The Causal Second Law

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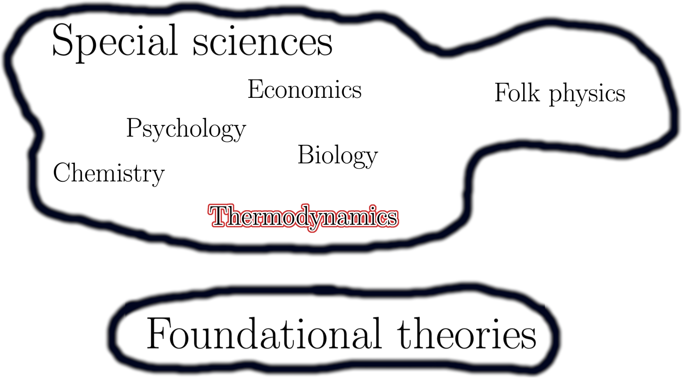
Wigner FK RMI Elméleti Osztály Szeminárium, 2026.02.27.

Introduction



Main claim: the Causal Second Law

If a special science satisfies certain key assumptions, then its causal regularities have an associated notion of entropy, and that this causal entropy cannot decrease from a robust cause to its effect.

Structure of the talk

- Context: special sciences and physics, causation, supervenience, measure-preservation
- The causal second law
- Strict increase of causal entropy, relaxing robustness
- Connection with experimental entropy and the phenomenological second law of thermodynamics
- Reversibility objection
- Entropy-from-cause-to-effect vs. entropy-in-time
- Further remarks, open questions

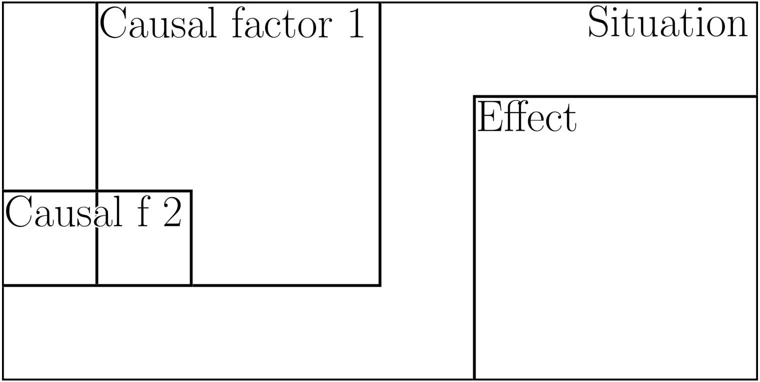
Special science causal regularity examples

Economics: printing large amounts of currency and keeping interest rates near zero in a modern economy regularly leads to inflation in six months.

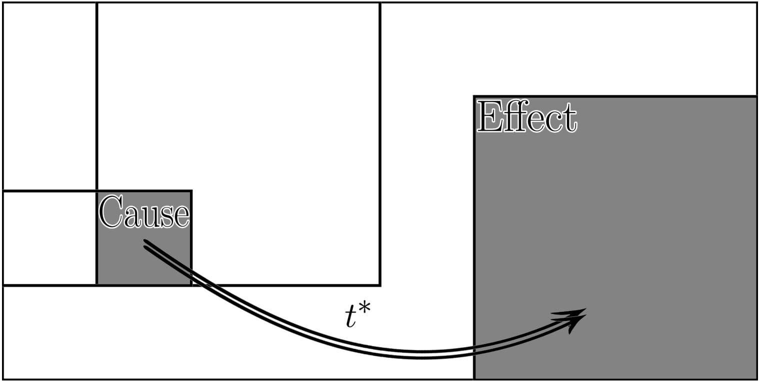
Psychology: sleep deprivation and prolonged social isolation in young adults regularly leads to dysregulated mood in two days.

Folk physics: Gas only in the left half of a box and an open valve regularly leads to gas spread-out the entire box in a minute.

Special science: states of affairs

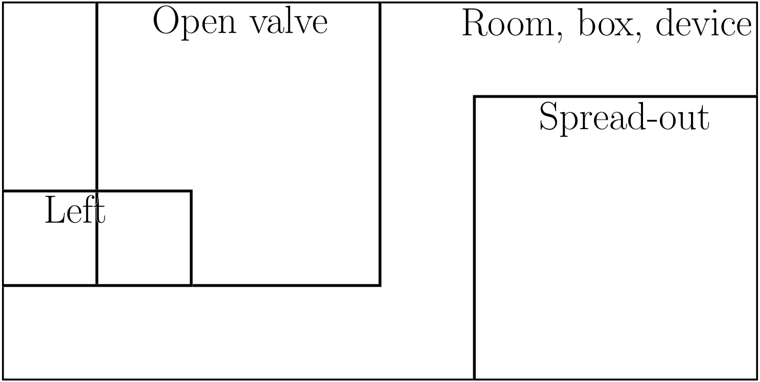


Special science: causal regularity



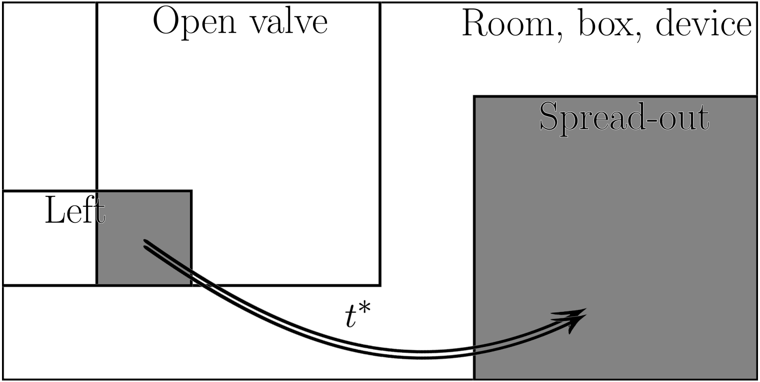
Folk physics example: states of affairs

Gas only in the left half of a box and an *open valve* regularly leads to *gas spread-out the entire box* in a minute.



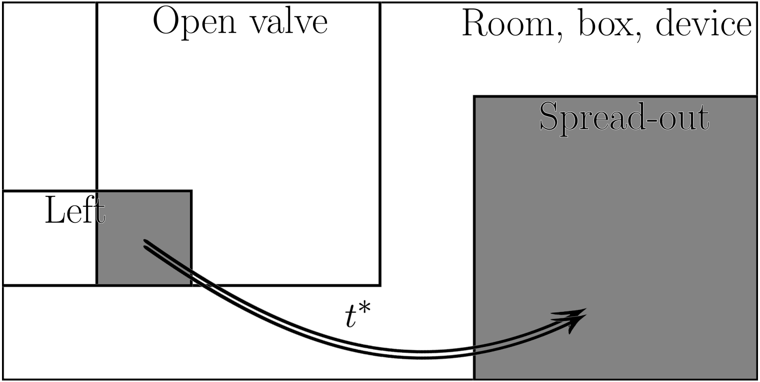
Folk physics example: causal regularity

Gas only in the left half of a box and an open valve *regularly leads to* gas spread-out the entire box *in a minute*.

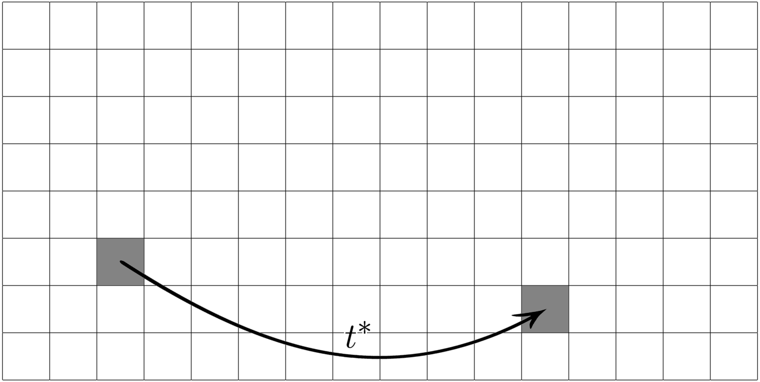


Folk physics example: robust causal regularity

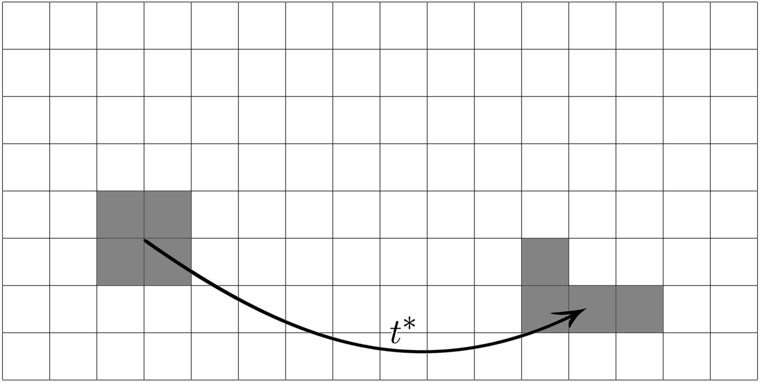
Robust cause: the cause (almost) always leads to the effect, in a time characteristic of the regularity.



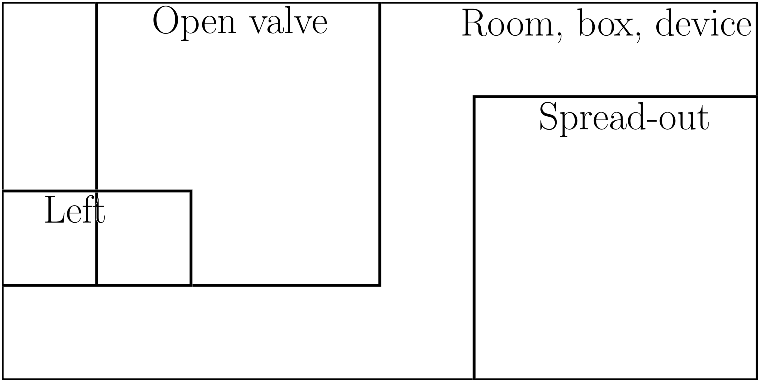
Dynamical system: (P, Σ, m, U_t)



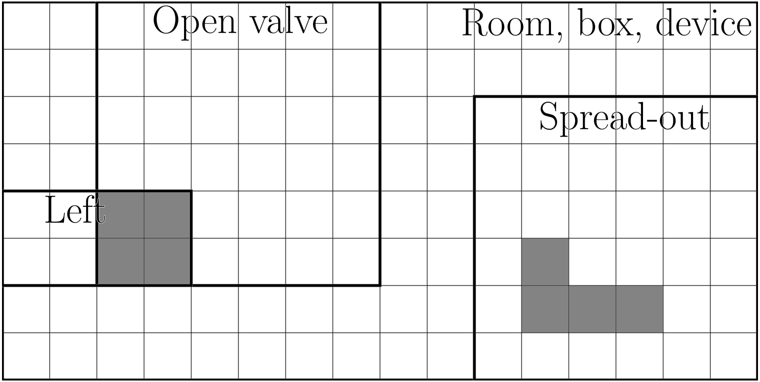
Dynamical system: (P, Σ, m, U_t)



Supervenience: special science states of affairs



Supervenience: special science states of affairs



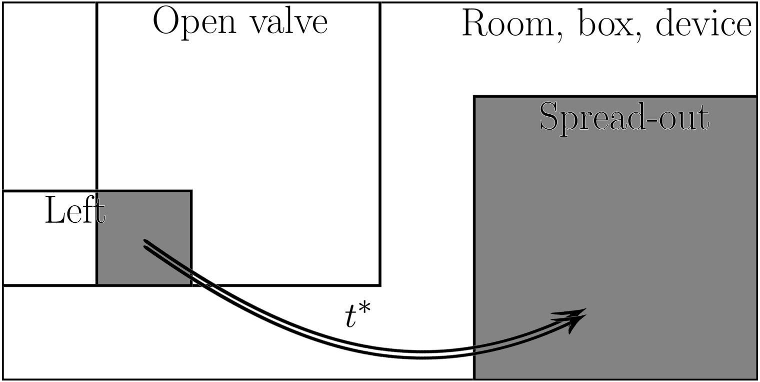
Supervenience: special science states of affairs

“Causal” dynamical system:

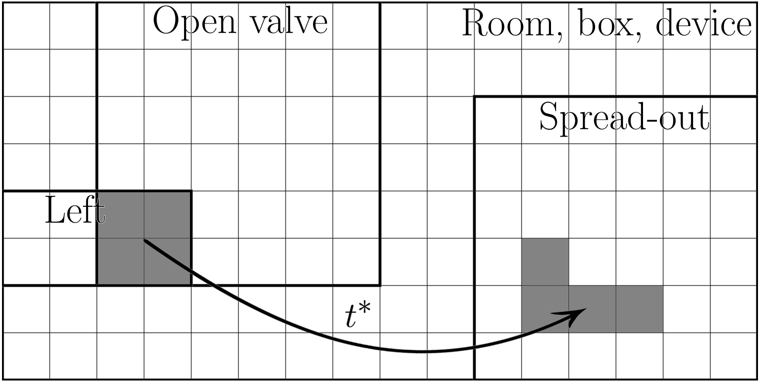
$$(P, \mathcal{R}, \Sigma, m, U_t)$$

(or simply: $(P, \Sigma_{\mathcal{R}}, m, U_t)$), where $\mathcal{R} \subseteq \Sigma$, and $(P, \mathcal{R})$ is a σ-algebra.

Supervenience: special science robust causation



Supervenience: special science robust causation



Supervenience: special science robust causal regularity

Robust cause: (almost) every physical state instantiating the cause evolves, in a characteristic time, to a physical state instantiating the effect.

Robust cause: For $C, E \in \mathcal{R}$,

$$C \overset{t^*}{\leadsto} E$$

if $m(\{p \in C \mid U_{t^*} p \in E\}) = m(C)$, $t^* > 0$.

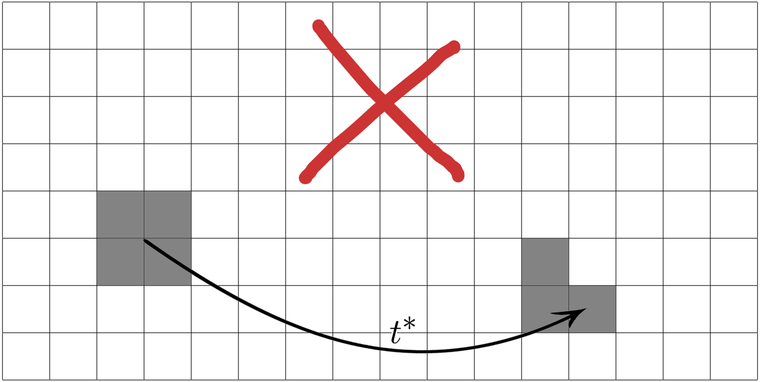
Assumption: measure preservation

A **measure-preserving** dynamical system is one in which the number (measure) of physical states in any set remains unchanged under the system's evolution over time.

$(P, \Sigma_{\mathcal{R}}, m, U_t)$ is **measure-preserving** if $\forall S \in \Sigma_{\mathcal{R}}$:
$m(U_{-t}S) = m(S)$.

Conservative dynamics of Hamiltonian formulations of all foundational theories of physics are measure preserving due to Liouville's well-known theorem.

Assumption: measure preservation

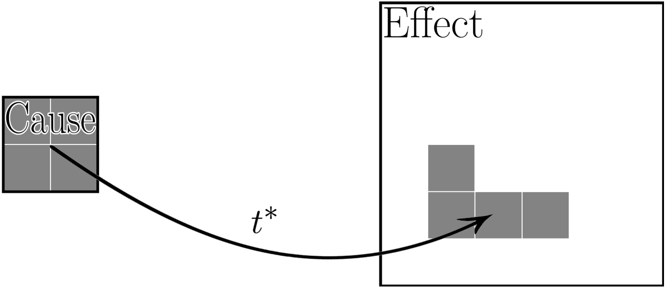


Causal entropy

Causal entropy of a special science states of affairs is the number (measure) of physical states instantiating it.

For a robust cause C and its effect E: $m(C)$ and $m(E)$.

The Causal Second Law



The Causal Second Law

Assume that the physical dynamics is measure-preserving. We showed:

Causal second law: the causal entropy cannot decrease from a robust cause to its effect.

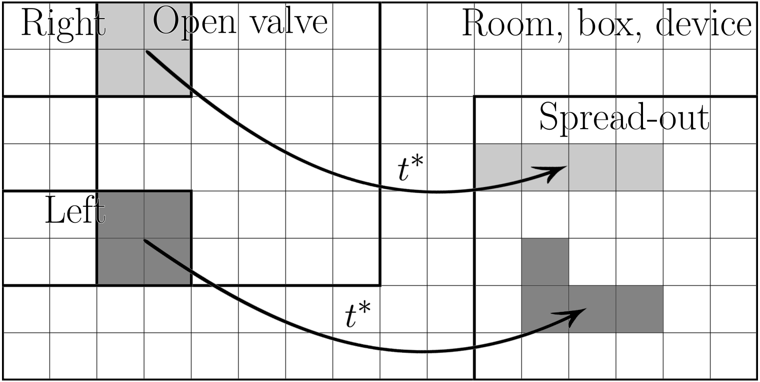
Causal second law: if $(P, \Sigma_{\mathcal{R}}, m, U_t)$ is measure-preserving, then if

$$C \overset{t^*}{\rightsquigarrow} E$$

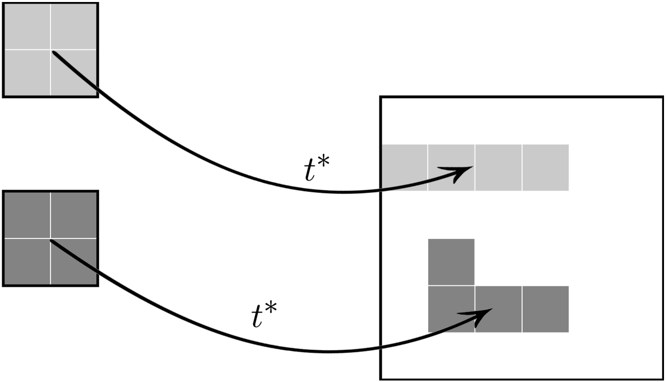
then

$$m(C) \leq m(E).$$

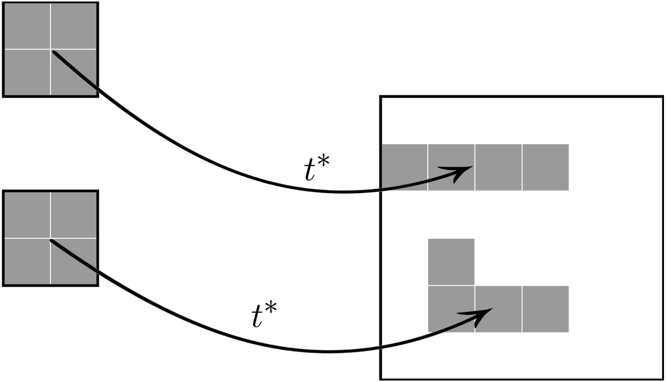
Strict increase of entropy 1: multiple potential causes



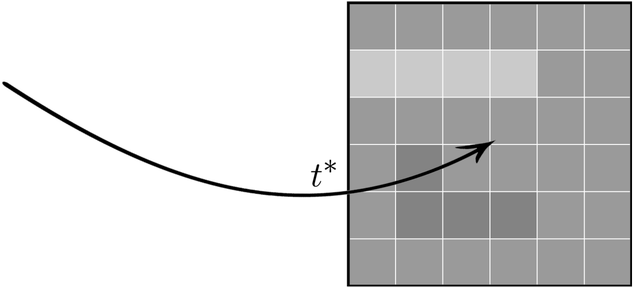
Strict increase of entropy 2: mismatching descriptive capabilities



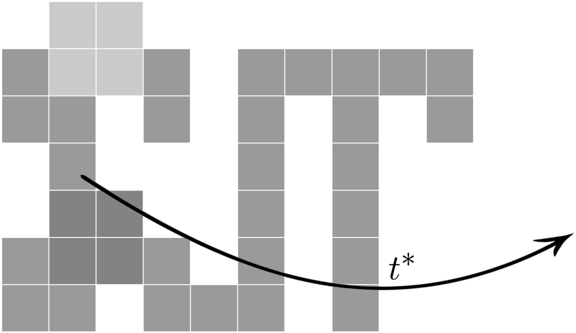
Strict increase of entropy 2: mismatching descriptive capabilities



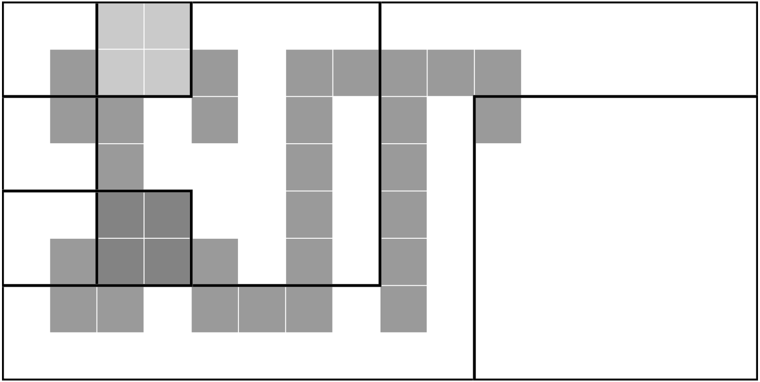
Strict increase of entropy 2: mismatching descriptive capabilities



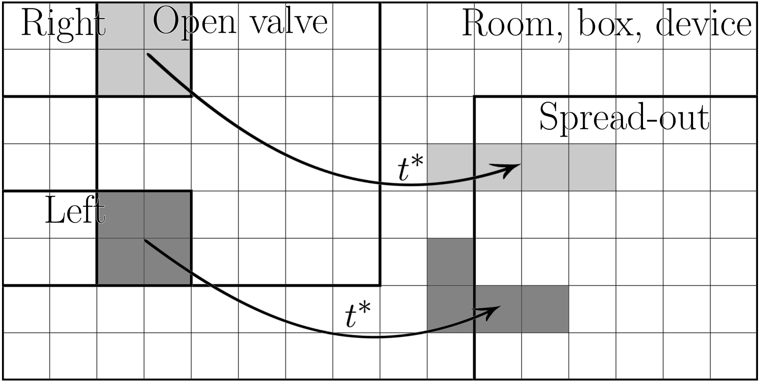
Strict increase of entropy 2: mismatching descriptive capabilities



Strict increase of entropy 2: mismatching descriptive capabilities



Portional causal regularity



Portional causal regularity

Portional causal regularity: an $0 < \alpha \leq 1$ portion of physical states instantiating the cause evolve, in a characteristic time, to physical states instantiating the effect.

The Causal Second Law for Portional Regularities

The causal second law also holds for portional causal regularities due to multiplicity of potential causes, mismatching descriptive capabilities of the special sciences and physics (as well as for other reasons not mentioned in this talk).

The Causal and the Thermodynamical Second Law

Jaynes' (1965) explication of the second law of thermodynamics renders it a special case of the causal second law, applied to the special science of classical thermodynamics.

The so-called Boltzmann entropy of thermodynamics is a special case of causal entropy.

Experimental entropy S_e

- The entropy of *phenomenological thermodynamics*: the quantity operationally fixed by macroscopic measurements (calorimetry, equations of state, steam tables).
- Defined for **equilibrium states** via reversible thermodynamic relations:
$$dS_e = \frac{dQ}{T}.$$
- Depends on which macroscopic variables $X = (X_1, \ldots, X_n)$ we treat as defining the thermodynamic state.

Bridge I: Gibbs entropy of canonical ensemble $= S_e$

P : $6N$-dimensional phase space, $H(\gamma)$: classical Hamiltonian, $W_N(\gamma)$: relative frequency of $\gamma \in P$.

Gibbs entropy:

$$S_G(W_N) = -k \int_P W_N(\gamma) \log W_N(\gamma)\, d\gamma.$$

Assumption. Equilibrium macroscopic quantities are correctly represented by the canonical distribution

$$W_N^{\mathrm{can}}(\gamma) = Z^{-1}\, e^{-\beta H(\gamma)}, \qquad \beta = (kT)^{-1}.$$

Bridge I: Gibbs entropy of canonical ensemble $= S_e$

Jaynes' identification (Sec. III–IV): After fixing an additive constant at one point, over all distributions W_N consistent with the measured energy U,

$$S_G(W_N) \leq S_e,$$

with equality *iff* $W_N = W_N^{\mathrm{can}}$.

Thus, for equilibrium states,

$$S_e = S_G(W_N^{\mathrm{can}}).$$

Bridge II: Gibbs entropy = $k \log(\text{typical phase volume})$

High-probability region $R(\epsilon)$ relative to W_N:

- For fixed small $0 < \epsilon < 1$, choose threshold C so that

$$R(\epsilon) \doteq \{\gamma \in P : \; W_N(\gamma) \geq C\}, \qquad \int_{R(\epsilon)} W_N \, d\gamma = 1 - \epsilon.$$

- Phase volume of $R(\epsilon)$:

$$m(R(\epsilon)) = \int_{R(\epsilon)} d\gamma.$$

Bridge II: Gibbs entropy $= k \log(\text{typical phase volume})$

Shannon AEP: For canonical W_N^{can}, in the thermodynamic limit $N \to \infty$ at fixed density,

$$S_G(W_N^{\mathrm{can}}) \approx k \log m(R(\epsilon)).$$

Important: the result is *independent* of the choice of ϵ.

We choose a suitably small ϵ, with which define $R \doteq R(\epsilon)$

Combining results: $S_e(X) \approx k \log m(R_X)$

For an equilibrium macrostate X:

1. Represent X by the canonical distribution $W_N^{\mathrm{can}}(X)$.
2. Define the high-probability region R_X relative to $W_N^{\mathrm{can}}(X)$.
3. $$S_e(X) = S_G(W_N^{\mathrm{can}}(X)) \,.$$
4. $$S_G(W_N^{\mathrm{can}}(X)) \approx k \log m(R_X).$$

Thus,

$$\boxed{S_e(X) \;\approx\; k \log m(R_X)}$$

Interpretation: S_e measures the log phase volume of "reasonably probable" microstates compatible with the macroscopic constraints X.

Setup for using the causal second law

Reproducible adiabatic process (box of gas):

- Initial equilibrium: macrostate X_0, region R_{X_0}.
- Perform adiabatic operation (e.g. open valve) over time t^*.
- Final equilibrium: macrostate X', region $R_{X'}$.

Reproducibility condition: the macroscopic transition from X_0 to X' is experimentally reproducible.

This translates to:

$$R_{X_0} \overset{t^*}{\rightsquigarrow} R_{X'}.$$

The phenomenological second law

Thus, from the causal second law:

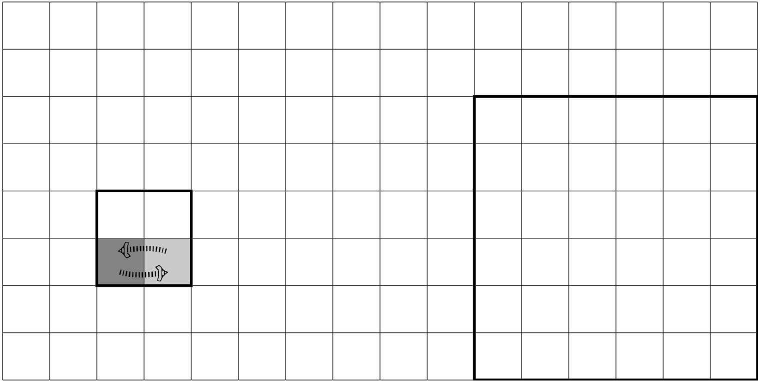
$$m(R_{X_0}) \leq m(R_{X'}).$$

Therefore,

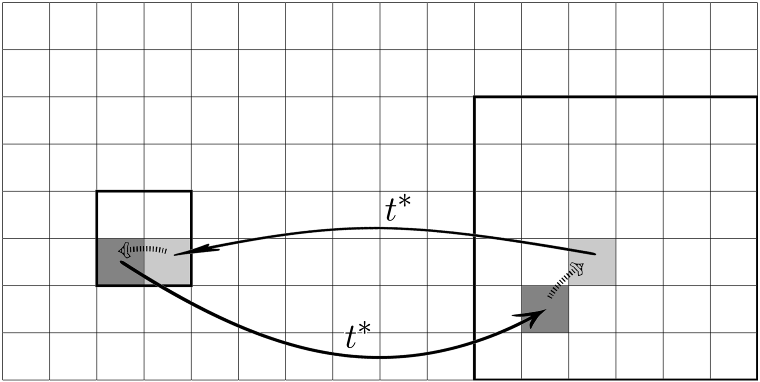
$$S_e(X_0) \leq S_e(X'),$$

which is the **phenomenological second law.**

Time reversal operator: $\forall p: \quad TTp = p$



Time reversal invariance: $\forall p, t^* : \; TU_{t^*}TU_{t^*}p = p$



Loschmidt's reversibility objection

If

(i) every macrostate has an entropy defined by its phase volume,

(ii) the entropy of a microstate equals that of the macrostate it instantiates,

(iii) the entropy of a microstate equals that of its time-reversed microstate,

(iv) the physical dynamics is time reversal invariant,

then for every trajectory $p_t = U_t p_0$ along which the entropy increases, there exists another trajectory $q_t = U_t q_0$ along which the entropy decreases. Ergo, the second law is either satisfied trivially (entropy always stays constant), or it cannot be exceptionless.

The Causal Second Law and the reversibility objection

Even if we accept assumptions (i)–(iii) of the reversibility objection (!), and even if we assume that the time-reversed TC (the set of physical states that are time reverses of physical states in C) and TE also correspond to special science descriptions (!), since the time reversal operator preserves the phase volume, it cannot be the case that almost all physical states evolve from TE to TC, since TE has a larger phase volume than TC, and the phase volume of TE is preserved by the measure-preserving dynamics. Hence, TE cannot be a robust cause of TC; therefore, we do not have an exception to the causal second law, which would only demand entropy increase if TE were a robust cause of TC.

Four causal asymmetries

(1) The cause precedes its effect in time.
(2) The cause leads to its effect.
(3) The causal entropy cannot decrease in time.
(4) The causal entropy cannot decrease from cause to effect.

Without measure-preservation: (1), (3), (4) are logically independent.

With measure-preservation: (1) $\leftrightarrow$ (3); (2) $\leftrightarrow$ (4); but (4) does not imply (1) or (3) (the causal second law does not imply a causal arrow of time).

Do special sciences need a reduction to thermodynamics?

If every special science had its own causal entropy that increases in time, then reduction to thermodynamics would no longer be necessary for understanding the arrow of time of the special sciences, for the sought type of entropic asymmetry would already be implied by the very existence of a regular connection between causes and effects of the special science in question.

Additional topics addressed in the manuscript

The manuscript further discusses:

- weakening determinism, multiple realization, supervenience;
- a discussion of description-relativity;
- the “occurrent” causal second law;
- a fully formalized mathematical framework within which all propositions can be rigorously expressed and proven.

Thanks!

Gyenis, Balazs (2026): The Causal Second Law, forthcoming in *Noûs*.

Preprint: https://philsci-archive.pitt.edu/28287/

Definitions

Dynamical system: (P, Σ, m, U_t).

"Causal" dynamical system: $(P, R, \mathcal{R}, \Sigma, m, U_t)$ (or simply: $(P, \Sigma_{\mathcal{R}}, m, U_t)$), where $R \subseteq P$, $\mathcal{R} \subseteq \Sigma$, and $(R, \mathcal{R})$ is a σ-algebra.

Robust causal regularity: $C \overset{t^*}{\rightsquigarrow} E$ if $m(\{p \in C \mid U_{t^*} p \in E\}) = m(C)$, $t^* > 0$.

$(P, \Sigma_{\mathcal{R}}, m, U_t)$ is **measure-preserving** if $\forall S \in \Sigma_{\mathcal{R}}$: $m(U_{-t} S) = m(S)$.

Proposition

Causal Second Law: if $(P, \Sigma_{\mathcal{R}}, m, U_t)$ is measure-preserving, then if $C \overset{t*}{\rightsquigarrow} E$ then $m(C) \leq m(E)$.

Definitions

α-**portional causal regularity**: $C \overset{t^*}{\rightsquigarrow}_{\alpha} E$ if $m(\{p \in C \mid U_{t^*} p \in E\}) = \alpha \cdot m(C)$, $t^* > 0$.

Let $m(P) = 1$, m measure preserving. It seems appropriate to further require $m(E) < \alpha$, since this condition is equivalent with:

$$\begin{aligned} m(E \mid U_{t^*} C) &> m(E), \\ m(U_{-t^*} E \mid C) &> m(U_{-t^*} E). \end{aligned}$$

But $m(E) < \alpha$ is automatically satisfied for non-vague causal claims (in general, I assume that for non-vague causal claims $0 < m(C), m(E) \ll 1$, and $0 \ll \alpha \leq 1$).

Definitions

Robust retro-causal regularity: $C \overset{-t^*}{\rightsquigarrow} E$ if $m(\{p \in C \mid U_{-t^*}p \in E\}) = m(C)$, $t^* > 0$.

Definitions

Let D be a set, $\mathcal{D}$ a σ-algebra on D, $0 \in T \subseteq \mathbb{R}^{\geq 0}$. A *total transition structure* is a tuple $(D, \mathcal{D}, T, \{\mu_u\}_{u \in T \times \mathcal{D} \times \mathcal{D}^{-\emptyset}})$ satisfying the following conditions:

(i) For all $C, E \in \mathcal{D}$, $C \neq \emptyset$ and all $t \in T$: $0 \leq \mu_{t,E,C} \leq 1$.

(ii) For every fixed $t \in T$ and $C \in \mathcal{D}^{-\emptyset}$, the map $\mu_{t,\cdot,C} : \mathcal{D} \to [0,1]$ defined by $E \mapsto \mu_{t,E,C}$ is a probability measure on $\mathcal{D}$.

(iii) For all $C \in \mathcal{D}^{-\emptyset}$: $\mu_{0,C,C} = 1$.

(iv) *Consistency (Law of Total Probability):* For any $t \in T$, $E \in \mathcal{D}$, and any disjoint non-empty sets $A, B \in \mathcal{D}$:

$$\mu_{t,E,A \cup B} \cdot \mu_{0,A \cup B,D} = \mu_{t,E,A} \cdot \mu_{0,A,D} + \mu_{t,E,B} \cdot \mu_{0,B,D}.$$

Definitions

When $(D, \mathcal{D}, T, \{\mu_u\}_{u \in T \times \mathcal{D} \times \mathcal{D}^{-\emptyset}})$ is a total transition structure and $U \subseteq T \times \mathcal{D} \times \mathcal{D}^{-\emptyset}$, I call $(D, \mathcal{D}, T, \{\mu_u\}_{u \in U})$ a *(partial) transition structure (on domain $\mathcal{D}$, T)*.

Definitions

Let $(D, \mathcal{D}, T, \{\mu_u\}_{u \in U})$ be a transition structure, $(t^*, E, C) \in U$, $t^* > 0$. I say that the *causal regularity from C to E is robust (with characteristic time t^*, and with respect to $(D, \mathcal{D}, T, \{\mu_u\}_{u \in U})$)* — in notation: $C \overset{t^*}{\rightsquigarrow} E$ — if $\mu_{t^*, E, C} = 1$.

Definitions

Let $(D, \mathcal{D}, T, \{\mu_u\}_{u \in U})$ be a partial transition structure and $(P, \Sigma, R, \mathcal{R}, T, m, U_t)$ a dynamical system. I say that $(D, \mathcal{D}, T, \{\mu_u\}_{u \in U})$ *history-supervenes on* $(P, \Sigma, R, \mathcal{R}, T, m, U_t)$ *(on domain* $\mathcal{R}, T$*)*, and I call $(P, \Sigma, R, \mathcal{R}, T, m, U_t)$ the *underlying* dynamical system if:

(i) *State-supervenience:* $\mathcal{D}$ is isomorphic to $\mathcal{R}$ as a σ-algebra (in particular, D corresponds to R under the isomorphism);

(ii) $0 < m(R) < \infty$ (hence, $m(C) < \infty$ for all $C \in \mathcal{R}$), and m *recovers* $\{\mu_u\}_{u \in U}$ in the sense that for all $(t, E, C) \in U$ (note: by definition, $E, C \in \mathcal{D}$ and hence, under the isomorphism assumed in (i), $E, C \in \mathcal{R}$) the so-called *Probability Rule* holds:

$$\mu_{t,E,C} \cdot m(C) = m(U_{-t} E \cap C). \tag{1}$$

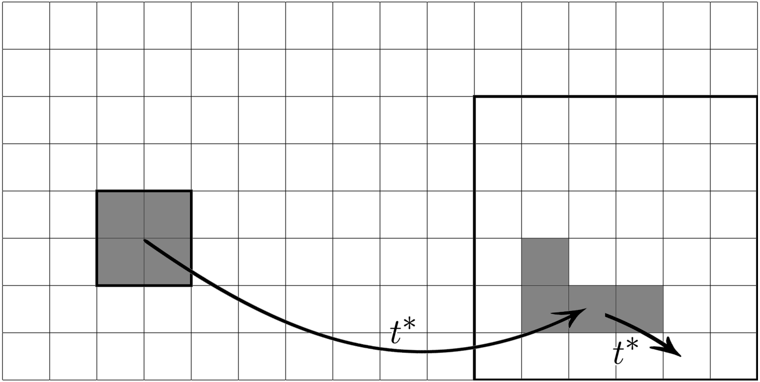
Proposition

Let $(D, \mathcal{D}, T, \{\mu_u\}_{u \in U})$ be a transition structure that history-supervenes on the dynamical system $(P, \Sigma, R, \mathcal{R}, T, m, U_t)$, and let $(t^*, E, C) \in U$ (correspondingly, $E, C \in \mathcal{R}$), $m(C) > 0$. Then $C \underset{\rightsquigarrow}{\overset{t^*}{\rightsquigarrow}} E$ iff $C \overset{t^*}{\rightsquigarrow} E$.

Proposition

Suppose $(P, \Sigma, R, \mathcal{R}, T, m, U_t)$ is a measure-preserving dynamical system, $(R, \mathcal{R}, T, \{\mu_u\}_{u \in U})$ history-supervenes on $(P, \Sigma, R, \mathcal{R}, T, m, U_t)$, $(0, C, R) \in U$, $(0, E, R) \in U$, and $C \overset{t^*}{\rightsquigarrow} E$. Then $\mu_{0,C,R} \leq \mu_{0,E,R}$.

No rebound



Strict increase of entropy 3: $TC = C$, $TE = E$, no rebound