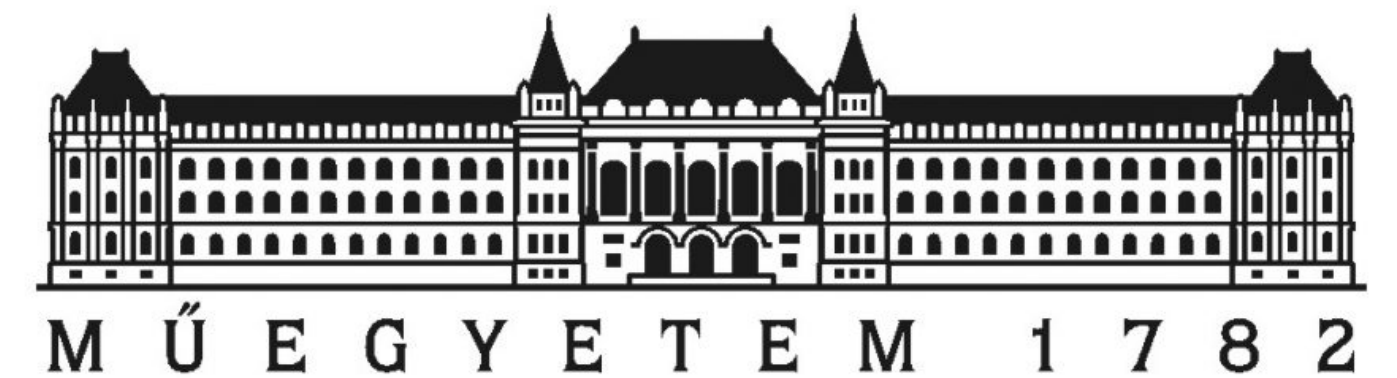


ENTANGLEMENT HAMILTONIAN AFTER A LOCAL QUENCH

Theory seminar @ Wigner Research Center for Physics
17 April 2026

R.B. and V. Eisler, *JHEP02(2026)232*



OUTLINE

- Entanglement Hamiltonian
 - Definitions and motivations
 - EH in $(1+1)D$ CFTs
 - EH for free fermions on a lattice
- EH after a local quench
 - CFT derivation
 - Continuum limit
- Conclusions

DEFINITIONS

ENTANGLEMENT: DEFINITIONS

□ Let us consider a bipartite quantum system $S = A \cup B$ in a pure state $|\Psi\rangle \in \mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$

- A pure state $|\Psi\rangle \in \mathcal{H}_{AB}$ is separable \Leftrightarrow if it can be written as Kronecker product of states $|\psi_A\rangle \in \mathcal{H}_A$ and $|\phi_B\rangle \in \mathcal{H}_B$,

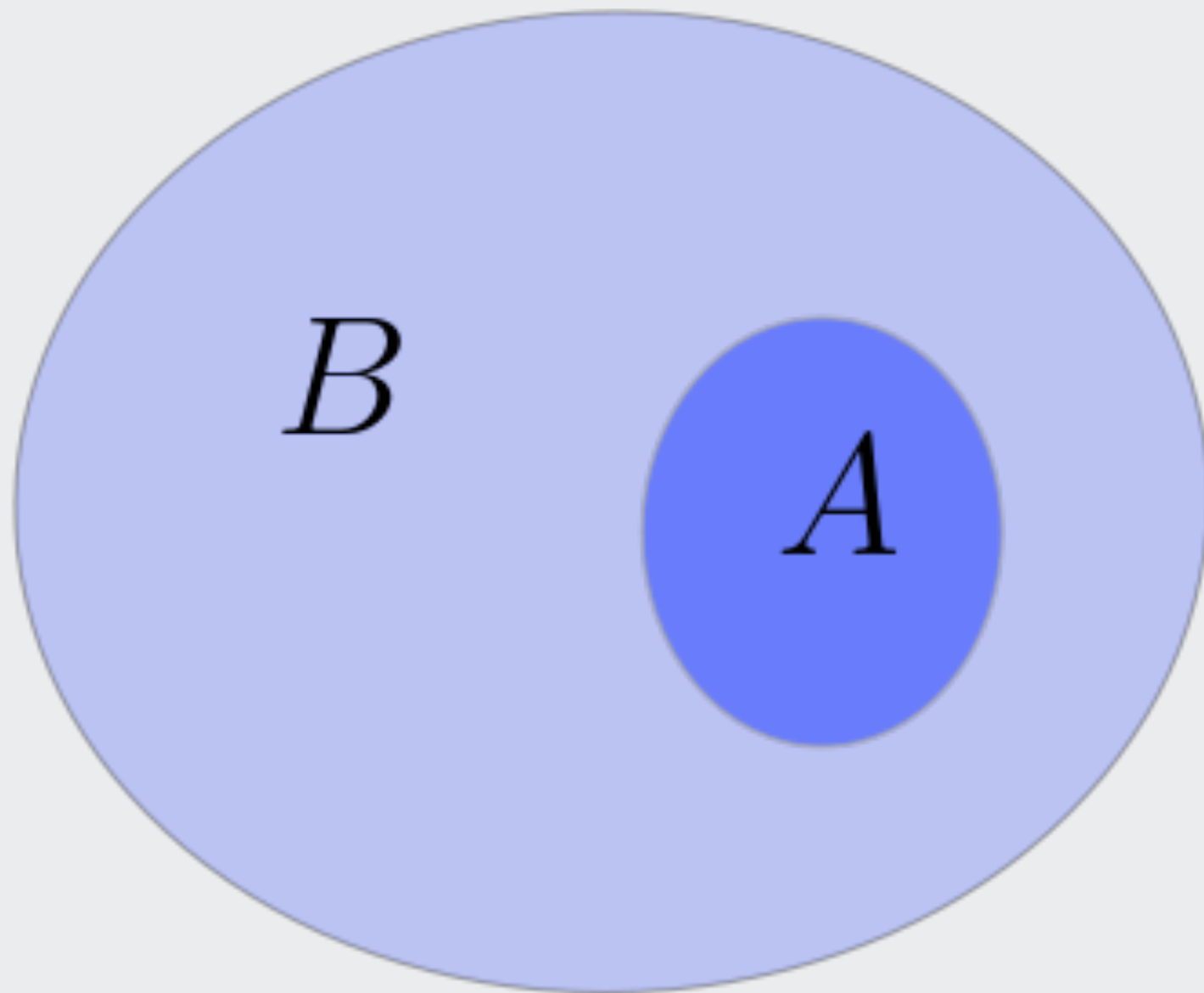
$$|\Psi\rangle = |\psi_A\rangle \otimes |\phi_B\rangle$$

otherwise it is said to be entangled.

- Schmidt decomposition: for a pure state $|\Psi\rangle \in \mathcal{H}_{AB}$, the entanglement properties of bipartition A:B are encoded in the Schmidt decomposition:

$$|\Psi\rangle = \sum_{\alpha=1}^{\chi_\alpha} \lambda_\alpha |\psi_A^\alpha\rangle \otimes |\phi_B^\alpha\rangle$$

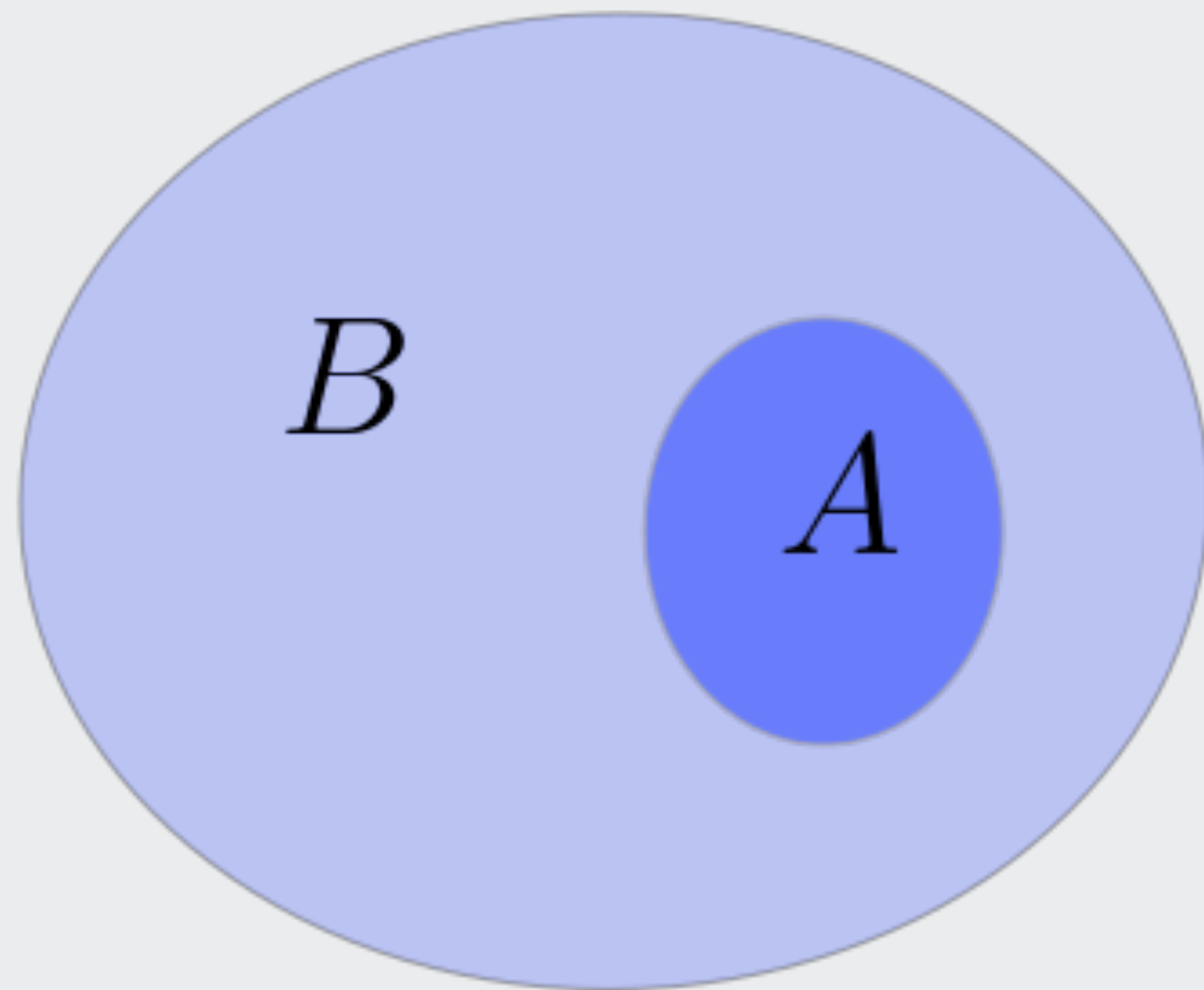
Schmidt values



ENTANGLEMENT HAMILTONIAN: DEFINITION

- Reduced density matrix of A: $\rho_A = \text{Tr}_B \rho$, with $\rho = |\psi\rangle\langle\psi|$.

$$\rho_A \propto e^{-\mathcal{H}} = \sum_{\alpha=1}^{\chi_\alpha} e^{-\varepsilon_\alpha} |\psi_A^\alpha\rangle\langle\psi_A^\alpha|$$



- \mathcal{H} is the *entanglement hamiltonian*.
- The spectrum of \mathcal{H} : *entanglement spectrum* $\{\varepsilon_\alpha\}$

$$\lambda_\alpha^2 \equiv e^{-\varepsilon_\alpha}$$

- Entanglement Entropy (EE): $S = -\text{Tr} \rho_A \log \rho_A$

BISOGNANO-WICHMANN THEOREM

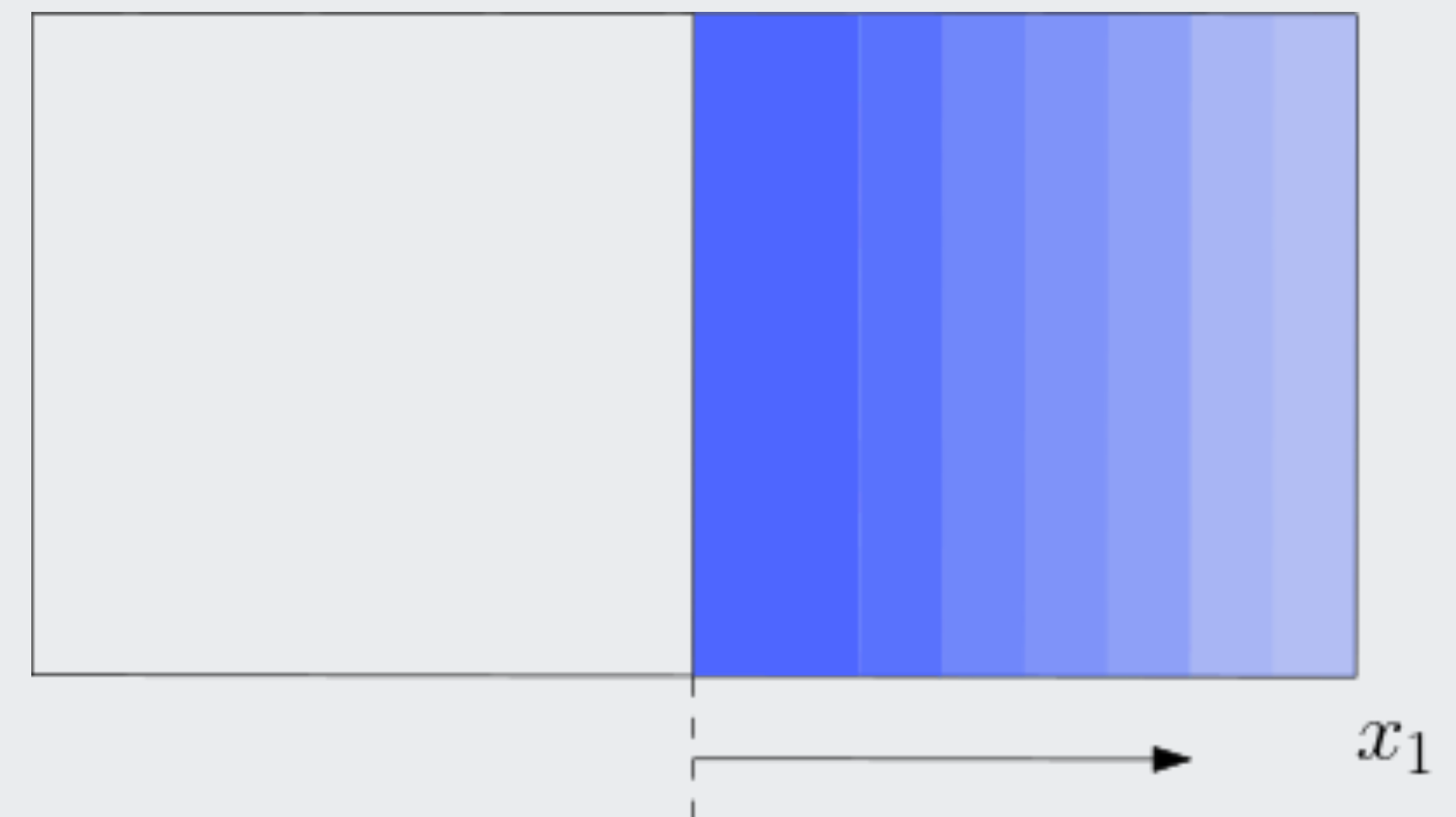
- Relativistic QFT in $(D+1)$ dimensions, spatial coordinates $x = \{x_1, x_2, \dots, x_D\}$, hamiltonian density $H(x)$.
- The partition A is the (right) half plane $x_1 > 0$.

- *Bisognano-Wichmann theorem*: the EH of the vacuum state is

$$\mathcal{H} = \frac{2\pi}{c} \int_{x \in A} dx x_1 H(x)$$

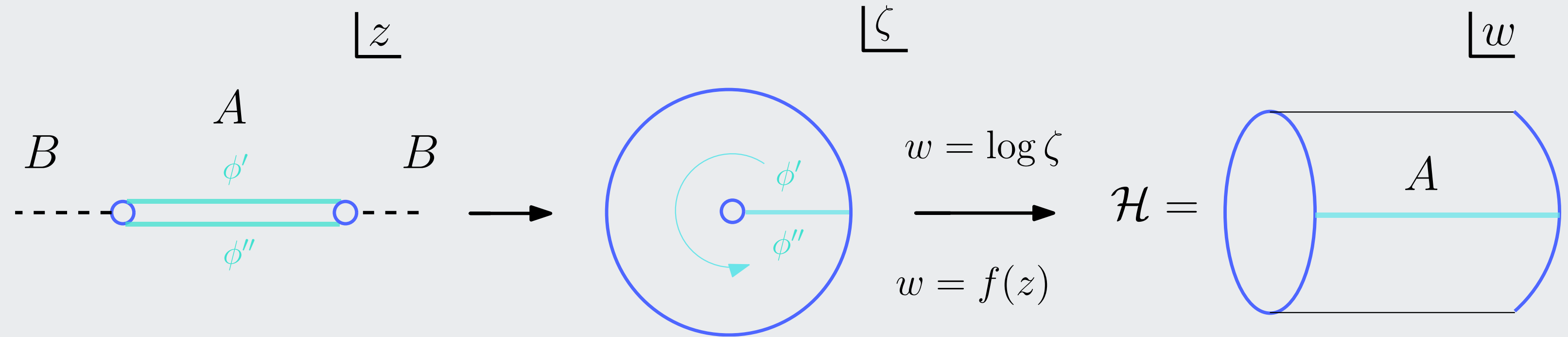
- The RDM has the form of a thermal state wrt to the original Hamiltonian, with a position-dependent inverse temperature $\frac{2\pi}{c}x_1$.

J. J. Bisognano and E. H. Wichmann, *J. Math. Phys.* **16**, 985–1007 (1975)
J. J. Bisognano and E. H. Wichmann, *J. Math. Phys.* **17**, 303–321 (1976)



MAPPING TO THE ANNULUS

- (1+1)D CFTs
(equilibrium case)

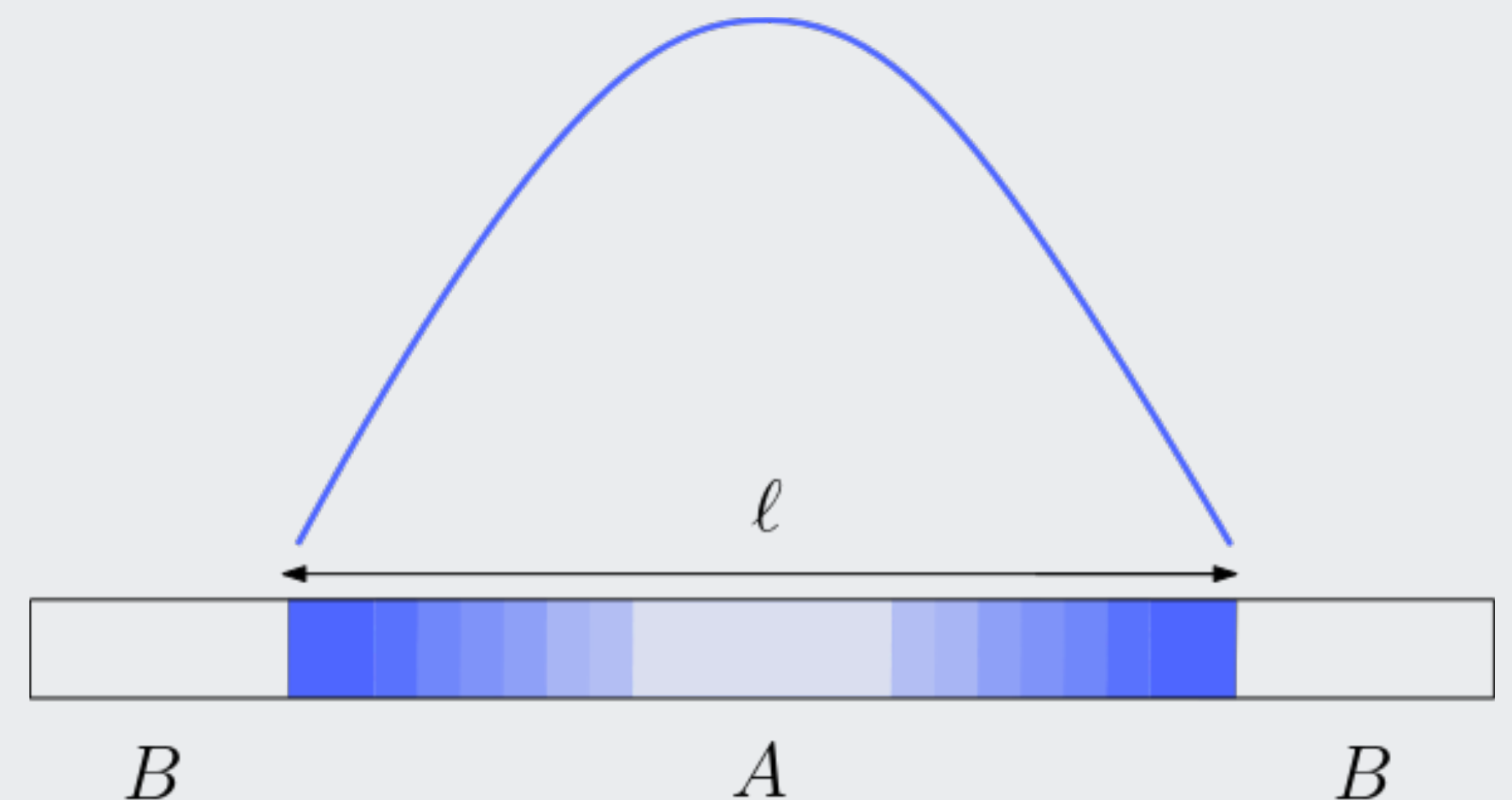


- ➔ ○ The EH can be written as a local integral over the energy-momentum tensor

$$\mathcal{H} = \frac{2\pi}{c} \int_A \beta(x) T_{00}(x) dx$$

- Example: finite interval $A = [0, \ell]$

$$\beta(x) \propto \frac{x}{\ell} \left(1 - \frac{x}{\ell} \right)$$



J. Cardy and E. Tonni, *J. Stat. Mech.* 123103 (2016)

MAPPING TO THE ANNULUS



- In the final strip geometry, the EH is given by the integral of the generator of translations in the imaginary time direction $\nu = \text{Im}w$.

$$\mathcal{H} = - \int_{\nu=\text{const}} T_{\nu\nu} du = \int_{f(C)} T(w) dw + \int_{\bar{f}(C)} \bar{T}(\bar{w}) d\bar{w}$$

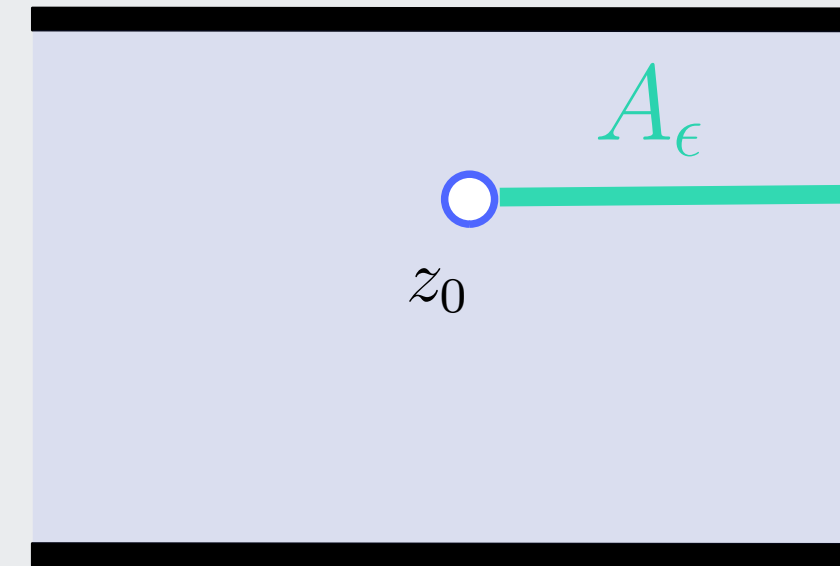
Back to original
Euclidean spacetime

$$\mathcal{H} = \int_C \frac{T(z)}{f'(z)} dz + \int_{\bar{C}} \frac{\bar{T}(\bar{z})}{\bar{f}'(\bar{z})} d\bar{z}$$

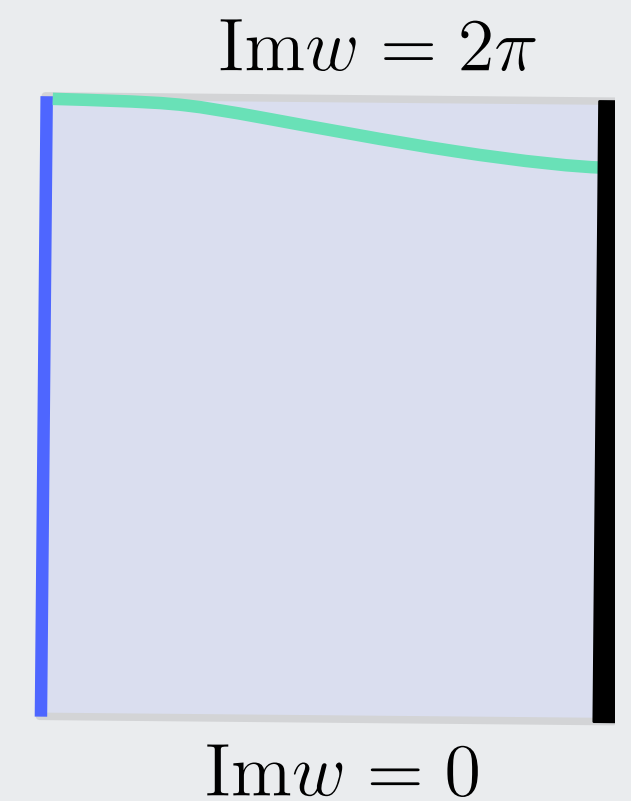
- (1+1)D CFTs out of equilibrium:

$$z \rightarrow w = u + iv = f(z)$$

z



w

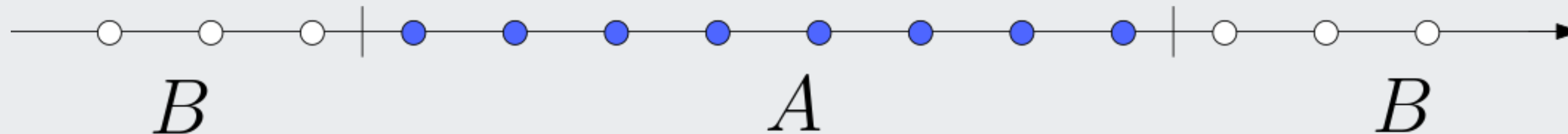


J. Cardy and E. Tonni, *J. Stat. Mech.* 123103 (2016)

EH ON THE LATTICE: FREE FERMIONS

- Free fermions hopping on a lattice: $\hat{H} = -\frac{1}{2} \sum_n t_n (c_n^\dagger c_{n+1} + c_{n+1}^\dagger c_n)$
- Subsystem $A = [1, \dots, L_A]$

$$\rho_A = \frac{e^{-\mathcal{H}}}{Z}, \quad \mathcal{H} = \sum_{i,j \in A}^{L_A} H_{ij} c_i^\dagger c_j = \sum_k^{L_A} \varepsilon_k \tilde{c}_k^\dagger \tilde{c}_k$$



EH ON THE LATTICE: FREE FERMIONS

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- Correlation matrix restricted to $A = [1, \dots, L_A]$: $(C_A)_{i,j} = \text{Tr}[\rho_A c_i^\dagger c_j] = \langle c_i^\dagger c_j \rangle, \quad i, j \in A$



$$H = \ln[(\mathbb{1} - C_A)/C_A]$$

$$\varepsilon_k = \ln[(1 - \zeta_k)/\zeta_k] \quad \text{eigenvalues}$$

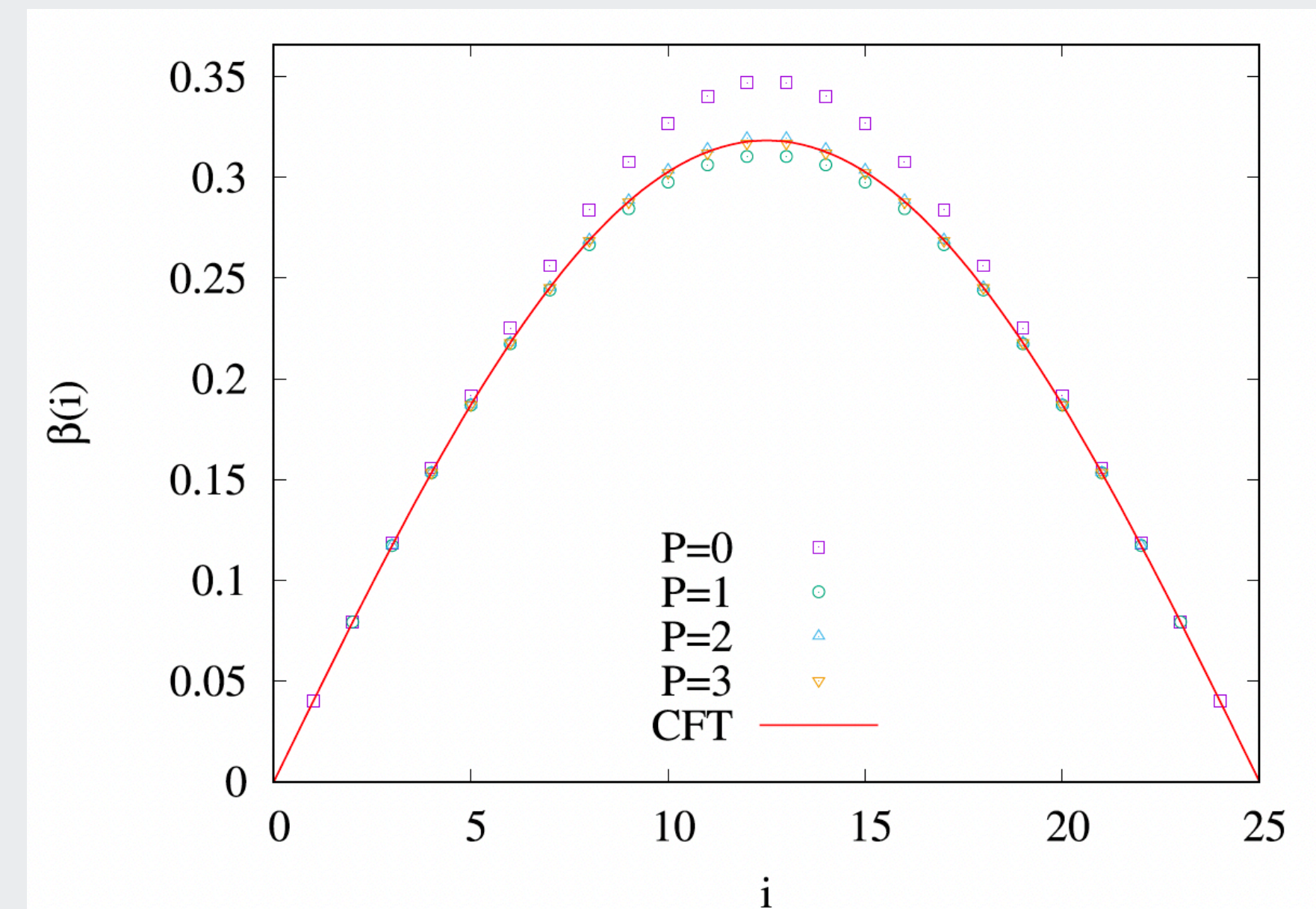
BW ON THE LATTICE?

- The lattice EH cannot be obtained as a simple discretization of the continuum result.
- Results:
 - The CFT result is recovered from the lattice EH including all the long range hopping terms in the continuum limit

V. Eisler, E. Tonni and I. Peschel, *J. Stat. Mech.* 073V101 (2019)

- *Example:* finite interval $A = [1, \dots, L]$ in a finite chain

$$\beta(x) = \frac{1}{\pi L} \sum_{p=0}^P (-1)^p (2p + 1) H_{i-p, i+p+1}$$



BW ON THE LATTICE?

- The lattice EH cannot be in general obtained as a simple discretization of the continuum result.
- Results:
 - The CFT result is recovered from the lattice EH including all the long range hopping terms in the continuum limit

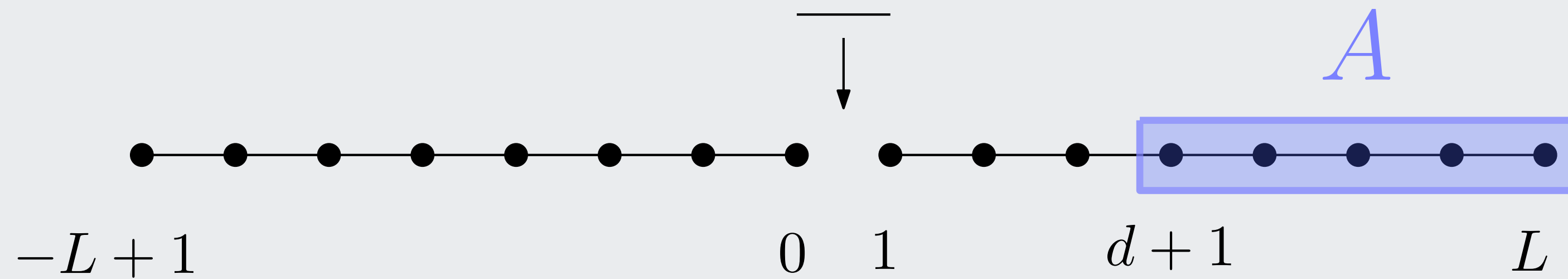
V. Eisler, E. Tonni and I. Peschel, *J. Stat. Mech.* 073101 (2019)

- A tridiagonal matrix exists that commutes with the lattice EH and has the form of the discretized CFT ansatz.

I. Peschel, *J. Stat. Mech.* P06004 (2004)

EH AFTER A LOCAL QUENCH

THE MODEL



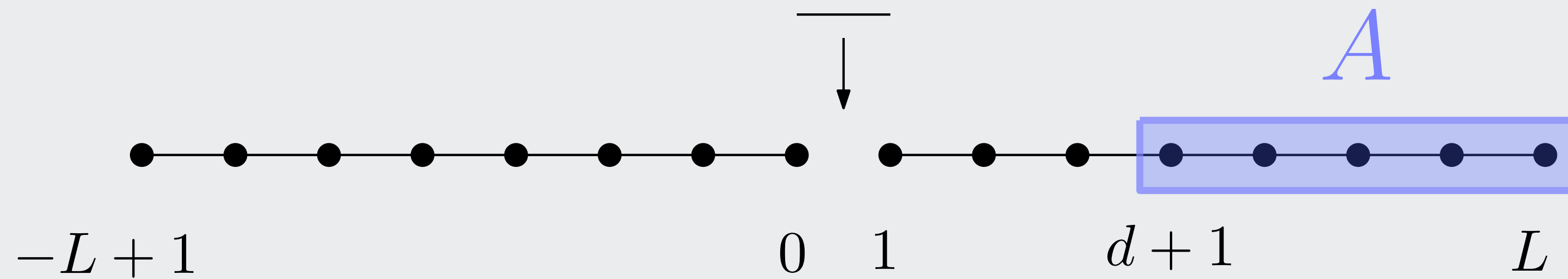
- Two half-chains are in their ground states at half filling, initially disconnected

$$\hat{H}_0 = \hat{H}_\ell + \hat{H}_r = -\frac{1}{2} \sum_{n=-L+1}^{-1} \left(c_n^\dagger c_{n+1} + c_{n+1}^\dagger c_n \right) - \frac{1}{2} \sum_{n=1}^{L-1} \left(c_n^\dagger c_{n+1} + c_{n+1}^\dagger c_n \right)$$

- $t = 0$: two halves joined \longrightarrow The system evolves under the action of the homogeneous Hamiltonian

$$\hat{H} = -\frac{1}{2} \sum_{n=-L+1}^{L-1} \left(c_n^\dagger c_{n+1} + c_{n+1}^\dagger c_n \right).$$

THE MODEL



- Two point correlation

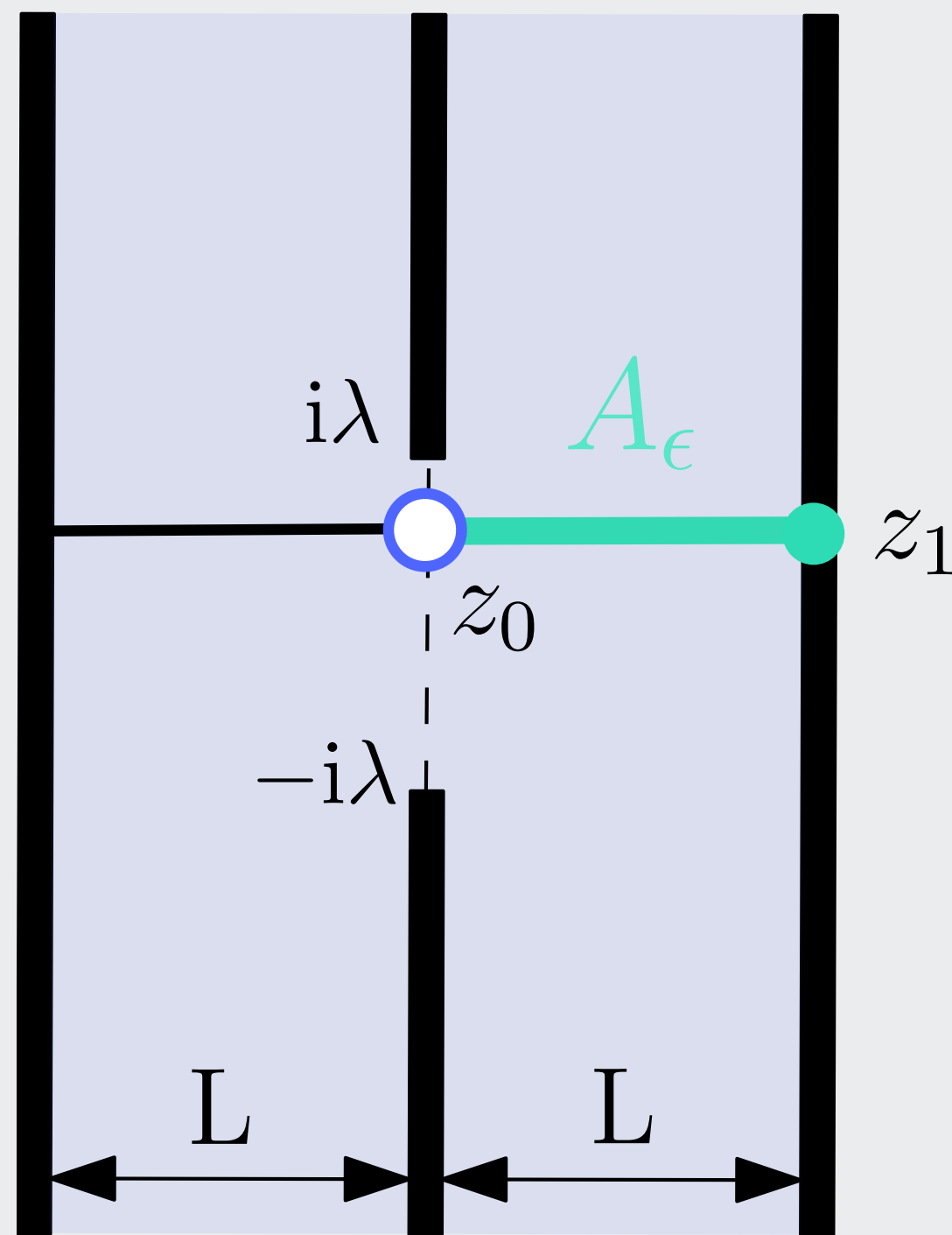
$$C_{m,n}(t) = \langle \psi(t) | c_m^\dagger c_n | \psi(t) \rangle, \quad |\psi(t)\rangle = e^{-i\hat{H}t} |\psi_0\rangle_\ell \otimes |\psi_0\rangle_r$$

- The EH associated to a subsystem $A = [d+1, L]$ is

$$\rho_A(t) = \frac{e^{-\mathcal{H}(t)}}{\mathcal{Z}}, \quad \mathcal{H}(t) = \sum_k H_{i,j}(t) c_i^\dagger c_j, \quad H(t) = \ln \left[\frac{1 - C_A(t)}{C_A(t)} \right].$$

CFT FORMULATION

- Path-integral representation of the time-evolved density matrix $\rho(t)$

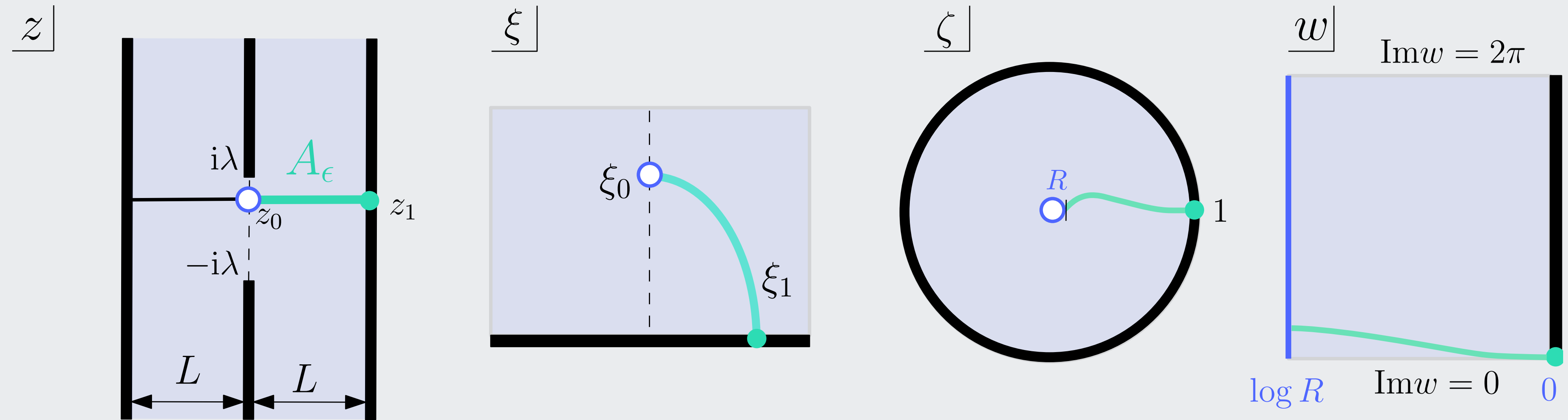


$$\langle \varphi'' | \rho(t) | \varphi' \rangle = Z^{-1} \langle \varphi'' | e^{-it\hat{H}} e^{-\lambda\hat{H}} | \varphi_\ell \otimes \varphi_r \rangle \langle \varphi_\ell \otimes \varphi_r | e^{it\hat{H}} e^{-\lambda\hat{H}} | \varphi' \rangle$$

Boundary states: $|\varphi_\ell \otimes \varphi_r\rangle = \lim_{\beta \rightarrow \infty} \frac{e^{-\beta(\hat{H}_\ell + \hat{H}_r)} e^{\beta E_0} |s\rangle}{\langle \varphi_\ell \otimes \varphi_r | s \rangle}$.

- Reduced density matrix:
 - Cut along $C = \{z = x + i\tau, x_0 < x < L\}$
 - Regularisation disk around entangling point

CONFORMAL MAPPING TO THE ANNULUS



- Conformal mapping to the annulus

$$\xi(z) = i \sqrt{\frac{\sin\left(\frac{\pi}{2L}(i\lambda + z)\right)}{\sin\left(\frac{\pi}{2L}(i\lambda - z)\right)}} \rightarrow \zeta(z) = \left(\frac{\xi_1 - \bar{\xi}_0}{\xi_1 - \xi_0}\right) \frac{\xi(z) - \xi_0}{\xi(z) - \bar{\xi}_0} \rightarrow w(z) = \log \left[\left(\frac{\xi_1 - \bar{\xi}_0}{\xi_1 - \xi_0}\right) \frac{\xi(z) - \xi_0}{\xi(z) - \bar{\xi}_0} \right]$$

CFT RESULT FOR THE EH

- The EH in the strip geometry is

$$\mathcal{H} = 2\pi \int_{w(C_\epsilon)} T(w)dw + 2\pi \int_{\overline{w(C_\epsilon)}} \bar{T}(\bar{w})d\bar{w}.$$

- The EH in the original double-pant geometry is

$$\mathcal{H} = \int_{C_\epsilon} \beta(z)T(z)dz + \int_{\bar{C}_\epsilon} \bar{\beta}(\bar{z})\bar{T}(\bar{z})d\bar{z},$$

$$\beta(z) = \frac{2\pi}{w'(z)}, \quad \bar{\beta}(\bar{z}) = \frac{2\pi}{\overline{w'(z)}}$$

$$\beta(z) = 2\pi \frac{(\xi(z) - \xi_0)(\xi(z) - \bar{\xi}_0)}{(\xi_0 - \bar{\xi}_0)\xi'(z)}.$$

- Analytical continuation $\tau \rightarrow it$

$$\mathcal{H} = \int_{A_\epsilon} \beta(x, t)T(x - t)dx + \int_{A_\epsilon} \bar{\beta}(x, t)\bar{T}(x + t)dx.$$

EH FOR THE HALF CHAIN

- Half-chain partition $x_0 = 0 \rightarrow z_0 = i\tau$

z

ξ

$\xi_0 = \xi(i\tau) = i \sqrt{\frac{\sinh \frac{\pi}{2L}(\lambda + \tau)}{\sinh \frac{\pi}{2L}(\lambda - \tau)}}$

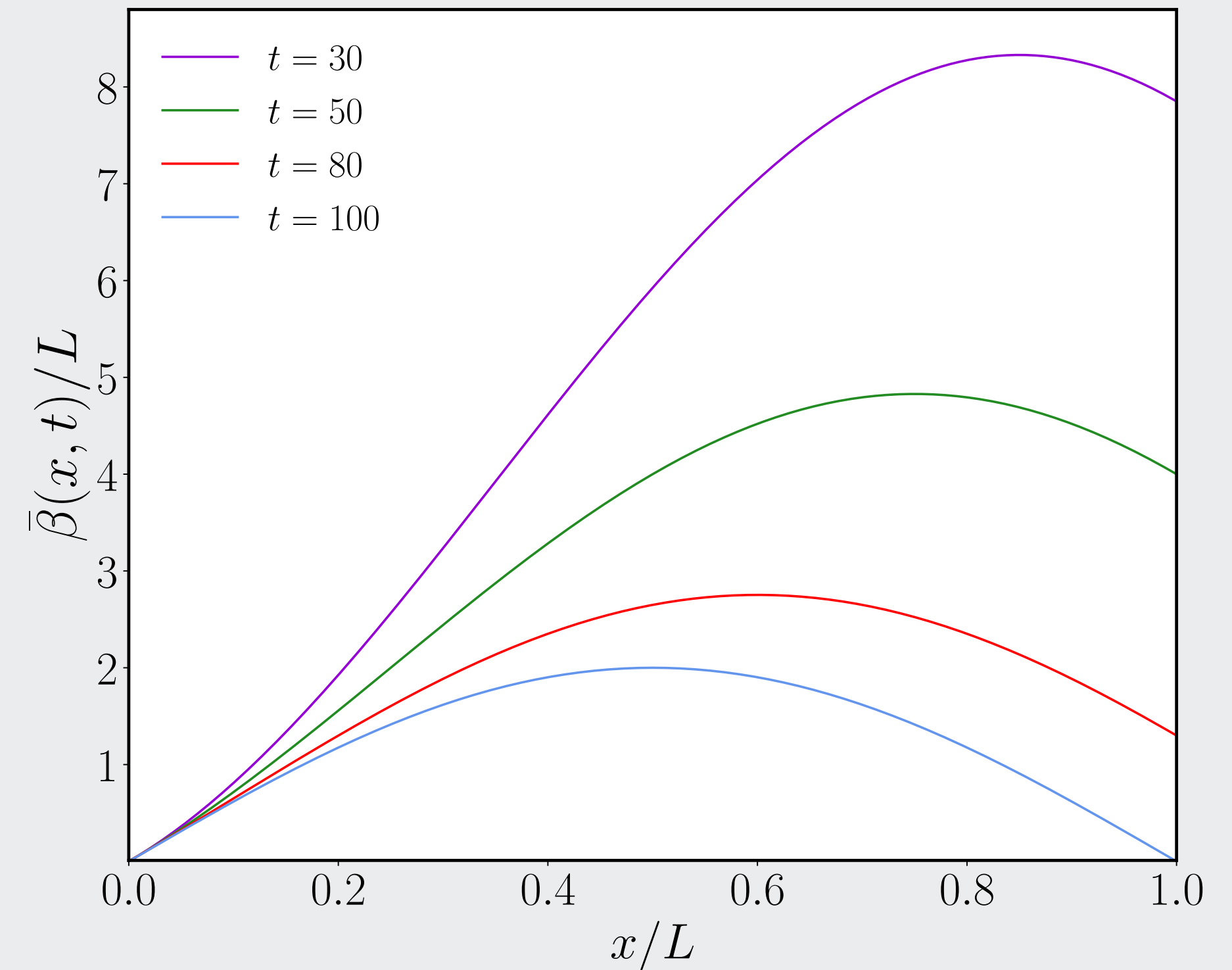
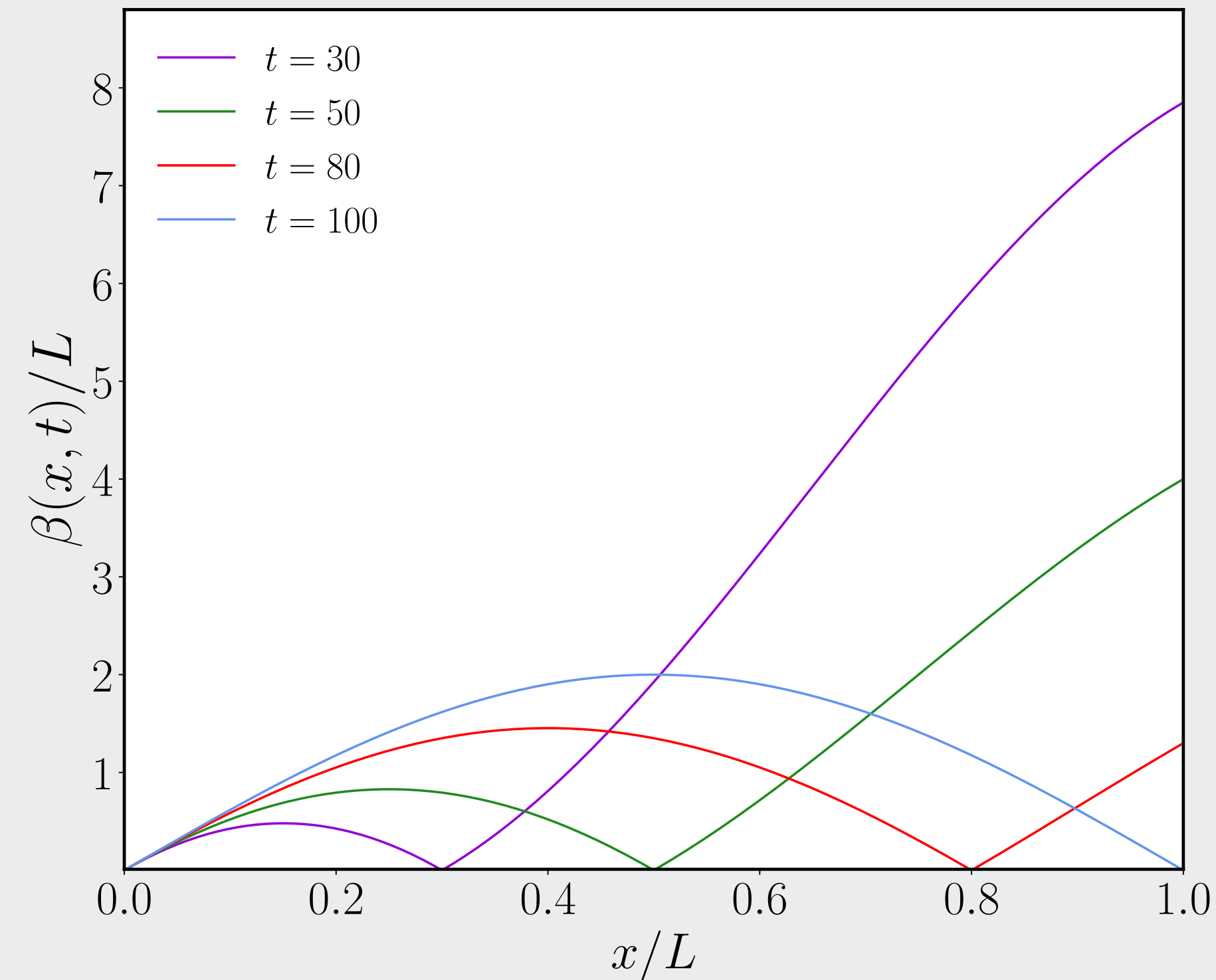
$\beta(z) = 4L \sin \left[\frac{\pi}{2L}(z - i\tau) \right] \sqrt{\frac{\cosh \frac{\pi\lambda}{L} - \cos \frac{\pi z}{L}}{\cosh \frac{\pi\lambda}{L} - \cosh \frac{\pi\tau}{L}}}$

- After analytic continuation, and in the limit $t \gg \lambda$,

$$t < L : \quad \beta(x, t) = 4L \left| \frac{\sin \frac{\pi x}{2L} \sin \frac{\pi}{2L}(x - t)}{\sin \frac{\pi t}{2L}} \right|,$$

$$\bar{\beta}(x, t) = 4L \left| \frac{\sin \frac{\pi x}{2L} \sin \frac{\pi}{2L}(x + t)}{\sin \frac{\pi t}{2L}} \right|$$

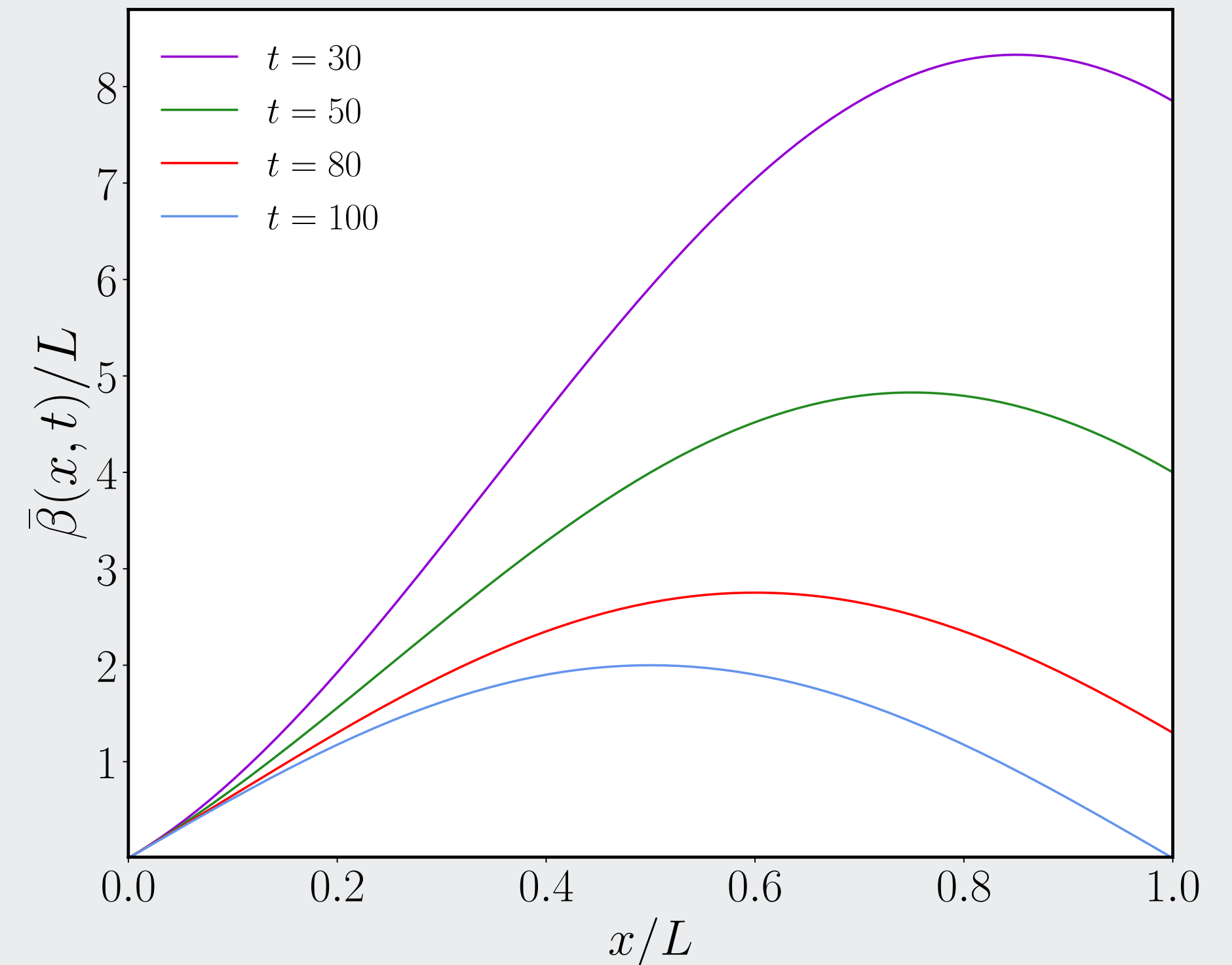
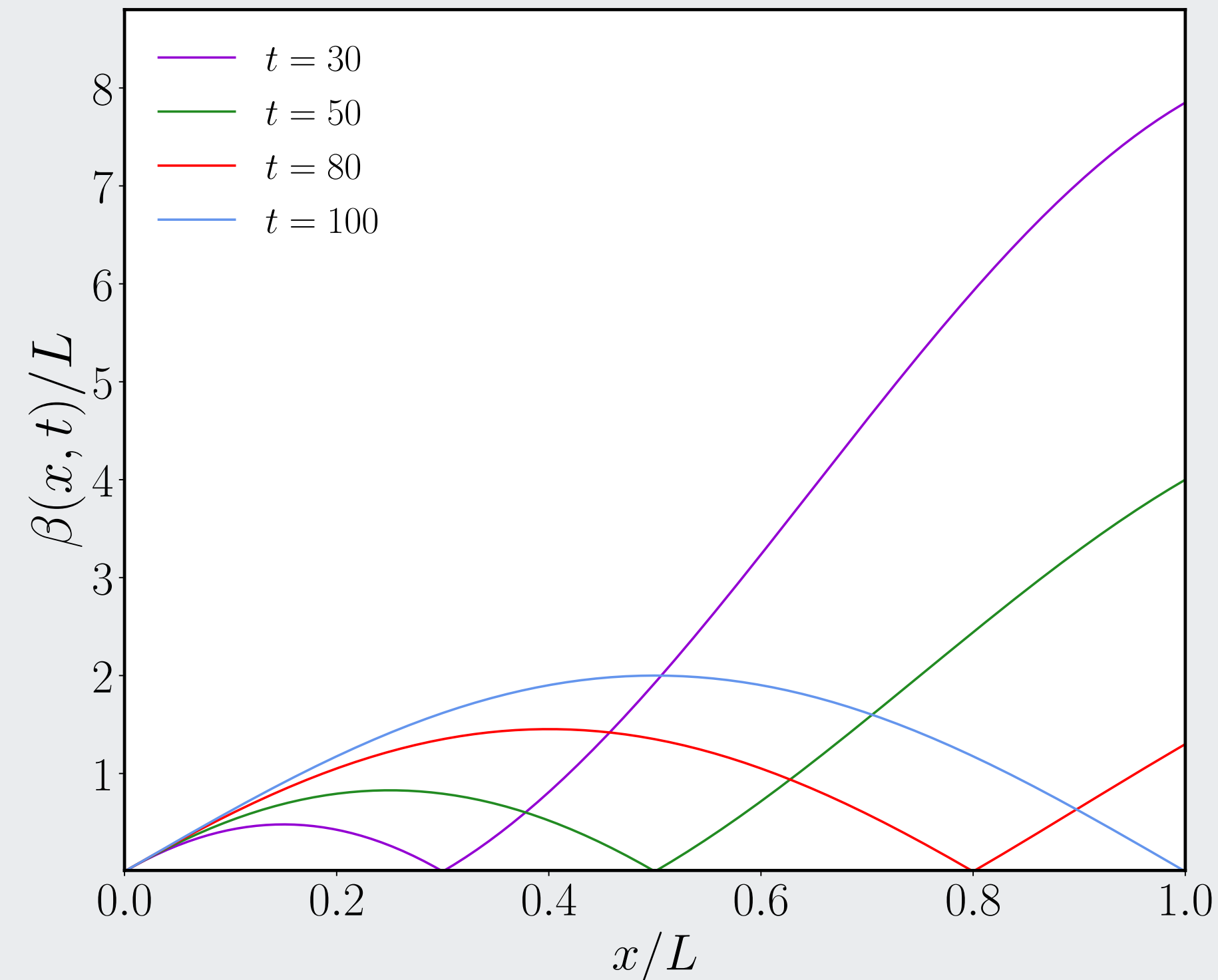
EH FOR THE HALF CHAIN



$$t < L : \quad \beta(x, t) = 4L \left| \frac{\sin \frac{\pi x}{2L} \sin \frac{\pi}{2L}(x - t)}{\sin \frac{\pi t}{2L}} \right|,$$

$$\bar{\beta}(x, t) = 4L \left| \frac{\sin \frac{\pi x}{2L} \sin \frac{\pi}{2L}(x + t)}{\sin \frac{\pi t}{2L}} \right|$$

EH FOR THE HALF CHAIN



- After reflection at $t = L$, we have the symmetry property

$$L < t < 2L : \quad \bar{\beta}(x, t) = \beta(x, 2L - t), \quad \beta(x, t) = \bar{\beta}(x, 2L - t)$$

ENTANGLEMENT ENTROPY

- The different number of roots of $\beta(x, t)$ and $\bar{\beta}(x, t)$ for $x > 0$ implies that the contribution to the entanglement entropy from the R-movers is three times the one from the L-movers.

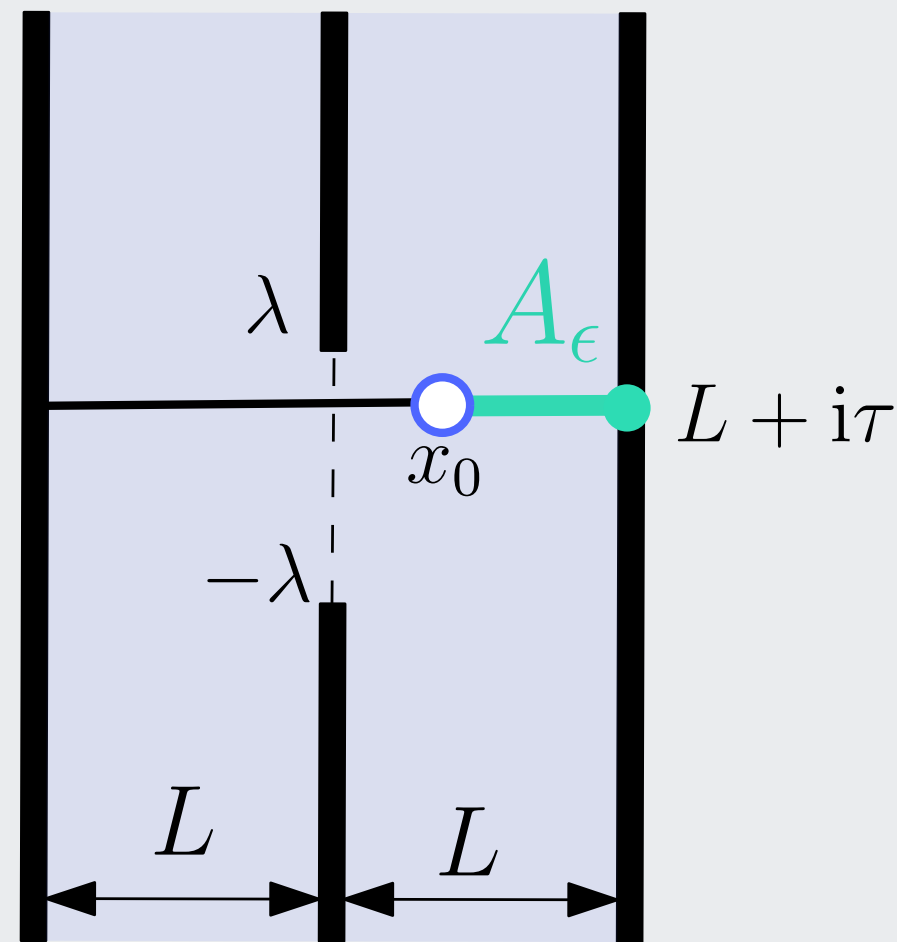
$$S \equiv S_R + S_L = \frac{\pi c}{6} \int_{A_e} \frac{dx}{\beta(x, t)} + \frac{\pi c}{6} \int_{A_e} \frac{dx}{\bar{\beta}(x, t)}$$

with

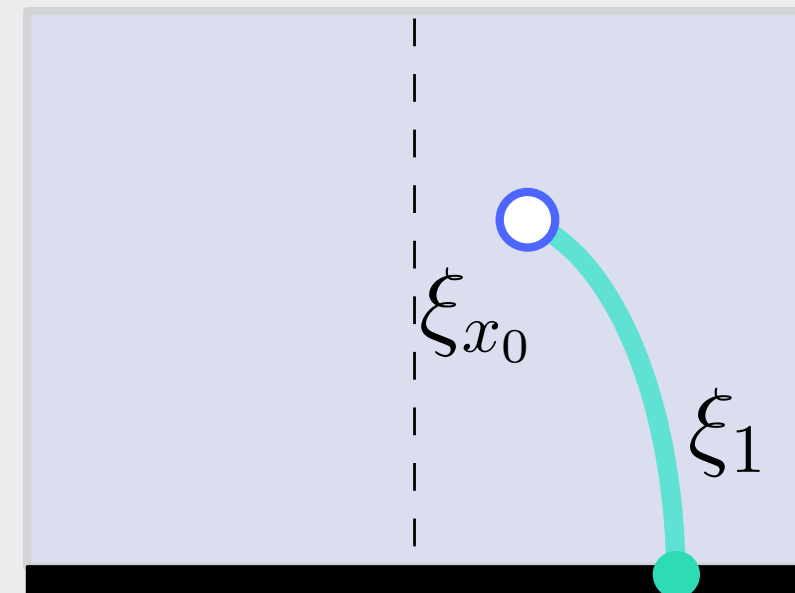
$$\left\{ \begin{array}{l} S_L = \frac{c}{12} \log \left| \frac{\sin \frac{\pi}{2L}(t + \epsilon)}{\sin \frac{\pi\epsilon}{2L} \cos \frac{\pi t}{2L}} \right| \\ S_R = \frac{c}{12} \log \left| \frac{\sin \frac{\pi(t-\lambda)}{2L} \sin \frac{\pi(t-\epsilon)}{2L}}{\sin \frac{\pi\lambda}{2L} \sin \frac{\pi\epsilon}{2L}} \right| + \frac{c}{12} \log \left| \frac{\sin \frac{\pi(t+\lambda)}{2L} \cos \frac{\pi t}{2L}}{\sin \frac{\pi\lambda}{2L}} \right| \end{array} \right. \xrightarrow{\lambda, \epsilon \ll 1} S_R = 3S_L$$

DECENTERED INTERVAL

z



ξ



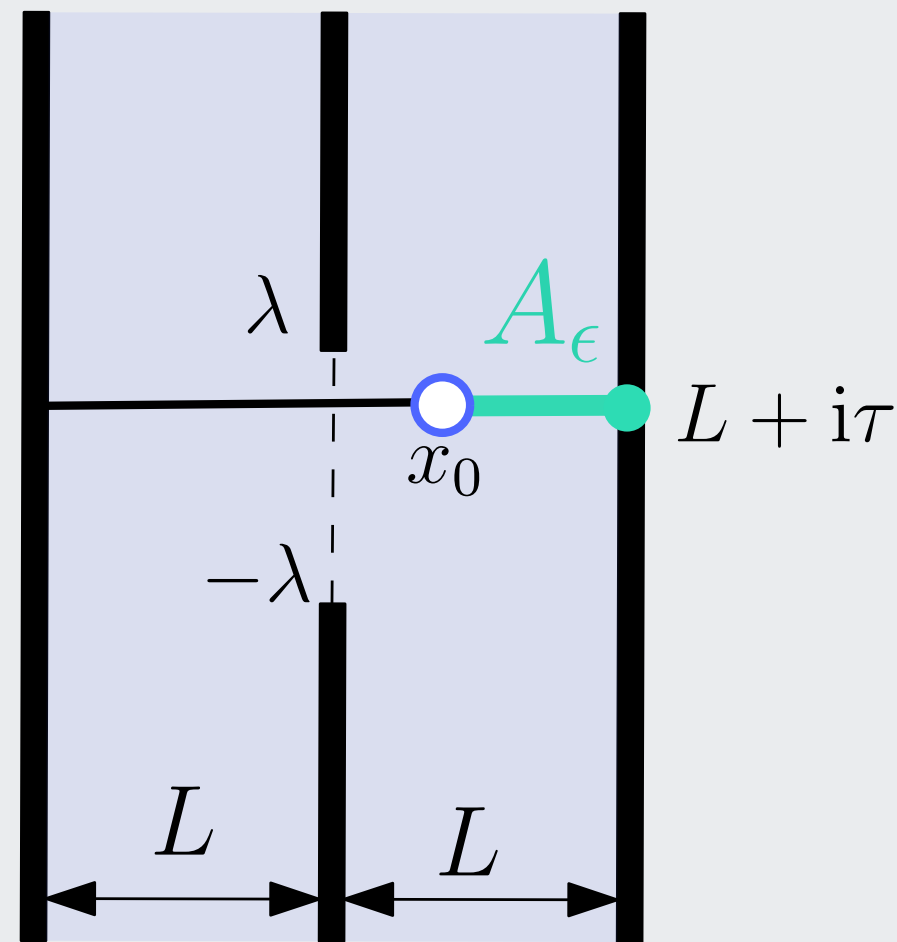
- The boundary point x_0 does not coincide with the quench location $x = 0$.

$$\beta(x, t) = \begin{cases} 2L \frac{\cos \frac{\pi x_0}{L} - \cos \frac{\pi x}{L}}{\sin \frac{\pi x_0}{L}} & t < x_0 < x \\ 4L \frac{\sin \frac{\pi}{2L}(x - x_0) \sin \frac{\pi}{2L}(t - x)}{\sin \frac{\pi}{2L}(t - x_0)} & x_0 < x < t \\ 4L \frac{\sin \frac{\pi}{2L}(x + x_0) \sin \frac{\pi}{2L}(x - t)}{\sin \frac{\pi}{2L}(t + x_0)} & x_0 < t < x \end{cases}$$

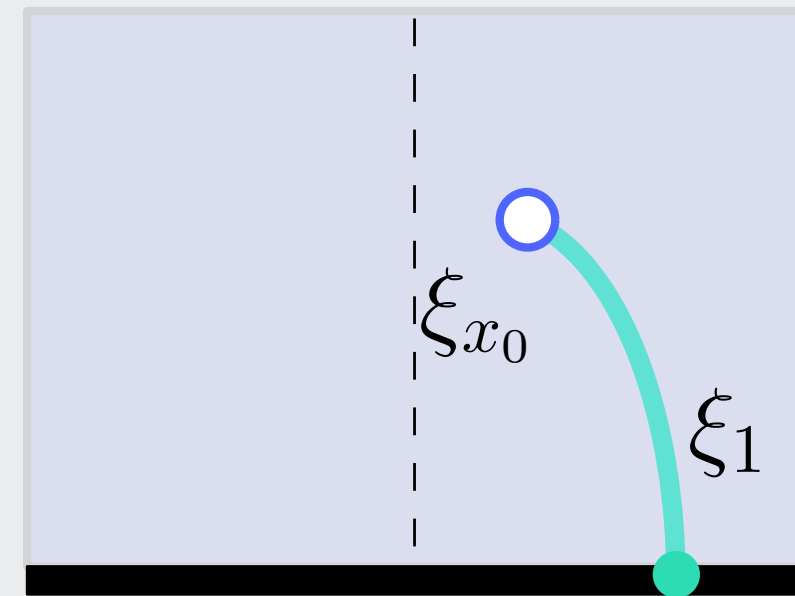
$$\bar{\beta}(x, t) = \begin{cases} 2L \frac{\cos \frac{\pi x_0}{L} - \cos \frac{\pi x}{L}}{\sin \frac{\pi x_0}{L}} & t < x_0 < x \\ 4L \frac{\sin \frac{\pi}{2L}(x - x_0) \sin \frac{\pi}{2L}(x + t)}{\sin \frac{\pi}{2L}(t + x_0)} & x_0 < x < t \\ 4L \frac{\sin \frac{\pi}{2L}(x - x_0) \sin \frac{\pi}{2L}(x + t)}{\sin \frac{\pi}{2L}(t + x_0)} & x_0 < t < x \end{cases}$$

DECENTERED INTERVAL

z



ξ



- The boundary point x_0 does not coincide with the quench location $x = 0$.

Equilibrium result

E. Tonni, J. Rodriguez-Laguna, and G. Sierra, *J. Stat. Mech.* (2018) 043105

$$\beta(x, t) = \begin{cases} 2L \frac{\cos \frac{\pi x_0}{L} - \cos \frac{\pi x}{L}}{\sin \frac{\pi x_0}{L}} & t < x_0 < x \\ 4L \frac{\sin \frac{\pi}{2L}(x - x_0) \sin \frac{\pi}{2L}(t - x)}{\sin \frac{\pi}{2L}(t - x_0)} & x_0 < x < t \\ 4L \frac{\sin \frac{\pi}{2L}(x + x_0) \sin \frac{\pi}{2L}(x - t)}{\sin \frac{\pi}{2L}(t + x_0)} & x_0 < t < x \end{cases}$$

$$\bar{\beta}(x, t) = \begin{cases} 2L \frac{\cos \frac{\pi x_0}{L} - \cos \frac{\pi x}{L}}{\sin \frac{\pi x_0}{L}} & t < x_0 < x \\ 4L \frac{\sin \frac{\pi}{2L}(x - x_0) \sin \frac{\pi}{2L}(x + t)}{\sin \frac{\pi}{2L}(t + x_0)} & x_0 < x < t \\ 4L \frac{\sin \frac{\pi}{2L}(x - x_0) \sin \frac{\pi}{2L}(x + t)}{\sin \frac{\pi}{2L}(t + x_0)} & x_0 < t < x \end{cases}$$

EH ON THE LATTICE: CONTINUUM LIMIT

- We write the lattice EH as

$$\mathcal{H} = \sum_j t_0(j) c_j^\dagger c_j + \sum_j \sum_{r \geq 1} \left[t_r(j + r/2) (c_j^\dagger c_{j+r} + c_{j+r}^\dagger c_j) + i s_r(j + r/2) (c_j^\dagger c_{j+r} - c_{j+r}^\dagger c_j) \right].$$

where

$$t_r(j + r/2) = \text{Re } H_{j,j+r}(t), \quad s_r(j + r/2) = \text{Im } H_{j,j+r}(t),$$

- Continuum limit: $aj \rightarrow x$ Fermi points: $\pm aq_F$

$$c_j \rightarrow \sqrt{a} \left(e^{iq_F x} \psi(z) + e^{-iq_F x} \bar{\psi}(\bar{z}) \right), \quad \begin{cases} \psi(z), \bar{\psi}(\bar{z}) & \text{R/L movers} \\ z = x - t, \quad \bar{z} = x + t \end{cases}$$

EH ON THE LATTICE: CONTINUUM LIMIT

- Continuum limit of the EH

$$\mathcal{H} = \int_A dx \left[\beta_0(x, t) T_{00}(z, \bar{z}) - \beta_1(x, t) T_{01}(z, \bar{z}) + \mu_0(x, t) \rho(z, \bar{z}) - \mu_1(x, t) j(z, \bar{z}) \right],$$

- density and current of the $U(1)$ charge

$$\rho(z, \bar{z}) = \psi^\dagger(z) \psi(z) + \bar{\psi}^\dagger(\bar{z}) \bar{\psi}(\bar{z}), \quad j(z, \bar{z}) = \psi^\dagger(z) \psi(z) - \bar{\psi}^\dagger(\bar{z}) \bar{\psi}(\bar{z}).$$

- weight functions

$$\beta_0(x, t) = -2a \sum_{r=1}^{\infty} r \sin(rq_F a) t_r(x) \quad \mu_0(x, t) = t_0 + 2 \sum_{r=1}^{\infty} \cos(rq_F a) t_r(x),$$

$$\beta_1(x, t) = 2a \sum_{r=1}^{\infty} r \cos(rq_F a) s_r(x) \quad \mu_1(x, t) = 2a \sum_{r=1}^{\infty} \sin(rq_F a) s_r(x).$$

RESULTS AT HALF FILLING

- Local quench at half-filling: $q_F a = \pi/2$
- The diagonals of $H(t)$ are diagonals are either purely real or imaginary,

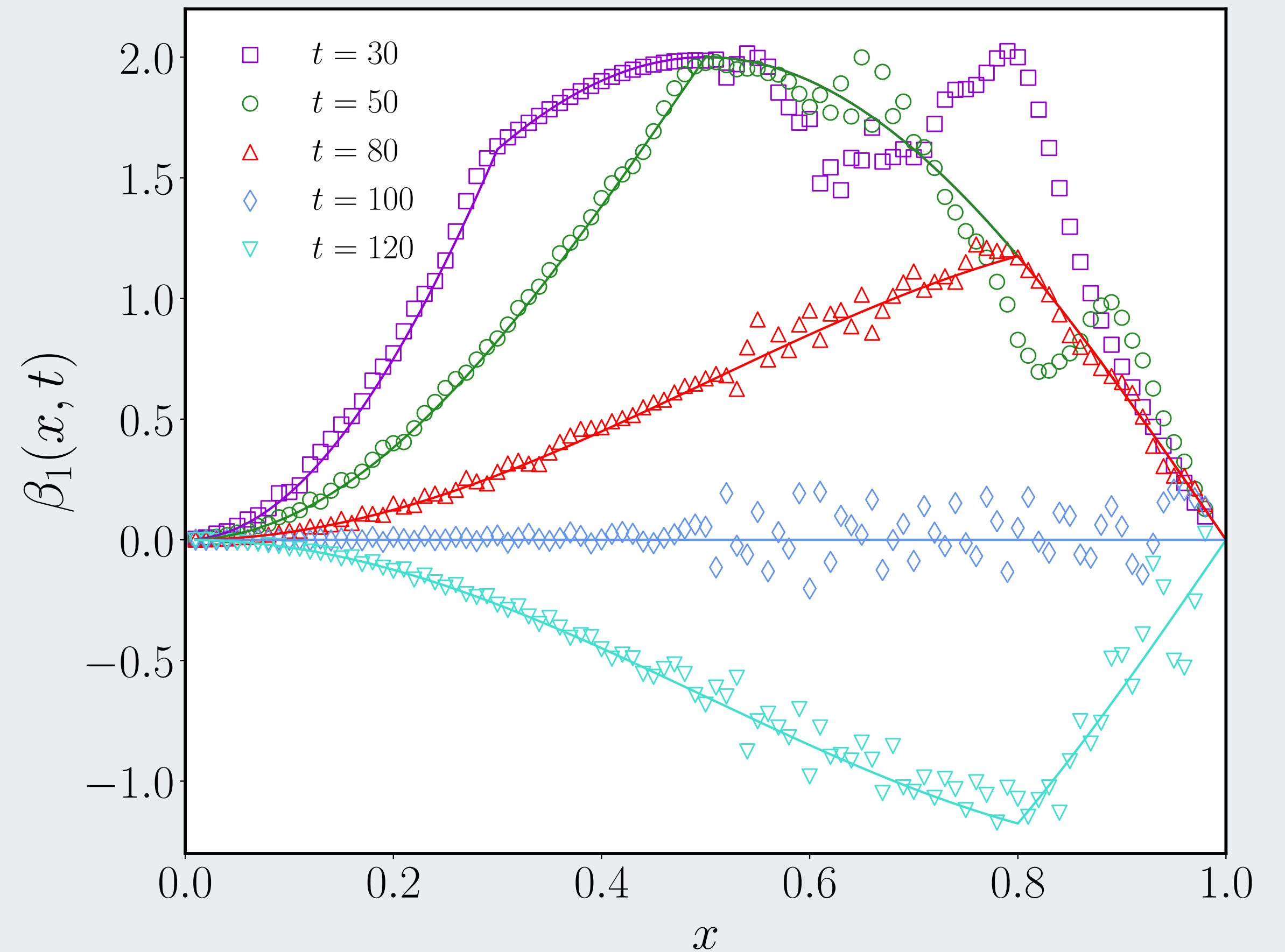
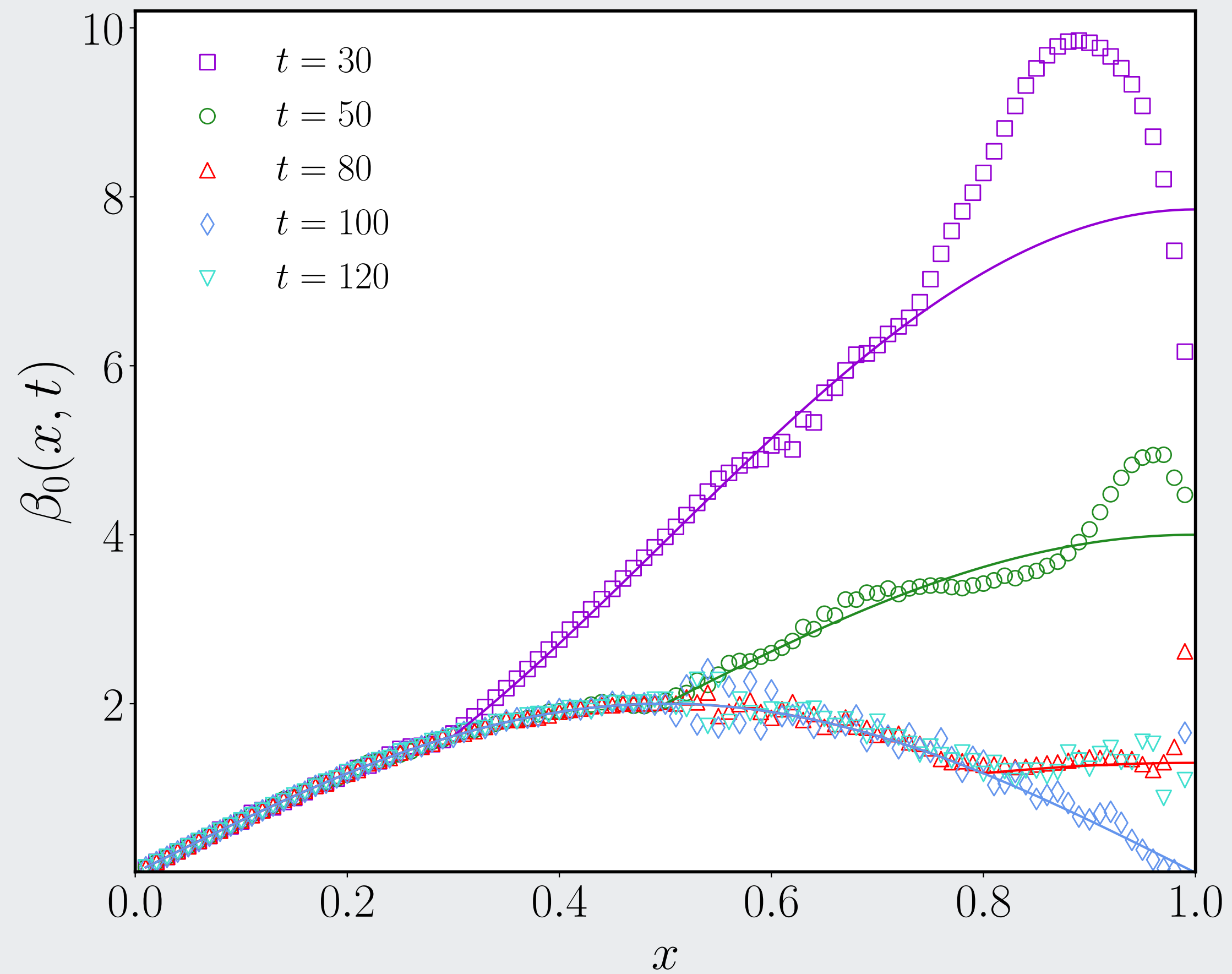
$$t_{2p}(x) \equiv 0, \quad s_{2p+1}(x) \equiv 0 \quad \longrightarrow \quad \mu_0(x, t) \equiv 0, \quad \mu_1(x, t) \equiv 0,$$

- Nonvanishing weights

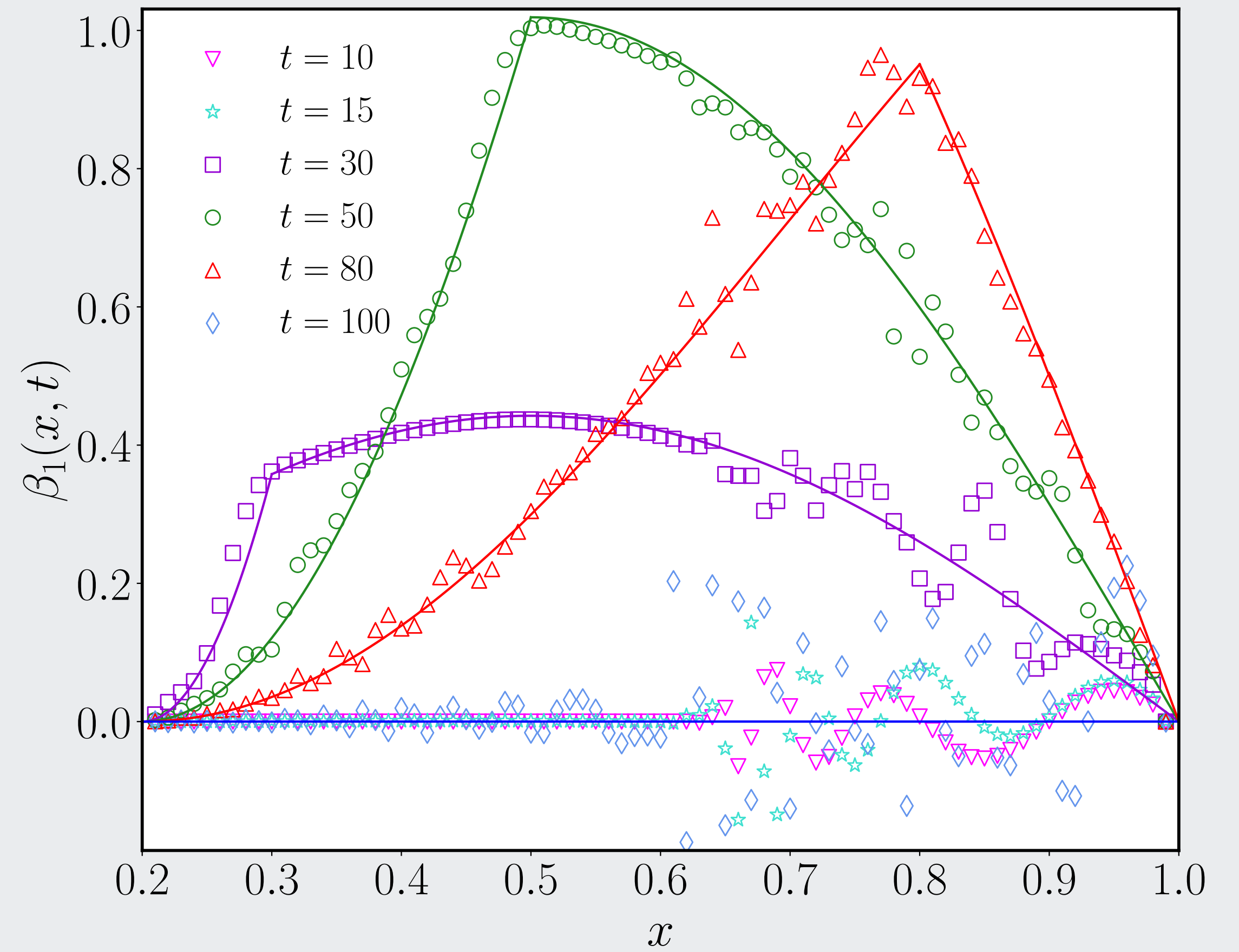
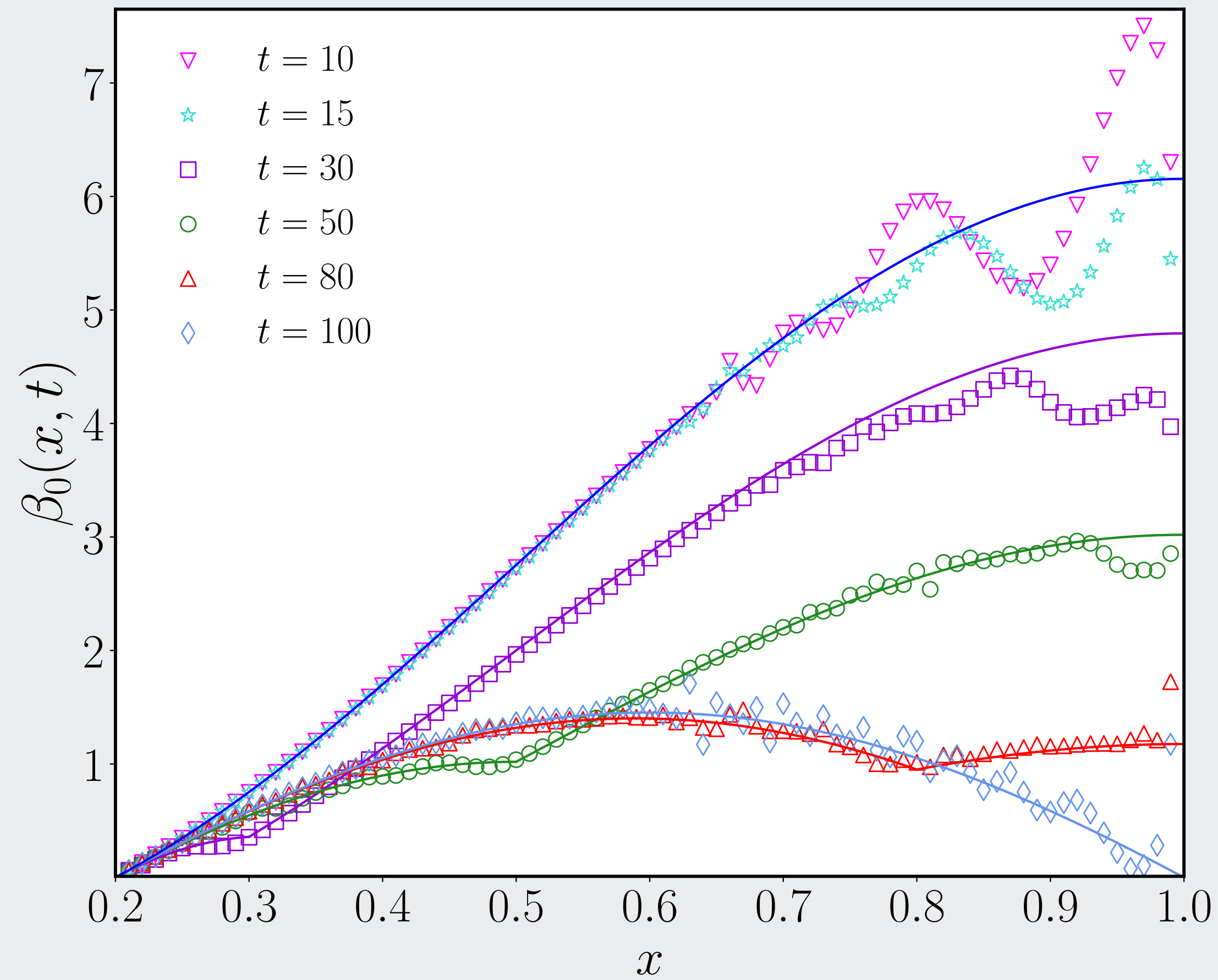
$$\beta_0(x, t) = -2a \sum_{p=0}^{\infty} (2p+1)(-1)^p t_{2p+1}(x), \quad \beta_1(x, t) = 2a \sum_{p=1}^{\infty} (2p)(-1)^p s_{2p}(x).$$

- Comparison with CFT expressions $\beta_0(x, t) = \frac{\beta(x, t) + \bar{\beta}(x, t)}{2}, \quad \beta_1(x, t) = \frac{\bar{\beta}(x, t) - \beta(x, t)}{2}.$

CENTERED INTERVAL



DECENTERED INTERVAL



CONCLUSIONS

- Main results:
 - Derivation of the CFT expression of the EH after a local quench between two finite subsystems
 - The CFT prediction for the EH after a local quench can be recovered via proper continuum limit procedure from the lattice EH.
- Further investigation:
 - Local approximation of lattice EH (commuting operators)
 - Different conformal boundary conditions

THANK YOU!