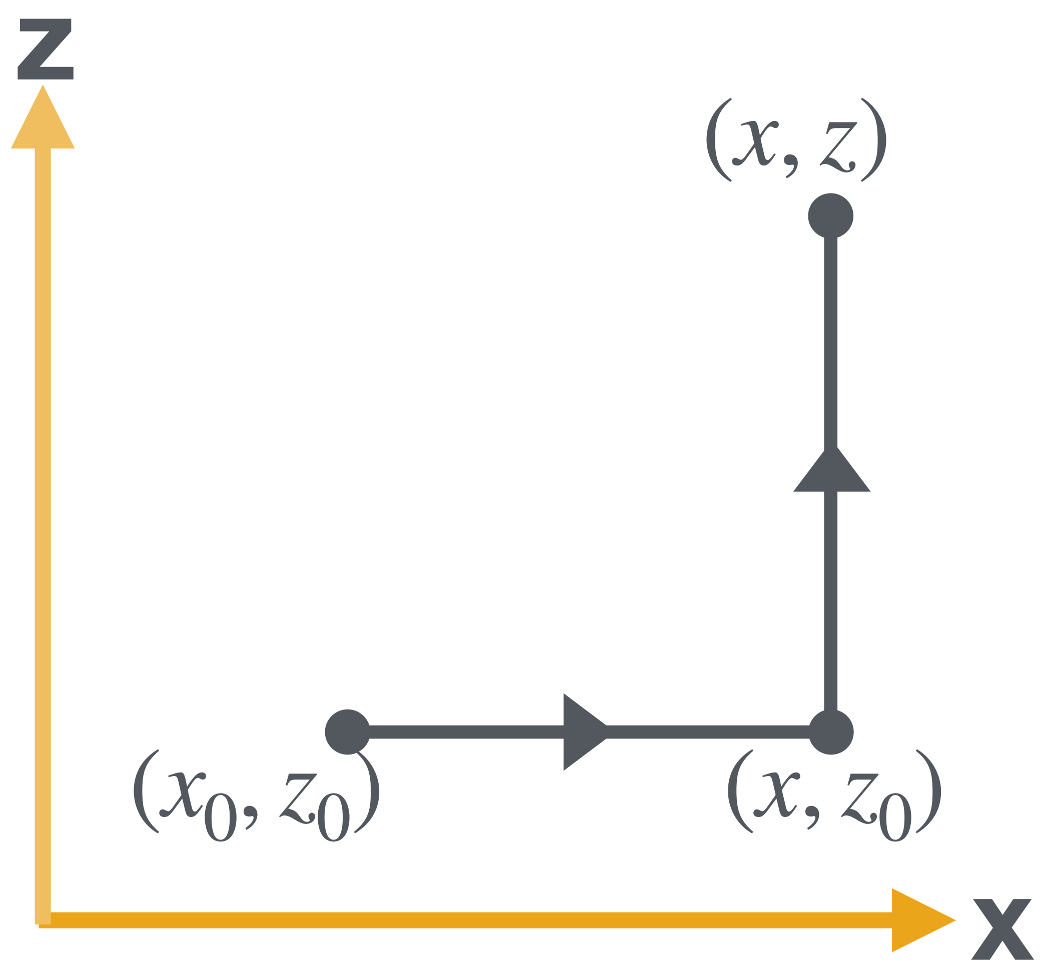
Calculation of Master Integrals

- Path-ordered exponential

$$d\overrightarrow{\tilde{J}} = \varepsilon(\hat{A}_x dx + \hat{A}_z dz)\overrightarrow{\tilde{J}}\ .$$

$$\overrightarrow{\tilde{J}}(x, z, \varepsilon) = P\exp\left\{\varepsilon\int_{x_0,z_0}^{x,z}(\hat{A}_x dx + \hat{A}_z dz)\right\}\overrightarrow{\tilde{J}_0}(x_0, z_0, \varepsilon)$$

- we did line integration on this path



Calculation of Master Integrals

- We have calculated explicitly boundary conditions and expanded above order by order in ε and solved all the MIs iteratively.

- We get $J_i(x', z') = F_i(x', z', x_0 = 0, z_0 = 0) = F_i(x', z', \{J_i(0,0)\})$, since we are taking our starting point of path ordered integration is $(x_0 = 0, z_0 = 0)$.

- since $J_i(0,0)$'s are unknown, we first solve for $J_i(1,1) = F_i(1,1,\{J_i(0,0)\} = B_i)$.

- To compute $J_i(1,1)$, we calculated all the MIs in $(x' \to 1, z' \to 1)$ limit in some frame.

- After getting all the constants, we solved the full result $J_i(x', z') = F_i(x', z', \{B_i\})$

Calculation of Master Integrals

- We got some Alphabets as arguments of iterative generalised PolyLog

$$r_i = \quad x', \ (1-x'), \ (1+x'), \ z', \ (1-z'), \quad \text{and}$$
$$(x')^{1/2}, \ (z')^{1/2}, \ (z'-x'), \ (z'+x'), \ (1+x'z'),$$
$$((1-z')^2+4x'z')^{1/2}, \ ((1+x')^2-4x'z')^{1/2},$$
$$(1-z')^2+4x'z', \ (1+x')^2-4x'z'$$

- we defined our set of GPL's using these alphabets as

$$G(r_1, r_2, r_3, \lambda) = \int_0^{\lambda} \frac{d\lambda_1}{r_1(\lambda_1)} \int_0^{\lambda_1} \frac{d\lambda_2}{r_2(\lambda_2)} \int_0^{\lambda_2} \frac{d\lambda_3}{r_3(\lambda_3)}$$

- we expressed our results in terms of these iterated integrals.

UV divergences

- Amplitude beyond LO have UV divergences.To cure this divergence , we need to first renormalize strong coupling constant through

$$\hat{a}_s S_\varepsilon \left(\frac{1}{\mu^2}\right)^{\varepsilon/2} = a_s(\mu_R^2)\left(\frac{1}{\mu_R^2}\right)^{\varepsilon/2} Z_{a_s}(\mu_R^2)$$

$$Z_{a_s}(\mu_R^2) = 1 + a_s\left(\frac{2\beta_0}{\varepsilon}\right) + a_s^2\left(\frac{4\beta_0^2}{\varepsilon^2} + \frac{\beta_1}{\varepsilon}\right) + O(a_s^3)$$

IR divergences

- KLN theorem: infrared (soft & collinear) singularities cancel when summed over initial and final states

$$(\Delta)\mathscr{C} = 1 + a_s(Soft_V + Collinear_V + Finite_V + Soft_R + Collinear_R + Finite_R) + O(a_s^2)$$

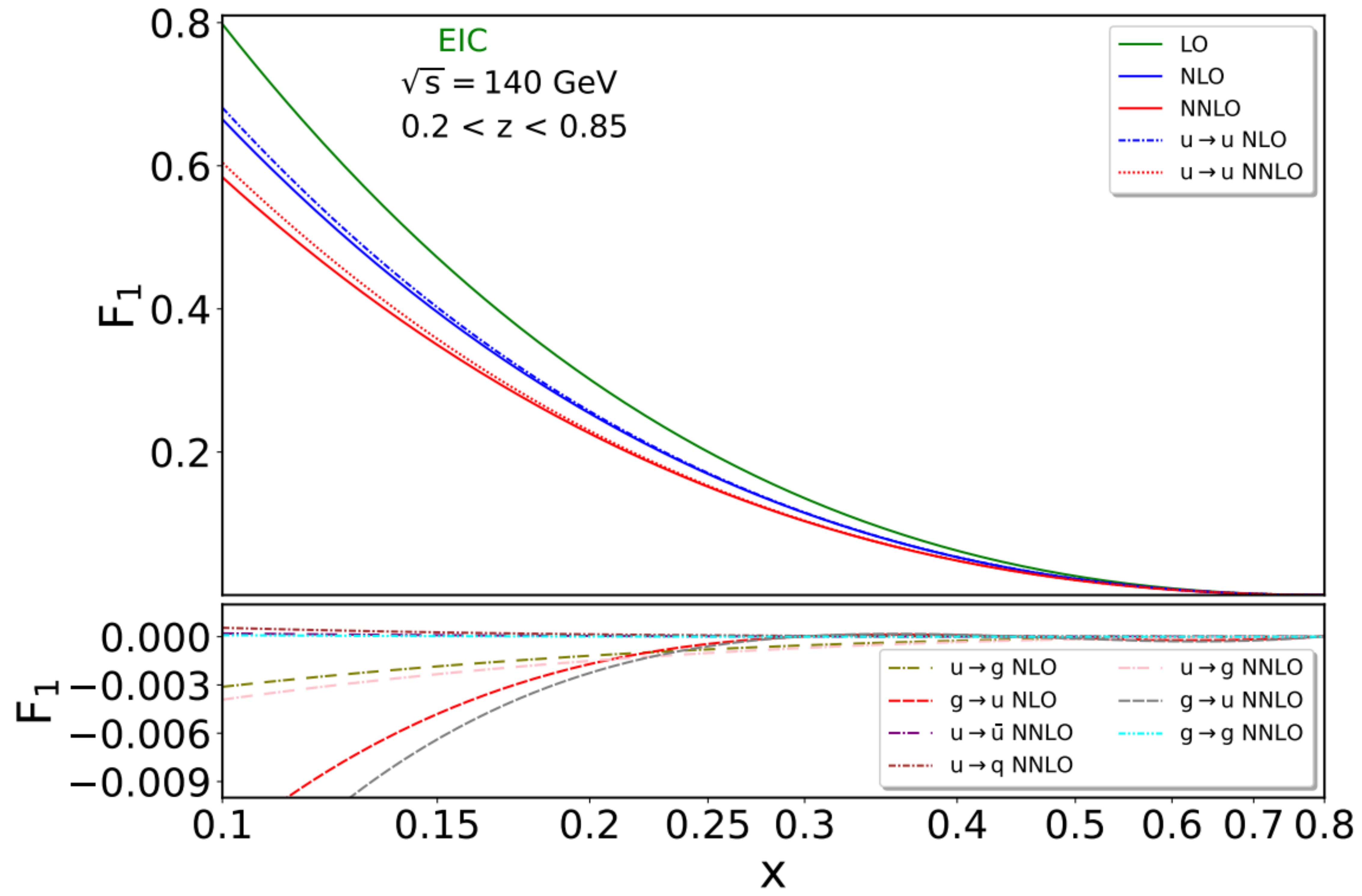
$$Soft_V + Soft_R = 0; \qquad Collinear_V + Collinear_R \neq 0;$$

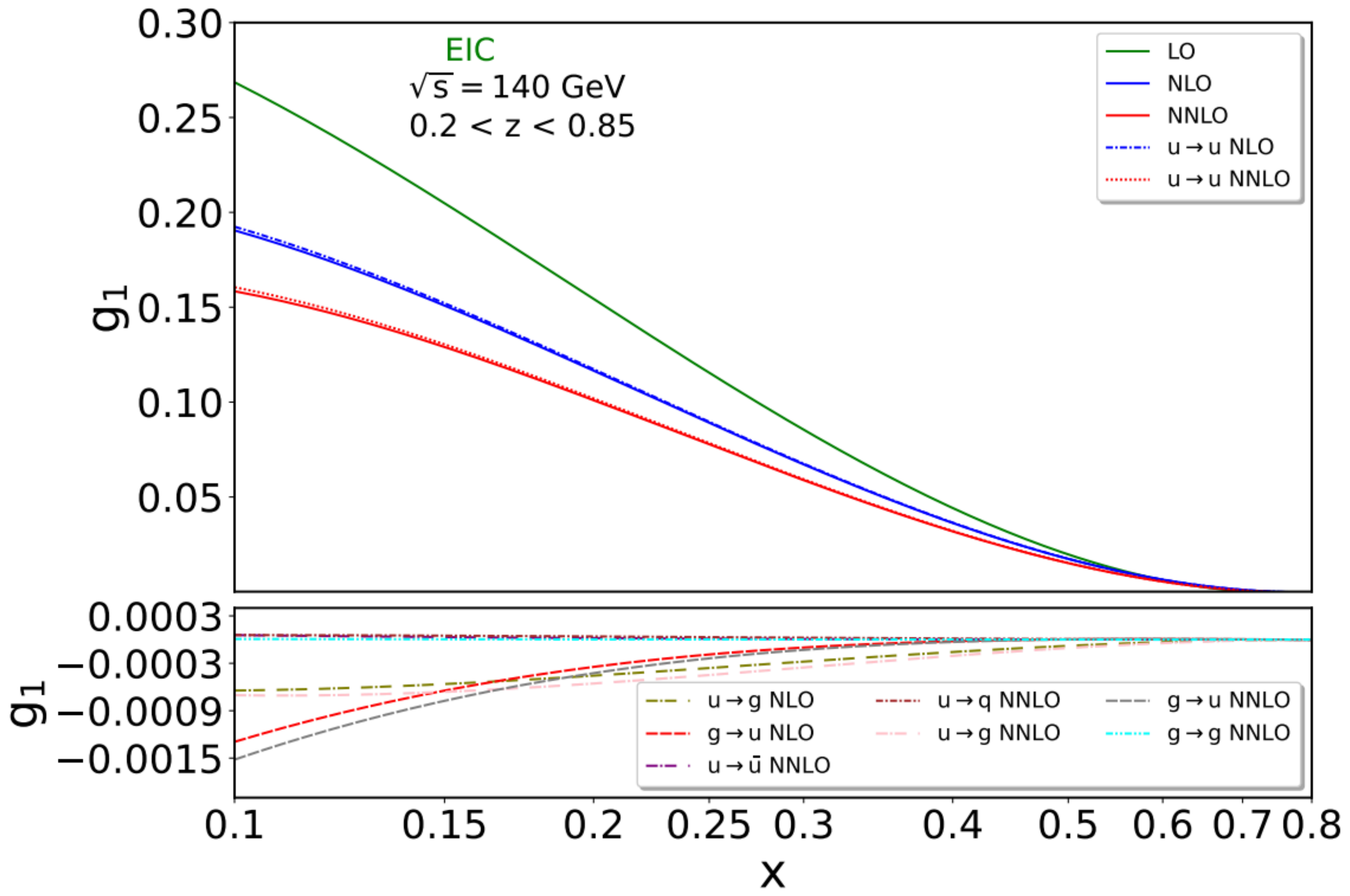
IR divergences

- To remove remaining collinear singularity, we will use the Mass Factorisation prescription.
- By using Mass factorisation, we can factorize our remaining collinear singularity from Coefficient function $(\Delta)\hat{\mathscr{C}} = (\Delta)\Gamma \otimes \mathscr{C} \otimes \tilde{\Gamma}$ into space and time-like AP-kernels.
- Divergences coming in $(\Delta)\Gamma$ and $\tilde{\Gamma}$ will be totally absorbed in bare Parton distribution functions $(\Delta)\hat{f}_a$ and Fragmentation function $\hat{D}_b$ respectively .
- After doing Mass factorization, we removed all the poles and got finite coefficient function.

Numerical Results

Channelwise-Comparison

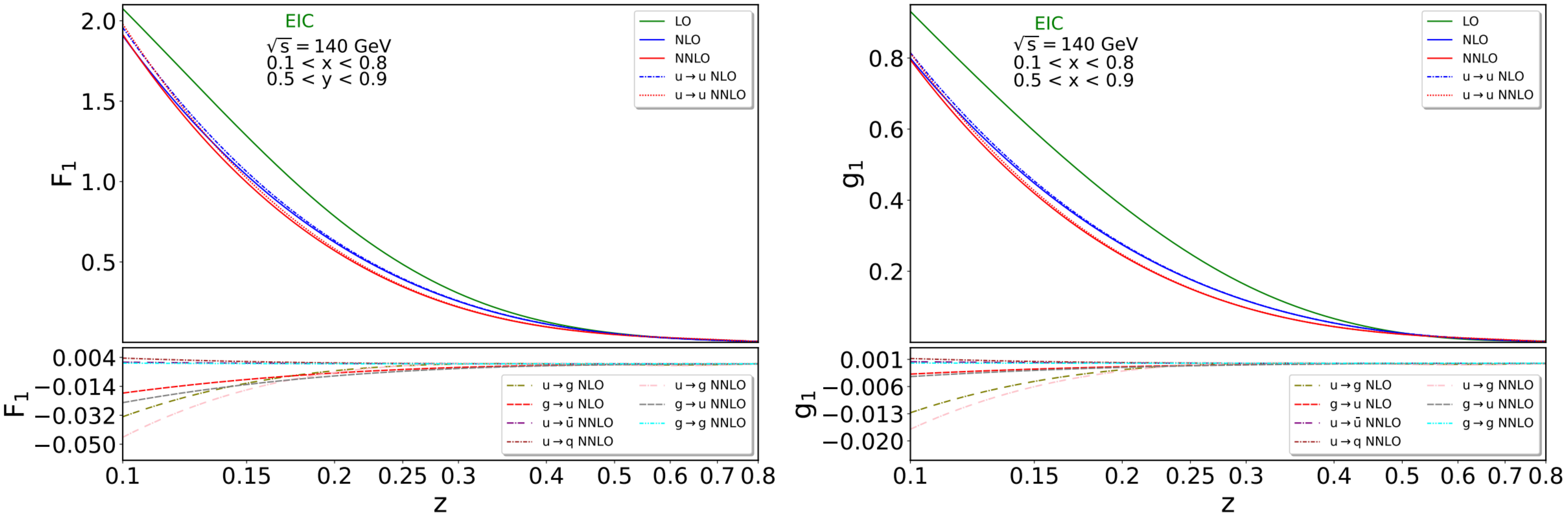




Contribution form all partonic channels to the Structure Function F_1 and g_1 with respect to x.

Numerical Results

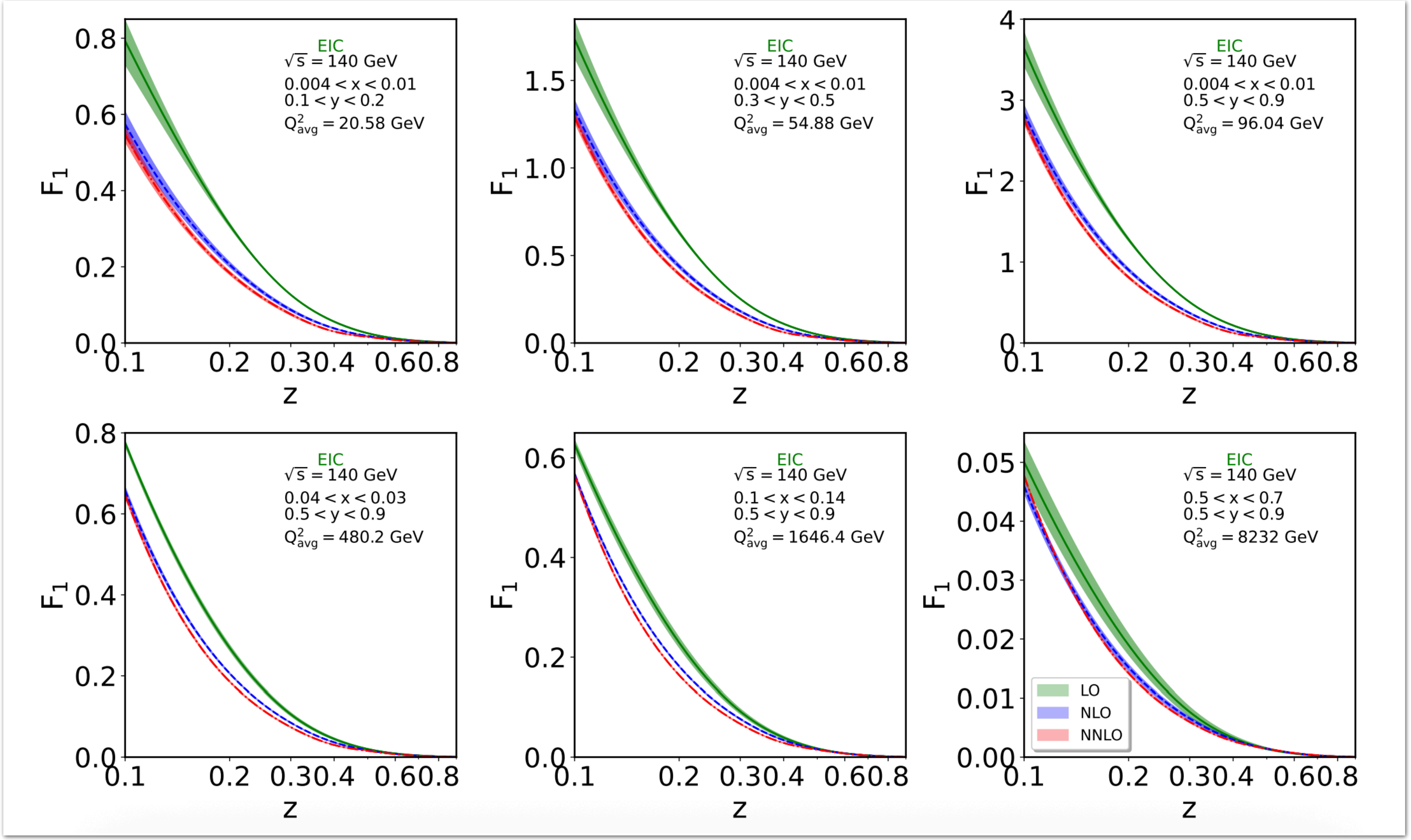
Channelwise-comparison



Contribution form all partonic channels to the Structure Function F_1 and g_1 with respect to z.

Numerical Results

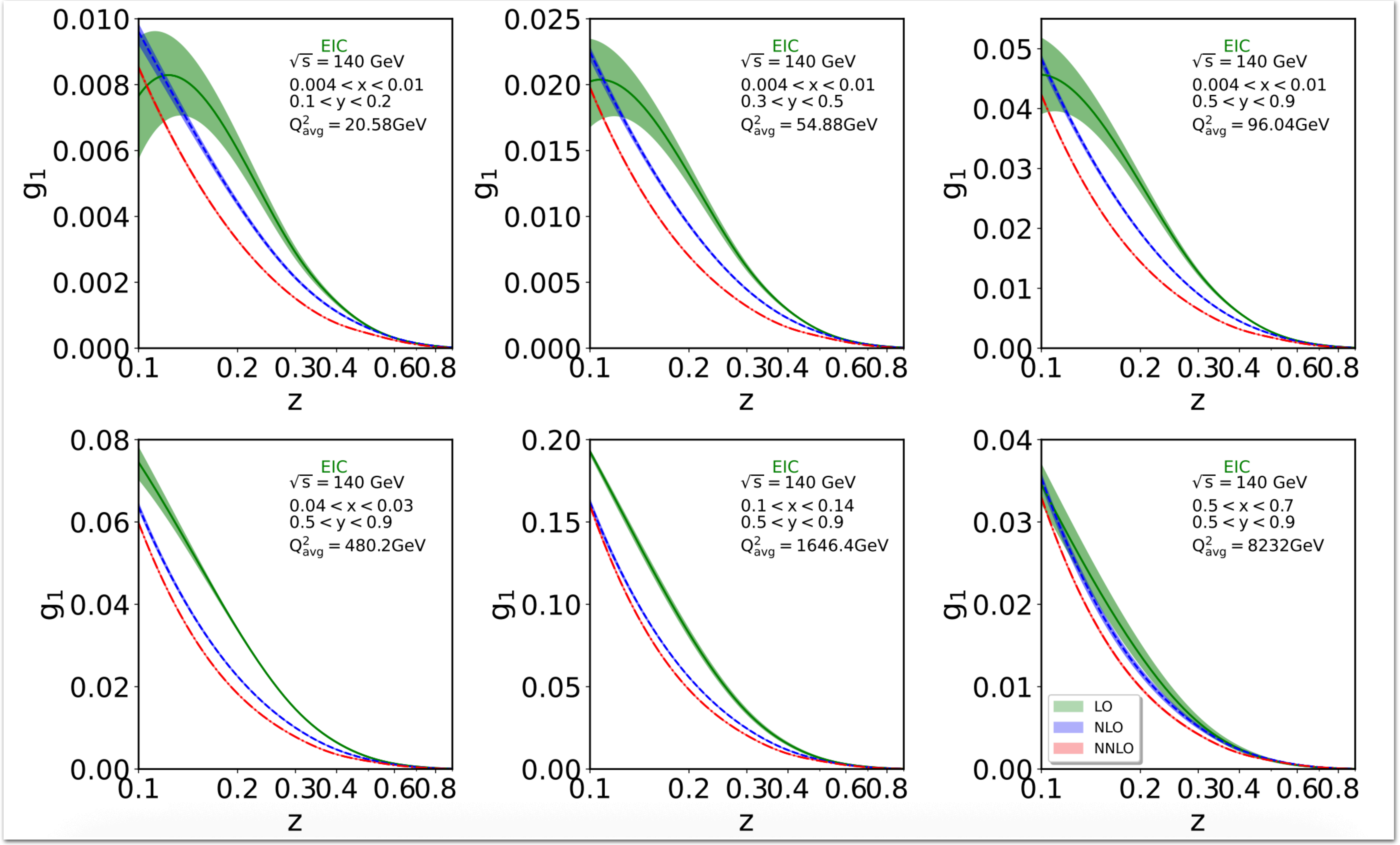
7-point scale variation



7-point scale variation of the Structure Function F_1 with respect to z at different Q^2.

Numerical Results

7-point scale variation



7-point scale variation of the Structure Function g_1 with respect to z at different Q^2.

Taking Forward …

- Next-to-Next-to-Leading Order QCD Corrections to Semi-Inclusive Deep-Inelastic Scattering [Goyal, Moch, VP, Rana, Ravindran ‘24]
- Semi-Inclusive Deep-Inelastic Scattering at Next-to-Next-to-Leading Order in QCD [Bonino, Gehrmann, Stagnitto ‘24]
- Next-to-Next-to-Leading Order QCD Corrections to Polarized Semi-Inclusive Deep-Inelastic Scattering [Bonino, Gehrmann , Löchner , Schönwald, Stagnitto ’24]
- Next-to-Next-to-Leading Order QCD Corrections to Polarized Semi-Inclusive Deep-Inelastic Scattering [Goyal, Lee, Moch, VP, Rana, Ravindran ‘24]
- NNLO phase-space integrals for semi-inclusive deep-inelastic scattering [Ahmed, Goyal, Hasan, Lee, Moch, VP, Rana, Rapakoulias, Ravindran ‘25]
- NNLO QCD corrections to unpolarized and polarized SIDIS [Goyal, Lee, Moch, VP, Rana, Ravindran ‘25]
- Single-valued representation of unpolarized and polarized semi-inclusive deep inelastic scattering at next-to-next-to-leading order [Haug, Wunder ‘25]

Next Correction ?

- NNLO QED and QCD $\otimes$ QED corrections to F_1, F_2 and g_1, thus providing a more precise theoretical description of SIDIS observables.
- The perturbative expansion of an observable $\mathbb{O}$ in SIDIS cross section in both the strong coupling and electromagnetic coupling is given by

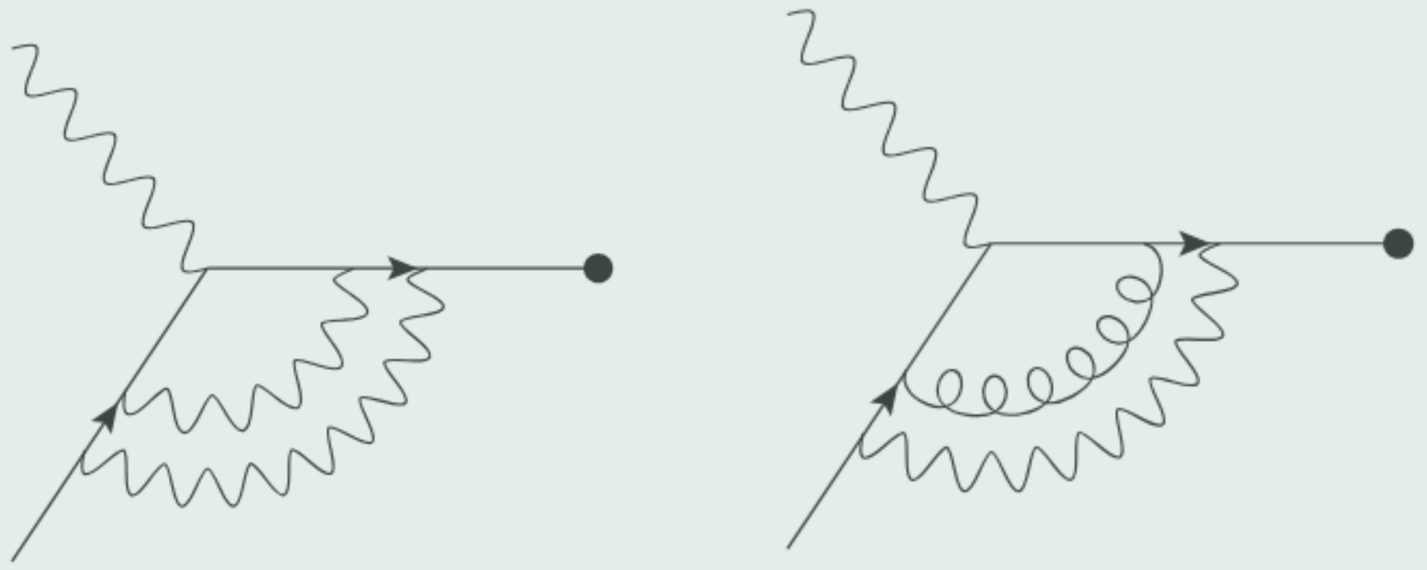
$$\sigma = \sigma^{(0,0)} + a_s\,\sigma^{(1,0)} + a_e\,\sigma^{(0,1)} + a_s^2\,\sigma^{(2,0)} + a_e^2\,\sigma^{(0,2)} + a_s a_e\,\sigma^{(1,1)}$$

where, $a_s = \alpha_s/(4\pi) = g_s^2/(16\pi^2)$ and $a_e = \alpha_e^2/4\pi = e^2/(16\pi^2)$.

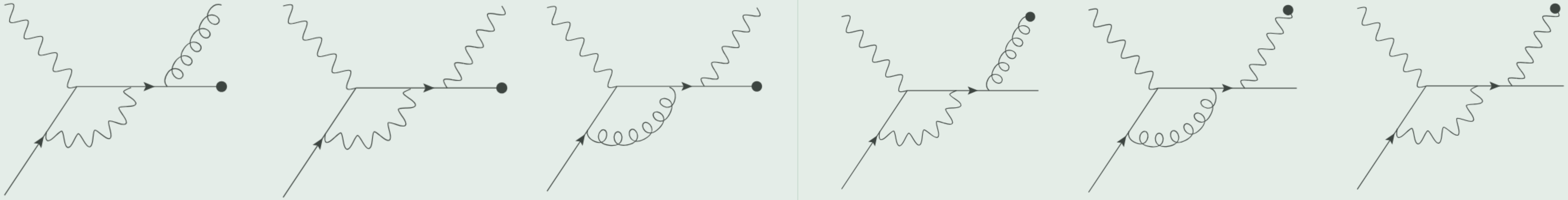
Relevant Processes

Next-to-Next Leading Order (NNLO) :

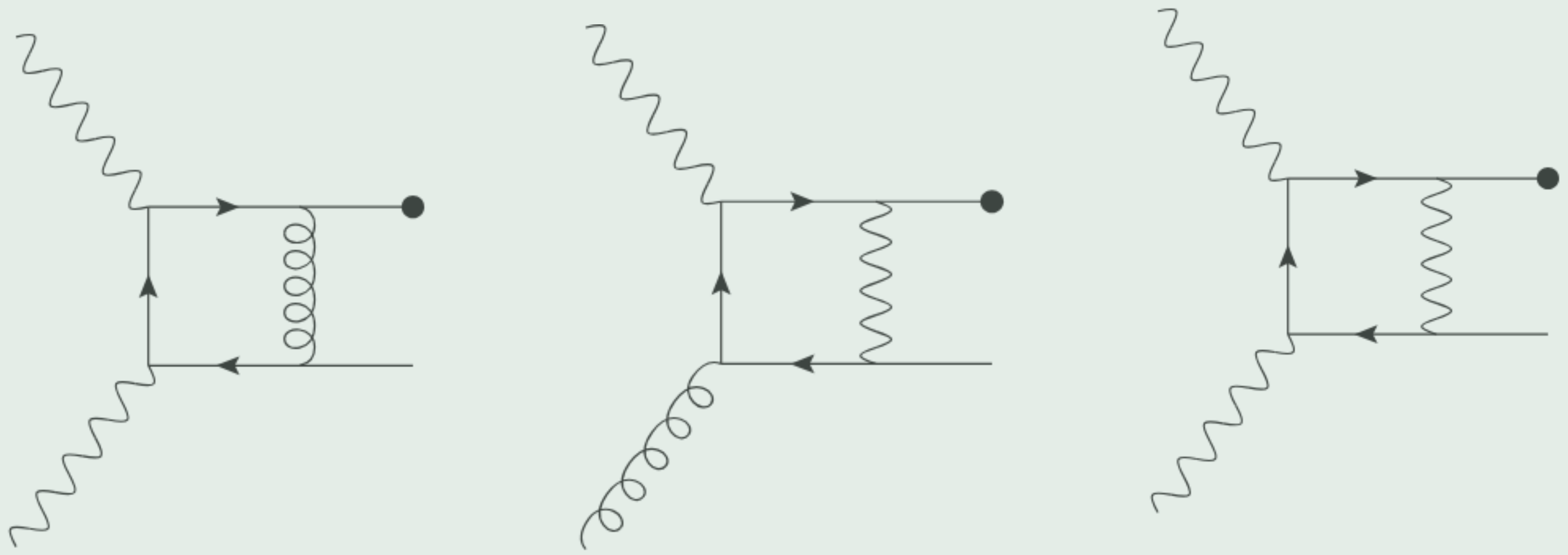
$q(\overline{q}) \ + \ \gamma^* \rightarrow q(\overline{q}) \ +$ 2 loop



$q(\overline{q}) \ + \ \gamma^* \rightarrow q(\overline{q}) \ + \ g/\gamma \ +$ 1 loop



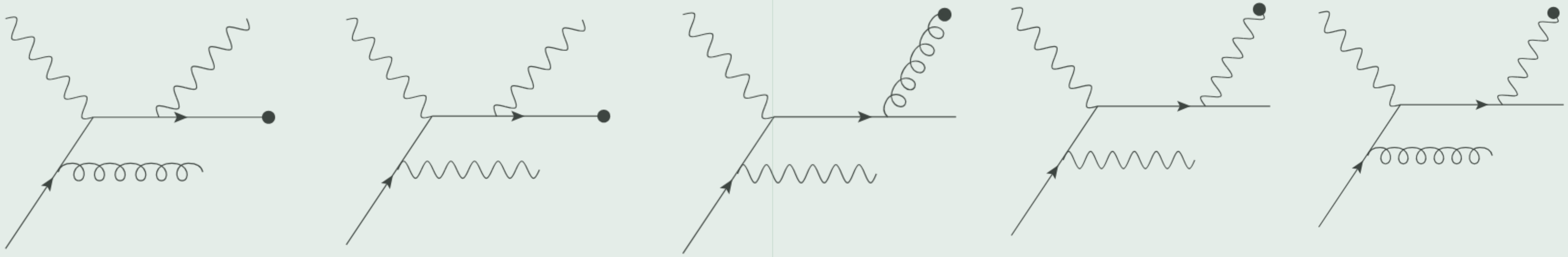
$g/\gamma \ + \ \gamma^* \rightarrow q \ + \ \overline{q} \ +$ 1 loop



Relevant Processes

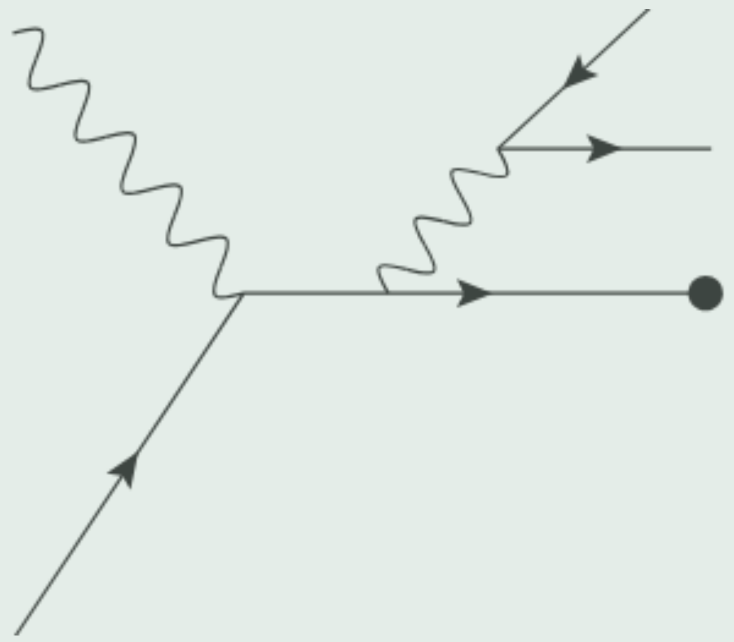
Next-to-Next Leading Order (NNLO) :

$$q(\overline{q}) \ + \ \gamma^* \rightarrow q(\overline{q}) \ + \ g/\gamma \ + \ g/\gamma$$



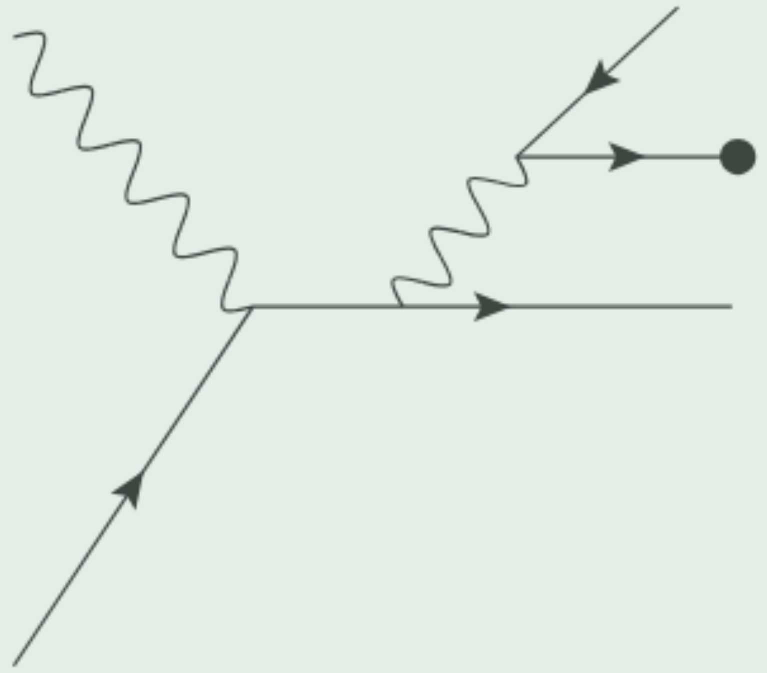
$$q(\overline{q}) \ + \ \gamma^* \rightarrow q(\overline{q}) \ + \ q \ + \ \overline{q}$$

$$q(\overline{q}) \ + \ \gamma^* \rightarrow q(\overline{q}) \ + \ q' \ + \ \overline{q}'$$



A-type

$$q(\overline{q}) \ + \ \gamma^* \rightarrow q(\overline{q}) \ + \ q \ + \ \overline{q}$$

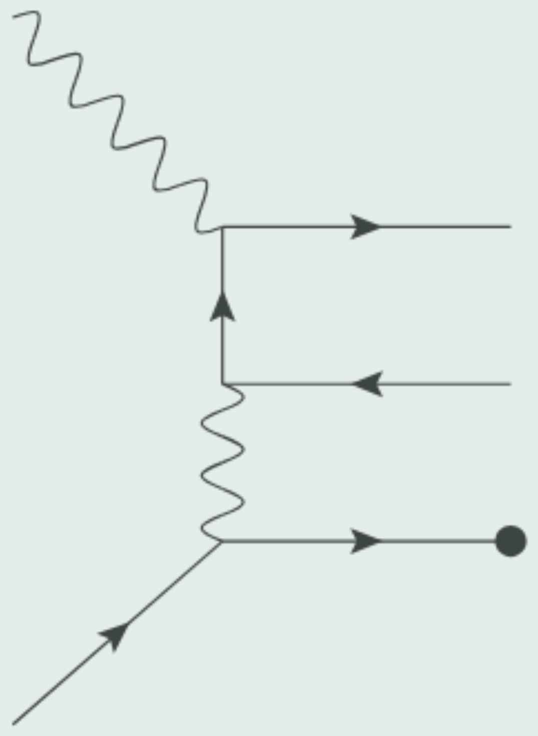


B-type

Relevant Processes

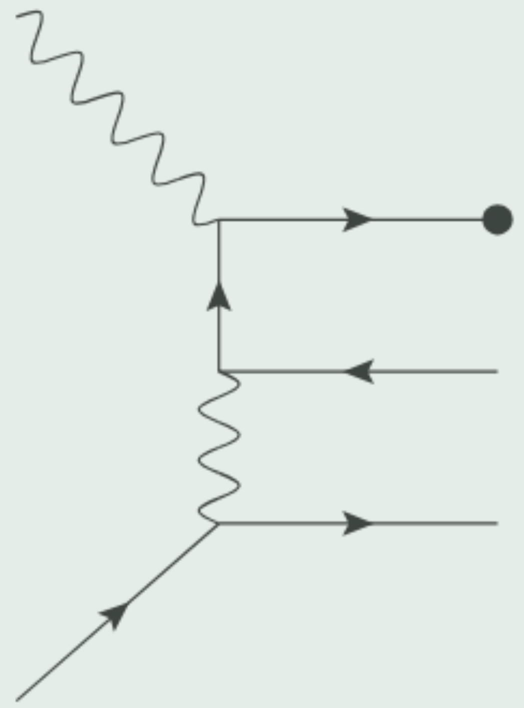
Next-to-Next Leading Order (NNLO) :

$$q(\overline{q}) \;+\; \gamma^* \rightarrow q(\overline{q}) \;+\; q \;+\; \overline{q}$$
$$q(\overline{q}) \;+\; \gamma^* \rightarrow q(\overline{q}) \;+\; q' \;+\; \overline{q}'$$



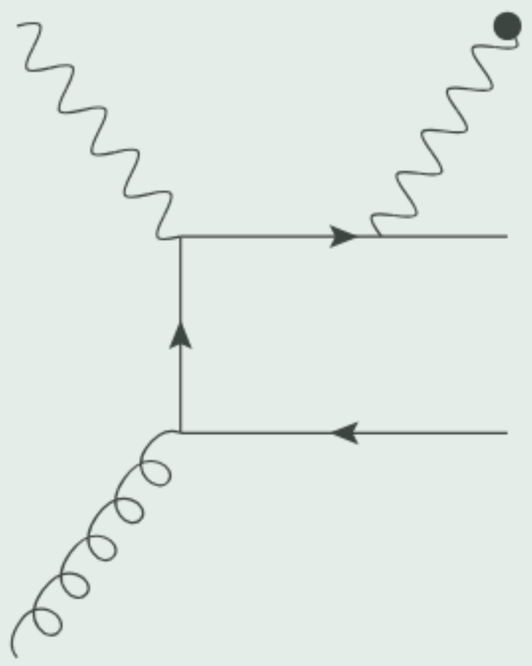
C-type

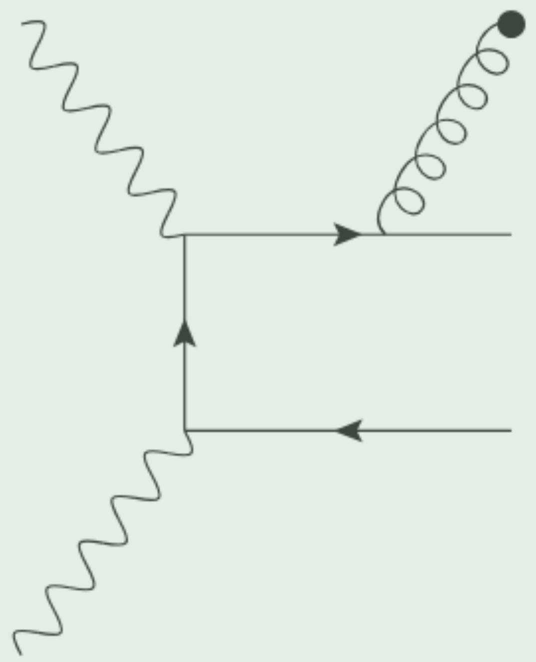
$$q(\overline{q}) \;+\; \gamma^* \rightarrow q(\overline{q}) \;+\; q \;+\; \overline{q}$$

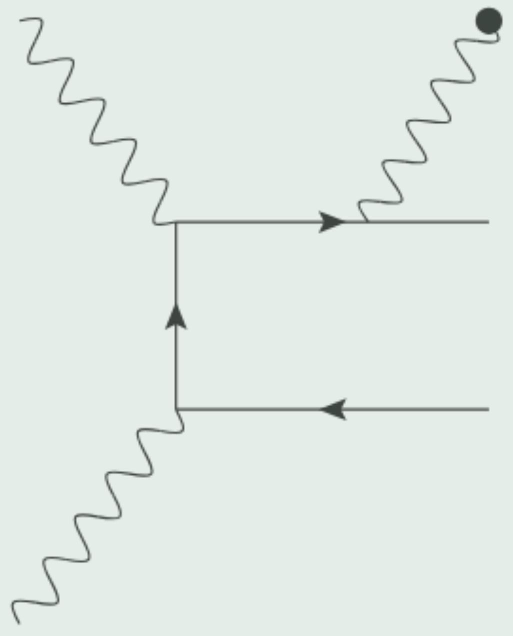


D-type

$$g/\gamma \;+\; \gamma^* \rightarrow q \;+\; \overline{q} \;+\; g/\gamma$$







Abelianization

- After adding all the channels , Soft singularity get cancelled out and to remove Collinear singularity we use mass-factorization. With the help of coupling constant renormalisation of a_s and a_e, we remove UV divergences.
- After getting Finite CFs of QCD,QED and mixed QCDQED, we checked the abelianization.

Mapping	Rule
$qq^{(1,0)} \to qq^{(0,1)}$ $qg^{(1,0)} \to q\gamma^{(0,1)}$ $gq^{(1,0)} \to \gamma q^{(0,1)}$	$C_f \to Q_{eu}^2$
$qq^{(2,0),NS} \to qq^{(0,2),NS}$ $qg^{(2,0)} \to q\gamma^{(0,2)}$ $gq^{(2,0)} \to \gamma q^{(0,2)}$ $gg^{(2,0)} \to \gamma\gamma^{(0,2)}$ $qQ^{(2,0)} \to qQ^{(0,2)}$ $qb^{(2,0)} \to qb^{(0,2)}$	$C_f \to Q_{eu}^2, C_a \to 0, T_f\ n_f \to N\ \sum Q_q^2, T_f \to N\ Q_{eu}^2$
$qq^{(2,0),PS} \to qq^{(0,2),PS}$	$C_f \to Q_{eu}^2, T_f\ \sum Q_q^2 \to N\ \sum Q_{eu}^4$
$qq^{(2,0),NS} \to qq^{(1,1)}$ $qQ^{(2,0)} \to qQ^{(1,1)}$	$C_f^2 \to 2\ C_f\ Q_{eu}^2$, rest color factor $\to 0$
$qg^{(2,0)} \to qg^{(1,1)}$ $qg^{(2,0)} \to q\gamma^{(1,1)}$ $gq^{(2,0)} \to gq^{(1,1)}$	$C_f^2 \to C_f\ Q_{eu}^2$, rest color factor $\to 0$
$\gamma q^{(0,2)} \to \gamma q^{(1,1)}$	$N\ Q_{eu}^6 \to C_f Q_{eu}^4$
$gg^{(2,0)} \to g\gamma^{(1,1)}$	$C_f \to Q_{eu}^2, C_a \to 0$
$\gamma\gamma^{(0,2)} \to \gamma g^{(1,1)}$	$Q_{eu}^4 \to C_f\ Q_{eu}^2$

Conclusions and Future Directions

- Our result shows what are the contributions all the channels are making. It shows $q \to q$ channel is dominating.

- Our result shows inclusion of NNLO corrections reduces the scale dependence when compared to the previous order.

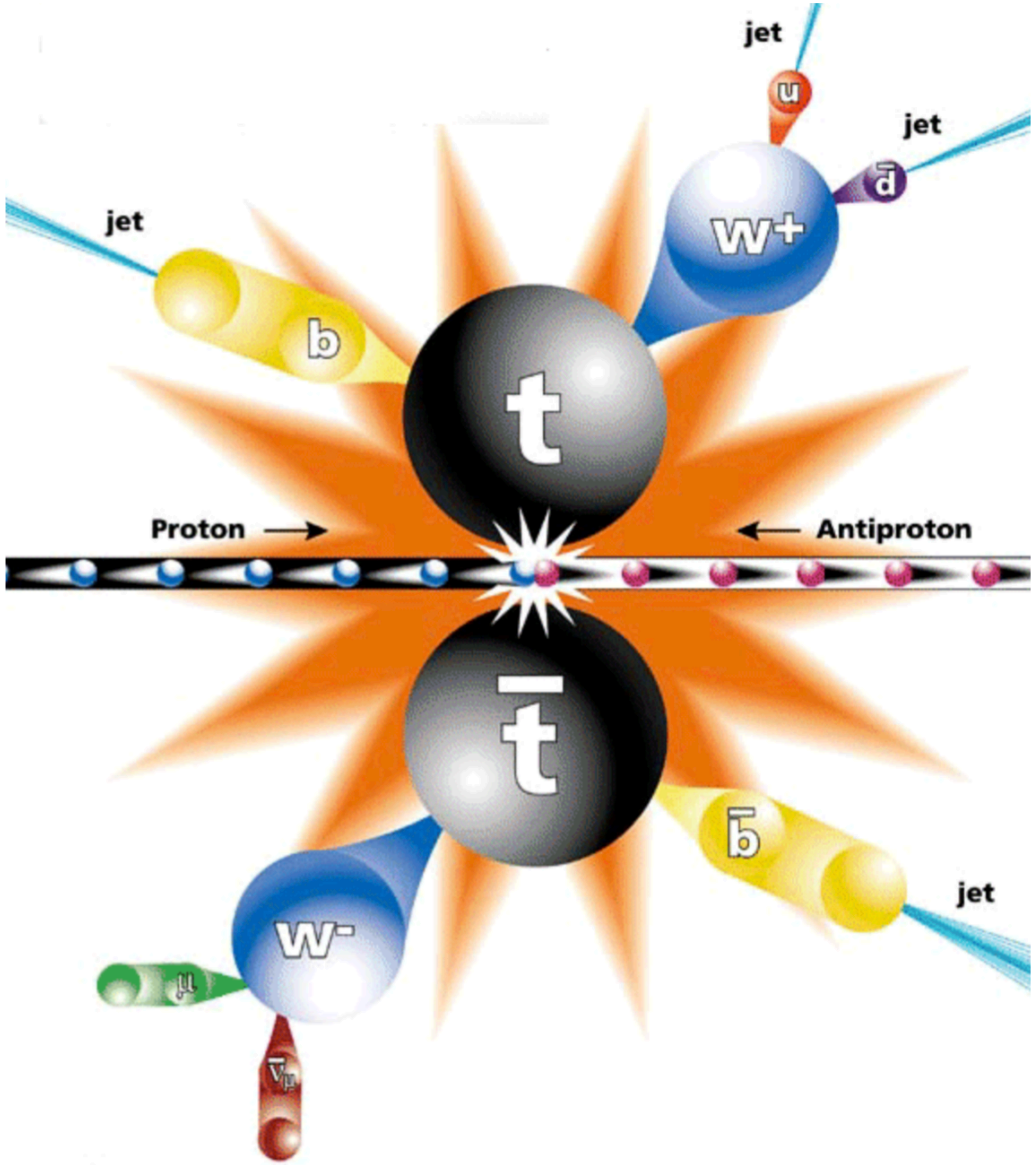
- Improvement in CFs gives us better control over Parton distribution functions as well as Fragmentation functions.

Conclusions and Future Directions

- Pure QED corrections may not be impactful but mixed QCDQED corrections are significant. Hence it necessary to add it.
- We could extract out Time-like splitting functions related to QED and mixed QCDQED which was unknown in literature. These Time-like splitting functions give us good constraint on less known fragmentation function.
- We could extract out Space-like Polarized splitting functions related to QED and mixed QCDQED which was unknown in literature. These Space-like splitting functions give us good constraint on Polarized PDFs.
- We are also currently investigating parallely the numerical impact of Neutral and Charged current intermediate processes. We expect improvement in result.
- We also plan to calculate QCD $\otimes$ EW correction to SIDIS.

Thank You !

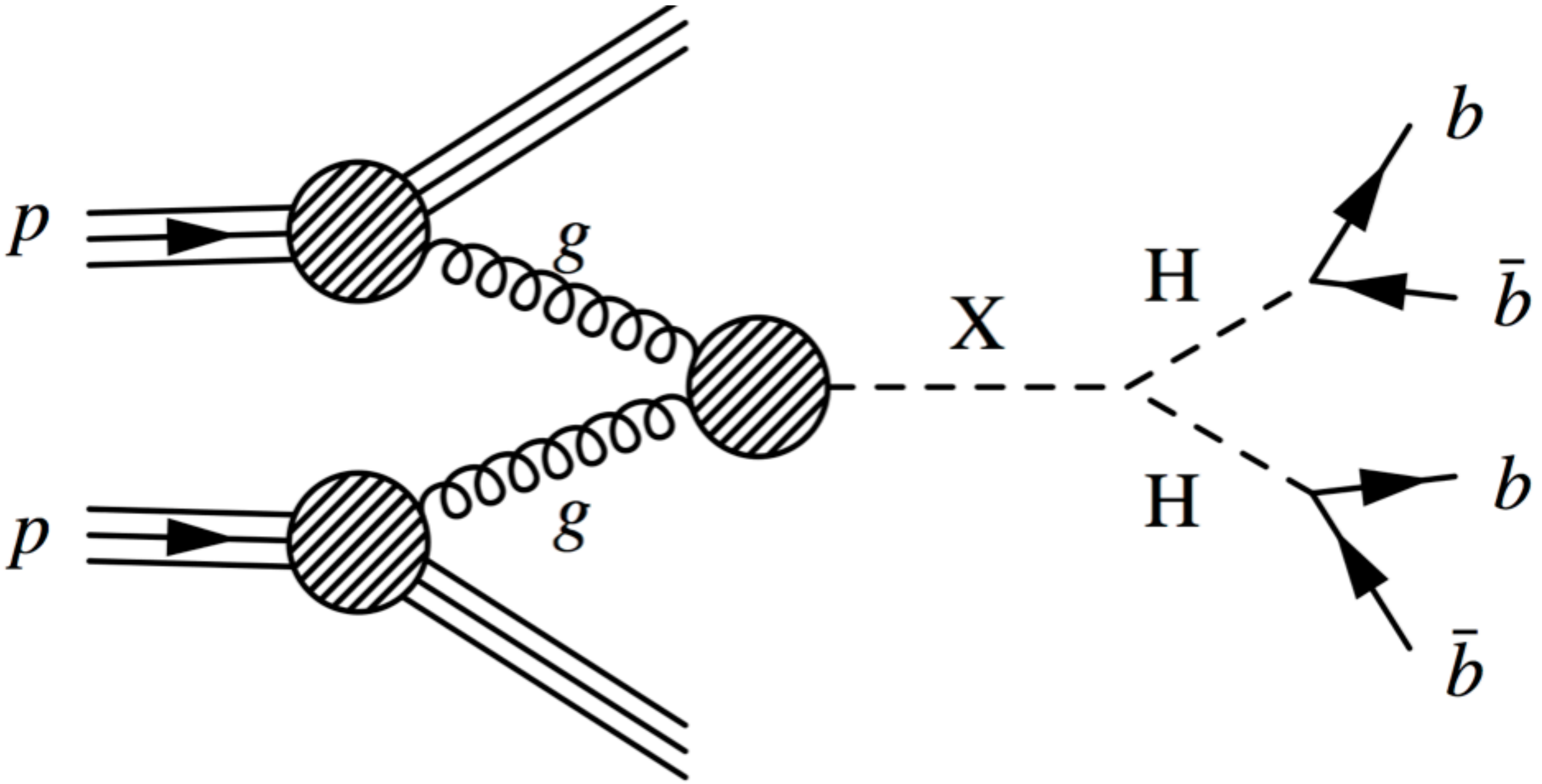
Top pair Production at the LHC



Parton Distribution Function

$$\Phi_{ab}(\mu_F^2, z) = \int \frac{dy}{y} f_a(y, \mu_F^2) f_b\left(\frac{z}{y}, \mu_F^2\right)$$

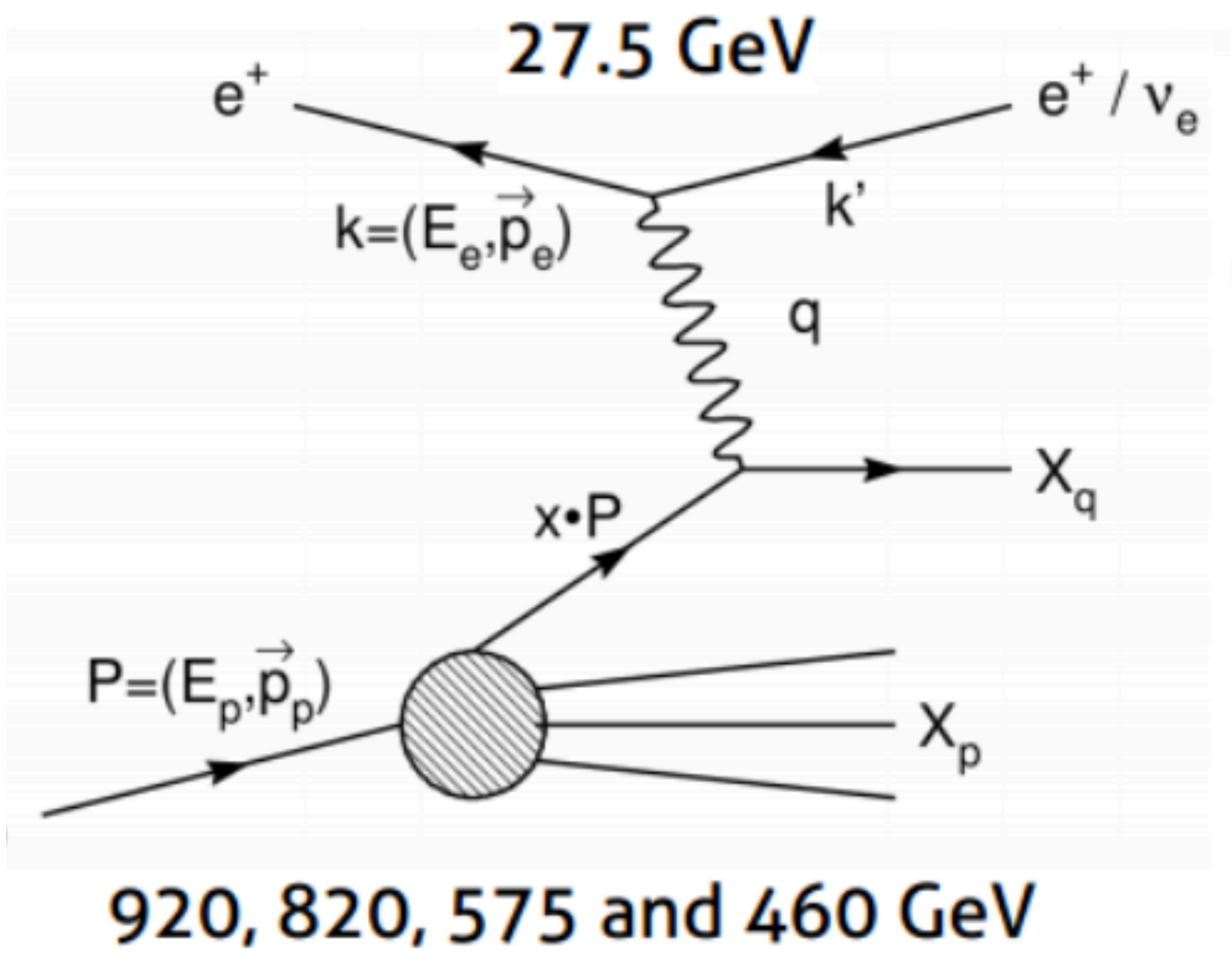
Higgs Production at the LHC



Parton Distribution Function

$$\Phi_{ab}(\mu_F^2, z) = \int \frac{dy}{y} f_a(y, \mu_F^2) f_b\left(\frac{z}{y}, \mu_F^2\right)$$

HERA — world only $e^{\pm}p$ collider



PDF Extraction

GRV, GJR ...

MRST, MSTW ...

CTEQ, CT# ...

NNPDF

ABM, ABKM

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Long List of 19 Pages

Calculation of Diagrams

- We start computation by generating the set of Feynman diagrams by using QGRAF to get expression in symbolic form.
- To apply the Feynman rules, perform SU(N) colour manipulation and d-dimensional Lorentz and spin algebra, we pass the resulting expression through various procedures based on FORM and Mathematica.
- Using the Reduze package, we find appropriate loop momentum shifts for each Feynman diagram beyond tree level to classify them in one of the integral families.

Theoretical Framework

- SFs are not calculable due to the non-perturbative nature of the incoming and outgoing hadrons.

- In the QCD improved parton model, they are related to PDFs, $(\Delta)f_{a/P}(x_1, \mu_F^2)$ and FFs, $D_{H/b}(z_1, \mu_F^2)$ through calculable coefficient functions (CFs), $(\Delta)\mathscr{C}_{J,ab}(x/x_1, z/z_1, Q^2, \mu_F^2)$ at factorization scale μ_F^2.

- Here, $x_1 = P_a/P$ is the fraction of the momentum carried away by incoming parton a from hadron P, and $z_1 = P_H/P_b$ is the fraction of the final state parton b's momentum carried away by the outgoing hadron H$'$.

Theoretical Framework

- Structure functions (SFs) are coefficients of hadronic tensors, being Lorentz invariant, depend on the scaling variables x, z and the invariant mass of the intermediate photon Q^2:

$$W_J = W_J(x, z, Q^2) \text{ and } g_J = g_J(x, z, Q^2) \text{ for } J = 1,2$$

- The (polarized) unpolarized $(\Delta)W_{\mu\nu}$ can be parametrize as,

$$W_{\mu\nu} = \sum_{J=1,2} W_J(x, z, Q^2) T_{J,\mu\nu}(P, q)$$

$$\Delta W_{\mu\nu} = \sum_{J=1,2} g_J(x, z, Q^2) S_{J,\mu\nu}(P, q)$$

Theoretical Framework

- The tensors $T_{J,\mu\nu}$ for the Unpolarized case,

$$T_{1,\mu\nu} = -\, g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}$$

$$T_{2,\mu\nu} = \frac{1}{M^2}\left(P_\mu - \frac{P\,.\,q}{q^2}q_\mu\right)\left(P_\nu - \frac{P\,.\,q}{q^2}q_\nu\right)$$

and $S_{J,\mu\nu}$ for the Polarized case,

$$S_{1,\mu\nu} = \frac{i}{P\,.\,q}\epsilon_{\mu\nu\sigma\lambda}\; q^\sigma S^\lambda$$

$$S_{2,\mu\nu} = \frac{i}{P\,.\,q}\epsilon_{\mu\nu\sigma\lambda}\; q^\sigma\left(S^\lambda - \frac{S\,.\,q}{P\,.\,q}P^\lambda\right)$$

with $S^2 = 1$ and $S \cdot P = 0$.

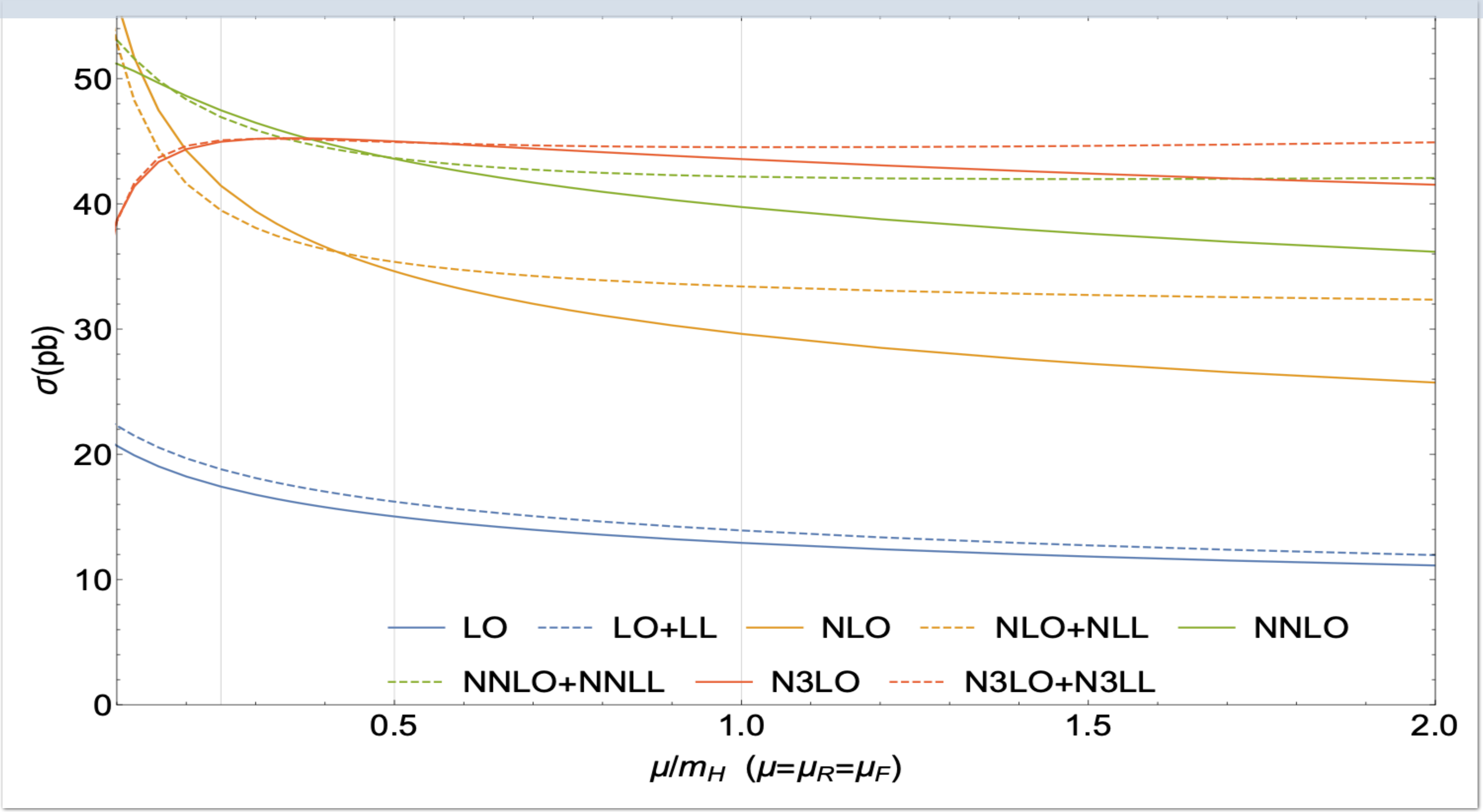
Theoretical Framework

- SFs are not calculable due to the non-perturbative nature of the incoming and outgoing hadrons.

- In the QCD improved parton model, they are related to PDFs, $(\Delta)f_{a/P}(x_1, \mu_F^2)$ and FFs, $D_{H/b}(z_1, \mu_F^2)$ through calculable coefficient functions (CFs), $(\Delta)\mathscr{C}_{J,ab}(x/x_1, z/z_1, Q^2, \mu_F^2)$ at factorization scale μ_F^2.

- Here, $x_1 = P_a/P$ is the fraction of the momentum carried away by incoming parton a from hadron P, and $z_1 = P_H/P_b$ is the fraction of the final state parton b's momentum carried away by the outgoing hadron H$'$.

Why Higher Orders are important ?



Higgs cross-section @ LHC through gluon fusion