Uniwersytet Wrocławski

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# BAYESIAN INFERENCE OF HYBRID NEUTRON STAR EQUATIONS OF STATE WITH A COLOR-SUPERCONDUCTING QUARK CORE

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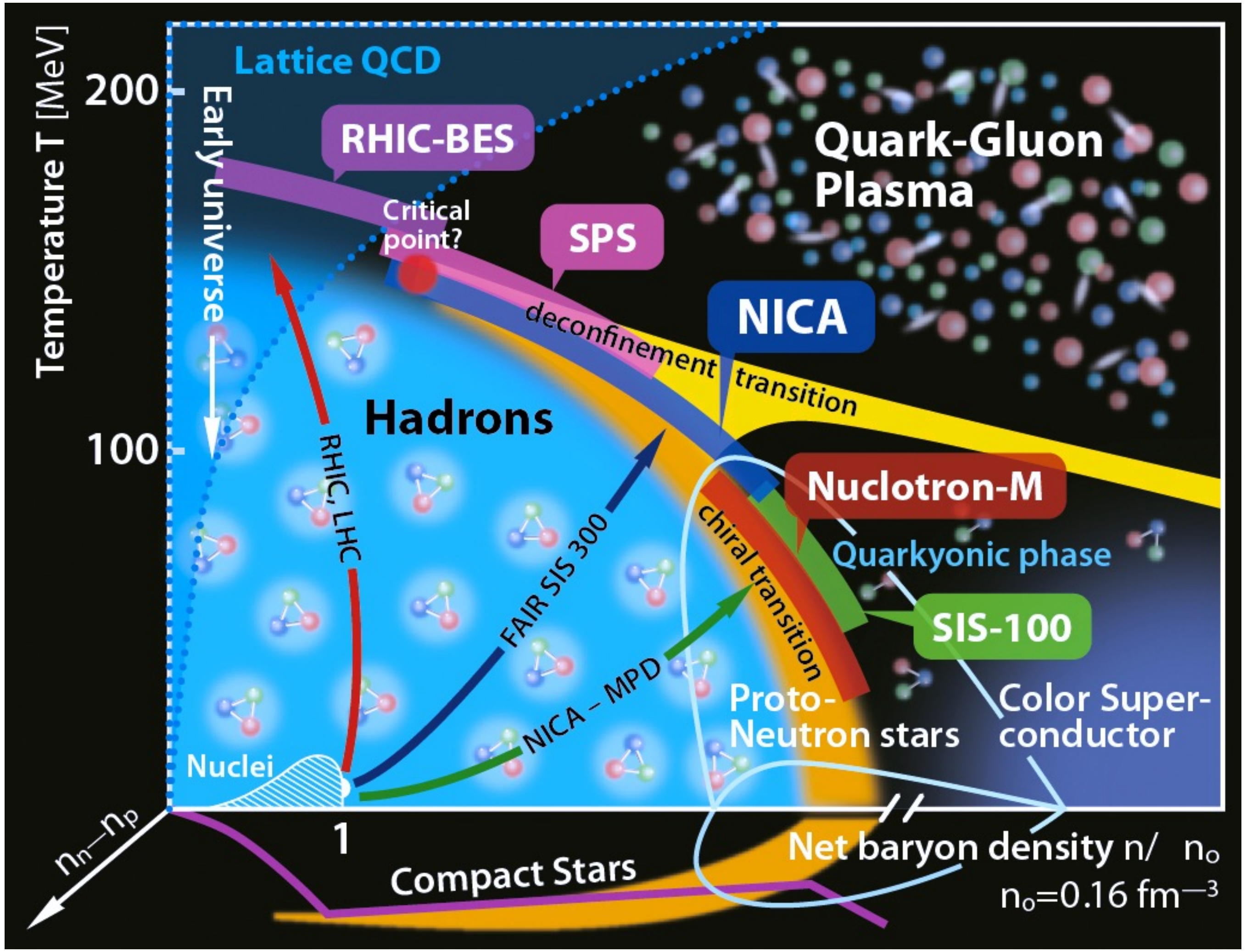
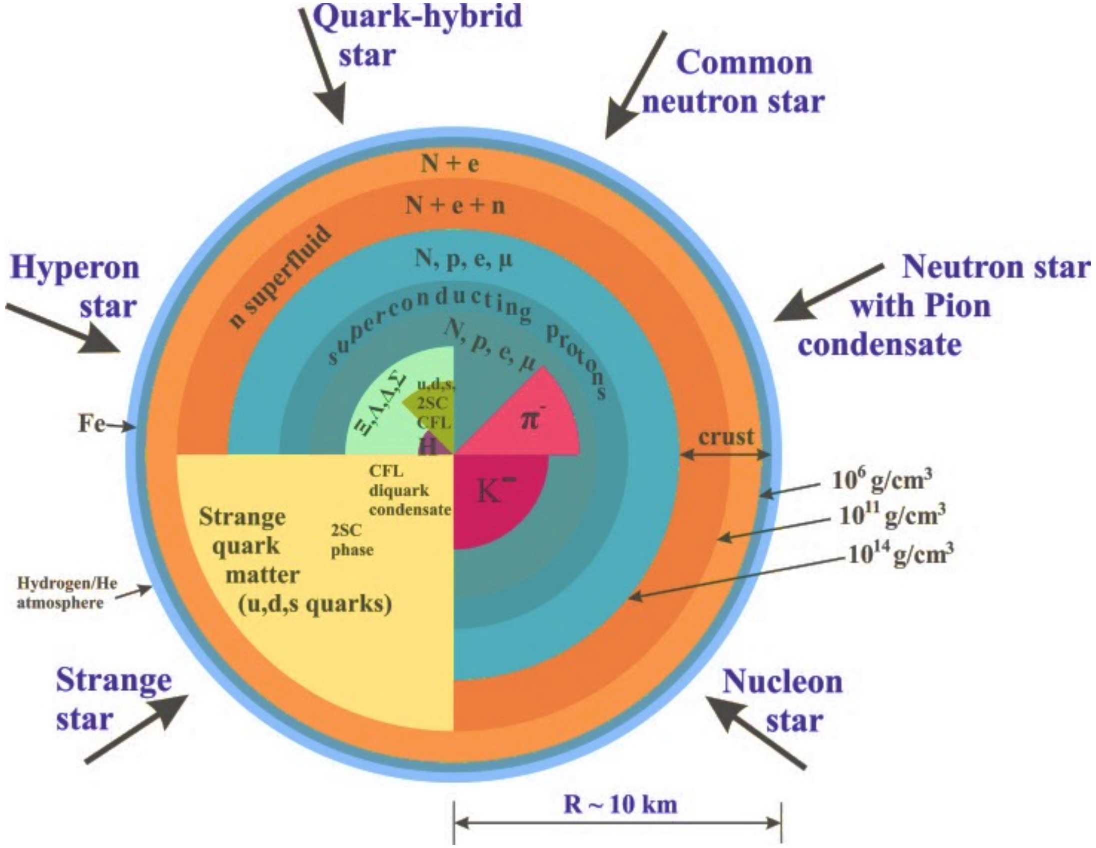
Ayriyan, Blaschke, Carlomagno, Contrera, Grunfeld. Universe 11(5), 141 (2025)
Ayriyan, Ivanytskyi, Blaschke. arXiv:2509.02554 [nucl-th]

# Neutron stars



# BAYESIAN INFERENCE FROM MULTI-MESSENGER ASTRONOMY

**Agnostic**
- Model-independent
- No evidence to physical quantities

**Physics Informed**
- Model-dependent
- Quantitative measure of physical parameters

**Bayes' theorem:**

Likelihood  Prior

$$p\,(H_1 \,|\, D, I\,) = \frac{p\,(D \,|\, H_1, I\,)\, p\,(H_1 \,|\, I\,)}{p\,(D \,|\, I\,)}$$

Posterior  Evidence

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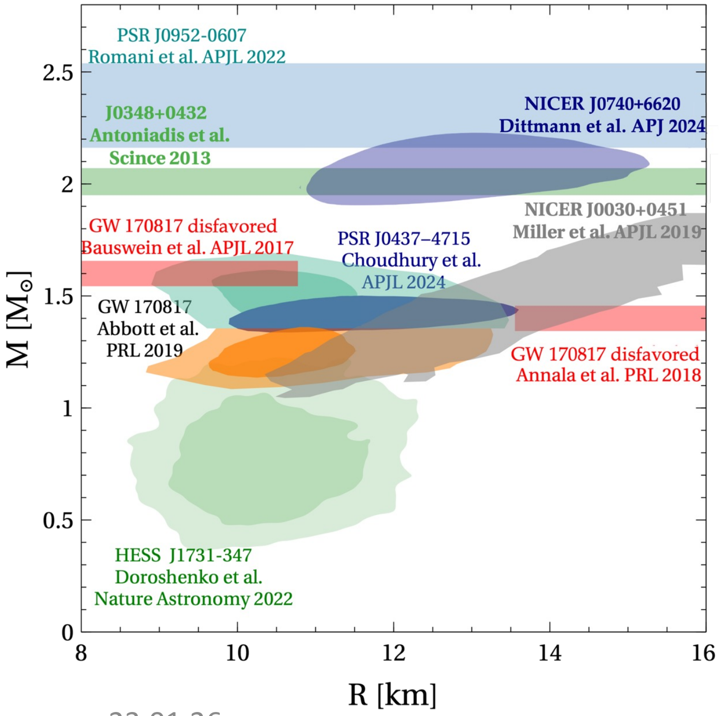
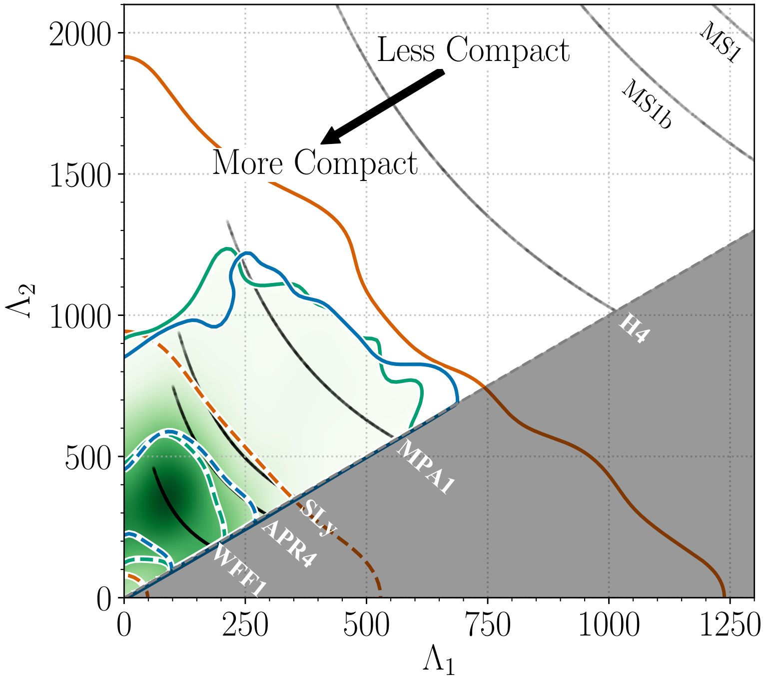


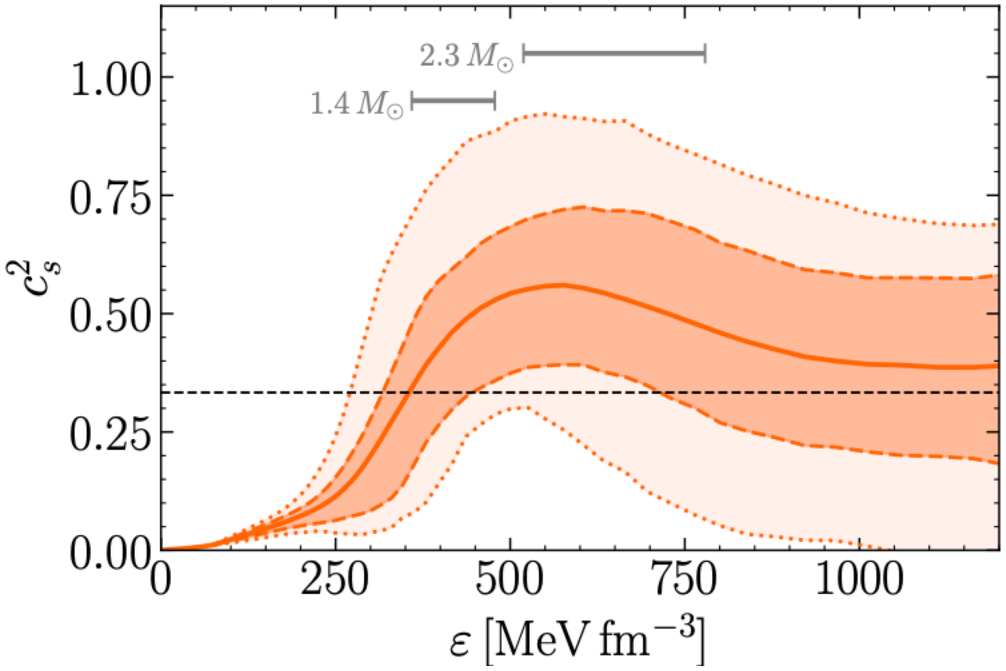

$$P\,(E_{MR}\,|\pi_i\,) = \int_{l_2} \mathcal{N}(\mu_R, \sigma_R, \mu_M, \sigma_M, \rho)d\tau + \int_{l_3} \mathcal{N}(\mu_R, \sigma_R, \mu_M, \sigma_M, \rho)d\tau$$

$$P\,(E_{GW}\,|\pi_i\,) = \int_{l_{22}} \beta(\Lambda_1(\tau), \Lambda_2(\tau))d\tau + \int_{l_{32}} \beta(\Lambda_1(\tau), \Lambda_2(\tau))d\tau + \int_{l_{23}} \beta(\Lambda_1(\tau), \Lambda_2(\tau))d\tau + \int_{l_{33}} \beta(\Lambda_1(\tau), \Lambda_2(\tau))d\tau$$

$$P\,(E_A\,|\pi_i\,) = \Phi(M_i, \mu_A, \sigma_A) = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{M_i - \mu_A}{\sqrt{2}\sigma_A}\right)\right]$$

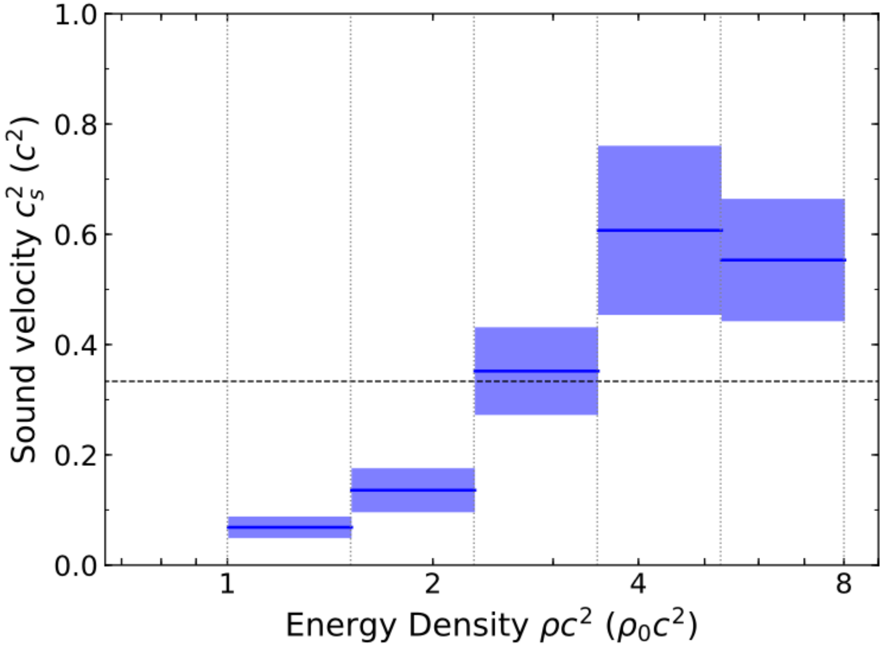
# Speed of Sound in Neutron Stars

Brandes, Weise & Kaiser. PRD108, 094014 (2023)




$$c_s^2(\varepsilon,\theta) = \frac{(\varepsilon_{i+1}-\varepsilon)c_{s,i}^2 + (\varepsilon-\varepsilon_i)c_{s,i+1}^2}{\varepsilon_{i+1}-\varepsilon_i}$$

Fujimoto, Fukushima & Murase. PRD101, 054016 (2020)



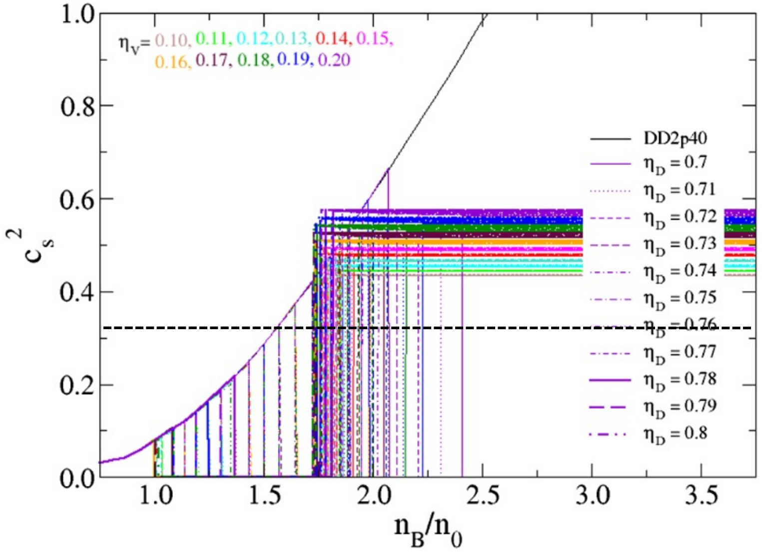

$$p_i = p_{i-1} + c_{s,i}^2(\rho_i - \rho_{i-1})$$

Annala et al. Nature Phys. 16, 907 (2020)
QM: $c_s^2 <= 1/3$
HM: $\max(c_s^2) > 0.5$

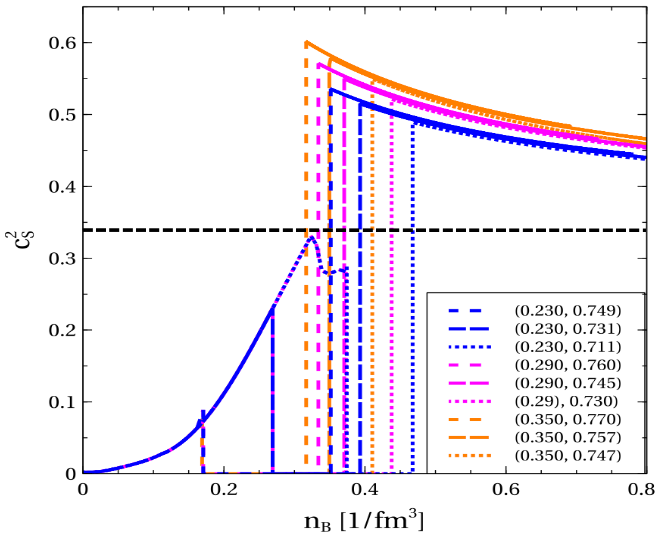
**CSC: $c_s^2 >> 1/3$**
**PhT is before or after the peak?**

$\rho_0 \approx 150$ MeV/fm$^3$

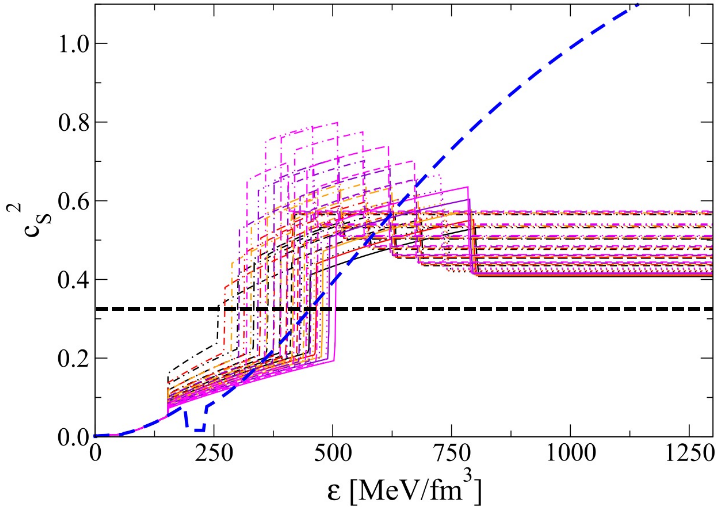
# POSSIBLE PHASE TRANSTITION TO CSC MATTER



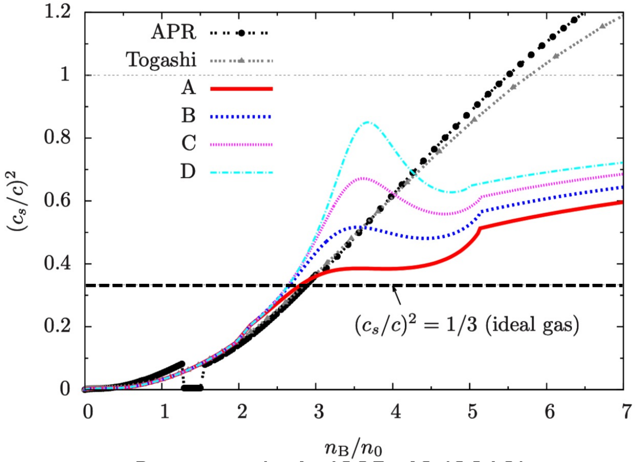
Shahrbaf et al. PRD107, 054011 (2023)



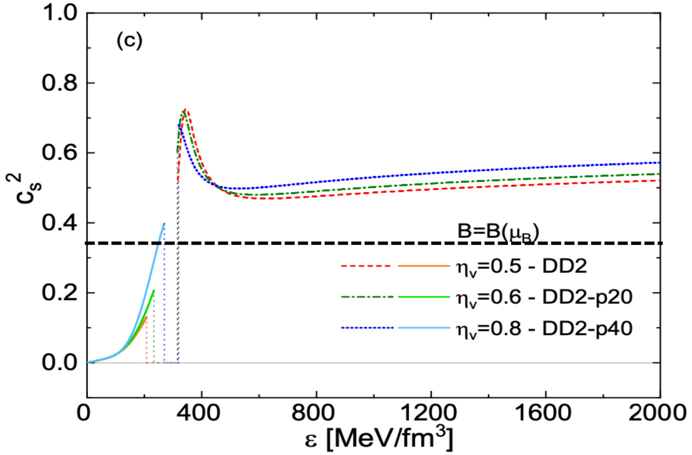
Gärtlein et al. PRD108, 114028 (2023)



Ayriyan et al. EPJA57, 318 (2021)



Baym et al., ApJ885, 42 (2019)



Contrera et al. PRC105, 045808 (2022)

# CONSEQUENCES

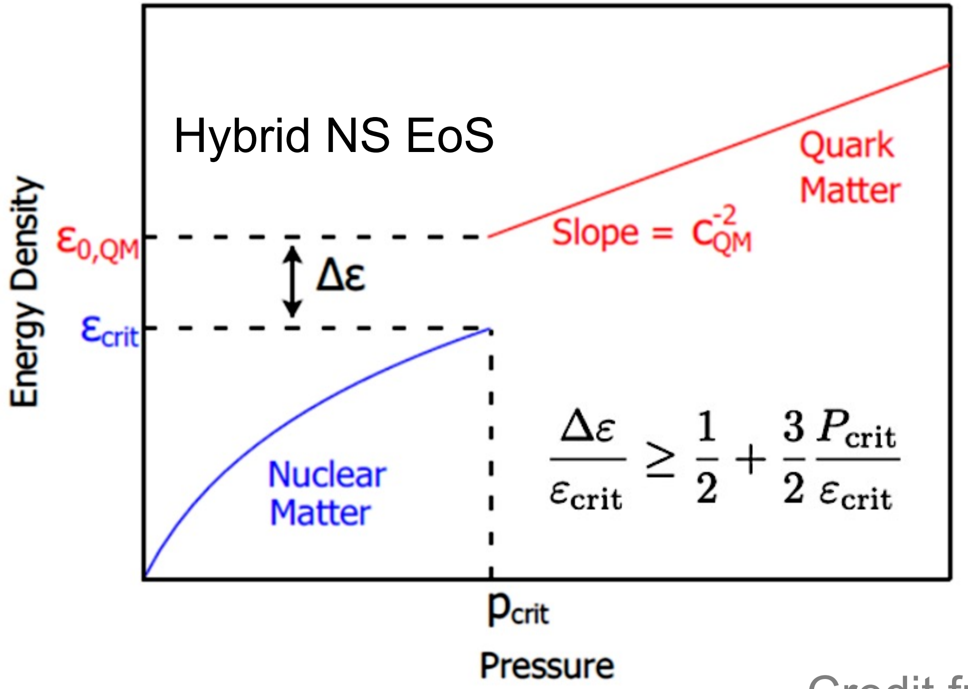
- Bayesian inference based on multi-messenger astronomy data supports a high peak in the speed of sound ($\max(c_s^2) > 0.45$) at $\varepsilon$=350-600 MeV/fm$^3$
- Conclusion that high sound speeds ($c_s^2 > 1/3$) indicates only hadronic phase was made without considering CSC matter, while phase transition to CSC shows $c_s^2 > 0.45$.
- Multi-messenger astronomy data meets CSC matter

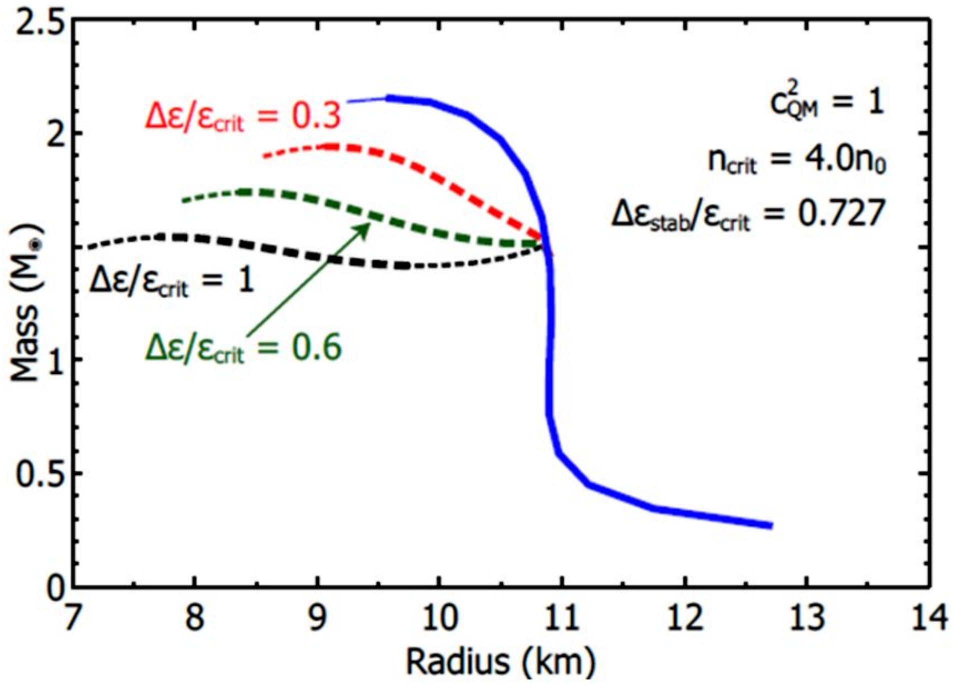
# NS MASS AND RADIUS

$$\begin{cases} \dfrac{dP(r)}{dr} = -\dfrac{GM(r)\varepsilon(r)}{r^2} \dfrac{\left(1+\dfrac{P(r)}{\varepsilon(r)}\right)\left(1+\dfrac{4\pi r^3 P(r)}{M(r)}\right)}{\left(1-\dfrac{2GM(r)}{r}\right)} \\ \dfrac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r). \end{cases}$$

$m(0) = 0$ (no mass at the center)
$P(0) = P_c$ (central pressure from the EoS)

$P(R) = 0$ (no pressure at the surface)
$M = m(R)$ (total graavitational mass)




Credit from Alford, Han &c Prakash. PRD88, 083013 (2013)

# BA FORMALISM

Bayes' theorem provides the posterior distribution of the parameters $(\eta_D, \eta_V)$:

$$p((\eta_D, \eta_V)|\mathcal{D}) = \frac{p(\mathcal{D}|(\eta_D, \eta_V))\, p(\eta_D, \eta_V)}{p(\mathcal{D})},$$

where: $p((\eta_D, \eta_V)|\mathcal{D})$ is the posterior distribution, $p(\mathcal{D}|(\eta_D, \eta_V))$ is the full likelihood, $p(\eta_D, \eta_V)$ is the prior distribution, $p(\mathcal{D})$ is the evidence.

The prior distribution is taken to be equiprobable:

$$p(\eta_D, \eta_V) = \frac{1}{|(\eta_D, \eta_V)|} = \frac{1}{N}$$

The full likelihood is obtained by production of the likelihoods for all independent observations $D_\alpha$:

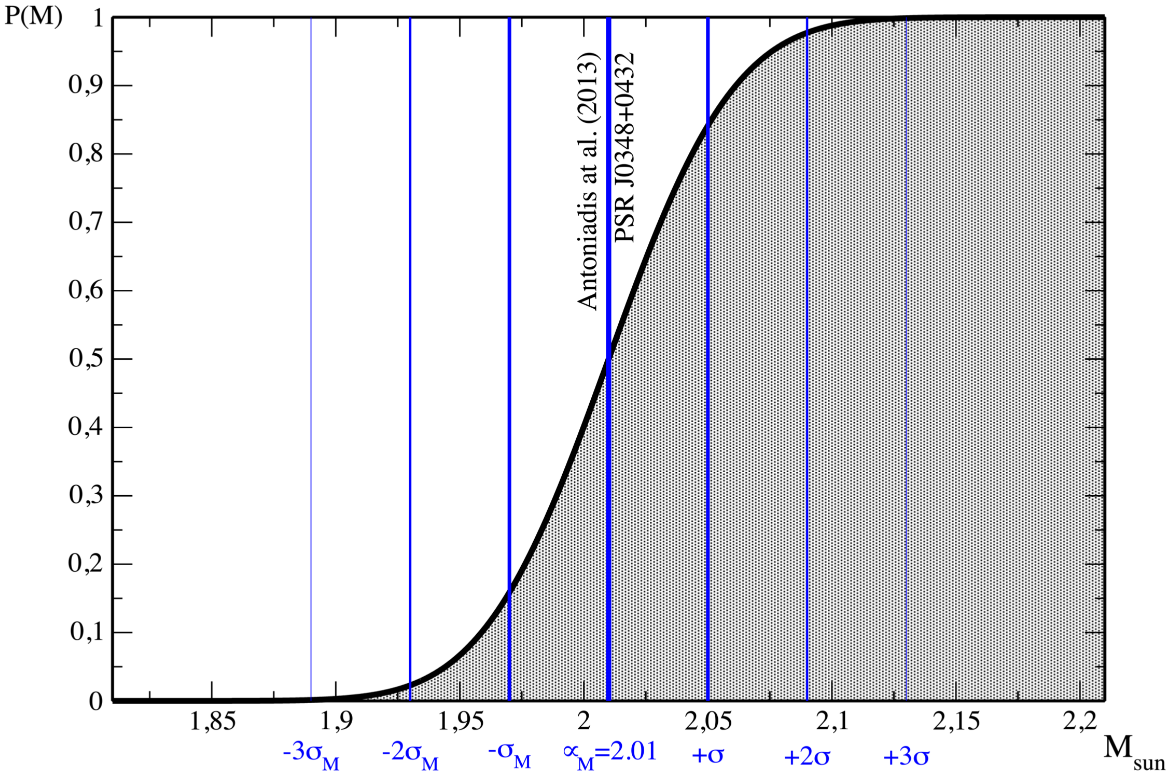
$$p(\mathcal{D}|(\eta_D, \eta_V)) = \prod_{\alpha}^{N} p(D_\alpha|(\eta_D, \eta_V)).$$

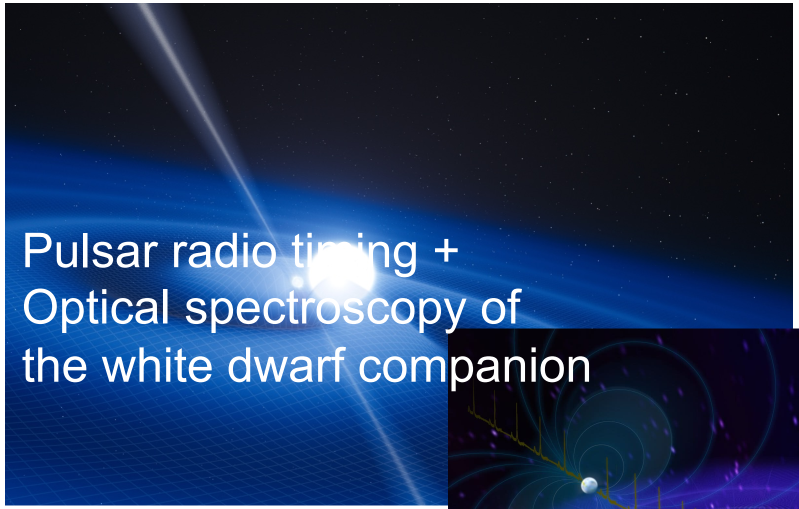
The evidence $p(\mathcal{D})$ is obtained by marginalizing over all possible values of $(\eta_D, \eta_V)$:

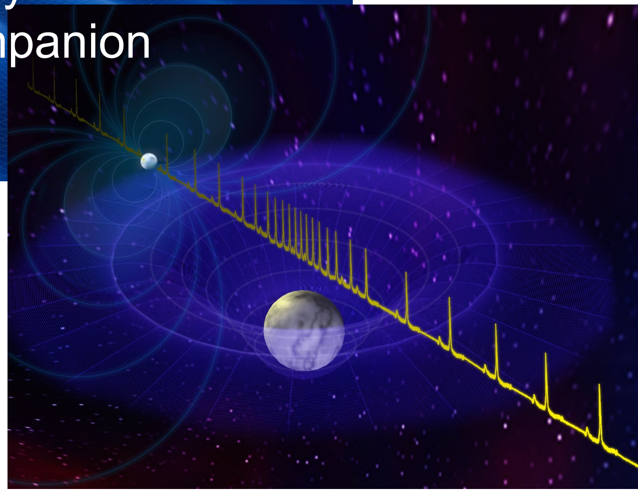
$$p(\mathcal{D}) = \sum_{(\eta_D, \eta_V)} p(\mathcal{D}|(\eta_D, \eta_V))\, p(\eta_D, \eta_V).$$

# Lower limit of maximum mass

$$p(D_M \,|\, (\eta_D, \eta_V)) = F_{\mathcal{N}}(M_{\max}(\eta_D, \eta_V);\, \mu_M, \sigma_M)$$



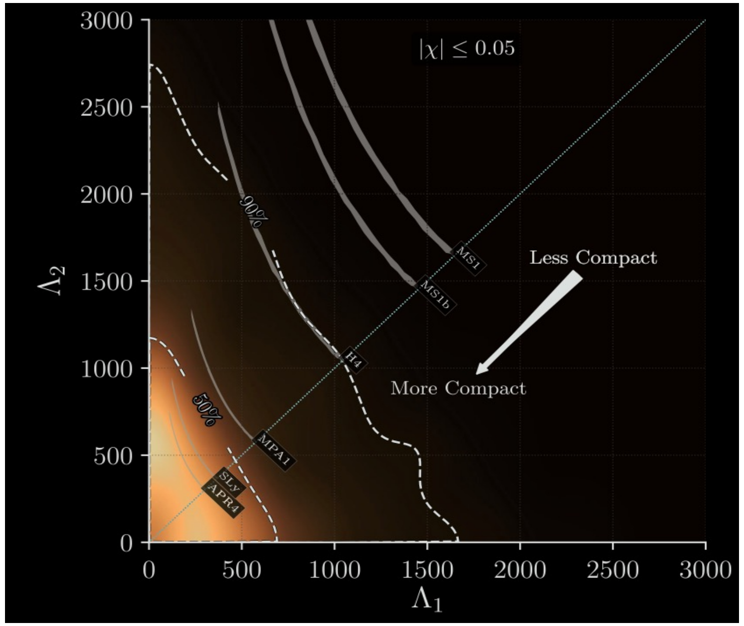



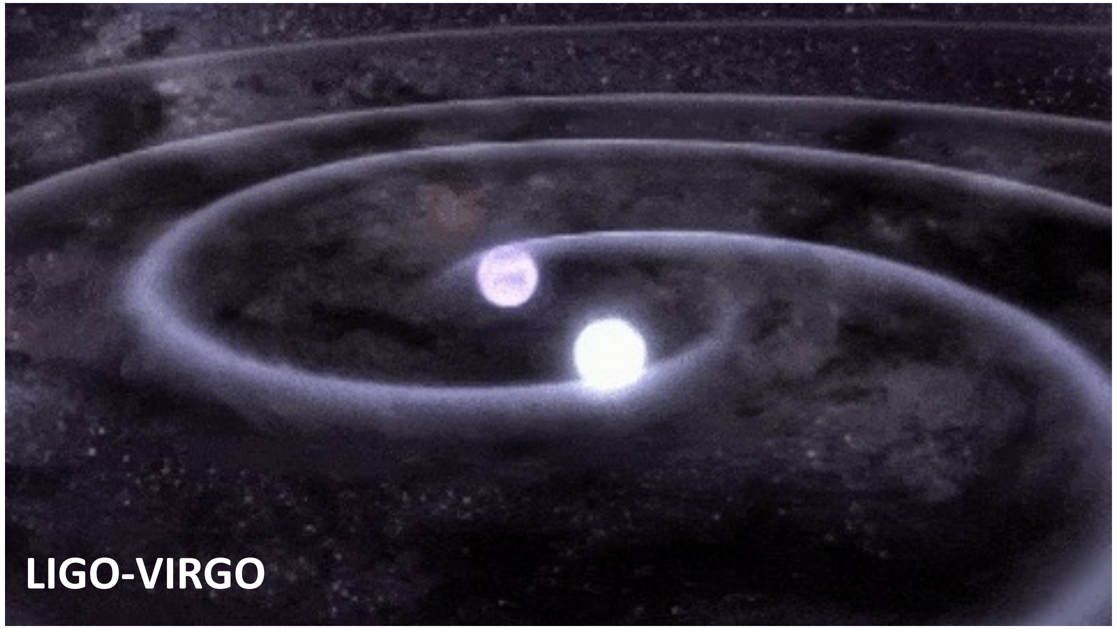
PSR J0348+0432 M=2.01+0.04−0.04 $M_\odot$
[Antoniadis et al. (2013)]

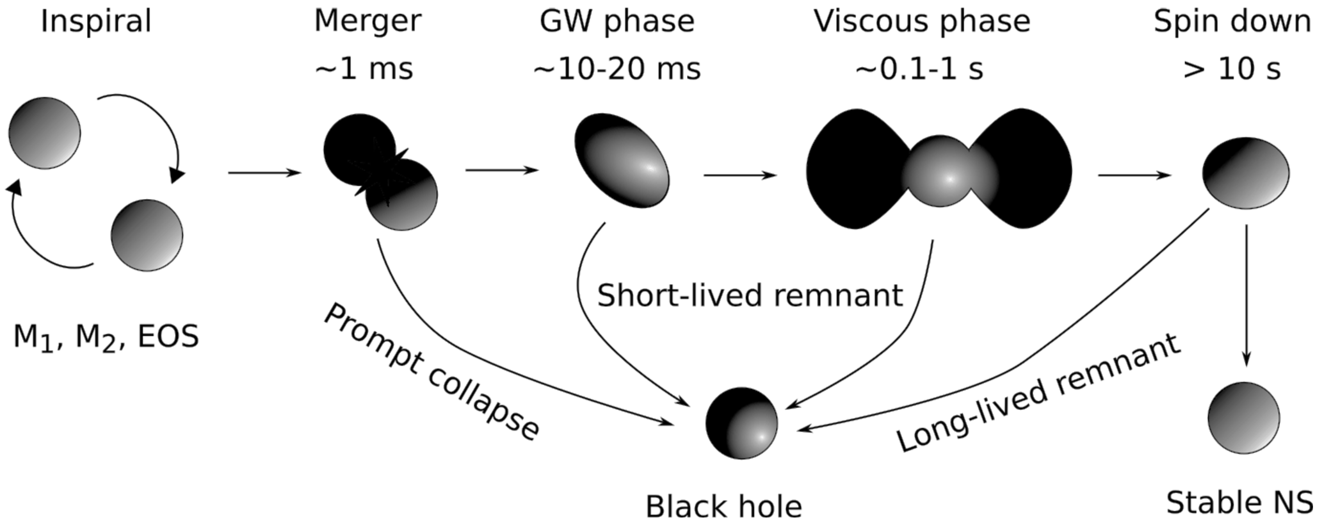
PSR J0952-0607 M= 2.35+0.17−0.17 $M_\odot$
[Romani et al. (2022) „Black Widow“ pulsar]
(**included to the extended set**)

# $\Lambda_1$-$\Lambda_2$ RELATION FROM GW170817

| | |
|---|---|
| Primary mass $m_1$ | $1.36–1.60\ M_\odot$ |
| Secondary mass $m_2$ | $1.17–1.36\ M_\odot$ |
| Chirp mass $\mathcal{M}$ | $1.188^{+0.004}_{-0.002} M_\odot$ |
| Mass ratio $m_2/m_1$ | $0.7–1.0$ |
| Total mass $m_{\rm tot}$ | $2.74^{+0.04}_{-0.01} M_\odot$ |
| Radiated energy $E_{\rm rad}$ | $> 0.025 M_\odot c^2$ |
| Luminosity distance $D_{\rm L}$ | $40^{+8}_{-14}$ Mpc |
| Viewing angle $\Theta$ | $\leq 55°$ |
| Using NGC 4993 location | $\leq 28°$ |
| Combined dimensionless tidal deformability $\tilde{\Lambda}$ | $\leq 800$ |
| Dimensionless tidal deformability $\Lambda(1.4 M_\odot)$ | $\leq 800$ |

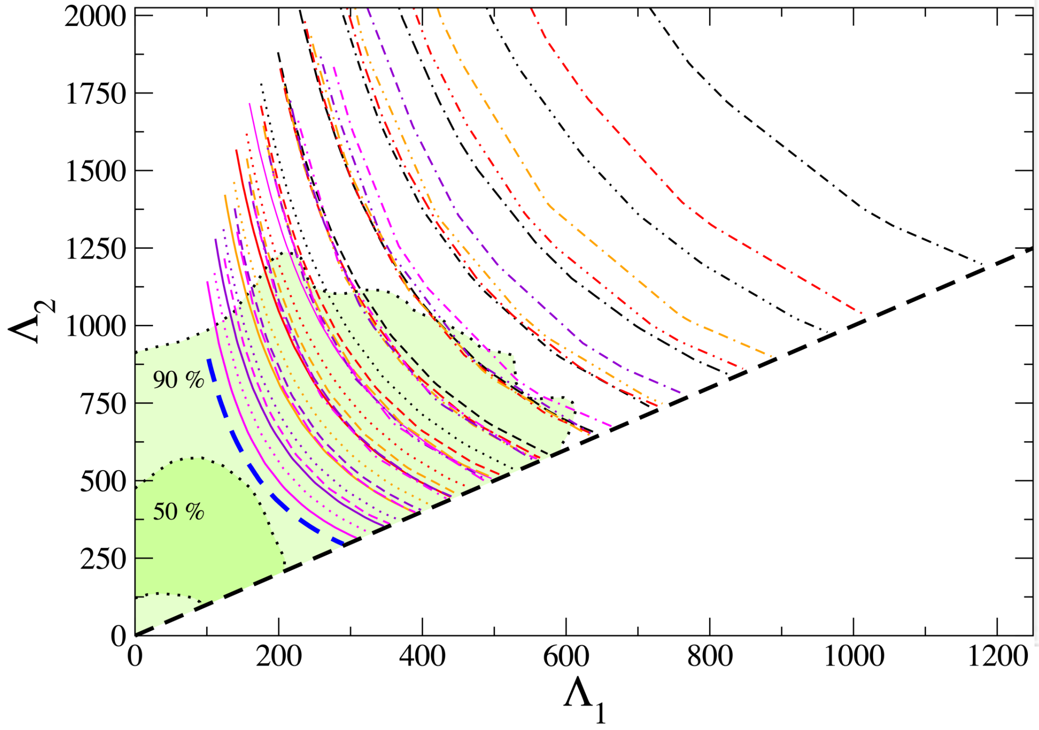


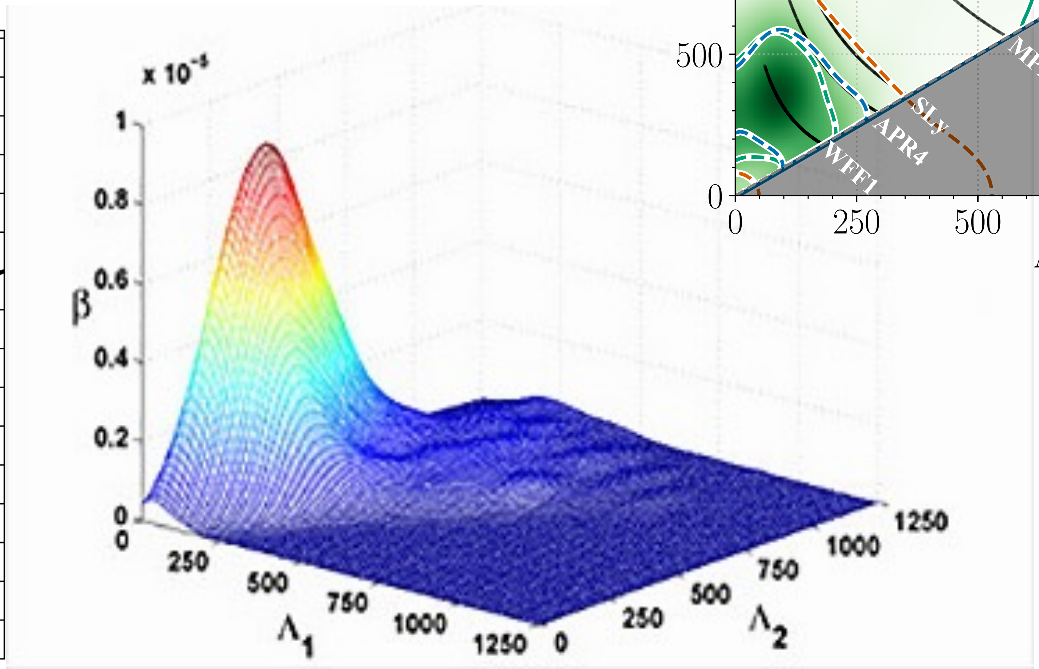



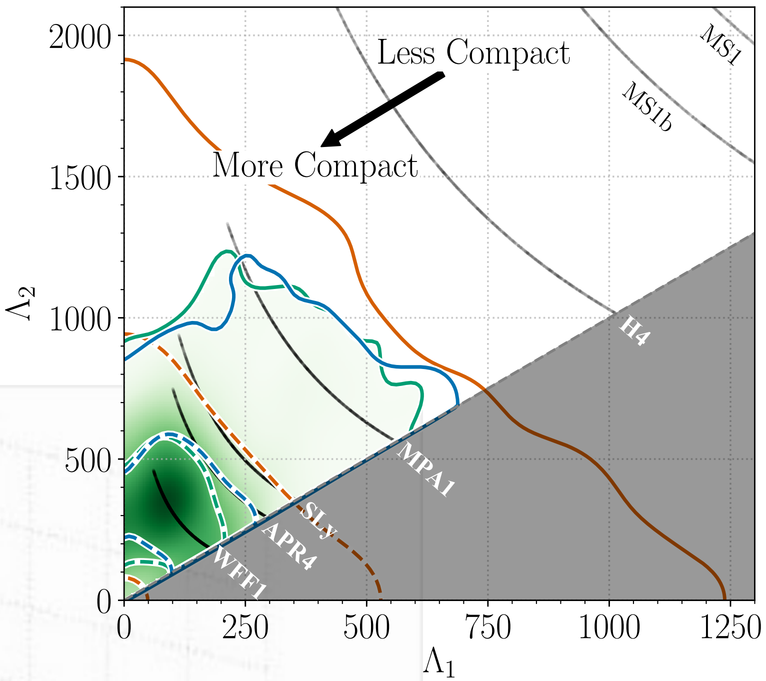



# $\Lambda_1$-$\Lambda_2$ RELATION FROM GW170817

$$p(D_{GW} \,|\, (\eta_D,\eta_V)) = \int_{\Lambda_1\Lambda_2} f_{GW}(\Lambda_1(\eta_D,\eta_V),\Lambda_2(\Lambda_1))$$

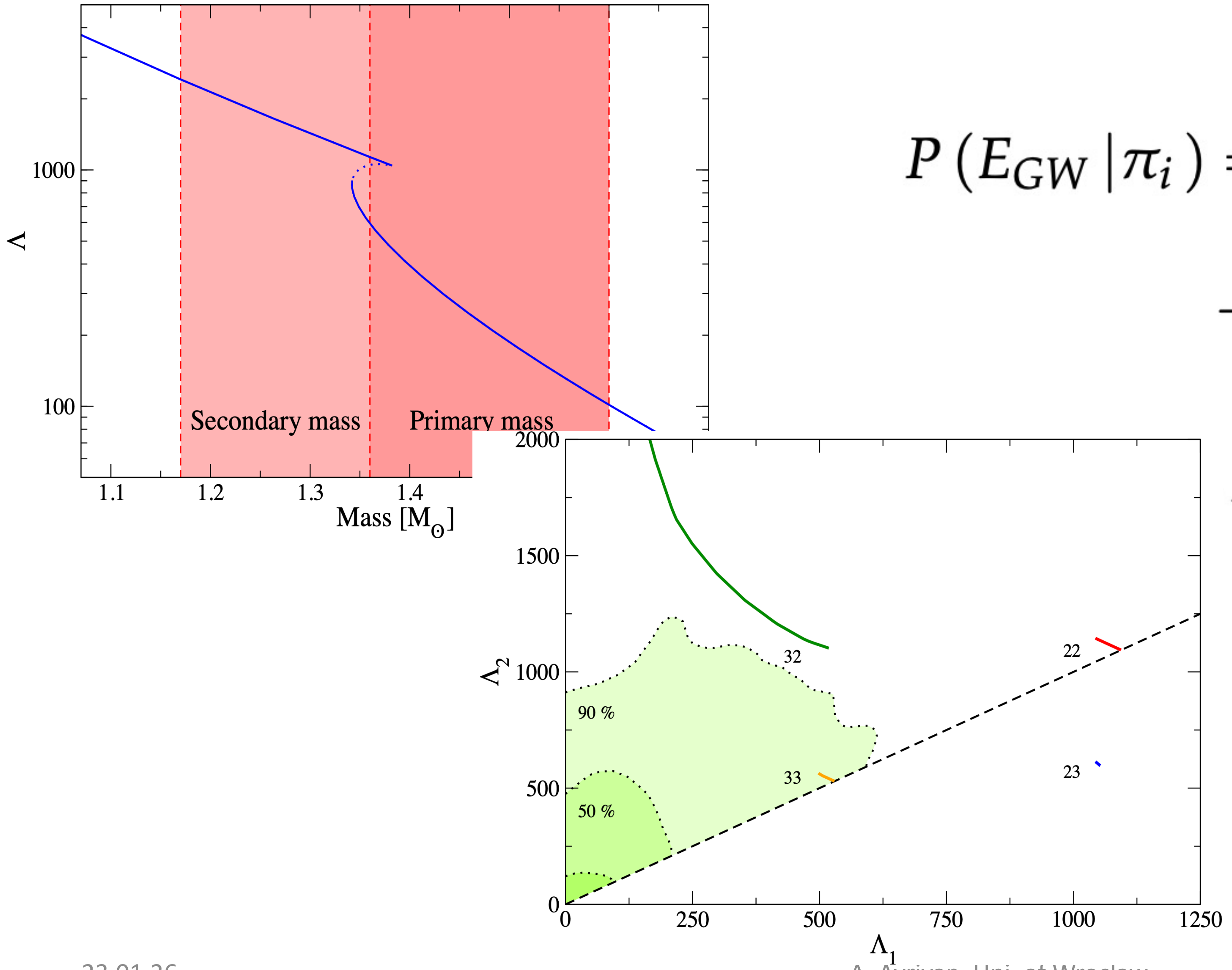


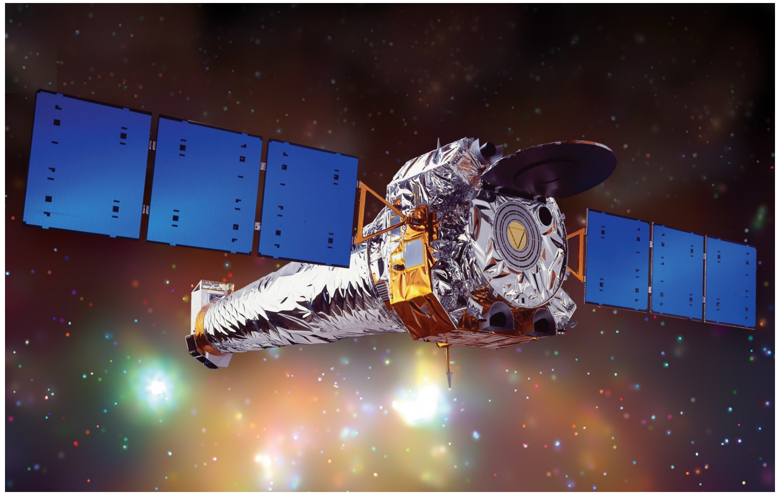

LIGO-P1800115-v12 https://dcc.ligo.org/LIGO-P1800115/public (2018)

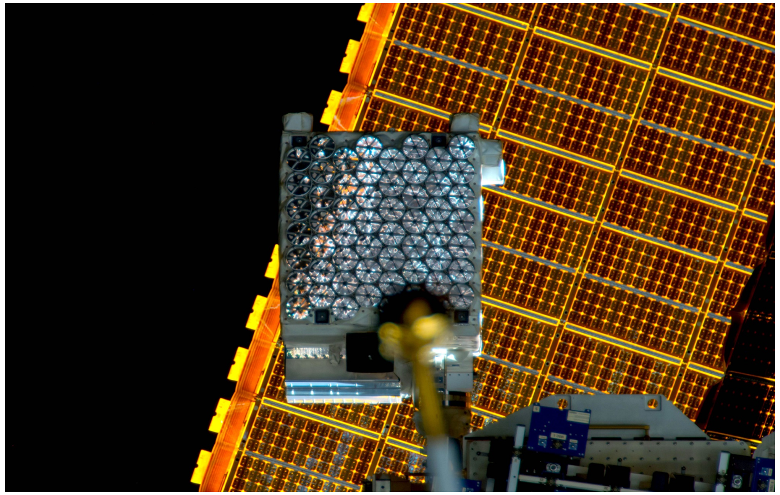
# $\Lambda_1$-$\Lambda_2$ RELATION FROM GW170817 (2)




$$P\left(E_{GW}\,|\pi_i\right) = \int_{l_{22}} \beta(\Lambda_1(\tau),\Lambda_2(\tau))d\tau + \int_{l_{32}} \beta(\Lambda_1(\tau),\Lambda_2(\tau))d\tau + \int_{l_{23}} \beta(\Lambda_1(\tau),\Lambda_2(\tau))d\tau + \int_{l_{33}} \beta(\Lambda_1(\tau),\Lambda_2(\tau))d\tau$$

# From X-ray Observations to Pulsar Mass and Radius





XMM-Newton

Chandra

NICER

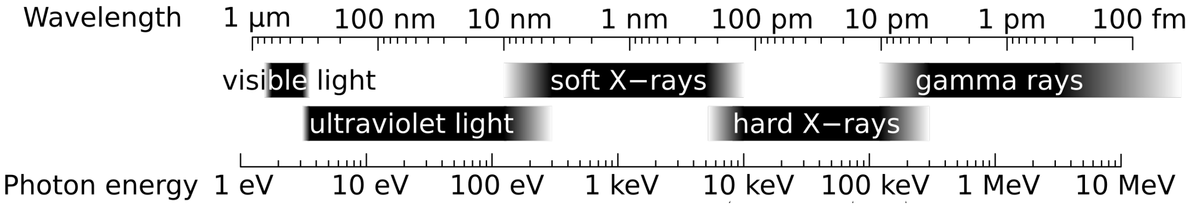
**Satellites**

- large mirror area
- produces quality spectra
- helps measure background

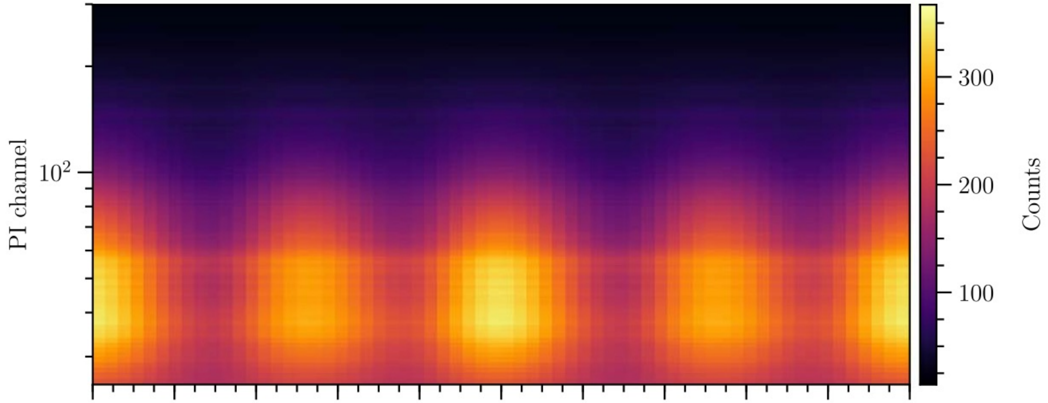
- ultra-high spatial resolution
- provides high-resolution spectra
- helps remove background

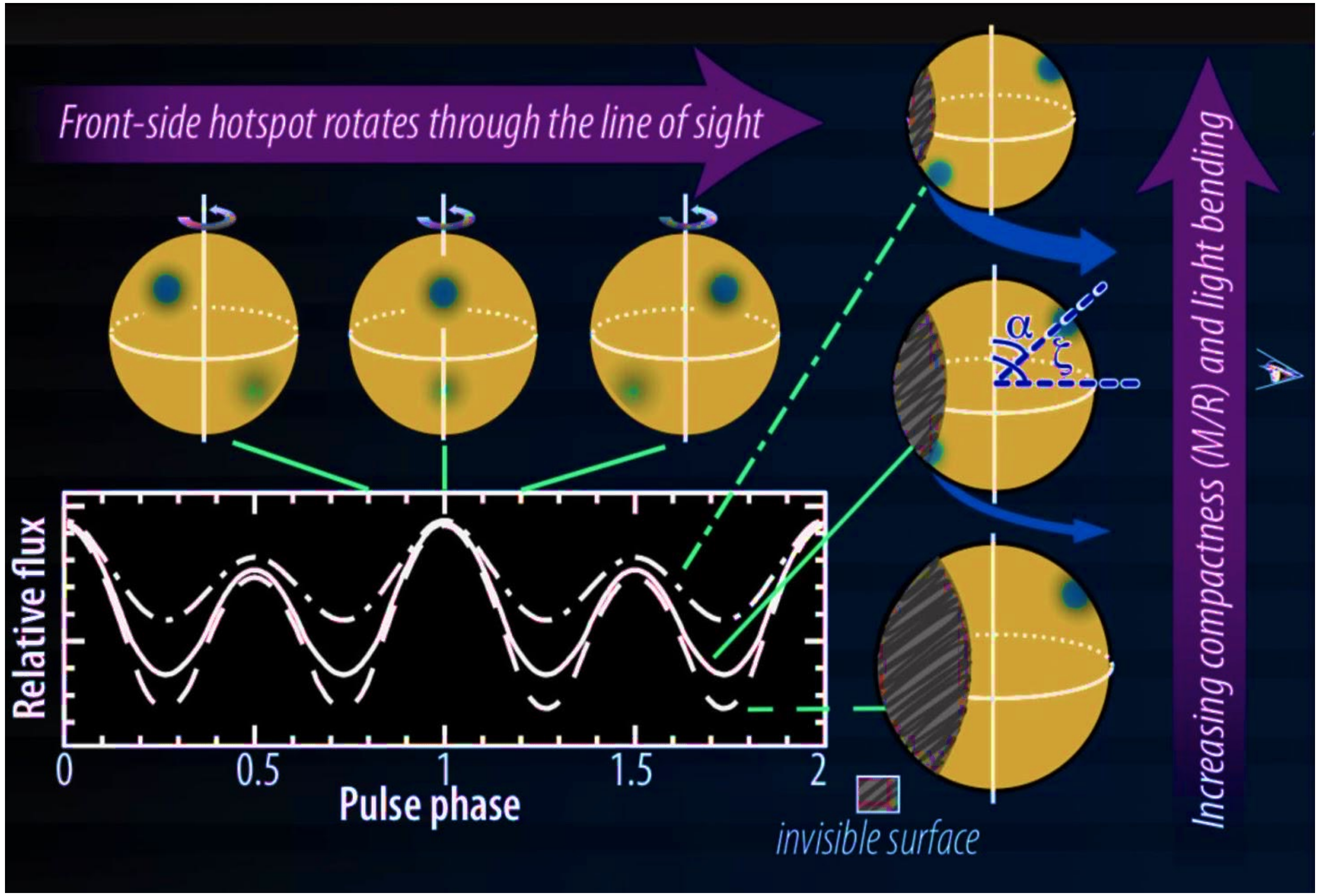
**Installed on ISS**

- focusing on single object
- pulse shape (timing)
- measures compactness



# From X-ray Observations to Pulsar Mass and Radius

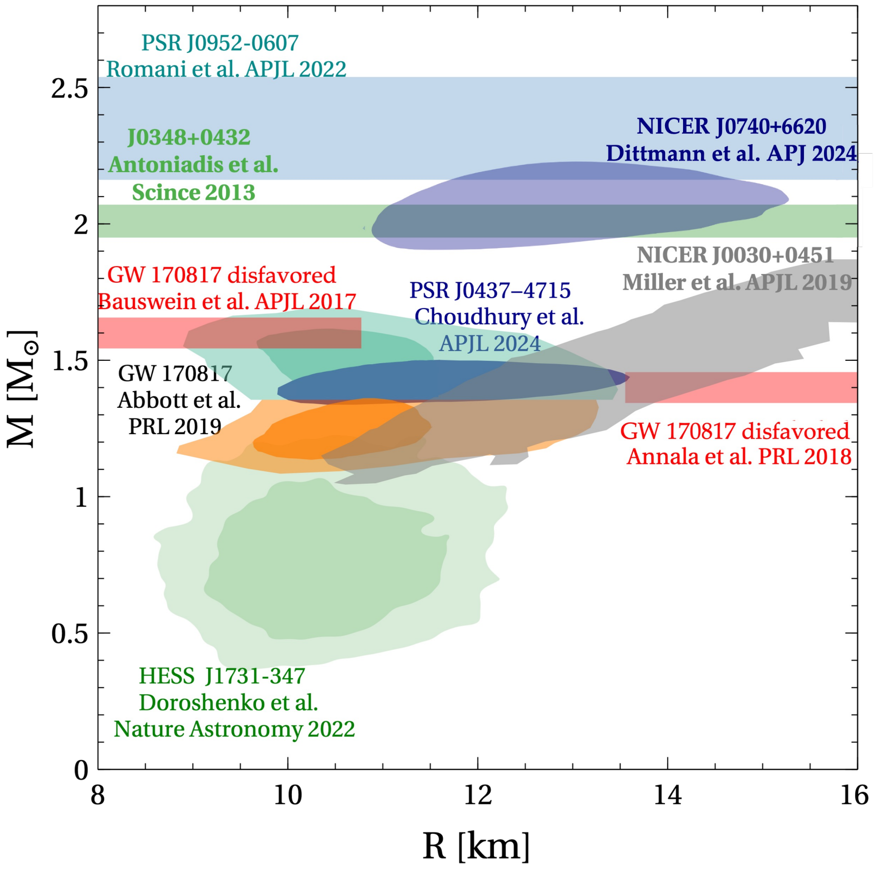



$$C = \frac{2GM}{Rc^2} \qquad z = \frac{1}{\sqrt{1-C}} - 1 \quad R_\infty = R\,(1+z)$$

# MASS-RADIUS CONSTRAINTS (2)

$$p(D_{MR}\,|\,(\eta_D,\eta_V)) = \int\limits_{MR} f_{MR}(M(\eta_D,\eta_V),\, R(M))$$




(A) PSR J0030+0451: M= 1.44+0.07−0.07 $M_\odot$
and R= 13.7+2.6−1.5 km

(B) PSR J0740+6620: M= 2.08+0.07−0.07 $M_\odot$
and R= 13.7+2.6−1.5 km

(C) PSR J0437-4715: M= 1.418+0.037−0.037 $M_\odot$
and R= 11.36+0.95−0.63 km

(D) HESS J1731-347: M= 0.77+0.20−0.17 $M_\odot$
and R= 10.04+0.86−0.78 km
(**included to the extended set**)

# HYBRID NS EOS: DD2 + 2FNLNJL

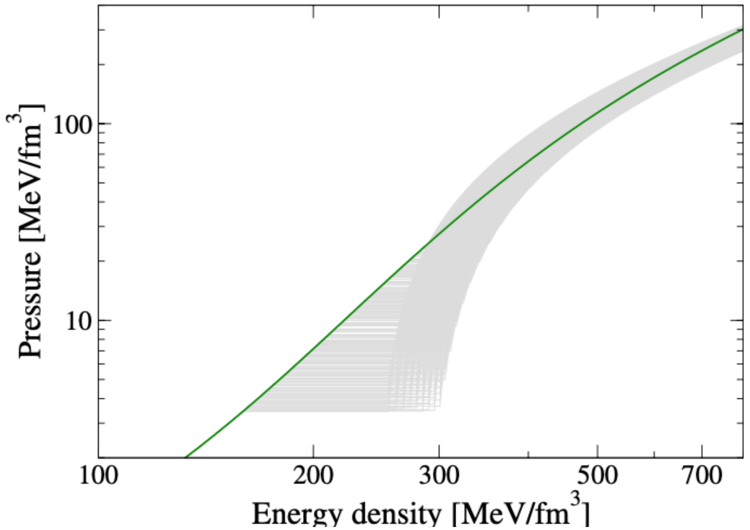
Two-flavor color-superconducting quark matter with 3DFF nonlocal chiral quark model defined by an effective Euclidean action functional that in the case of two light flavors is given by

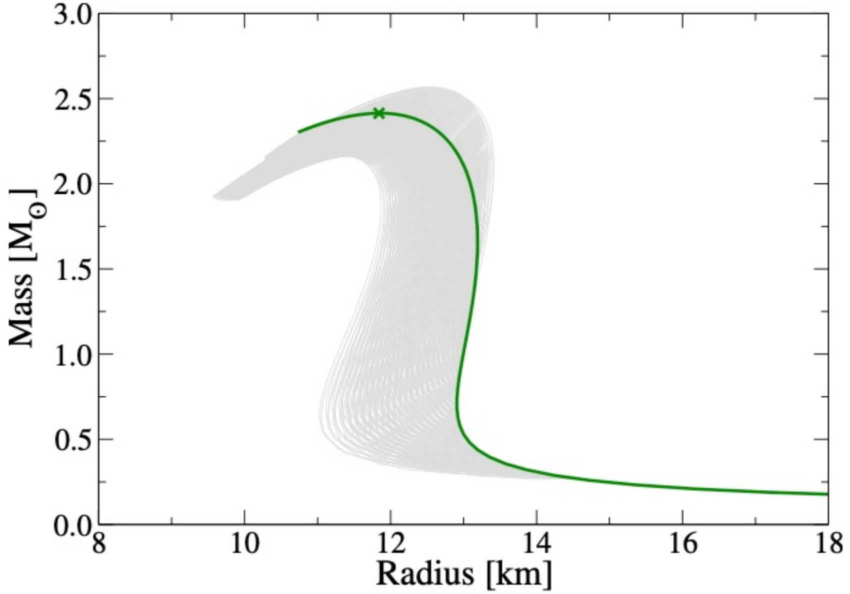
$$S_E = \int d^4x \left\{ \bar{\psi}(x)(-i\partial\!\!\!/ + \hat{m} - \gamma_0\hat{\mu})\psi(x) - \frac{G_S}{2}\left[j_S^f(x) j_S^f(x)\right.\right.$$
$$\left.\left. + \eta_D \left[j_D^a(x)\right]^\dagger j_D^a(x) - \eta_V j_V^\mu(x)\, j_V^\mu(x)\right]\right\}.$$

$$j_S^f(x) = \int d^4z\, g_S(z)\, \bar{\psi}(x+\frac{z}{2})\, \Gamma_f\, \psi(x-\frac{z}{2}),$$
$$j_D^a(x) = \int d^4z\, g_D(z)\, \bar{\psi}_C(x+\frac{z}{2})\, i\gamma_5\tau_2\lambda_a\, \psi(x-\frac{z}{2}),$$
$$j_V^\mu(x) = \int d^4z\, g_V(z)\, \bar{\psi}(x+\frac{z}{2})\, i\gamma^\mu\, \psi(x-\frac{z}{2}),$$

For the nonlocality a Gaussian ansatz is employed

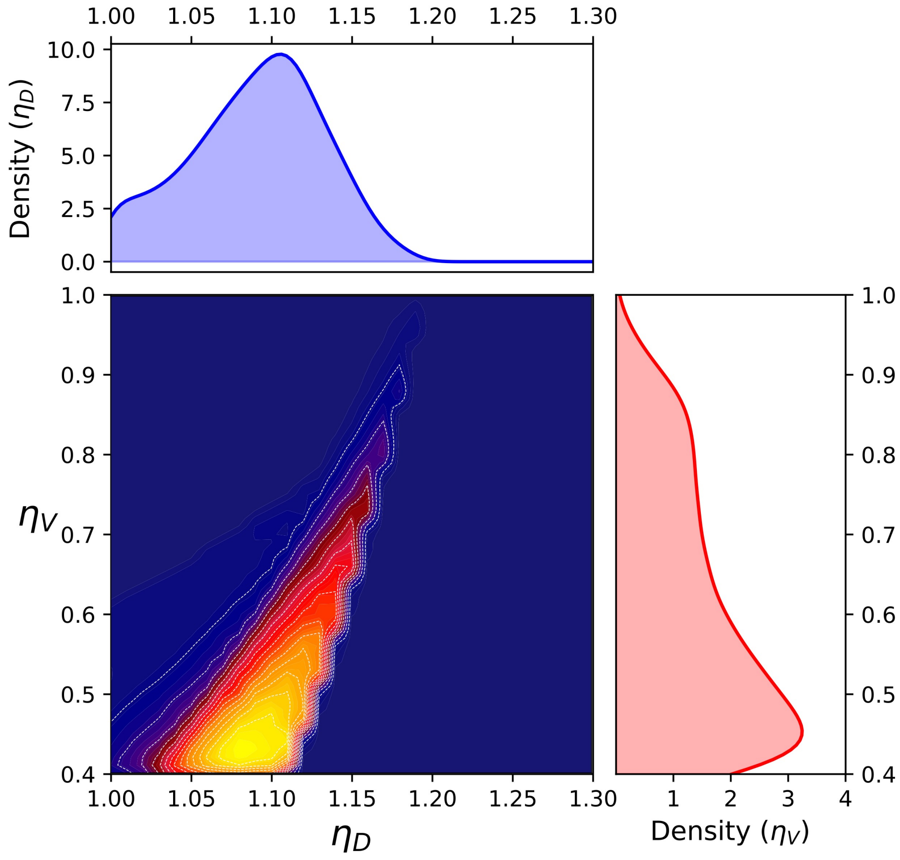
$$g_i(\vec{p}) = \exp(-\vec{p}^2/\Lambda_i^2), \quad i = S, D,$$

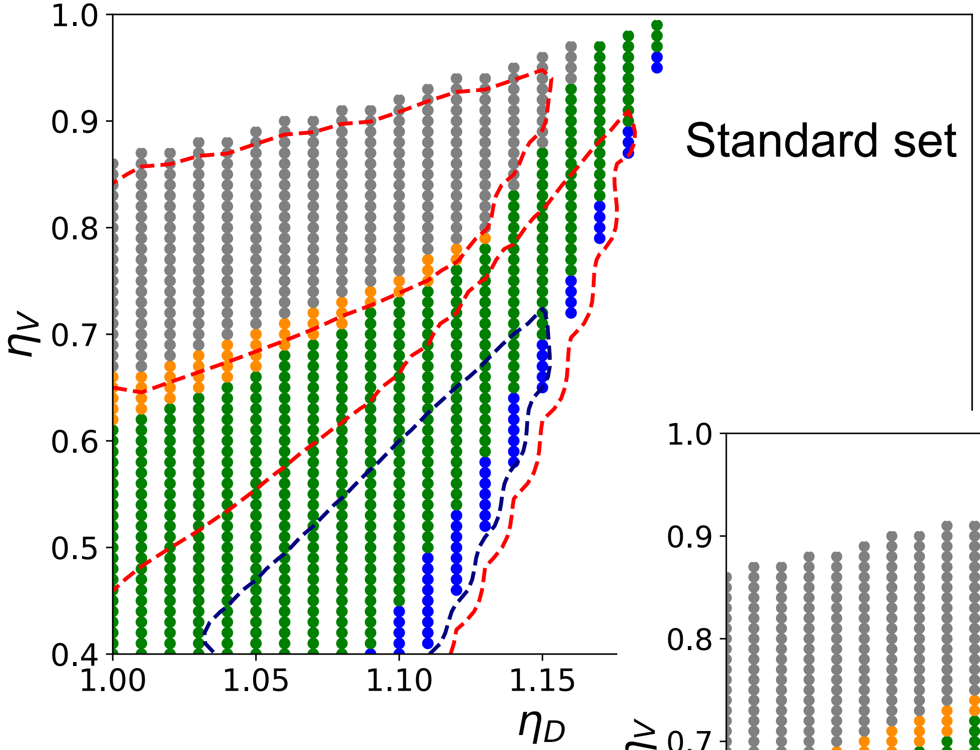


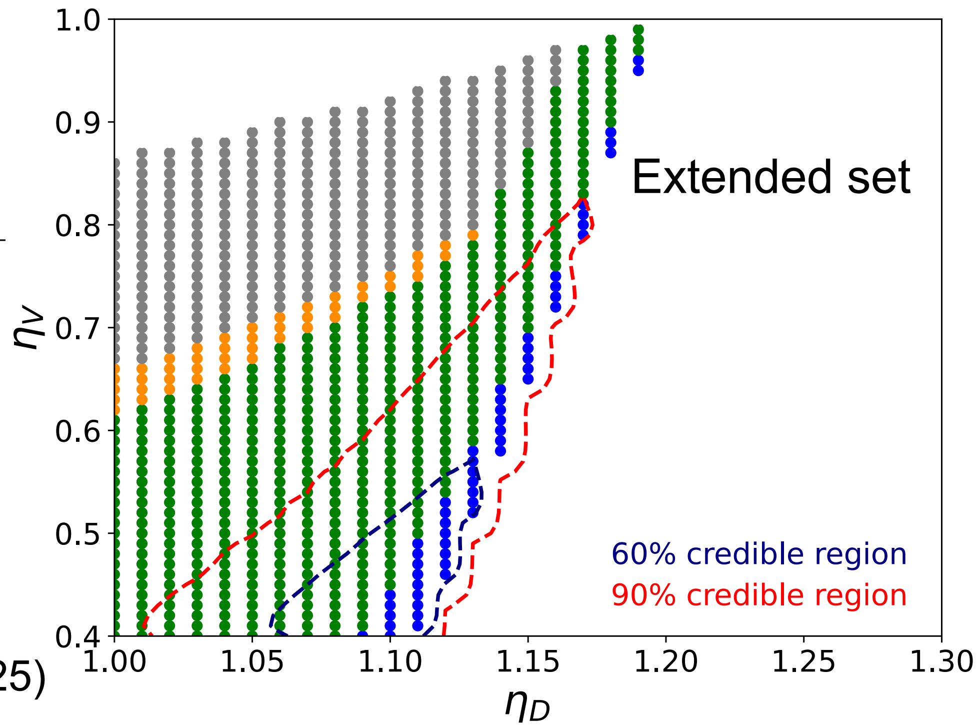




Ayriyan, Blaschke, Carlomagno, Contrera, Grunfeld. Universe 11(5), 141 (2025)

# BA RESULTS (1)

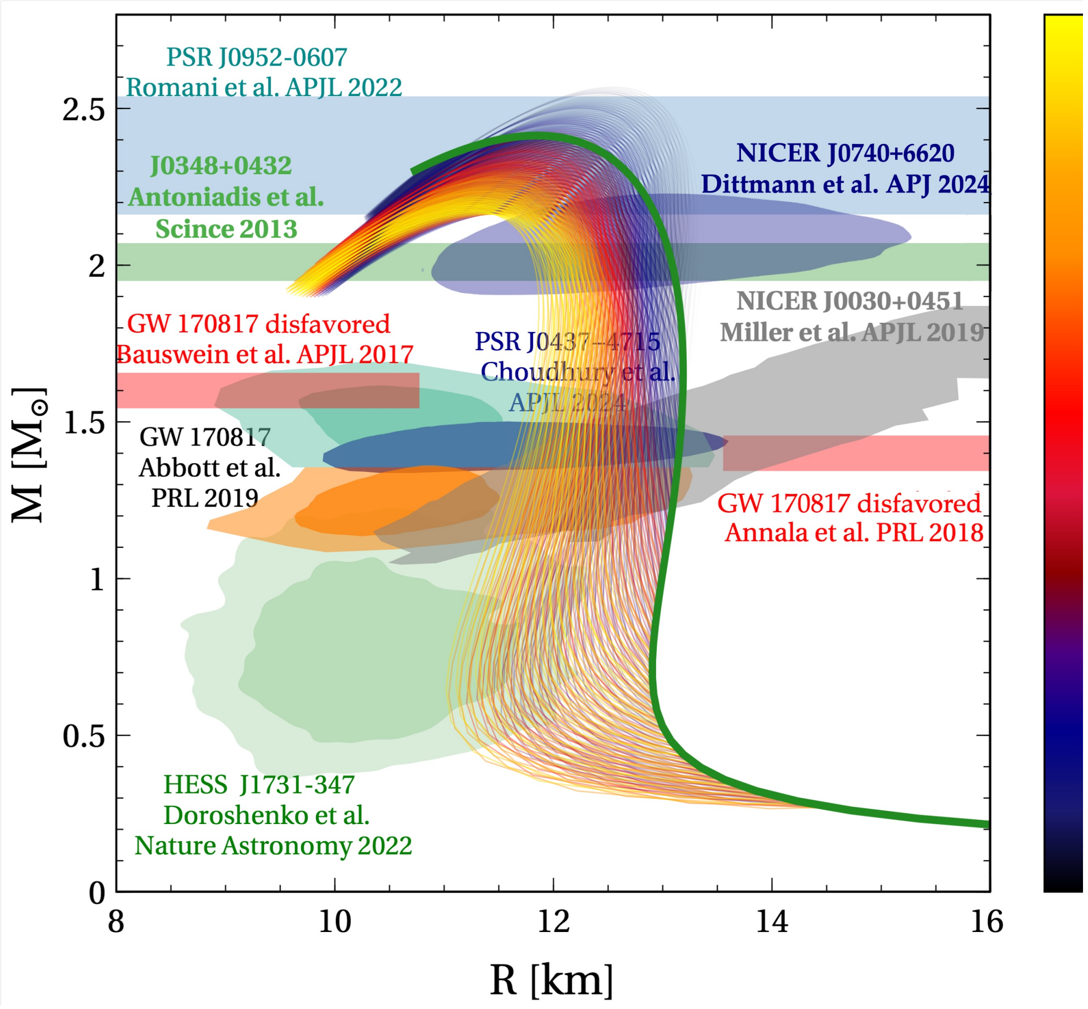


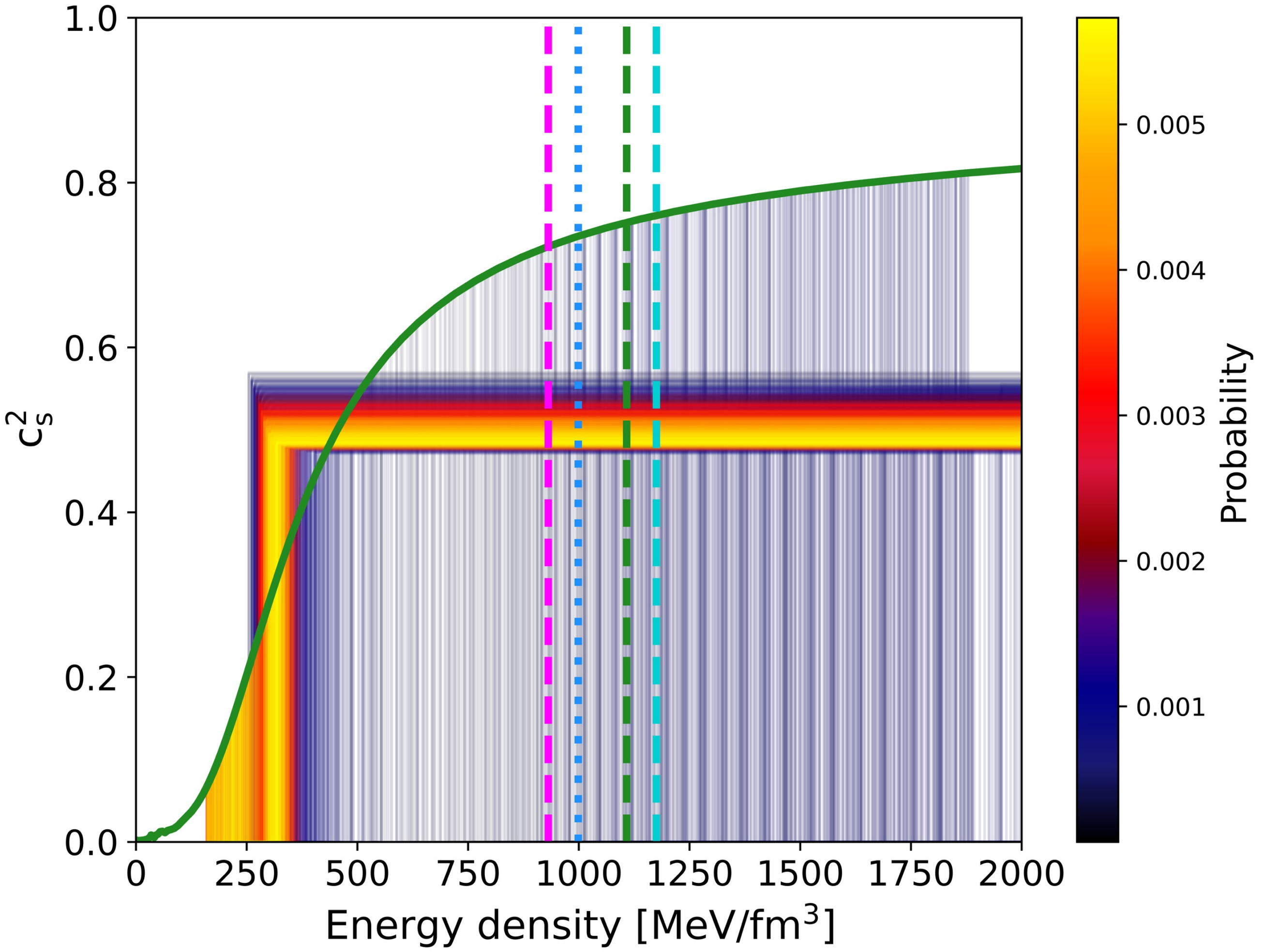




Ayriyan, Blaschke, Carlomagno, Contrera, Grunfeld. Universe 11(5), 141 (2025)

# BA RESULTS (2)




Ayriyan, Blaschke, Carlomagno, Contrera, Grunfeld. Universe 11(5), 141 (2025)

# HYBRID NS EOS: DD2Y + 3FNLNJL

Three-flavor nonlocal NJL model with vector repulsion and CFL color-superconductivity

$$\mathcal{L} = \bar{q}(i\not\partial - m + \mu\gamma_0)q + G_S \sum_{a=\overline{0,8}} s_a s_a - G_V j_\mu j^\mu + 3G_D \sum_{a,b=2,5,7} d^+_{ab} d_{ab}$$

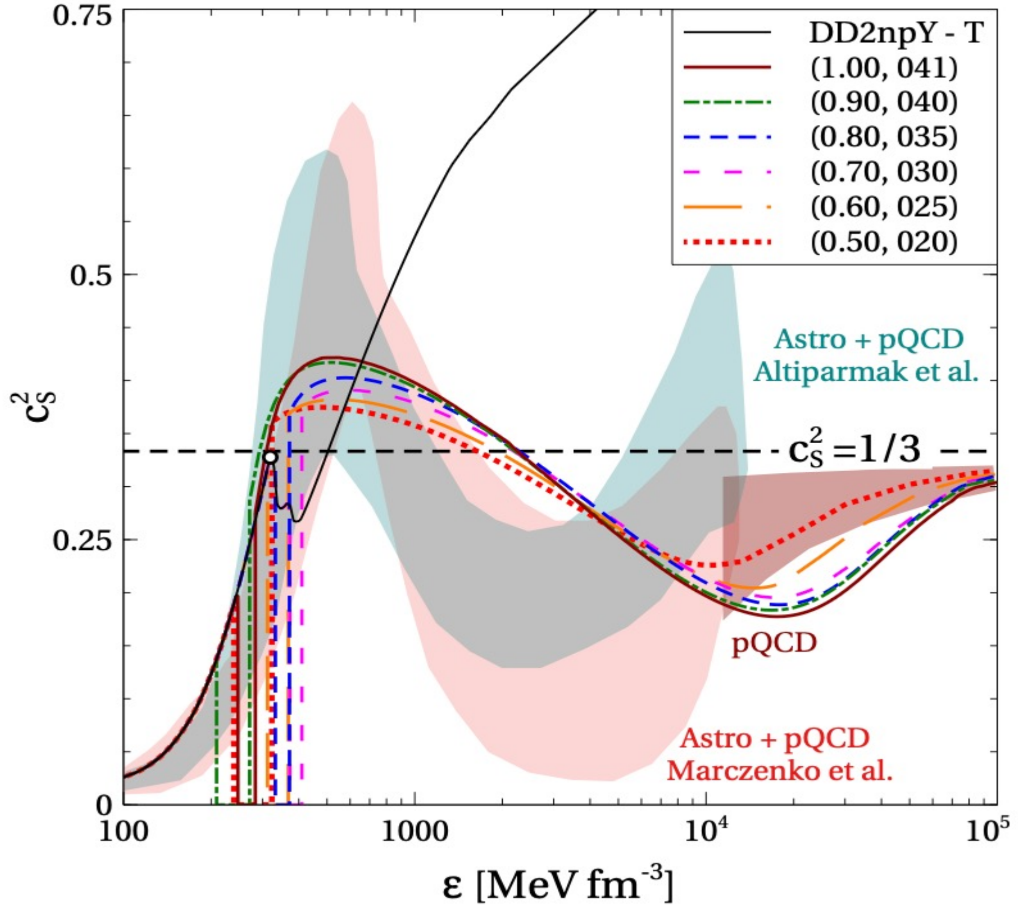
$$s_a(x) = \int dz\ g(z)\bar{q}\left(x+\frac{z}{2}\right)\tau_a q\left(x-\frac{z}{2}\right),$$

$$j_\mu(x) = \int dz\ g(z)\bar{q}\left(x+\frac{z}{2}\right)\gamma_\mu q\left(x-\frac{z}{2}\right),$$

$$d_{ab}(x) = \int dz\ g(z)\bar{q}\left(x+\frac{z}{2}\right)i\gamma_5\tau_a\lambda_b q^c\left(x-\frac{z}{2}\right)$$

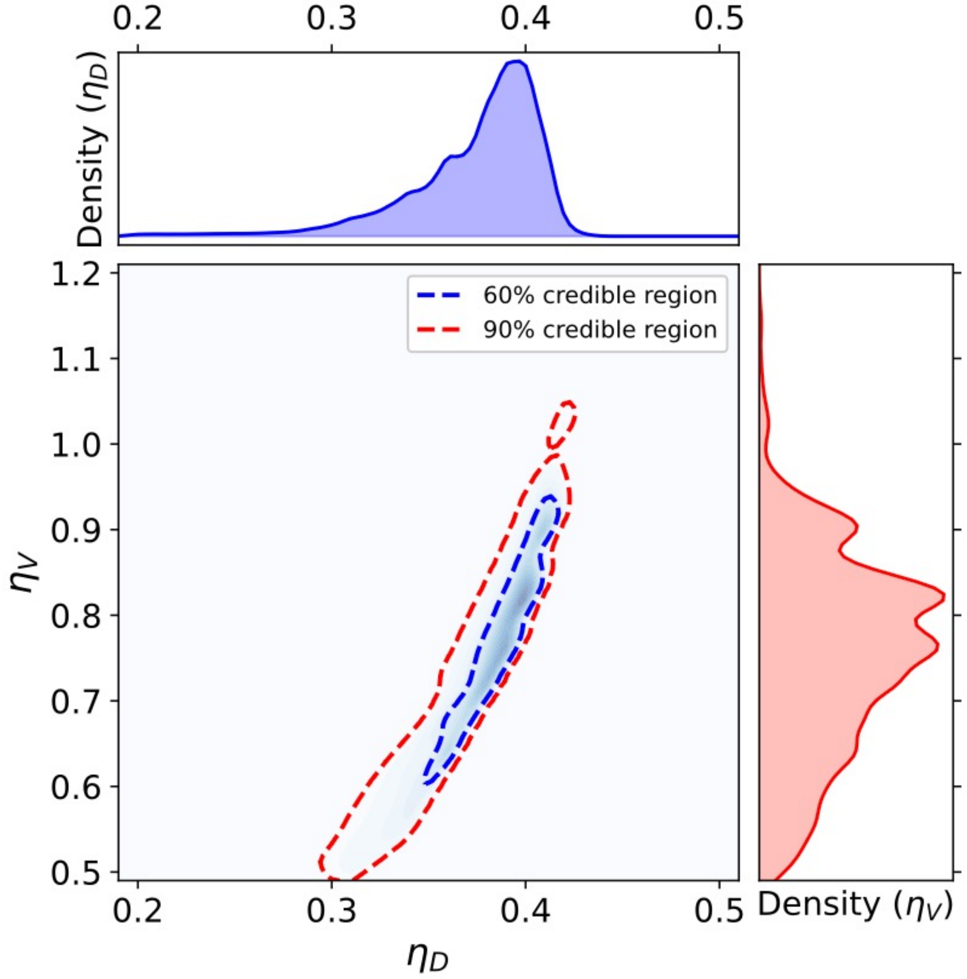
For the nonlocality an ansatz is employed

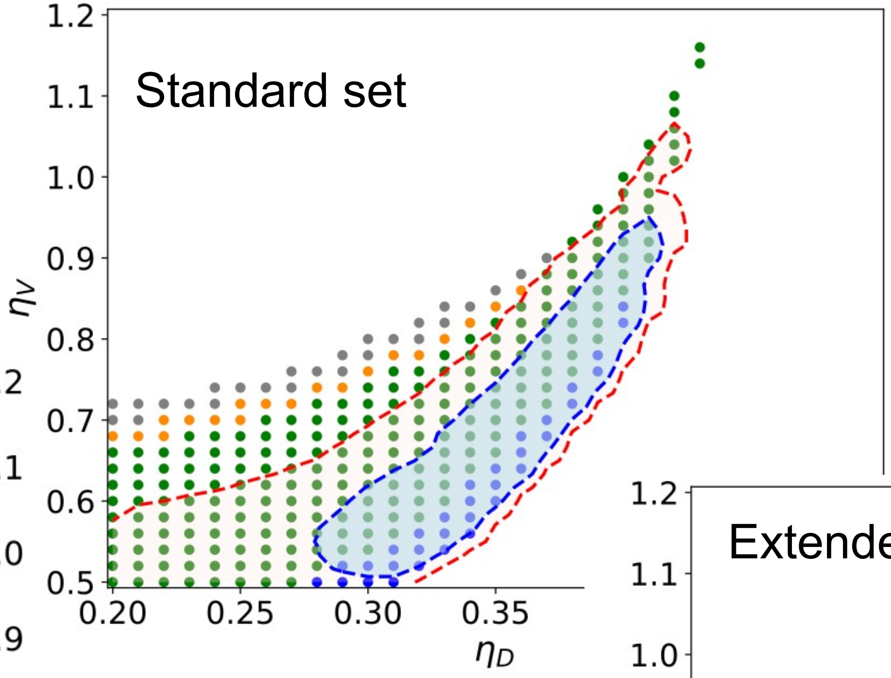
$$g_{\mathbf{k}} = \exp(-\mathbf{k}^2/\Lambda^2)$$



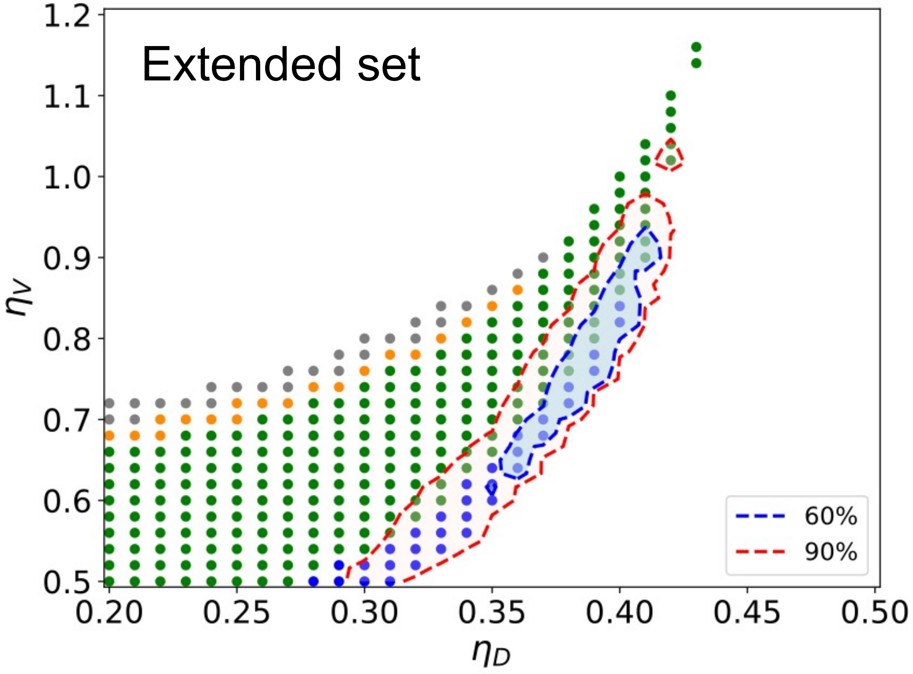

Ivanytskyi. Phys.Rev.D 111 (2025) 3, 034004 (arXiv:2409.05859)

# BA RESULTS (1)



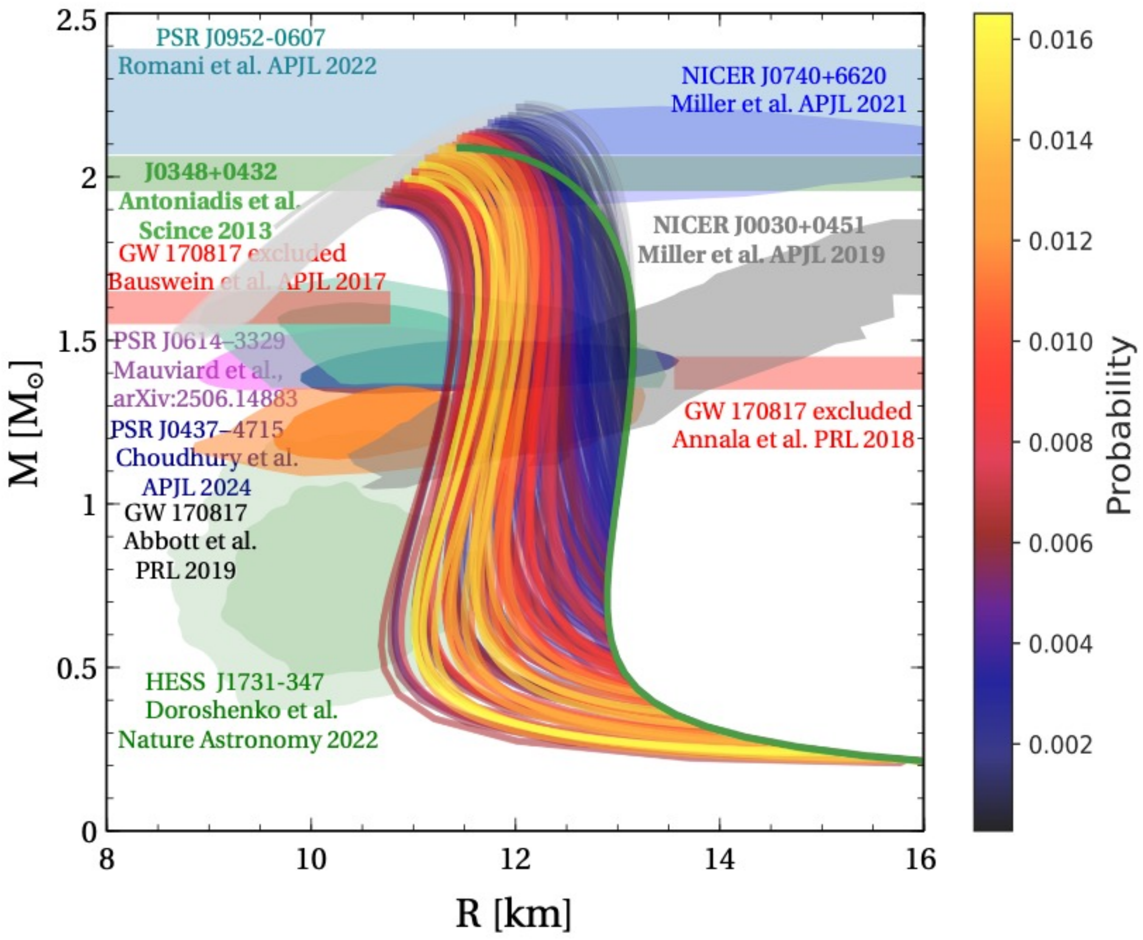



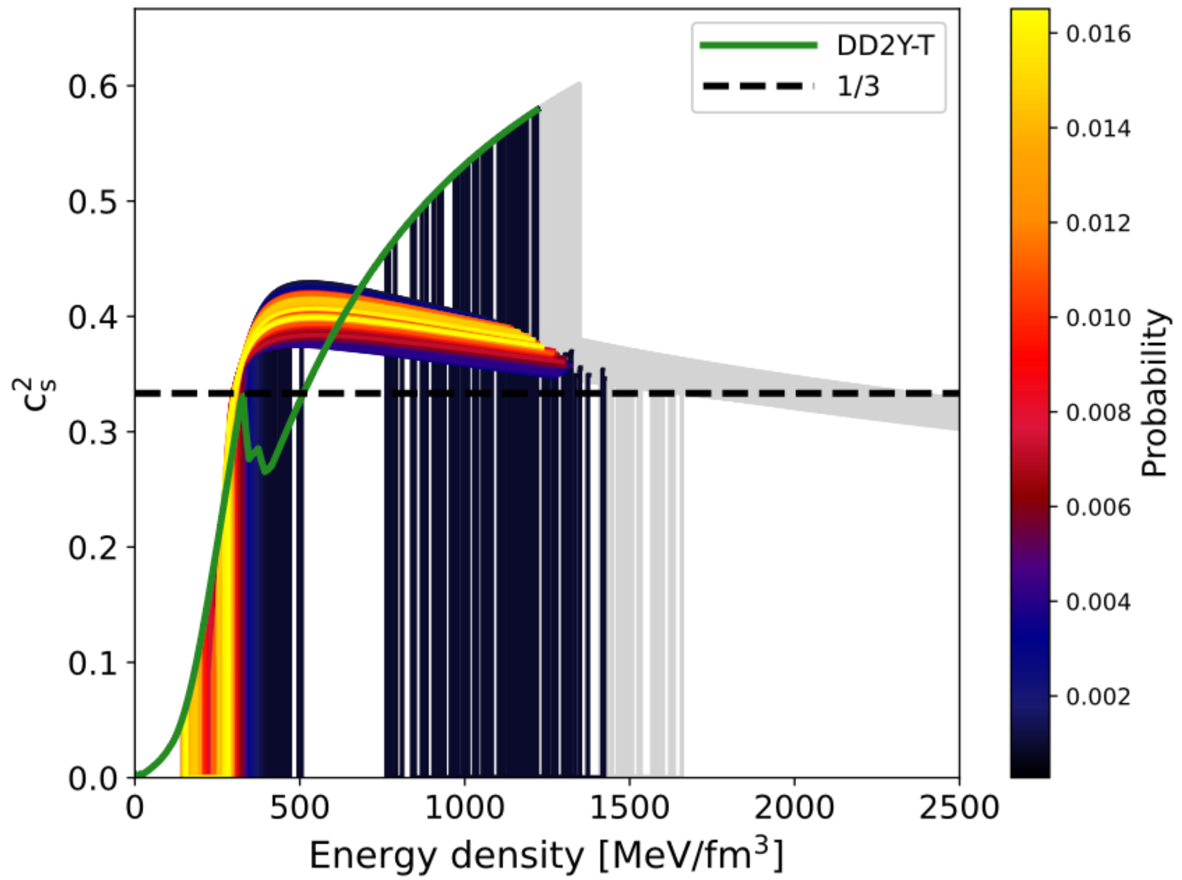

NSs with CFL quark cores are one-two orders of magnitude more favourable than purely hadronic ones.




Ayriyan, Ivanytskyi, Blaschke. arXiv:2509.02554 [nucl-th]

# BA RESULTS (2)




Ayriyan, Ivanytskyi, Blaschke. arXiv:2509.02554 [nucl-th]

# Summary of Results

- Milt-messenger astronomy data supports color superconducting matter
- Stable hybrid stars found only in a limited region of the EOS parameter space.
- Credibility regions (60% and 90%) were identified in the ($\eta_V$, $\eta_D$) parameter space measuring Lagrangian parameters.
- Favor hybrid EOS with early deconfinement
- Disfavor hybrid EOS with very high maximum masses due to insufficient compactness from tidal deformability constraints.
- Color superconductivity is essential to achieve small radii (R1.4~12 km) consistent with HESS J1731.
- Strong vector interactions are required only when imposing the high-mass constraint from PSR J0952-0607.
- The presence of a quark core in neutron stars is one to two orders of magnitude more probable than their absence.

**Ayriyan, Blaschke, Carlomagno, Contrera, Grunfeld. Universe 11(5), 141 (2025)**

**Ayriyan, Ivanytskyi, Blaschke. arXiv:2509.02554 [nucl-th]**