Identification of cosmic muons and determination of their momentum distribution using a semiconductor detector

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Muographers - June 3, 2026

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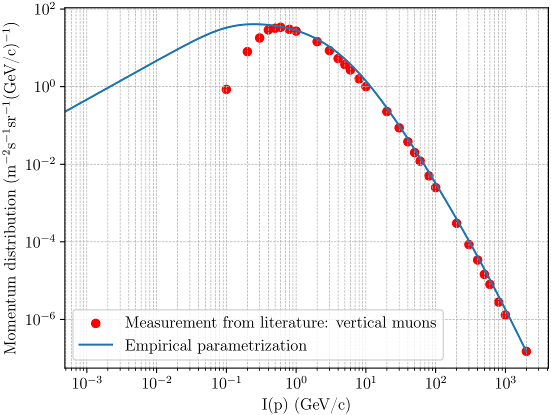
Problem statement

- The empirical parametrization does not match the experimental results across all momentum ranges!
- Problem: the identification of low-momentum muons is challenging.



Empirical parametrization: arxiv.org/pdf/1606.06907.

Measurements from literature: http://crd.yerphi.am/Muons#ref_3.

Setup for measurement

- Detector type: High Purity Germanium detector (HPGe) $\rightarrow$ gamma spectroscopy
- High voltage unit
- Signal amplifier
- Multi channel analyser

- **Question**:
  - How do we measure the energy deposit spectra?
  - Why with this detector?

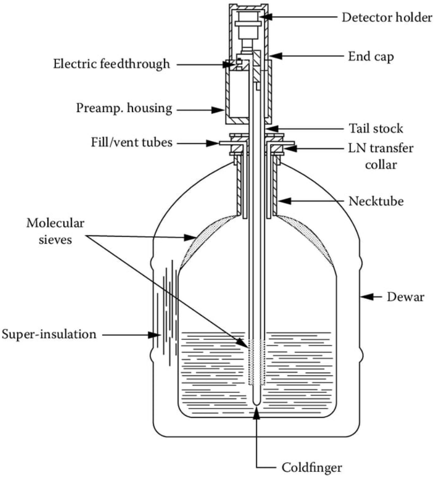
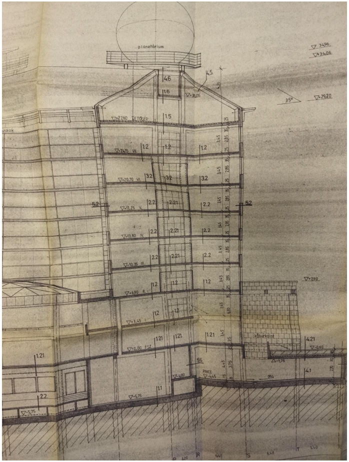


Image source: ResearchGate

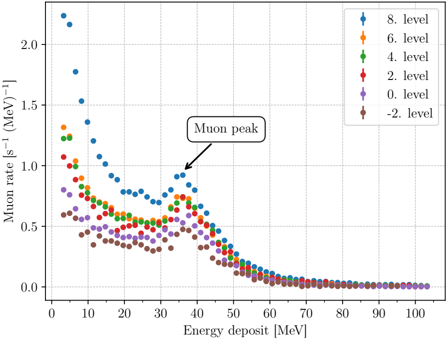
Experimental procedure

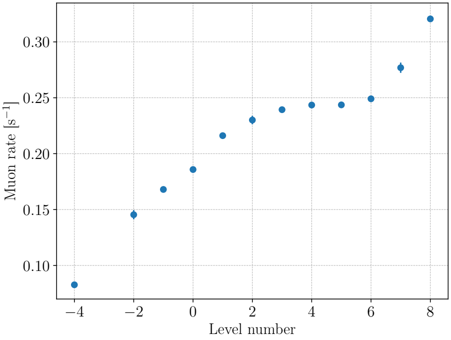
- **Aim**: measurement of the energy deposit spectra at each floor → rate can be calculated.
- **Necessary steps:** energy calibration, usage of a signal attenuator.



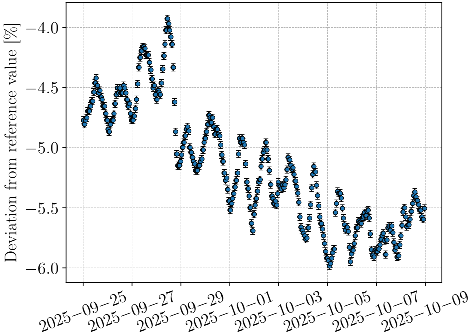
Measured energy deposit spectra

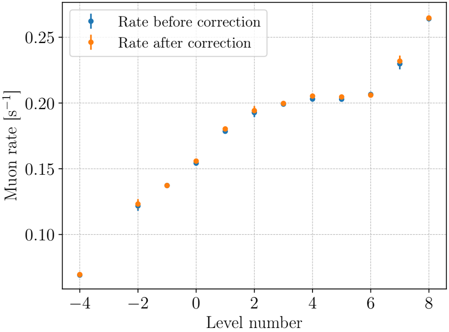
- Measured energy deposit spectra → rate of muons on different floors of the building.





- Rate of muons can vary with the **space weather** → correction is needed.
- Used data: Uragan-hodoscope [1].

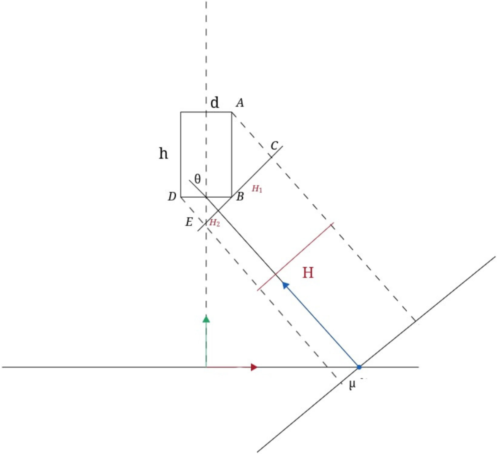


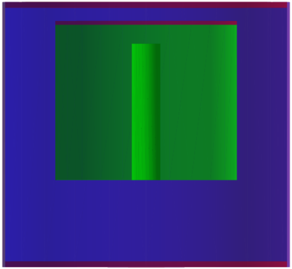


[1] nevod.mephi.ru/uragan-data.

Simulation

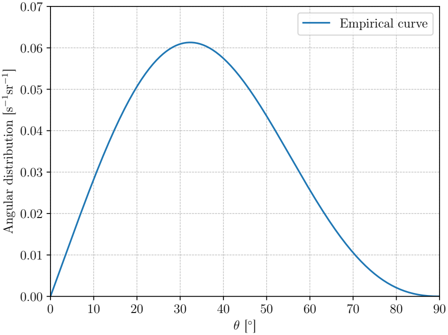
- Used simulation library: Geant4.

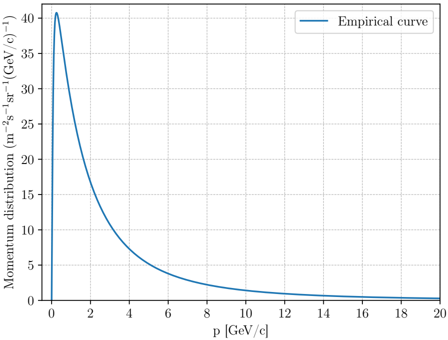




Angular and momentum distribution

- Used form of muon kinematic distributions:





$$\Phi(\theta) = I_0 d \cos^2(\theta) \sin(\theta)(h \sin(\theta) + d \cos(\theta))$$

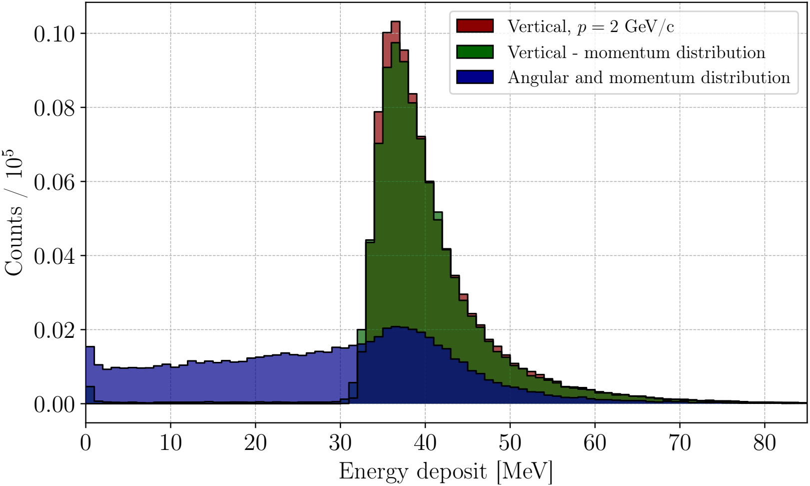
- d, h : diameter, height of the detector;
- I_0, E_0, ε : fitted parameters;
- N : normalization factor;
- p, m : momentum and mass of the muon;

$$I(p) = I_0\, N \left(E_0 + \sqrt{p^2 + m^2}\right)^{-n} \times \left(1 + \frac{\sqrt{p^2 + m^2}}{\varepsilon}\right)^{-1} \frac{p}{\sqrt{p^2 + m^2}} \text{(2)}$$

(2) arxiv.org/pdf/1606.06907.

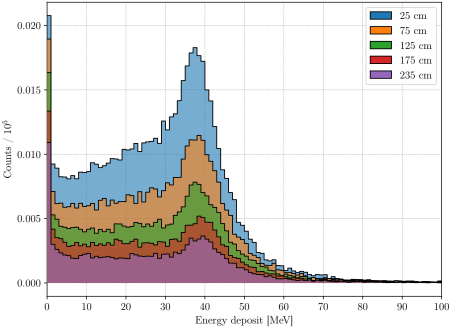
Simulation

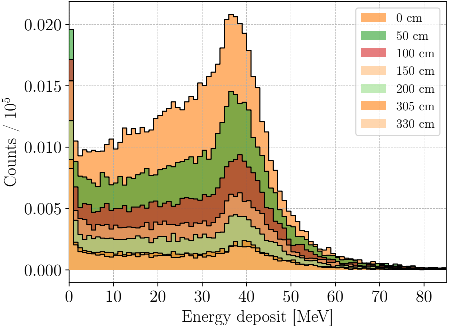
- Simulation without shielding, with varied initial states:



Simulation

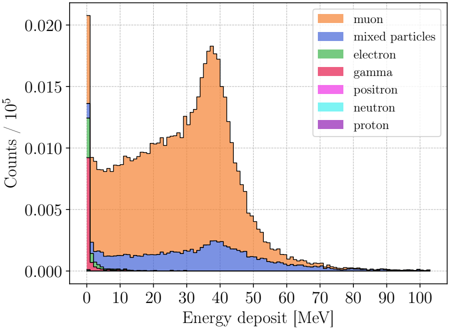
- Simulation results with different shielding thickness:

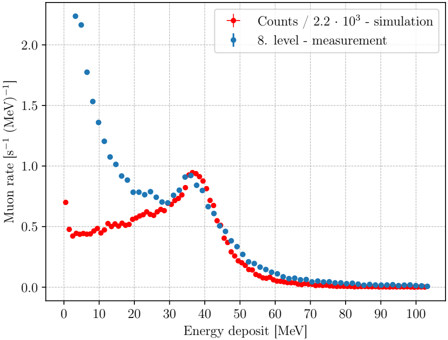




Simulation

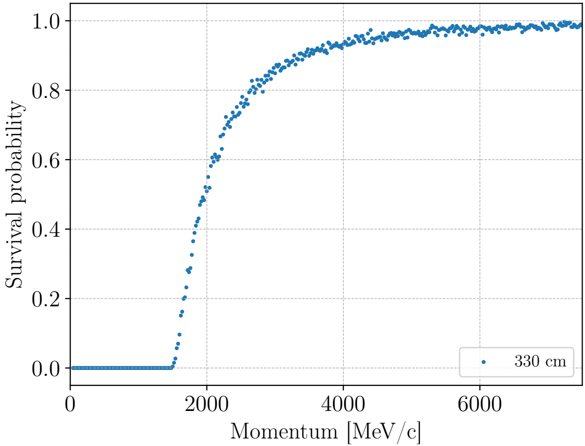
- Comparison between simulation and measurements:
  - The positions of the peaks shift
  - The simulation lacks the large rate at low energies





Analysis of experimental and simulated results

- Calculation the momentum distribution of muons.
- Calculation of the minimum momentum threshold for each shielding thickness.
- Then: $f\left(\frac{p_{i+1}+p_i}{2}\right) = \frac{R_{i+1}-R_i}{p_{i+1}-p_i} = \frac{\Delta R_i}{\Delta p_i}$, R_i, p_i muon rates and survival momenta at the $i-th$ level.



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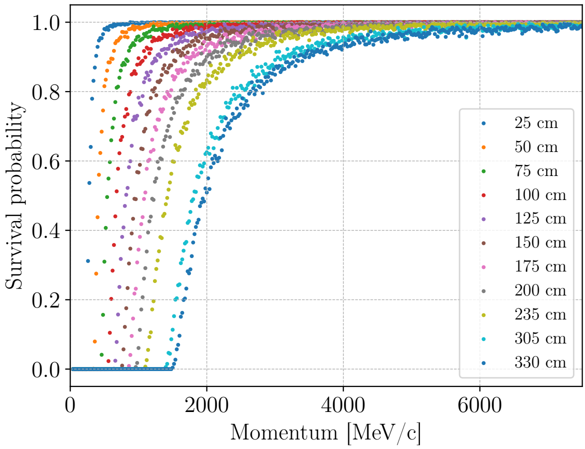
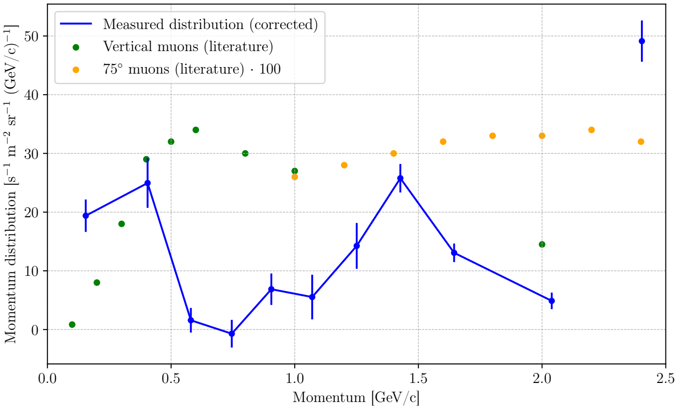
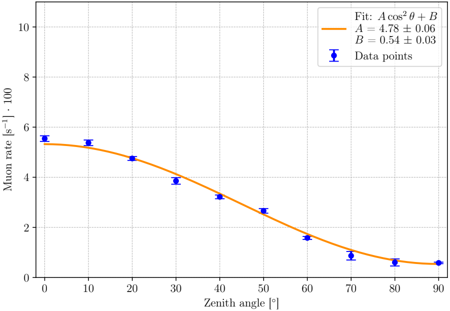


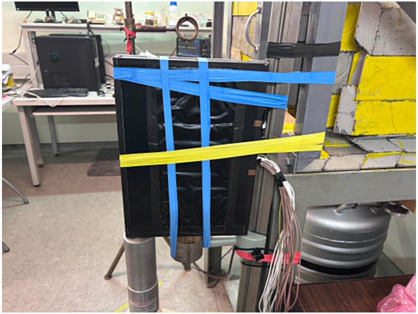
- Dividing by the surface area and the mean solid angle, we get the momentum distribution.



Future plans

- Future plans with muons:
  - Measurement of vertical muons with coincidence method using scintillation pads.
  - Determination of angular distribution.





Thank you for your attention!

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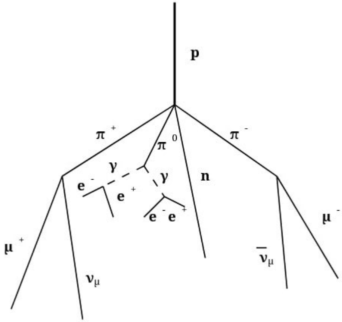
- Where do cosmic particles come from?
  - **From outer space!**
- What types are there?
  - **Primaries:** p^+, ${}^4_2\mathrm{He}$, heavy nuclei
  - **Secondaries:** π^+, π^-, π^0, K^+, K^-, K^0, μ^-, μ^+, e^-, e^+, ...



Energy distribution $\rightarrow$ momentum distribution

- Dimension of energy distribution: $[I] = \mathrm{s}^{-1}\mathrm{m}^{-2}\mathrm{sr}^{-1}(\mathrm{MeV})^{-1}$
- Theoretical parametrization for energy distribution:
  $I(E) = I_0 N (E_0 + E)^{-n}(1 + E/\varepsilon)^{-1}$
- Normalization factor: $\frac{1}{N} = \int_{E_c}^{\infty} I_0 (E_0 + E)^{-n}(1 + E/\varepsilon)^{-1} dE$. $\rightarrow$ Now $E_c = 0$ MeV.
- The applied fitting parameters:

N	I_0 ($\mathrm{m}^{-2}\mathrm{s}^{-1}\mathrm{sr}^{-1}$)	n	E_0 (GeV/c)	$1/\varepsilon$ (GeV^{-1})
37.71 ± 1.02	70.7 ± 0.2	3.01 ± 0.01	4.29 ± 0.04	$1/854$

Parameters of the table: arxiv.org/pdf/1606.06907.

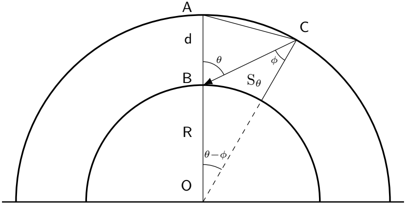
- Location of the measurements: Tsukuba, Japan (36.2° N, 140.1° W)
- Converting into momentum distribution:

$$E = \sqrt{m^2 + p^2}, \quad \text{if we work with c} = 1 \text{ unit,}$$

$$I(E) = \frac{dN}{dE}, \quad \text{and} \quad \exists N(E(p)) \quad \longrightarrow \quad \frac{dN}{dp} = \frac{dN}{dE} \cdot \frac{dE}{dp},$$

$$\frac{dE}{dp} = \frac{p}{\sqrt{p^2 + m^2}},$$

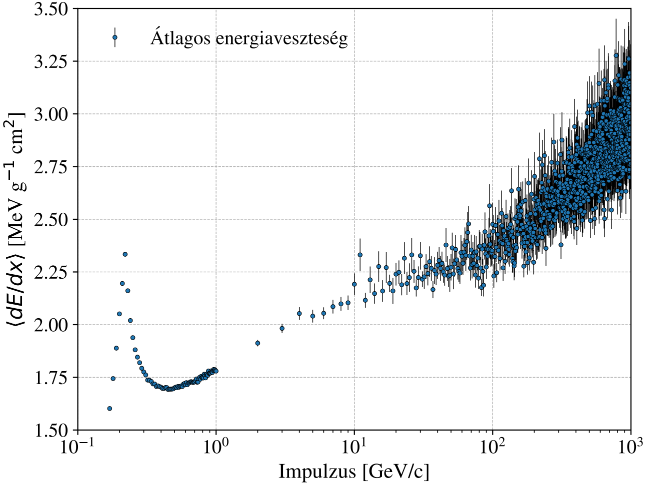
Why $\cos^2(\theta)$?



- This is just an approximation! In reality, the exponent is close to 2.
- It is closely related to the momentum distribution:
    - Its shape can be discussed as a function of momentum distribution, derived from pion and proton momentum distributions.

Energy deposit in concrete shielding

- The average energy deposit in concrete can be verified by simulation as a function of momentum, yielding the Bethe-Bloch curve.



- The shape of the Bethe-Bloch curve:

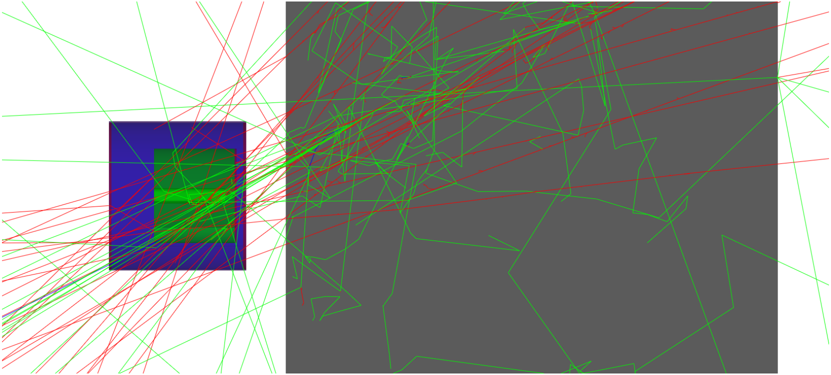
$$\left\langle \frac{dE}{dx} \right\rangle = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta}{2} \right].$$

Parameter	Name	Value	Dimension
K	constant	0.307	[MeV·cm^2/g]
m_e	electron mass	0.511	[MeV]
$T_{\max}$	maximum transferable energy to an electron	305	[MeV]
I	mean excitation potential	$6 \cdot 10^{-4}$	[MeV]
z	charge of the incident particle	1	-
Z	atomic number of the medium	32	-
A	atomic mass of the medium	73	-
β	relative velocity	0.9986	-
c	speed of light	1	-
γ	Lorentz factor	18.9	-
δ	density correction factor	5.35	-

- The average energy deposit:

$$\Delta E = \left\langle \frac{dE}{dx} \right\rangle \cdot \rho \cdot \Delta x.$$

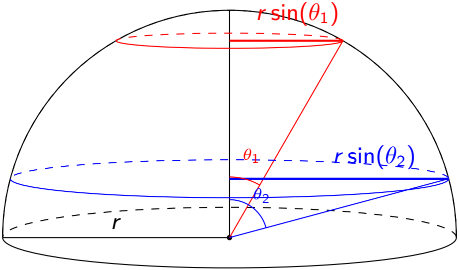
Muon interactions in concrete



- Above we can see some muon interactions traversing through concrete shielding.

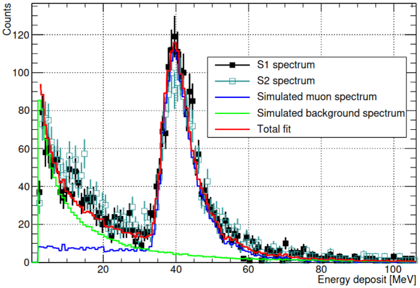
Angular distribution

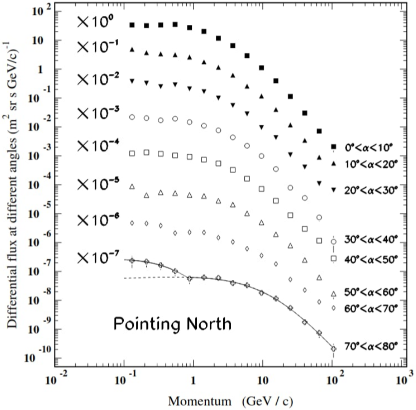
- Why does a $\sin(\theta)$ factor appear? The radius is proportional to it.



Low energy/momentum region

- These regions are non-trivial regarding both energy deposit and momentum distribution.
- Muon identification is challenging in the $[0, 1]$ GeV/c range.
- Below $\Delta E < 25$ MeV, the energy deposits of other particles also appear.





Left figure: arxiv.org/pdf/2206.13061.
RIght figure: ResearchGate