Entanglement and quench dynamics in the thermally perturbed tricritical fixed point

Csilla Király[1,2,3], Máté Lencsés[3]

1. Department of Theoretical Physics, Institute of Physics, Budapest University of Technology and Economics, Budapest, Hungary
2. BME-MTA Statistical Field Theory 'Lendület' Research Group, Budapest University of Technology and Economics, Budapest, Hungary
3. HUN-REN Wigner Research Centre for Physics, Budapest, Hungary

What is our goal?

- statistical physics: systems with many degrees of freedom $\longrightarrow$ exponential complexity

What is our goal?

- statistical physics: systems with many degrees of freedom $\longrightarrow$ exponential complexity
- the microscopical description unmanageable

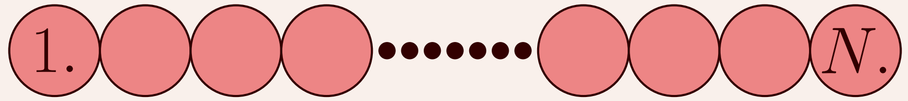
What is our goal?

- statistical physics: systems with many degrees of freedom $\longrightarrow$ exponential complexity
- the microscopical description unmanageable
- goal: collective, universal behavior

What is our goal?

- statistical physics: systems with many degrees of freedom $\longrightarrow$ exponential complexity
- the microscopical description unmanageable
- goal: collective, universal behavior $\Longrightarrow$ spin chains: 1D quantum systems

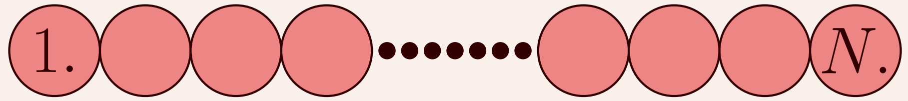
Spin chain with N sites



$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \otimes \mathcal{H}_N$$

$$\forall i: \;\; \dim \mathcal{H}_i = d \;\; \Rightarrow \;\; \dim \mathcal{H} = d^N$$

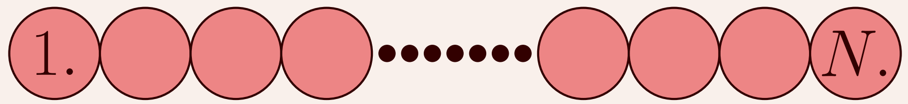
Spin chain with N sites



$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \otimes \mathcal{H}_N$$
$$\forall i: \ \dim \mathcal{H}_i = d \ \Rightarrow \ \dim \mathcal{H} = d^N$$

- the pure state of the system: $|\psi\rangle = \sum_{\alpha_1, \alpha_2, \ldots, \alpha_N = 1}^{d} \psi^{\alpha_1 \alpha_2 \ldots \alpha_N} |\alpha_1\rangle \otimes |\alpha_2\rangle \otimes \ldots \otimes |\alpha_N\rangle$

Spin chain with N sites

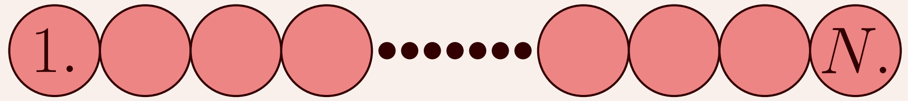


$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \otimes \mathcal{H}_N$$
$$\forall i: \ \dim\mathcal{H}_i = d \ \Rightarrow \ \dim\mathcal{H} = d^N$$

- the pure state of the system: $|\psi\rangle = \sum_{\alpha_1,\alpha_2,\ldots,\alpha_N=1}^{d} \psi^{\alpha_1\alpha_2\ldots\alpha_N} |\alpha_1\rangle \otimes |\alpha_2\rangle \otimes \ldots \otimes |\alpha_N\rangle$

$\hookrightarrow d^N$ pieces of $\psi^{\alpha_1\alpha_2\ldots\alpha_N} \Rightarrow$ exact calculations are limited

Spin chain with N sites



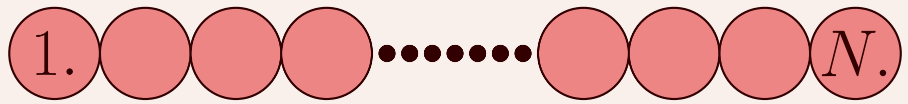
$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \otimes \mathcal{H}_N$$
$$\forall i: \ \dim \mathcal{H}_i = d \ \Rightarrow \ \dim \mathcal{H} = d^N$$

- the pure state of the system: $|\psi\rangle = \sum_{\alpha_1, \alpha_2, \ldots, \alpha_N = 1}^{d} \psi^{\alpha_1 \alpha_2 \ldots \alpha_N} |\alpha_1\rangle \otimes |\alpha_2\rangle \otimes \ldots \otimes |\alpha_N\rangle$

$\hookrightarrow d^N$ pieces of $\psi^{\alpha_1 \alpha_2 \ldots \alpha_N} \Rightarrow$ exact calculations are limited

- can we find a more efficient way to describe these states?

Spin chain with N sites



$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \otimes \mathcal{H}_N$$

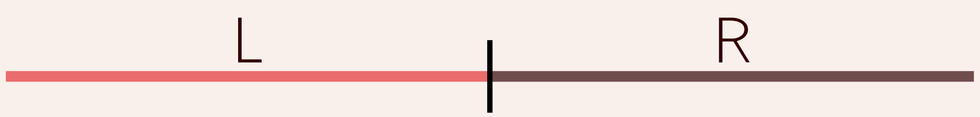
$$\forall i: \ \dim\mathcal{H}_i = d \ \Rightarrow \ \dim\mathcal{H} = d^N$$

- the pure state of the system: $|\psi\rangle = \sum_{\alpha_1,\alpha_2,\ldots,\alpha_N=1}^{d} \psi^{\alpha_1\alpha_2\ldots\alpha_N} |\alpha_1\rangle \otimes |\alpha_2\rangle \otimes \ldots \otimes |\alpha_N\rangle$

$\hookrightarrow d^N$ pieces of $\psi^{\alpha_1\alpha_2\ldots\alpha_N} \Rightarrow$ exact calculations are limited

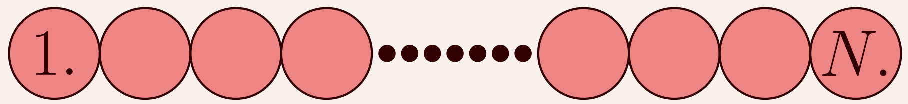
- can we find a more efficient way to describe these states?

Area law and Schmidt states



$$\mathcal{H} = \mathcal{H}_L \otimes \mathcal{H}_R \qquad |\psi\rangle \in \mathcal{H}$$

Spin chain with N sites



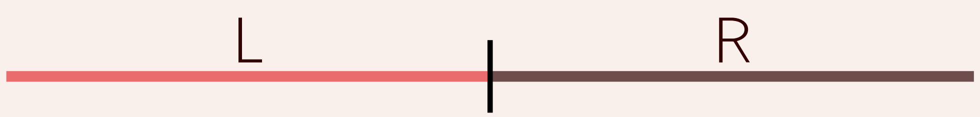
$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \otimes \mathcal{H}_N$$
$$\forall i: \ \dim \mathcal{H}_i = d \ \Rightarrow \ \dim \mathcal{H} = d^N$$

- the pure state of the system: $|\psi\rangle = \sum_{\alpha_1, \alpha_2, \ldots, \alpha_N = 1}^{d} \psi^{\alpha_1 \alpha_2 \ldots \alpha_N} |\alpha_1\rangle \otimes |\alpha_2\rangle \otimes \ldots \otimes |\alpha_N\rangle$

$\hookrightarrow d^N$ pieces of $\psi^{\alpha_1 \alpha_2 \ldots \alpha_N} \Rightarrow$ exact calculations are limited

- can we find a more efficient way to describe these states?

Area law and Schmidt states

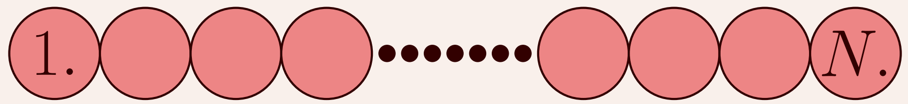


$$\mathcal{H} = \mathcal{H}_L \otimes \mathcal{H}_R \qquad |\psi\rangle \in \mathcal{H}$$

Schmidt decomposition: $|\psi\rangle = \sum_{\alpha=1}^{\chi} \Lambda_\alpha |\alpha\rangle_L \otimes |\alpha\rangle_R$, where $\forall \alpha: \ \Lambda_\alpha \geq 0$

$$\sum_\alpha \Lambda_\alpha^2 = 1$$

Spin chain with N sites



$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \dots \otimes \mathcal{H}_N$$

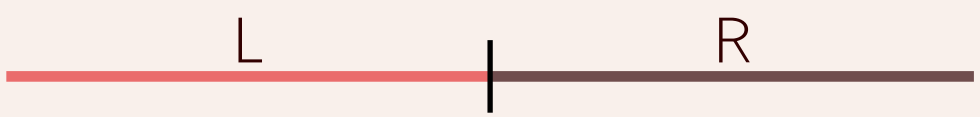
$$\forall i: \ \dim\mathcal{H}_i = d \ \Rightarrow \ \dim\mathcal{H} = d^N$$

- the pure state of the system: $|\psi\rangle = \sum_{\alpha_1,\alpha_2,\dots,\alpha_N=1}^{d} \psi^{\alpha_1\alpha_2\dots\alpha_N} |\alpha_1\rangle \otimes |\alpha_2\rangle \otimes \dots \otimes |\alpha_N\rangle$

$\hookrightarrow d^N$ pieces of $\psi^{\alpha_1\alpha_2\dots\alpha_N} \Rightarrow$ exact calculations are limited

- can we find a more efficient way to describe these states?

Area law and Schmidt states



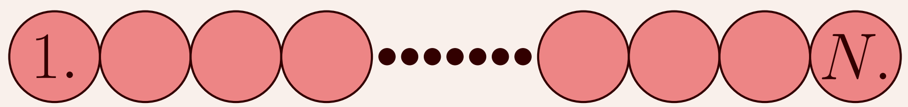
$$\mathcal{H} = \mathcal{H}_L \otimes \mathcal{H}_R \qquad |\psi\rangle \in \mathcal{H}$$

Schmidt decomposition: $|\psi\rangle = \sum_{\alpha=1}^{\chi} \Lambda_\alpha |\alpha\rangle_L \otimes |\alpha\rangle_R$, where $\forall\, \alpha: \ \Lambda_\alpha \geq 0$

$$\sum_\alpha \Lambda_\alpha^2 = 1$$

- entanglement entropy: $S = -\sum_\alpha \Lambda_\alpha^2 \log(\Lambda_\alpha^2)$

Spin chain with N sites



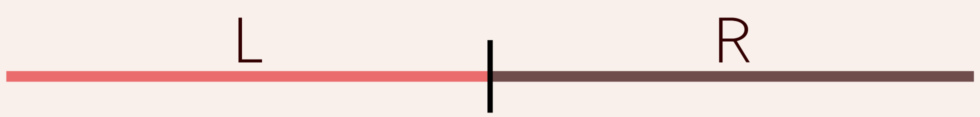
$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \otimes \mathcal{H}_N$$
$$\forall i: \ \dim \mathcal{H}_i = d \quad \Rightarrow \quad \dim \mathcal{H} = d^N$$

- the pure state of the system: $$|\psi\rangle = \sum_{\alpha_1, \alpha_2, \ldots, \alpha_N = 1}^{d} \psi^{\alpha_1 \alpha_2 \ldots \alpha_N} |\alpha_1\rangle \otimes |\alpha_2\rangle \otimes \ldots \otimes |\alpha_N\rangle$$

$\hookrightarrow d^N$ pieces of $\psi^{\alpha_1 \alpha_2 \ldots \alpha_N} \Rightarrow$ exact calculations are limited

- can we find a more efficient way to describe these states?

Area law and Schmidt states



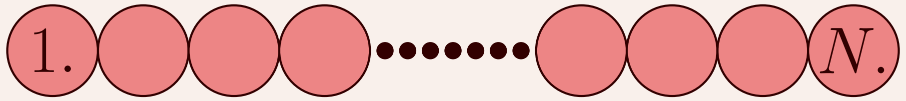
$$\mathcal{H} = \mathcal{H}_L \otimes \mathcal{H}_R \qquad |\psi\rangle \in \mathcal{H}$$

Schmidt decomposition: $|\psi\rangle = \sum_{\alpha=1}^{\chi} \Lambda_\alpha |\alpha\rangle_L \otimes |\alpha\rangle_R$, where $\forall\, \alpha: \ \Lambda_\alpha \geq 0$

$$\sum_\alpha \Lambda_\alpha^2 = 1$$

- entanglement entropy: $S = -\sum_\alpha \Lambda_\alpha^2 \log\left(\Lambda_\alpha^2\right)$
- area law: $\sum_{\alpha=1}^{\chi} \Lambda_\alpha |\alpha\rangle_L \otimes |\alpha\rangle_R \approx |\psi\rangle$, where $\chi \ll d^N$

Spin chain with N sites



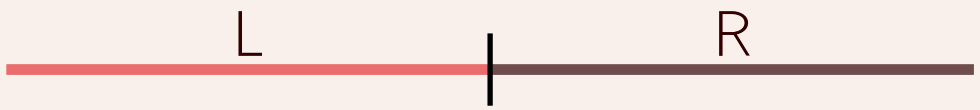
$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \otimes \mathcal{H}_N$$
$$\forall i: \ \dim\mathcal{H}_i = d \ \Rightarrow \ \dim\mathcal{H} = d^N$$

- the pure state of the system: $|\psi\rangle = \sum_{\alpha_1,\alpha_2,\ldots,\alpha_N=1}^{d} \psi^{\alpha_1\alpha_2\ldots\alpha_N} |\alpha_1\rangle \otimes |\alpha_2\rangle \otimes \ldots \otimes |\alpha_N\rangle$

$\hookrightarrow d^N$ pieces of $\psi^{\alpha_1\alpha_2\ldots\alpha_N} \Rightarrow$ exact calculations are limited

- can we find a more efficient way to describe these states?

Area law and Schmidt states



$$\mathcal{H} = \mathcal{H}_L \otimes \mathcal{H}_R \qquad |\psi\rangle \in \mathcal{H}$$

Schmidt decomposition: $|\psi\rangle = \sum_{\alpha=1}^{\chi} \Lambda_\alpha |\alpha\rangle_L \otimes |\alpha\rangle_R$, where $\forall\, \alpha: \ \Lambda_\alpha \geq 0$

$$\sum_\alpha \Lambda_\alpha^2 = 1$$

- entanglement entropy: $S = -\sum_\alpha \Lambda_\alpha^2 \log\left(\Lambda_\alpha^2\right)$
- area law: $\sum_{\alpha=1}^{\chi} \Lambda_\alpha |\alpha\rangle_L \otimes |\alpha\rangle_R \approx |\psi\rangle$, where $\chi \ll d^N$

$\Longrightarrow$ This property allows for the Matrix Product State (MPS) representation of one-dimensional quantum states.

Matrix product state

$$|\psi\rangle = \sum_{j_1,\ldots,j_N} A^{[1]j_1} A^{[2]j_2} \ldots\ldots A^{[N]j_N} |j_1, \ldots, j_N\rangle$$

Matrix product state

$$|\psi\rangle = \sum_{j_1,\dots,j_N} A^{[1]j_1} A^{[2]j_2} \dots\dots A^{[N]j_N} |j_1, \dots, j_N\rangle$$

- linear algebra theorem $\longrightarrow A^{[n]j_n} = \Gamma^{[n]j_n} \Lambda^{[n]}$

$$\Rightarrow |\psi\rangle = \sum_{j_1,\dots,j_N} \Gamma^{[1]j_1} \Lambda^{[1]} \Gamma^{[2]j_2} \Lambda^{[2]} \dots \Gamma^{[N]j_N} \Lambda^{[N]} |j_1, j_2, \dots, j_N\rangle$$

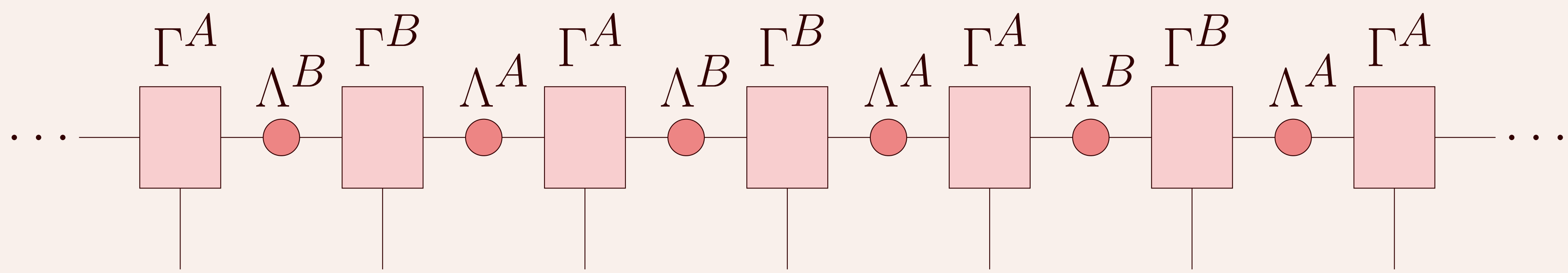
Matrix product state

$$|\psi\rangle = \sum_{j_1,\dots,j_N} A^{[1]j_1}A^{[2]j_2}.....A^{[N]j_N}\,|j_1,...,j_N\rangle$$

- linear algebra theorem $\longrightarrow A^{[n]j_n} = \Gamma^{[n]j_n}\Lambda^{[n]}$

$$\Rightarrow |\psi\rangle = \sum_{j_1,\dots,j_N} \Gamma^{[1]j_1}\Lambda^{[1]}\Gamma^{[2]j_2}\Lambda^{[2]}...\Gamma^{[N]j_N}\Lambda^{[N]}\,|j_1,j_2,...,j_N\rangle$$

- if the spin chain is infinite and translational invariant: $\Lambda^A, \Lambda^B, \Gamma^A, \Gamma^B$



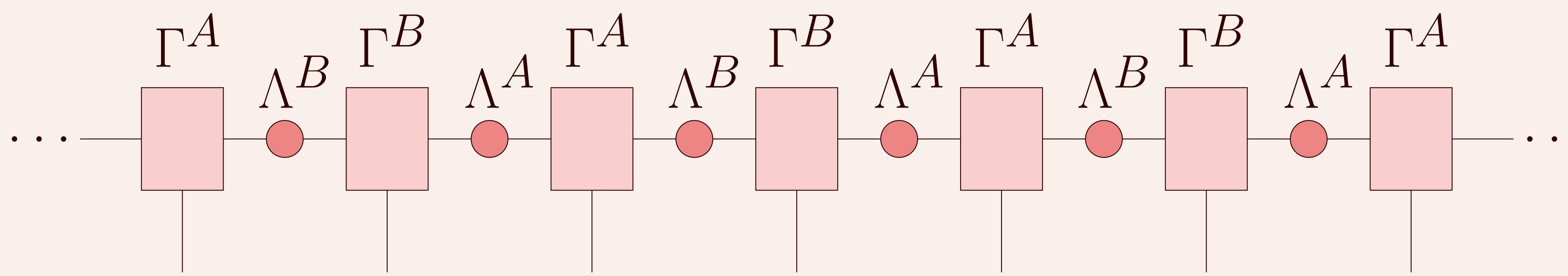
Matrix product state

$$|\psi\rangle = \sum_{j_1,\dots,j_N} A^{[1]j_1} A^{[2]j_2} \dots\dots A^{[N]j_N} \, |j_1, \dots, j_N\rangle$$

- linear algebra theorem $\longrightarrow$ $A^{[n]j_n} = \Gamma^{[n]j_n} \Lambda^{[n]}$

$$\Rightarrow |\psi\rangle = \sum_{j_1,\dots,j_N} \Gamma^{[1]j_1} \Lambda^{[1]} \Gamma^{[2]j_2} \Lambda^{[2]} \dots \Gamma^{[N]j_N} \Lambda^{[N]} \, |j_1, j_2, \dots, j_N\rangle$$

- if the spin chain is infinite and translational invariant: Λ^A, Λ^B, Γ^A, Γ^B



Time evolution with infinite Time Evolving Block Decimation (iTEBD) algorithm

$$|\psi(N\delta t)\rangle = \prod_{i=1}^{N} U(\delta t) \, |\psi(0)\rangle \, , \text{ where } U(\delta t) = e^{-iH\delta t}$$

and $\delta t \in \mathbb{R}$ or $\delta t = -i\delta\tau \quad (\delta\tau \in \mathbb{R})$

- imaginary time evolution $\longrightarrow$ define ground state
- real time evolution $\longrightarrow$ post-quench dynamics

Blume–Capel spin-1 quantum chain

$$H_{BC} = \xi \sum_{i=1}^{N} \left[\alpha (S_i^x)^2 + \beta S_i^z + \gamma (S_i^z)^2 - S_i^x S_{i+1}^x \right], \quad \text{spins: } 0, \pm 1$$

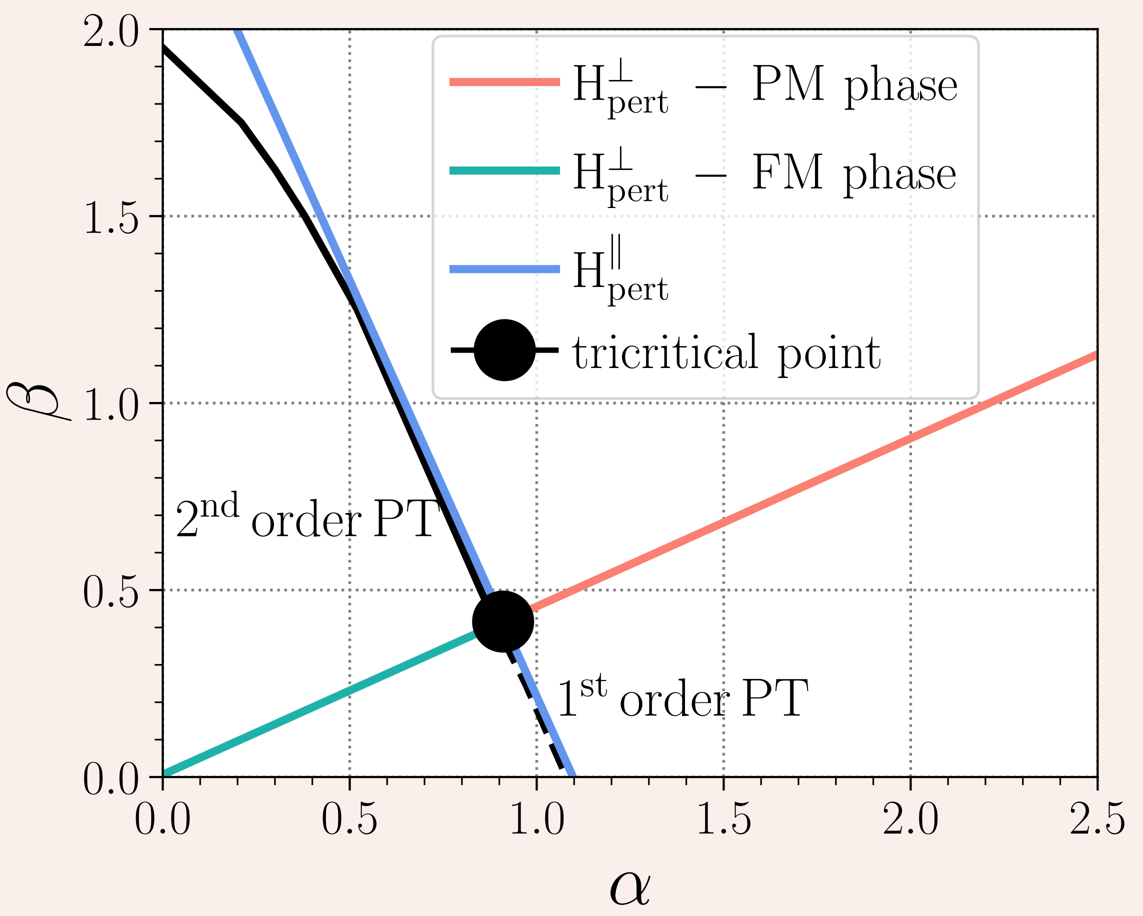
- tricritical point: $\alpha_{tc} = 0,910207(4)$, $\beta_{tc} = 0,415685(6)$, $\gamma = 0$ [G. von Gehlen, 1989]
- matrices: $S_i^x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}_i, \quad S_i^z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}_i$

Blume–Capel spin-1 quantum chain

$$H_{BC} = \xi \sum_{i=1}^{N} \left[\alpha (S_i^x)^2 + \beta S_i^z + \gamma (S_i^z)^2 - S_i^x S_{i+1}^x\right], \quad \text{spins: } 0, \pm 1$$

- tricritical point: $\alpha_{tc} = 0,910207(4)$, $\beta_{tc} = 0,415685(6)$, $\gamma = 0$ [G. von Gehlen, 1989]
- matrices: $S_i^x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}_i, \quad S_i^z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}_i$

Phase diagram

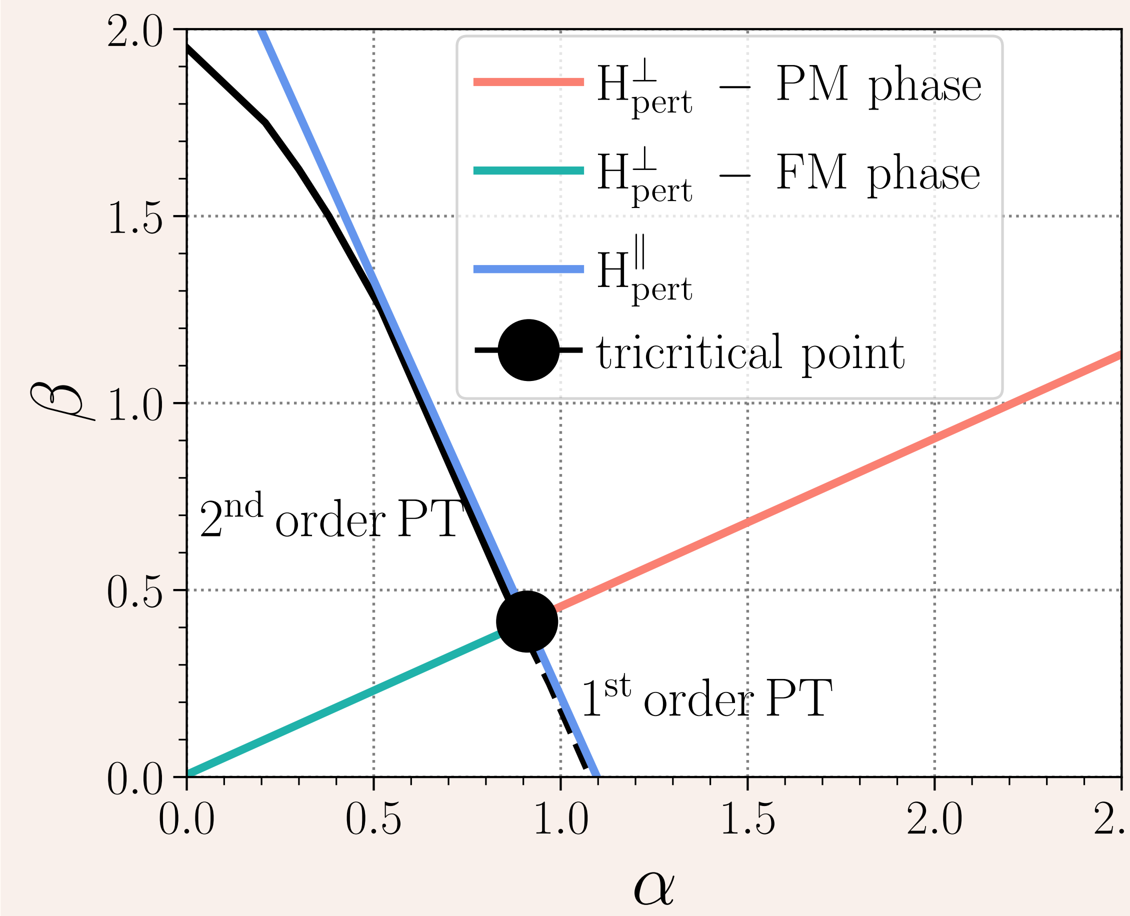


Blume–Capel spin-1 quantum chain

$$H_{BC} = \xi \sum_{i=1}^{N} \left[\alpha (S_i^x)^2 + \beta S_i^z + \gamma (S_i^z)^2 - S_i^x S_{i+1}^x \right], \quad \text{spins: } 0, \pm 1$$

- tricritical point: $\alpha_{tc} = 0,910207(4)$, $\beta_{tc} = 0,415685(6)$, $\gamma = 0$ [G. von Gehlen, 1989]
- matrices: $S_i^x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}_i$, $S_i^z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}_i$

Phase diagram



The relation between the scaling limit of the spin chains and the field theories

- spin chains: 1D quantum systems $\longrightarrow$ special property: critical points can be described with CFT
- scaling limit: at a critical point, the correlation length is divergent, so the lattice spacing becomes irrelevant, and the spin chain converges to a continuous quantum field theory in the scaling limit.

$\Longrightarrow$ **This connection makes it possible to obtain field theory results from a spin chain.**

- measurable quantities (spin chain operators) $\longleftrightarrow$ QFT operators $\Longleftarrow$ symmetries+dimensional analysis

E_7 quantum field theory

field	name	weights	η	N
σ	leading magnetic field	$\left(\frac{3}{80},\frac{3}{80}\right)$	$-\sigma$	μ
ϵ	thermal field	$\left(\frac{1}{10},\frac{1}{10}\right)$	ϵ	$-\epsilon$

$$H_{E7} = H_{\mathsf{TIM}} + \tilde{\lambda} \int dx\, \epsilon(x)$$

exact	numerical	parity	excitation
m_1	1	odd	kink
$m_2 = 2m_1 \cos\frac{5\pi}{18}$	$1.285(6)$	even	particle
$m_3 = 2m_1 \cos\frac{\pi}{9}$	$1.879(4)$	odd	kink
$m_4 = 2m_1 \cos\frac{\pi}{18}$	$1.969(6)$	even	particle
$m_5 = 4m_1 \cos\frac{\pi}{18}\cos\frac{5\pi}{18}$	$2.532(1)$	even	particle
$m_6 = 4m_1 \cos\frac{2\pi}{9}\cos\frac{\pi}{9}$	$2.879(4)$	odd	kink
$m_7 = 4m_1 \cos\frac{\pi}{18}\cos\frac{\pi}{9}$	$3.701(7)$	even	particle

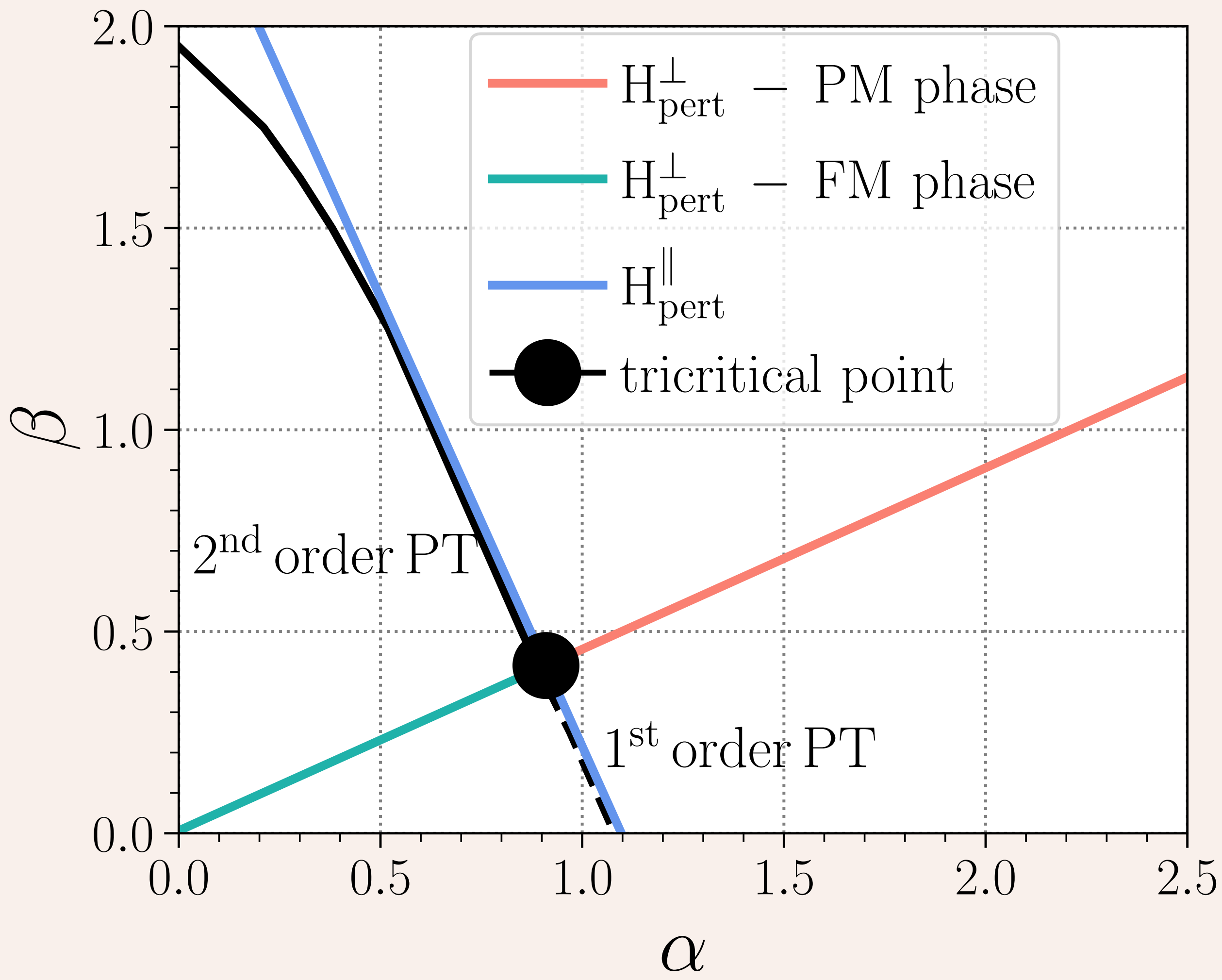
E_7 quantum field theory

field	name	weights	η	N
σ	leading magnetic field	$\left(\frac{3}{80},\frac{3}{80}\right)$	$-\sigma$	μ
ϵ	thermal field	$\left(\frac{1}{10},\frac{1}{10}\right)$	ϵ	$-\epsilon$

$$H_{E7} = H_{\mathsf{TIM}} + \tilde{\lambda}\int dx\,\epsilon(x)$$

exact	numerical	parity	excitation
m_1	1	odd	kink
$m_2 = 2m_1\cos\frac{5\pi}{18}$	1.285(6)	even	particle
$m_3 = 2m_1\cos\frac{\pi}{9}$	1.879(4)	odd	kink
$m_4 = 2m_1\cos\frac{\pi}{18}$	1.969(6)	even	particle
$m_5 = 4m_1\cos\frac{\pi}{18}\cos\frac{5\pi}{18}$	2.532(1)	even	particle
$m_6 = 4m_1\cos\frac{2\pi}{9}\cos\frac{\pi}{9}$	2.879(4)	odd	kink
$m_7 = 4m_1\cos\frac{\pi}{18}\cos\frac{\pi}{9}$	3.701(7)	even	particle

Phase diagram



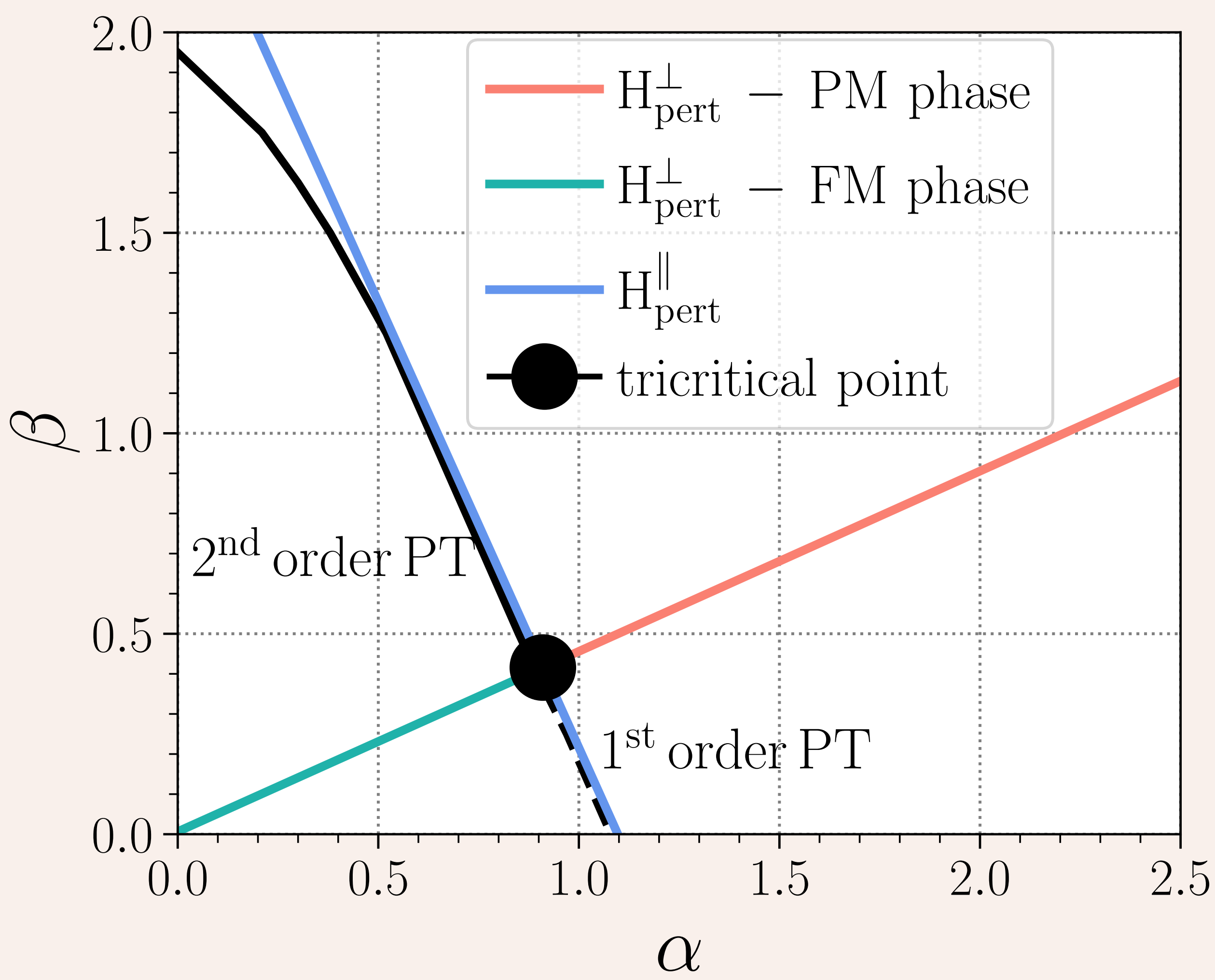
E_7 quantum field theory

field	name	weights	η	N
σ	leading magnetic field	$\left(\frac{3}{80}, \frac{3}{80}\right)$	$-\sigma$	μ
ϵ	thermal field	$\left(\frac{1}{10}, \frac{1}{10}\right)$	ϵ	$-\epsilon$

$$H_{E7} = H_{\mathsf{TIM}} + \tilde{\lambda} \int dx\, \epsilon(x)$$

exact	numerical	parity	excitation
m_1	1	odd	kink
$m_2 = 2m_1 \cos\frac{5\pi}{18}$	1.285(6)	even	particle
$m_3 = 2m_1 \cos\frac{\pi}{9}$	1.879(4)	odd	kink
$m_4 = 2m_1 \cos\frac{\pi}{18}$	1.969(6)	even	particle
$m_5 = 4m_1 \cos\frac{\pi}{18} \cos\frac{5\pi}{18}$	2.532(1)	even	particle
$m_6 = 4m_1 \cos\frac{2\pi}{9} \cos\frac{\pi}{9}$	2.879(4)	odd	kink
$m_7 = 4m_1 \cos\frac{\pi}{18} \cos\frac{\pi}{9}$	3.701(7)	even	particle

Phase diagram



The perturbation

$$H_{E7} = \xi \left\{ \sum_i \left[\alpha_{tc} (S_i^x)^2 + \beta_{tc} S_i^z - S_i^x S_{i+1}^x \right] + \lambda H^{\perp} \right\}$$

$$H^{\perp} = \sum_i h_i^{\perp} = \sum_i \left(-\sin\theta (S_i^x)^2 + \cos\theta S_i^z \right)$$

with $\quad \tan\theta \approx 2.224$

E_7 quantum field theory

field	name	weights	η	N
σ	leading magnetic field	$\left(\frac{3}{80}, \frac{3}{80}\right)$	$-\sigma$	μ
ϵ	thermal field	$\left(\frac{1}{10}, \frac{1}{10}\right)$	ϵ	$-\epsilon$

$$H_{E7} = H_{\mathsf{TIM}} + \tilde{\lambda} \int dx\, \epsilon(x)$$

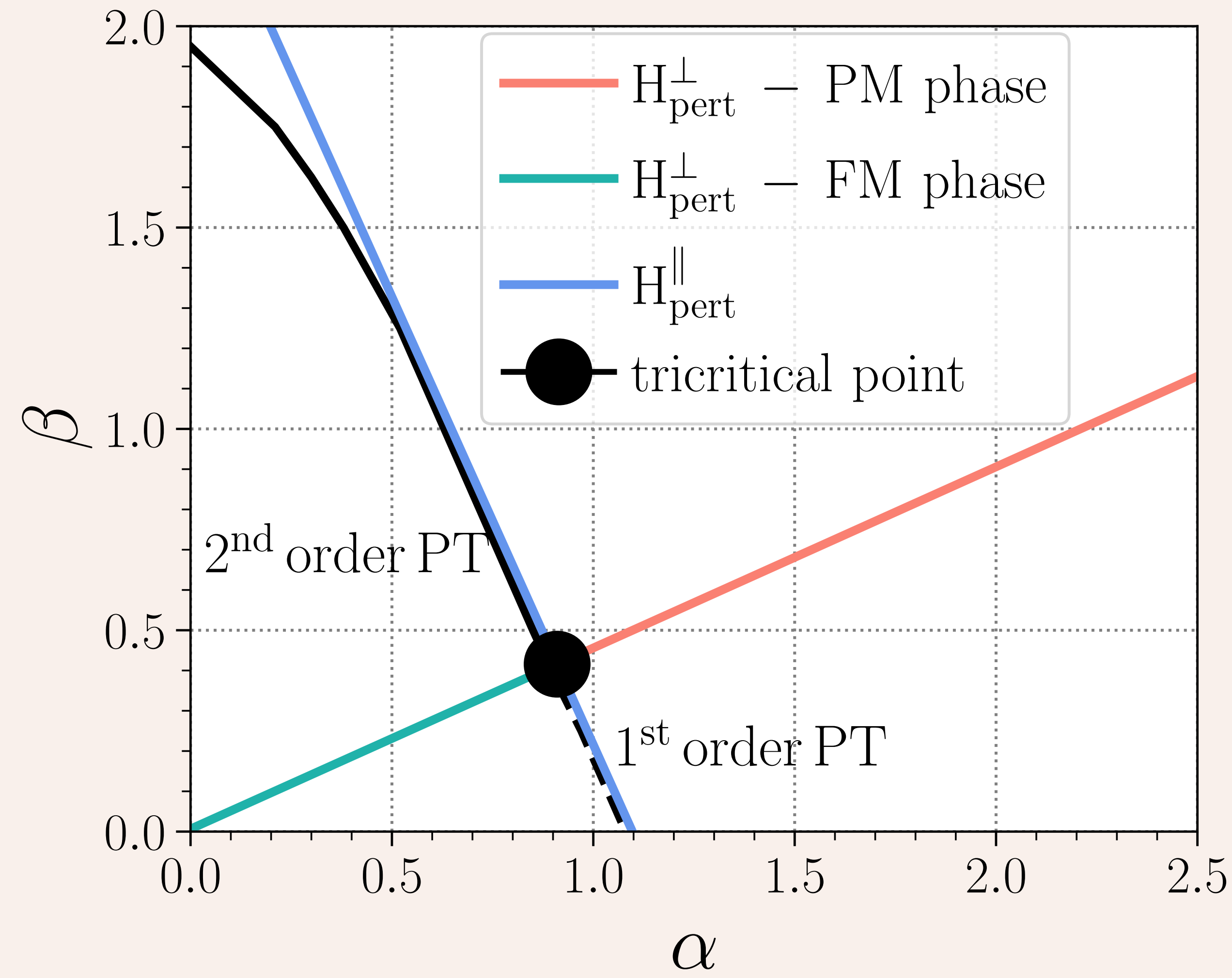
exact	numerical	parity	excitation
m_1	1	odd	kink
$m_2 = 2m_1 \cos\frac{5\pi}{18}$	1.285(6)	even	particle
$m_3 = 2m_1 \cos\frac{\pi}{9}$	1.879(4)	odd	kink
$m_4 = 2m_1 \cos\frac{\pi}{18}$	1.969(6)	even	particle
$m_5 = 4m_1 \cos\frac{\pi}{18} \cos\frac{5\pi}{18}$	2.532(1)	even	particle
$m_6 = 4m_1 \cos\frac{2\pi}{9} \cos\frac{\pi}{9}$	2.879(4)	odd	kink
$m_7 = 4m_1 \cos\frac{\pi}{18} \cos\frac{\pi}{9}$	3.701(7)	even	particle

Operator correspondences

$$\sigma(x = na) = a^{-3/40} \bar{s} S_n^x$$

$$\epsilon(x = na) = a^{-1/5} \bar{e} \mathcal{E}_n \approx a^{-1/5} \bar{e} h_n^{\perp}$$

Phase diagram



The perturbation

$$H_{E7} = \xi \left\{ \sum_i \left[\alpha_{tc} (S_i^x)^2 + \beta_{tc} S_i^z - S_i^x S_{i+1}^x \right] + \lambda H^{\perp} \right\}$$

$$H^{\perp} = \sum_i h_i^{\perp} = \sum_i \left(-\sin\theta (S_i^x)^2 + \cos\theta S_i^z \right)$$

with $\tan\theta \approx 2.224$

Post-quench dynamics

Why study quantum quenches?

- in real systems the parameters change over time
- equilibrium → non-equilibrium dynamics
- quench = controlled, well-defined non-equilibrium protocol

Why study quantum quenches?

- in real systems the parameters change over time
- equilibrium → non-equilibrium dynamics
- quench = controlled, well-defined non-equilibrium protocol

The derivation of the post-quench dynamics of expectation value of an operator in the QFT

$H_{\text{pre}} = H_{\text{CFT}} + \tilde{\lambda} \int dx\, \mathcal{O}^{(q)}(x) \quad \longrightarrow \quad$ ground state: $|\Omega\rangle_{\text{pre}}$

Why study quantum quenches?

- in real systems the parameters change over time
- equilibrium → non-equilibrium dynamics
- quench = controlled, well-defined non-equilibrium protocol

The derivation of the post-quench dynamics of expectation value of an operator in the QFT

$H_{\mathsf{pre}} = H_{\mathsf{CFT}} + \tilde{\lambda} \int dx\, \mathcal{O}^{(q)}(x) \quad \longrightarrow \quad$ ground state: $|\Omega\rangle_{\mathsf{pre}}$

$H_{\mathsf{post}} = H_{\mathsf{CFT}} + (\tilde{\lambda} + \tilde{\delta}_\lambda) \int dx\, \mathcal{O}^{(q)}(x) \quad \longrightarrow \quad |\Psi(t)\rangle = e^{-iH_{\mathsf{post}}t} |\Omega\rangle_{\mathsf{pre}}$

Why study quantum quenches?

- in real systems the parameters change over time
- equilibrium → non-equilibrium dynamics
- quench = controlled, well-defined non-equilibrium protocol

The derivation of the post-quench dynamics of expectation value of an operator in the QFT

$$H_{\text{pre}} = H_{\text{CFT}} + \tilde{\lambda} \int dx\, \mathcal{O}^{(q)}(x) \quad \longrightarrow \quad \text{ground state: } |\Omega\rangle_{\text{pre}}$$

$$H_{\text{post}} = H_{\text{CFT}} + (\tilde{\lambda} + \tilde{\delta}_\lambda) \int dx\, \mathcal{O}^{(q)}(x) \quad \longrightarrow \quad |\Psi(t)\rangle = e^{-iH_{\text{post}}t} |\Omega\rangle_{\text{pre}}$$

$$H_{\text{pre}} = H_{\text{post}} + (\underbrace{-\tilde{\delta}_\lambda}_{\text{small}}) \int dx\, \mathcal{O}^{(q)}(x)$$

Why study quantum quenches?

- in real systems the parameters change over time
- equilibrium → non-equilibrium dynamics
- quench = controlled, well-defined non-equilibrium protocol

The derivation of the post-quench dynamics of expectation value of an operator in the QFT

$$H_{\text{pre}} = H_{\text{CFT}} + \tilde{\lambda} \int dx\, \mathcal{O}^{(q)}(x) \quad \longrightarrow \quad \text{ground state: } |\Omega\rangle_{\text{pre}}$$

$$H_{\text{post}} = H_{\text{CFT}} + (\tilde{\lambda} + \tilde{\delta}_\lambda) \int dx\, \mathcal{O}^{(q)}(x) \quad \longrightarrow \quad |\Psi(t)\rangle = e^{-iH_{\text{post}}t} |\Omega\rangle_{\text{pre}}$$

$$\hookrightarrow H_{\text{pre}} = H_{\text{post}} + \underbrace{(-\tilde{\delta}_\lambda)}_{\text{small}} \int dx\, \mathcal{O}^{(q)}(x)$$

$\Longrightarrow$ Rayleigh-Schrödinger perturbation theory : $\quad |\Omega\rangle_{\text{pre}} = |\Omega\rangle_{\text{post}} + 2\pi\delta_\lambda \sum_{l \neq \Omega} \frac{\delta(p_l^{\text{post}})}{E_l^{\text{post}}} \left(F_l^{\mathcal{O}^{(q)}} \right)^* |l\rangle_{\text{post}} + O(\delta_\lambda^2)\,,$

Why study quantum quenches?

- in real systems the parameters change over time
- equilibrium → non-equilibrium dynamics
- quench = controlled, well-defined non-equilibrium protocol

The derivation of the post-quench dynamics of expectation value of an operator in the QFT

$$H_{\text{pre}} = H_{\text{CFT}} + \tilde{\lambda} \int dx\, \mathcal{O}^{(q)}(x) \longrightarrow \text{ground state: } |\Omega\rangle_{\text{pre}}$$

$$H_{\text{post}} = H_{\text{CFT}} + (\tilde{\lambda} + \tilde{\delta}_\lambda) \int dx\, \mathcal{O}^{(q)}(x) \longrightarrow |\Psi(t)\rangle = e^{-iH_{\text{post}}t}\, |\Omega\rangle_{\text{pre}}$$

$$\hookrightarrow H_{\text{pre}} = H_{\text{post}} + \underbrace{(-\tilde{\delta}_\lambda)}_{\text{small}} \int dx\, \mathcal{O}^{(q)}(x)$$

$$\Longrightarrow \text{Rayleigh-Schrödinger perturbation theory}: \quad |\Omega\rangle_{\text{pre}} = |\Omega\rangle_{\text{post}} + 2\pi\delta_\lambda \sum_{l \neq \Omega} \frac{\delta(p_l^{\text{post}})}{E_l^{\text{post}}} \Big(\overbrace{F_l^{\mathcal{O}^{(q)}}}^{{}_{\text{post}}\langle\Omega|\,\mathcal{O}^{(q)}(0)\,|l\rangle_{\text{post}}}\Big)^{*} |l\rangle_{\text{post}} + O(\delta_\lambda^2)\,,$$

where
$$\sum_{l \neq \Omega} := \sum_{n=0}^{\infty} \int_{p_1 < p_2 < \cdots < p_n} \prod_{j=1}^{n} \frac{dp_j}{2\pi \cdot \tilde{e}(p_j)} = \sum_a \int \frac{dp_a}{2\pi \cdot \tilde{e}(p_a)} + \sum_{a,b} \int \frac{dp_a}{2\pi \cdot \tilde{e}(p_a)} \frac{dp_b}{2\pi \cdot \tilde{e}(p_b)} + \ldots$$

Why study quantum quenches?

- in real systems the parameters change over time
- equilibrium → non-equilibrium dynamics
- quench = controlled, well-defined non-equilibrium protocol

The derivation of the post-quench dynamics of expectation value of an operator in the QFT

$$H_{\text{pre}} = H_{\text{CFT}} + \tilde{\lambda} \int dx\, \mathcal{O}^{(q)}(x) \quad \longrightarrow \quad \text{ground state: } |\Omega\rangle_{\text{pre}}$$

$$H_{\text{post}} = H_{\text{CFT}} + (\tilde{\lambda} + \tilde{\delta}_\lambda) \int dx\, \mathcal{O}^{(q)}(x) \quad \longrightarrow \quad |\Psi(t)\rangle = e^{-iH_{\text{post}}t} |\Omega\rangle_{\text{pre}}$$

$$H_{\text{pre}} = H_{\text{post}} + \underbrace{(-\tilde{\delta}_\lambda)}_{\text{small}} \int dx\, \mathcal{O}^{(q)}(x)$$

$$\Longrightarrow \text{Rayleigh-Schrödinger perturbation theory}: \quad |\Omega\rangle_{\text{pre}} = |\Omega\rangle_{\text{post}} + 2\pi\delta_\lambda \sum_{l\neq\Omega} \frac{\delta(p_l^{\text{post}})}{E_l^{\text{post}}} \Big(\underbrace{F_l^{\mathcal{O}^{(q)}}}_{{}_{\text{post}}\langle\Omega|\mathcal{O}^{(q)}(0)|l\rangle_{\text{post}}}\Big)^* |l\rangle_{\text{post}} + O(\delta_\lambda^2),$$

where
$$\sum_{l\neq\Omega} := \sum_{n=0}^{\infty} \int_{p_1<p_2<\dots<p_n} \prod_{j=1}^{n} \frac{dp_j}{2\pi\cdot\tilde{e}(p_j)} = \sum_a \int \frac{dp_a}{2\pi\cdot\tilde{e}(p_a)} + \sum_{a,b} \int \frac{dp_a}{2\pi\cdot\tilde{e}(p_a)} \frac{dp_b}{2\pi\cdot\tilde{e}(p_b)} + \dots$$

What we want to calculate: $\langle\mathcal{O}(t)\rangle_{\text{post}} = \langle\Psi(t)|\mathcal{O}(0)|\Psi(t)\rangle$

Why study quantum quenches?

- in real systems the parameters change over time
- equilibrium → non-equilibrium dynamics
- quench = controlled, well-defined non-equilibrium protocol

The derivation of the post-quench dynamics of expectation value of an operator in the QFT

$H_{\text{pre}} = H_{\text{CFT}} + \tilde{\lambda} \int dx\, \mathcal{O}^{(q)}(x) \longrightarrow$ ground state: $|\Omega\rangle_{\text{pre}}$

$H_{\text{post}} = H_{\text{CFT}} + (\tilde{\lambda} + \tilde{\delta}_\lambda) \int dx\, \mathcal{O}^{(q)}(x) \longrightarrow \boxed{|\Psi(t)\rangle = e^{-iH_{\text{post}}t} |\Omega\rangle_{\text{pre}}}$

$\hookrightarrow H_{\text{pre}} = H_{\text{post}} + \underbrace{(-\tilde{\delta}_\lambda)}_{\text{small}} \int dx\, \mathcal{O}^{(q)}(x)$

$\Longrightarrow$ Rayleigh-Schrödinger perturbation theory :

$$\boxed{|\Omega\rangle_{\text{pre}} = |\Omega\rangle_{\text{post}} + 2\pi\delta_\lambda \sum_{l\neq\Omega} \frac{\delta(p_l^{\text{post}})}{E_l^{\text{post}}} \Big(\overbrace{F_l^{\mathcal{O}^{(q)}}}^{{}_{\text{post}}\langle\Omega|\,\mathcal{O}^{(q)}(0)\,|l\rangle_{\text{post}}}\Big)^* |l\rangle_{\text{post}} + O(\delta_\lambda^2)\,,}$$

where $\displaystyle \sum_{l\neq\Omega} := \sum_{n=0}^{\infty} \int_{p_1<p_2<\cdots<p_n} \prod_{j=1}^{n} \frac{dp_j}{2\pi\cdot\tilde{e}(p_j)} = \sum_a \int \frac{dp_a}{2\pi\cdot\tilde{e}(p_a)} + \sum_{a,b} \int \frac{dp_a}{2\pi\cdot\tilde{e}(p_a)} \frac{dp_b}{2\pi\cdot\tilde{e}(p_b)} + \ldots$

What we want to calculate: $\langle\mathcal{O}(t)\rangle_{\text{post}} = \boxed{\langle\Psi(t)|\,\mathcal{O}(0)\,|\Psi(t)\rangle}$

The time evolution of the expectation value of an operator after a global quench

$$\boxed{\frac{\Delta\mathcal{O}}{\bar{\mathcal{O}}} = \frac{\tilde{\delta}_\lambda}{\tilde{\lambda}} \cdot \left[C^{\mathcal{O},\mathcal{O}^{(q)}} + C^{\mathcal{O}^{(q)}} \sum_a \frac{2}{r_a^2} \cdot \hat{F}_a^{\mathcal{O}^{(q)}} \cdot \hat{F}_a^{\mathcal{O}} \cdot \cos(m_a t) + \ldots \right] + O(\tilde{\delta}_\lambda^2)}$$

The time evolution of the expectation value of an operator after a global quench

$$\boxed{\frac{\Delta\mathcal{O}}{\bar{\mathcal{O}}} = \frac{\tilde{\delta}_\lambda}{\tilde{\lambda}} \cdot \left[\overset{\frac{\Delta_{\mathcal{O}}}{2-\Delta_q}}{\downarrow} C^{\mathcal{O},\mathcal{O}^{(q)}} + C^{\mathcal{O}^{(q)}} \sum_a \frac{2}{r_a^2} \cdot \hat{F}_a^{\mathcal{O}^{(q)}} \cdot \hat{F}_a^{\mathcal{O}} \cdot \cos(m_a t) + \dots \right] + O(\tilde{\delta}_\lambda^2)}$$

The time evolution of the expectation value of an operator after a global quench

$$\frac{\Delta_{\mathcal{O}}}{2-\Delta_q}$$

$$\boxed{\frac{\Delta\mathcal{O}}{\bar{\mathcal{O}}} = \frac{\tilde{\delta}_\lambda}{\tilde{\lambda}} \cdot \left[C^{\mathcal{O},\mathcal{O}^{(q)}} + C^{\mathcal{O}^{(q)}} \sum_a \frac{2}{r_a^2} \cdot \hat{F}_a^{\mathcal{O}^{(q)}} \cdot \hat{F}_a^{\mathcal{O}} \cdot \cos(m_a t) + \dots \right] + O(\tilde{\delta}_\lambda^2)}$$

it doesn't matter, because we don't know the normalization

The time evolution of the expectation value of an operator after a global quench

$$\frac{\Delta_{\mathcal{O}}}{2-\Delta_q}$$

$$\boxed{\frac{\Delta\mathcal{O}}{\bar{\mathcal{O}}} = \frac{\tilde{\delta}_\lambda}{\tilde{\lambda}} \cdot \left[C^{\mathcal{O},\mathcal{O}^{(q)}} + C^{\mathcal{O}^{(q)}} \sum_a \frac{2}{r_a^2} \cdot \hat{F}_a^{\mathcal{O}^{(q)}} \cdot \hat{F}_a^{\mathcal{O}} \cdot \cos(m_a t) + \dots \right] + O(\tilde{\delta}_\lambda^2)}$$

it doesn't matter, because we don't know the normalization

$$\hat{F}_a^{\mathcal{O}} = \frac{F_a^{\mathcal{O}}}{{}_{\text{post}}\langle\Omega|\,\mathcal{O}(0)\,|\Omega\rangle_{\text{post}}} = \frac{{}_{\text{post}}\langle\Omega|\,\mathcal{O}(0)\,|\mathcal{E}_1\rangle_a}{{}_{\text{post}}\langle\Omega|\,\mathcal{O}(0)\,|\Omega\rangle_{\text{post}}}$$

we put an a-type particle
in the vacuum with energy $\mathcal{E}_1$

The time evolution of the expectation value of an operator after a global quench

$$\frac{\Delta_{\mathcal{O}}}{2-\Delta_q}$$

$$\left(\hat{F}_{ab}^{\mathcal{O}^{(q)}}\right)^* \cdot \hat{F}_{ab}^{\mathcal{O}}$$

$$\boxed{\frac{\Delta\mathcal{O}}{\bar{\mathcal{O}}} = \frac{\tilde{\delta}_\lambda}{\tilde{\lambda}} \cdot \left[C^{\mathcal{O},\mathcal{O}^{(q)}} + C^{\mathcal{O}^{(q)}} \sum_a \frac{2}{r_a^2} \cdot \hat{F}_a^{\mathcal{O}^{(q)}} \cdot \hat{F}_a^{\mathcal{O}} \cdot \cos(m_a t) + \dots \right] + O(\tilde{\delta}_\lambda^2)}$$

it doesn't matter, because we don't know the normalization

$$\hat{F}_a^{\mathcal{O}} = \frac{F_a^{\mathcal{O}}}{{}_{\text{post}}\langle\Omega|\,\mathcal{O}(0)\,|\Omega\rangle_{\text{post}}} = \frac{{}_{\text{post}}\langle\Omega|\,\mathcal{O}(0)\,|\mathcal{E}_1\rangle_a}{{}_{\text{post}}\langle\Omega|\,\mathcal{O}(0)\,|\Omega\rangle_{\text{post}}}$$

we put an a-type particle in the vacuum with energy $\mathcal{E}_1$

The time evolution of the expectation value of an operator after a global quench

$$\frac{\Delta_{\mathcal{O}}}{2-\Delta_q}$$

$$\left(\hat{F}^{\mathcal{O}^{(q)}}_{ab}\right)^* \cdot \hat{F}^{\mathcal{O}}_{ab}$$

$$\boxed{\frac{\Delta\mathcal{O}}{\bar{\mathcal{O}}} = \frac{\tilde{\delta}_\lambda}{\tilde{\lambda}} \cdot \left[C^{\mathcal{O},\mathcal{O}^{(q)}} + C^{\mathcal{O}^{(q)}} \sum_a \frac{2}{r_a^2} \cdot \hat{F}^{\mathcal{O}^{(q)}}_a \cdot \hat{F}^{\mathcal{O}}_a \cdot \cos(m_a t) + \dots \right] + O(\tilde{\delta}^2_\lambda)}$$

it doesn't matter, because we don't know the normalization

$$\hat{F}^{\mathcal{O}}_a = \frac{F^{\mathcal{O}}_a}{{}_{\text{post}}\langle\Omega|\,\mathcal{O}(0)\,|\Omega\rangle_{\text{post}}} = \frac{{}_{\text{post}}\langle\Omega|\,\mathcal{O}(0)\,|\mathcal{E}_1\rangle_a}{{}_{\text{post}}\langle\Omega|\,\mathcal{O}(0)\,|\Omega\rangle_{\text{post}}}$$

we put an a-type particle in the vacuum with energy $\mathcal{E}_1$

Quench protocol on the spin chain

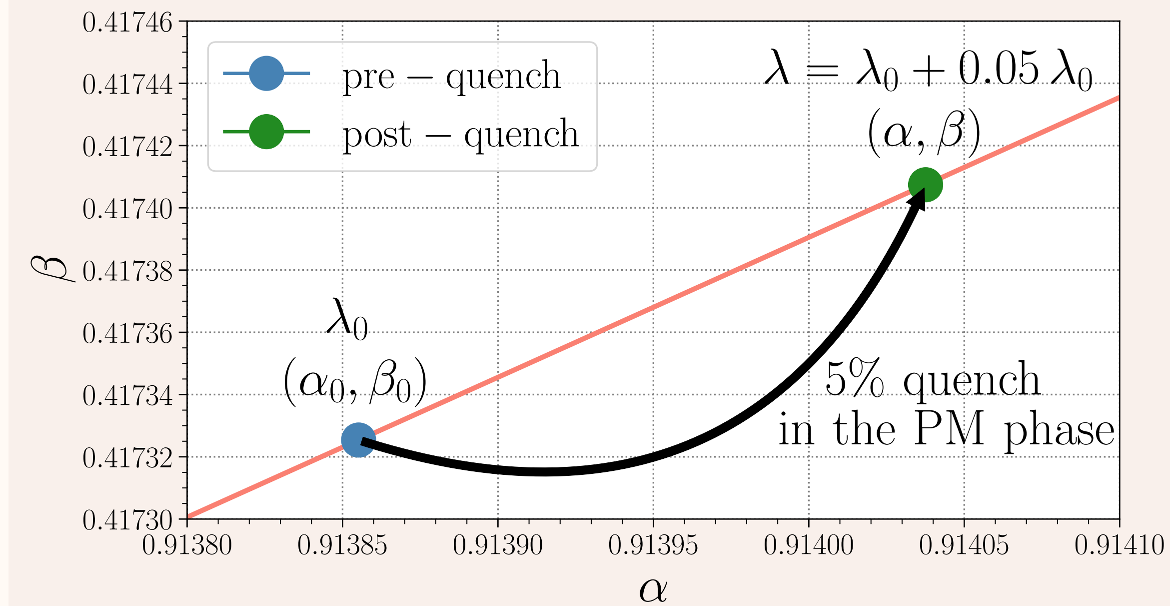
- pre-quench Hamilton: $H_{\text{pre}} = H_{\text{TBC}} + \lambda_0 H^{\perp}_{\text{pert}}$
- post-quench Hamilton: $H_{\text{post}} = H_{\text{TBC}} + \lambda H^{\perp}_{\text{pert}}$,

where $\lambda = \lambda_0 + \delta_\lambda$ with $\delta_\lambda = 0.05\lambda_0$

- we define the ground state of H_{pre} by using the iTEBD algorithm
- after the quench we time evolve this ground state with H_{post}

$\Rightarrow$ we can study post-quench dynamics

$E_7 \to E_7$ quench



S-matrix

$$S_{ab}(\theta) = (-1)^{\delta_{ab}} \prod_{\alpha \in \mathcal{A}_{ab}} (-f_\alpha(\theta))^{p_\alpha} \,,$$

$$f_\alpha(\theta) = \frac{\tanh \frac{1}{2}\left(\theta + i\pi \frac{\alpha}{18}\right)}{\tanh \frac{1}{2}\left(\theta - i\pi \frac{\alpha}{18}\right)}$$

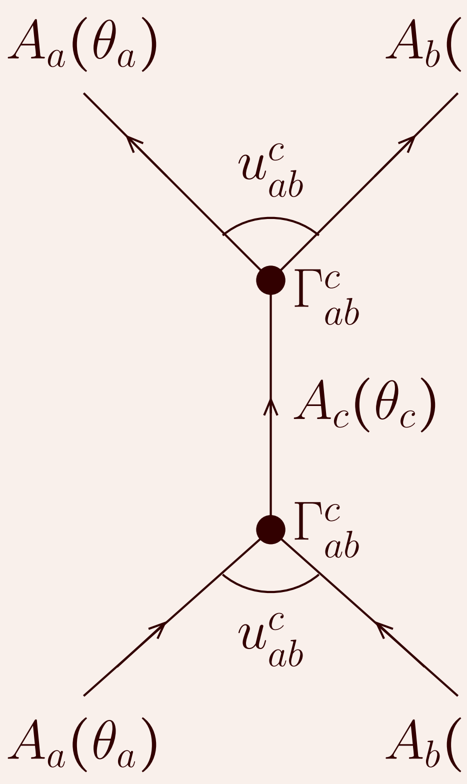
S-matrix

$$S_{ab}(\theta) = (-1)^{\delta_{ab}} \prod_{\alpha \in \mathcal{A}_{ab}} (-f_\alpha(\theta))^{p_\alpha} ,$$

$$f_\alpha(\theta) = \frac{\tanh \frac{1}{2}\left(\theta + i\pi \frac{\alpha}{18}\right)}{\tanh \frac{1}{2}\left(\theta - i\pi \frac{\alpha}{18}\right)}$$

$$u_{ab}^c = \arccos\left(\frac{m_c^2 - m_a^2 - m_b^2}{2 m_a m_b}\right)$$

$$\Gamma_{ab}^c = \sqrt{\frac{1}{i} \operatorname{Res}_{\theta = i u_{ab}^c} S_{ab}(\theta)}$$



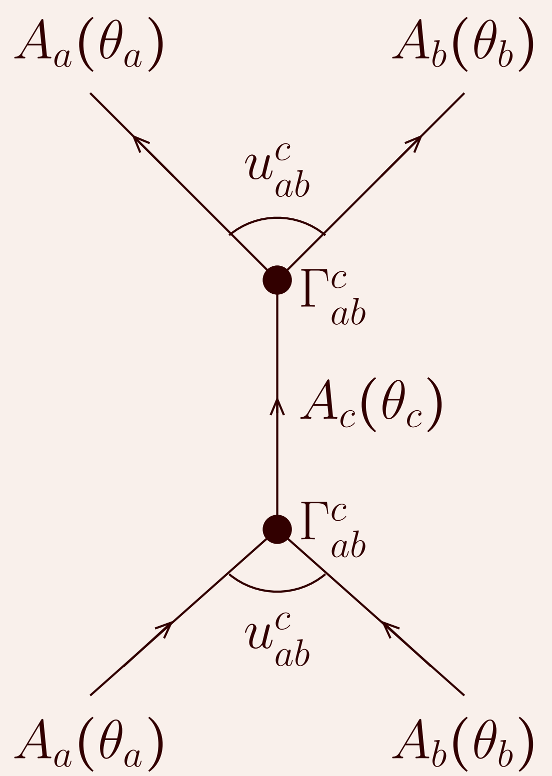
S-matrix

$$S_{ab}(\theta) = (-1)^{\delta_{ab}} \prod_{\alpha \in \mathcal{A}_{ab}} (-f_\alpha(\theta))^{p_\alpha} \,,$$

$$f_\alpha(\theta) = \frac{\tanh \frac{1}{2}\left(\theta + i\pi \frac{\alpha}{18}\right)}{\tanh \frac{1}{2}\left(\theta - i\pi \frac{\alpha}{18}\right)}$$

$$u_{ab}^{c} = \arccos\left(\frac{m_c^2 - m_a^2 - m_b^2}{2 m_a m_b}\right)$$

$$\Gamma_{ab}^{c} = \sqrt{\frac{1}{i} \operatorname{Res}_{\theta = i u_{ab}^{c}} S_{ab}(\theta)}$$



S-matrix amplitudes in E_7 field theory

$a\ b$	S_{ab}
1 1	$\overset{\mathbf{2}}{(10)}\overset{\mathbf{4}}{(2)}$
1 2	$\overset{\mathbf{1}}{(13)}\overset{\mathbf{3}}{(7)}$
1 3	$\overset{\mathbf{2}}{(14)}\overset{\mathbf{4}}{(10)}\overset{\mathbf{5}}{(6)}$
1 4	$\overset{\mathbf{1}}{(17)}\overset{\mathbf{3}}{(11)}\overset{\mathbf{6}}{(3)}(9)$
1 5	$\overset{\mathbf{3}}{(14)}\overset{\mathbf{6}}{(8)}(6)^2$
1 6	$\overset{\mathbf{4}}{(16)}\overset{\mathbf{5}}{(12)}\overset{\mathbf{7}}{(4)}(10)^2$
1 7	$\overset{\mathbf{6}}{(15)}(9)(5)^2(7)^2$
2 2	$\overset{\mathbf{2}}{(12)}\overset{\mathbf{4}}{(8)}\overset{\mathbf{5}}{(2)}$
2 3	$\overset{\mathbf{1}}{(15)}\overset{\mathbf{3}}{(11)}\overset{\mathbf{6}}{(5)}(9)$
2 4	$\overset{\mathbf{2}}{(14)}\overset{\mathbf{5}}{(8)}(6)^2$
2 5	$\overset{\mathbf{2}}{(17)}\overset{\mathbf{4}}{(13)}\overset{\mathbf{7}}{(3)}(7)^2(9)$
2 6	$\overset{\mathbf{3}}{(15)}(7)^2(5)^2(9)$
2 7	$\overset{\mathbf{5}}{(16)}\overset{\mathbf{7}}{(10)^3}(4)^2(6)^2$
3 3	$\overset{\mathbf{2}}{(14)}\overset{\mathbf{7}}{(2)}(8)^2(12)^2$

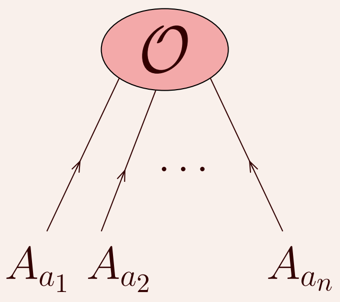
$a\ b$	S_{ab}
3 4	$\overset{\mathbf{1}}{(15)}(5)^2(7)^2(9)$
3 5	$\overset{\mathbf{1}}{(16)}\overset{\mathbf{6}}{(10)^3}(4)^2(6)^2$
3 6	$\overset{\mathbf{2}}{(16)}\overset{\mathbf{5}}{(12)^3}\overset{\mathbf{7}}{(8)^3}(4)^2$
3 7	$\overset{\mathbf{3}}{(17)}\overset{\mathbf{6}}{(13)^3}(3)^2(7)^4(9)^2$
4 4	$\overset{\mathbf{4}}{(12)}\overset{\mathbf{5}}{(10)^3}\overset{\mathbf{7}}{(4)}(2)^2$
4 5	$\overset{\mathbf{2}}{(15)}\overset{\mathbf{4}}{(13)^3}\overset{\mathbf{7}}{(7)^3}(9)$
4 6	$\overset{\mathbf{1}}{(17)}\overset{\mathbf{6}}{(11)^3}(3)^2(5)^2(9)^2$
4 7	$\overset{\mathbf{4}}{(16)}\overset{\mathbf{5}}{(14)^3}(6)^4(8)^4$
5 5	$\overset{\mathbf{5}}{(12)^3}(2)^2(4)^2(8)^4$
5 6	$\overset{\mathbf{1}}{(16)}\overset{\mathbf{3}}{(14)^3}(6)^4(8)^4$
5 7	$\overset{\mathbf{2}}{(17)}\overset{\mathbf{4}}{(15)^3}\overset{\mathbf{7}}{(11)^5}(5)^4(9)^3$
6 6	$\overset{\mathbf{4}}{(14)^3}\overset{\mathbf{7}}{(10)^5}(12)^4(16)^2$
6 7	$\overset{\mathbf{1}}{(17)}\overset{\mathbf{3}}{(15)^3}\overset{\mathbf{6}}{(13)^5}(5)^6(9)^3$
7 7	$\overset{\mathbf{2}}{(16)^3}\overset{\mathbf{5}}{(14)^5}\overset{\mathbf{7}}{(12)^7}(8)^8$

$\hookrightarrow$ $\overset{\mathbf{a}}{(\alpha)}^{p_\alpha}$: pole α with multiplicity p_α corresponds to the bound state $A_{\mathbf{a}}$

What are the form factors and why we need to calculate them?

$$\langle \mathcal{O}(x,t)\mathcal{O}(0,0)\rangle = \sum_{n=0}^{\infty} \int_{\theta_1>\theta_2>\dots>\theta_n} \frac{d\theta_1}{2\pi}\dots\frac{d\theta_n}{2\pi} \times |\langle 0|\mathcal{O}(0,0)|A_{a_1}(\theta_1)\dots A_{a_n}(\theta_n)\rangle|^2 e^{-|N|\sum_{k=1}^{n} m_k \cosh\theta_k}$$

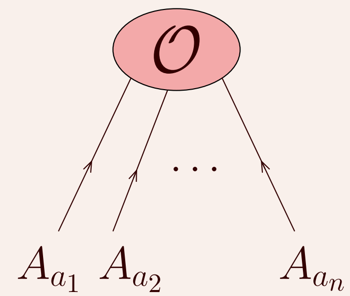
$$F^{\mathcal{O}}_{a_1,\dots,a_n}(\theta_1,\dots,\theta_n) = \langle 0|\mathcal{O}(0,0)|A_{a_1}(\theta_1)\dots A_{a_n}(\theta_n)\rangle$$



What are the form factors and why we need to calculate them?

$$\langle\mathcal{O}(x,t)\mathcal{O}(0,0)\rangle = \sum_{n=0}^{\infty}\int_{\theta_1>\theta_2>\dots>\theta_n}\frac{d\theta_1}{2\pi}\dots\frac{d\theta_n}{2\pi}\times|\langle 0|\mathcal{O}(0,0)|A_{a_1}(\theta_1)\dots A_{a_n}(\theta_n)\rangle|^2 e^{-|N|\sum_{k=1}^{n} m_k\cosh\theta_k}$$

$$F^{\mathcal{O}}_{a_1,\dots,a_n}(\theta_1,\dots,\theta_n) = \langle 0|\mathcal{O}(0,0)|A_{a_1}(\theta_1)\dots A_{a_n}(\theta_n)\rangle$$



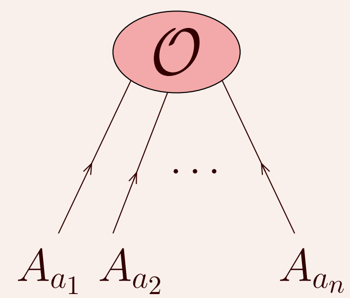
Form factor axioms

- theory $\Rightarrow$ Lorentz invariance $\Rightarrow$ dependence only on the rapidity differences
- S-matrix $\Rightarrow$ monodromy properties:

What are the form factors and why we need to calculate them?

$$\langle \mathcal{O}(x,t)\mathcal{O}(0,0)\rangle = \sum_{n=0}^{\infty}\int_{\theta_1>\theta_2>\dots>\theta_n}\frac{d\theta_1}{2\pi}\dots\frac{d\theta_n}{2\pi}\times|\langle 0|\mathcal{O}(0,0)|A_{a_1}(\theta_1)\dots A_{a_n}(\theta_n)\rangle|^2 e^{-|N|\sum_{k=1}^{n} m_k\cosh\theta_k}$$

$$F^{\mathcal{O}}_{a_1,\dots,a_n}(\theta_1,\dots,\theta_n) = \langle 0|\mathcal{O}(0,0)|A_{a_1}(\theta_1)\dots A_{a_n}(\theta_n)\rangle$$



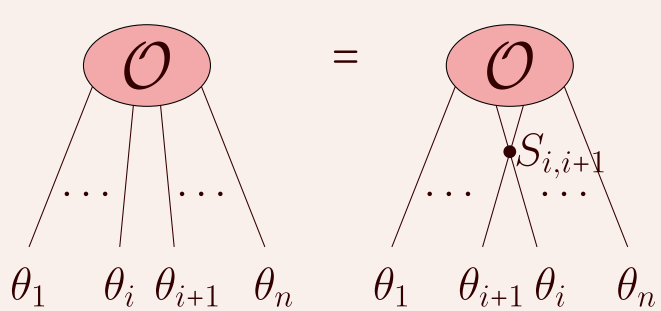
Form factor axioms

- theory ⇒ Lorentz invariance ⇒ dependence only on the rapidity differences
- S-matrix ⇒ monodromy properties:

exchange property

$$F^{\mathcal{O}}_{a_1\dots a_i a_{i+1}\dots a_n}(\theta_1,\dots,\theta_i,\theta_{i+1},\dots,\theta_n) =$$

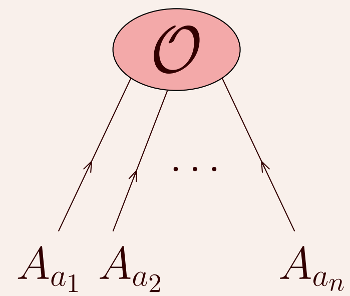
$$= S_{a_i a_{i+1}}(\theta_i-\theta_{i+1})F^{\mathcal{O}}_{a_1\dots a_{i+1}a_i\dots a_n}(\theta_1,\dots,\theta_{i+1},\theta_i,\dots,\theta_n)$$



What are the form factors and why we need to calculate them?

$$\langle \mathcal{O}(x,t)\mathcal{O}(0,0)\rangle = \sum_{n=0}^{\infty}\int_{\theta_1>\theta_2>\dots>\theta_n}\frac{d\theta_1}{2\pi}\dots\frac{d\theta_n}{2\pi}\times|\langle 0|\mathcal{O}(0,0)|A_{a_1}(\theta_1)\dots A_{a_n}(\theta_n)\rangle|^2 e^{-|N|\sum_{k=1}^{n} m_k\cosh\theta_k}$$

$$F^{\mathcal{O}}_{a_1,\dots,a_n}(\theta_1,\dots,\theta_n) = \langle 0|\mathcal{O}(0,0)|A_{a_1}(\theta_1)\dots A_{a_n}(\theta_n)\rangle$$



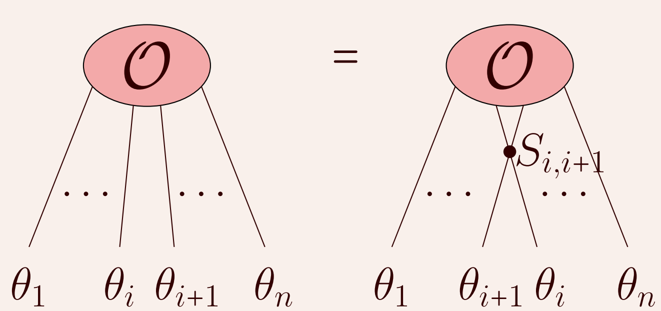
Form factor axioms

- theory $\Rightarrow$ Lorentz invariance $\Rightarrow$ dependence only on the rapidity differences
- S-matrix $\Rightarrow$ monodromy properties:

exchange property

$$F^{\mathcal{O}}_{a_1\dots a_i a_{i+1}\dots a_n}(\theta_1,\dots,\theta_i,\theta_{i+1},\dots,\theta_n) =$$

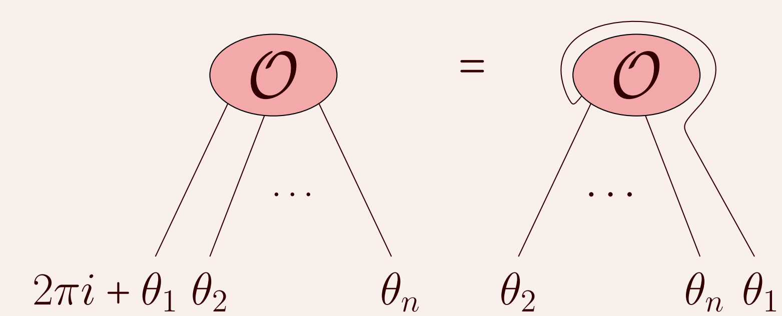
$$= S_{a_i a_{i+1}}(\theta_i-\theta_{i+1})F^{\mathcal{O}}_{a_1\dots a_{i+1}a_i\dots a_n}(\theta_1,\dots,\theta_{i+1},\theta_i,\dots,\theta_n)$$



cyclic property / crossing symmetry

$$F^{\mathcal{O}}_{a_1\dots a_n}(\theta_1+2\pi i,\theta_2,\dots,\theta_n) =$$

$$= F^{\mathcal{O}}_{a_2\dots a_n a_1}(\theta_2,\dots,\theta_n,\theta_1)$$



Form factor residue equations

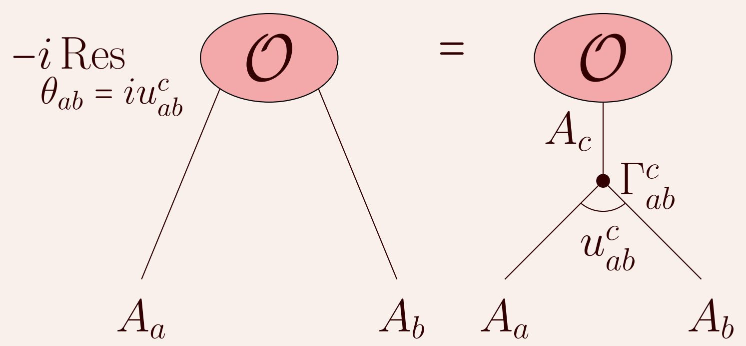
- FFs satisfy a set of equations which specify their monodromy properties (Watson's equations) and their pole structure (residue equations) $\longrightarrow$ altogether, these equations are enough to (almost) entirely fix the FFs

Form factor residue equations

- FFs satisfy a set of equations which specify their monodromy properties (Watson's equations) and their pole structure (residue equations) $\longrightarrow$ altogether, these equations are enough to (almost) entirely fix the FFs

bound state

$$-i\,\mathrm{Res}_{\theta_{ab}=iu_{ab}^{c}}\,F_{ab}^{\mathcal{O}}(\theta_{ab}) = \Gamma_{ab}^{c}F_{c}^{\mathcal{O}}$$

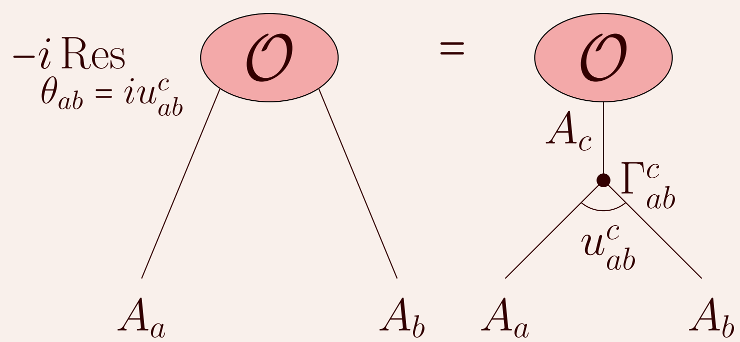


Form factor residue equations

- FFs satisfy a set of equations which specify their monodromy properties (Watson's equations) and their pole structure (residue equations) $\longrightarrow$ altogether, these equations are enough to (almost) entirely fix the FFs

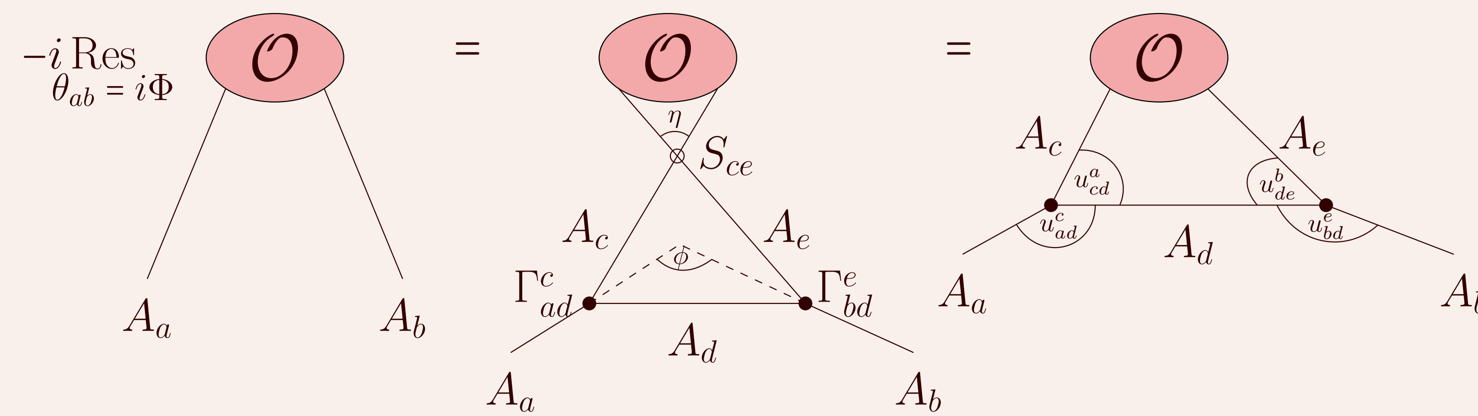
bound state

$$-i\,\mathrm{Res}_{\theta_{ab}=iu^c_{ab}}\,F^{\mathcal{O}}_{ab}(\theta_{ab}) = \Gamma^c_{ab}F^{\mathcal{O}}_c$$



double pole

$$-i\lim_{\theta_{ab}\to i\phi}(\theta_{ab}-i\phi)F^{\mathcal{O}}_{ab}(\theta_{ab}) = \Gamma^c_{ad}\Gamma^e_{bd}F^{\mathcal{O}}_{ce}(i\eta)$$

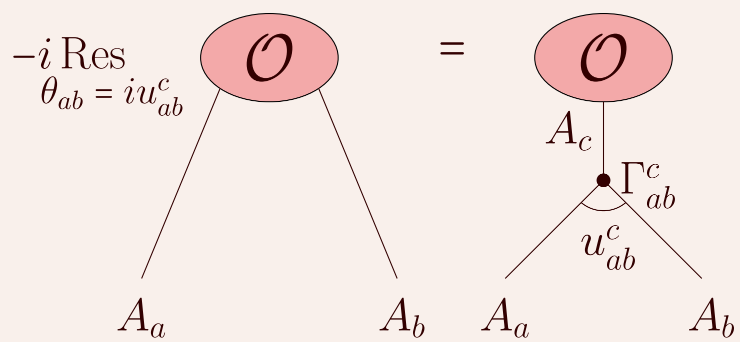


Form factor residue equations

- FFs satisfy a set of equations which specify their monodromy properties (Watson's equations) and their pole structure (residue equations) $\longrightarrow$ altogether, these equations are enough to (almost) entirely fix the FFs

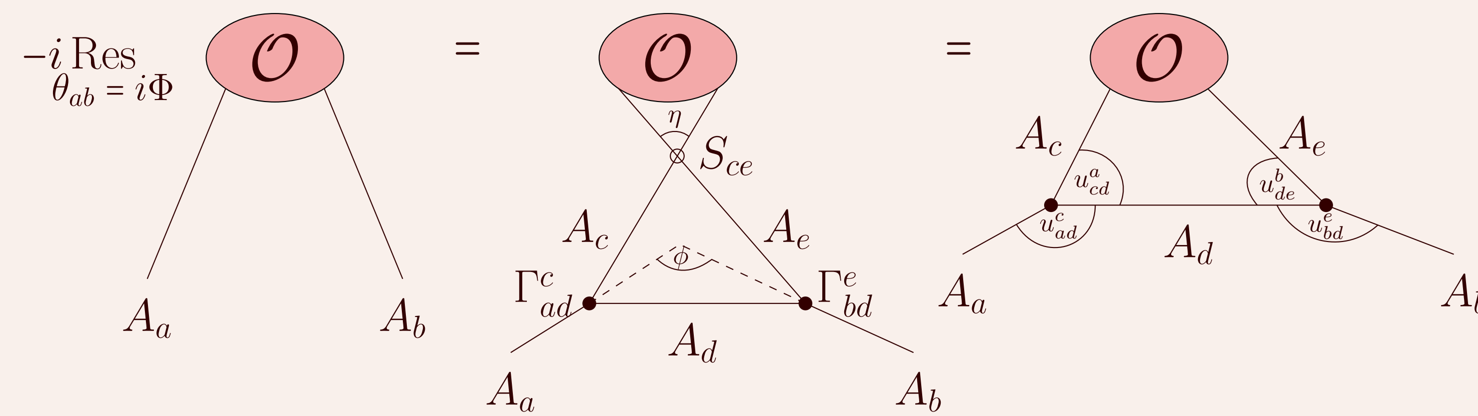
bound state

$$-i\,\mathrm{Res}_{\theta_{ab}=iu^c_{ab}}\,F^{\mathcal{O}}_{ab}(\theta_{ab}) = \Gamma^c_{ab}F^{\mathcal{O}}_c$$



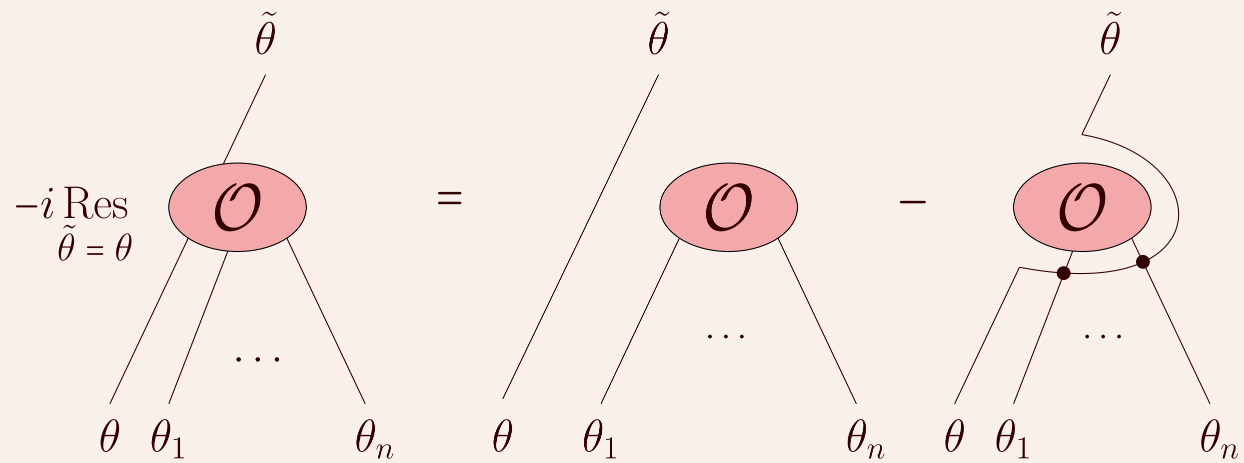
double pole

$$-i\lim_{\theta_{ab}\to i\phi}(\theta_{ab}-i\phi)F^{\mathcal{O}}_{ab}(\theta_{ab}) = \Gamma^c_{ad}\Gamma^e_{bd}F^{\mathcal{O}}_{ce}(i\eta)$$



kinematical pole

$$-i\lim_{\tilde{\theta}\to\theta}(\tilde{\theta}-\theta)F^{\mathcal{O}}_{a\bar{a}a_1\ldots a_n}(\tilde{\theta}+i\pi,\theta,\theta_1,\ldots,\theta_n) = \left(1-e^{2\pi i\omega_a}\prod_{j=1}^{n}S_{aa_j}(\theta-\theta_j)\right)F^{\mathcal{O}}_{a_1\ldots a_n}(\theta_1,\ldots,\theta_n)$$



Two-particle form factors

- in integrable models: n-particle FFs $\longrightarrow$ two-particle FFs

Two-particle form factors

- in integrable models: n-particle FFs $\longrightarrow$ two-particle FFs
- monodromy equations $\Rightarrow$ $F_{ab}^{min}(\theta)$ (free from poles)

Two-particle form factors

- in integrable models: n-particle FFs $\longrightarrow$ two-particle FFs
- monodromy equations $\Rightarrow$ $F_{ab}^{min}(\theta)$ (free from poles)

$\Rightarrow$ general solution: $F_{ab}^{\mathcal{O}}(\theta) = \dfrac{Q_{ab}^{\mathcal{O}}(\theta)}{D_{ab}(\theta)} F_{ab}^{min}(\theta)\,, \qquad Q_{ab}^{\mathcal{O}}(\theta),\ D_{ab}(\theta) :$ polynomials in $\cosh\theta$

Two-particle form factors

- in integrable models: n-particle FFs $\longrightarrow$ two-particle FFs
- monodromy equations $\Rightarrow$ $F_{ab}^{min}(\theta)$ (free from poles)

$\Rightarrow$ general solution: $F_{ab}^{\mathcal{O}}(\theta) = \dfrac{Q_{ab}^{\mathcal{O}}(\theta)}{D_{ab}(\theta)} F_{ab}^{min}(\theta)\,, \qquad Q_{ab}^{\mathcal{O}}(\theta),\ D_{ab}(\theta)$: polynomials in $\cosh\theta$

- $D_{ab}(\theta)$: fixed by the singularities of the S-matrix

Two-particle form factors

- in integrable models: n-particle FFs $\longrightarrow$ two-particle FFs
- monodromy equations $\Rightarrow$ $F_{ab}^{min}(\theta)$ (free from poles)

$\Rightarrow$ general solution: $F_{ab}^{\mathcal{O}}(\theta) = \dfrac{Q_{ab}^{\mathcal{O}}(\theta)}{D_{ab}(\theta)} F_{ab}^{min}(\theta)\,,\qquad Q_{ab}^{\mathcal{O}}(\theta),\ D_{ab}(\theta):$ polynomials in $\cosh\theta$

- $D_{ab}(\theta)$: fixed by the singularities of the S-matrix
- $Q_{ab}^{\mathcal{O}}(\theta)$: fixed by the asymptotic behavior of the form factor of the operator $\mathcal{O}$:

$$\lim_{|\theta|\to\infty} F_{ab}^{\mathcal{O}}(\theta) \sim e^{y_{\mathcal{O}}|\theta|} \qquad y_{\mathcal{O}} \le \Delta_{\mathcal{O}} \quad \text{(conform dimension)}$$

$\hookrightarrow$ this is the only term in the FF that contains information about the operator

Two-particle form factors

- in integrable models: n-particle FFs $\longrightarrow$ two-particle FFs
- monodromy equations $\Rightarrow$ $F_{ab}^{min}(\theta)$ (free from poles)

$\Rightarrow$ general solution: $F_{ab}^{\mathcal{O}}(\theta) = \dfrac{Q_{ab}^{\mathcal{O}}(\theta)}{D_{ab}(\theta)} F_{ab}^{min}(\theta)\,,\qquad Q_{ab}^{\mathcal{O}}(\theta),\ D_{ab}(\theta)$: polynomials in $\cosh\theta$

- $D_{ab}(\theta)$: fixed by the singularities of the S-matrix
- $Q_{ab}^{\mathcal{O}}(\theta)$: fixed by the asymptotic behavior of the form factor of the operator $\mathcal{O}$:

$$\lim_{|\theta|\to\infty} F_{ab}^{\mathcal{O}}(\theta) \sim e^{y_{\mathcal{O}}|\theta|} \qquad y_{\mathcal{O}} \leq \Delta_{\mathcal{O}} \quad \text{(conform dimension)}$$

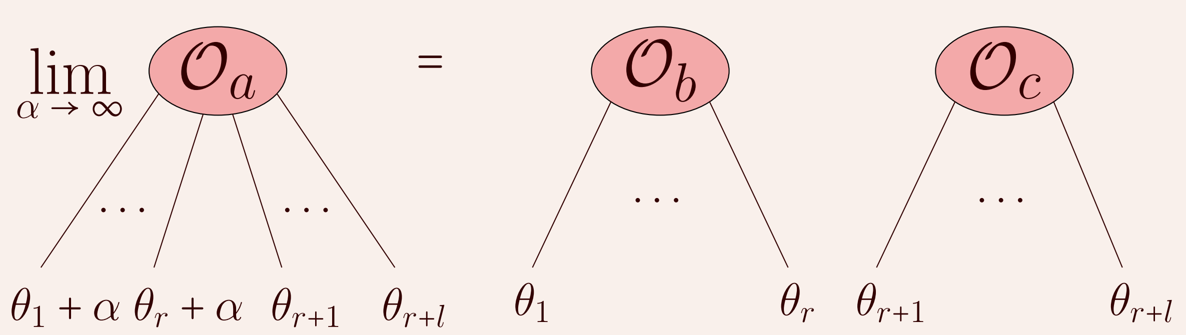
$\hookrightarrow$ this is the only term in the FF that contains information about the operator

cluster equation

$$\lim_{\alpha\to\infty} F^{\mathcal{O}_a}_{a_1\ldots a_{r+l}}(\theta_1+\alpha,\ldots,\theta_r+\alpha,\theta_{r+1},\ldots,\theta_{r+l}) =$$

$$= F^{\mathcal{O}_b}_{a_1\ldots a_r}(\theta_1,\ldots,\theta_r) F^{\mathcal{O}_c}_{a_{r+1}\ldots a_{r+l}}(\theta_{r+1},\ldots,\theta_{r+l})$$

$\longrightarrow$ useful for instance to fix the value of the one-particle FF from the asymptotics of the two-particle FF



What did we calculate?

$$\frac{\Delta\mathcal{O}}{\bar{\mathcal{O}}} \approx \frac{\tilde{\delta}_\lambda}{\tilde{\lambda}} \cdot \left[C^{\mathcal{O},\mathcal{O}^{(q)}} + C^{\mathcal{O}^{(q)}} \sum_a \frac{2}{r_a^2} \cdot \hat{F}_a^{\mathcal{O}^{(q)}} \cdot \hat{F}_a^{\mathcal{O}} \cdot \cos(m_a t) \right]$$

$\mathcal{O} = \sigma\,,\ \epsilon$

$\sigma \longrightarrow S^x$

$\epsilon \longrightarrow H^{\perp}_{\mathrm{pert}}$

FFs were known

Results I.

What did we calculate?

$$\frac{\Delta\mathcal{O}}{\bar{\mathcal{O}}} \approx \frac{\tilde{\delta}_\lambda}{\tilde{\lambda}} \cdot \left[C^{\mathcal{O},\mathcal{O}^{(q)}} + C^{\mathcal{O}^{(q)}} \sum_a \frac{2}{r_a^2} \cdot \hat{F}_a^{\mathcal{O}^{(q)}} \cdot \hat{F}_a^{\mathcal{O}} \cdot \cos(m_a t) \right]$$

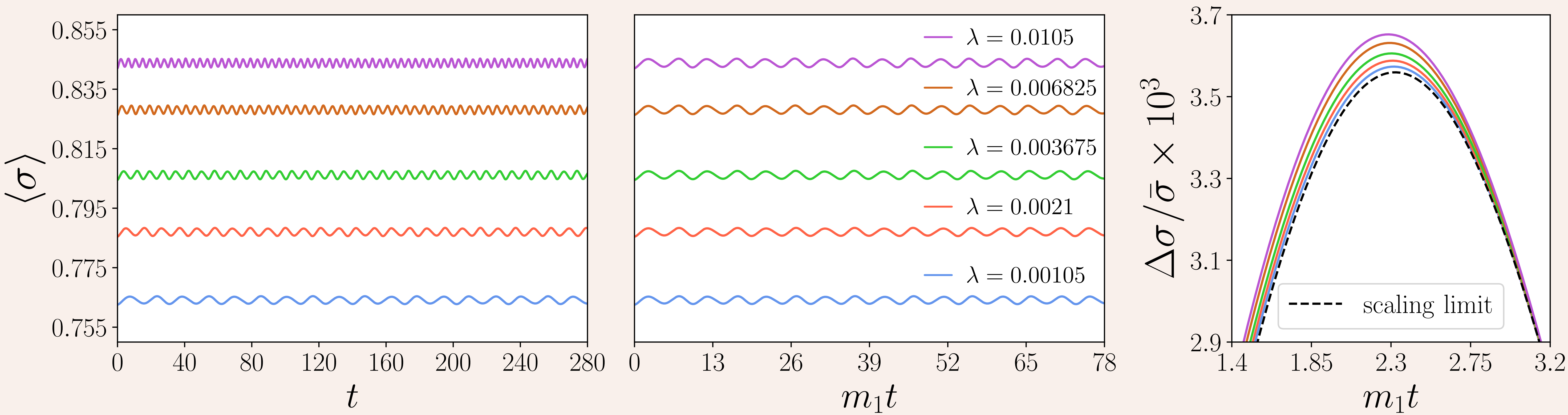
$\longrightarrow \mathcal{O} = \sigma\,,\ \epsilon$

FFs were known

$\sigma \longrightarrow S^x$

$\epsilon \longrightarrow H^{\perp}_{\mathsf{pert}}$

- simulations for various λ-s $\Rightarrow$ extrapolate to the scaling limit



$m_1 = \xi\,|\lambda|^{5/9}$

Entropy and replica trick

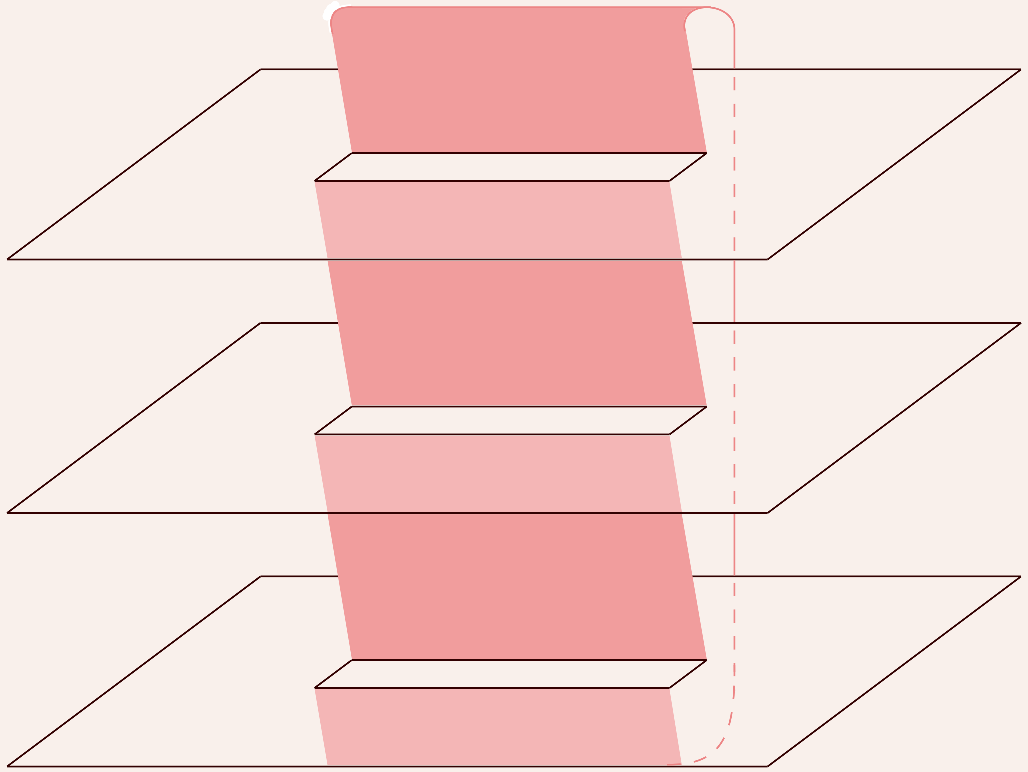
$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

$\mathbf{n}^{\text{th}}$ Rényi entropy: $S_n^{AB} = \dfrac{1}{1-n} \log \mathrm{Tr}_A \rho_A^n$

Neumann entropy: $S_1^{AB} = \lim\limits_{n \to 1} S_n^{AB}$

Entropy and replica trick

$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$



n$^{\text{th}}$ Rényi entropy: $S_n^{AB} = \frac{1}{1-n} \log \mathrm{Tr}_A \rho_A^n$

Neumann entropy: $S_1^{AB} = \lim_{n \to 1} S_n^{AB}$

Replica trick $\Rightarrow \mathrm{Tr}_A \rho_A^n = \langle \bar{\mathcal{T}}_n(x) \mathcal{T}_n(0) \rangle$

$\mathcal{T}_n$ is the **branch point twist field**

with conformal dimension: $\Delta^{\mathcal{T}_n} = \frac{c}{24}\left(n - \frac{1}{n}\right)$

Entropy and replica trick

$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

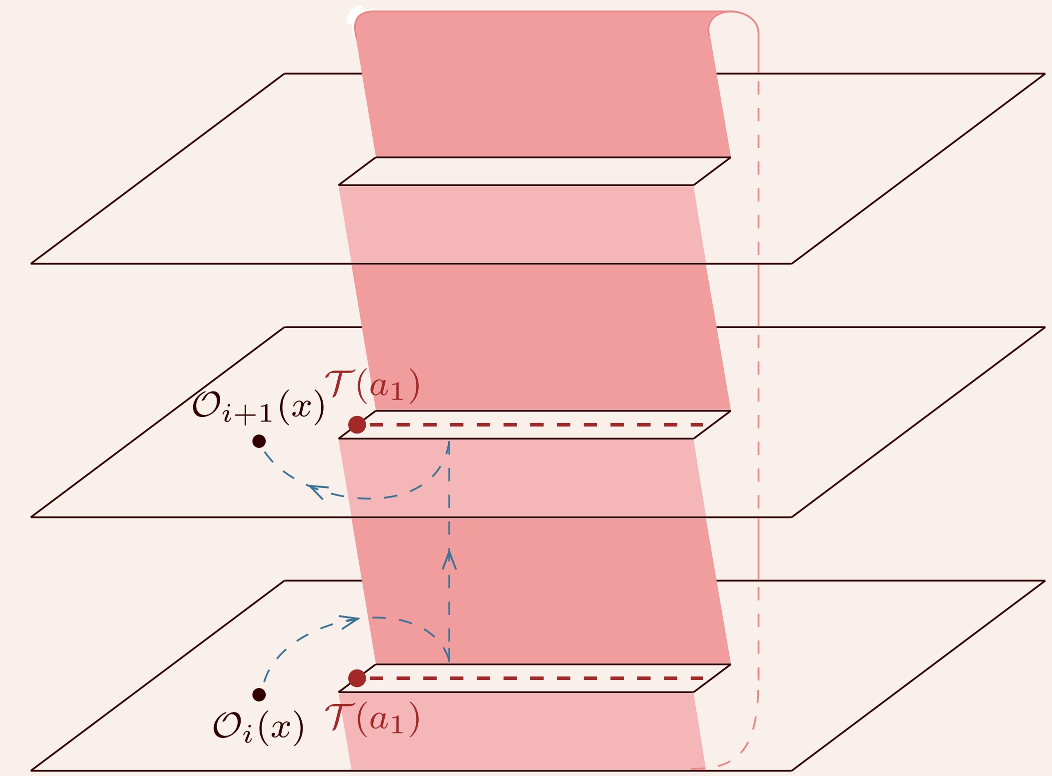
n$^{\text{th}}$ Rényi entropy: $S_n^{AB} = \dfrac{1}{1-n} \log \mathrm{Tr}_A \rho_A^n$

Neumann entropy: $S_1^{AB} = \lim\limits_{n \to 1} S_n^{AB}$

Replica trick $\Rightarrow$ $\mathrm{Tr}_A \rho_A^n = \langle \bar{\mathcal{T}}_n(x) \mathcal{T}_n(0) \rangle$

$\mathcal{T}_n$ is the **branch point twist field**

with conformal dimension: $\Delta^{\mathcal{T}_n} = \dfrac{c}{24}\left(n - \dfrac{1}{n}\right)$



Entropy and replica trick

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

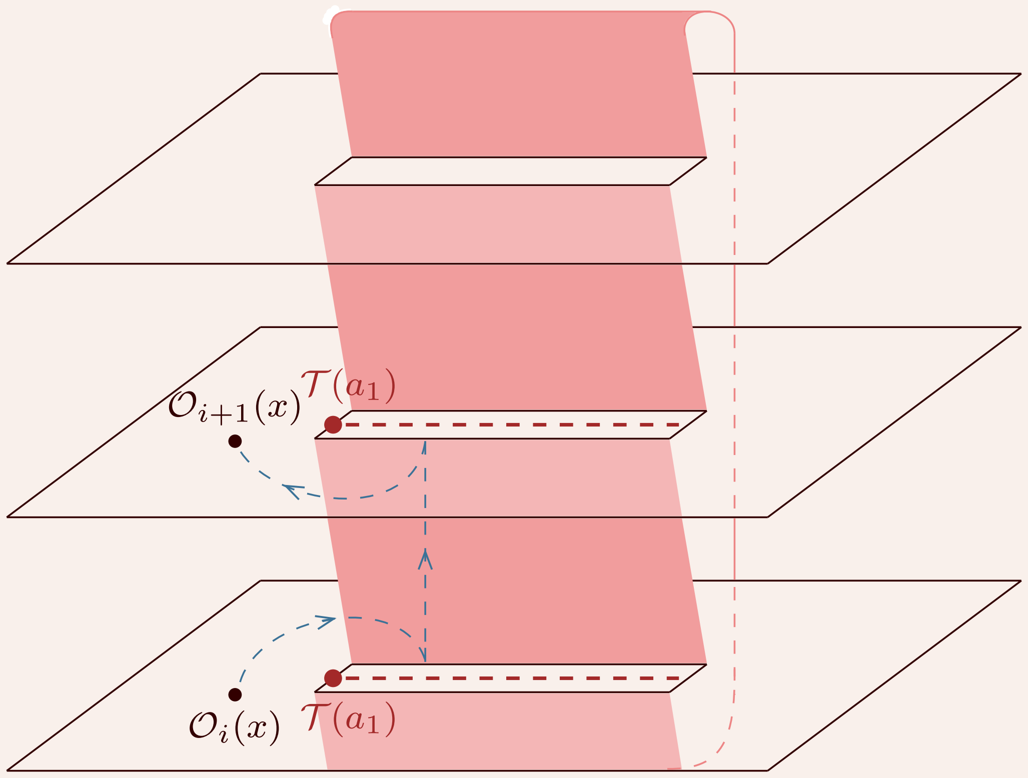
n^th^ Rényi entropy: $S_n^{AB} = \frac{1}{1-n} \log \mathrm{Tr}_A \rho_A^n$

Neumann entropy: $S_1^{AB} = \lim_{n\to 1} S_n^{AB}$

Replica trick $\Rightarrow$ $\mathrm{Tr}_A \rho_A^n = \langle \bar{\mathcal{T}}_n(x) \mathcal{T}_n(0) \rangle$

$\mathcal{T}_n$ is the **branch point twist field**

with conformal dimension: $\Delta^{\mathcal{T}_n} = \frac{c}{24}\left(n - \frac{1}{n}\right)$



Branch point twist field form factors

$$F^{\mathcal{T}_n}_{(s_1,a_1)(s_2,a_2)}(\theta) = \frac{Q^{s_1 s_2}_{a_1 a_2}(\theta;n)}{2n K_{s_1 s_2}(\theta;n)^{\delta_{a_1 a_2}} D_{a_1 a_2}(\theta;n)^{\delta_{s_1 s_2}}} \times \frac{F^{\mathcal{T}_n}_{\mathrm{min}|(s_1,a_1)(s_2,a_2)}(\theta;n)}{F^{\mathcal{T}_n}_{\mathrm{min}|(s_1,a_1)(s_2,a_2)}(i\pi;n)}$$

$K_{s_1 s_2}$ - new kin. poles

$D_{a_1 a_2}$ - S-matrix poles

$Q^{s_1 s_2}_{a_1 a_2}$ - from asymptotics

Entropy and replica trick

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

n

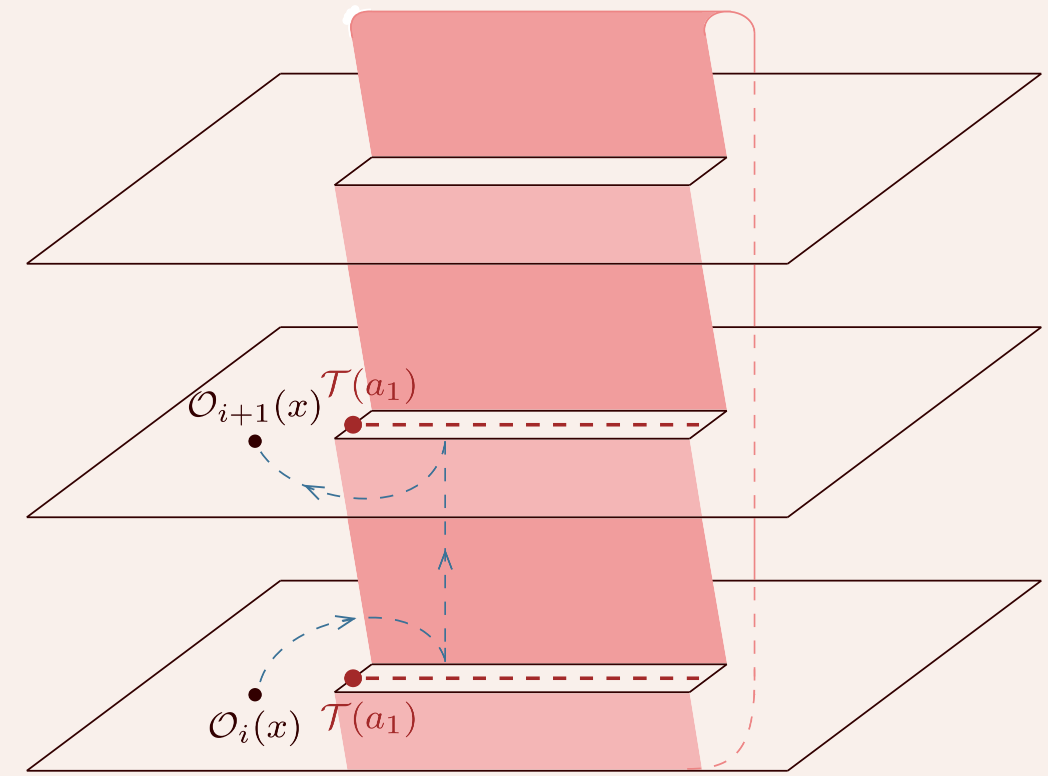
$\mathbf{n^{th}}$ **Rényi entropy:** $S_n^{AB} = \dfrac{1}{1-n} \log \mathrm{Tr}_A \rho_A^n$

Neumann entropy: $S_1^{AB} = \lim\limits_{n\to 1} S_n^{AB}$

Replica trick $\Rightarrow \mathrm{Tr}_A \rho_A^n = \langle \bar{\mathcal{T}}_n(x) \mathcal{T}_n(0) \rangle$

$\mathcal{T}_n$ is the **branch point twist field**

with conformal dimension: $\Delta^{\mathcal{T}_n} = \dfrac{c}{24}\left(n - \dfrac{1}{n}\right)$



Branch point twist field form factors

$$F^{\mathcal{T}_n}_{(s_1,a_1)(s_2,a_2)}(\theta) = \frac{Q^{s_1 s_2}_{a_1 a_2}(\theta; n)}{2n K_{s_1 s_2}(\theta; n)^{\delta_{a_1 a_2}} D_{a_1 a_2}(\theta; n)^{\delta_{s_1 s_2}}} \times \frac{F^{\mathcal{T}_n}_{\mathrm{min}|(s_1,a_1)(s_2,a_2)}(\theta; n)}{F^{\mathcal{T}_n}_{\mathrm{min}|(s_1,a_1)(s_2,a_2)}(i\pi; n)}$$

$K_{s_1 s_2}$ - new kin. poles

$D_{a_1 a_2}$ - S-matrix poles

$Q^{s_1 s_2}_{a_1 a_2}$ - from asymptotics

- TFFF bootstrap program for $n = 1, 2, 3, 4$

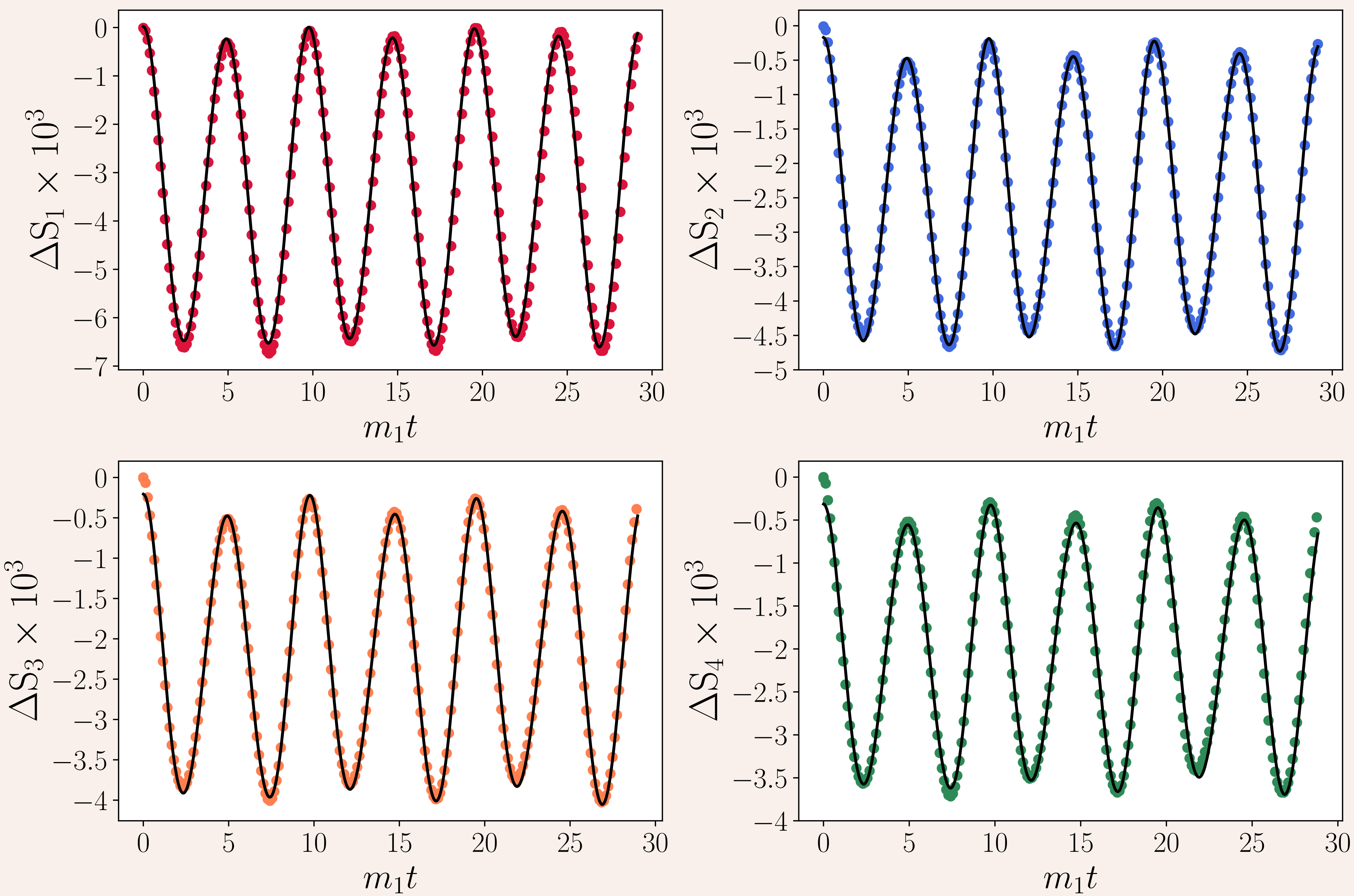
n	1	2	3	4
$\Delta^{\mathcal{T}_n}$ from Δ-theorem	$0.056170(n-1) + \mathcal{O}((n-1)^2)$	0.043278	0.078553	0.107572
exact result from CFT	$0.058333(n-1) + \mathcal{O}((n-1)^2)$	0.04375	0.077778	0.109375

Post-quench dynamics for entropies

$$\frac{\Delta \mathcal{T}_n}{\bar{\mathcal{T}}_n} = \frac{\delta_\lambda}{\lambda}\left[C^{\mathcal{T}_n,\epsilon} + nC^{\epsilon} \sum_a \frac{2}{r_a^2} \cdot \hat{F}_a^{\epsilon} \cdot \hat{F}_a^{\mathcal{T}_n} \cdot \cos\left(m_a t\right) \right] \quad \Rightarrow \quad S_n(t) - S_n(0) = \frac{1}{1-n} \cdot \frac{\Delta \mathcal{T}_n}{\bar{\mathcal{T}}_n}$$

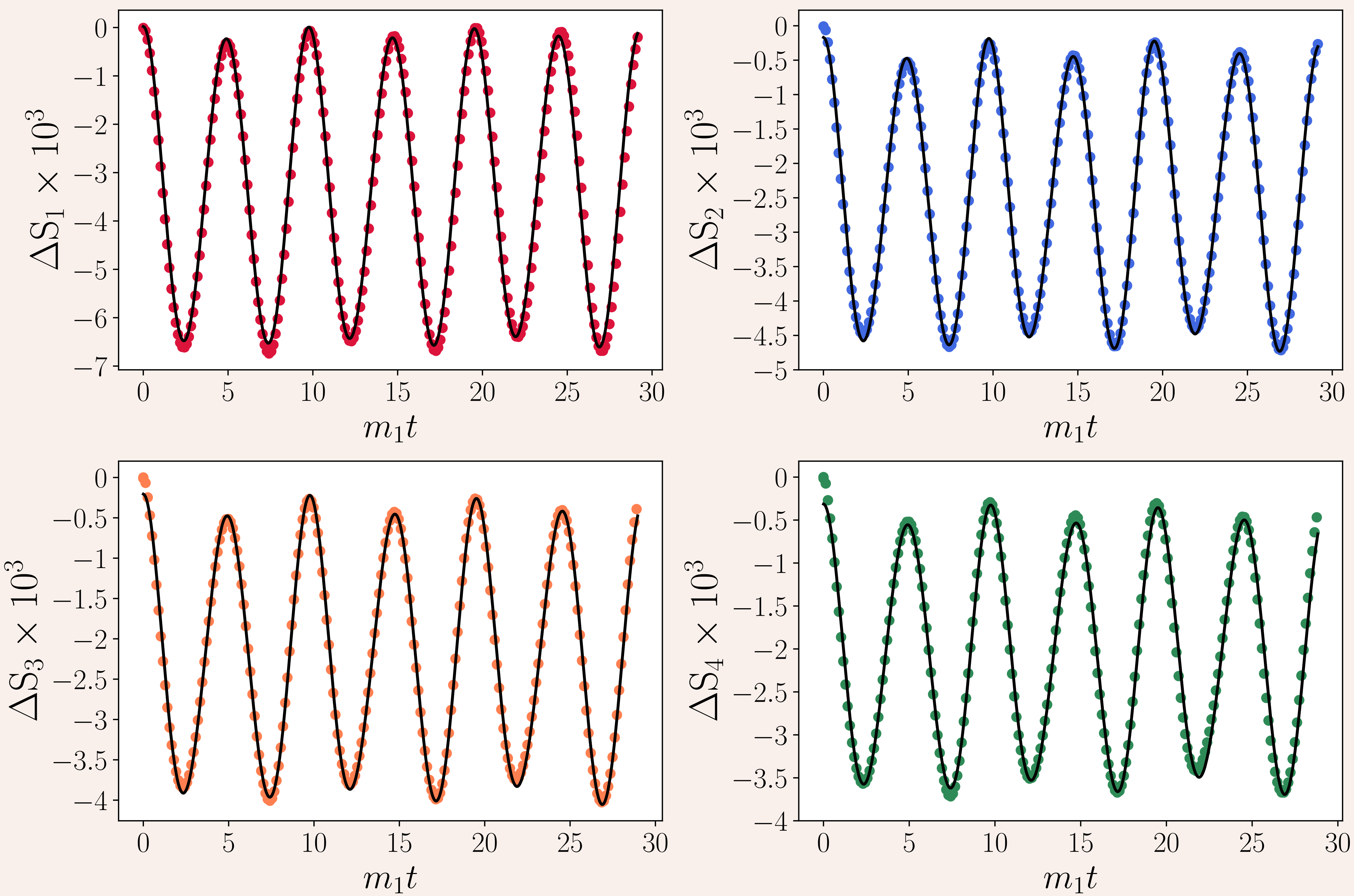
Post-quench dynamics for entropies

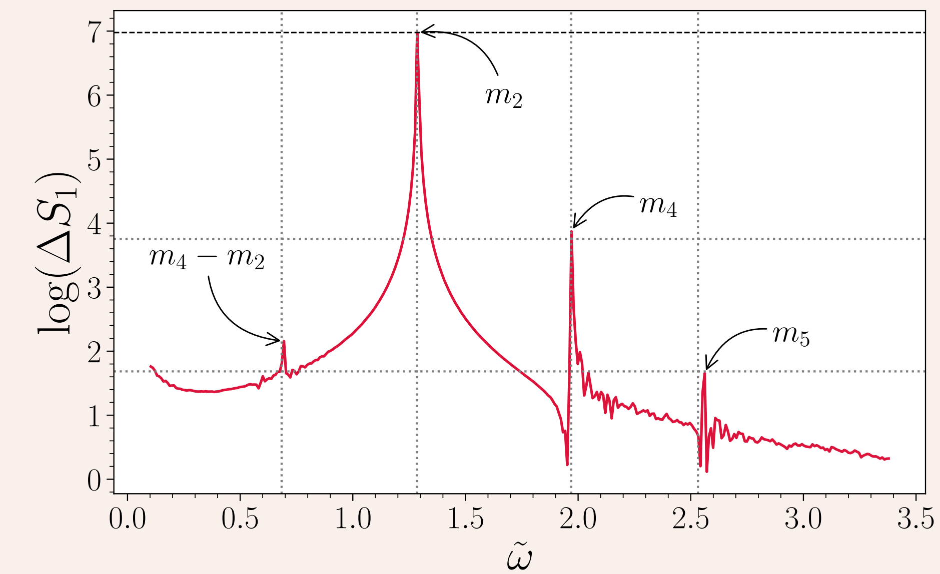
$$\frac{\Delta \mathcal{T}_n}{\bar{\mathcal{T}}_n} = \frac{\delta_\lambda}{\lambda}\left[C^{\mathcal{T}_n,\epsilon} + nC^{\epsilon}\sum_a \frac{2}{r_a^2}\cdot \hat{F}_a^{\epsilon}\cdot \hat{F}_a^{\mathcal{T}_n}\cdot \cos\left(m_a t\right)\right] \quad \Rightarrow \quad S_n(t) - S_n(0) = \frac{1}{1-n}\cdot\frac{\Delta\mathcal{T}_n}{\bar{\mathcal{T}}_n}$$



Post-quench dynamics for entropies

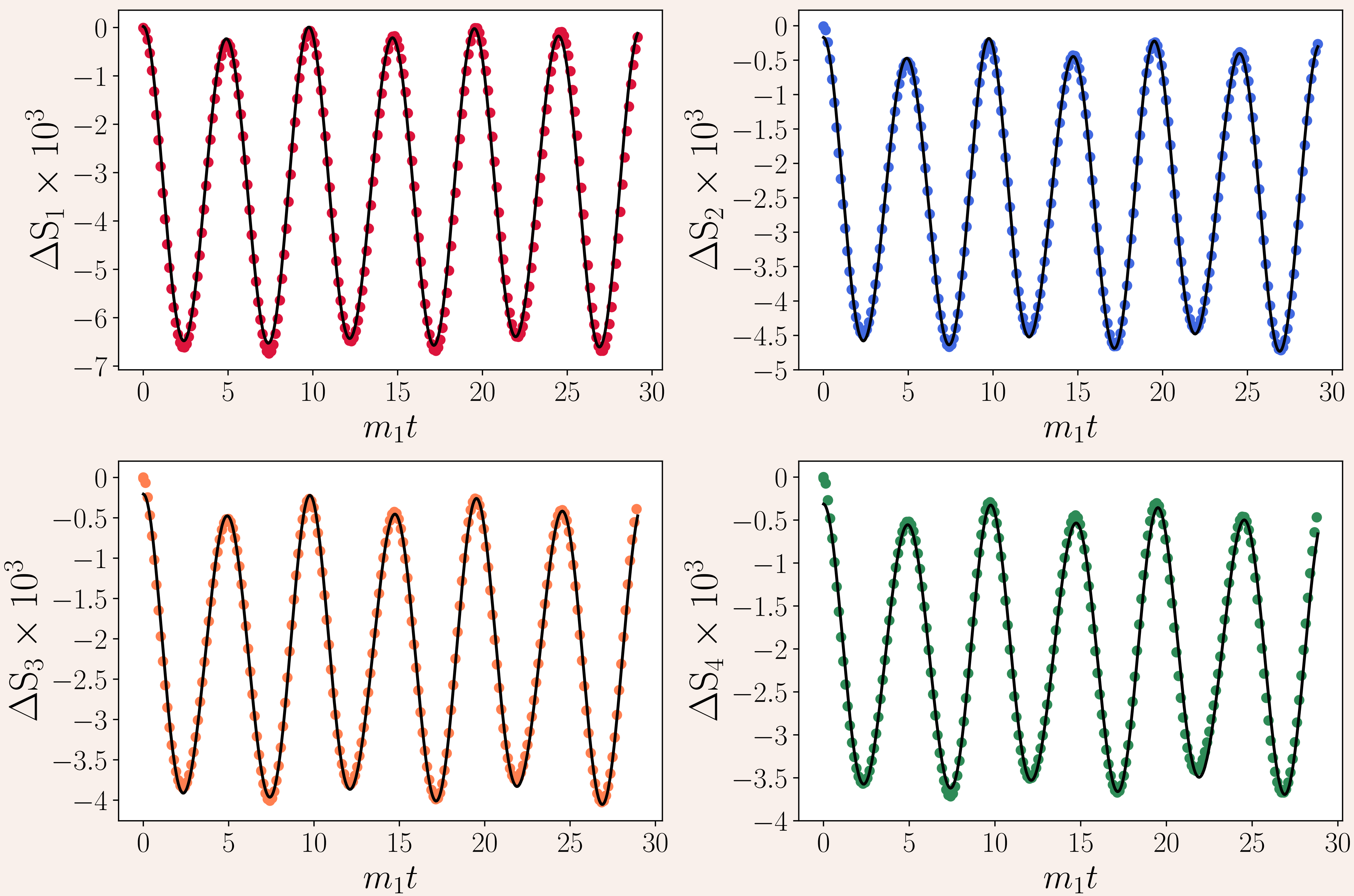
$$\frac{\Delta \mathcal{T}_n}{\bar{\mathcal{T}}_n} = \frac{\delta_\lambda}{\lambda}\left[C^{\mathcal{T}_n,\epsilon} + nC^{\epsilon}\sum_a \frac{2}{r_a^2}\cdot \hat{F}_a^{\epsilon}\cdot \hat{F}_a^{\mathcal{T}_n}\cdot \cos\left(m_a t\right)\right] \quad \Rightarrow \quad S_n(t) - S_n(0) = \frac{1}{1-n}\cdot\frac{\Delta \mathcal{T}_n}{\bar{\mathcal{T}}_n}$$

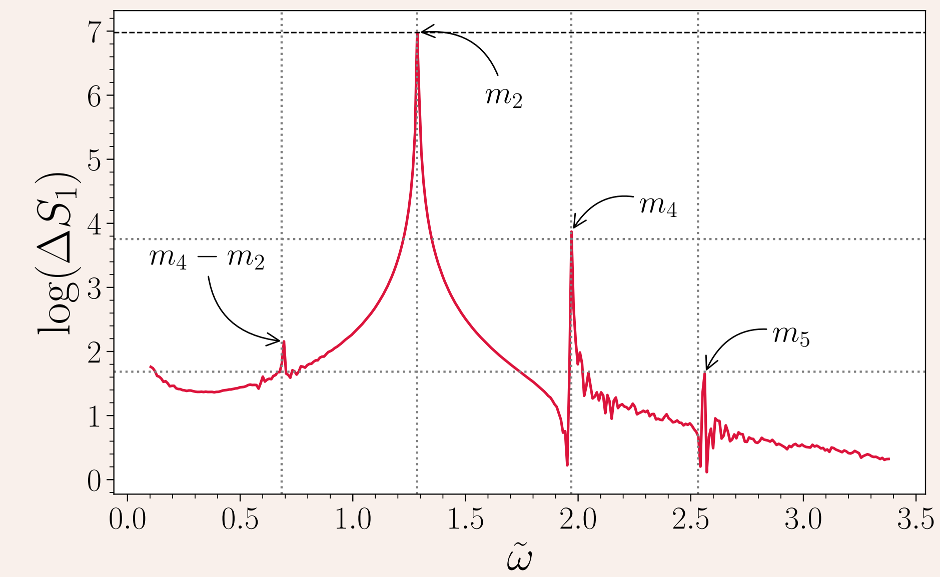




Post-quench dynamics for entropies

$$\frac{\Delta \mathcal{T}_n}{\bar{\mathcal{T}}_n} = \frac{\delta_\lambda}{\lambda}\left[C^{\mathcal{T}_n,\epsilon} + nC^{\epsilon}\sum_a \frac{2}{r_a^2}\cdot \hat{F}_a^{\epsilon}\cdot \hat{F}_a^{\mathcal{T}_n}\cdot \cos\left(m_a t\right)\right] \quad \Rightarrow \quad S_n(t) - S_n(0) = \frac{1}{1-n}\cdot\frac{\Delta \mathcal{T}_n}{\bar{\mathcal{T}}_n}$$





$\mathcal{O}$	$C^{\mathcal{O},\epsilon}$ theory	$C^{\mathcal{O},\epsilon}$ fit
σ FM	0.04167	0.04053
ϵ FM	0.11111	0.10301
ϵ PM	0.11111	0.11030
$\mathcal{T}_1$ PM	−0.06482	−0.06624
$\mathcal{T}_2$ PM	0.04861	0.04957
$\mathcal{T}_3$ PM	0.08642	0.08674
$\mathcal{T}_4$ PM	0.12153	0.12164

Work in progress

- calculation of dynamical structure factors to verify the odd particles of the E_7 model
- examination of the confinement on the spin chain to verify kinks

Future plans

- define the Kramers-Wannier duality in the Blume–Capel model

$\Rightarrow$ determine the right perturbation operator

$\Rightarrow$ find the exact tricritical point