µTRec: A Muon Trajectory Reconstruction Algorithm for Enhanced Scattering Tomography

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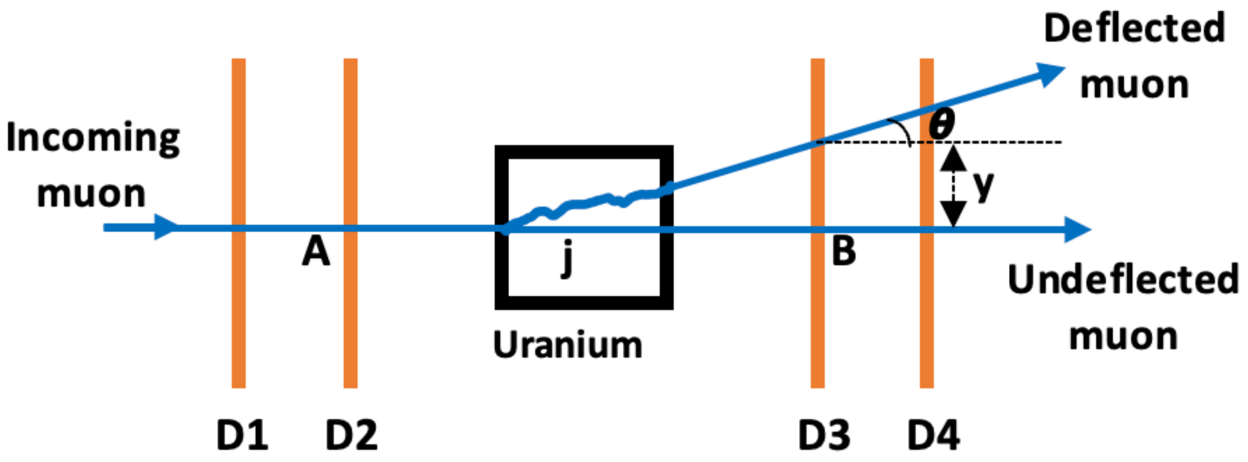
Overview

- Introduction
- Background
- μTRec algorithm
- Results
- Conclusion

How Muon Scattering Tomography Works?

GEANT4 → **Python (GPU/JAX accelerated)**



D1, D2, D3, D4 are four position sensitive detectors

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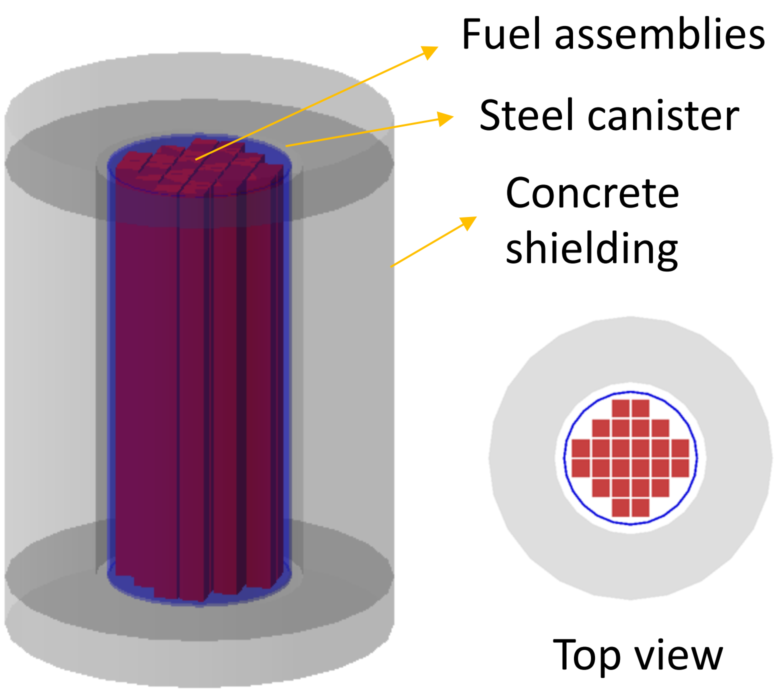
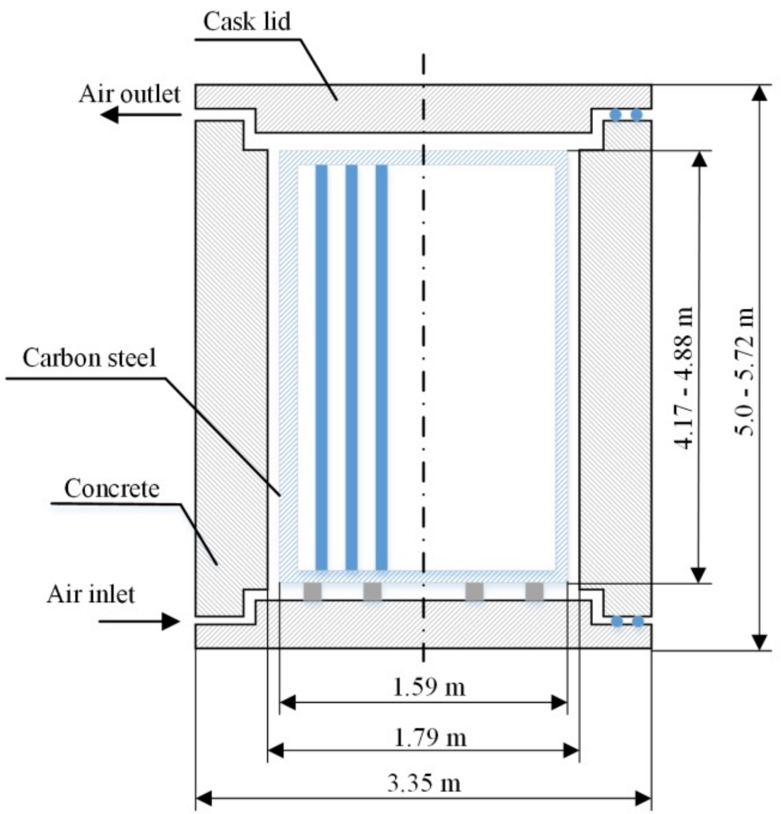
Dry Storage cask

- Steel canister shields gamma rays
- Concrete shields neutrons
- **Once sealed, it can't be opened**
- Need for non-invasive interior monitoring



3D view of **VSC-24 cask**

Geometry

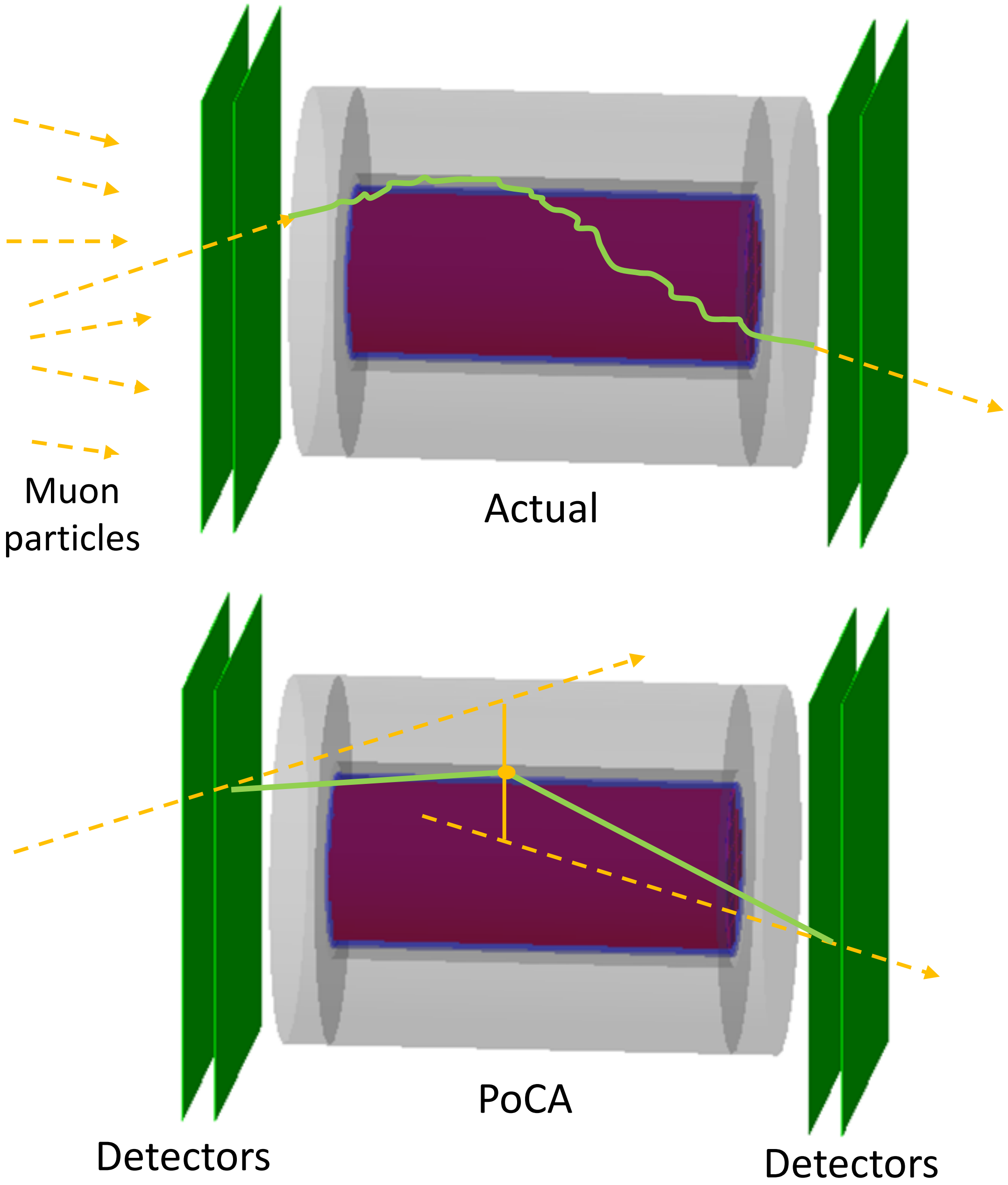


Chtazidakis, 2016

VSC-24 sketch

Background

- **Widely used algorithms:**
  1. Point of Closest Approach (PoCA)
  2. Maximum Likelihood Estimation Maximization (MLEM)
- **Shared limitation:** assume straight-line tracks for multiple Coulomb scattering (MCS)
- **Result:** Slower and less reliable reconstructions
- **Operational impact:**
  - Detecting a missing fuel assembly in a dry storage cask (DSC) needs hours of cosmic-muon data
  - Monitoring hundreds of DSCs takes weeks–months
- **Need:** a new physics-informed algorithm that models full MCS (curved trajectories) for faster, facility-scale monitoring



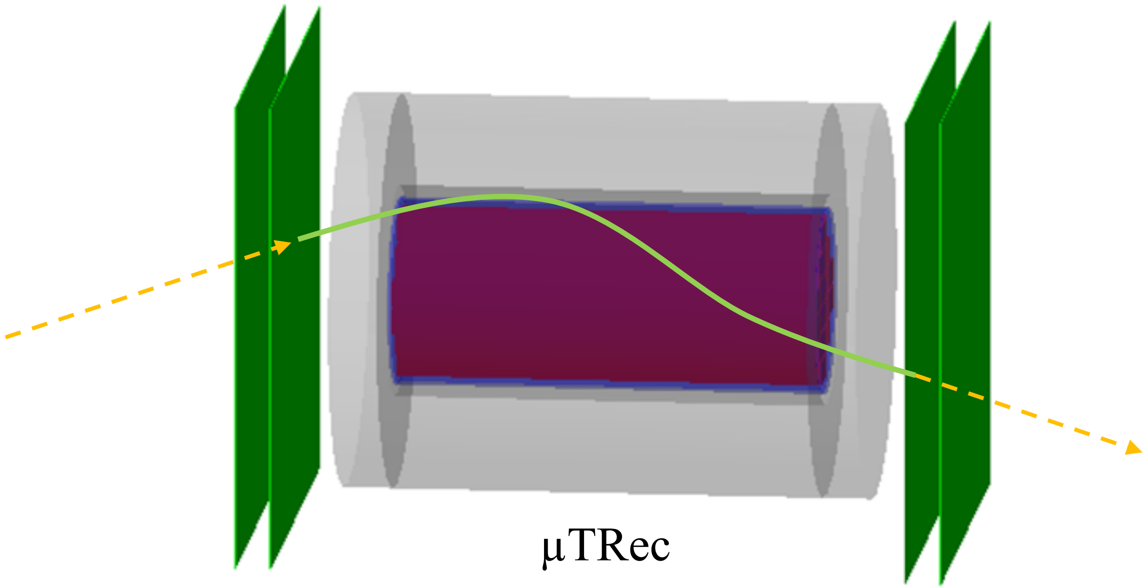
μTRec

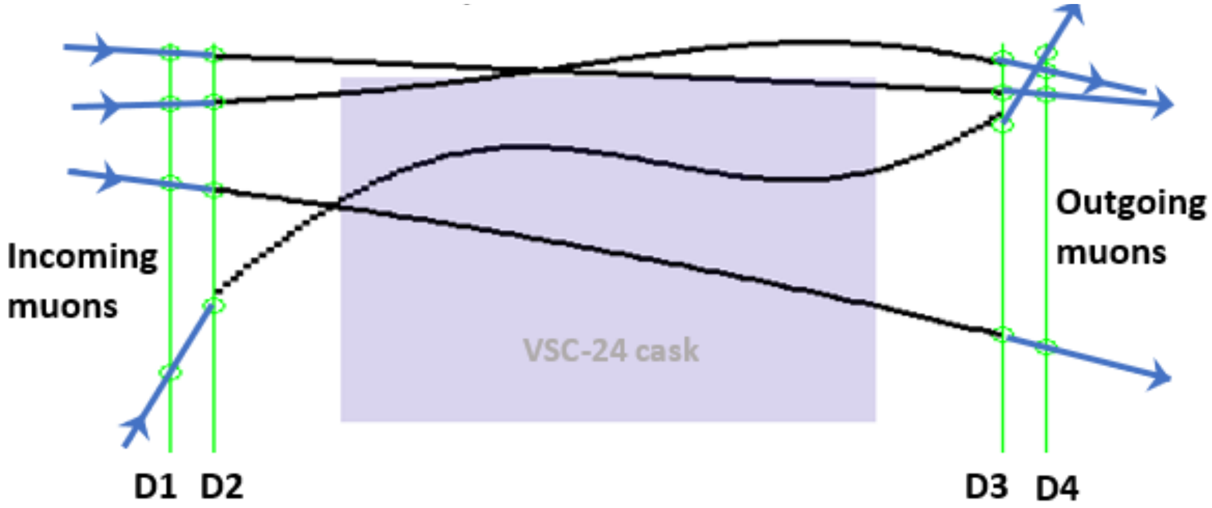
- Physics-based algorithm coupled with **Bayesian theory,**

$$P(A|B) = \frac{P(B|A).P(A)}{P(B)}$$

- A **Gaussian approximation** of multiple Coulomb scattering (MCS)

- The estimated muon path asymptotically approaches the incoming and outgoing muon trajectories





Trajectories of four randomly selected muons passing through the dry storage cask

Method - $\mu TRec$

Derivation for trajectory equation

- The muon state is represented using a two-dimensional state vector that encapsulates both the transverse displacement and angular deflection at z.

$$\mathbf{Y} = \begin{pmatrix} y \\ \theta \end{pmatrix}$$

- **Step 1: Bayesian theory**

Probability of a muon displaying a specific displacement and angle at y1 , given the exit point 2.

$$P(y_1 \mid y_2) = \frac{P(y_2 \mid y_1) \cdot P(y_1 \mid y_0)}{P(y_2)}$$

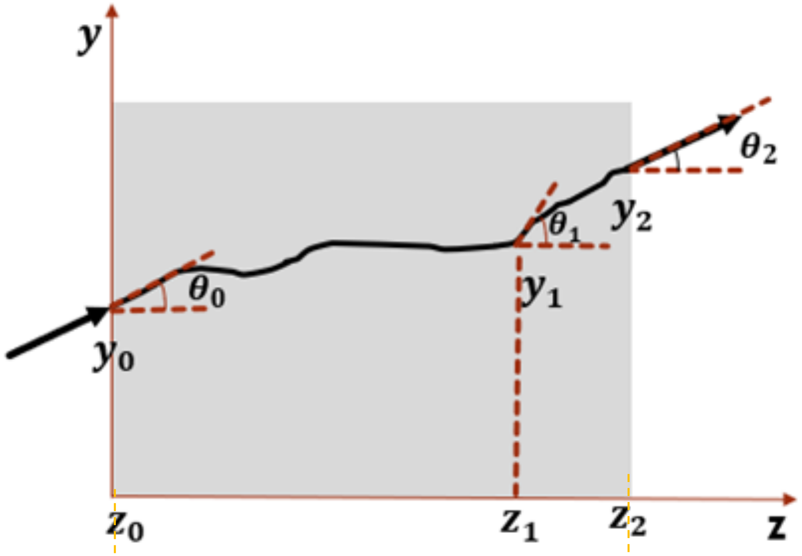
- **Step 2: Gaussian approximation for MCS:** $\mathcal{P}(Y) \sim \mathcal{N}(0, \Sigma)$

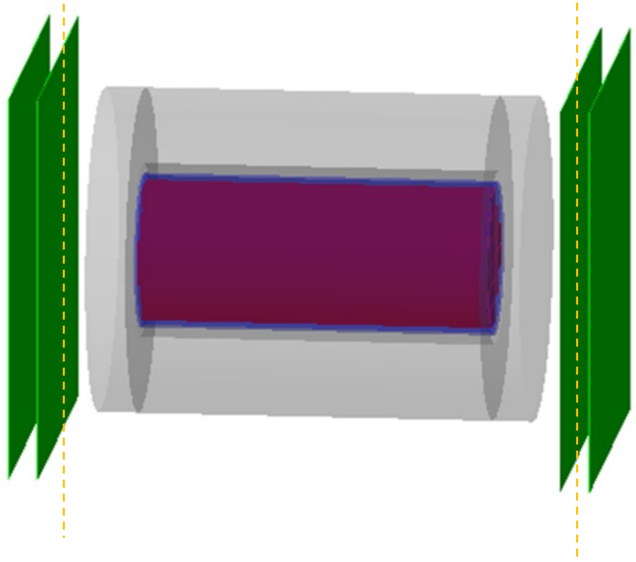
The Gaussian approximation of Y follows a zero-mean bivariate normal distribution.

$$P(y_1 \mid y_0) = \frac{1}{C_1} exp\left(-\frac{1}{2}(y_1 - R_0 y_0)^T \Sigma_1^{-1} (y_1 - R_0 y_0)\right)$$

$$P(y_2 \mid y_1) = \frac{1}{C_2} exp\left(-\frac{1}{2}(y_2 - R_2 y_1)^T \Sigma_2^{-1} (y_2 - R_2 y_1)\right)$$

Illustration of muon scattering occurring within the y-z plane.





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Method - $\mu TRec$ *(continuation)*

Derivation for trajectory equation

The general form

$$P(Y) = \frac{1}{2\pi\sqrt{|\Sigma|}} \exp\left(-\frac{1}{2} Y^T \Sigma^{-1} Y\right)$$

where Σ is the covariance matrix

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_y^2 & \sigma_{y\theta_y} \\ \sigma_{y\theta_y} & \sigma_{\theta_y}^2 \end{bmatrix},$$

where

σ_y^2 = scattering displacement variance

σ_θ^2 = scattering angle variance

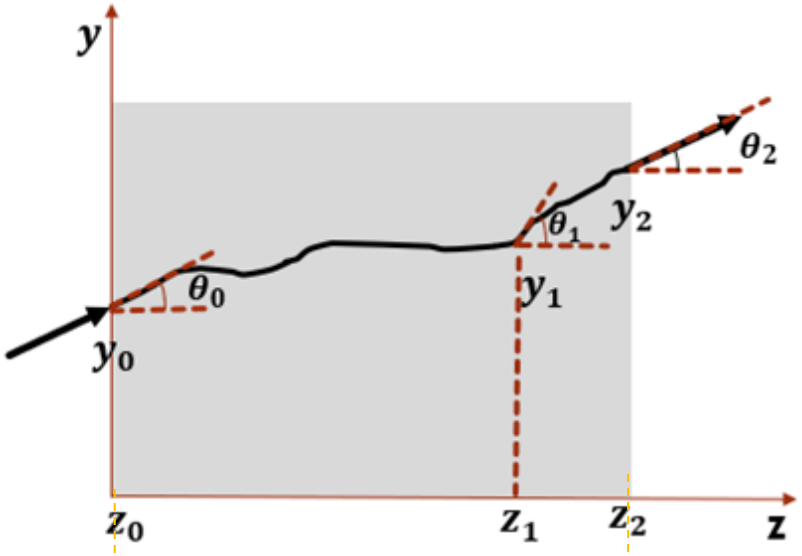
$\sigma_{y\theta}^2$ = covariance between the displacement and the scattering angle

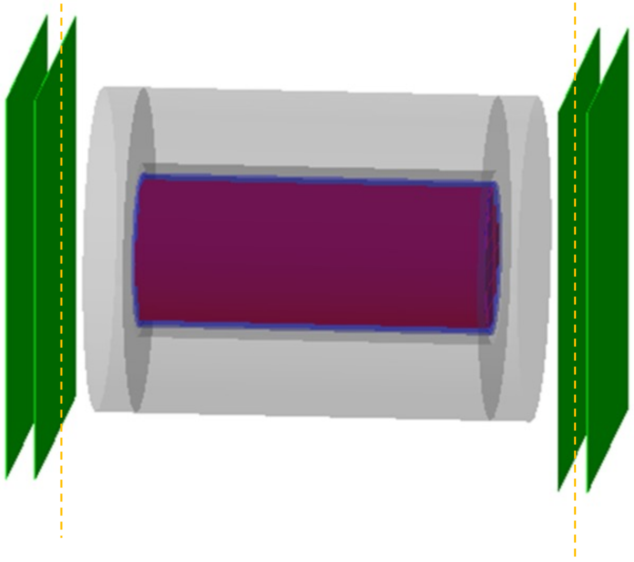
$$\sigma_{y_1}^2(z_0, z_1) = E_0^2 \left(1 + 0.038 \ln \frac{z_1 - z_0}{X_0}\right)^2 \times \int_{z_0}^{z_1} \frac{(z_1 - z)^2}{\beta^2(z) p^2(z)} \frac{dz}{X_0}$$

$$\sigma_{\theta_1}^2(z_0, z_1) = E_0^2 \left(1 + 0.038 \ln \frac{z_1 - z_0}{X_0}\right)^2 \times \int_{z_0}^{z_1} \frac{1}{\beta^2(z) p^2(z)} \frac{dz}{X_0}$$

$$\sigma_{y_1\theta_1}^2(z_0, z_1) = E_0^2 \left(1 + 0.038 \ln \frac{z_1 - z_0}{X_0}\right)^2 \times \int_{z_0}^{z_1} \frac{z_1 - z}{\beta^2(z) p^2(z)} \frac{dz}{X_0}$$

Illustration of muon scattering occurring within the y-z plane.





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Method - $\mu TRec$ *(continuation)*

Muon energy loss

- The Bethe-Bloch formula:

$$\frac{dE}{dx} = 4\pi N_A r_e^2 m_e c^2 Z_1 \frac{Z_2}{A_2}\frac{1}{\beta^2}\left[\ln\left(\frac{2m_e c^2\beta^2\gamma^2 T_{\max}}{I^2}\right) - \beta^2 - \frac{\delta}{2}\right] \quad (\mathrm{MeV/cm})$$

- Assumptions
  - Energy range relevant to cosmic ray muons **1 GeV < E < 100 GeV** and mean energy **3–4 GeV**
  - Most materials**, radiative effects** account for **less than 1%** and **nuclear loss rate is negligible**

- For practical purposes $\frac{dE}{dx} = -a$

For the given VSC-24 geometry

***a* = 5 MeV/cm**

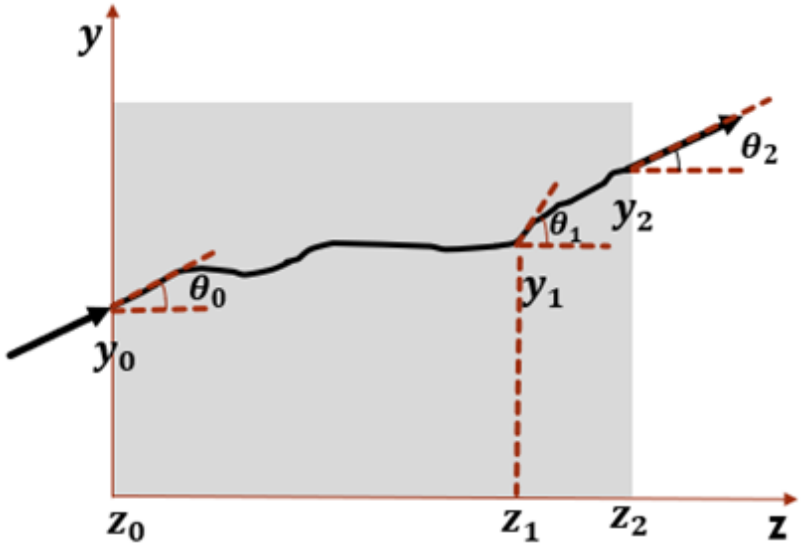

Illustration of muon scattering occurring within the y-z plane.

Method - $\mu TRec$ *(continuation)*

Derivation for trajectory equation

Finally , we get the trajectory equation :

$$Y = (\Sigma_1^{-1} + R_2^T \Sigma_2^{-1} R_2)^{-1} (\Sigma_1^{-1} R_0 y_0 + R_2^T \Sigma_2^{-1} y_2)$$

Similarly,

$$X = (\Sigma_1^{-1} + R_2^T \Sigma_2^{-1} R_2)^{-1} (\Sigma_1^{-1} R_0 x_0 + R_2^T \Sigma_2^{-1} x_2)$$

Variables:

$$R_0 = \begin{bmatrix} 1 & z_1 - z_0 \\ 0 & 1 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 1 & z_2 - z_1 \\ 0 & 1 \end{bmatrix},$$

$$\Sigma_1 = \begin{bmatrix} \sigma_{y_1}^2 & \sigma_{y_1 \theta_{y_1}} \\ \sigma_{y_1 \theta_{y_1}} & \sigma_{\theta_1}^2 \end{bmatrix}, \quad \Sigma_2 = \begin{bmatrix} \sigma_{y_2}^2 & \sigma_{y_2 \theta_{y_2}} \\ \sigma_{y_2 \theta_{y_2}} & \sigma_{\theta_2}^2 \end{bmatrix}.$$

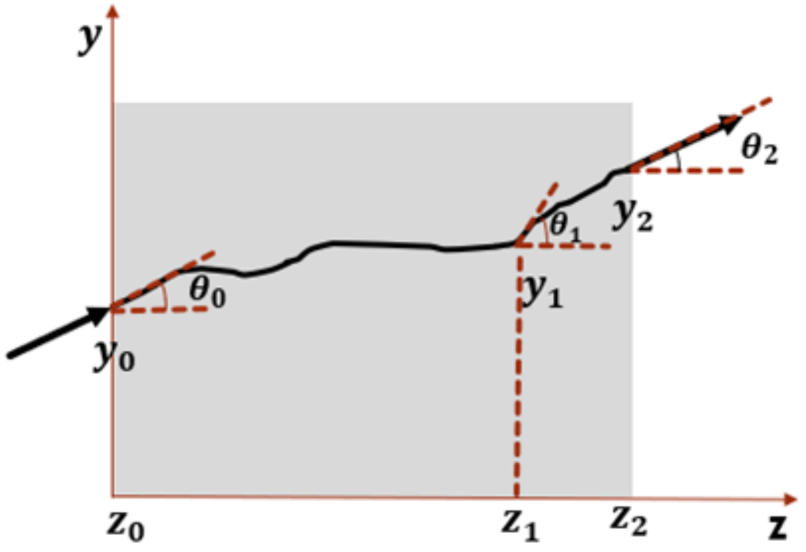

Illustration of muon scattering occurring within the y-z plane.

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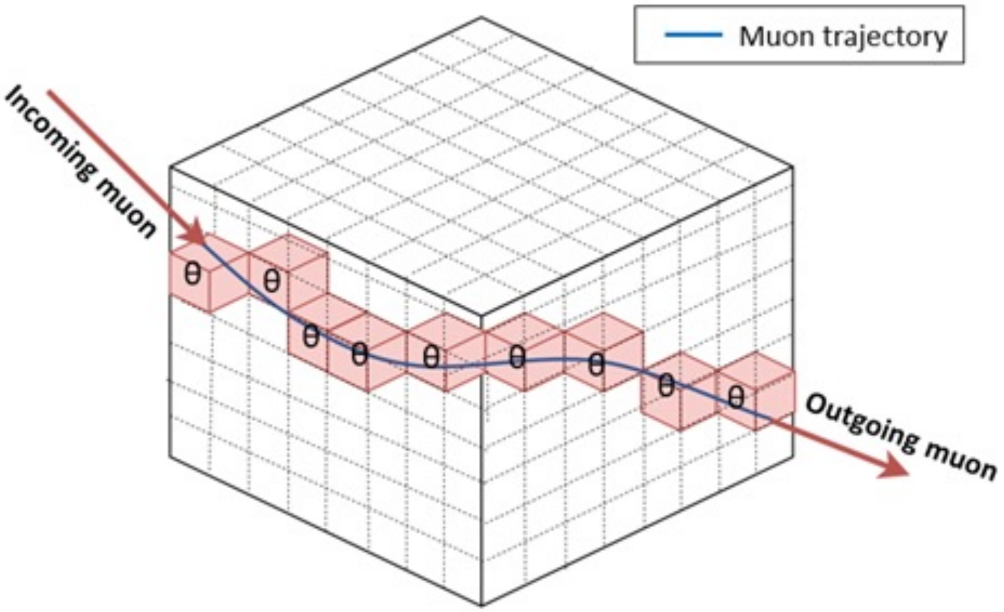
Scattering angle mapping



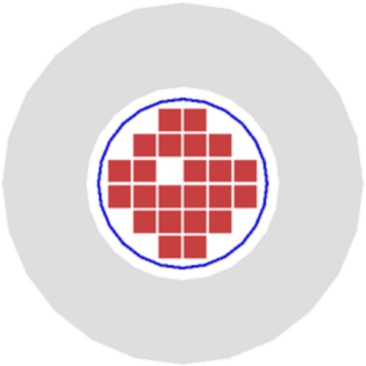
1. Collect incident and exiting muon points.
   (p1; p2) from entry plane,
   (p3; p4) from exit plane.
2. Compute scattering angles.
3. Estimate the muon trajectory using a Gaussian MCS model and Bayesian formalism.
4. **Discretize the reconstruction volume into n × n × n voxels.**
5. Reconstruct image using the computed voxel statistics.

$$\theta = \sqrt{\frac{\theta_x^2 + \theta_y^2}{2}},$$

$$\text{Voxel value} = \frac{1}{N}\sum_{i=1}^{N} \theta_i$$

Pseudocode for the µTRec algorithm

Results



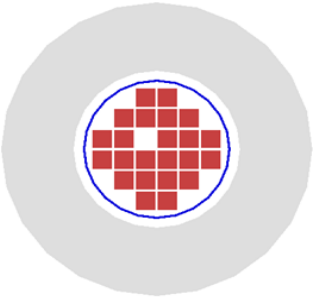
$$\mathrm{SNR} = \frac{\mathrm{mean}(8\ \text{assemblies surrounding missing one})}{\mathrm{std}(8\ \text{assemblies surrounding missing one})}$$

$$\mathrm{CNR} = \frac{\mathrm{mean}(8\ \text{assemblies}) - \mathrm{mean}(\text{missing one})}{\max(\mathrm{std}(8\ \text{assemblies}), \mathrm{std}(\text{missing one}))}$$

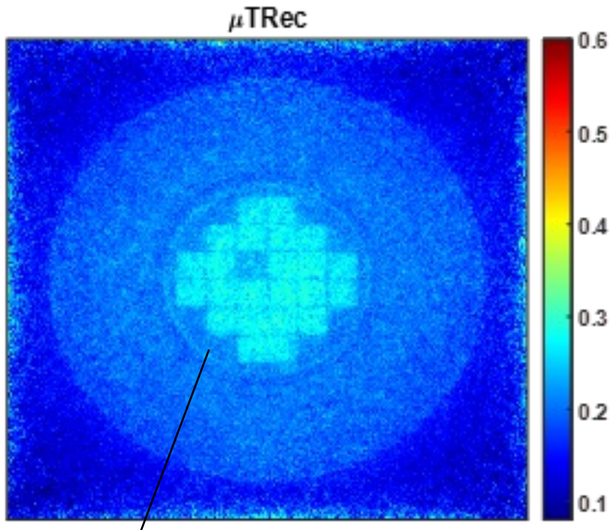
$$\mathrm{DP} = \mathrm{SNR} \times \mathrm{CNR}$$

SNR: signal-to-noise ratio
CNR: contrast-to-noise ratio
DP: detection power

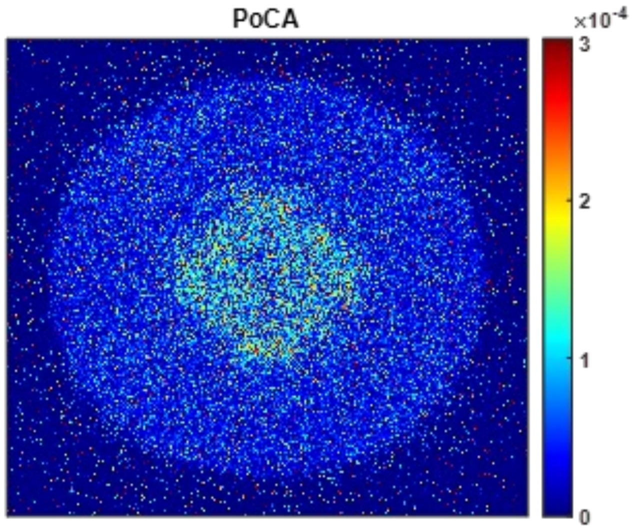
Results



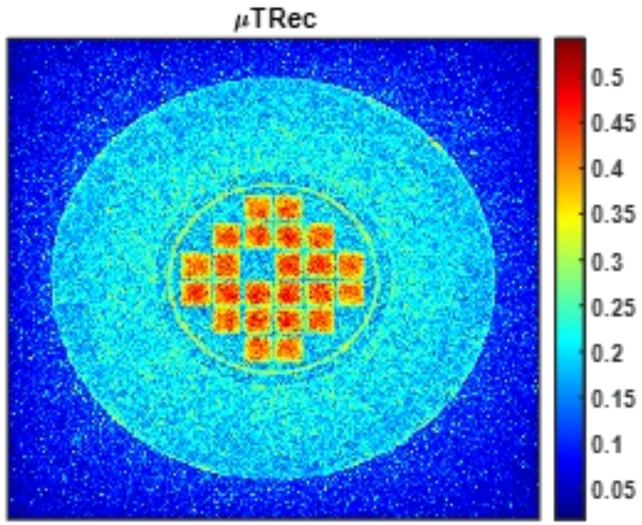
Expected



Outline of a steel canister



a) Cosmic ray muons



b) Laser-generated muons

Simulated reconstruction of DSC for finding a missing fuel assembly using **1 million** a) cosmic ray muons comparing µTRec & PoCA algorithms, and b) laser-generated muons, at **1 cm resolution**. Vertical scale depicting average scattering angle along the length of DSC.

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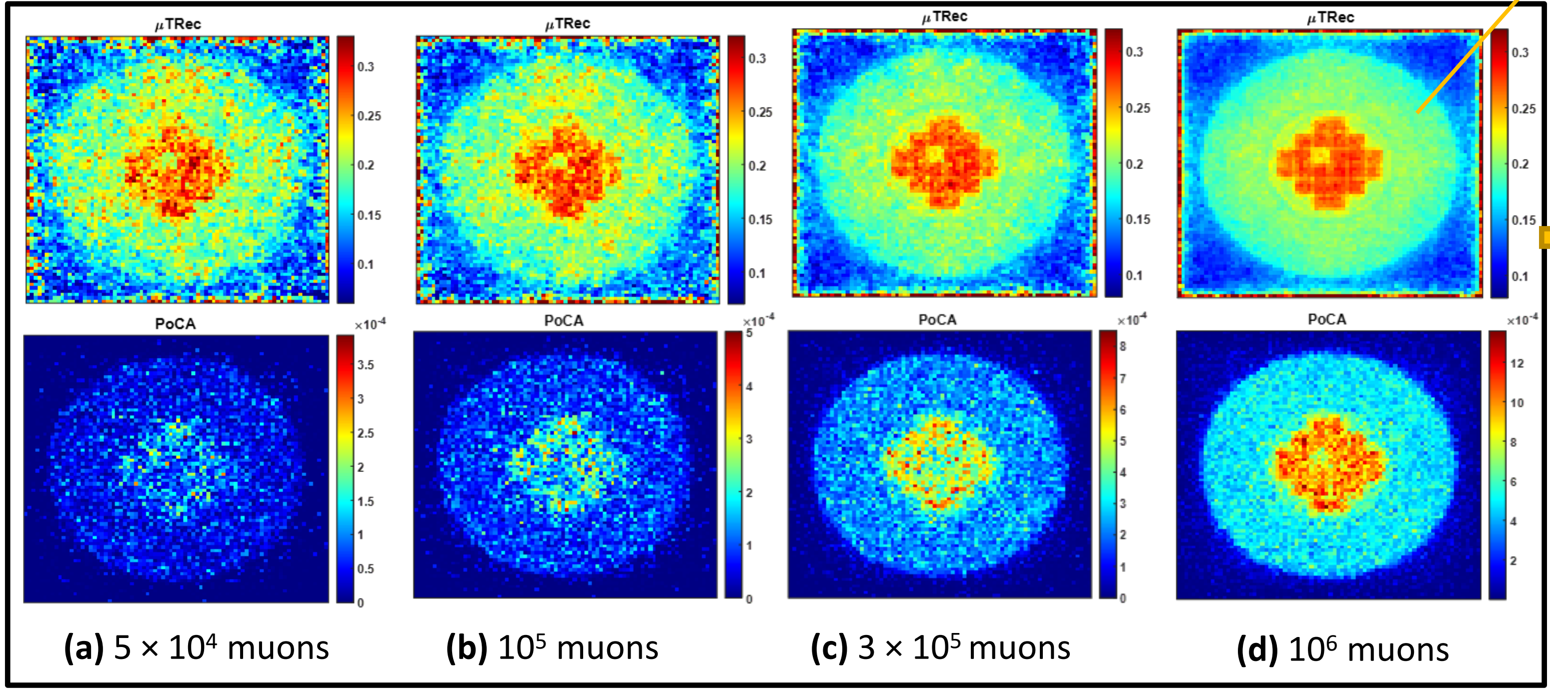
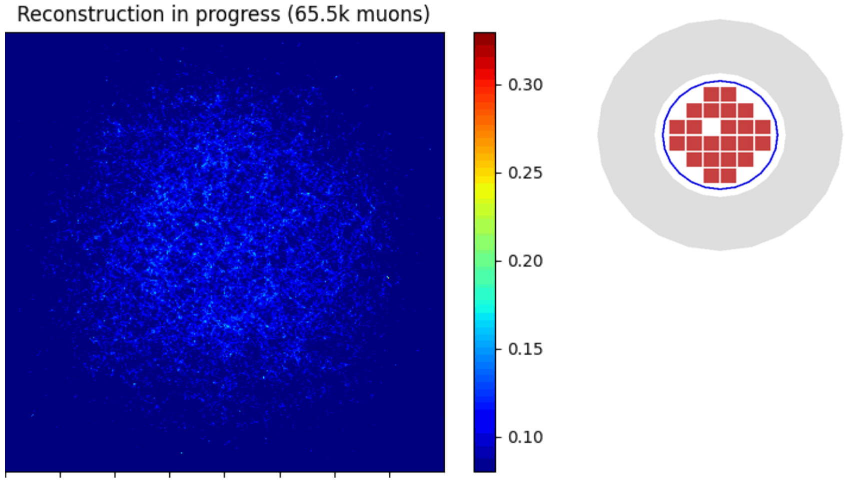
Results

Cosmic muon source



Voxel size = 5 cm



Muon flux	μTRec			PoCA		
	SNR	CNR	DP	SNR	CNR	DP
5×10^4	**9.07**	**1.25**	**11.37**	1.73	−0.06	−0.10
10^5	12.40	1.55	19.23	2.49	0.17	0.42
3×10^5	16.73	1.73	28.90	4.21	0.48	2.03
10^6	19.30	1.85	35.77	**8.69**	**1.37**	**11.90**

Quantitative comparison of μTRec and PoCA at varying muon flux levels using SNR, CNR and DP metrics

Only **1/20th** of muon flux required for similar quantitative comparison!

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Conclusion

- **µTRec Algorithm**
  - Physics-informed muon trajectory reconstruction algorithm
  - Models multiple Coulomb scattering using a Gaussian approximation
  - Uses Bayesian inference to estimate the most probable muon path
  - Accounts for constant muon energy loss along the trajectory
- **Dry Storage Cask Imaging**
  - Achieves equivalent detection quality using only 1/20 of the muon flux required by PoCA
  - Fine features: The 2.5 cm thick steel canister successfully localized using a single view and a muon flux of 1 million
  - Reduced flux suitable for near real time imaging

My publications

Articles

- **Reshma Ughade** and Stylianos Chatzidakis, “Non-intrusive monitoring of sealed microreactor cores using physics-informed scattering tomography with momentum measurements,” arXiv, 2026
- **Reshma Ughade** and Stylianos Chatzidakis, “µTRec: A Muon Trajectory Reconstruction Algorithm for Enhanced Scattering Tomography,” Journal of Applied Physics, vol. 138, p. 064909, 2025. Selected as “Editor’s pick” for noteworthy contribution
- **Reshma Ughade**, J. Bae, and S. Chatzidakis, “Performance evaluation of cosmic ray muon trajectory estimation algorithms,” AIP Advances, vol. 13, no. 12, p. 125 301, Dec. 2023

Book chapter

- J. Bae, S. Chatzidakis, and **Reshma Ughade**, “Gamma-Ray and Cosmic Ray Muon Modalities for Cargo Inspection,” Emerging Radiation Detection: Technology and Applications, K. Iniewski and H. Gadey, Eds., Cham: Springer Nature Switzerland, 2024

Conference papers

- **Reshma Ughade** and S. Chatzidakis, “Efficient low-flux muon tomography using the µTRec algorithm,” IEEE Nuclear Science Symposium, Medical Imaging Conference and International Symposium on Room Temperature Semi-conductor Detectors (NSS MIC RTSD), Nov. 2025
- **Reshma Ughade** and Stylianos Chatzidakis ‘Integrating detector resolution into µTRec algorithm for spent fuel cask imaging’. American Nuclear Society (ANS) conference, Washington, D.C., November 2025.
- **Reshma Ughade**, J. H. Bae, P. Cantonwine, and S. Chatzidakis, “3D Analysis for Generalized Muon Trajectory Estimation Algorithm,” in Transactions of the American Nuclear Society, Vol. 131 (2024).
- **Reshma Ughade** and S. Chatzidakis, “On-the-fly 3D muon path estimation algorithm for muon imaging applications,” IEEE Nuclear Science Symposium, Medical Imaging Conference and International Symposium on Room Temperature Semi-conductor Detectors (NSS MIC RTSD), Nov. 2023
- **Reshma Ughade**, J. Bae, and S. Chatzidakis, “Assessment of performance for algorithms estimating cosmic ray muon trajectories,” Transactions of the American Nuclear Society, vol. 129, pp. 369–372, 2023
- **Reshma Ughade** and S. Chatzidakis, “A physics-based muon trajectory estimation algorithm for muon tomographic applications,” APS April Meeting Abstracts, 2023

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Thank you for your attention

QUESTIONS?

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