Mesoscopic radiation reaction in scattering of light by active boundary surfaces.

Varró Sándor
ELI-ALPS (Attosecond Light Pulse Source), Research Institute, Szeged, Hungary.

Outline.

Introduction. Motivations.

Scattering on a single active boundary.
Rectangular waves.

[Haar’s orthogonal function system and wavelets.]

Scattering on two parallel current sheets.
Delay differential equations, singularly pertubed systems.

[Some similarities with classical radiation reaction problems.]

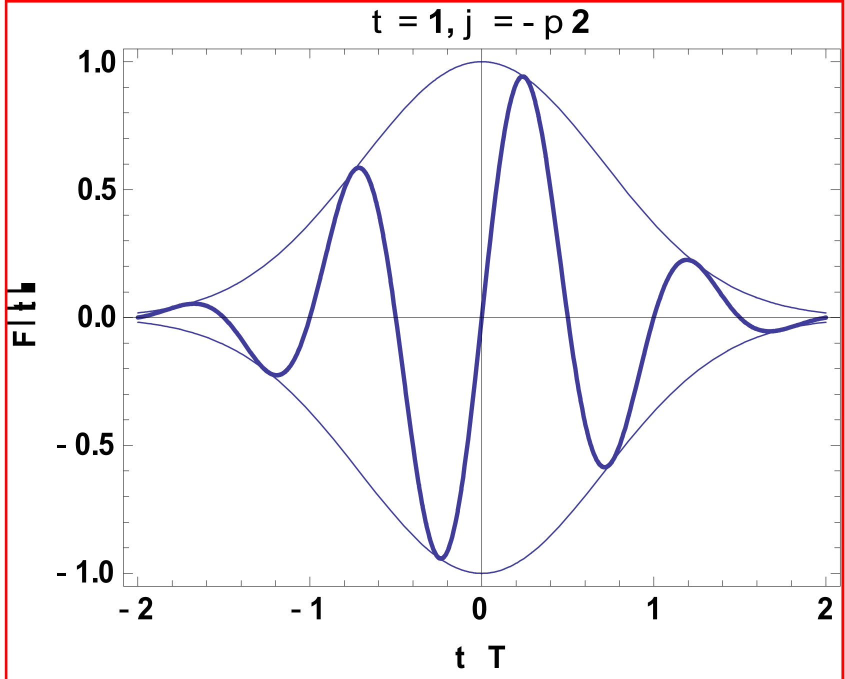
Introducion. Motivations. Strong-field pulse-shape multiphoton effects.

$$\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{e}F_0 f(t - \boldsymbol{n}\cdot\boldsymbol{r}/c)\cos(\omega_0 t - \boldsymbol{k}\cdot\boldsymbol{r} + \varphi)$$

$$\frac{\upsilon_{osc}}{c} = \frac{eF_0}{mc\omega_0} \approx 10^{-9}\sqrt{I}\cdot\lambda_0$$

$$F(t) = F_0 \exp(-t^2/2\tau^2)\cos(\omega_0 t + \varphi)$$

$$F_0 \approx 27\sqrt{I}$$



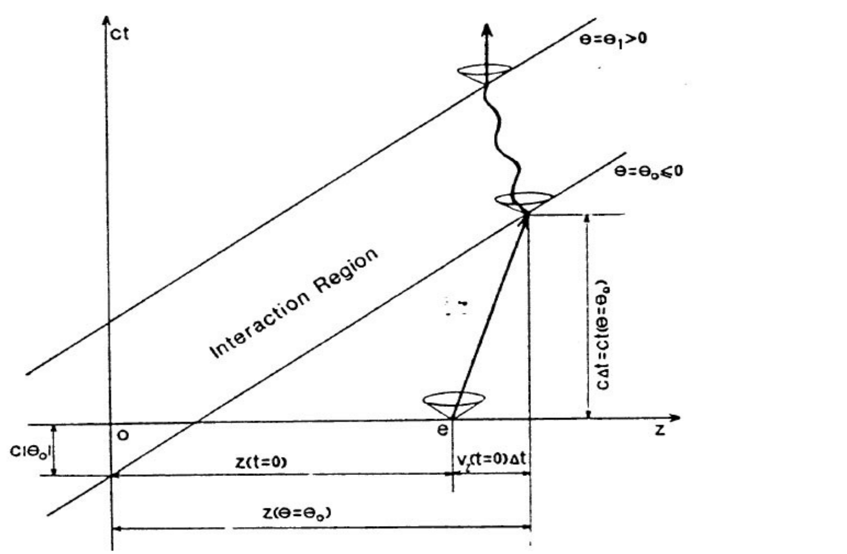
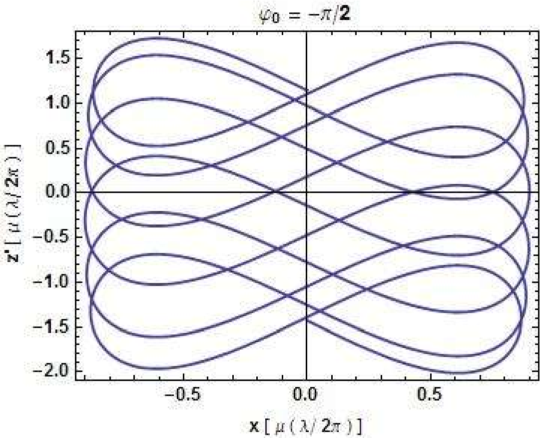
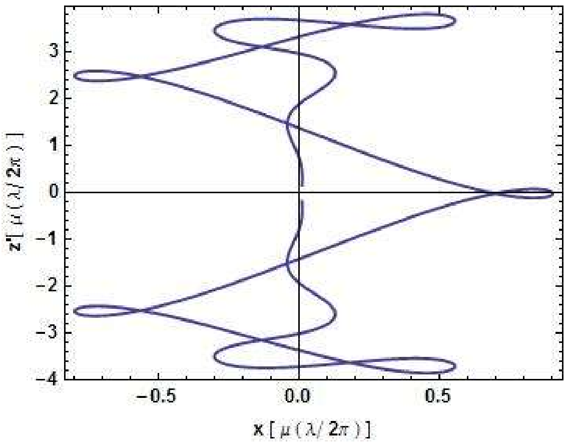


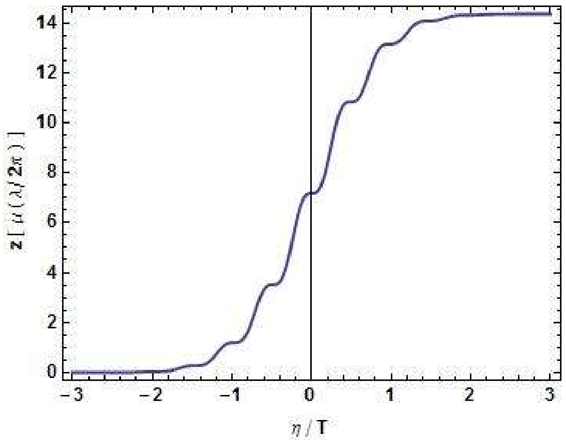
Fig. 1. Interrelation between the initial conditions for the electron, moving in the Redmond configuration of the two external fields, expressed on the one hand as a function of the light-cone coordinate

In very high-order multiphoton processes (optical tunneling, high-harmonic generation; attosecond pulses, laser acceleration of charged particles) the carrier-envelope phase difference plays a crucial role. Its stability, and measurement (diagnostics) is important.

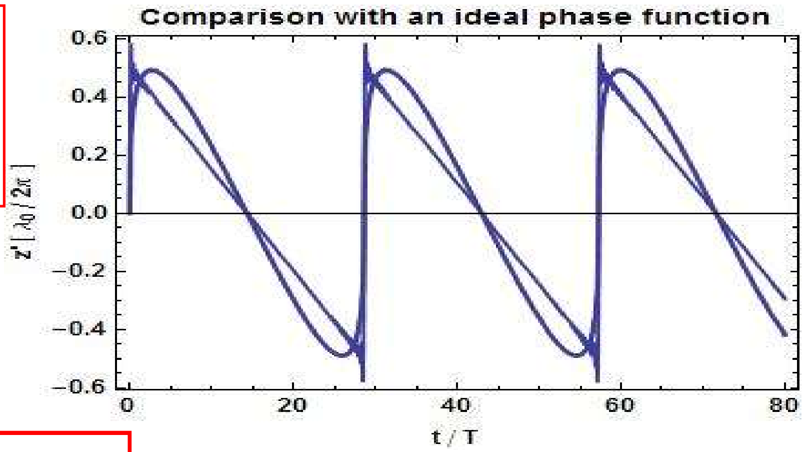
Extreme radiation, from terahertz to xuv from ultrarelativistic motion:







With these examples for the trajectories of an electron moving in a high-intensity laser field, we illustrate the very high non-linearity, which, of course, manifest itself in „extreme radiation".



$$\beta = \frac{\mu_0^2/4}{1+\mu_0^2/4}$$

$$\omega = \omega_0(1-\beta)$$

$$z'(t) = (\lambda_0/2\pi)\sum_k [J_k(k\beta)/k]\sin 2k\omega\cdot t$$

See eg.g: Varró S; Intensity effects and absolute phase effects in nonlinear laser-matter interactions; In *Laser Pulse Phenomena and Applications* (Ed. Duarte F J); Chapter 12, pp 243-266 . Lecture Notes (in Hungarian) Theor. Physics . SZTE (2012).

One-sidedness or two-sidedness of X-rays? Sommerfeld's current sheet model [1915].

1915. № 6.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 46.

1. *Über das Spektrum der Röntgenstrahlung;*
von A. Sommerfeld.

§ 5. Die Formänderung eines einseitigen Impulses beim Durchgang durch eine dünne Schicht freier Elektronen.

The change of shape of a one-sided pulse by passing through a thin layer of free electrons.

Fall.

Eine dünne Schicht des Raumes (Fig. 3) sei mit Elektronen erfüllt; sie habe die Dicke h und erstrecke sich senkrecht zur Einfallsrichtung des Röntgenimpulses, der x-Achse, ins Unendliche. Diese Schicht heiße das Gebiet 2 des Raumes und umgebe die Ebene $x = 0$, in welche sie für $h = 0$ übergeht. Die Gebiete 1 und 3 seien reines Vakuum und nur durch 2 begrenzt. Der Röntgenimpuls sei ein „ebener" Impuls, d. h. die Feldgrößen desselben sollen außer von t nur von x abhängen.

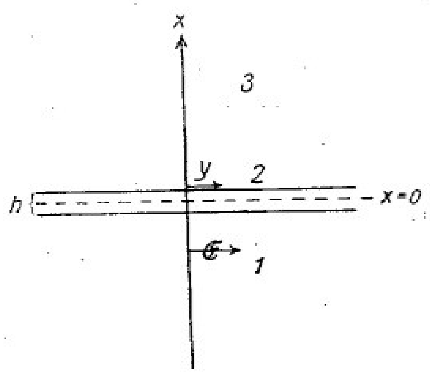
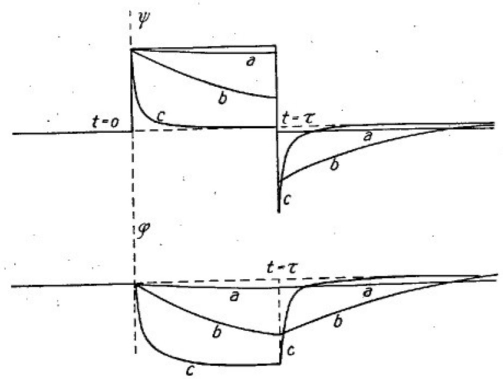


Fig. 3.

In Fig. 4 stellen wir ψ, d. h. den zeitlichen Verlauf der durchgelassenen Störung, für verschiedene Werte von α dar. Da α eine reziproke Zeit ist, betrachten wir die drei Fälle

$$\alpha\tau \ll 1, \quad \alpha\tau = 1, \quad \alpha\tau \gg 1.$$



Damping removes small frequencies by transmission:

$$J = \left(\frac{2E}{\pi}\right)^2 \frac{\sin^2 n\tau/2}{\alpha^2 + n^2}. \quad (24)$$

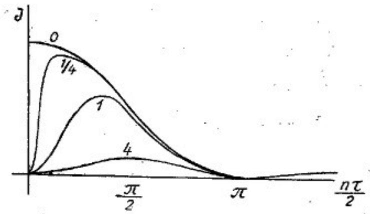


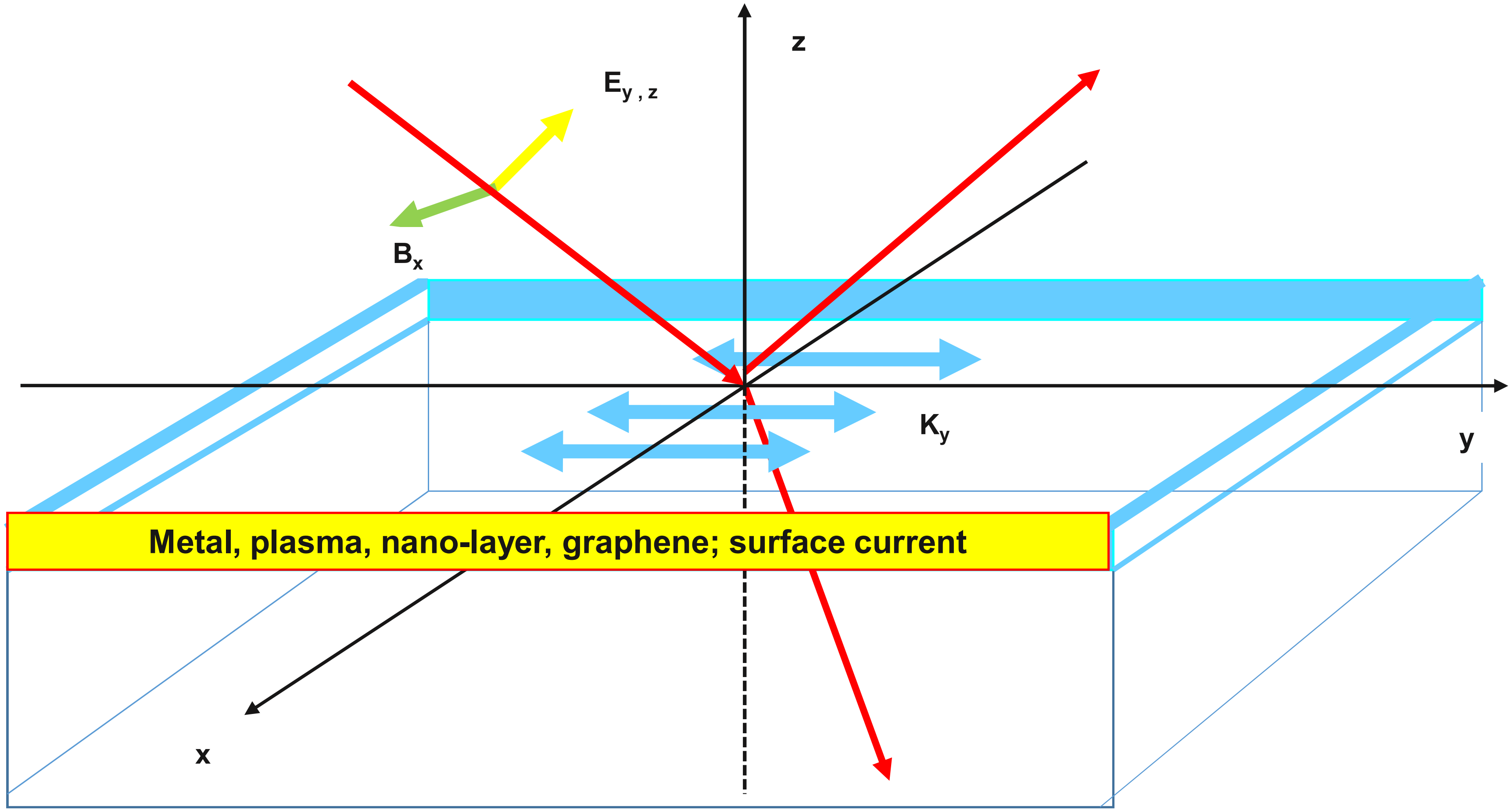
Fig. 6.

Das Ergebnis ist außerordentlich einfach und durchsichtig. Der ganze Einfluß der Umwandlung unseres Impulses besteht darin, daß im Nenner zu n^2 die Größe α^2 hinzutritt. In der Tat ergibt sich mit $\alpha^2 = 0$ wieder das ursprüngliche Spektrum des einseitigen Impulses, wie es in § 2 unter a) entwickelt wurde, nur daß dort $E = 1$ gesetzt wurde. Das

Scattering by a single layer (2003-). Treated by Sommerfeld's model.
a) Generalization to oblique incidence.
b) Generalization to dielectric-metal-dielectric structure.
c) Generalization to relativistic kinematics of te electrons.
d) Generalization to relativistic massless charges (graphene).

Modification of the Fresnel formulae.
Frozen-in Electromagnetic Pulse [„EMP"].
Generation of rectangular optical waves;
A brief sketch on Haar's orthogonal function system and wavelets.

Scattering geometry. A p-polarized wave impinges on a layer at the plane z=0. For a very thin layer the matching equation leads naturally to the appearance of „collective damping" or „mesocopic radiation reaction" in the equation of motion of the surface current.



See e.g.: S. V., Linear and nonlinear absolute phase effects in interactions of ulrashort laser pulses with a metal nano-layer or with a thin plasma layer. *Laser and Particle Beams* 25, 379–390 (2007).

Matching conditions for the fields at the „active boundary surfaces".

$$\boldsymbol{E}_1 = \boldsymbol{E}_{inc} + \boldsymbol{E}_{refl}$$

$$\boldsymbol{E}_3 = \boldsymbol{E}_{trans}$$

(Similarly for B magnetic induction)

$$[E_{1y} - E_{3y}]_{z=0} = 0, \quad [B_{1x} - B_{3x}]_{z=0} = (4\pi / c) K_{2y}$$

Three unknown scalars.: reflected: f_1, transmitted: g_3, surface current: K_2

$$f_1(t') = (1/(c_1 + c_3))[(c_3 - c_1)F(t') - c_3(4\pi / c)K_{2y}(t')]$$

$$c_3 g_3(t') = E_{3y} = (2c_1c_3 / (c_1 + c_3))[F(t') - (2\pi / c)K_{2y}(t')]$$

$$c_{1,3} \equiv cos\,\theta_{1,3} / n_{1,3}$$

$$K_{2y} = e\eta\delta'_y$$

(Displacement)' of charge element

Surface charge density

[Relativistic equation of motion for the displacement of the charge element.]

$$d^2\delta_\mu / d\tau^2 = (e / mc) F^{(total)}_{\mu\nu} d\delta^\nu / d\tau$$

Equation of motion for the displacement of the current elements (electrons).

$$t' \equiv t - \frac{1}{c} y \sin\theta \;\rightarrow\; \eta \equiv t' - \frac{1}{c}\vec{n}\cdot\vec{\delta}(t')$$

This quantity turns out to be the proper time.

$$\vec{E} = \vec{\varepsilon} g_3 \;,\; \vec{B} = \vec{n}\times\vec{E} \;,\; \vec{\varepsilon} = (0,\cos\theta,\sin\theta) \;,\; \vec{n} = (0,\sin\theta,-\cos\theta)$$

$$g_3 = F - \frac{m}{e}\frac{\Gamma}{\gamma}\dot{\delta}_y \quad and \quad f_1 = -\frac{m}{e}\frac{\Gamma}{\gamma}\dot{\delta}_y \;,\; where\; \dot{\delta}_y \equiv \frac{d\delta_y}{d\eta}$$

$$\dot{\delta}_y = \dot{\delta}_\perp \cos\theta + (\dot{\delta}_\perp^2/2c)\sin\theta \;,\; where \quad \delta_\perp \equiv \vec{\varepsilon}\cdot\vec{\delta}$$

$$\ddot{\delta}_\perp / c = \frac{e}{mc}F(\eta) - \Gamma\frac{(\dot{\delta}_\perp/c)\cos\theta + (\dot{\delta}_\perp^2/2c^2)\sin\theta}{(1+\dot{\delta}_\perp^2/2c^2)}$$

Relativistic

$$d^2\delta_\perp / d\eta^2 = \frac{e}{m}F(\eta) - (\Gamma\cos\theta)(d\delta_\perp/d\eta)$$

Non-Relativistic limit

Summary and simple illustrations. I. The physical meaning of the damping.

$$K_{y2} = e(d\delta_{y2}/dt)l_2 n_{e2},\ \ (4\pi/2c)K_{y2} = (m/e)\Gamma_2(d\delta_{y2}/dt)$$

$$\Gamma_2 \equiv 2\pi(e^2/mc)l_2 n_{e2},\ \ \Gamma_2 = (\omega_{p2}/\omega_0)^2(\pi l_2/\lambda_0)\omega_0$$

Damping factor in two equivalent forms.

This is the structure of the (non-rel.) equation, where F represents the incoming radiation. Γ represents the „collective damping", or, in other words the „mesoscopic radiation reaction". E.g.:

$$\frac{d\upsilon}{dt} = \frac{e}{m}F - \Gamma\upsilon$$

$$F(t) \approx F_0 \exp(-t^2/2\tau^2)\cos(\omega_0 t + \varphi)$$

This damping is not a result of the back-reaction of the self-field of an isolated electron. Its appearance comes from the „counter-field" of the other (closely or remotely spaced) electrons. That is why we should better call it „collective damping", however, the term „mesocopic radiation reaction" can also do.

Summary and simple illustrations. II. Fourier components of the reflected and and transmitted fields. The last equation shows the flux-conservation.

$$f_1(\omega) = -\frac{F(\omega)}{\gamma - i(v/b)}\left[\gamma + \frac{c_3 - c_1}{c_3 + c_1} i(v/b)\right]$$

$$v \equiv \omega / \omega_0$$

$$\gamma \equiv \Gamma_2 / \omega_0$$

$$g_3(\omega) = -\frac{2c_1}{c_1 + c_3}\frac{i(v/b)F(\omega)}{\gamma - i(v/b)}$$

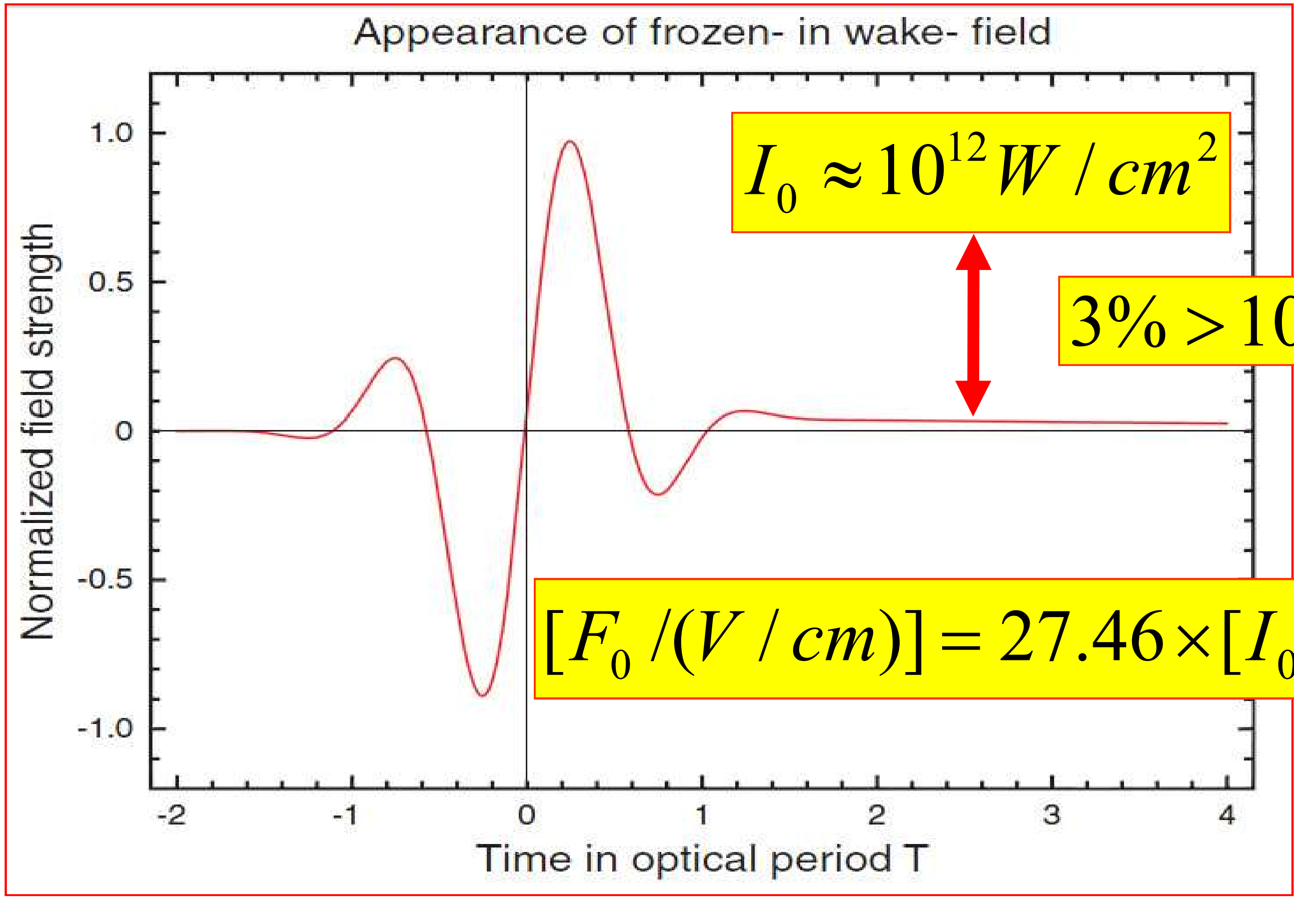
$$b \equiv 2c_1c_3/(c_1 + c_3)$$

$$c_1 = \cos\theta_1 / n_1$$

$$c_1 |f_1(\omega)|^2 + c_3 |g_3(\omega)|^2 = c_1 |F(\omega)|^2$$

The Brewster angle is defined by $c_3=c_1$, i.e. $\cos\theta_3 / n_3 = \cos\theta_1 / n_1$, at this angle of incidencel (for p-polarization) there is no reflexion; $f_1 = 0$, according to the usual Fresnel formula. In our case (owing to the „active boundary"; simply, the surface current) f_1 is nonzero.

Summary and simple illustrations. III. An earlier example: Quasi-static [~ rectified] electromagnetic pulse from reflection on an evaporated gold nano-layer.



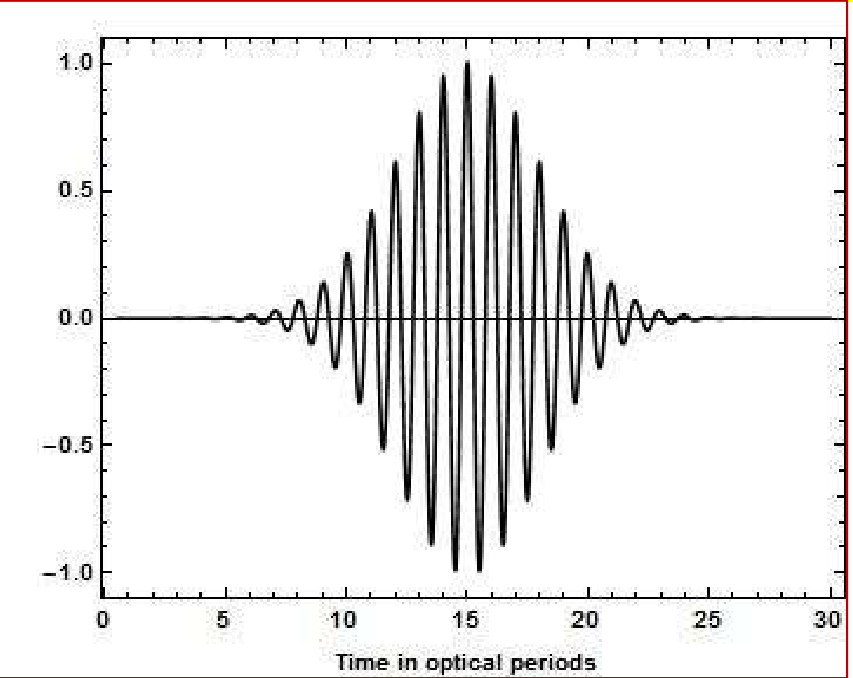
$$I_0 \approx 10^{12} W / cm^2$$

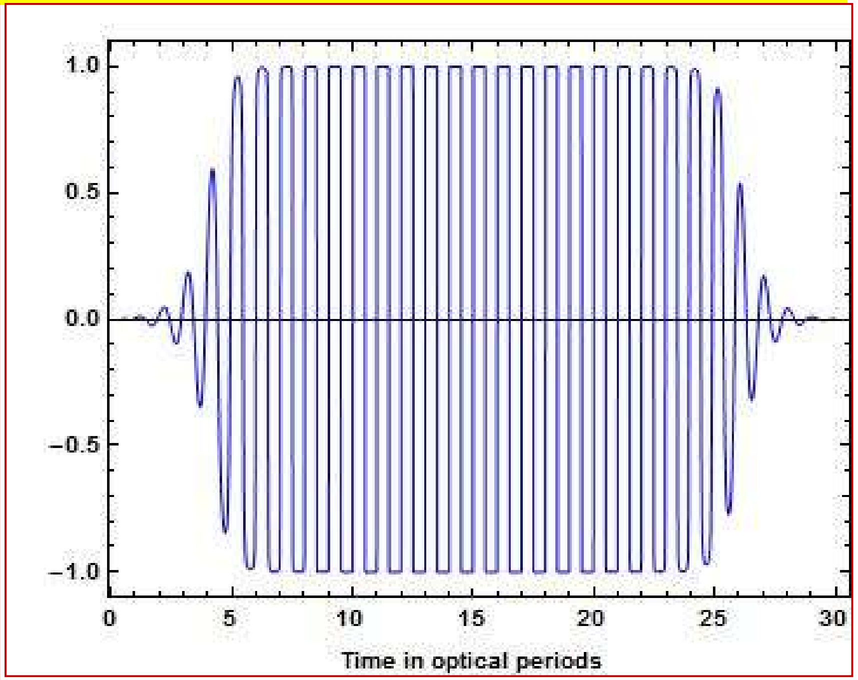
$$3\% > 100000 V / cm$$

$$[F_0 /(V / cm)] = 27.46 \times [I_0 /(W / cm^2)]^{1/2}$$

S. V., Linear and nonlinear absolute phase effects in interactions of ulrashort laser pulses with a metal nano-layer or with a thin plasma layer. *Laser and Particle Beams* 25, 379–390 (2007).

Summary and simple illustrations. IV. Generation of rectangular sequence of reflected optical pulses at Brewster angle. In this case the usual Fresnel formula gives zero. The radiation field stems from the velocity term. [Ebben az esetben a szokásos Fresnel-formula 0-t ad. Nálunk itt a forrás SEBESSÉG, NEM GYORSULÁS!]. Rademacher function (‚telegraph signal').





$$E^{sc} \sim f_1(t') = (1/(c_1 + c_3))[(c_3 - c_1)F(t') - c_3(4\pi/c)K_{2y}(t')]$$

$$A_\mu^{scatt}(y) = 4\pi e \int d\tau G(y - x(\tau)) u_\mu(\tau)$$

usual

~ Velocity of the charge element

„Ultrarelativistic kinematics” (zero-mass charge), violent nonlinearity, and ‘relativistic clipping’ (graphene).

$$\frac{dp_y(t')}{dt'} = \frac{2c_1c_3}{c_1+c_3}\left[eF(t') - \frac{2\pi e^2}{c}\eta \mathrm{v}_y(t')\right]$$

EOM, where t’ = retarded time at the surface

$$E(\boldsymbol{p}) = \mathrm{v}_\mathrm{F}\,|\,\boldsymbol{p}\,|$$

$$\mathrm{v}_y = \frac{\partial E(\boldsymbol{p})}{\partial p_y} = \frac{\mathrm{v}_\mathrm{F}}{|\,\boldsymbol{p}\,|}p_y$$

E-p dispersion relation, v-p connection

$$\pi\frac{\mathrm{v}_\mathrm{F}}{c}\cdot\frac{2mc^2}{\hbar\omega_0}\cdot\mu_0$$

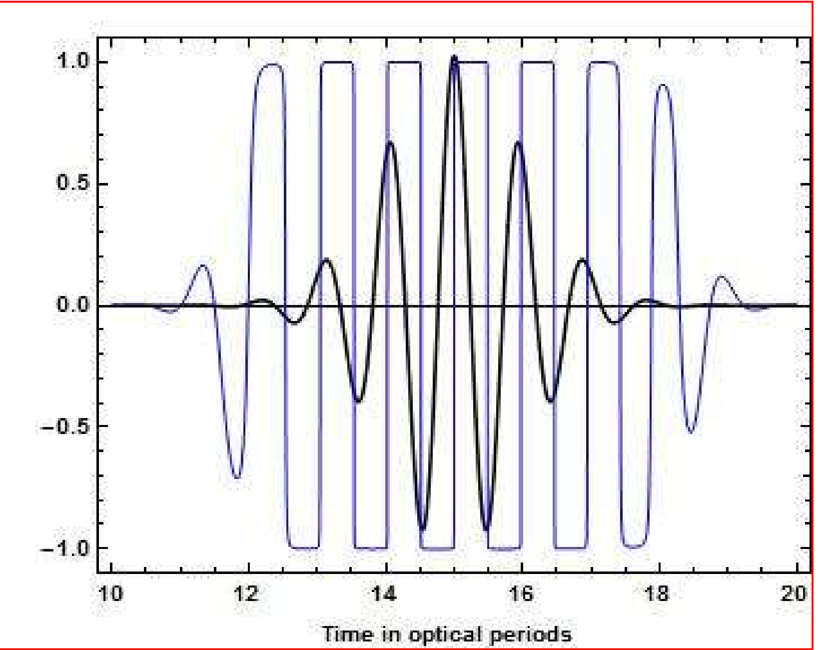
$$\frac{\upsilon_{osc}}{c} = \mu_0 \equiv \frac{eF_0}{mc\omega_0} = 8.5\times10^{-10}\sqrt{I_0}\cdot\lambda_0$$

The effective intensity parameter 10^4 times larger than the usual μ_0 parameter ‘dimensionless vector potential’ for ‘ordinary’ electrons in optical fields.

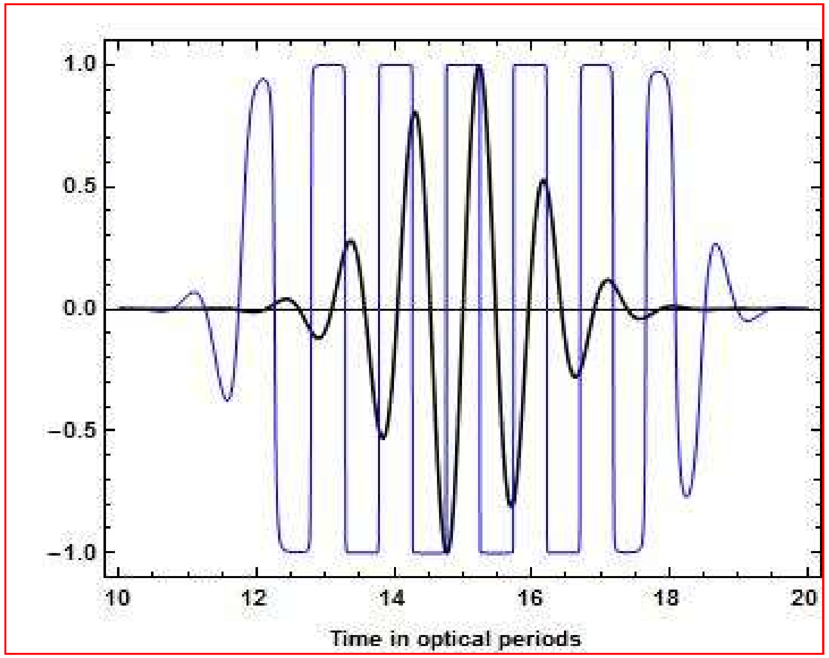
The velocity ‘saturates’ along the polarization direction, the signal is ‘clipped’.

S. V., Graphene-based carrier-envelope phase difference meter. In *AIP* Conf. *Proceedings 1462, 128-131 (2012)*.
Varró S, Generation of rectangular optical waves by relativistic clipping. *Laser Physics* 23 (2013) 056006 (6pp)

Generation of rectangular sequence of reflected optical pulses at Brewster angle. 'Relativistic clipping.' The rectangular structure follows the CE-phase.



Cosine incoming pulse.

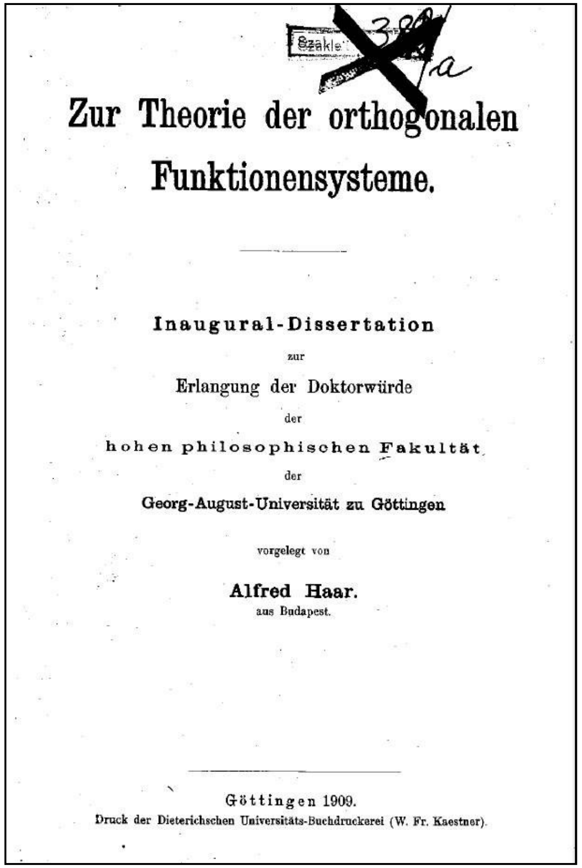


Sine incoming pulse.

S. V., Graphene-based carrier-envelope phase difference meter. In *AIP* Conf. *Proceedings 1462, 128-131 (2012)*.
[*Light at Extreme Intensities* (LEI-2011, 14-18. November 2011. Szeged, Hungary), Book of Abs. p. 43 (2011).]

Varró S, Generation of rectangular optical waves by relativistic clipping. *Laser Physics* 23 (2013) 056006 (6pp)

On the system of orthogonal functions, constructed by Alfréd Haar in 1909. I.



Zur Theorie der orthogonalen Funktionensysteme.

Inaugural-Dissertation

zur

Erlangung der Doktorwürde

der

hohen philosophischen Fakultät

der

Georg-August-Universität zu Göttingen

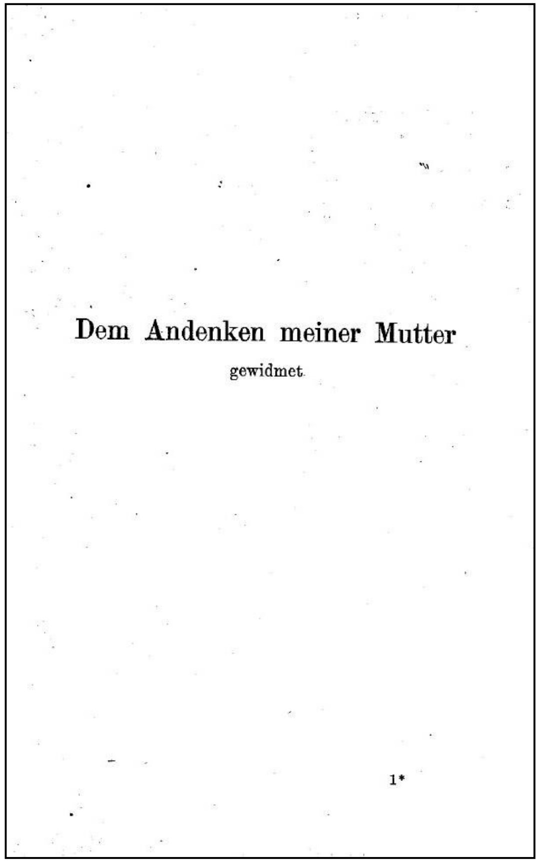
vorgelegt von

Alfred Haar.

aus Budapest.

Göttingen 1909.

Druck der Dieterichschen Universitäts-Buchdruckerei (W. Fr. Kaestner).



Dem Andenken meiner Mutter

gewidmet.

1*

„Is there, at all, an orthogonal system of functions, which is so constructed that *every continuous function can be expanded into a uniformly convergent Fourier-like series with respect to this system*? In Chapter three we shall get aquaintance of a whole class of orthogonal systems which have this properties.”

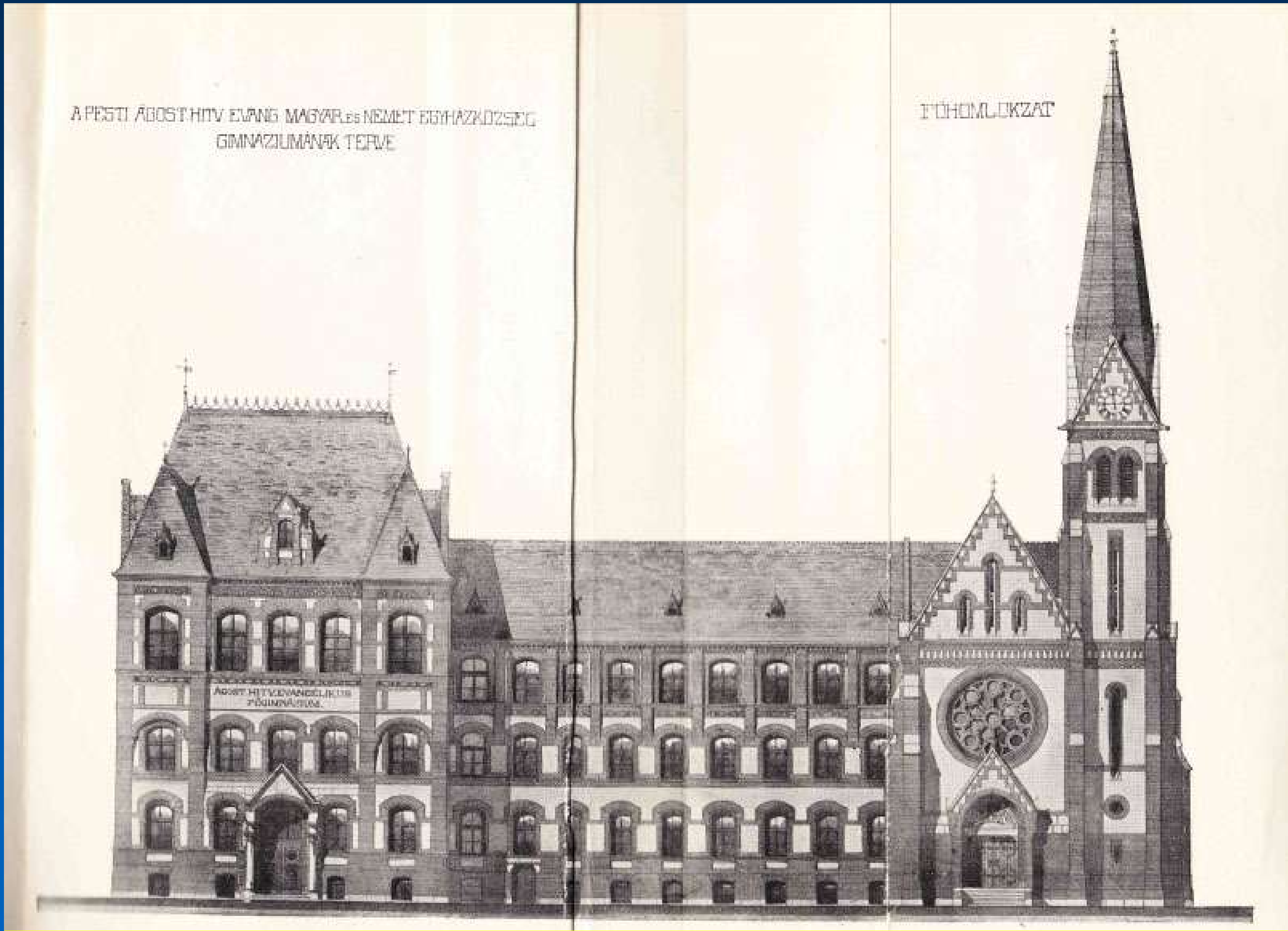
[3] Haar A, Zur Theorie der orthogonalen Funktionensysteme (Inaugural Dissertation zur Erlangung der Doktorwürde der hohen philosophischen Fakultät der Georg-August-Universität zu Göttingen, vorgelegt von Alfréd Haar aus Budapest). 1909, 1-49; [3a] Haar A, Zur Theorie… (Erste Mitteilung). Mathematische Annalen 69, 331-371 (1910); [3b] Haar A, Zur Theorie… (Zweite Mitteilung). Mathematische Annalen 71, 38-53 (1911).

Haar Alfréd. 1885. október 11., Budapest --- 1933. március 16., Szeged.

„Én Haar Alfréd, 1885. október 11-én születtem Budapesten, Haar Ignác földbirtokos és neje, szül. Fuchs Emma fiaként. Szülővárosomban az evangélikus gimnáziumba jártam, amelyet 1903-ban az érettségi bizonyítvánnyal hagytam el. 1904 húsvétja óta a budapesti egyetemen matematikai, fizikai és csillagászati előadásokat hallgattam. 1905 őszén a göttingai egyetemre iratkoztam be.

Budapesten Beke, báró Eötvös, Fröchlich, Kürschák, Rados, Scholtz, Göttingában Carathéodory, Hilbert, Klein, Minkowski, Prandtl, Runge, Schwarzschild, Voigt, Zermelo előadásain ill. szemináriumain vettem részt. Mindezeket az urakat el nem múló hálám illeti. Különösképpen mélyen lekötelezve érzem magam azonban Hilbert titkos tanácsos úrnak azért a sokoldalú ösztönzésért és sokrétű tanításért, amelyet egész göttingai tanulmányi időm alatt előadásai és személyes érintkezés révén nyújtott; állandó ösztönző érdeklődéséért legmélyebb hálámat fejezem ki.”

Haar A, *Összegyűjtött Munkái*. Sajtó alá rendezte Szőkefalvi-Nagy Béla (Akadémia Kiadó, Budapest, 1959). Bevezetés, Haar Alfréd rövid életrajza.



A Budapesti Ágost. Hitv. Evang. FŐGYMNASIUM ÉRTESÍTŐJE Az 1903/1904-iki iskolai évről.
Közzéteszi GÓBI IMRE, E. I. Igazgató (Budapest, Franklin-Társulat Könyvnyomdája, 1904)

Haar Alfréd. 1885. október 11., Budapest --- 1933. március 16., Szeged.

„Ich, Alfred Haar, wurde als Sohn des Gutbesitzers Ignatz Haar und seiner Ehegattin Emma geb. Fuchs am 11. Oktober 1885. in Budapest geboren. In meiner Heimatsstadt besuchte ich das evangelische Gymnasium, das ich 1903 mit dem Zeugnis der Reife verließ. Von Ostern 1904 hörte ich an der Universität in Budapest Vorlesungen über Mathematik, Physik und Astronomie. Im Herbst 1905 bezog ich die Universität in Göttingen.

Teilgenommen habe ich an Vorlesungen resp. Seminaren der Herren Beke, Baron Eötvös, Fröchlich, Kürschák, Rados, Scholtz, in Budapest. Charathéodory, Hilbert, Klein, Minkowski, Prandtl, Runge, Schwarzschild, Voigt, Zermelo, in Göttingen.

Allen diesen Herren gilt mein dauernder Dank. Insbesondere fühle ich mich aber Herrn Geheimrat Hilbert für die vielseitige Anregung und mannigfaltige Belehrung, die er mir während meiner ganzen Göttinger Studienzeit in Vorlesungen und im persönlichen Verkehr zuteilwerden ließ tief verpfichtet; für seine stetes förderndes Interesse spreche ich ihm meinen innigsten Dank aus.”

[1] Haar A, *Gesammelte Arbeiten*. Herausgegeben von Béla Szőkefalvi-Nagy (Akadémia Kiadó, Budapest, 1959). Einleitung, Kurzer Lebenslauf von Alfred Haar.

From left to right: Alfréd Haar, Franz Hilbert, Hermann Minkowski, unknown, Käthe Hilbert, David Hilbert, Ernst Hellinger.

On the system of orthogonal functions, constructed by Alfréd Haar in 1909. II.

Das orthogonale Funktionensystem χ.

Das vollständige orthogonale Funktionensystem χ, den einfachsten Repräsentanten jener Klasse von Orthogonalsystemen, definieren wir wie folgt:

Es sei $\chi_0(s) = 1$ im ganzen Intervall $[0, 1]$ einschließlich der Grenzen; sodann sei:

$$\begin{aligned} \chi_1(s) &= 1 \quad \text{für} \quad 0 \leqq s < \tfrac{1}{2}, \\ &= -1 \quad \text{für} \quad \tfrac{1}{2} < s \leqq 1 . \end{aligned}$$

Wir setzen ferner:

$$\begin{aligned} \chi_2^{(1)}(s) &= \sqrt{2} \quad \text{und} \quad \chi_2^{(2)}(s) = 0 \quad \text{für} \quad 0 \leqq s < \tfrac{1}{4}, \\ &= -\sqrt{2} \qquad\qquad\quad\; = 0 \quad \text{„} \quad \tfrac{1}{4} < s < \tfrac{1}{2}, \\ &= 0 \qquad\qquad\qquad\; = \sqrt{2} \quad \text{„} \quad \tfrac{1}{2} < s < \tfrac{3}{4}, \\ &= 0 \qquad\qquad\qquad\; = -\sqrt{2} \quad \text{„} \quad \tfrac{3}{4} < s \leqq 1 . \end{aligned}$$

Auf diese Weise fahren wir fort; allgemein definieren wir die Funktionen unseres Systems folgendermaßen: Wir teilen das Intervall $[0, 1]$ in 2^n gleiche Teile und bezeichnen diese Teilintervalle der Reihe nach mit $i_n^{(1)}, i_n^{(2)}, \cdots, i_n^{(2^n)}$. Wir setzen nun:

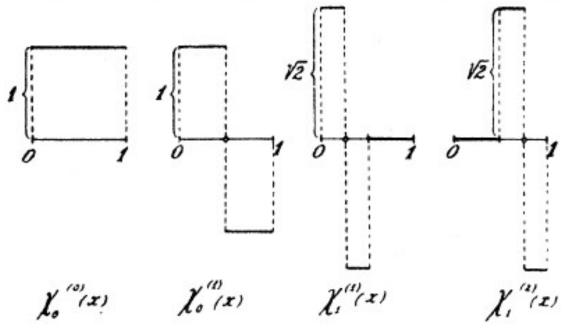
$$\begin{aligned} \chi_n^{(k)} &= 0 \quad \text{innerhalb der Intervalle} \quad i_n^{(1)}, i_n^{(2)}, \cdots, i_n^{(2k-2)}; \\ &= \sqrt{2^{n-1}} \quad \text{innerhalb des Intervalles} \quad i_n^{(2k-1)}; \\ &= -\sqrt{2^{n-1}} \quad \text{innerhalb des Intervalles} \quad i_n^{(2k)}; \\ &= 0 \quad \text{innerhalb der Intervalle} \quad i_n^{(2k+1)}, \cdots, i_n^{(2^n)} \\ &\qquad (k = 1, 2, \cdots, 2^{n-1}). \end{aligned}$$

‘Haar wavelets’.
A ‘Haar – féle ortogonális függvényrendszer’

On the system of orthogonal functions, constructed by Alfréd Haar in 1909. III.

E függvények a [0, 1] szakaszon vannak értelmezve, és így jelöljük őket:

$\chi_0^{(0)}(x) \equiv 1; \quad \chi_0^{(1)}(x); \quad \chi_1^{(1)}(x), \quad \chi_1^{(2)}(x); \quad \ldots; \quad \chi_n^{(1)}(x), \ldots, \chi_n^{(2^n)}(x); \quad \ldots.$



37. ábra

Értelmezésük a következő:

$$\chi_0^{(1)}(x) = \begin{cases} 1, & \text{ha } 0 \leqq x < \frac{1}{2}, \\ 0, & \text{ha } x = \frac{1}{2}, \\ -1, & \text{ha } \frac{1}{2} < x \leqq 1, \end{cases}$$

$$\chi_1^{(1)}(x) = \begin{cases} \sqrt{2}, & \text{ha } 0 \leqq x < \frac{1}{4}, \\ -\sqrt{2}, & \text{ha } \frac{1}{4} < x \leqq \frac{1}{2}, \\ 0 & \text{a többi pontokban,} \end{cases} \qquad \chi_1^{(2)}(x) = \begin{cases} \sqrt{2}, & \text{ha } \frac{1}{2} \leqq x < \frac{3}{4}, \\ -\sqrt{2}, & \text{ha } \frac{3}{4} < x \leqq 1, \\ 0 & \text{a többi pontokban,} \end{cases}$$

3. 4. Haar-féle ortogonális rendszer

További példaként ortogonális függvényrendszerre ismerjük meg a több szempontból is nevezetes Haar-féle ortogonális rendszert, melyet HAAR ALFRÉD*) doktori disszertációjában közölt, 1909-ben.

*) HAAR ALFRÉD (1885 – 1933) a kolozsvári, majd a szegedi egyetem tanára volt. Egyike volt azoknak a magyar matematikusoknak, akiknek művei nagy hatást gyakoroltak a matematika legújabb fejlődésére. Különösen fontos eredménye az ún. „Haar-féle mérték” felfedezése.

Szőkefalvi-Nagy Béla, Valós Függvények és Függvénysorok. Második, Átdolgozott Kiadás (Tankönyvkiadó, Bp., 1961)

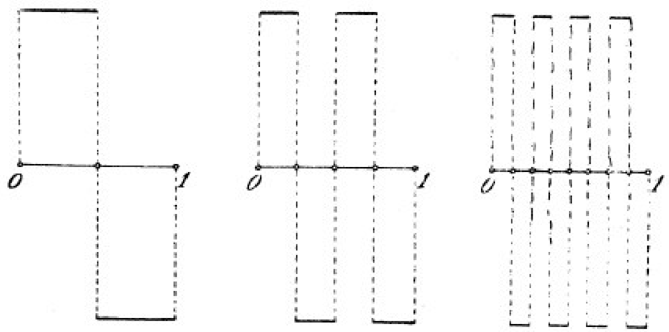
On the system of orthogonal functions, constructed by Alfréd Haar in 1909. IV.

3. 5. Rademacher-féle rendszer

Különösen bizonyos valószínűségszámítási alkalmazásai miatt bír jelentőséggel a következő, Rademacher-féle ortogonális rendszer a [0, 1] intervallumon:

$$r_0(x) = 1, \quad r_k(x) = \operatorname{sgn}(\sin 2^k \pi x)\,^{*}) \qquad (k = 1, 2, \ldots).$$

Ha tehát a [0, 1] intervallumot 2^k egyenlő részre osztjuk, akkor $r_k(x)$ ezekben felváltva a +1 és a −1 értékeket veszi fel, a 0, 1 végpontokban és az osztópontokban pedig az értéke 0 ($k \geqq 1$).



$r_1(x)$ $r_2(x)$ $r_3(x)$

38. ábra

Ez a rendszer ortogonális. Legyen $m > n \geqq 0$. Ekkor minden olyan I szakasz, amely $r_n(x)$ két szomszédos ugrási helye közé esik, és amelyen tehát $r_n(x)$ állandóan +1 vagy −1, az $r_m(x)$ függvénynek páros számú (pontosan 2^{m-n}) konstansintervallumát tartalmazza: ezek felén az $r_m(x)$ értéke +1, felén pedig −1, és így

$$r_n(x) = sign[\sin(2^n \pi x)]$$

The Rademacher system is orthogonal, but not complete.

On the system of orthogonal functions, constructed by Alfréd Haar in 1909. V.

The n-th Rademacher function:

$$r_n(x) = sign[\sin(2^n \pi x)]$$

The Walsh sytem $\{w_n(x)\}$ can be derived in the following way.

$$w_0(x) = 1 \quad in \quad (0,1);$$

Let

$$n = 2^{\nu_1} + \ldots + 2^{\nu_p} \quad (n \geq 1)$$

be the dyadic representation of n. Then

$$w_n(x) = r_{\nu_1+1}(x) r_{\nu_2+1}(x) \cdots r_{\nu_p+1}(x)$$

See e.g. Paley R E A C, A remarkable series of orthogonal functions (I). *Proc. London Math. Soc.* 34, 241-264 (1931).

The Walsh sytem is an orthonormal sytem on (0,1). This system is complete (in contrast to the Rademacher system.

Fundamental Papers in Wavelet Theory

Edited by Christopher Heil and David F. Walnut

PRINCETON UNIVERSITY PRESS. PRINCETON AND OXFORD

Preface

“This volume traces the development of modern wavelet theory by collecting into one place many of the fundamental original papers in signal processing, physics, and mathematics that stimulated the rise of wavelet theory, along with many major papers in the early development of the subject. The volume is a sourcebook for some of the most significant research papers in the subject and provides a way for a researcher who understands wavelets in his or her own milieu to get a glimpse of the development of the subject from another perspective.

A key feature of this volume is the appearance for the first time of a translation from the German of the original 1910 paper of Alfred Haar in which he described the orthonormal basis that now bears his name, along with translations of six papers heretofore available only in French. We are greatly indebted to John Horváth, Robert Ryan, and Georg Zimmermann for these translations.

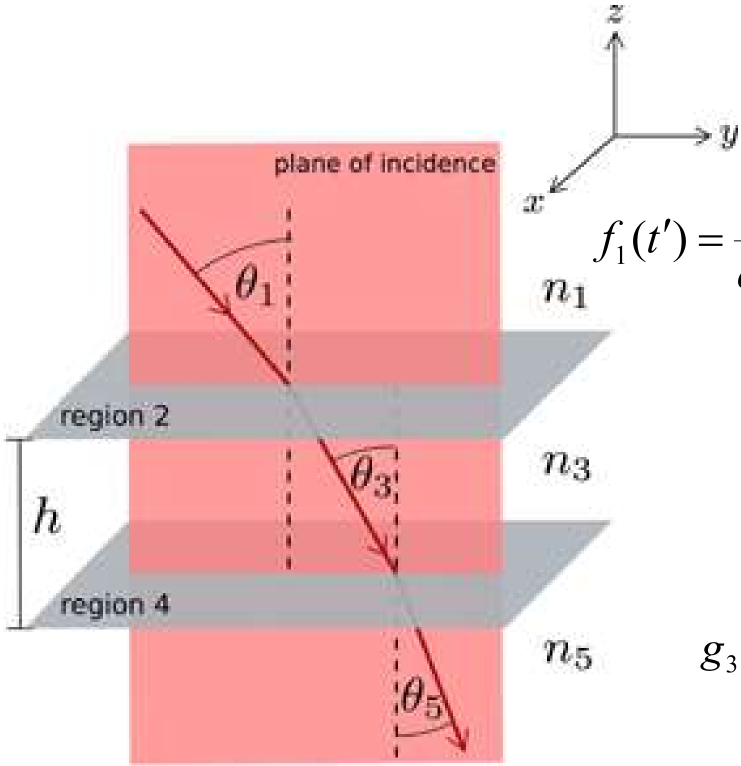
….”

Christopher Heil, Atlanta, Georgia
David Walnut, Fairfax, Virginia
July 2004”

Scattering on two parallel current sheets.
Delay differential equations, singularly pertubed systems.

[Some similarities with classical radiation reaction problems.].

The geometry of the scattering on two parallel current sheets. Delay times.



$$\Delta t_3 = n_3 h \cos\theta_3 / c = \tau$$

$$\Delta t_5 = n_5 h \cos\theta_5 / c$$

$$f_1(t') = \frac{1}{c_1 + c_3}[(c_3 - c_1)F(t') - c_3(4\pi / c)K_{y2}(t') + 2c_3 f_3(t')]$$

$$g_3(t') = \frac{2c_1}{c_1 + c_3}[F(t') - (4\pi / 2c)K_{y2}(t')] + \frac{c_1 - c_3}{c_1 + c_3} f_3(t')$$

The reflected wave f_1 and the transmitted wave g_5 can be expressed by three unknown functions: f_3 and the two velocities.

[4] Polner M, Varró S and Vörös-Kiss A, The role of the time delay in the reflection and transmission of ultrashort electromagnetic pulses on a system of parallel current sheets. Physica Scripta 94, 045205 (2019). E-print: arXiv: 1812.03857v1 [physics.class-ph].

The hybrid system of two delay differential equations for the electron motion in the layers and a recurrence relation for the scattered fields.

$$f_3(t') = \frac{c_5 - c_3}{c_5 + c_3} \cdot \frac{c_1 - c_3}{c_1 + c_3} f_3(t' - 2\Delta t_3)$$
$$+ \frac{c_5 - c_3}{c_5 + c_3} \cdot \frac{2c_1}{c_1 + c_3} [F(t' - 2\Delta t_3) - (m/e)\Gamma_2 \delta'_{y2}(t' - 2\Delta t_3)]$$
$$- \frac{2c_5}{c_5 + c_3}(m/e)\Gamma_4 \delta'_{y4}(t' - \Delta t_3)$$

Recurrence relation for f_3.

$$\delta''_{y2}(t') = \frac{2c_1c_3}{c_1 + c_3}[(e/m)F(t') - \Gamma_2 \delta'_{y2}(t') + (e/m) f_3(t')]$$

Delay differential equations for the two velocities.

$$\delta''_{y4}(t') = \frac{2c_1c_3}{c_1 + c_3}[(e/m)F(t' - \Delta t_3) - \Gamma_2 \delta'_{y2}(t' - \Delta t_3)] + \frac{c_1 - c_3}{c_1 + c_3} c_3 (e/m) f_3(t' - \Delta t_3) +$$
$$+ c_3 (e/m) f_3(t' + \Delta t_3)$$

The hybrid system of delay differential eqations. Singularly perturbed system.

$$x(t) = (\delta'_{y2}(t), \delta'_{y4}(t), f_3(t))^T$$

Hybrid system of delay differential equations:

$$\begin{pmatrix} x_1'(t) \\ x_2'(t) \\ 0 \end{pmatrix} = Ax(t) + Bx(t-\tau) + Cx(t-2\tau) + h(t)$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The determinant of this matrix is zero.

$$\frac{d}{dt}(E(\varepsilon)x(t)) = Ax(t) + Bx(t-\tau) + Cx(t-2\tau) + h(t)$$

$$E(\varepsilon) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \varepsilon \end{pmatrix}$$

$$\varepsilon \to 0^+$$

In the limit epsilon going to zero, the perturbed solution goes over to the solution of the unperturbed system.

$$\det E(\varepsilon) = \varepsilon > 0$$

The characteristic matrix Δ of the of the Laplace transform of the above system of differential equations:

The roots of the characteristic equation (44) determine the invariant manyfold and the stability properties of the system.

$$\Delta(\varepsilon, s) = sE(\varepsilon) - A - Be^{-\tau s} - Ce^{-2\tau s}$$

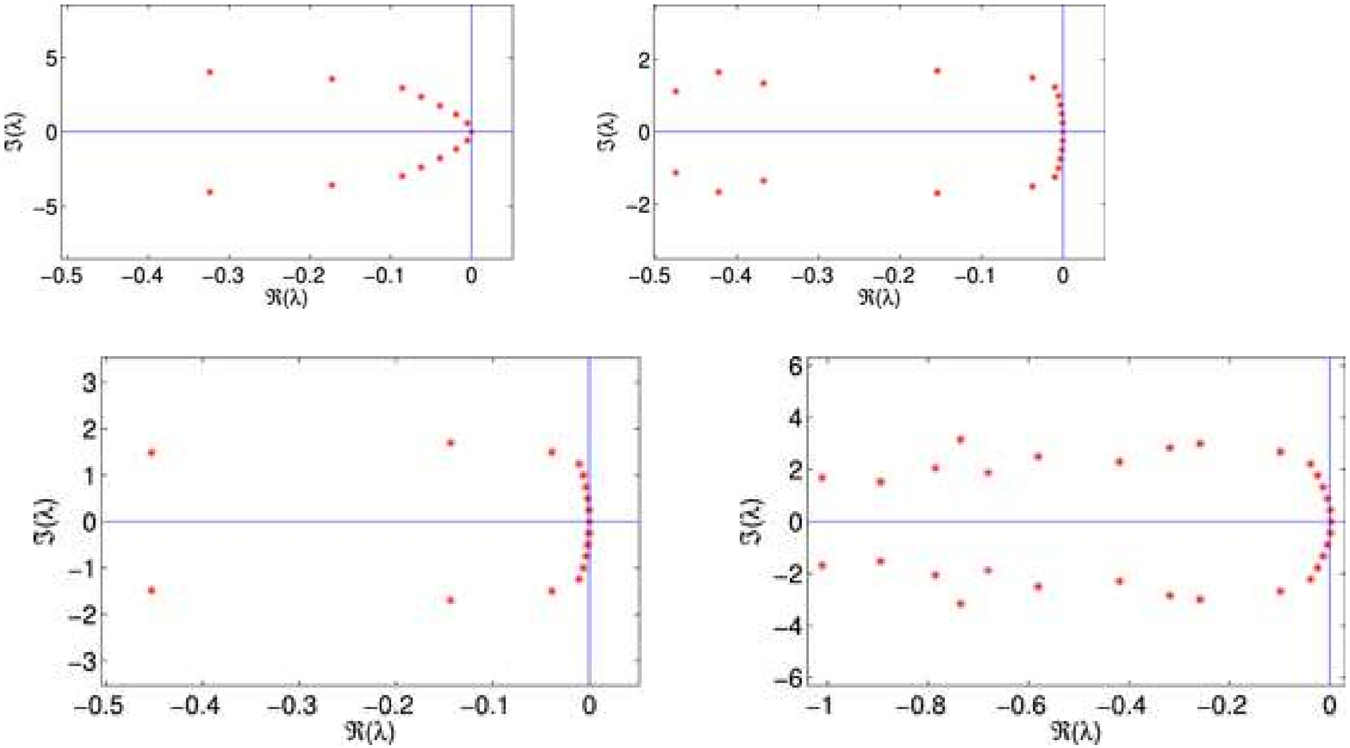
$$\det \Delta(\varepsilon, s) = 0$$

Illustrations for the roots of the characteristic equation (44).

Lemma 3.1. For the roots of the characteristic equation (44), the following hold for all $\varepsilon \geq 0$

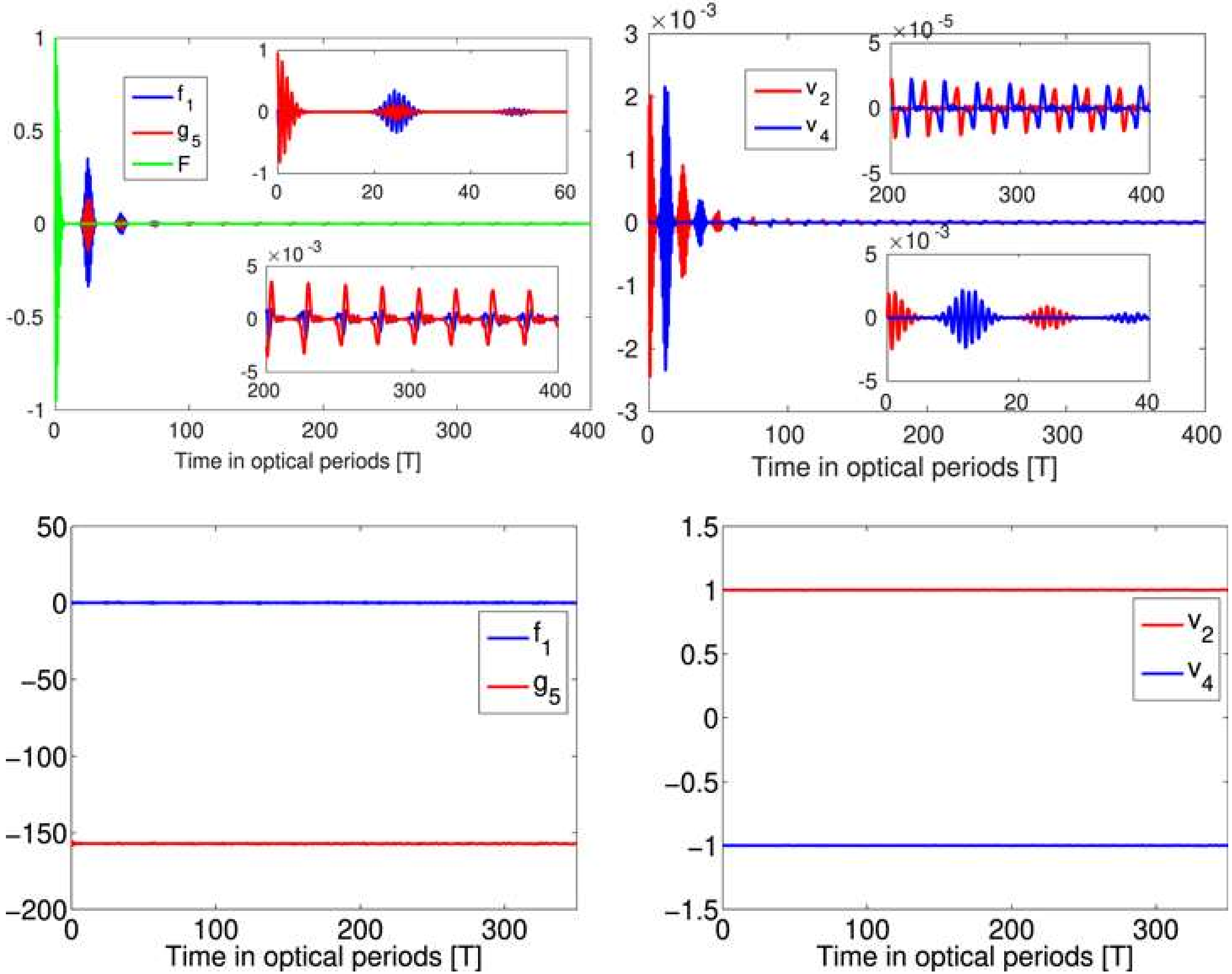
$$(a)\ \ \lambda = 0 \in \sigma(\varepsilon),\ \ \textit{simple root}$$

$$(b)\ \ \forall \lambda \in \sigma(\varepsilon) - \{0\},\ \ \mathrm{Im}(\lambda) < 0$$

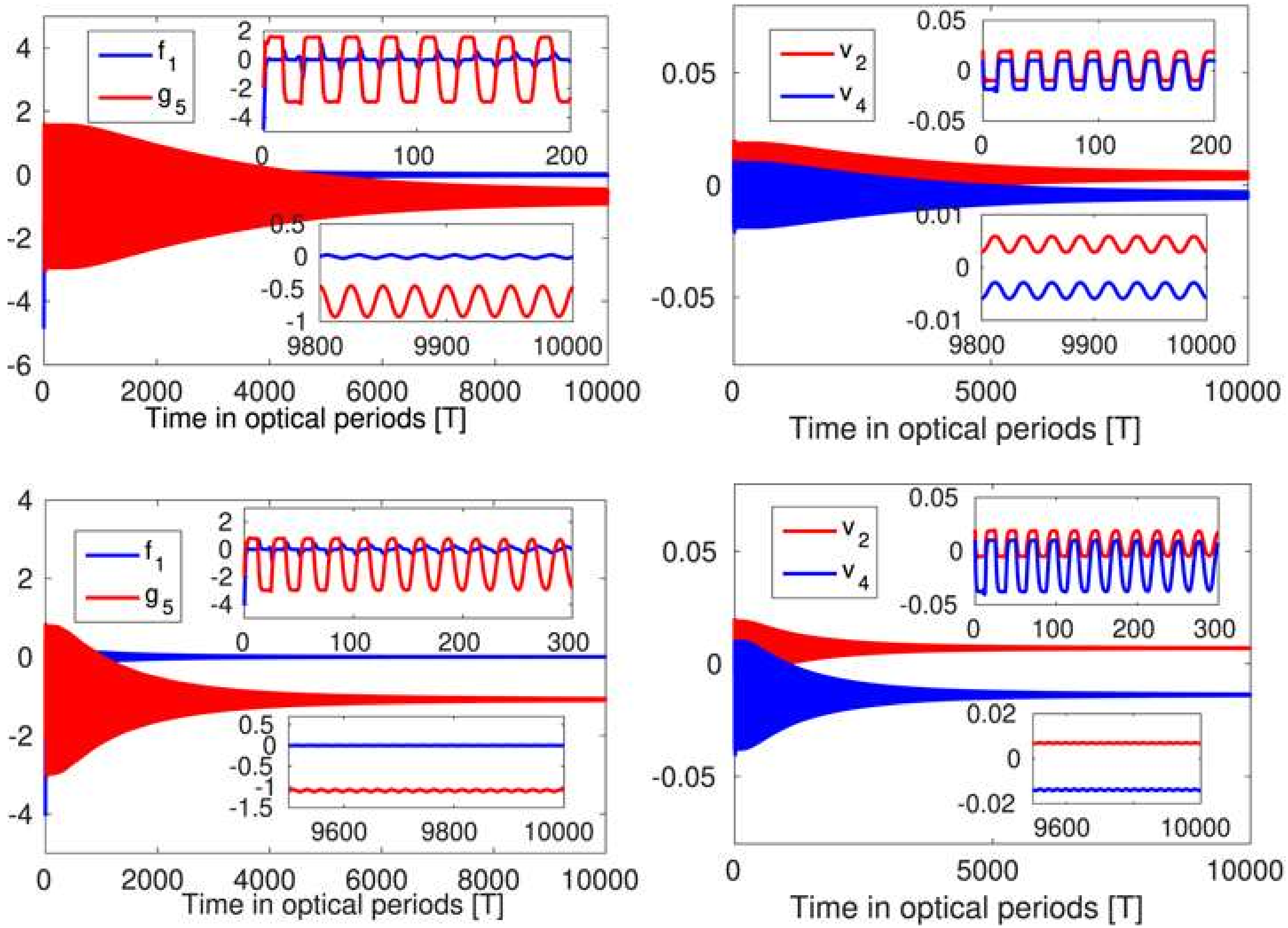


[4] Polner M, Varró S and Vörös-Kiss A, The role of the time delay in the reflection and transmission of ultrashort electromagnetic pulses on a system of parallel current sheets. Physica Scripta 94, 045205 (2019). E-print: arXiv: 1812.03857v1 [physics.class-ph].

Illustrations for the temporal behaviour of the currents and the fields. I.



Illustrations for the temporal behaviour of the currents and the fields. II.



Note: Radiation reaction. „LAD”: Lorentz-Abraham-Dirac equation. Runaway solutions, etc…

Classical theory of radiating electrons

By P. A. M. Dirac, F.R.S., *St John's College, Cambridge*

(*Received* 15 *March* 1938)

Summary

The object of the paper is to set up in the classical theory a self-consistent scheme of equations which may be used to calculate all the results that can be obtained from experiment about the interaction of electrons and radiation. The electron is treated as a point charge and the difficulties of the infinite Coulomb energy are avoided by a procedure of direct omission or subtraction of unwanted terms, somewhat similar to what has been used in the theory of the positron. The equations obtained are of the same form as those already in current use, but in their physical interpretation the finite size of the electron reappears in a new sense, the interior of the electron being a region of space through which signals can be transmitted faster than light.

P. A. M. Dirac, Classical theory of radiating electrons (1938).

R e f e r e n c e s

Fokker, A. D. 1929 *Z . Phys.* 58, 386-93.
Frenkel, J . 1925 *Z. Phys.* 32, 518-34.
Oppenheimer, J . R. 1935 *Phys. Rev.* 47, 44-52.
Page, L. 1918 *Phys. Rev.* 11, 376-400.
— 1924 *Phys. Rev.* 24, 296-305.
Schott, G. A. 1915 *Phil. Mag.* 29, 49-62.

Some similarities of our analysis with the treatment of the classical radiation reaction problem concerning the early electron models. I: Singular perturbation method (Herbert SPOHN, 2000).

EUROPHYSICS LETTERS 1 May 2000

Europhys. Lett., **50** (3), pp. 287–292 (2000)

Lorentz-Abraham point charge:

$$m\dot{\upsilon}^{\mu} = eF^{\mu\nu}\upsilon_{\nu} + \frac{e^2}{6\pi c^3}\left[\ddot{\upsilon}^{\mu} - \frac{1}{c^2}\dot{\upsilon}^{\lambda}\dot{\upsilon}_{\lambda}\upsilon^{\mu}\right]$$

The critical manifold of the Lorentz-Dirac equation

H. SPOHN(*)
Zentrum Mathematik and Physik Department
TU München, D-80290 München, Germany

Abstract. – „We investigate the solutions to the Lorentz-Dirac equation and show that its solution flow has a structure identical to the one of renormalization group flows in critical phenomena. The physical solutions of the Lorentz-Dirac equation lie on the critical surface. The critical surface is repelling, *i.e.* any slight deviation from it is amplified and as a result the solution runs away to infinity. On the other hand, Dirac's asymptotic condition (acceleration vanishes for long times) forces the solution to be on the critical manifold. The critical surface can be determined perturbatively. Thereby one obtains an effective second-order equation, which we apply to various cases, in particular to the motion of an electron in a Penning trap."

Point electron: Laue (1909). Dirac (1938). Remedied by Landau and Lifshitz (1975):

$$\begin{aligned} m\dot{v}^{\mu} = {} & eF^{\mu\nu}(x)v_{\nu} + (e^2/6\pi c^3)\{(e/m)v^{\sigma}(\partial_{\sigma}F^{\mu\nu}(x))v_{\nu} + \\ & +(e/m)^2F^{\mu\alpha}(x)F_{\alpha}{}^{\nu}(x)v_{\nu} + (e/mc)^2F^{\sigma\alpha}(x)F_{\alpha}{}^{\nu}(x)v_{\sigma}v_{\nu}v^{\mu}\}. \end{aligned}$$

Fritz Rohrlich,
Causality and the Arrow of Classical Time. *Stud. Hist. Phil. Mod. Phys.* 31, 1-13 (2000).

On page 8 Rohrlich writes:
"… Unfortunately, during much of the half-century following Dirac's work, some physicists tried to repair' the LAD equations instead of recognising that its pathologies are symptoms of the inapplicability of classical physics to point particles. Such particles must be treated by quantum mechanics and are outside the validity limits of any classical theory. Therefore, this 'repair work' led to a useless literature but was unfortunately quite voluminous.

Equally unfortunate was the fact that an old calculation made near the beginning of the twentieth century had been almost forgotten. That was a calculation made by Sommerfeld (1904) for a surface-charged sphere of total electric charge *e* (i.e. not a point charge) moving with non-relativistic velocity: he took the *self-interaction* due to the self-field into account. That calculation was apparently repeated by Page (1918), who published only the result. That result replaced the equation (1) by the self-force

$$F_s = m_e[\upsilon(t-\tau_a)-\upsilon(t)]/\tau_a \qquad (3)$$

where m_e is the electromagnetic contribution to the mass, $(2/3)e^2/(ac^2)$, *v* is the velocity, *a* is the radius of the sphere, and τ_a is the time it takes a light ray to traverse the diameter of the sphere…."

The pesent dia was inserted on 07 Feb 2026 (after the Sminar talk). Rohrich's text has only been red to the participans of the Seminar o6. 02. 2026.

Some similarities of our analysis with the treatment of the classical radiation reaction problem concerning the early electron models. II. <u>Delay differential equation</u>.

In the two-layer problem let us take $n_5 = n_3 = n_1$ **Two coupled (delayed) eq:**

$$\delta''_{y2}(t') = c_1[(e/m)F(t') - \Gamma_2\delta'_{y2}(t') - \Gamma_4\delta'_{y4}(t' - \Delta t_3)]$$

$$\Gamma \equiv 2\pi(e^2/mc)l \cdot n_e$$

$$\delta''_{y4}(t') = c_1[(e/m)F(t' - \Delta t_3) - \Gamma_2\delta'_{y2}(t' - \Delta t_3) - \Gamma_4\delta'_{y4}(t')]$$

Extended (charged sphere) electron model. Lorentz (1892), Abraham (1903-4), Hertz P. (1903), Sommerfeld (1904),

$$F_s = m_e[\upsilon(t - \tau_a) - \upsilon(t)]/\tau_a$$

$$m_e = (2/3)e^2/(ac^2)$$

Manifestly covariant form. Caldirola (1956), Yaghjian (1992)…

$$F_s^\mu = m_e[\upsilon^\mu(\tau - \tau_a) + \upsilon^\mu(\tau)\upsilon_\nu(\tau)\upsilon^\nu(\tau - \tau_a)]/\tau_a$$

[Power expansion with respect to the delay (proportional with the radius), the Landau-Lifshitz eq. results.]

Rohrlich F, Causality and the arrow of time. Stud. Hist. Phil. Mod. Phys. 31, 1-13 (2000).
[5] Spohn H, The critical manifold of the Lorentz-Dirac equation. Europhys. Lett. 50, 287 (2000).
[5a] Rohrlich F, The correct equation of motion of a classical point charge. Phys. Lett. A 283, 276-278 (2001).
[5b] Yaghjian A D, Relativistic Dynamics of a Charged Sphere. Updating the Lorentz-Abraham Model, 2nd ed. Lect. Notes Phys. 686 (Springer, New York 2006). For the basic references see this book.

Some similarities of our analysis with the treatment of the classical radiation reaction problem concerning the early electron models. Sommerfeld's Huygens-construction; shadow

Physics. — "*Simplified Deduction of the Field and the Forces of an Electron, moving in any given way.*" By Prof. A. SOMMERFELD. (Communicated by Prof. H. A. LORENTZ).

§ 1. *Summary.*

In the "Göttinger Nachrichten" [1]) I communicated a general representation of the field of an electron, moving in any given way, which seems to be simpler than the formulae, hitherto known, which are based on the work of H. A. LORENTZ. This is the difference: My formulae express the potentials by a *simple integral, extending over the past time* and containing only the varying distances of the point in question *from the centre of the electron*, supposed to be spherical, whereas the formulae hitherto known are *double or triple integrals*, extending over the *space*, charged with electricity, and containing the distance of the point in question *from the position of the charge at a certain former time*. It may be remarked, that

[1]) Nachrichten d. K. Gesellschaft d. Wissenschaften 1904 Heft 2; in the following to be cited as "first paper".

This is the paper in which Sommerfeld also describes the „Cherenkov effect" in 1904 (see next page).

Some similarities of our analysis with the treatment of the classical radiation reaction problem concerning the early electron models. Sommerfeld's Huygens-construction; shadow

It follows: The approximate formulae mentioned before, which I have derived formerly supposing two roots τ_1 τ_2 to exist, hold good absolutely if the velocity is less than that of light; in the opposite case they are to be replaced by 0 out of the shadow of motion, and they are to be completed by a member similarly formed within the shadow of motion.

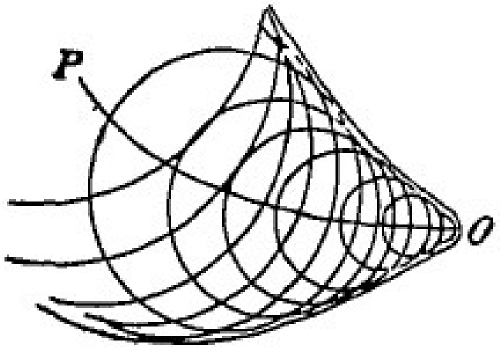


Fig. 2.

Fig. 2 explains, what shadow of motion means. Here the momentary position O of the electron and its preceding path OP is marked. Round every point P of the path the sphere may be constructed with the radius $c\tau$, where τ denotes the time, in which the electron gets from that point to O. The envelope of these spheres

Note*: „An extensible model of the electron”. By P. A. M. Dirac, F. R. S. (1962). *

An extensible model of the electron

By P. A. M. Dirac, F.R.S.
St John's College, University of Cambridge

(*Received 5 February* 1962)

It is proposed that the electron should be considered classically as a charged conducting surface, with a surface tension to prevent it from flying apart under the repulsive forces of the charge. Such an electron has a state of stable equilibrium with spherical symmetry, and if disturbed its shape and size oscillate. The equations of motion are deduced from an action principle and a Hamiltonian formalism is obtained. The energy of the first excited state with spherical symmetry is worked out according to the Bohr–Sommerfeld method of quantization, and is found to be about 53 times the rest-energy of the electron. It is suggested that this first excited state may be considered as a muon. The present theory has no electron spin, so it cannot agree accurately with experiment.

The model

The concept of an electron of finite size is an old one, first proposed by Abraham and Lorentz. It is the most natural concept that makes the total energy of the Coulomb field of the electron finite.

References

Dirac, P. A. M. 1958 *Proc. Roy. Soc.* A, **246**, 326.
Lees, A. 1939 *Phil. Mag.* **28**, 385.

This paper, containing the „Dirac-bubble” is a standard reference in papers dealing with the „quark bag model”.:

Chodos A, Jaffe R L, Johnson K, Thorn C B and Weisskopf V F, New extended model of hadrons. Physical Review D 9 (12), 3471-3495 (1974).

Gnädig P, Hasenfratz P, Kuti J, Szalay A S, The quark bag model with surface tension. Physics Letters B 64 (1), 62-66 (1976).

Hasenfratz P and Kuti J, The quark bag model. Physics Reports 40 (2), 75-179 (1978).

In mentioning the work of Lorentz and Abraham, the usage of the mere expression „proposed” is a massive understatement. Abraham (Sommerfeld) worked out this model in very details (ellipsoids also) and the radiation reaction was calculated, too…

***The pesent dia was inserted on 07 Feb 2026 (after the Sminar talk). Its inserion has been motivated by the discussion with participans of the Seminar o6. 02. 2026.**

“Tief ist der Brunnen von Vergangenheit. Sollte man ihn nich unergründlich nennen?”
[Thomas Mann, Joseph und seine Brüder.]

“Finally, I wish to mention the much broader and perhaps most important complex of problems whether physics is to become a job for technicians, highly trained to be sure and skilful, but of limited horizon and integrated into efficiently organized teams. Or is there room for some of the excitement of the discovery and the birth of new ideas? Is physics part of our western culture? Does it make sense to see and reveal beauty in the laws of nature or is that hopelessly old-fashioned?
When Galileo discovered the moons of Jupiter, people wrote poems about this event. Thus was the imagination of the whole man enthralled by a scientific discovery in the 17th century. How far we are today from such an attitude!” (1966)

[Josef-Maria Jauch (1914-1974)]

Taken from:Löwdin P.-O., and Béné G J, Josef-Maria Jauch in Memorian. EPN 0512p7. (1974).

Appendix

Mesoscopic radiation reaction in scattering of light by active boundary surfaces.

Sándor Varró
ELI-ALPS (Attosecond Light Pulse Source) Research Institute, Szeged, Hungary.

ABSTRACT. We discuss the dynamics of reflection and transmission of light pulses at active boundary surfaces being very thin conducting layers (metal, transient plasma, graphene) represented by current sheets (which may be embedded in dielectrics). The boundary conditions contain the unknown surface current components, whose electrons move under the action of the Lorentz force, stemming from the superposition of the incoming field and the unknown reflected and transmitted fields. This model gives a special closed set of coupled Maxwell-Lorentz equations, which we have solved. In these equations a damping term (being proportional with the velocity of the electrons) automatically appears, due to the matching conditions satisfied by the complete radiation field at the boundary surface. One may say that this damping term represents a sort of mesoscopic radiation reaction. The solutions of these equations have allowed us to predict various new phenomena, like the modification of the Fresnel coefficients, the appearance of frozen-in electromagnetic pulses [1], and the generation of rectangular optical waves [2]. The latter effect may perhaps be relevant for a physical realization of Haar wavelets [3] in the optical range. We also show our results on the time-dependent reflection and refraction of electromagnetic pulses on a system of two parallel current sheets [4]. Here the problem has been reduced to a hybrid system of two delay differential equations for the electron motion in the layers and a recurrence relation for the scattered fields. The solution has been given as a limit of a singularly pertubed system. Finally, we highlight some similarities of our analysis with the treatment of the classical radiation reaction problem concerning the early electron models [5].

Mesoscopic radiation reaction in scattering of light by active boundary surfaces.

Sándor Varró
ELI-ALPS (Attosecond Light Pulse Source) Research Institute, Szeged, Hungary.

References.

[1] Varró S : Linear and nonlinear absolute phase effects in interactions of ulrashort laser pulses with a metal nano-layer or with a thin plasma layer. Laser and Particle Beams 25, 379 (2007). E-print: arXiv: physics/0610266 [physics.plasm-ph].

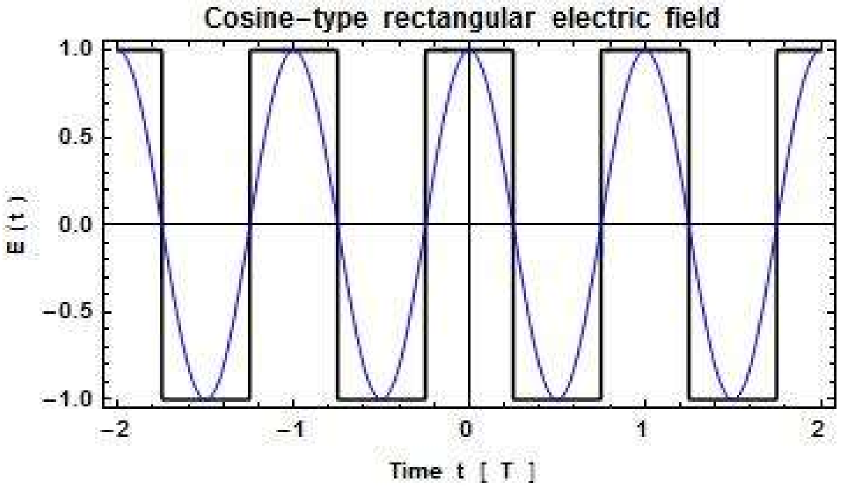
[2] Varró S; Graphene-based carrier-envelope phase difference meter. AIP Conf. Proceedings 1462, 128 (2012). [2a] Varró S, Generation of rectangular optical waves by relativistic clipping. Laser Physics 23, 056006 (2013). E-print: arXiv:1301.6103 [physics.optics].

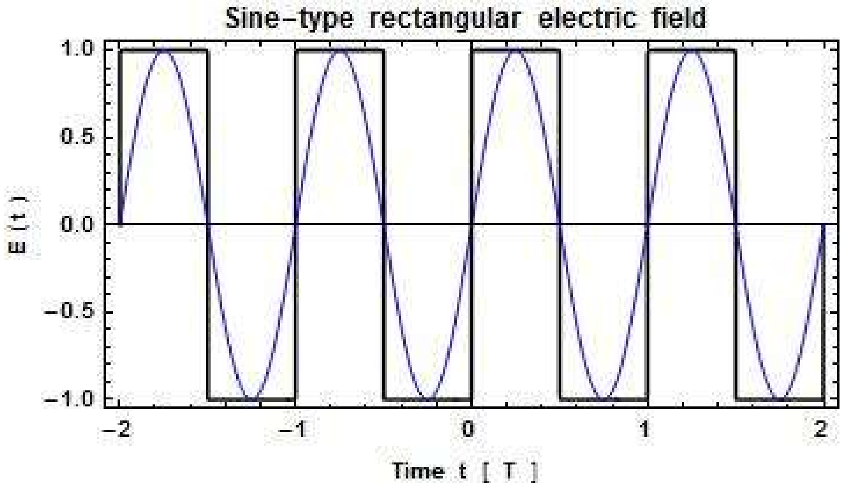
[3] Haar A, Zur Theorie der orthogonalen Funktionensysteme (Inaugural Dissertation zur Erlangung der Doktorwürde der hohen philosophischen Fakultät der Georg-August-Universität zu Göttingen, vorgelegt von Alfréd Haar aus Budapest). 1909, 1-49; [3a] Haar A, Zur Teorie… (Erste Mitteilung). Mathematische Annalen 69, 331-371 (1910); [3b] Haar A, Mathematische Annalen 71, 38-53 (1911).

[4] Polner M, Varró S and Vörös-Kiss A, The role of the time delay in the reflection and transmission of ultrashort electromagnetic pulses on a system of parallel current sheets. Physica Scripta 94, 045205 (2019). E-print: arXiv: 1812.03857v1 [physics.class-ph].

[5] Spohn H, The critical manifold of the Lorentz-Dirac equation. Europhys. Lett. 50, 287 (2000).
[5a] Rohrlich F, The correct equation of motion of a classical point charge. Phys. Lett. A 283, 276-278 (2001).
For the basic references see: [5b] Yaghjian A D, Relativistic Dynamics of a Charged Sphere. Updating the Lorentz-Abraham Model, 2nd ed. Lect. Notes Phys. 686 (Springer, New York 2006).

Az ideális (periodikus, Rademacher) négyszög-impulzus Fourier-sora.



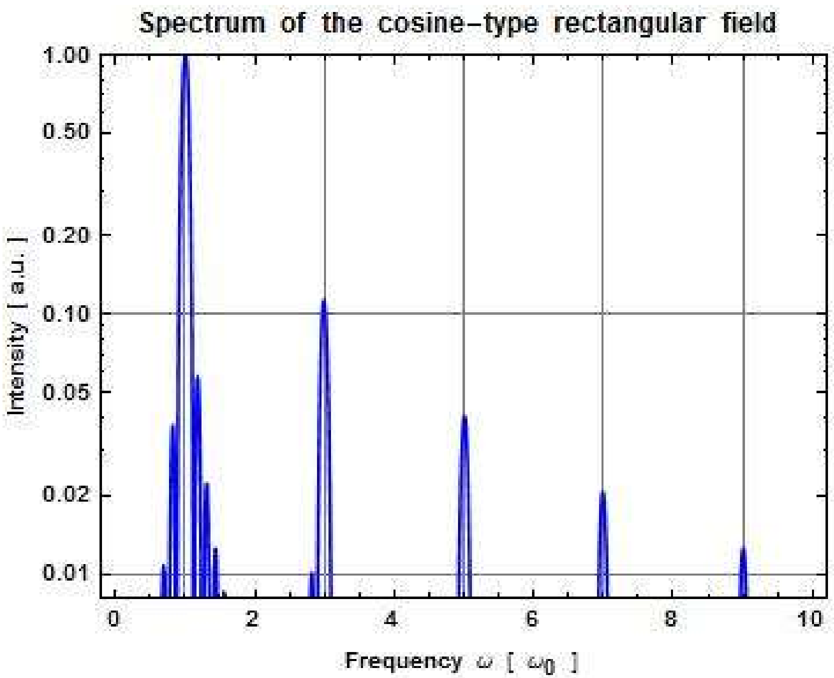


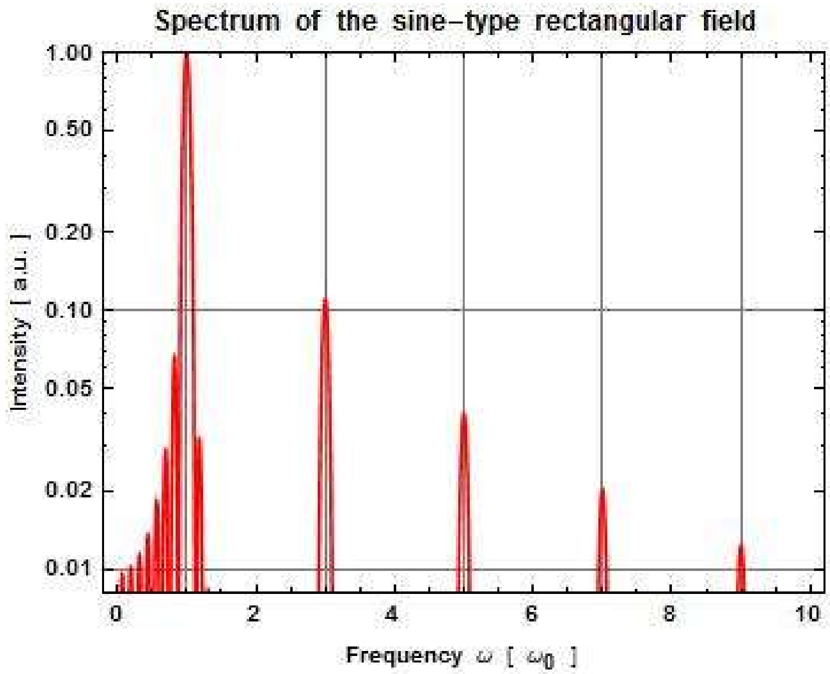
$$E(t) = E_0 \sum_{k=1}^{\infty} (-1)^k (2k-1)^{-1} \cos[(2k-1)\omega_0 (t - t_0)]$$

S. V., Graphene-based carrier-envelope phase difference meter. In *AIP* Conf. *Proceedings 1462, 128-131 (2012)*.
[*Light at Extreme Intensities* (LEI-2011, 14-18. November 2011. Szeged, Hungary), Book of Abs. p. 43 (2011).]

Varró S, Generation of rectangular optical waves by relativistic clipping. *Laser Physics* 23 (2013) 056006 (6pp)

The calculated spectrum of the rectangular signal (scattered radiation).



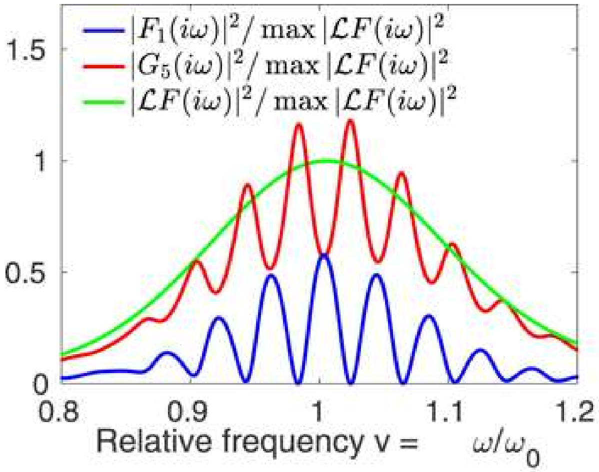


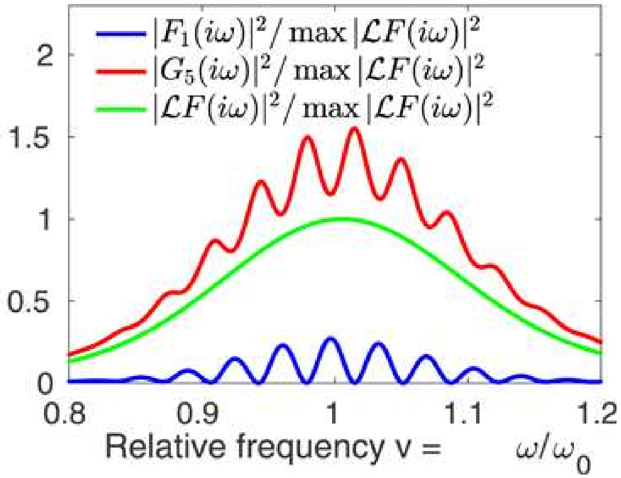
$$|\tilde{E}(\omega)|^2_{\omega\approx n\omega_0} \propto 1/n^2, \quad n = 2k-1$$

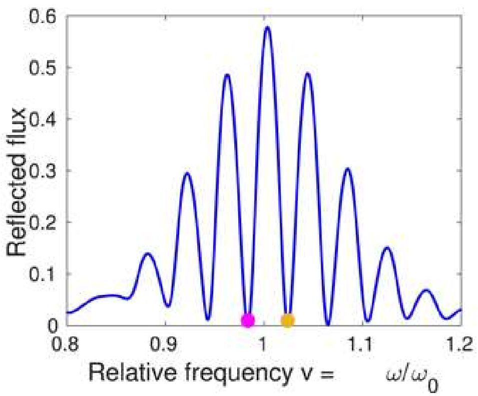
S. V., Graphene-based carrier-envelope phase difference meter. In *AIP* Conf. *Proceedings 1462, 128-131 (2012)*.
[*Light at Extreme Intensities* (LEI-2011, 14-18. November 2011. Szeged, Hungary), Book of Abs. p. 43 (2011).]

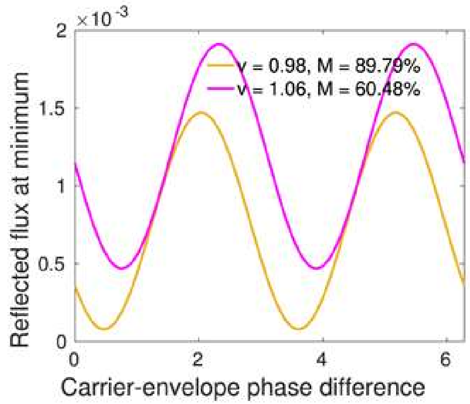
Varró S, Generation of rectangular optical waves by relativistic clipping. *Laser Physics* 23 (2013) 056006 (6pp)

Illustration of the change of the spectrum. Carrier-envelope phase dependence









[4] Polner M, Varró S and Vörös-Kiss A, The role of the time delay in the reflection and transmission of ultrashort electromagnetic pulses on a system of parallel current sheets. Physica Scripta 94, 045205 (2019). E-print: arXiv: 1812.03857v1 [physics.class-ph].