

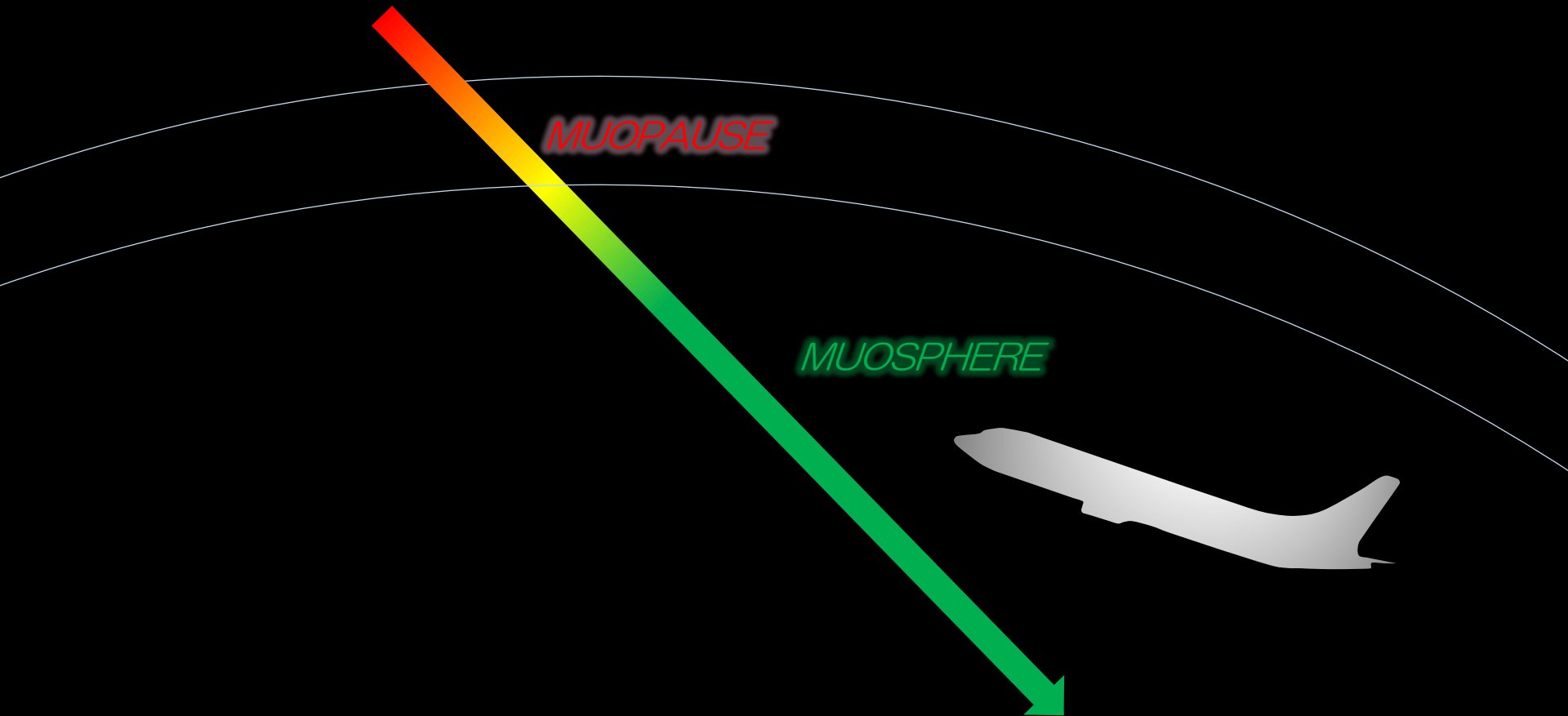
Muometric Positioning, Navigation and Timing

Hiroyuki K.M. Tanaka

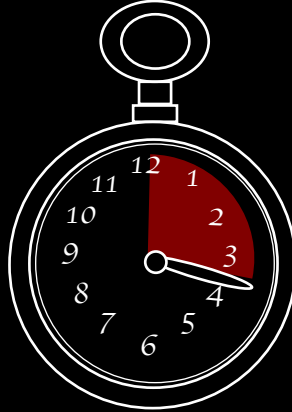
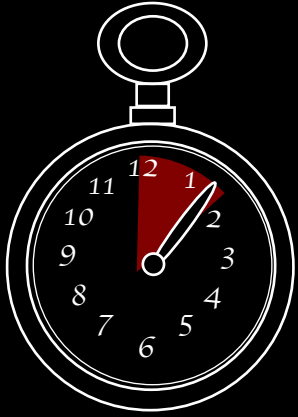


1. COSMIC MUONS

Muons are secondary cosmic rays produced when primary cosmic rays collide with the Earth's atmospheric nuclei. Cosmic-ray muon flux is generated in a region ranging from the upper troposphere to the lower stratosphere. This region is called the **muopause**, and cosmic-ray muon flux is predominant in the region below the muopause, which is called the **muosphere**.



2.Ultimate Inertia



Acceleration of matter in general is defined by the **variation in velocity** (Δv) divided by **time** (t). Therefore, as the cosmic muon's time dilates, the acceleration ($a = dv/dt$) applied to the cosmic muon appears to decrease; this "decrease rate" is inversely proportional to the γ factor when it is observed in the lab frame

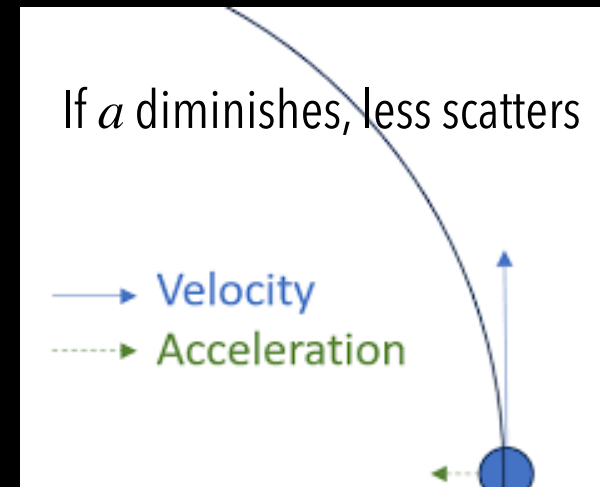
$$\mathbf{F} = \frac{d\mathbf{p}}{d\tau} = \gamma \frac{d\mathbf{p}}{dt}$$

shows that as matter approaches the speed of light, more force is needed to accelerate this matter.

$$= m\gamma \frac{dv}{dt} = \gamma \mathbf{f}$$

\mathbf{f} : lab frame three-force

\mathbf{F} : spatial components of the four-force



The signal scattering angle provides a good indicator to evaluate how straightforward its pathway has been.

If the scattering angle is very small, the scattering angle is directly proportional to the average perpendicular force.



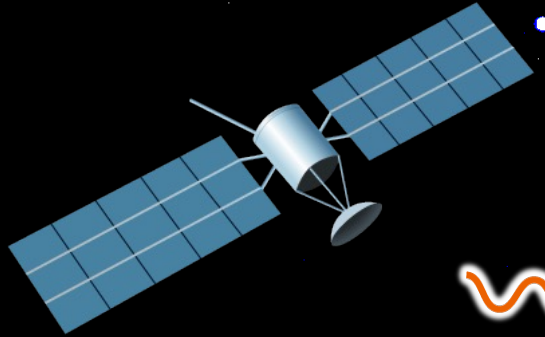
It is expected that the scattering angles observed in the lab frame is inversely proportional to the Lorentz factor.

$$\theta \propto \frac{1}{\gamma}$$

In fact, the root mean square scattering angle of muons traveling at nearly the speed of light (in the lab system) after passing through the material with a given thickness (LX_0^{-1} , normalized to the radiation length) is **inversely proportional to the time dilation factor:**

$$\theta_{\text{RMS}} = 13.6 \text{ [MeV]} \frac{m (LX_0^{-1})^{1/2} [1 + 0.0038 \ln (LX_0^{-1})]}{\gamma}$$

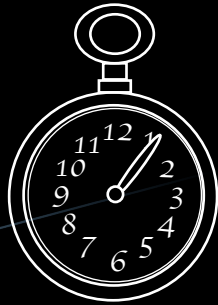
3. 21st Century Sextant



A modern type of sextant has been devised that measures the **direction of arrival of cosmic signals with greater penetrating power than light**, and it can be used for local positioning indoors or underground where the light of the sun or stars cannot reach.

UTC + Global Coordinate

Realtime Connection



Traceable Navigation
within Global Coordinates

By identifying
muons
using
a **precise clock**



mDetect's muon detector

UTC + Global Coordinate

With this new kind of the sextant, the time and position of an underground robot can be **wirelessly linked to the satellite system in real time via muons and the grand master clock** located aboveground so that the time and the underground position is outputted in terms of UTC and the world coordinate system in the NMEA format in real time. This makes indoor/underground motion globally traceable. For this...

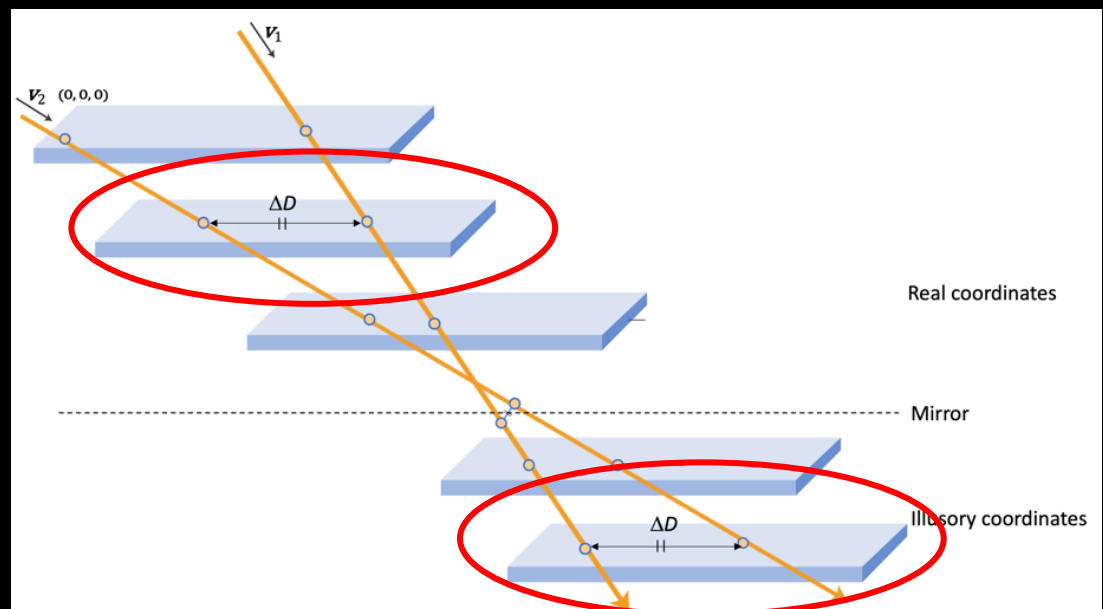
We use two muon signal vectors

4. Mathematical mirror ambiguity in Coordination

However, when coordinating, mathematical mirror ambiguity that occurs because the geometric relationships are identical on both sides of the shortest line segment connecting the two skew signal vector lines ("Mirror").

However, by applying real-world constraints, we can automatically filter out the mathematically valid but physically impossible coordinates. For example, even though the receiver is supposed to be located on the ground, one solution places the receiver underground or floating high in the sky, we can safely reject this solution.

In order to know this "illusory" solution, we must know the position of the "Mirror".



5. 4D Formulation to find the "Mirror"

$$\hat{G}\mathbf{u} = \mathbf{b} \text{ in } \mathcal{W} = \{(x, y, z, t)\} \in \mathbb{R}^4(+, +, +, +)$$

\hat{G}

World operator, and its eigenvalue is $\lambda = 1 \pm \cos\theta$, indicating the angle between dual planes \mathcal{D}_1 and \mathcal{D}_2 of muon signal vectors \mathbf{v}_1 and \mathbf{v}_2

$$\mathbf{u} = \begin{bmatrix} ct_1 \\ ct_2 \end{bmatrix}$$

Bi-parametric time vector which represents the temporal evolution of cosmic muon signals in a single reference frame.

The magnitude of \mathbf{u} represents the potential for coordinating power or the coordination effort

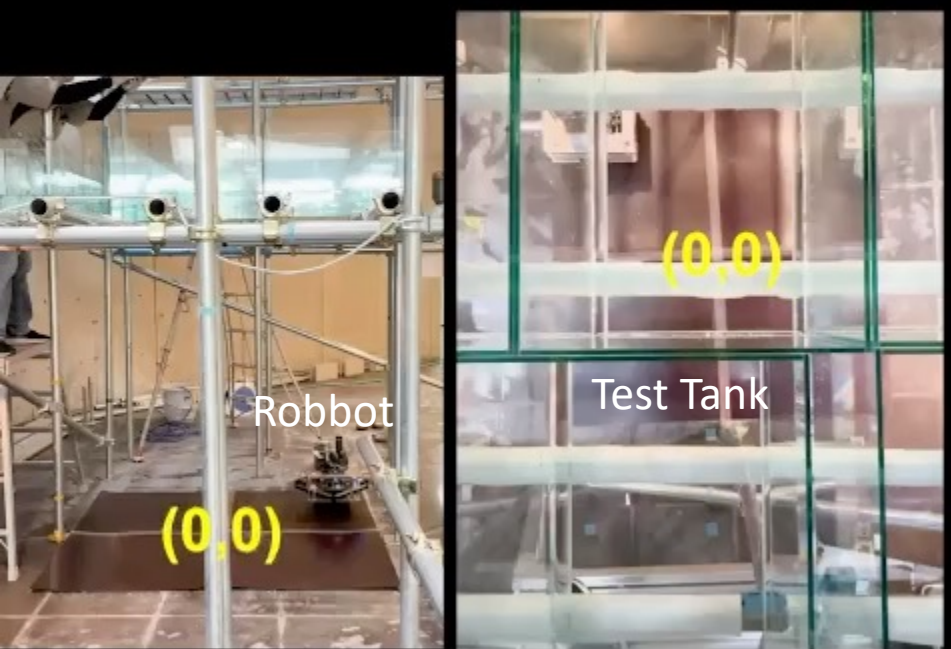
\mathbf{b}

Vector related with the reference location

If we solve this, we'll get the space-time parameters required for the muon signals to travel from the reference to this mirror position.

6. Demonstration of Navigation @ West Tokyo Underground Muography Laboratory

\$GNRMC,022339.66,A,3545.3723098,N,13917.2063955,E,000.0,191.7,071025,,,A*46
UTC Clock: 2025-10-07 Tue 02:23:39.665 35.7562052 139.2867733



↑ Side View

↑ Top View



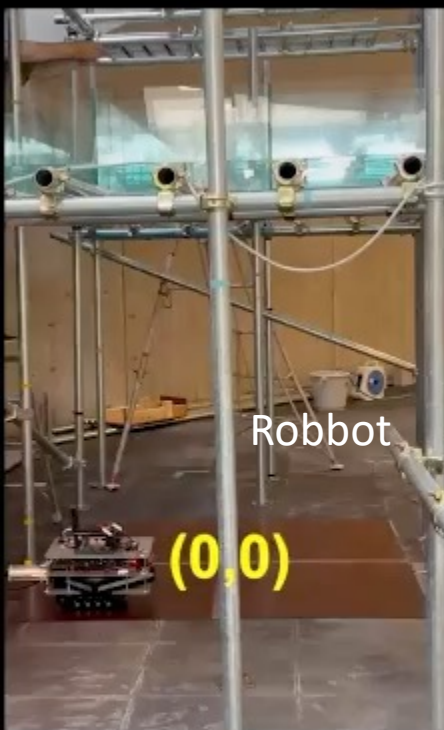
↑ When The Muon Hit
NMEA RMC Message
UTC Time And Decimal Degrees

The robot was operated in our underground lab, and water tanks were placed above the robot for the current demonstration.

muPS Navigation Experiment
GPS Data Transferred via CAT
Running in a Square
Clockwise Rotation ①

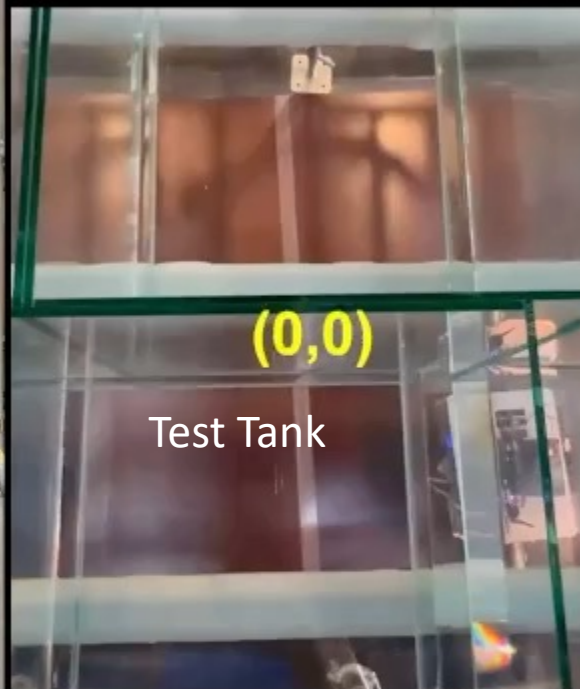
The positioning results were outputted within the UTC and world coordinate in real time in the NMEA format. The actual discrepancy between UTC and the underground robot time after CTC synchronization was measured to be typically a few hundred ns (1 microsecond at the maximum.)

Positioning results can also be outputted on a map in real time so that the trajectories and the moving directions of the robot can be visually monitored on a map.



Robot

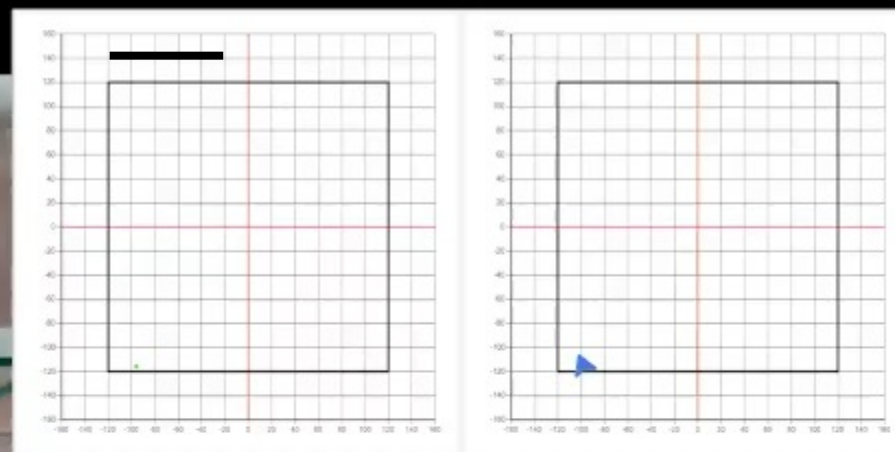
$(0,0)$



$(0,0)$

Test Tank

1 m



↑ Navigation Screen
(Position Data Calculated by RPi)

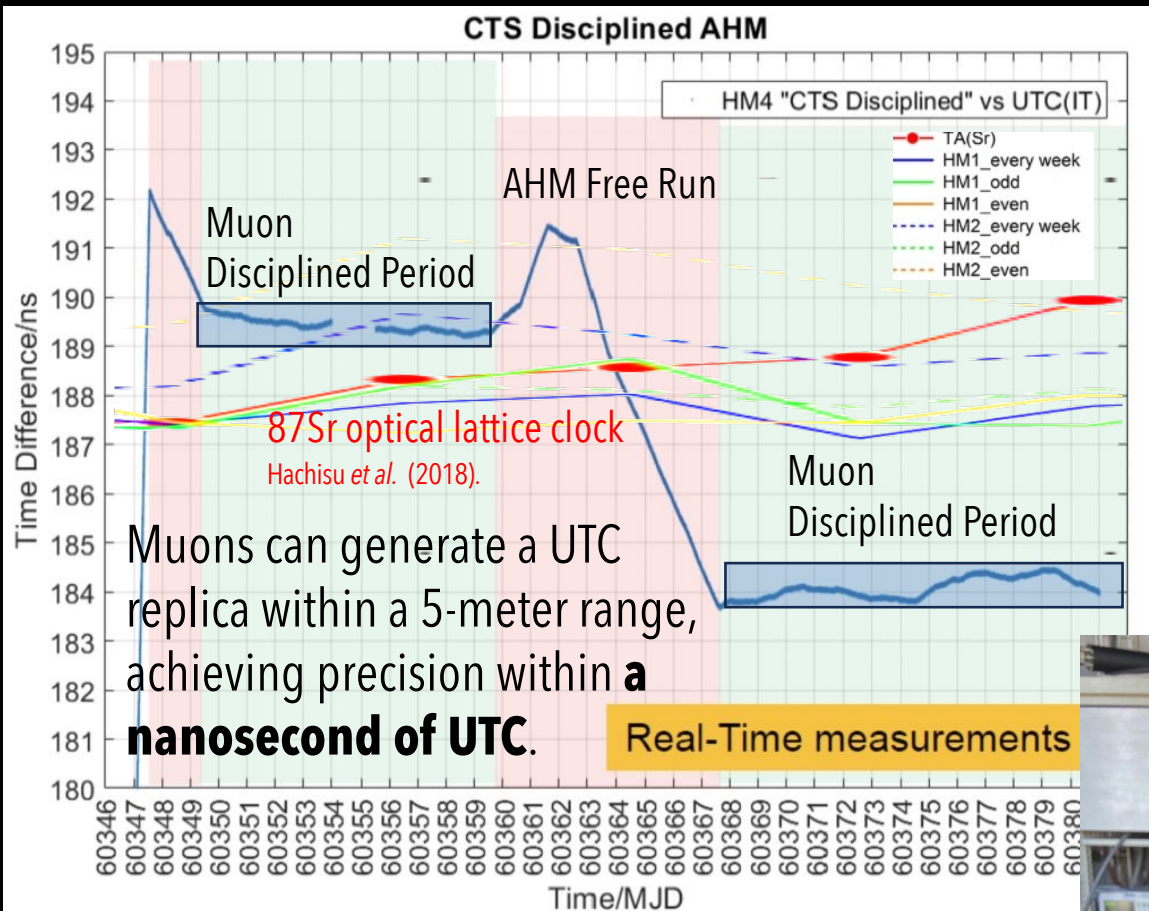
muPS Navigation Experiment
Fill The Tank With Water
(Water Depth 10cm)
Pattern④ 2nd Time

↑ Side View

↑ Top View

(Hit Rate: 4.775 per 10 sec.)

7. Another Demonstration of Precise Timing @ Italian National Institute of Metrology



Muons can generate a UTC replica within a 5-meter range, achieving precision within a **nanosecond of UTC**.

At the RadioNavigation Laboratory of Italian National Institute of Metrology, **tests** were conducted to discipline the AHM to UTC and to remotely disseminate a UTC replica leveraging the short-term characteristics of the AHM alongside the medium-to-long-term stability of UTC.

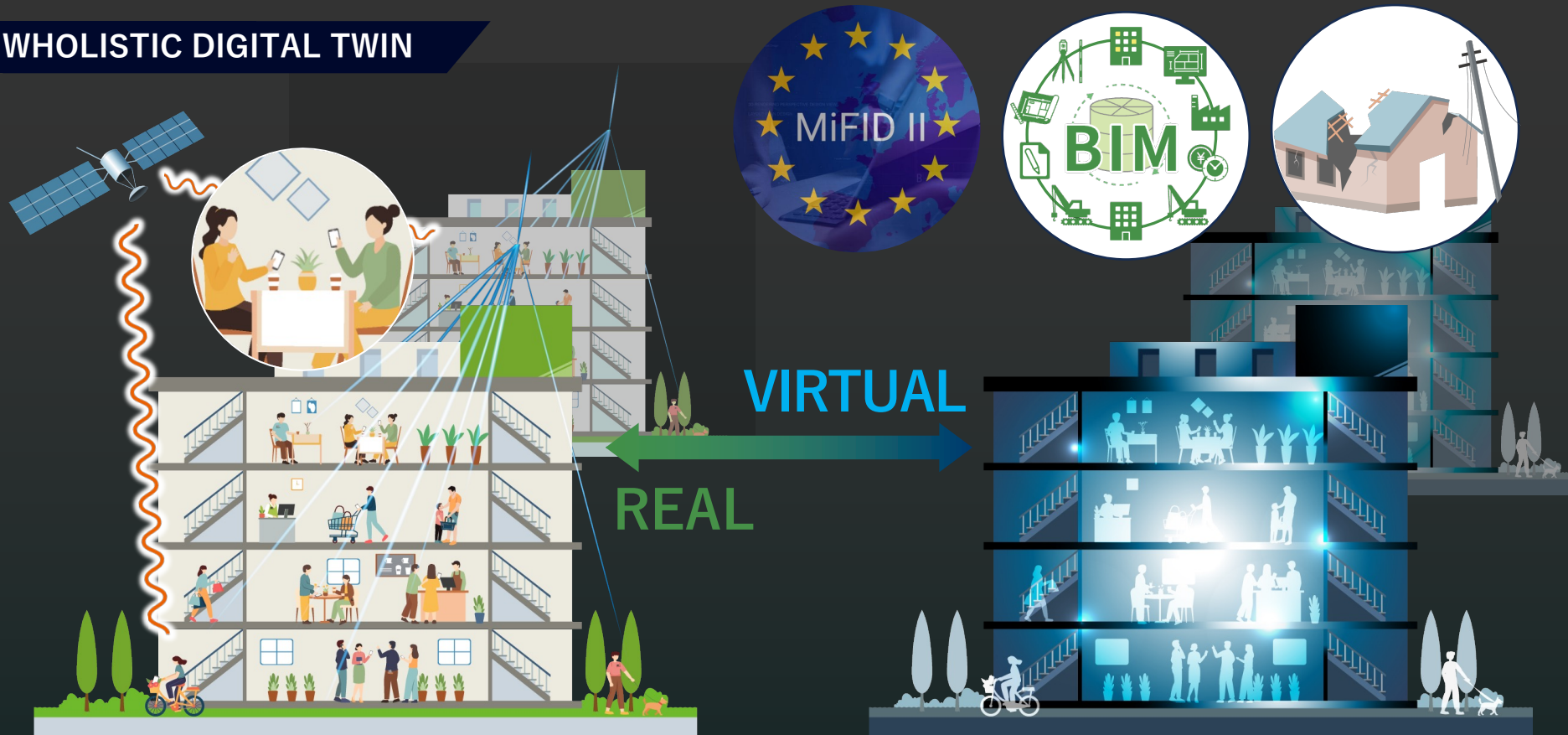
G. Cerretto, M. Sellone, E. Cantoni, C. E. Calosso, I. Gnesi, and H.K.M. Tanaka. "Muochrony: timing with muons. First experimental results at INRIM on the synchronization of atomic clocks and dissemination of reference timescales". ESA 9th International Colloquium on Scientific and Fundamental Aspects of GNSS, Wrocław, Poland, 25-27 September, 2024 BEST PRESENTATION AWARD

INRIM VMI Team



8. One of the Applications of muPS

WHOLISTIC DIGITAL TWIN



Satellite-muPS Digital Twin

A UTC underground/indoor ▶ MiFID2

B 3D modeling within the World Coordinate System

C Visualization of the human flux and material flux within the World Coordinate System

Universally Traceable Digital Twin can be constructed.

One of the aims of science is to **expand the capabilities and toolsets for human beings**, with technology and know-how, to observe objects, phenomenon, and cycles of nature in new and innovative ways.