Experiences with tomographic inversion of subsurface muographic measurements
(bias and artifacts)

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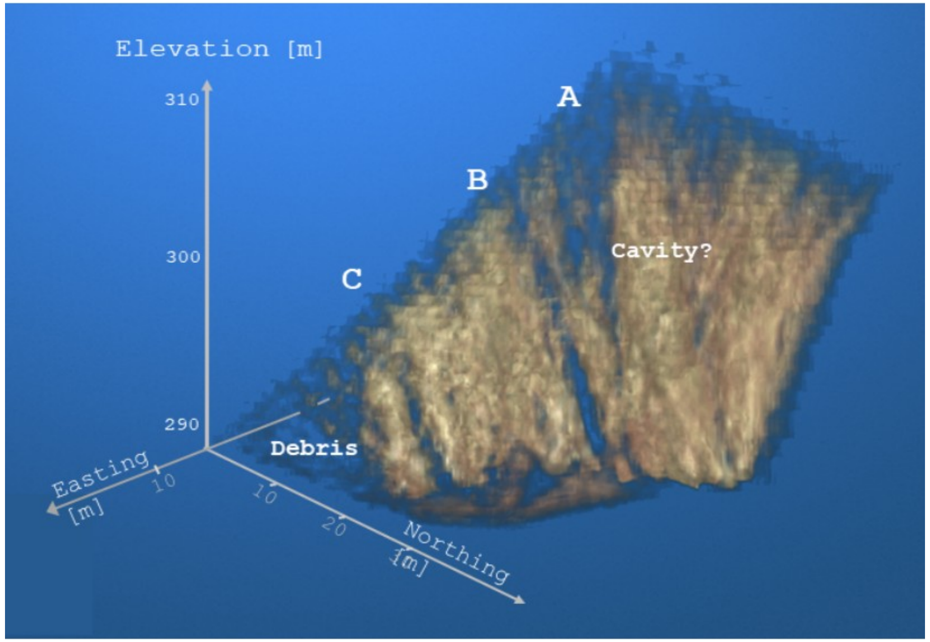

Key words:
Regularization, Bias, Covariance, Artifacts

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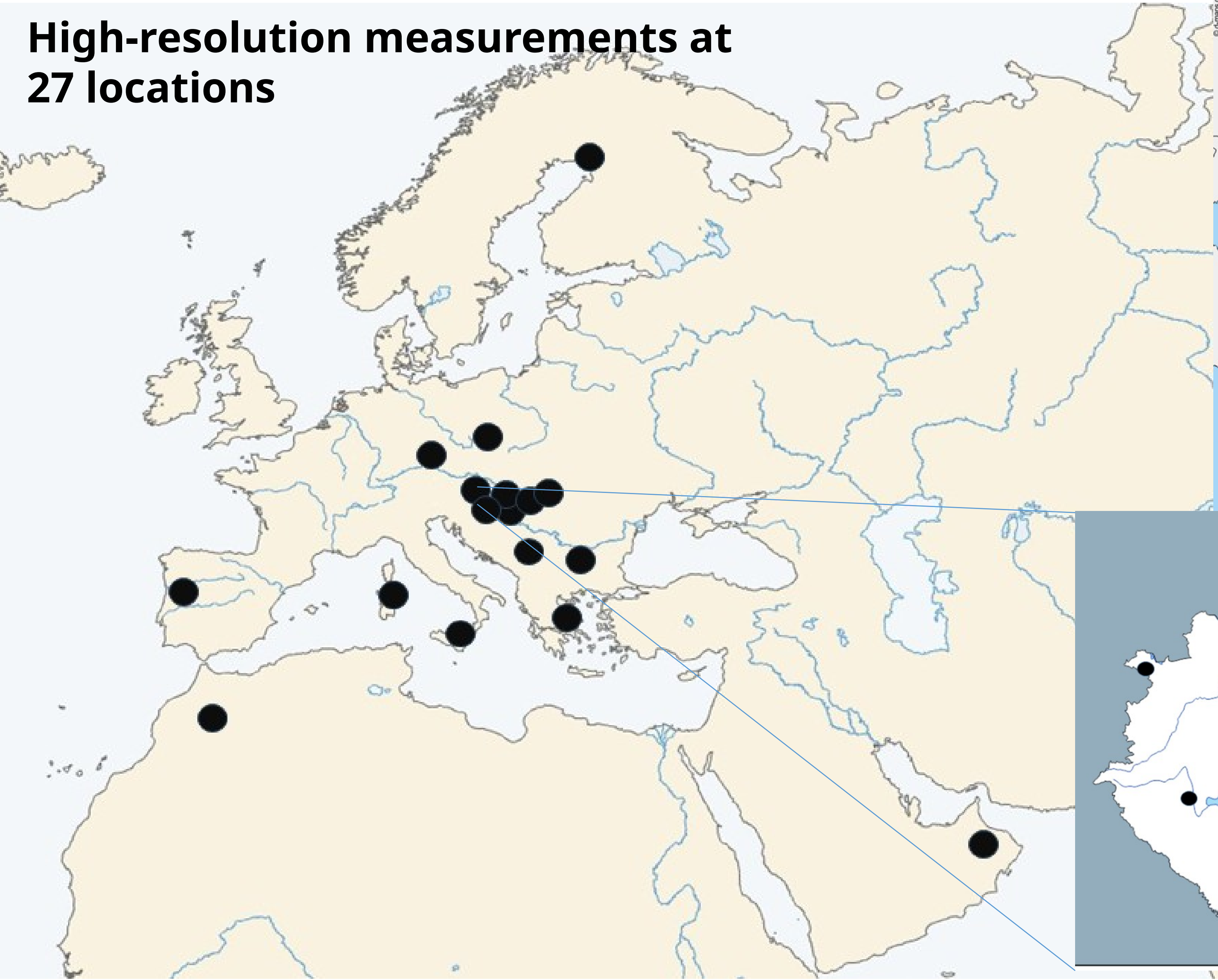
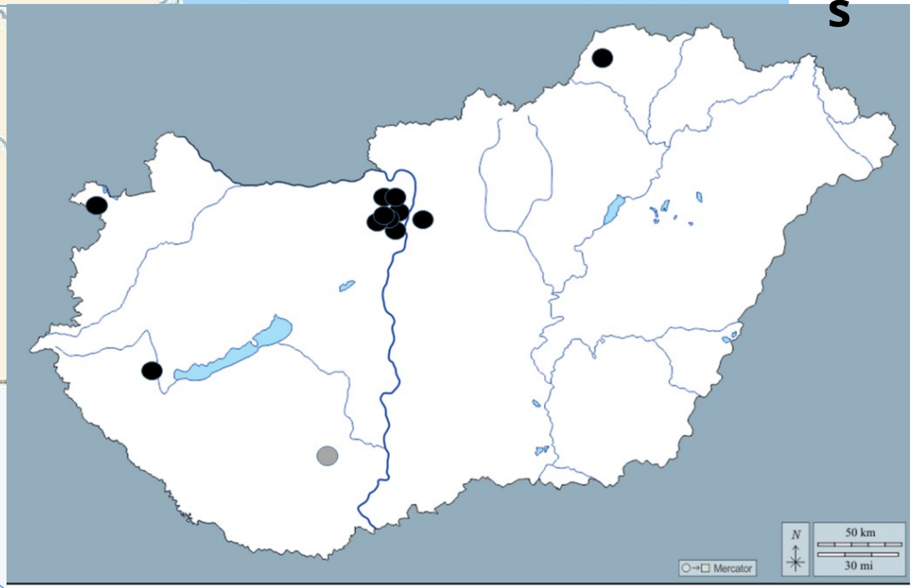
Muon tomography: experiences (Muographic projects)

High-resolution measurements at 27 locations

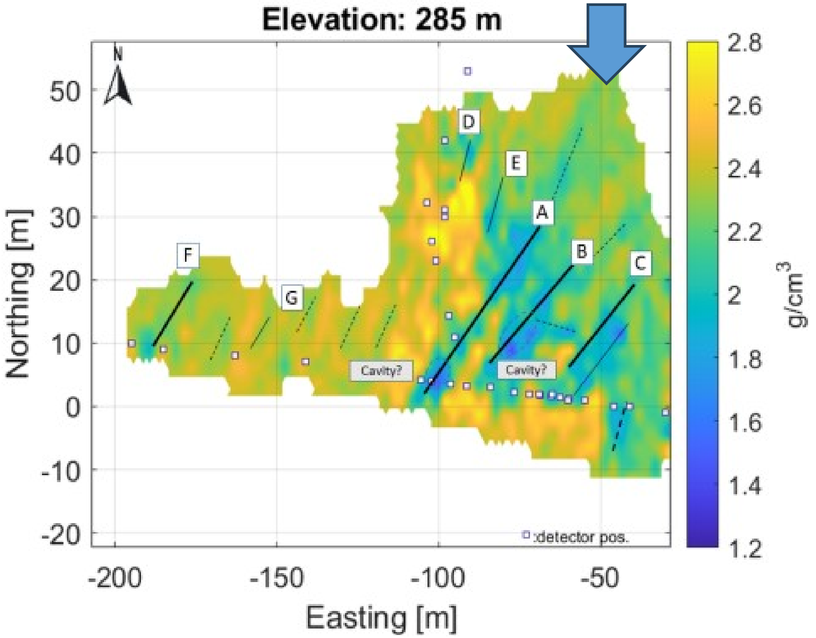


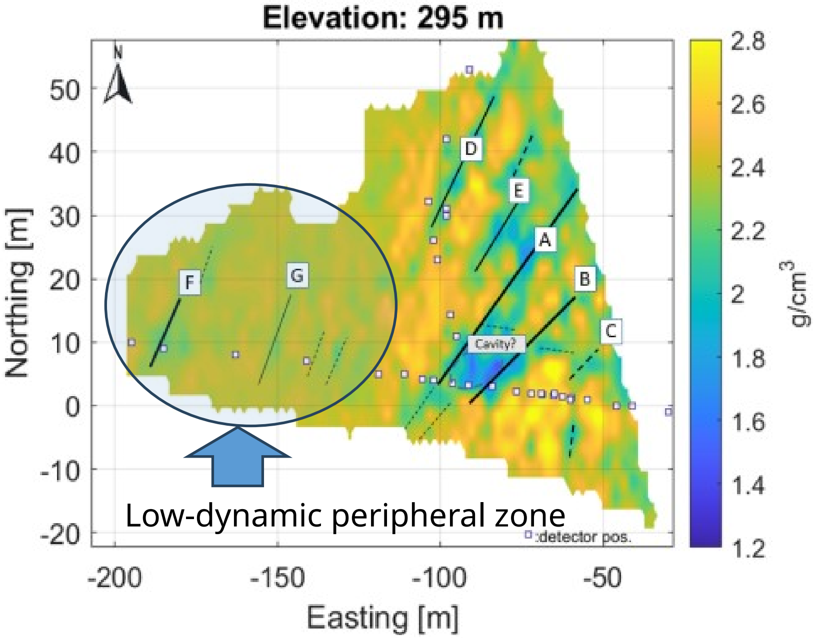
Hundreds of measurement campaigns

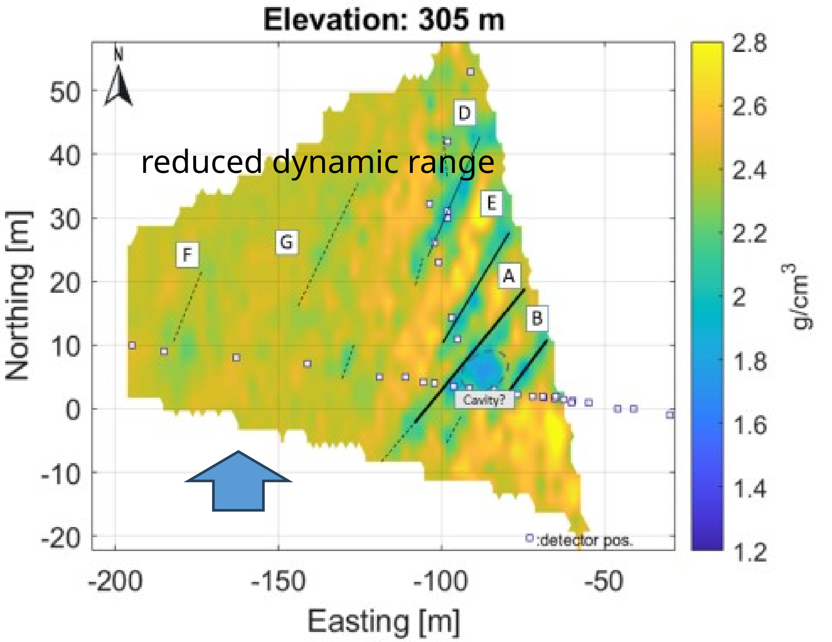
High resolution measurements In extreme environments

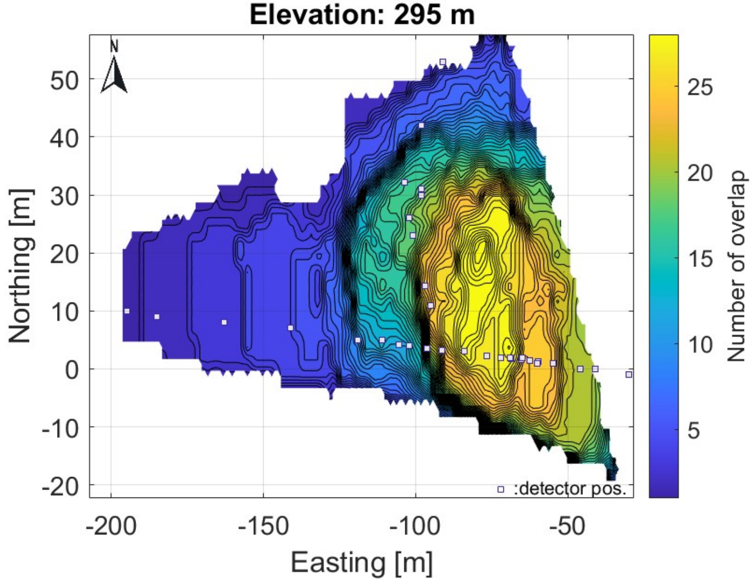


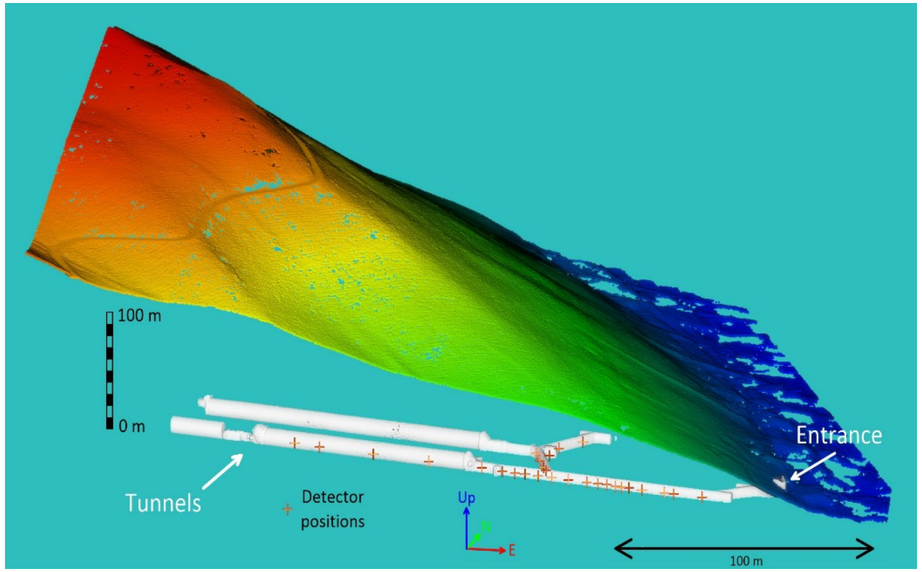
Muon tomography: Muographic projects (Budapest, Hármashatár-hill)

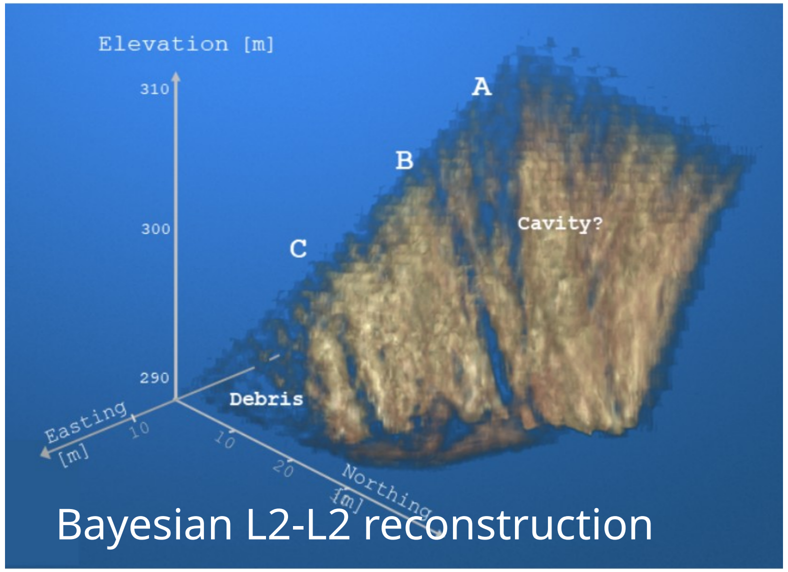




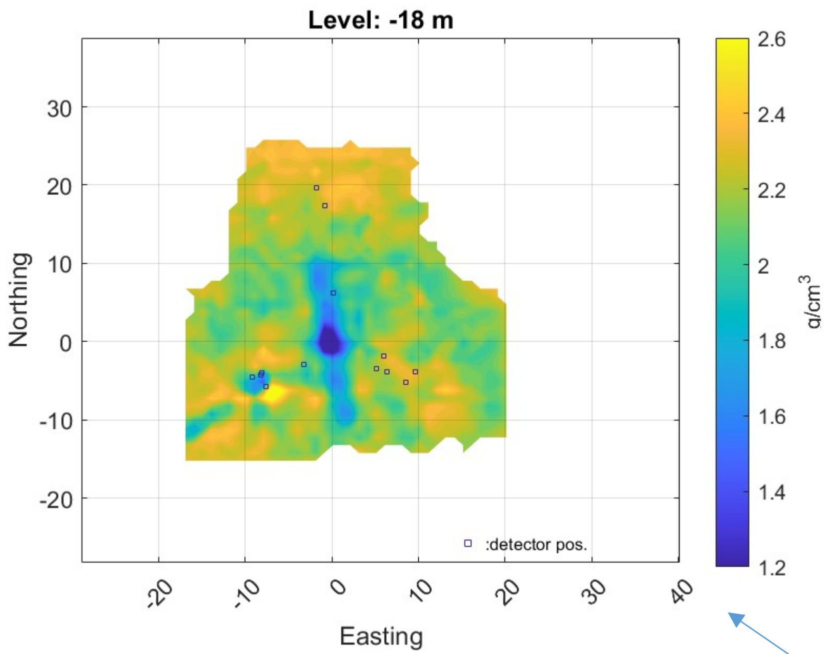


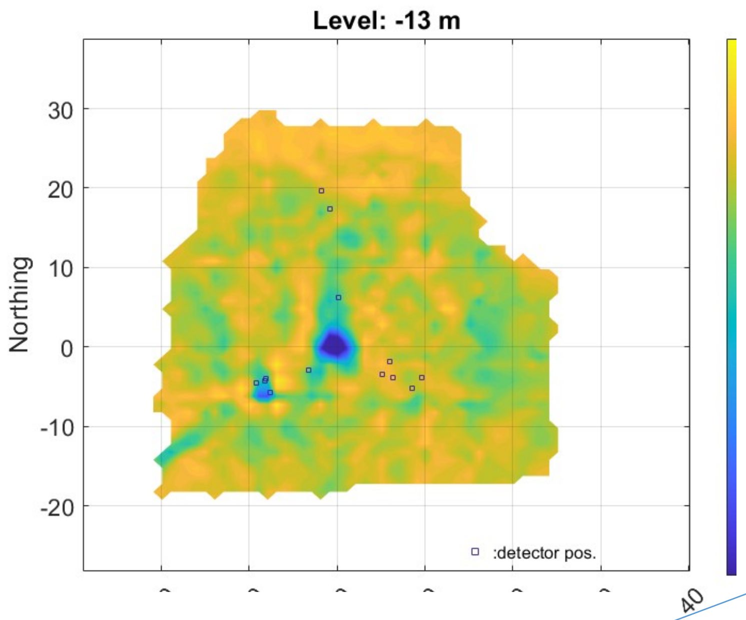


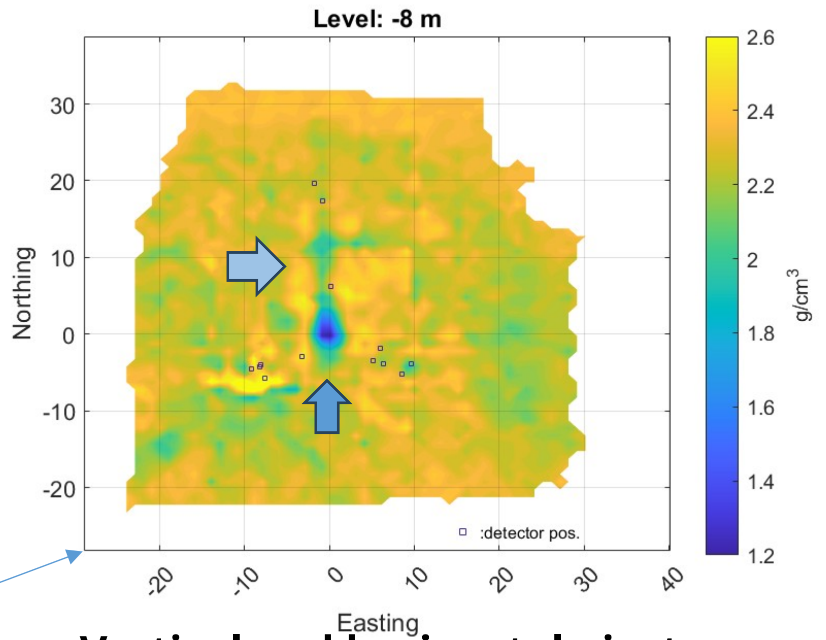




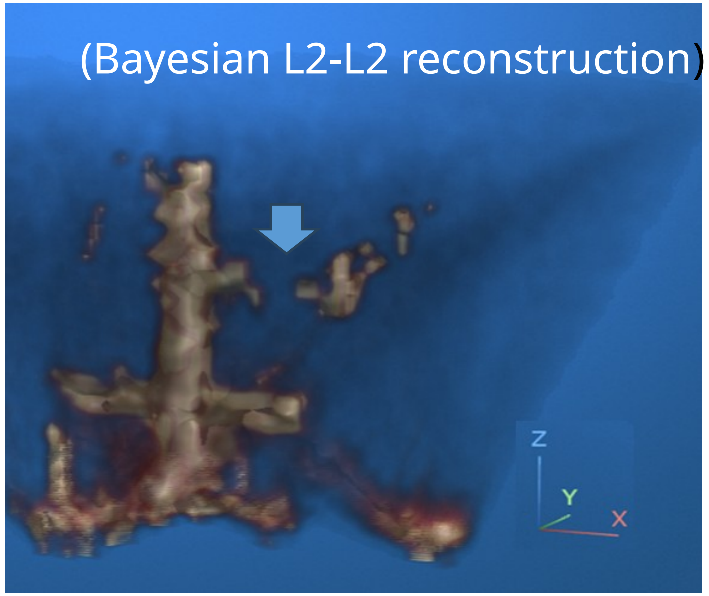
Muon tomography: Muographic projects: Jánossy Underground Lab.



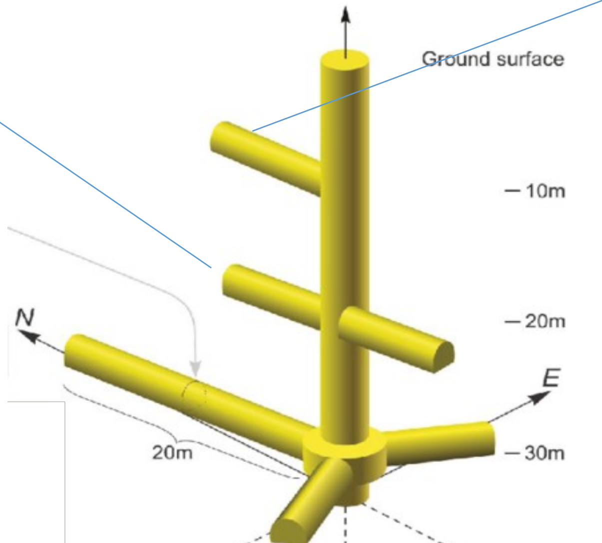




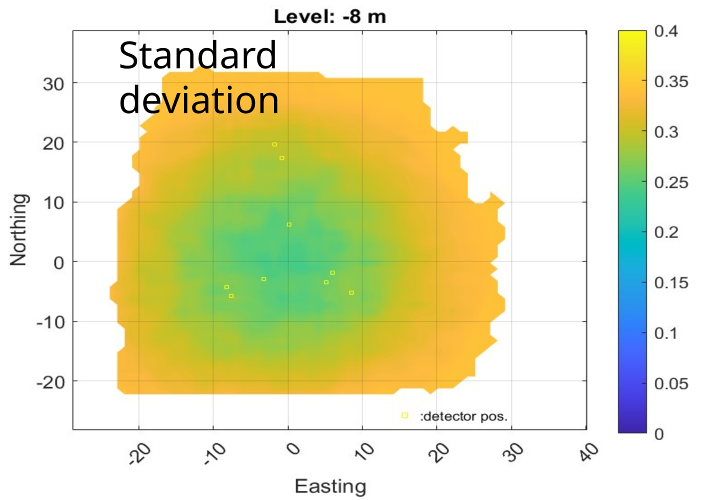
Vertical and horizontal ojects on 3D tomographic results



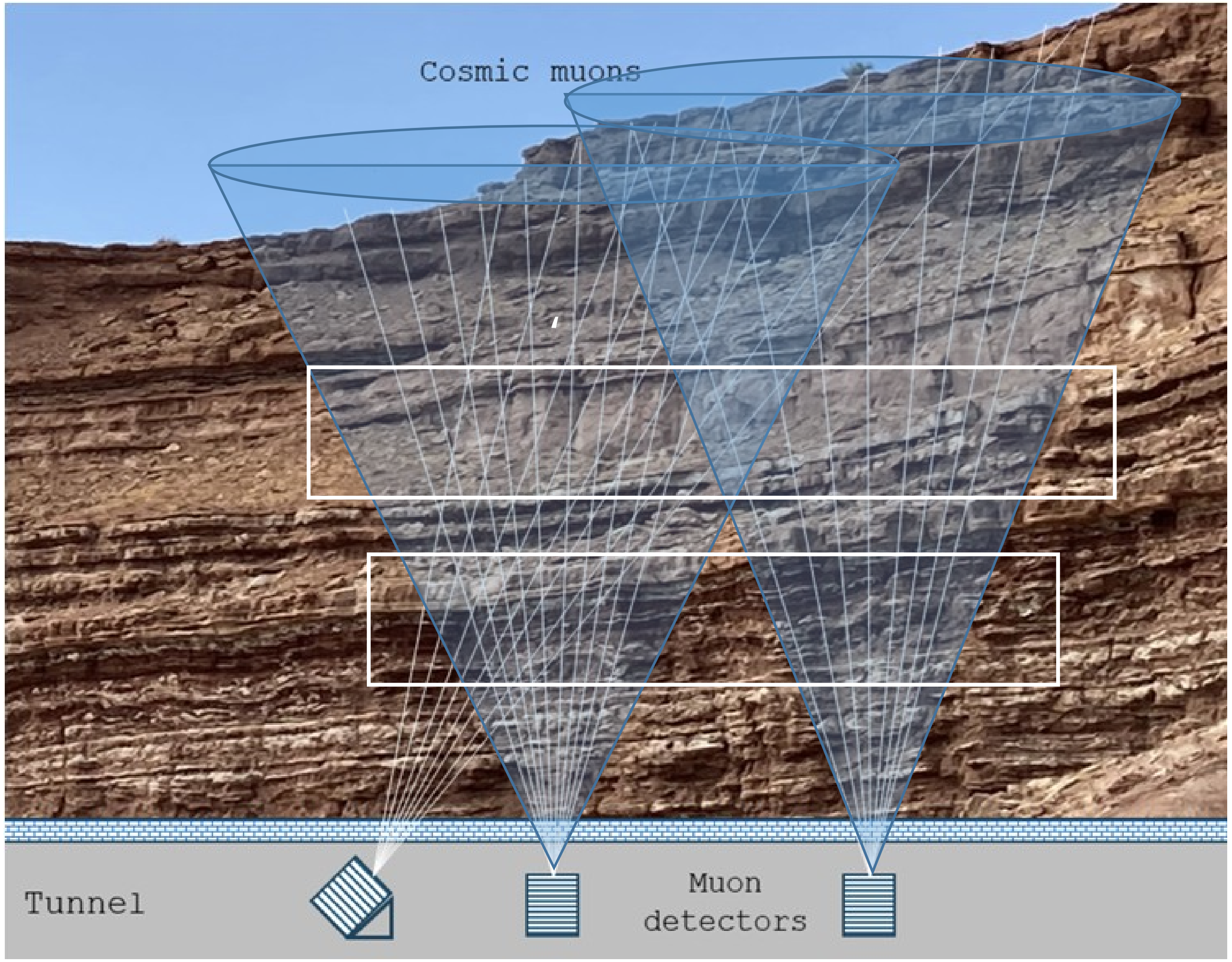
(Bayesian L2-L2 reconstruction)



Reference measuring site with known geometry



Muon tomography: What is causing the problem?



Limited number of measuring position
with limited measirement time

Limited azimuth range
(sampling problem)

Interchangeable subsets in the solution set (on the same set of trajectory)

Jacobian (permutation) symmetry
(equivalences)

High-dimensional null space
(with vertically oriented base vectors)

Nullity>Rank situation

Singular pseudo inverse

Regularization is necessary, with all that it entails

Muon tomography: Tomogrpahic inverse problem

Linearized direct problem of discrete tomography

Pixel-voxel transformation

$$\mathrm{F}\varrho = \boldsymbol{\gamma}$$

Density length error distribution

$$\mathcal{N}(\mathbf{0}, \boldsymbol{C}_\gamma)$$

prior distribution

$$\mathcal{N}(\boldsymbol{\varrho}_0, \boldsymbol{C}_0)$$

(informative or noninformative, generally smooth)

ML-term (loss)

Prior-term

$$\hat{\boldsymbol{\varrho}} = \min\_arg_{\boldsymbol{\varrho}}\{(\boldsymbol{\gamma} - \mathrm{F}\boldsymbol{\varrho})^T \boldsymbol{C}_\gamma^{-1} (\boldsymbol{\gamma} - \mathrm{F}\boldsymbol{\varrho}) + (\boldsymbol{\varrho} - \boldsymbol{\varrho}_0)^T \boldsymbol{C}_0^{-1} (\boldsymbol{\varrho} - \boldsymbol{\varrho}_0)\}$$

$$\mathbf{R} = \mathbf{F}^{\mathbf{T}} \mathbf{C}_\gamma^{-1} \mathbf{F}$$

Fisher-matrix

$$\hat{\boldsymbol{\varrho}} = (\mathbf{R} + \mathbf{C}_0^{-1})^{-1} \left(\mathbf{F}^T \mathbf{C}_\gamma^{-1} \boldsymbol{\gamma} + \mathbf{C}_0^{-1} \boldsymbol{\varrho}_0\right)$$

$$\mathbb{E}(\hat{\boldsymbol{\varrho}}) = (\mathbf{R} + \mathbf{C}_0^{-1})^{-1} (\mathbf{R}\varrho + \mathbf{C}_0^{-1} \boldsymbol{\varrho}_0)$$

The result vector is the weighted sum of the two types of information (flexibility)

From the null space, prior selects one element ~ NS(prior)

Unique solution with prior dependent bias

Muon tomography: Error term: bias and variance

The source of bias is always a real an

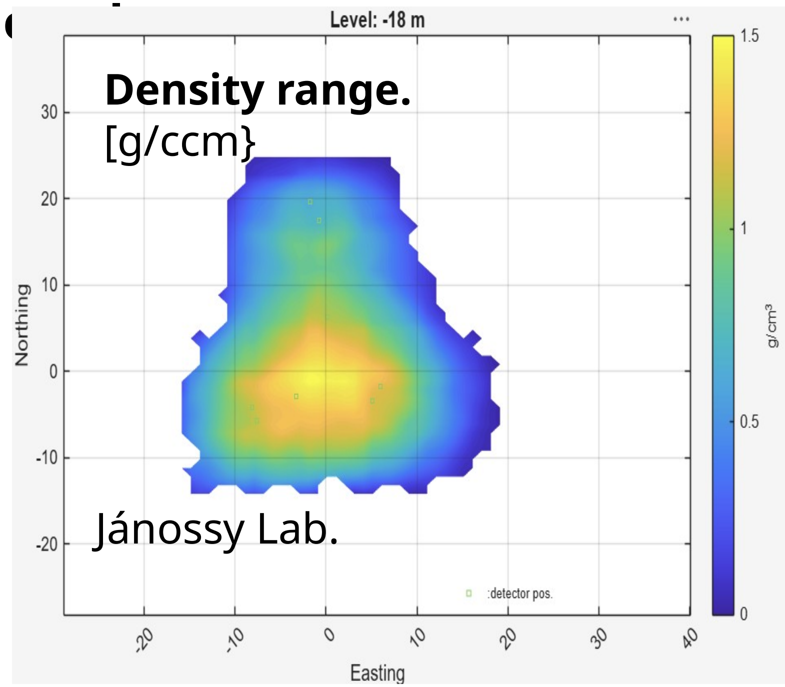
Bias

Posterior distribution

$$\delta\hat{\varrho} = \left(R + C_0^{-1}\right)^{-1} C_0^{-1}(\varrho_0 - \varrho)$$

$$C_{\hat{\varrho}} = \left(R + C_0^{-1}\right)^{-1}$$

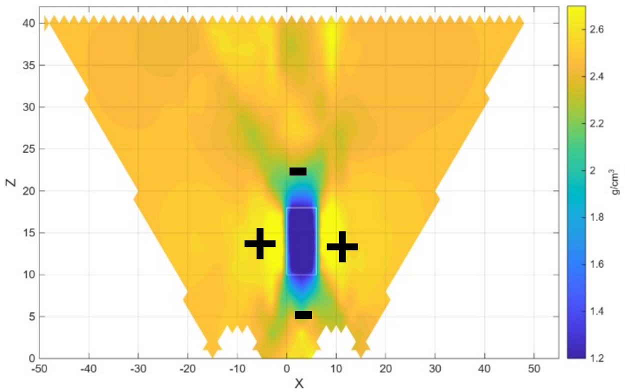
Parameter Covariance-matrix



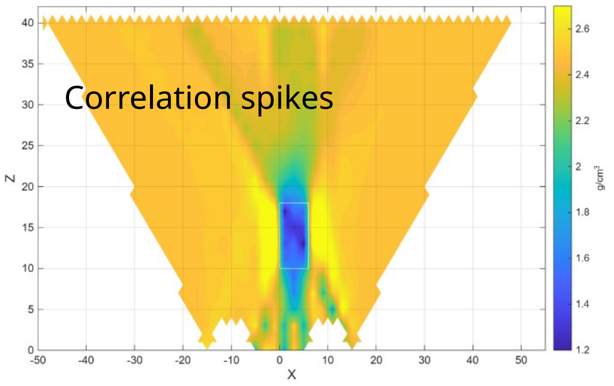
Formally, variance and bias are linked through the covariance matrix „Trade-off"

„point" spread function (on Simulated 2D inversion)

Localized inversion



Non localized inversion



Result of regularization: Smoothing around the prior: **dynamic range compression** + **Fisher-matrix baked correlations**

Anomaly leak along the trajectories
Anomaly at the detector position: Compensation artifact

Muon tomography: Algorithms and priors

Key inversion algorithms

- **SVD: Spectral Value Decomposition**
- **Tikhonov-type regularization**
- **Bayes-type estimation (Geologic prior)**
- **Filtered Back Projection**
- **Expectation Maximalization (MLEM)**
- **Early stop Iterations**

+ bias reduction post process

Prior

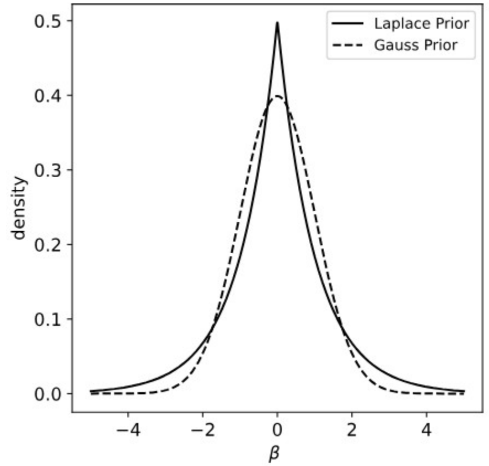
- **Minimum norm**
- **Smoothness (spatial correlations)**
- **Prior density distribution (large scale geology)**
- **Z dependent**
- **Fisher inforn**

Because of fan-shaped imaging

Metric and pdf

Gauss or Mixed Gaussian L2 norm

Laplacean L1 norm



Sparsity handling

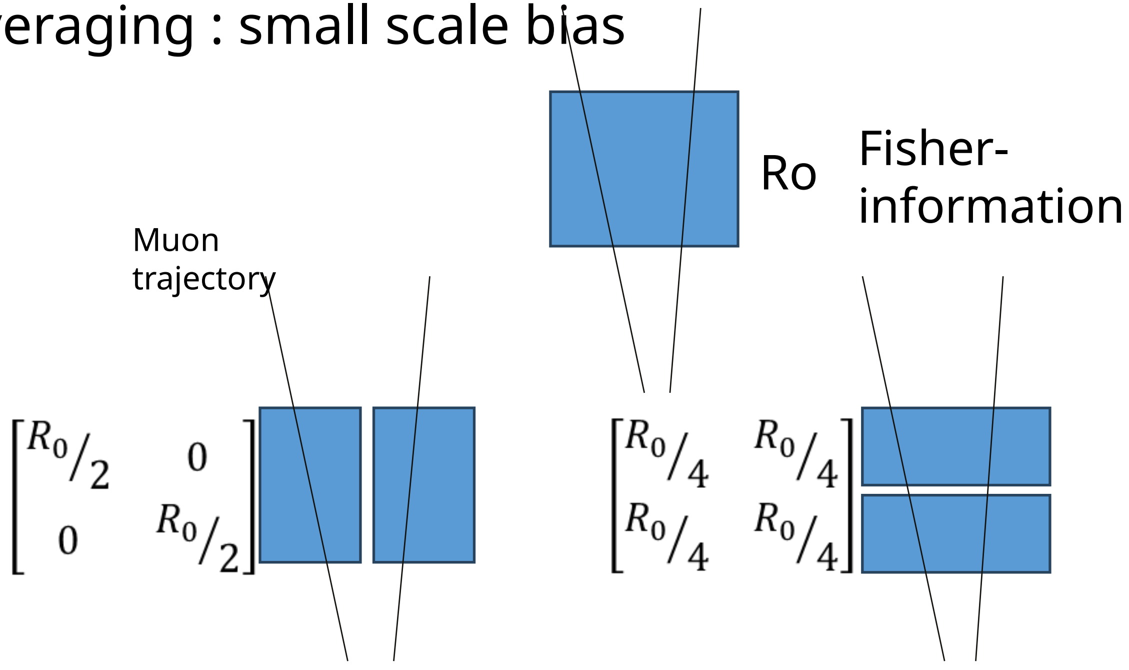
Muon tomography: Voxel grid and adaptive prior

The prior distribution relates to voxel based densities, so it depends on the voxel size.

Dependence requires an appropriate averaging model.

(scalable prior)

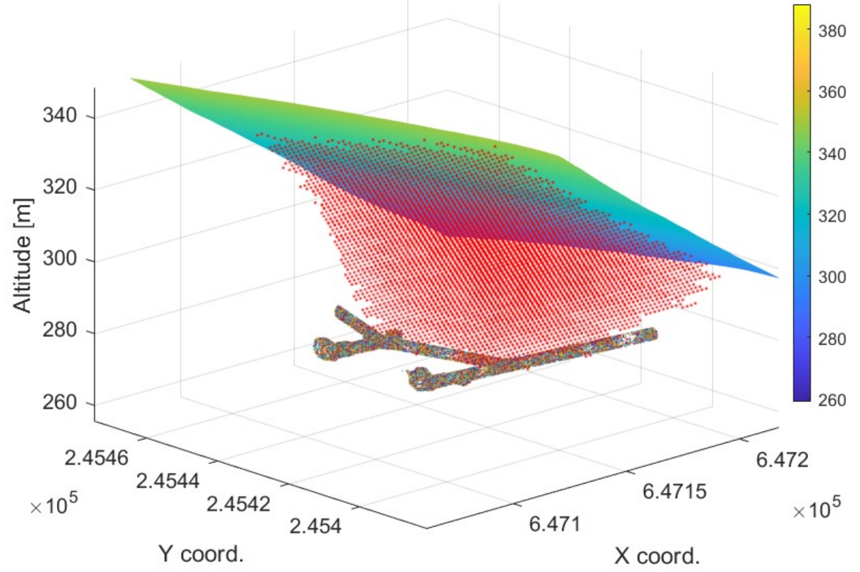
Averaging : small scale bias



$$\begin{bmatrix} R_0/_2 & 0 \\ 0 & R_0/_2 \end{bmatrix} \qquad \begin{bmatrix} R_0/_4 & R_0/_4 \\ R_0/_4 & R_0/_4 \end{bmatrix}$$

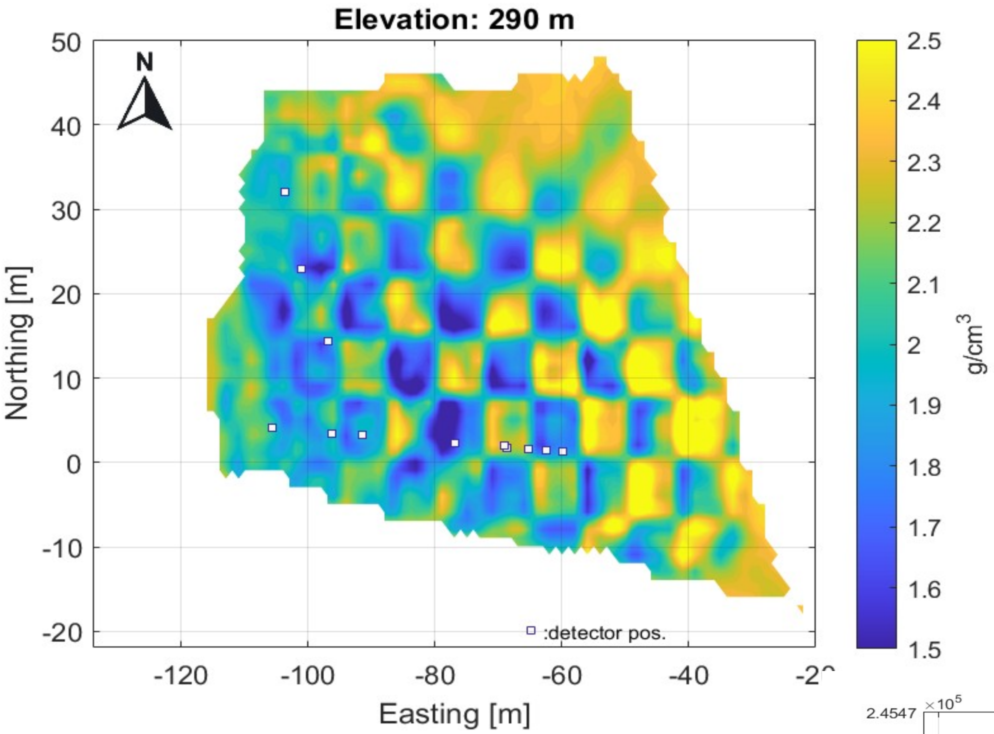
Information loss and voxel split

Voxel grid – Hármashatár hill

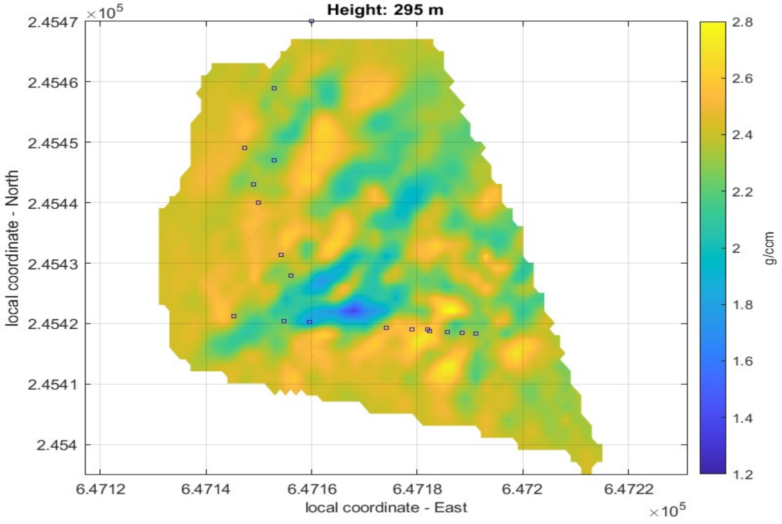


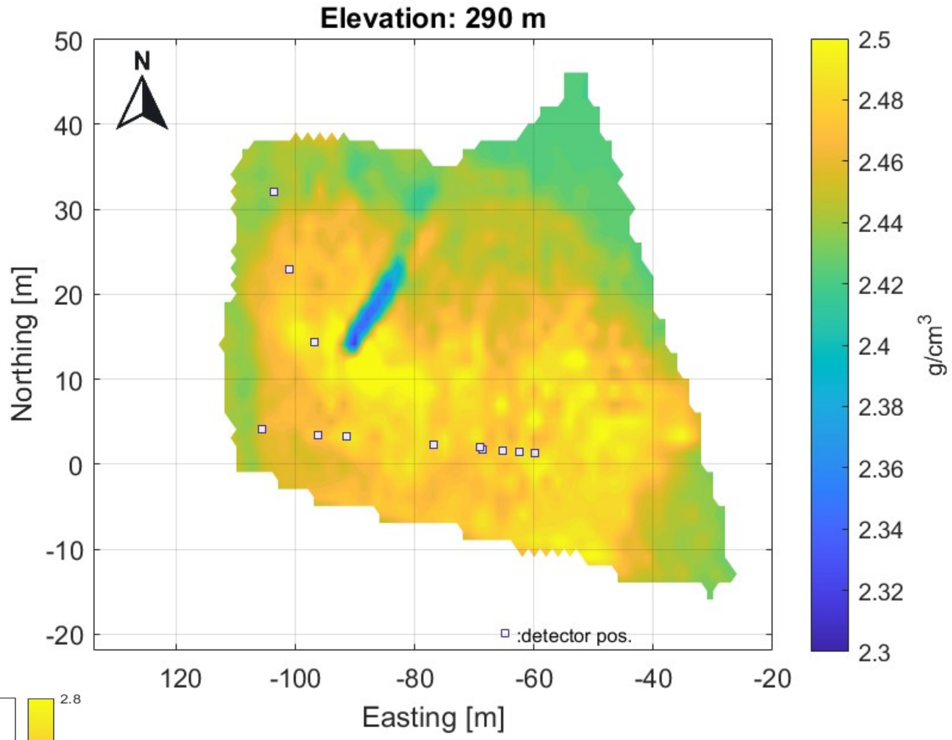
An overly refined model increases the correlation among voxel elements. Changing the correlation structure (associated with the mapping) can reduce the rank of the Fisher matrix

Muon tomography: Simulation and tests



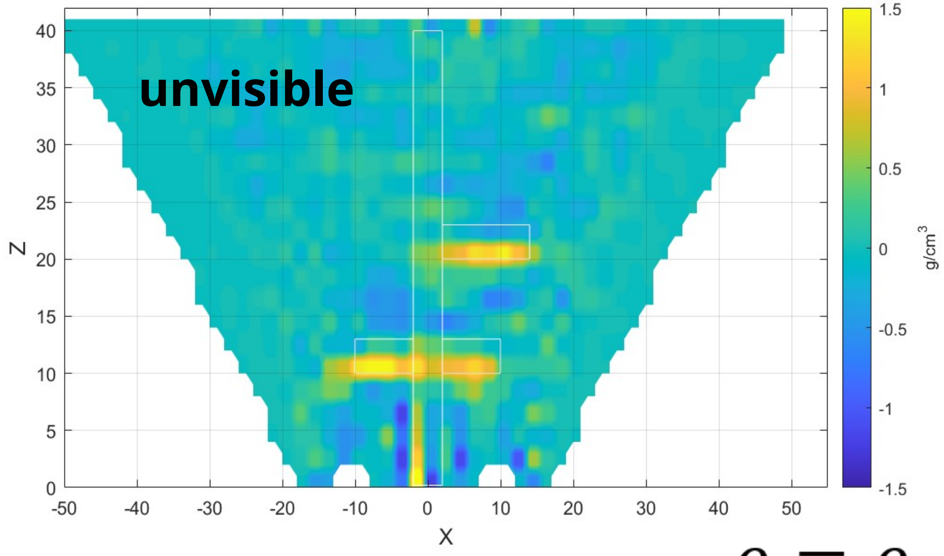
Chessboard-test
For focus range definition
(Bias mapping)

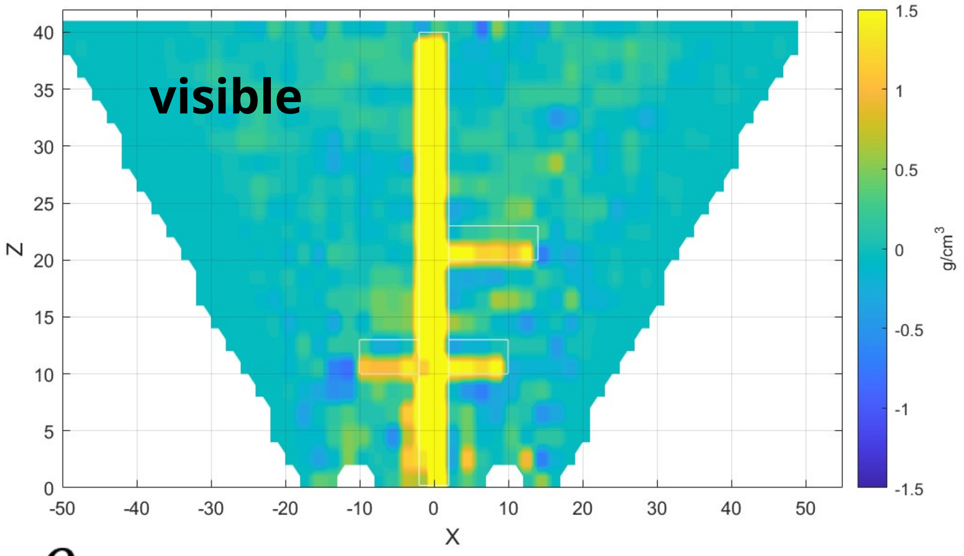




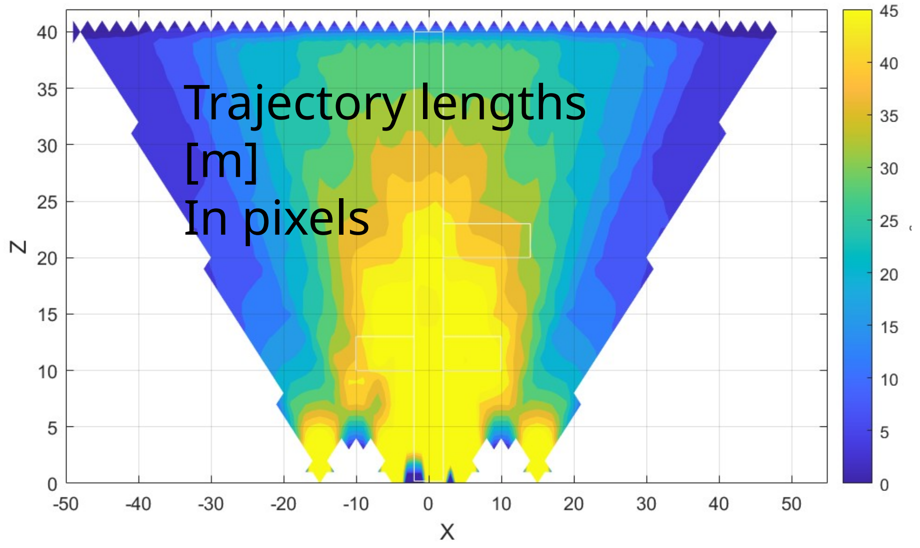
Fault-test for detectability
(Hármashatár hill)
The detection limit changes with the regularization parameters

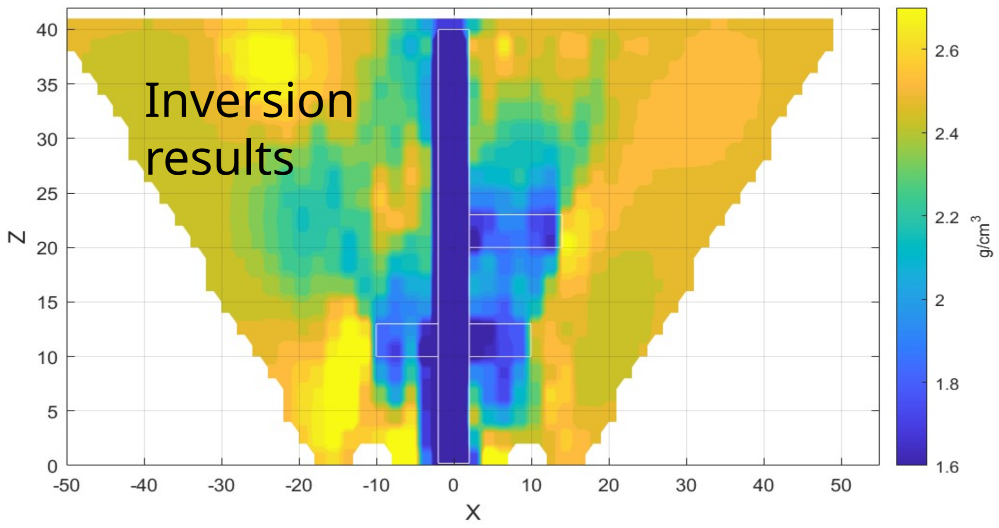
Muon tomography: Horizontal and vertical object distortion





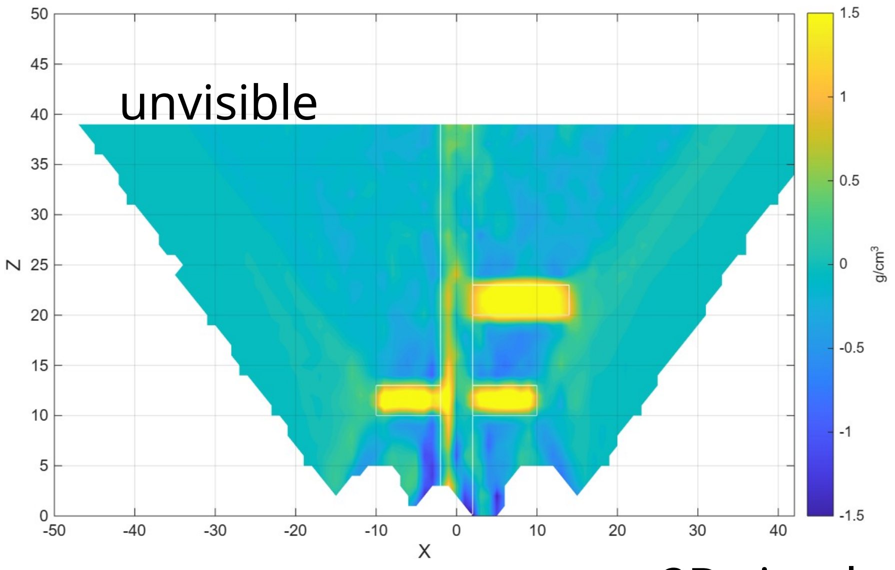
$$\varrho = \varrho_{visible} + \varrho_{unvisible}$$

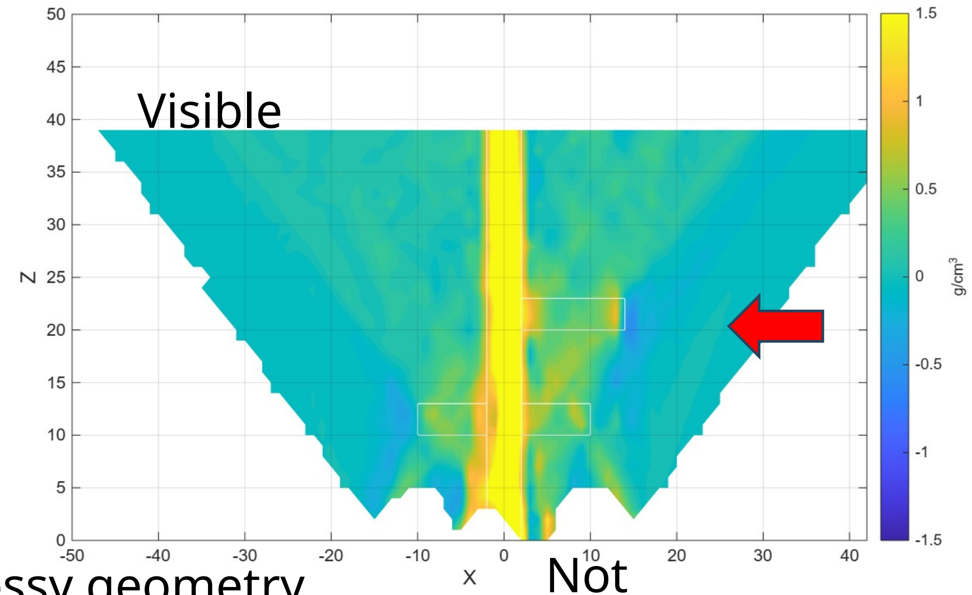




2D simulation in Janossy geometry (2D is the worst case in term of artifact)

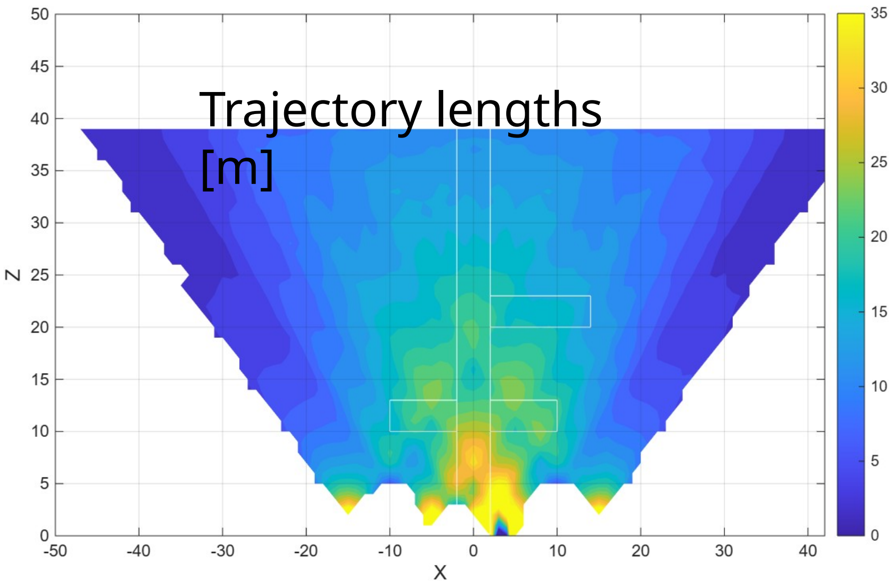
Muon tomography: Horizontal and vertical object distortion

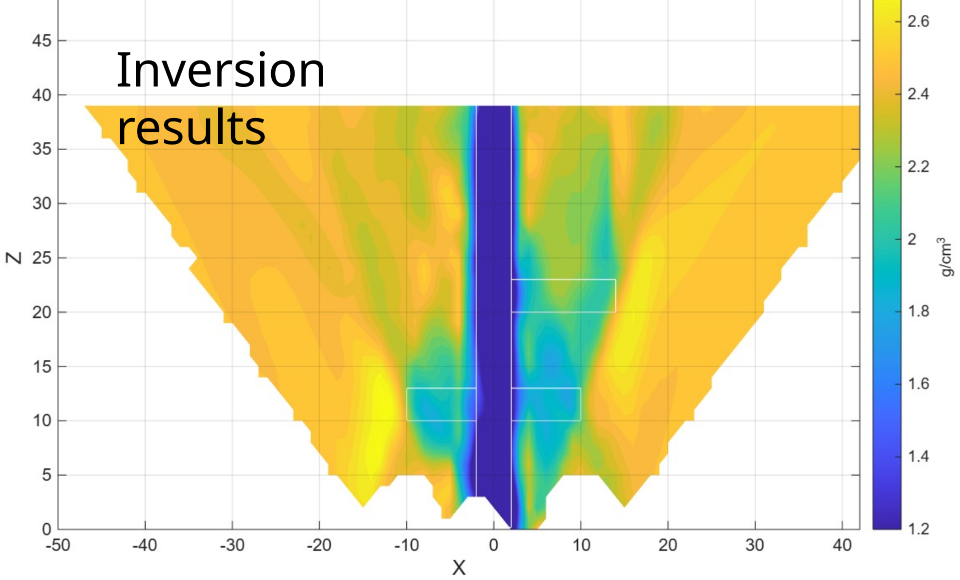




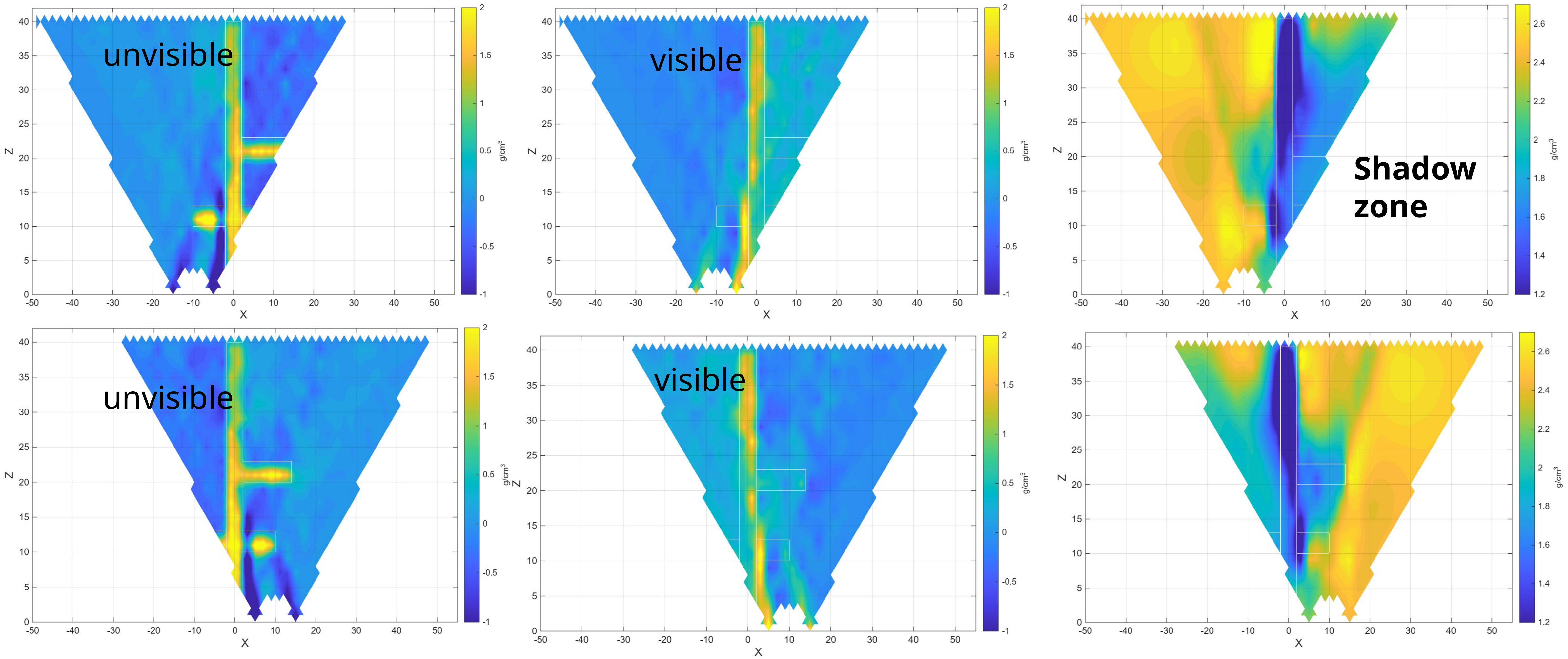
Not localizable

2D simulation in Janossy geometry





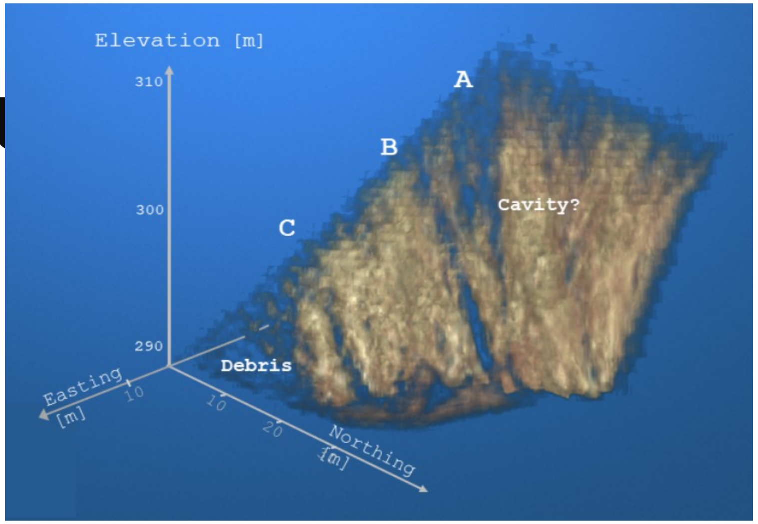
Muon tomography: One-sided measurements



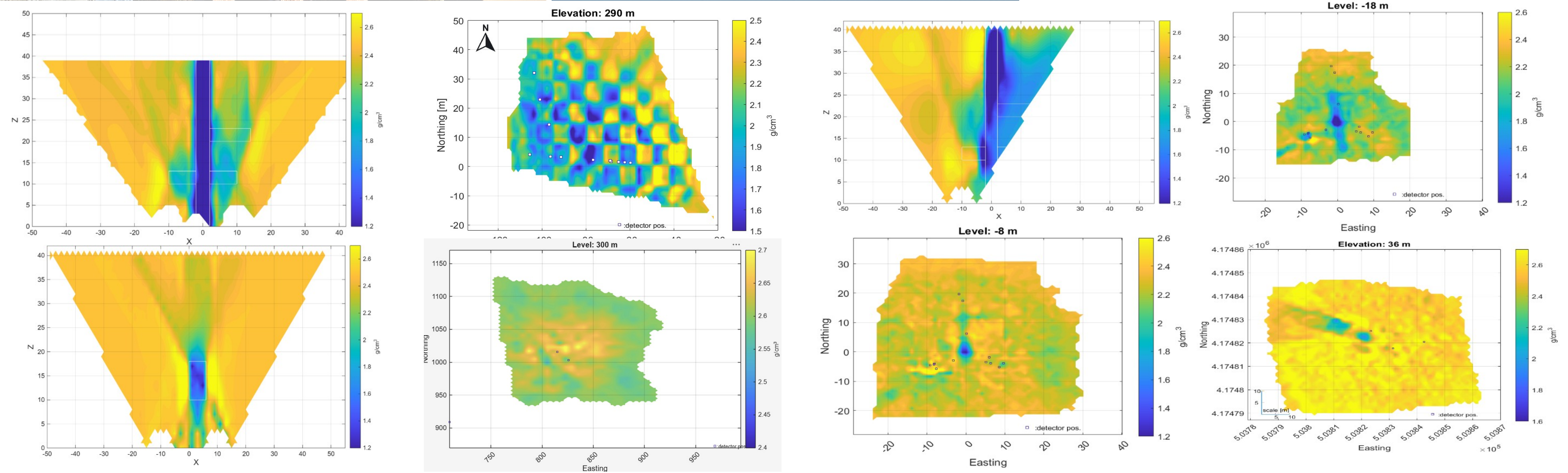
One-sided mapping – apparently tilted object + shadow zone

Muon tomography

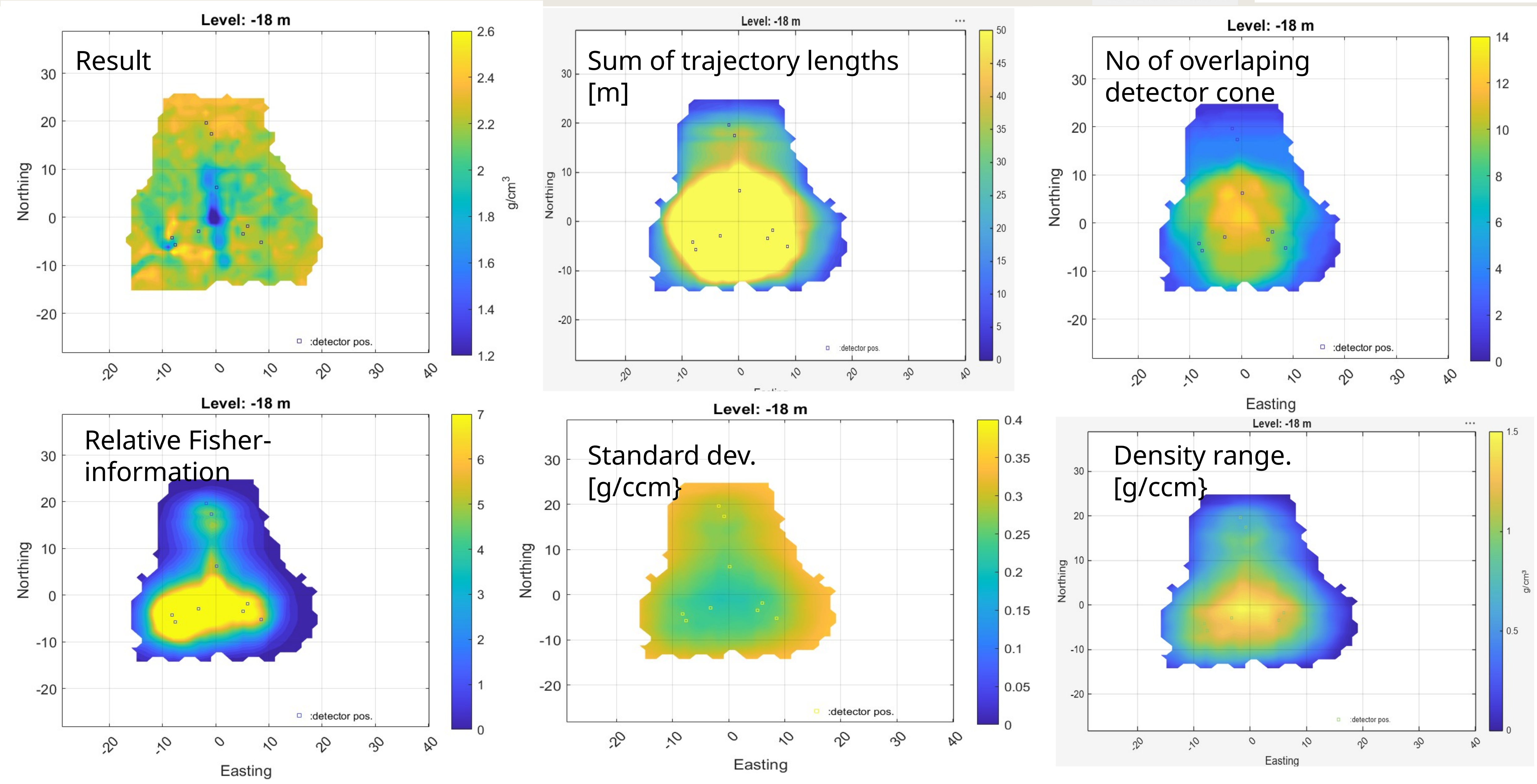
Thank you for yo attention!



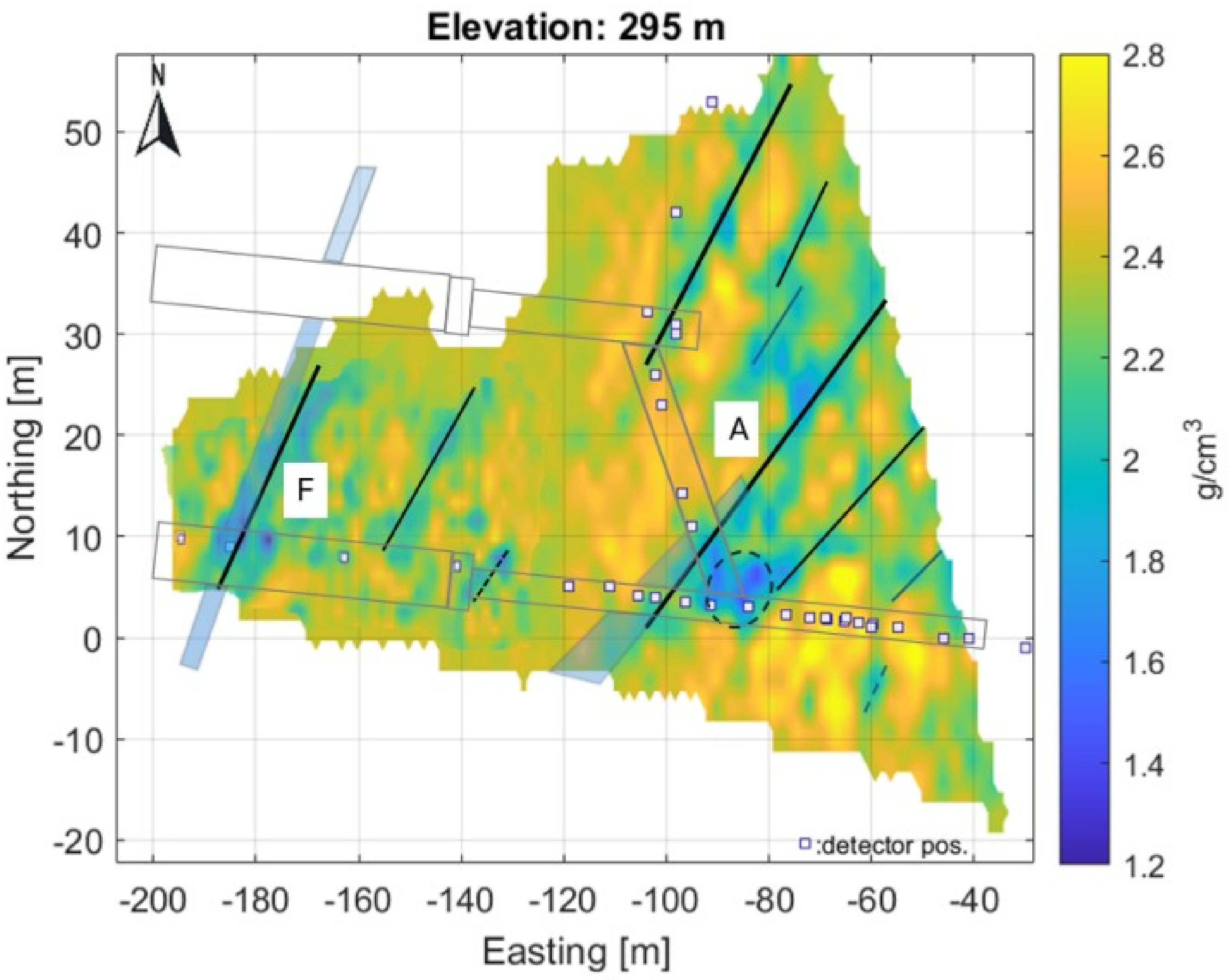
An album of characteristic bias and artefacts

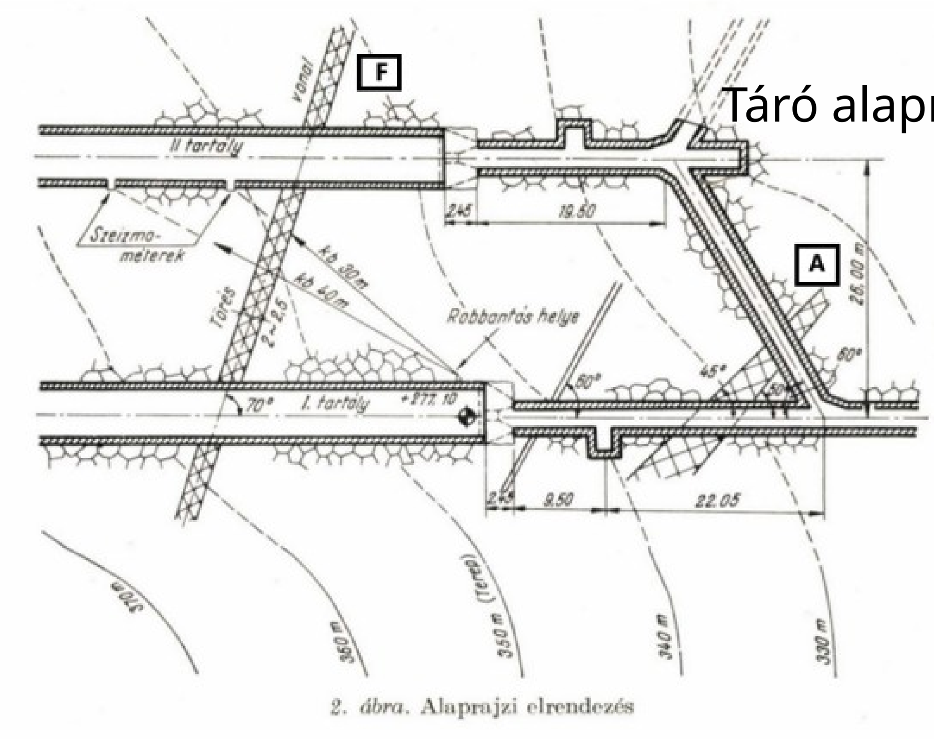


Muon tomography: Focus range - Information distribut



Muon tomography: Extra





Táró alaprajza

Justification of fault indication