Chemical Freeze-Out as Mott Localization of Clusters

David Blaschke

Uniwersytet Wrocławski

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BTSLXX Symposium

HUN-REN Wigner RCP,
Budapest 11.-12. June 2026

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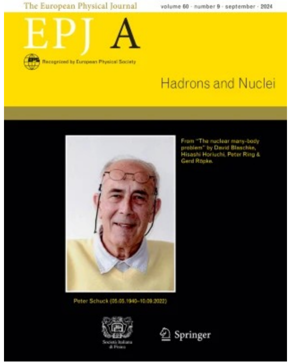
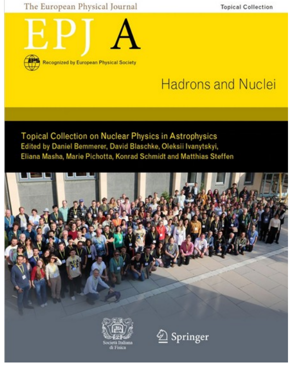
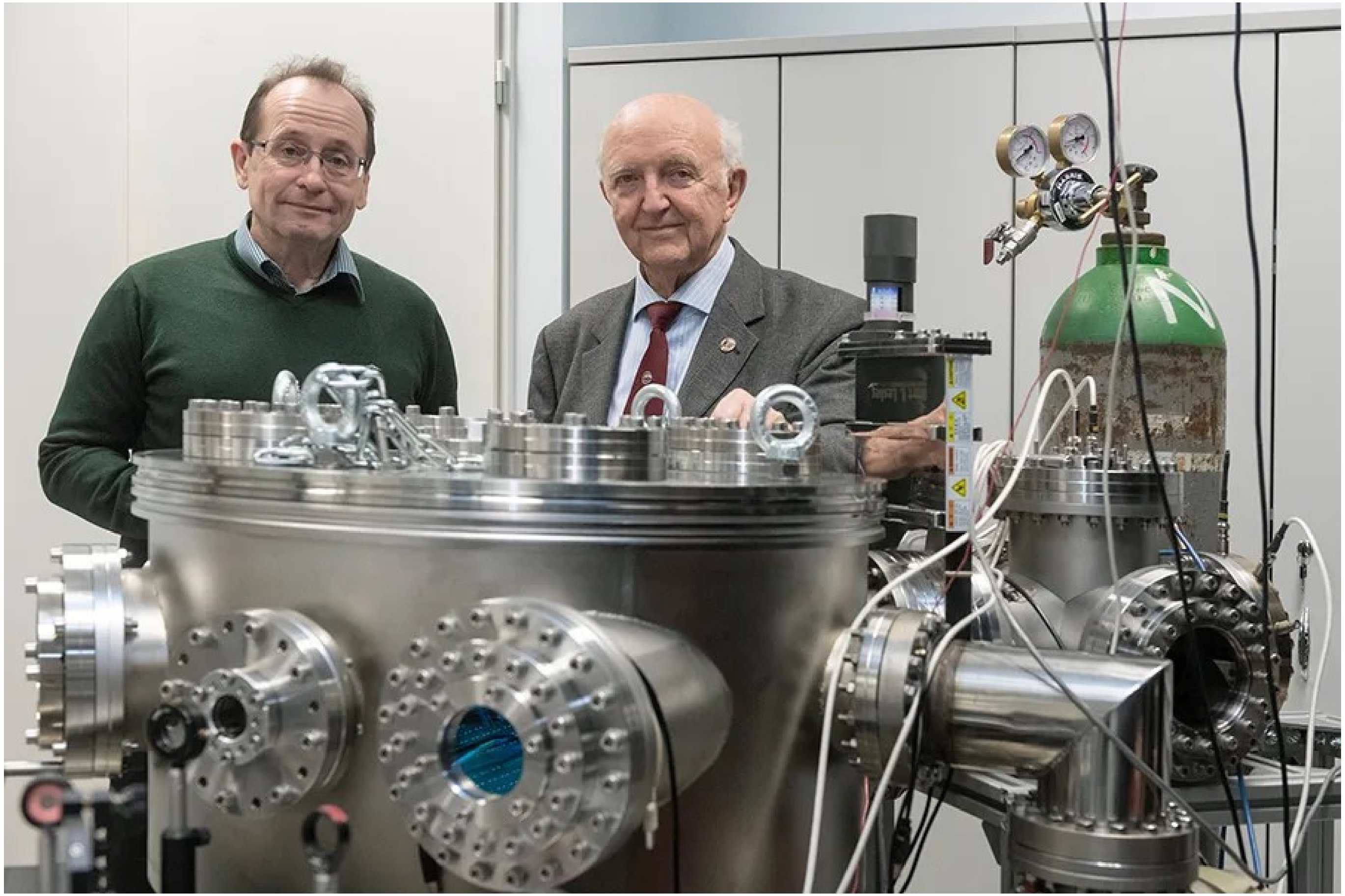
Editor in Chief, European Physical Journal A





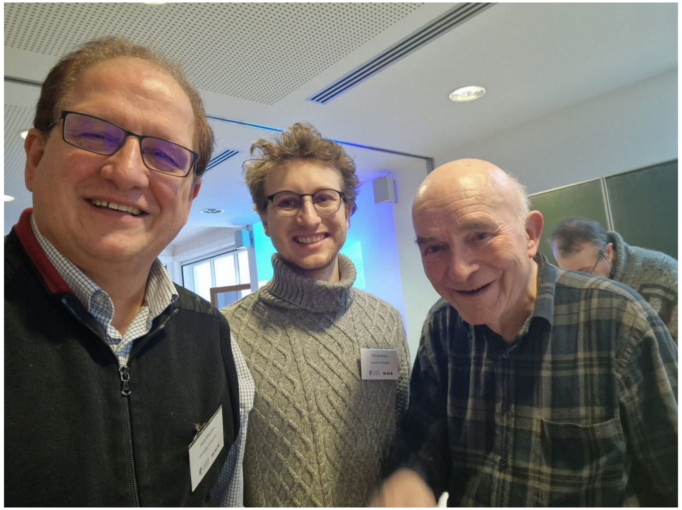
In 2013, Tamas as EiC of EPJA invited me to the Editorial Board, now I am EiC!

Laser-induced Fusion Plasmas … The NAPLIFE Project



PiConGPU simulations of Laser-induced plasmas at HZDR
arXiv:2605.16206

DB,Optolowicz, Meyer-Ter-Vehn at Hirschegg Workshop on High-Energy Density Matter, 01/2026



Particles and Plasmas
In Strong Fields

Dresden-Rossendorf
& Görlitz

June 22 – 25, 2025

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WILHELM UND ELSE
HERAEUS-STIFTUNG

Hungarian-German
Seminar on:

„Particles and Plasmas in Strong Fields“
Dresden and Görlitz, June 22-25, 2025

J. Rafelski, R. Redmer, T.S. Biro,
G.G. Barnaföldi, ?, B. Mueller

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Special Issue

Particles and Plasmas in Strong Fields

Guest Editors

Prof. Dr. David Blaschke
Prof. Dr. Tamás S. Biró
Prof. Dr. Ralf Schützhold

Particles

Quark Clusters and Hadronisation

T. BIRÓ
H.W. BARZ
B. LUKÁCS
J. ZIMÁNYI

KFKI-1981-90

ENTROPY AND HADROCHEMICAL COMPOSITION IN HEAVY ION COLLISION

Hungarian Academy of Sciences

CENTRAL RESEARCH INSTITUTE FOR PHYSICS

BUDAPEST

Proc. Budapest'02 Workshop on Quark & Hadron Dynamics (2002) 000–000

Budapest'02 Workshop on Quark & Hadron Dynamics
Budapest, Hungary
March 3–7, 2002

Faces of quark matter

J. Zimányi, P. Lévai, T.S. Biró

Research Institute for Particle and Nuclear Physics
POB. 49. Budapest, 1525, Hungary

Quark Clusters and Hadronisation

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PHYSICAL REVIEW D VOLUME 34, NUMBER 11 1 DECEMBER 1986

Pauli quenching effects in a simple string model of quark/nuclear matter

G. Röpke and D. Blaschke
Department of Physics, Wilhelm-Pieck-Universtiy, 2500 Rostock, German Democratic Republic

H. Schulz
Central Institute for Nuclear Research, Rossendorf, 8051 Dresden, German Democratic Republic
and The Niels Bohr Institute, 2100 Copenhagen, Denmark

(Received 16 December 1985)

Quark Clustering and Chemical Freeze-Out

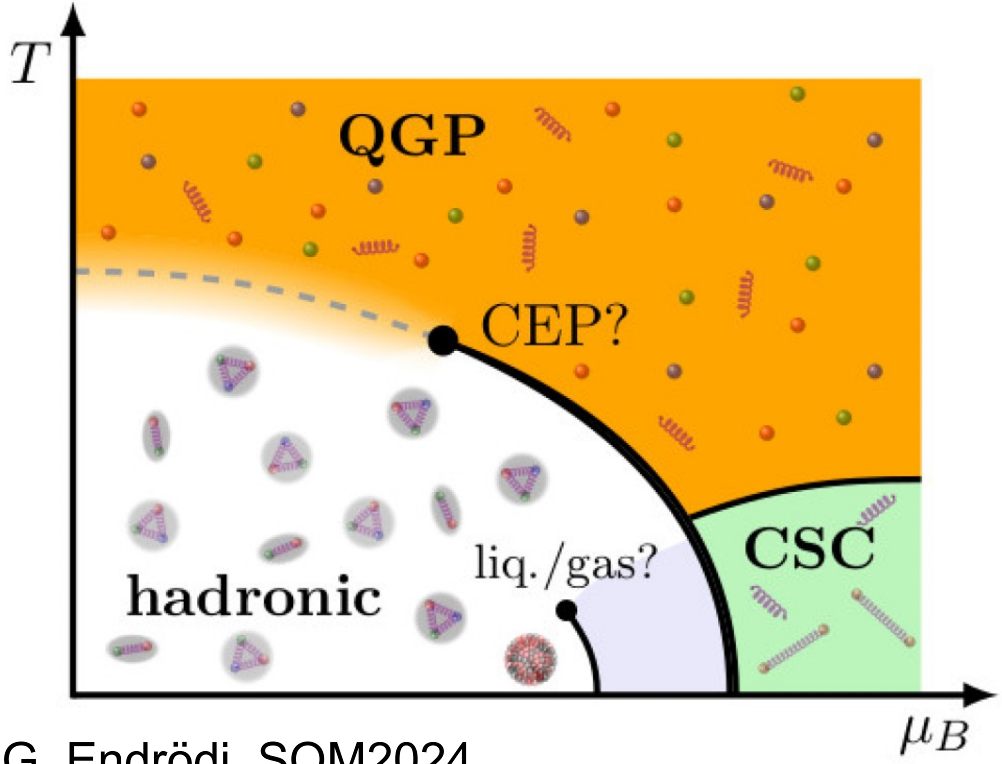
David Blaschke

Contents:

1. **Introduction: Phase diagram & CFO**
   Particle Clustering & Mott transition
2. **Hypothesis:**
   CFO = inverse Mott transition
3. **Applications:**
   - High-energy HIC: Hadronization
   - Low-energy HIC: Clusters
   - Cosmology: r-process elements

Peter Senger, Tamas Biro, Jozef Zimanyi (2002) →

Exploring the QCD Phase Diagram



G. Endrödi, SQM2024

The Goal: Proving the phase structure!

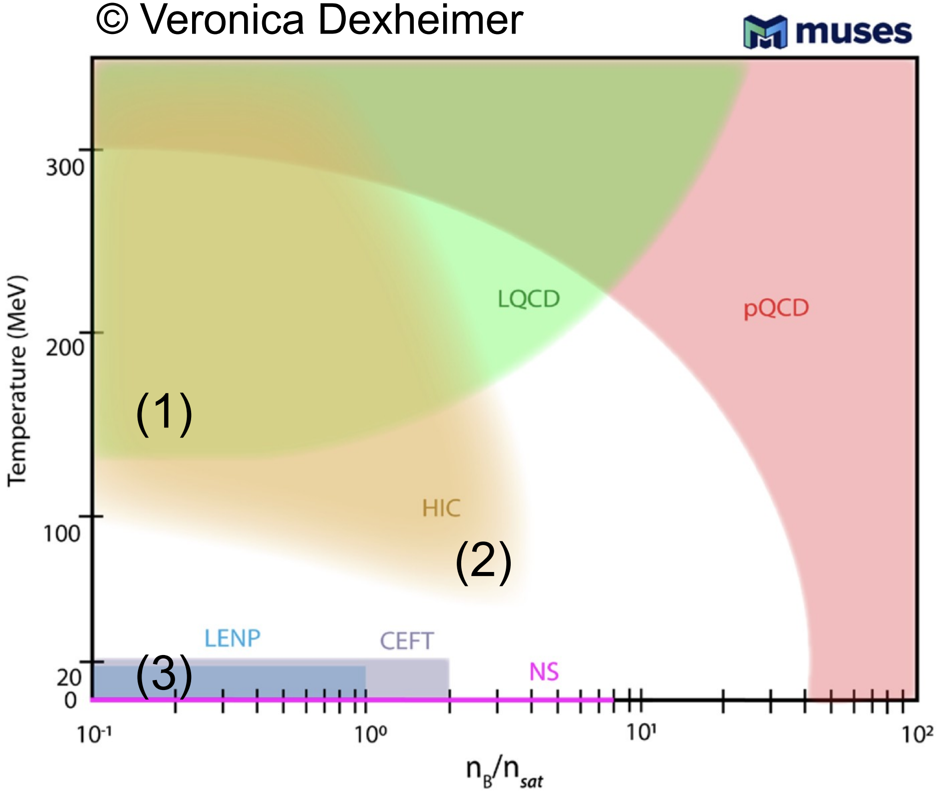
Lattice QCD results only for pseudocritical temperature Tc near μ ~ 0 (sign problem)

Liquid-gas PT indicated in experiment

Other structures are so far model dependent conjectures!!

Exploring the QCD Phase Diagram

© Veronica Dexheimer



Living Reviews in Relativity (2024)27:3
https://doi.org/10.1007/s41114-024-00049-6

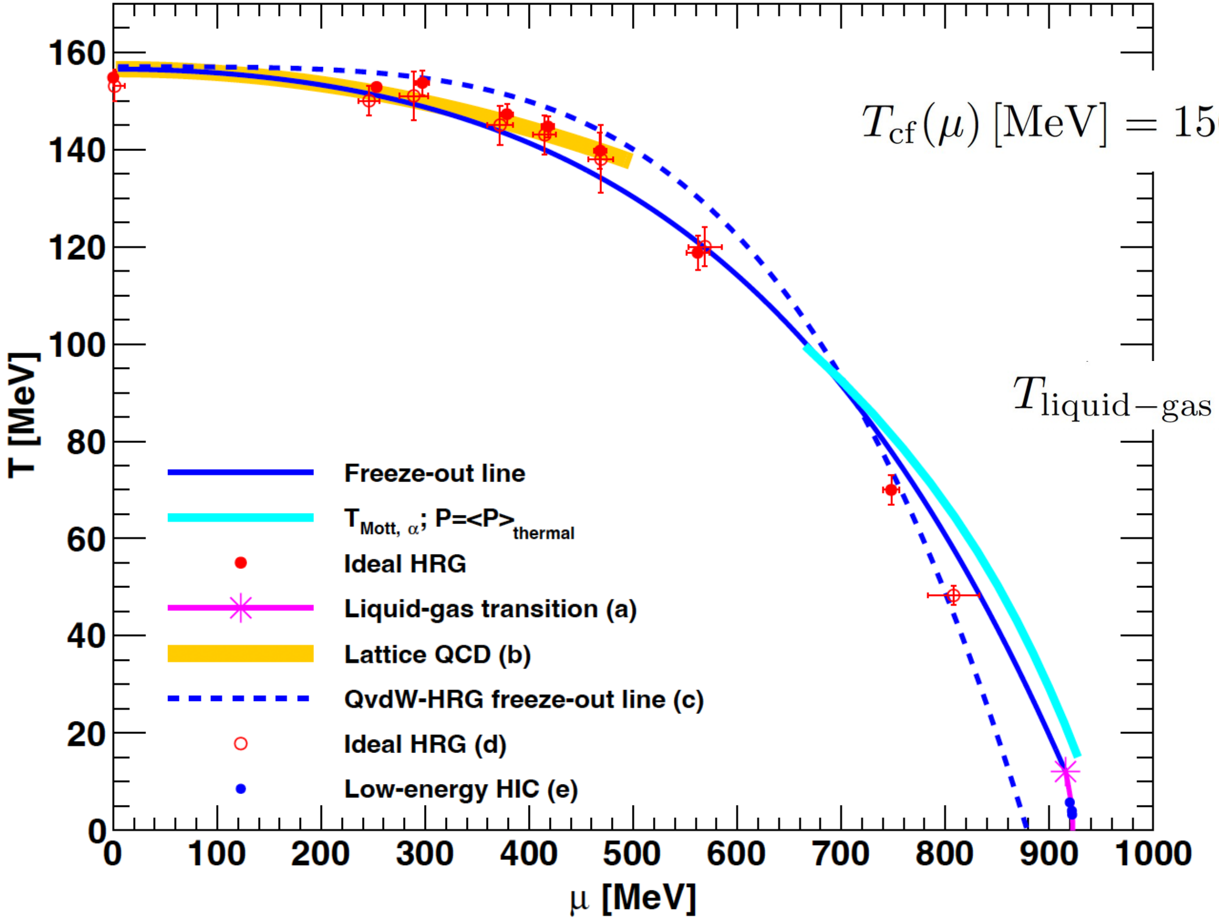
This Talk:
Chemical freeze-out as inverse Mott effect: Mott „localization" = collapse of the cluster wave functions in expanding, cooling matter

(1) QCD transition = hadron dissociation / localization in meson dominated matter at high temperatures for T>130 MeV

(2) Freeze-out of alpha clusters as „inverse Mott effect" signals chemical freezeout in baryon-dominated region for T<100 MeV

(3) Heavy r-process elements by freezeout from a (cosmological) thermal source

(1)+(2) Statistical Model Fit for CFO, T – μ Diagram



The freeze-out line (blue):

$$T_{\mathrm{cf}}(\mu)\,[\mathrm{MeV}] = 156.5 - 76.68\,(\mu\,[\mathrm{GeV}])^2 - 139.7\,(\mu\,[\mathrm{GeV}])^4$$

interpolates between lattice QCD chiral restoration crossover (yellow) and CEP of the liquid-gas transition

$$T_{\mathrm{liquid-gas}} = 12.1\,\mathrm{MeV},\ \mu_{\mathrm{liquid-gas}} = 915.61\,\mathrm{MeV}$$

Mott dissociation line for alphas is well correlated in baryon-dominant region

(a) Typel et al., PRC 81, 015803 (2010)

(c) Poberezhnyuk et al., PRC 100, 054904 (2019)

(e) Natowitz et al., PRL 104, 202501 (2010)

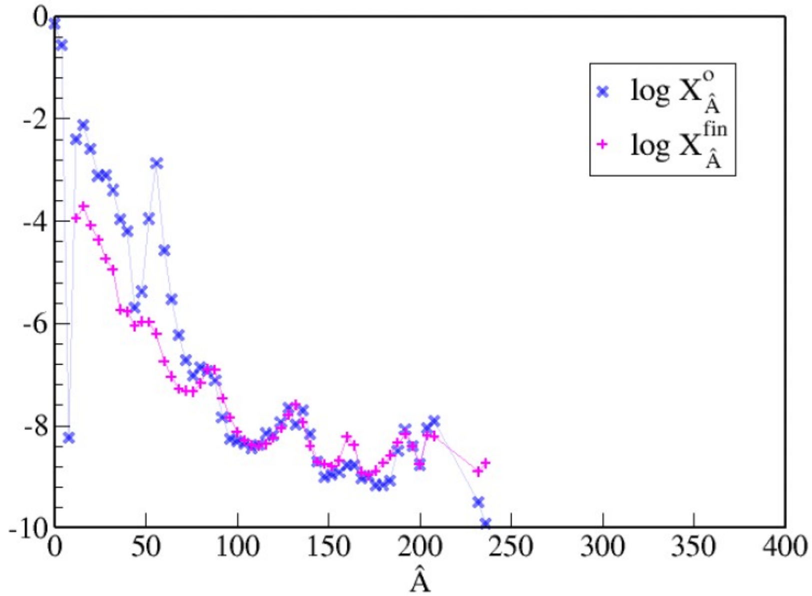
D. Blaschke, B. Dönigus, S. Liebing, G. Röpke, arXiv:2408.01399, PLB accepted

(3) Primordial black-hole formation and heavy r-process element synthesis from the cosmological QCD transition

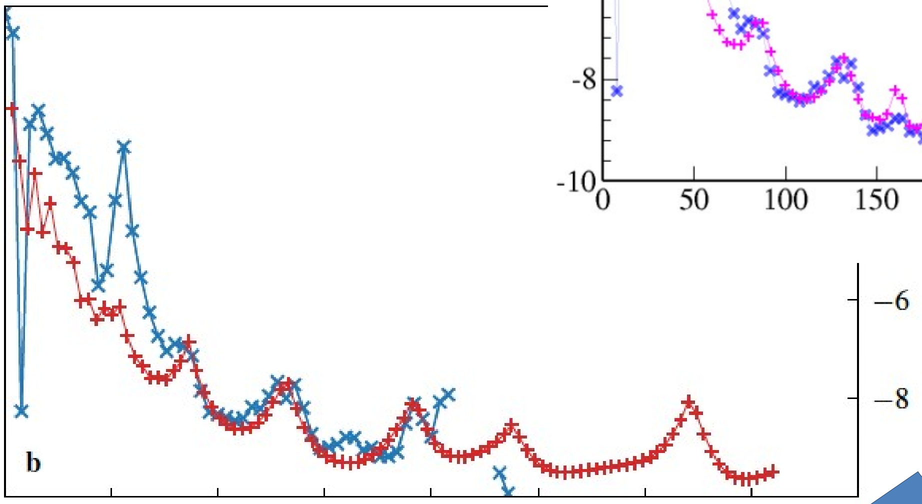
Two aspects of an inhomogeneous early Universe

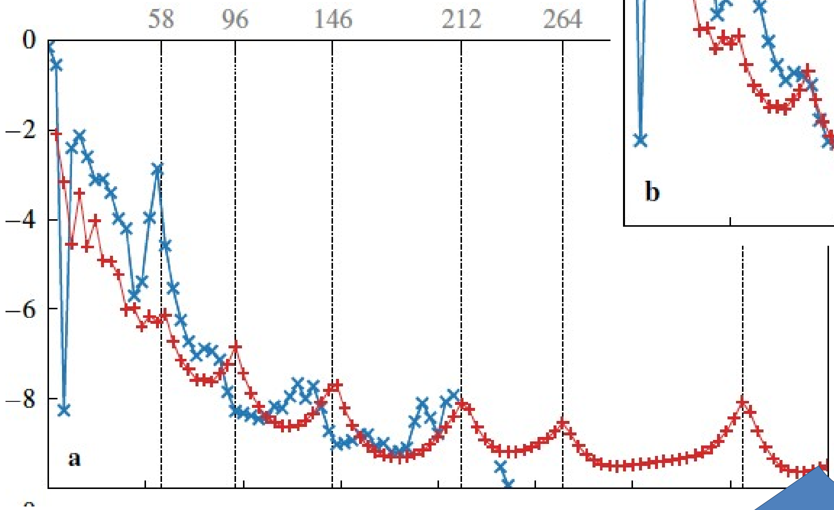
Maël Gonin[1,2], Günther Hasinger[1,2,3], David Blaschke[4,5,6], Oleksii Ivanytskyi[7], and Gerd Röpke[8]

EPJA 61 (2025) 170; arXiv:2505.05463



Primordial accumulated mass fraction (red plusses) compared to the solar one (blue crosses)

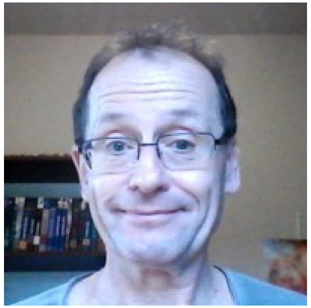




Modified by: 1) neutron evaporation ...

… and 2) fission (A~330 → 165) and 3) alpha decays (A>212 → A~200)

Zimanyi School 2025

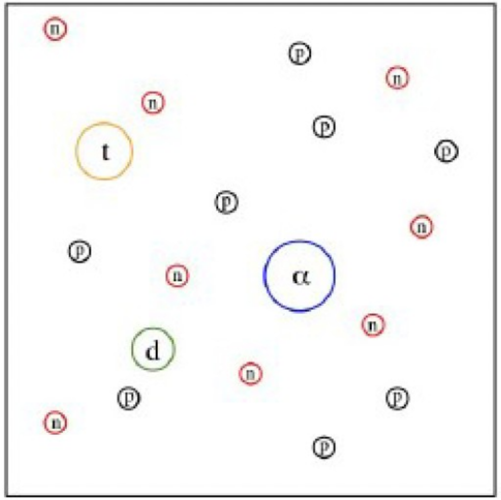


Mott dissociation for bound states in a plasma

Chemical picture:

Ideal mixture of reacting components

Mass action law



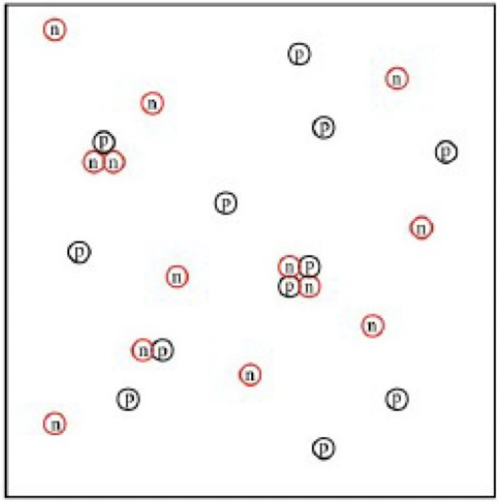
Interaction between the components

internal structure: Pauli principle

Physical picture:

"elementary" constituents

and their interaction



Quantum statistical (QS) approach,

quasiparticle concept, virial expansion

Mott dissociation for bound states in a plasma

Effective wave equation for deuterons in nuclear matter

In-medium two-particle wave equation in mean-field approximation

$$\left(\frac{p_1^2}{2m_1}+\Delta_1+\frac{p_2^2}{2m_2}+\Delta_2\right)\Psi_{d,P}(p_1,p_2)+\sum_{p_1',p_2'}(1-f_{p_1}-f_{p_2})V(p_1,p_2;p_1',p_2')\Psi_{d,P}(p_1',p_2')$$

$$= E_{d,P}\Psi_{d,P}(p_1,p_2)$$

Add self-energy

Pauli-blocking

Fermi distribution function

$$f_p=\left[e^{(p^2/2m-\mu)/k_BT}+1\right]^{-1}$$

Thouless criterion

$$E_d(T,\mu)=2\mu$$

BEC-BCS crossover:
Alm et al.,1993

Mott dissociation for bound states in a plasma

Effective wave equation for deuterons in nuclear matter

[Derivation of a „Plasma Hamiltonian“ from a Bethe-Goldstone Eq. for two-particle states]

In-medium two-particle wave equation in mean-field approximation

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Add self-energy

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[R. Zimmermann et al. (5-men-work), Phys. Stat. Sol. (b) 90 (1978) 175]

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Mott dissociation for clusters in nuclear matter

Nuclear Physics **A379** (1982) 536–552

PARTICLE CLUSTERING AND MOTT TRANSITIONS IN NUCLEAR MATTER AT FINITE TEMPERATURE (I). Method and general aspects

G. RÖPKE

Sektion Physik, Wilhelm-Pieck-University, Rostock, GDR

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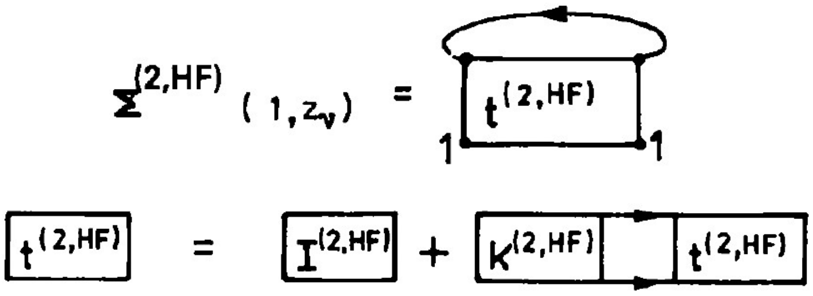

Che-Ming Ko
& Peter Levai

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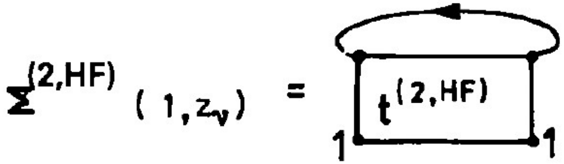

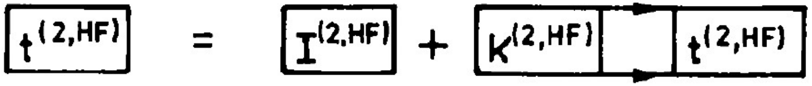

$$t^{(2,\mathrm{HF})}(1234,\Omega_\lambda)=\sum_\alpha \frac{\phi_\alpha^{\mathrm{HF}}(12)\phi_\alpha^{\mathrm{HF}}(34)}{E_\alpha^{\mathrm{HF}}-\hbar\Omega_\lambda}(1+\tfrac{1}{2}(f(1)+f(2)+f(3)+f(4)))$$
$$\times(\hbar\Omega_\lambda-E(1)-\Delta^{\mathrm{HF}}(1)-E(2)-\Delta^{\mathrm{HF}}(2))$$
$$\times(E_\alpha^{\mathrm{HF}}-E(3)-\Delta^{\mathrm{HF}}(3)-E(4)-\Delta^{\mathrm{HF}}(4))\,,$$

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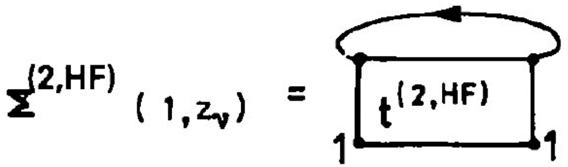

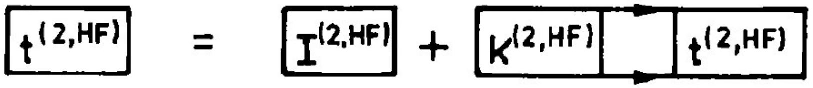

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$$\{E(1)+E(2)-E_\alpha^{\mathrm{HF}}\}\phi_\alpha^{\mathrm{HF}}(12)+\sum_{1'2'}V(12,\,1'2')\phi_\alpha^{\mathrm{HF}}(1'2')$$
$$=\sum_{1'2'}\{\tfrac{1}{2}(f(1)+f(2)+f(1')+f(2'))V(12,\,1'2')$$
$$-(\Delta^{\mathrm{HF}}(1)+\Delta^{\mathrm{HF}}(2))\,\delta_{11'}\,\delta_{22'}\}\phi_\alpha^{\mathrm{HF}}(1'2')$$
$$=-\sum_{1'2'}H^{(2,\mathrm{HF})}_{\mathrm{nucl.matter}}\,(12,\,1'2')\phi_\alpha^{\mathrm{HF}}(1'2')\,.$$

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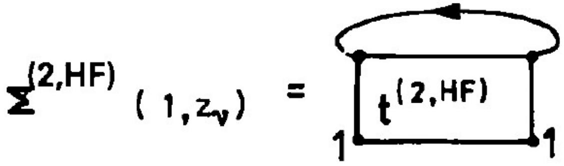
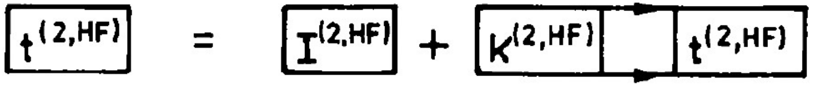
$$\Delta E_\alpha^{\rm HF} = \sum_{121'2'} \phi_\alpha^0(12)\phi_\alpha^{0*}(1'2')H^{(2,\rm HF)}_{\rm nucl.\,matter}(121'2')\,,$$





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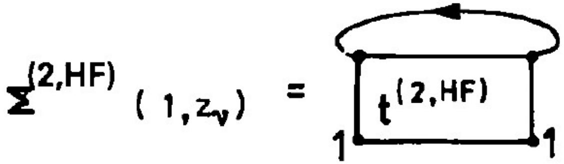
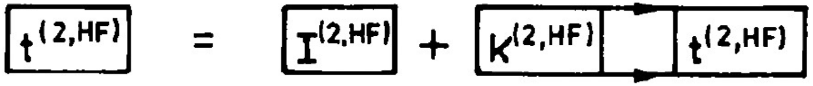
$$\Delta E_{\mathrm{d},P}^{\mathrm{HF}} = \tfrac{3}{2} t_0\rho_{\mathrm{nucl}} - \sqrt{2}\, t_0\rho_{\mathrm{nucl}}(1+x_0)\left(1+\frac{\pi}{\alpha^2\Lambda^2}\right)^{-3/2} \exp\left[-\frac{P^2}{16\alpha^2}\left(1+\frac{\pi}{\Lambda^2\alpha^2}\right)^{-1}\right]$$

$$= 2\Delta^{\mathrm{HF}}(0) + \Delta_{\mathrm{d},P}^{\mathrm{Pauli}}\rho_{\mathrm{nucl}}\,, \qquad (2.$$





$$t^{(2,\mathrm{HF})}(1234, \Omega_\lambda) = \sum_\alpha \frac{\phi_\alpha^{\mathrm{HF}}(12)\phi_\alpha^{\mathrm{HF}}(34)}{E_\alpha^{\mathrm{HF}} - \hbar\Omega_\lambda}\left(1+\tfrac{1}{2}(f(1)+f(2)+f(3)+f(4))\right)$$
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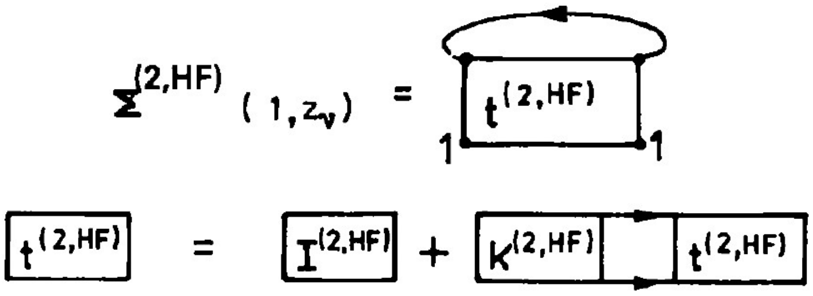
$$\Delta E^{HF}_{d,P} = \tfrac{3}{2} t_0\rho_{nucl} - \sqrt{2}\, t_0\rho_{nucl}(1+x_0)\left(1+\frac{\pi}{\alpha^2\Lambda^2}\right)^{-3/2} \exp\left[-\frac{P^2}{16\alpha^2}\left(1+\frac{\pi}{\Lambda^2\alpha^2}\right)^{-1}\right]$$

$$= 2\Delta^{HF}(0) + \Delta^{Pauli}_{d,P}\,\rho_{nucl},$$

$$\alpha = \hbar(|E_d^0|/M)^{1/2} = 0.23\ \mathrm{fm}^{-1}.$$

$$\Lambda = (2\pi\beta\hbar^2/M)^{1/2}$$

$$\rho^{(0)}_{nucl}(\beta,\mu) = \frac{4}{\Lambda^3}\exp(\beta\mu), \quad t_0 = -1057.3\ \mathrm{MeV}\cdot\mathrm{fm}^3 \text{ and } x_0 = 0.56.$$



$$t^{(2,HF)}(1234,\Omega_\lambda) = \sum_\alpha \frac{\phi_\alpha^{HF}(12)\phi_\alpha^{HF}(34)}{E_\alpha^{HF} - \hbar\Omega_\lambda}\left(1+\tfrac{1}{2}(f(1)+f(2)+f(3)+f(4))\right)$$

$$\times (\hbar\Omega_\lambda - E(1) - \Delta^{HF}(1) - E(2) - \Delta^{HF}(2))$$

$$\times (E_\alpha^{HF} - E(3) - \Delta^{HF}(3) - E(4) - \Delta^{HF}(4)),$$

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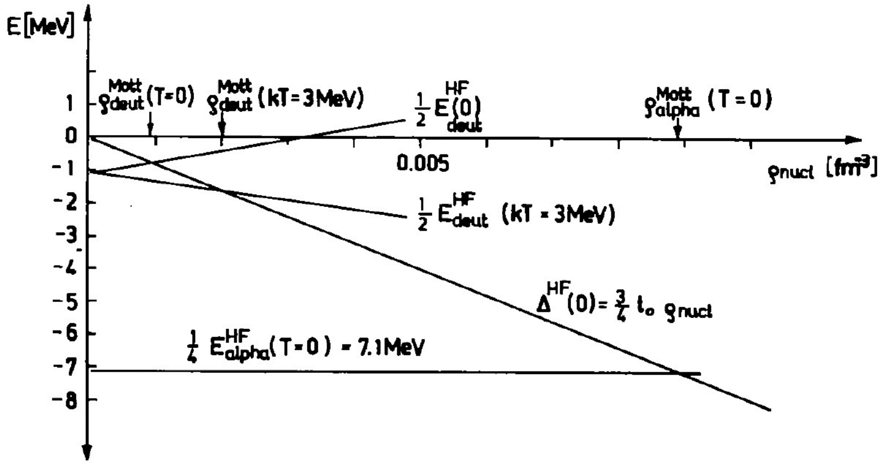
Mott dissociation for clusters in nuclear matter



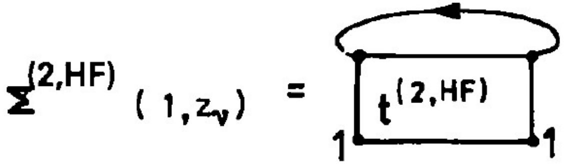
$$\Delta E_\alpha^{\mathrm{HF}} = \sum_{121'2'} \phi_\alpha^0(12)\phi_\alpha^{0*}(1'2')H^{(2,\mathrm{HF})}_{\mathrm{nucl.\,matter}}(121'2')\,,$$

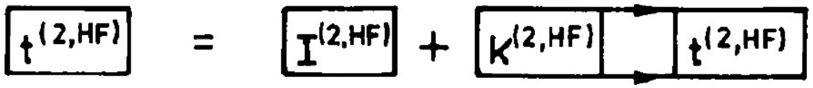
$$\Delta E^{\mathrm{HF}}_{\mathrm{d},P} = \tfrac{3}{2}t_0\rho_{\mathrm{nucl}} - \sqrt{2}t_0\rho_{\mathrm{nucl}}(1+x_0)\left(1+\frac{\pi}{\alpha^2\Lambda^2}\right)^{-3/2}\exp\left[-\frac{P^2}{16\alpha^2}\left(1+\frac{\pi}{\Lambda^2\alpha^2}\right)^{-1}\right]$$

$$= 2\Delta^{\mathrm{HF}}(0) + \Delta^{\mathrm{Pauli}}_{\mathrm{d},P}\,\rho_{\mathrm{nucl}}\,, \qquad \alpha = \hbar(|E_\mathrm{d}^0|/M)^{1/2} = 0.23\ \mathrm{fm}^{-1}.$$

$$\Lambda = (2\pi\beta\hbar^2/M)^{1/2}$$

$$\rho^{(0)}_{\mathrm{nucl}}(\beta,\mu) = \frac{4}{\Lambda^3}\exp(\beta\mu)\,, \quad t_0 = -1057.3\ \mathrm{MeV}\cdot\mathrm{fm}^3 \ \text{and} \ x_0 = 0.56.$$





$$t^{(2,\mathrm{HF})}(1234,\Omega_\lambda) = \sum_\alpha \frac{\phi_\alpha^{\mathrm{HF}}(12)\phi_\alpha^{\mathrm{HF}}(34)}{E_\alpha^{\mathrm{HF}} - \hbar\Omega_\lambda}(1+\tfrac{1}{2}(f(1)+f(2)+f(3)+f(4)))$$

$$\times(\hbar\Omega_\lambda - E(1) - \Delta^{\mathrm{HF}}(1) - E(2) - \Delta^{\mathrm{HF}}(2))$$

$$\times(E_\alpha^{\mathrm{HF}} - E(3) - \Delta^{\mathrm{HF}}(3) - E(4) - \Delta^{\mathrm{HF}}(4))\,,$$

$$\{E(1)+E(2)-E_\alpha^{\mathrm{HF}}\}\phi_\alpha^{\mathrm{HF}}(12) + \sum_{1'2'} V(12,1'2')\phi_\alpha^{\mathrm{HF}}(1'2')$$

$$= \sum_{1'2'}\{\tfrac{1}{2}(f(1)+f(2)+f(1')+f(2'))V(12,1'2')$$

$$-(\Delta^{\mathrm{HF}}(1)+\Delta^{\mathrm{HF}}(2))\,\delta_{11'}\,\delta_{22'}\}\phi_\alpha^{\mathrm{HF}}(1'2')$$

$$= -\sum_{1'2'} H^{(2,\mathrm{HF})}_{\mathrm{nucl.\,matter}}(12,1'2')\phi_\alpha^{\mathrm{HF}}(1'2')\,.$$

Mott dissociation for clusters in nuclear matter

Present state-of-the-art:

G. Röpke, Nuclear matter equation of state including two-, three-, and four-nucleon correlations, Phys. Rev. C 92 (5) (2015) 054001, https://doi.org/10.1103/PhysRevC.92.054001, arXiv:1411.4593.

$$E_{A,\nu}(P) = E^0_{A,\nu}(P) + \Delta E^{\mathrm{SE}}_{A,\nu}(P) + \Delta E^{\mathrm{Pauli}}_{A,\nu}(P) + \Delta E^{\mathrm{Coulomb}}_{A,\nu}(P).$$

The cluster binding energies $E^{\mathrm{bind}}_{A,\nu}(P;T,n_B,Y_p,T_{\mathrm{eff}})$ are defined as

$$E^{\mathrm{bind}}_{A,\nu}(P;T,n_B,Y_p) = -[E_{A,\nu}(P;T,n_B,Y_p) - E^{\mathrm{cont}}_{A,\nu}(P;T,n_B,Y_p)]$$

$$E^{\mathrm{cont}}_{A,\nu}(P;T,n_B,Y_p) = N E_n(P/A;T,n_B,Y_p) + Z E_p(P/A;T,n_B,Y_p)$$

the in-medium dispersion relations for nucleons ($\tau = n, p$) are defined as

$$E_\tau(p;T,n_B,Y_p) = \sqrt{\left[m_\tau c^2 - S(T,n_B,Y_p)\right]^2 + \hbar^2 c^2 p^2} + V_\tau(T,n_B,Y_p) - m_\tau c^2$$

$$S_i(T,n_B,Y_p) = (4463 - 6.610\,T - 0.1703\delta^2 + 4.112\delta^4)n_B \times \frac{1 + c_1 n_b + c_2 n_B^2}{1 + c_3 n_b + c_4 n_B^2},$$

$$V_p(T,n_B,Y_p) = (3403 + 0.000052\,T - 486.6\,\delta - 2.420\,\delta^2)n_B \times \frac{1 + d_1 n_b + d_2 n_B^2}{1 + d_3 n_b + d_4 n_B^2}$$

Scalar and vector mean fields from RDF EoS DD2
S. Typel et al., Phys. Rev. C 81, 015803 (2010)

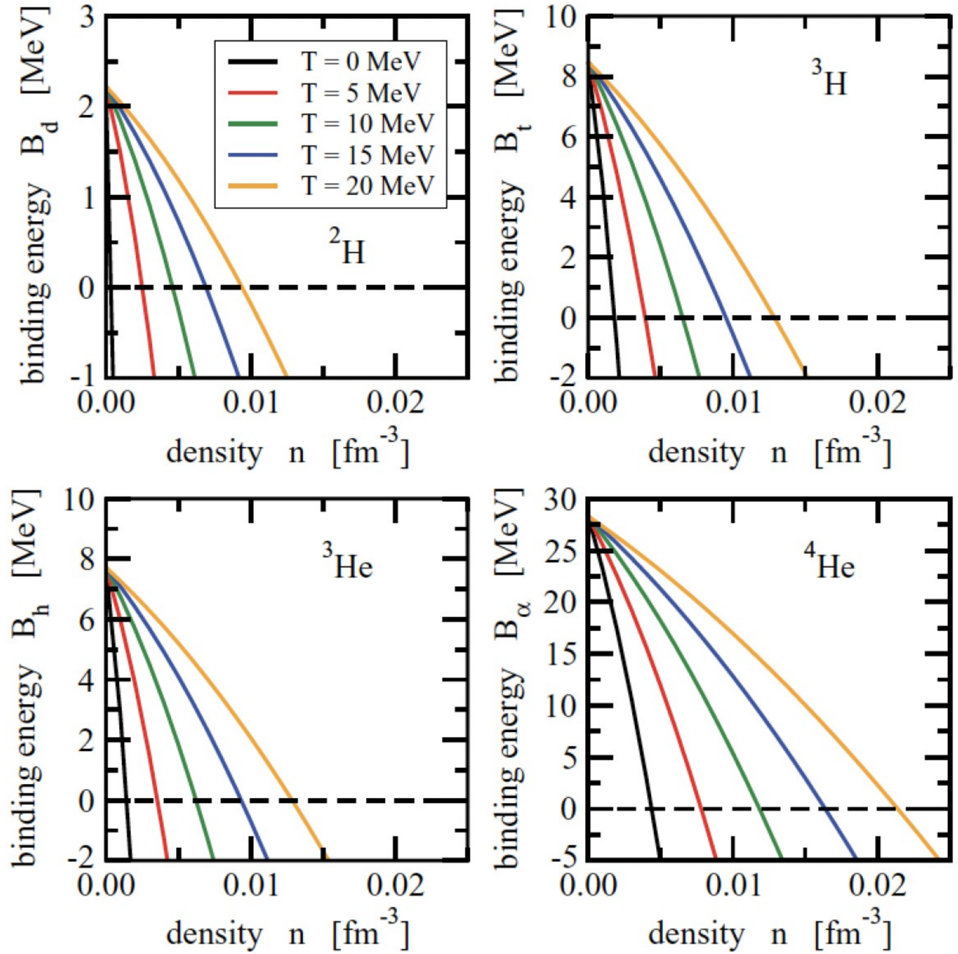
Parameter fit provided by G. Röpke et al.,
Phys. Part. Nucl. Lett. 15, 225 (2018)

$$c_1 = 20.56 - 0.04099\,T - 0.3394\,\delta^2 + 0.9972\,\delta^4,$$
$$c_2 = 15.98 + 0.8664\,T - 2.020\,\delta^2 - 3.018\,\delta^4,$$
$$c_3 = 24.27 - 0.07417\,T - 0.5427\,\delta^2 + 1.196\,\delta^4,$$
$$c_4 = 114.6 + 1.350\,T + 2.674\,\delta^2 + 0.7268\,\delta^4,$$

$$d_1 = 0.6629 - 0.006142\,T - 1.141\,\delta - 0.7176\,\delta^2,$$
$$d_2 = 10.7780 + 0.004432\,T + 0.8020\,\delta + 0.4576\,\delta^2,$$
$$d_3 = 3.433 + 0.000104\,T - 1.549\,\delta - 0.3360\,\delta^2,$$
$$d_4 = 23.01 - 0.03302\,T - 5.923\,\delta + 0.05090\,\delta^2,$$

with the isospin asymmetry $\delta = (1 - 2Y_p)$.

Binding energies for light clusters in T – n plane



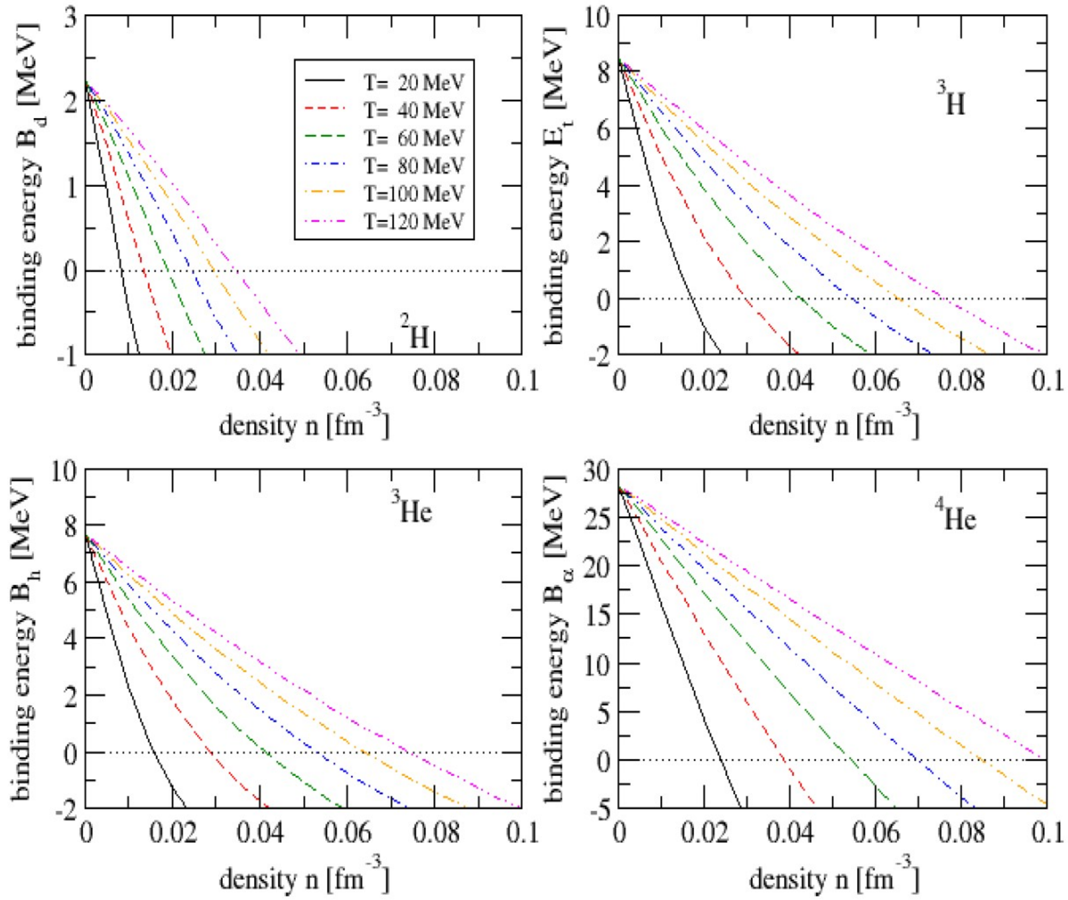
Vanishing binding energies
Indicate Mott effect for the
Light clusters!

Mott-lines in the T-μ plane
can be extracted, where the
Binding energy vanishes

Here lower temperatures:
0 < T[MeV] < 20

S. Typel et al., PRC 81,
015803 (2010)

Binding energies for light clusters in T – n plane

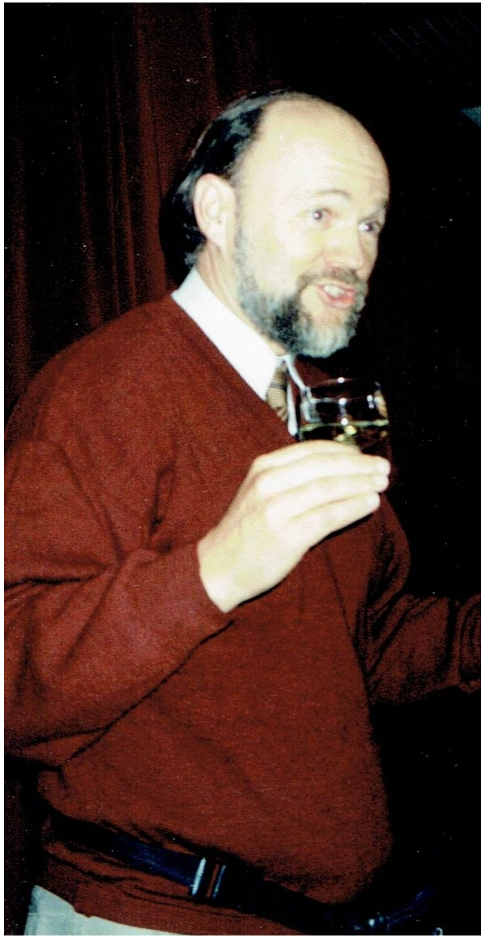


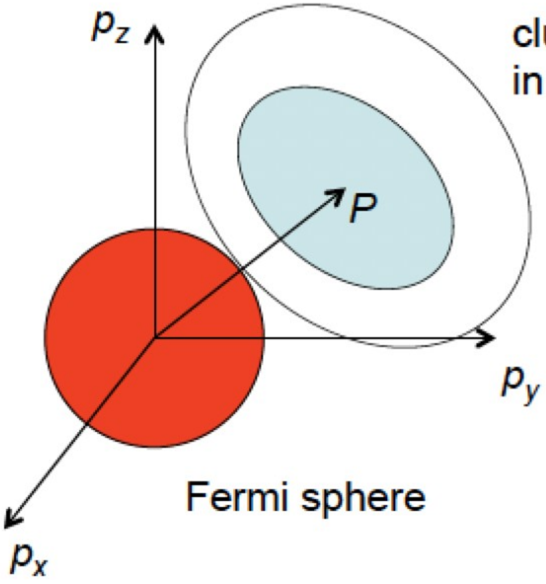
Mott-lines in the T-μ plane can be extracted, where The binding energy vanishes

Here higher temperatures:

20 < T[MeV] < 120

Pauli blocking: phase space occupation





cluster wave function (deuteron, alpha,…)
in momentum space

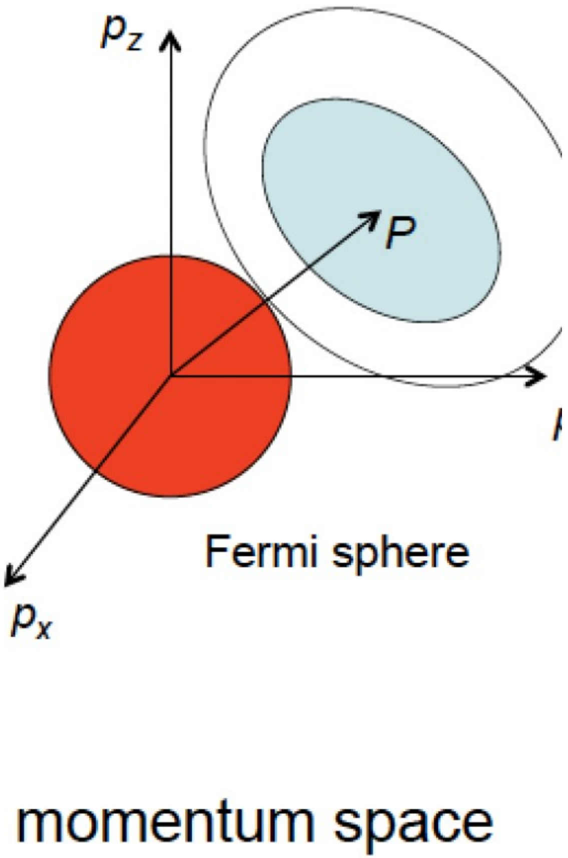
P - center of mass momentum

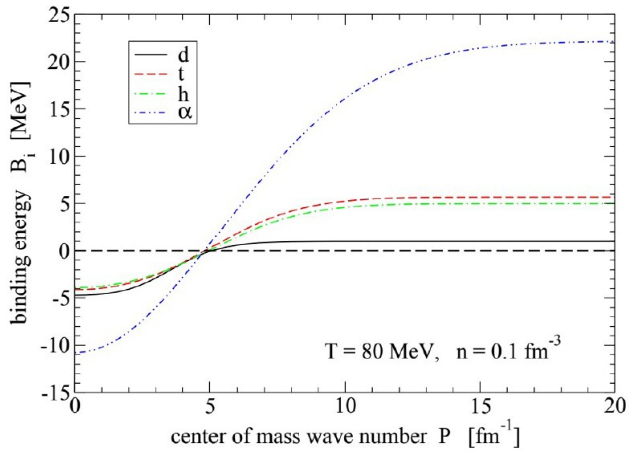
The Fermi sphere is forbidden,
deformation of the cluster wave function
in dependence on the c.o.m. momentum P

momentum space

The deformation is maximal at $P = 0$.
It leads to the weakening of the interaction
(disintegration of the bound state).

Momentum-dependent binding energies

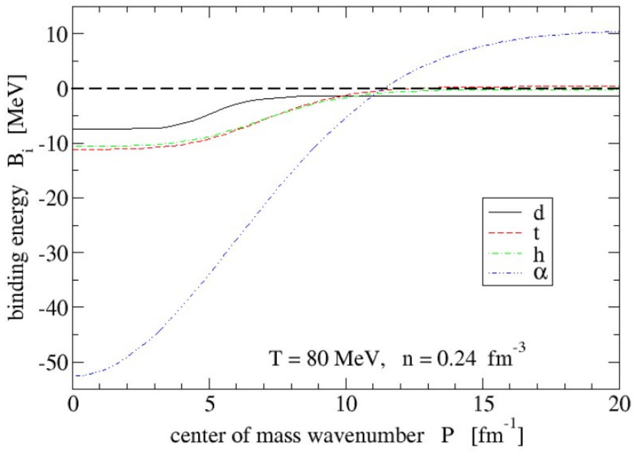


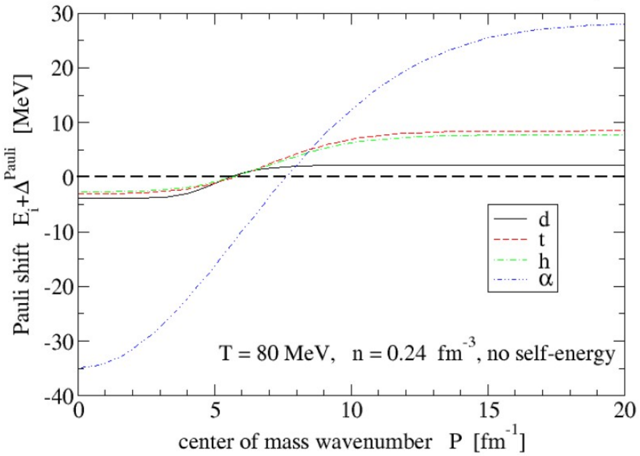


The light clusters that underwent a Mott Dissociation for low momenta become "resurrected" at high momenta relative to the medium !

The minimal momentum where this Occurs is called "Mott momentum";
It depends on temperature and density

Binding energies without selfenergy shift,
Only Pauli blocking shift accounted for

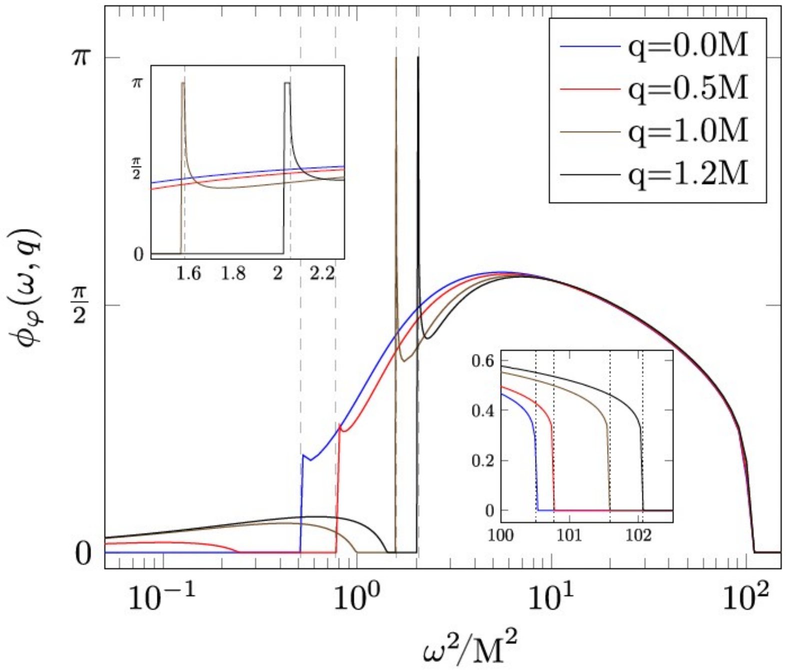


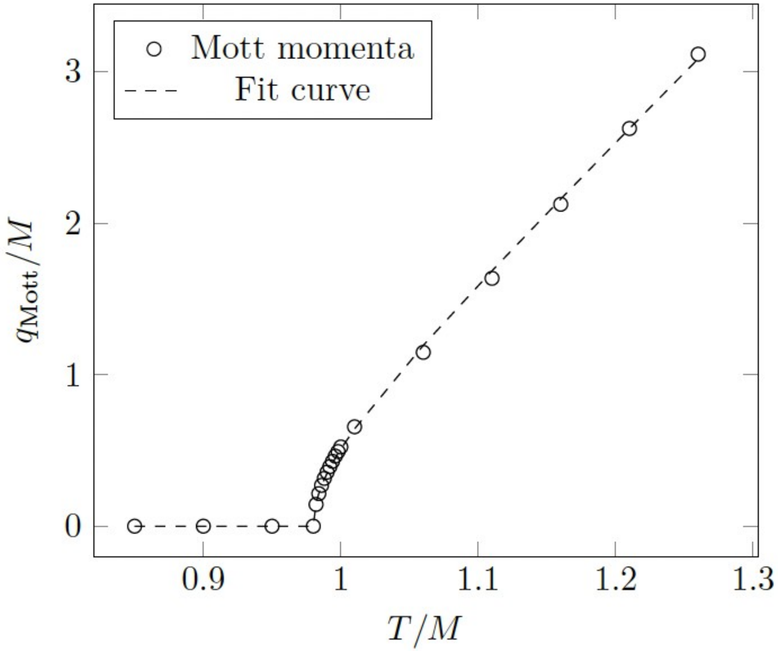


Mott momentum for bound states in matter

Based on the Pauli principle (100th anniversary), the Mott momentum is a general effect for bound states in matter, e.g. excitons in graphene!

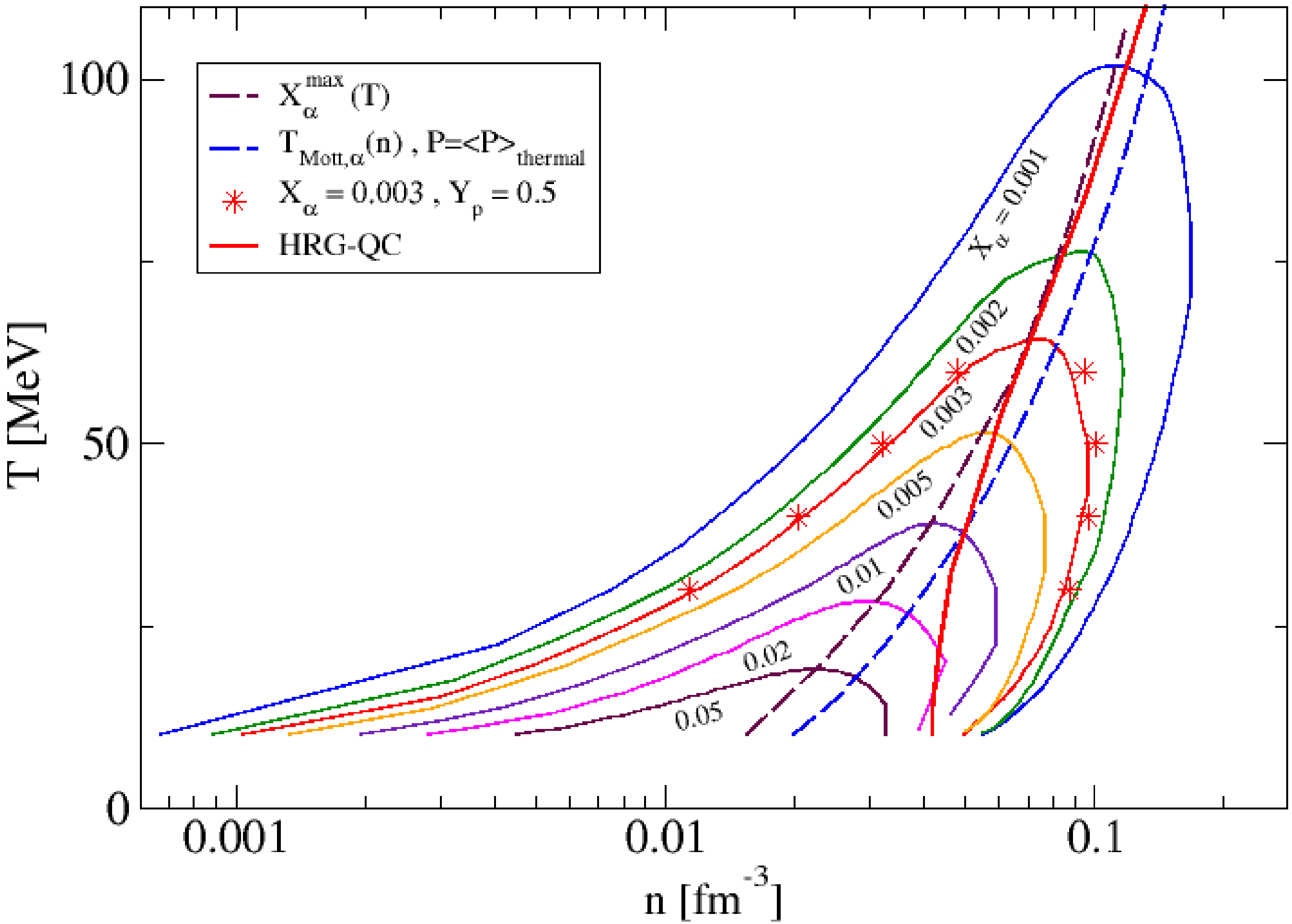
Biplab Mahato, D.B., D. Ebert,
Beth-Uhlenbeck equation for the thermodynamics of fluctuations in a generalized 2+1D Gross-Neveu model, Phys. Rev. D 112, 036013 (2025)





See also: Pauli potential effects on the tetraquark spectrum [Morgan Kuchta, Master Thesis, University of Wroclaw (2024)]

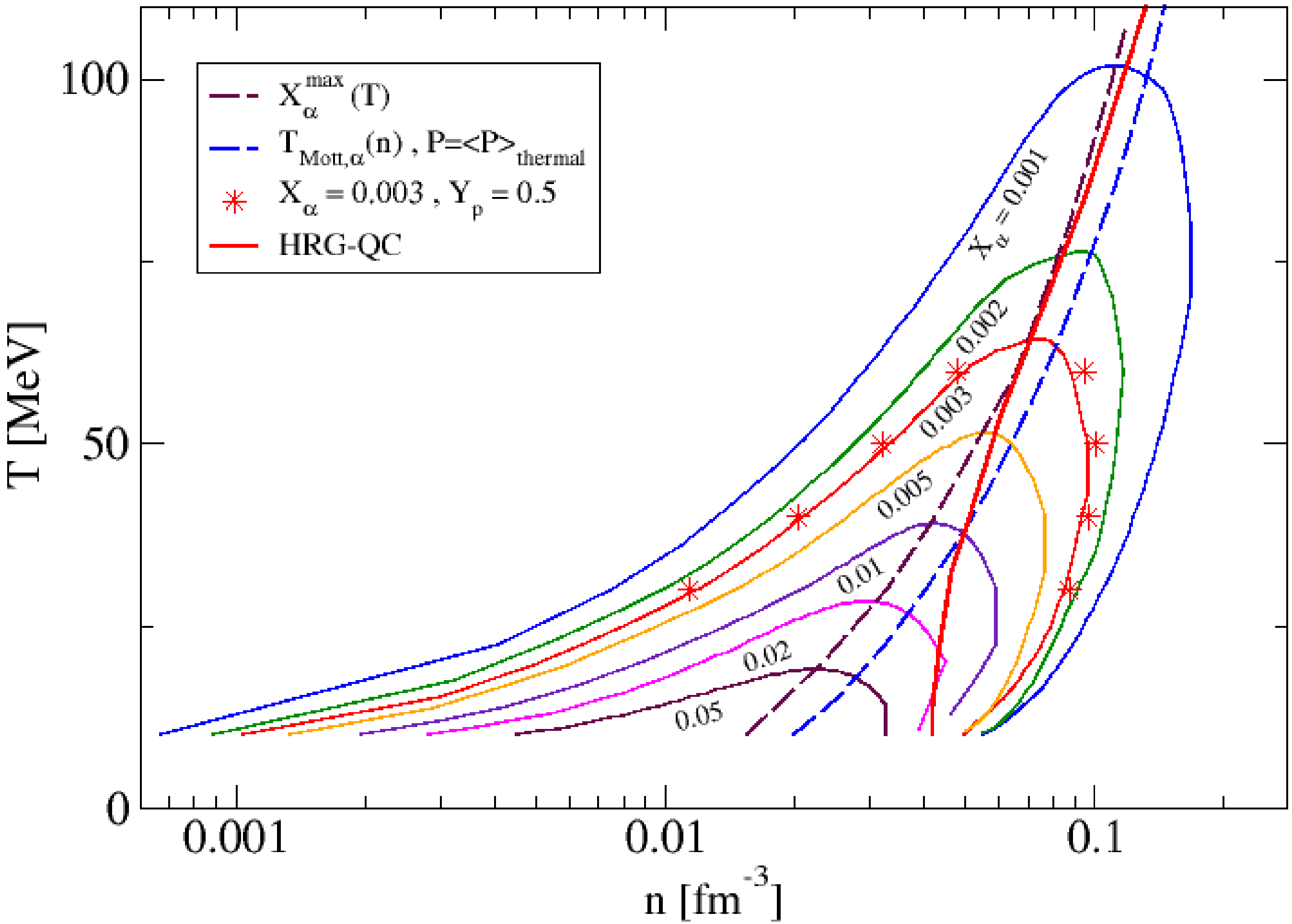
CFO in the Temperature – density plane



The mass action law of the chemical picture (nuclear statistical equilibrium) is modified by quantum effects (compositeness):

D. Blaschke, B. Dönigus, S. Liebing, G. Röpke, arXiv:2408.01399, PLB 860 (2025) 139206

CFO in the Temperature – density plane



The mass action law of the chemical picture (nuclear statistical equilibrium) is modified by quantum effects (compositeness):

Pauli blocking →

→ Mott dissociation

D. Blaschke, B. Dönigus, S. Liebing, G. Röpke, arXiv:2408.01399, PLB 860 (2025) 139206

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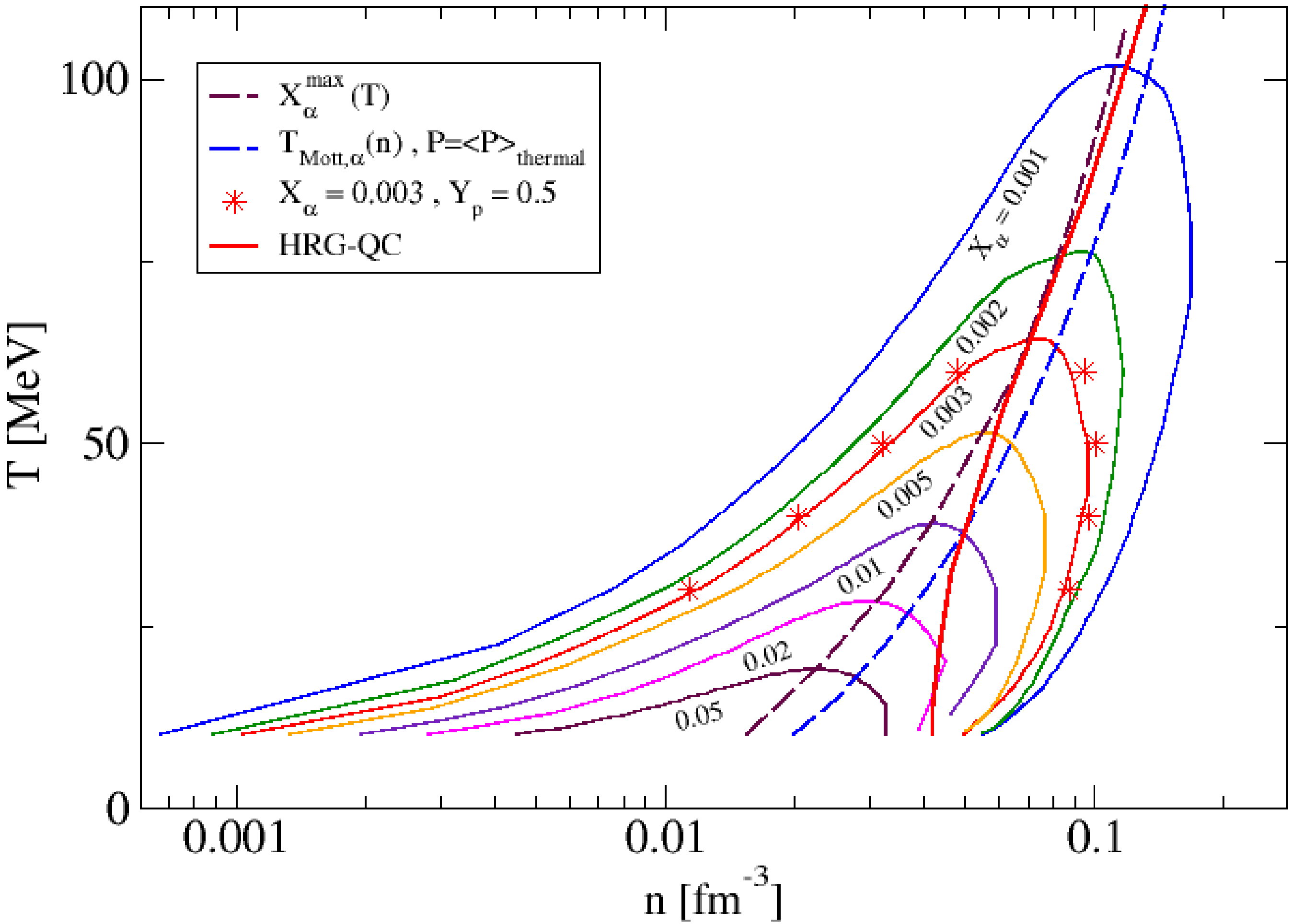


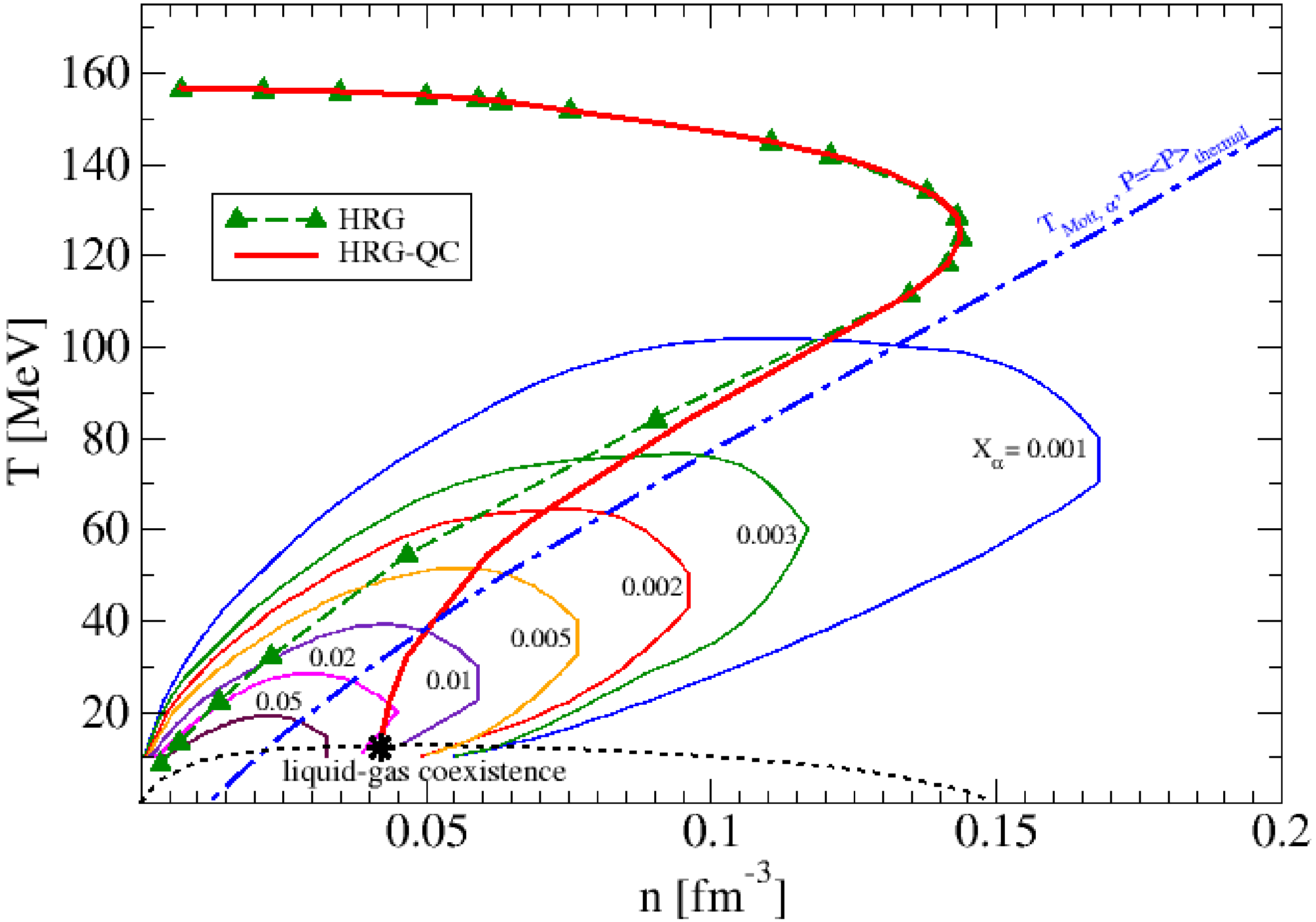
The mass action law of the chemical picture (nuclear statistical equilibrium) is modified by quantum effects (compositeness):

Pauli blocking →
→ Mott dissociation

Mott-line for alpha clusters (equivalent to the line of maximum alpha fraction) is well correlated with the Chemical Freeze-Out (CFO) line

D. Blaschke, B. Dönigus, S. Liebing, G. Röpke, arXiv:2408.01399, PLB 860 (2025) 139206

CFO in the Temperature – density plane



Main result:

Chemical freeze-out may be interpreted as „inverse“ Mott transition:

Strong localization effect of nucleon-nucleon correlations in bound states (clusters) entails freeze out of the nuclear composition

„collapse of wave function“

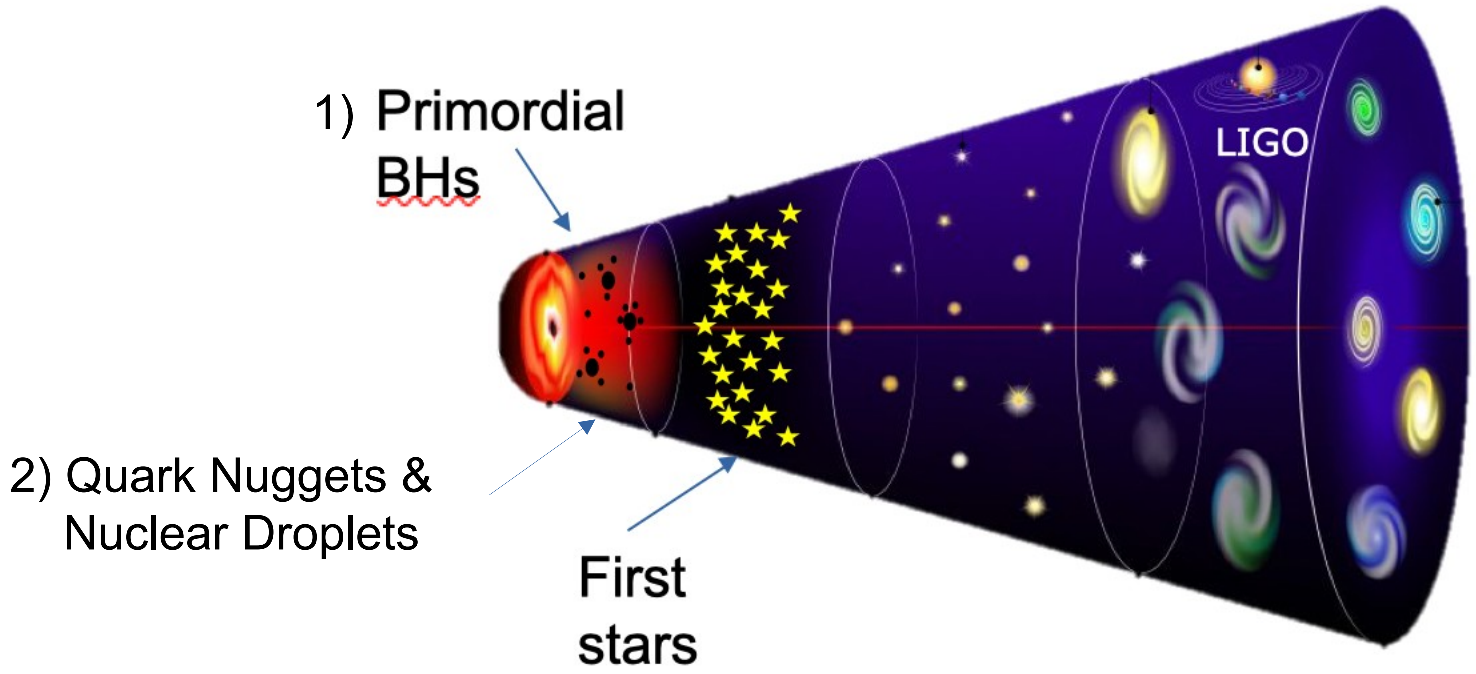
D. Blaschke, B. Dönigus, S. Liebing, G. Röpke, arXiv:2408.01399, PLB 860 (2025) 139206

Madeleine Soyeur
Burkhard Kämpfer

Bela Lukacs, NN,
Tamas Biro

PBH Formation and CFO of heavy elements?

JWST results – primordial black holes !



QCD hadronization transition plays key role for PBH formation !

Cluster virial expansion for quark-hadron matter

The cluster decomposition of the thermodynamic potential is given as

$$\Omega_{\rm total}(T,\mu,\phi,\bar\phi) = \Omega_{PNJL}(T,\mu,\phi,\bar\phi) + \Omega_{\rm pert}(T,\mu,\phi,\bar\phi) + \Omega_{MHRG}(T,\mu,\phi,\bar\phi),$$

where the first two terms describe the quark and gluon degrees of freedom via the mean–field thermodynamic potential for quark matter in a gluon background field $\mathcal{U}$

$$\Omega_{PNJL}(T,\mu,\phi,\bar\phi) = \Omega_Q(T,\mu,\phi,\bar\phi) + \mathcal{U}(T,\phi,\bar\phi)$$

with a perturbative correction $\Omega_{\rm pert}(T,\mu,\phi,\bar\phi)$.
The Mott-Hadron-Resonance-Gas (MHRG) part for the multi-quark clusters is

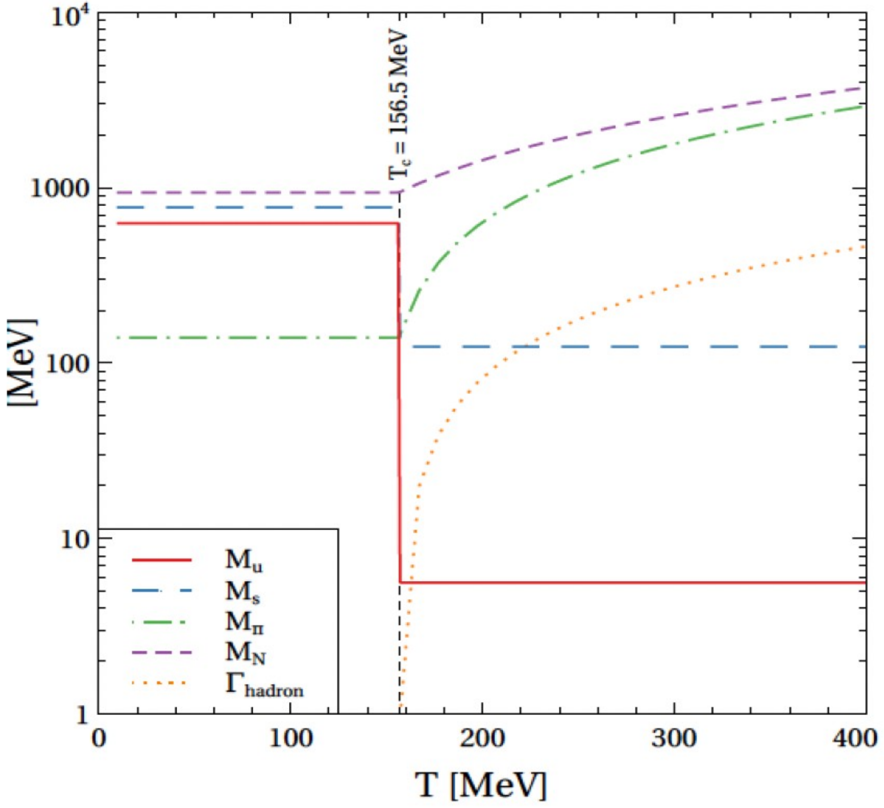
$$\Omega_{MHRG}(T,\mu,\phi,\bar\phi) = \sum_{i=M,B,\ldots} \Omega_i(T,\mu,\phi,\bar\phi),$$

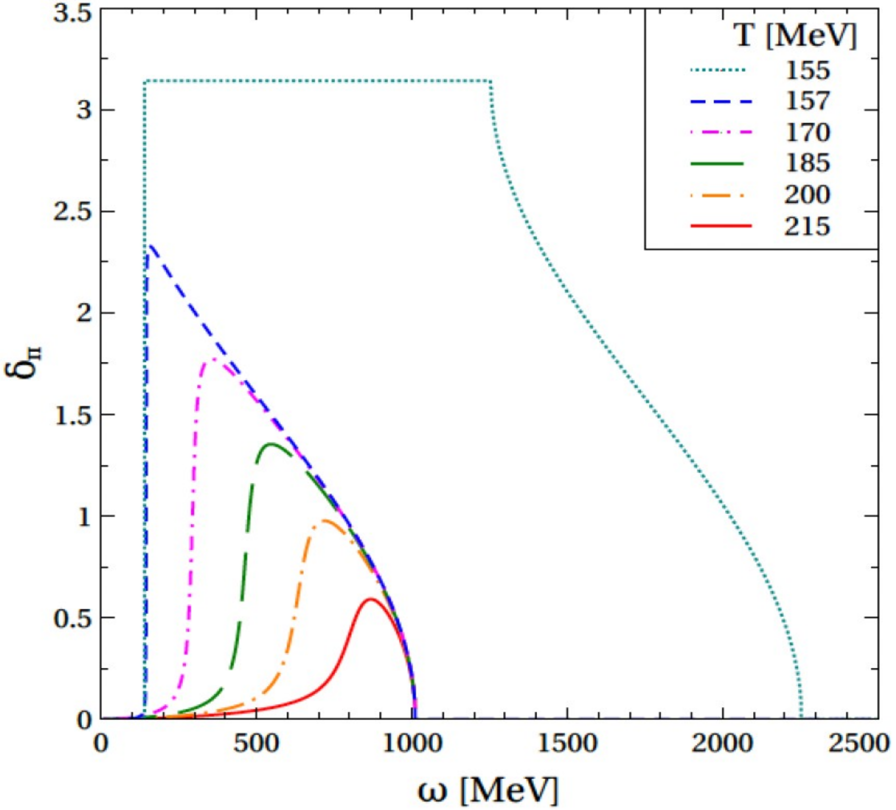
where the multi–quark states are described by the GBU formula:

$$\begin{aligned} n &= -\frac{\partial\Omega}{\partial\mu} = \sum_a a\, n_a(T,\mu) \\ &= \sum_a a\, d_a \int \frac{d\omega}{\pi} \int \frac{d^3q}{(2\pi)^3} \left\{ f_\phi^{(a),+} - \left[f_\phi^{(a),-} \right]^* \right\} 2\sin^2\delta_a(\omega,q) \frac{\partial\delta_a(\omega,q)}{\partial\omega}\ , \end{aligned}$$

where d_i is the degeneracy factor, a is the number of valence quarks in the cluster an $f_\phi^{(a),+}$, $\left[f_\phi^{(a),-} \right]^*$ are the Polyakov-loop modified distribution functions.
Analogous for the entropy density $s = -\partial\Omega/\partial T$.

Inputs: mass spectrum & phase shifts (models)





D.B., M. Ciernak, O. Ivanytskyi and G. Röpke, arXiv:2308.07950; EPJA 60 (2024) 14

Inputs: mass spectrum (Particle Data Tables)

Mesons

PDG mesons	d_i	$M_{\rm PDG}$ [MeV]	M_i [MeV]	$M^{<}_{{\rm th},i}$ [MeV]	$M^{>}_{{\rm th},i}$ [MeV]
π^+/π^0	3	140	140	1254	11.2
K^+/K^0	4	494	494	1397	129.6
η	1	548	878	1349	90.1
ρ^+/ρ^0	9	775	783	1254	11.2
ω	9	783	783	1254	11.2
K^{*+}/K^{*0}	12	895	$806^{*)}$	2651	140.8
η'	1	960	878	1349	90.1
a_0	3	980	$1095^{*)}$	2508	22.4
f_0	1	980	$1095^{*)}$	2508	22.4
ϕ	3	1020	1069	1540	248
..					
$\pi_2(1880)$	15	1895	$1095^{*)}$	2508	22.4
$f_2(1950)$	5	1944	$1095^{*)}$	2508	22.4
$a_4(2040)$	27	1996	$1095^{*)}$	2508	22.4
$f_2(2010)$	5	2011	$1095^{*)}$	2508	22.4
$f_4(2050)$	9	2018	$1095^{*)}$	2508	22.4
$K_4^*(2045)$	36	2045	$1238^{*)}$	2651	140.8
$\phi(2170)$	3	2175	$1381^{*)}$	2794	259.2
$f_2(2300)$	5	2297	$1095^{*)}$	2508	22.4
$f_2(2340)$	5	2339	$1095^{*)}$	2508	22.4

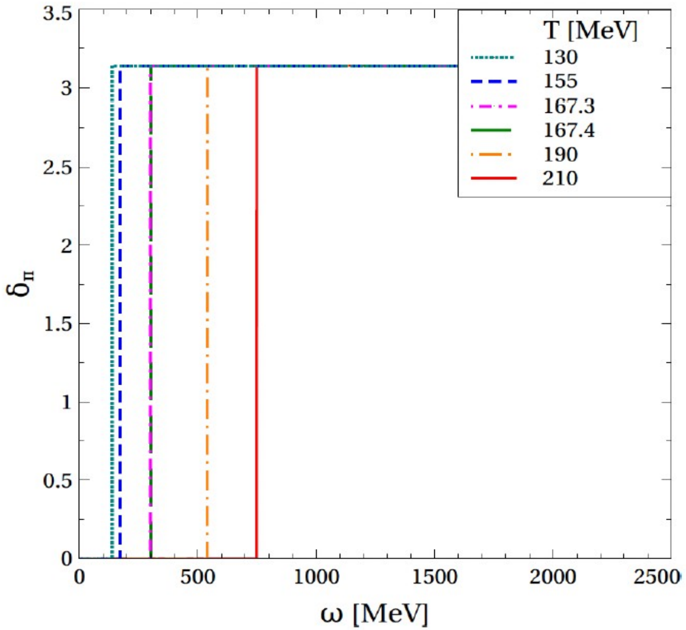
Baryons

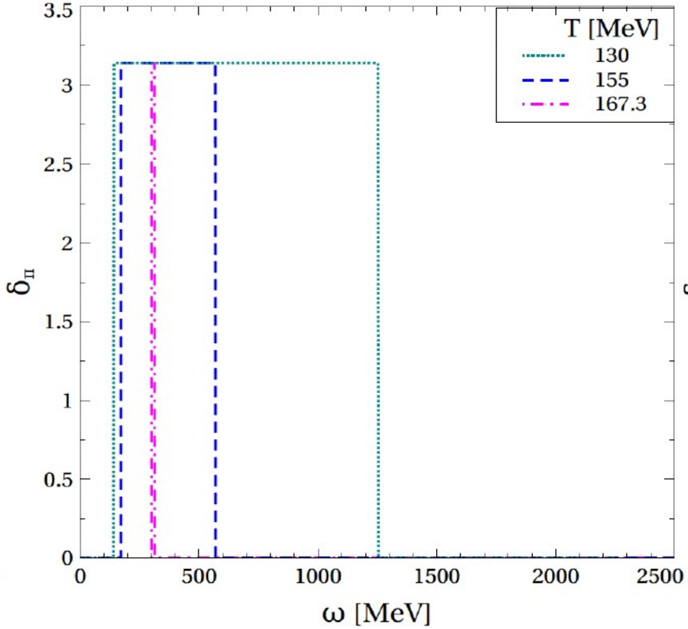
PDG baryons	d_i	$M_{\rm PDG}$ [MeV]	M_i [MeV]	$M^{<}_{{\rm th},i}$ [MeV]	$M^{>}_{{\rm th},i}$ [MeV]
n/p	4	939	939	1881	16.8
Λ	2	1116	1082	2024	135.2
Σ	6	1193	1082	2024	135.2
Δ	16	1232	$1251^{**)}$	3135	28
Ξ^0	2	1315	1225	2167	253.6
Ξ^-	2	1322	1225	2167	253.6
$\Sigma(1385)$	6	1385	$1394^{**)}$	3278	146.4
$\Lambda(1405)$	2	1405	$1394^{**)}$	3278	146.4
$N(1440)$	4	1440	$1251^{**)}$	3135	28
..					
$N(2195)$	36	2220	$1251^{**)}$	3135	28
$\Sigma(2250)$	6	2250	$1394^{**)}$	3278	146.4
$\Omega^-(2250)$	2	2252	$1680^{**)}$	3564	383.2
$N(2250)$	20	2275	$1251^{**)}$	3135	28
$\Lambda(2350)$	10	2350	$1394^{**)}$	3278	146.4
$\Delta(2420)$	48	2420	$1251^{**)}$	3135	28
$N(2600)$	24	2600	$1251^{**)}$	3135	28

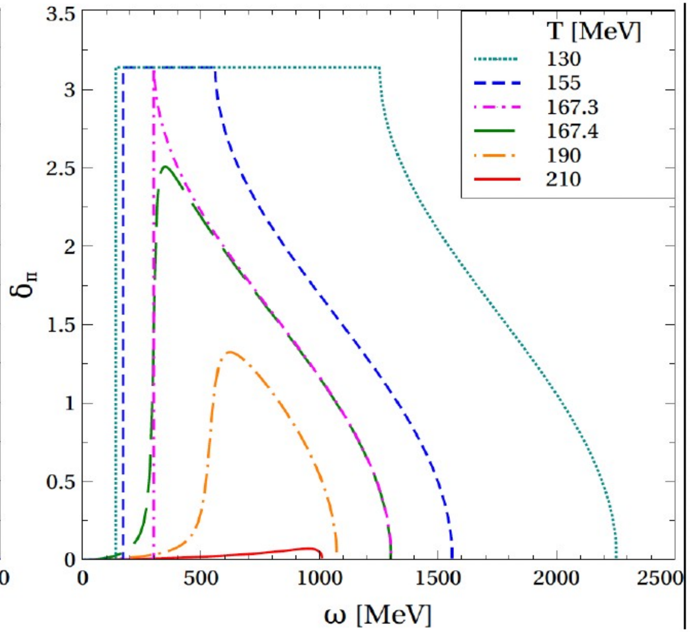
… and colored clusters (model) !

D.B., M. Ciernak, O. Ivanytskyi and G. Röpke, arXiv:2308.07950; EPJA 60 (2024) 14

Inputs for the phase shifts (models)





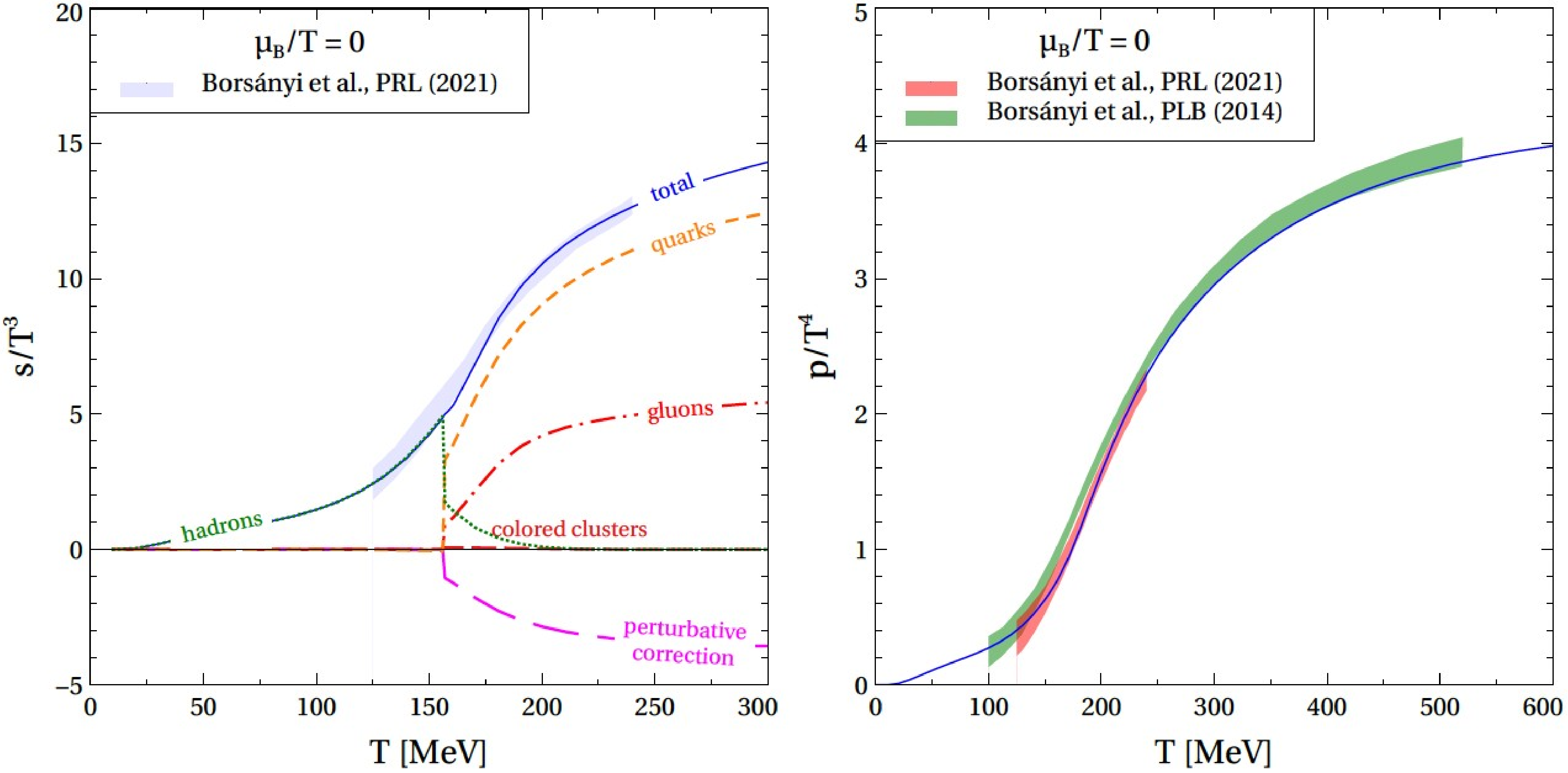


Step-up (SU) model → Hadron Resonance Gas

Step-up-step-down model → Mott Hadron Resonance Gas (MHRG)

Step-up-continuum model

D.B., M. Ciernak, O. Ivanytskyi and G. Röpke, arXiv:2308.07950; EPJA 60 (2024) 14



$\mu_B/T = 0$
Borsányi et al., PRL (2021)
total
quarks
gluons
hadrons
colored clusters
perturbative correction
s/T^3
T [MeV]
$\mu_B/T = 0$
Borsányi et al., PRL (2021)
Borsányi et al., PLB (2014)
p/T^4
T [MeV]

„Sudden Switch“ from HRG to QGP

Chemical Freeze-out: „inverse“ Mott effect – hadron localization = collapse of the wave function

$$H_{\rm exp}(T_{{\rm cf},i}) = \tau_i^{-1}(T_{{\rm cf},i})$$

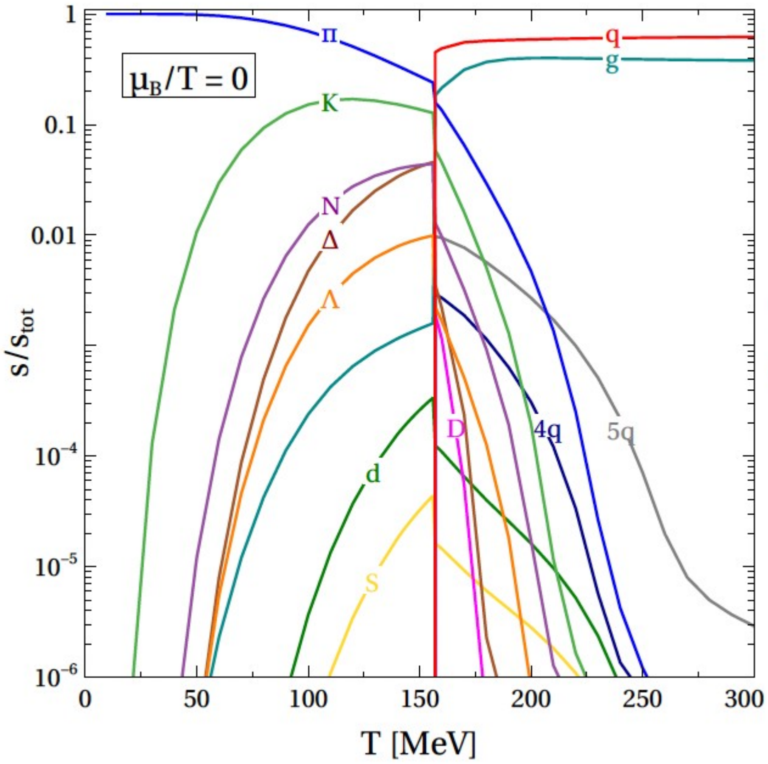
$$H_{\rm exp} = \frac{1}{\tau_{\rm exp}} = \frac{s^{1/3}}{a}$$

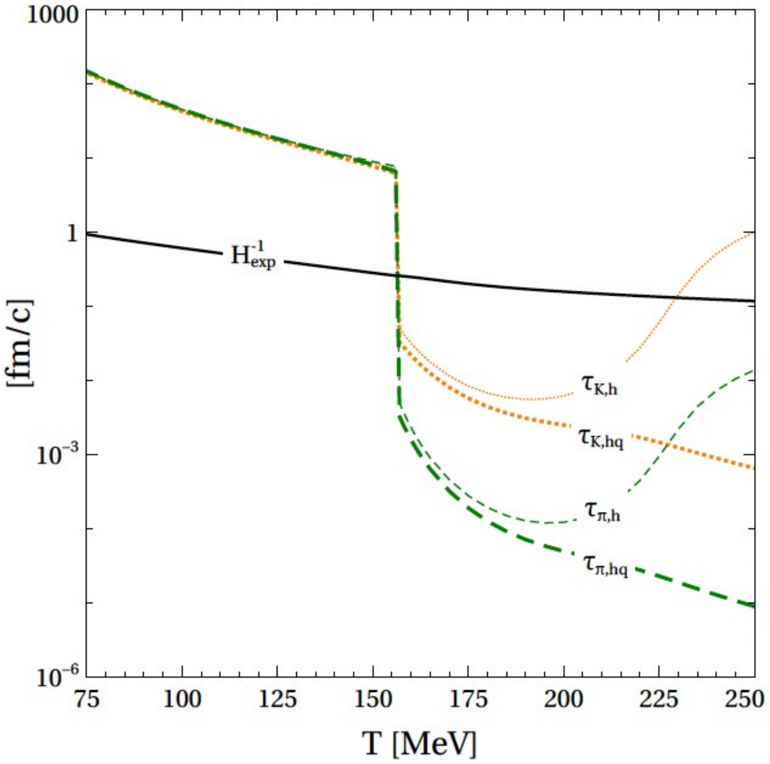
$$\tau_i^{-1} = \sum_j \sigma_{ij} v_{\rm rel} n_j$$

$$\sigma_{ij} = \lambda \langle r_i^2\rangle \langle r_j^2\rangle$$

$$\langle r^2\rangle_\pi^{1/2} \sim |T - T_{{\rm Mott},\pi}|^{-1/2}$$

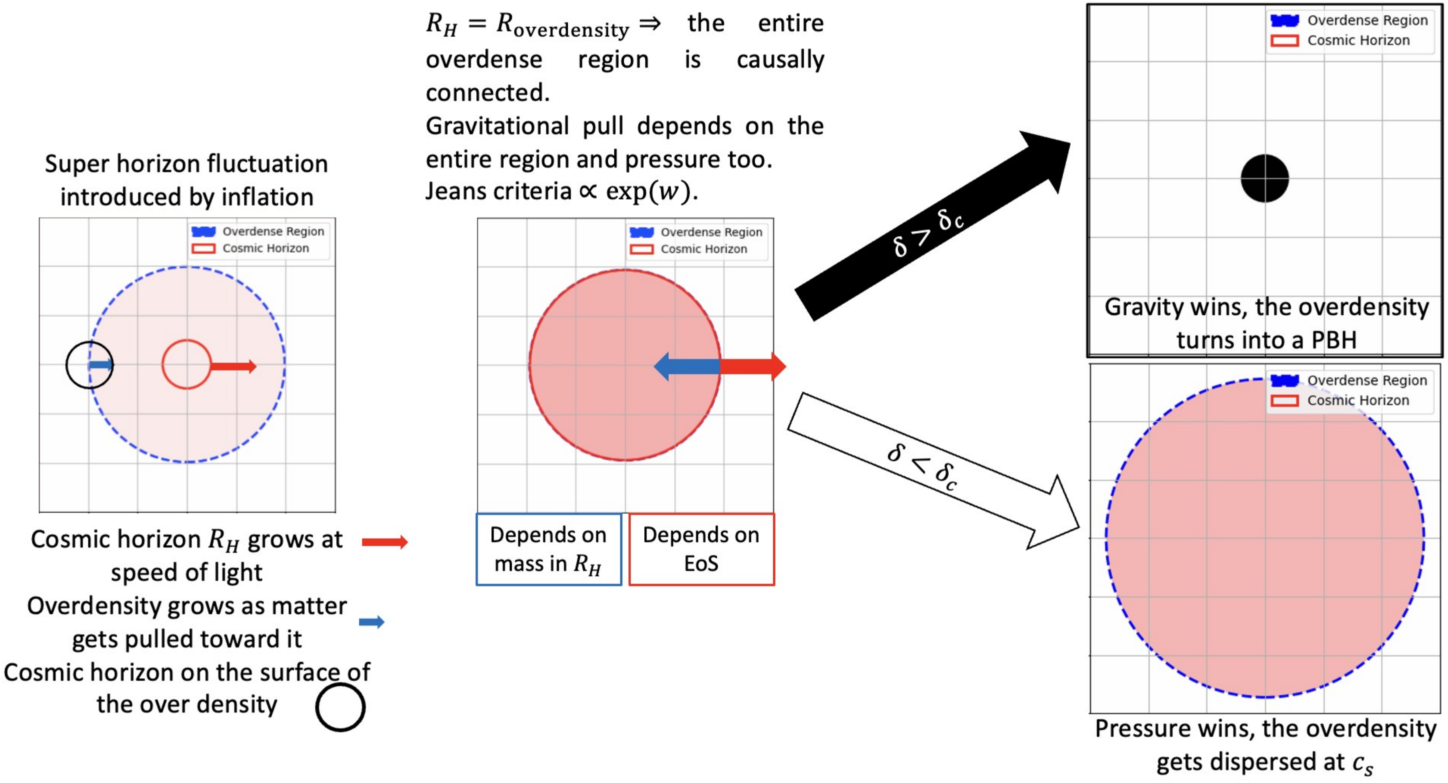
$$\langle r^2\rangle_\pi \sim E_{B,\pi}(T)^{-1/2}$$





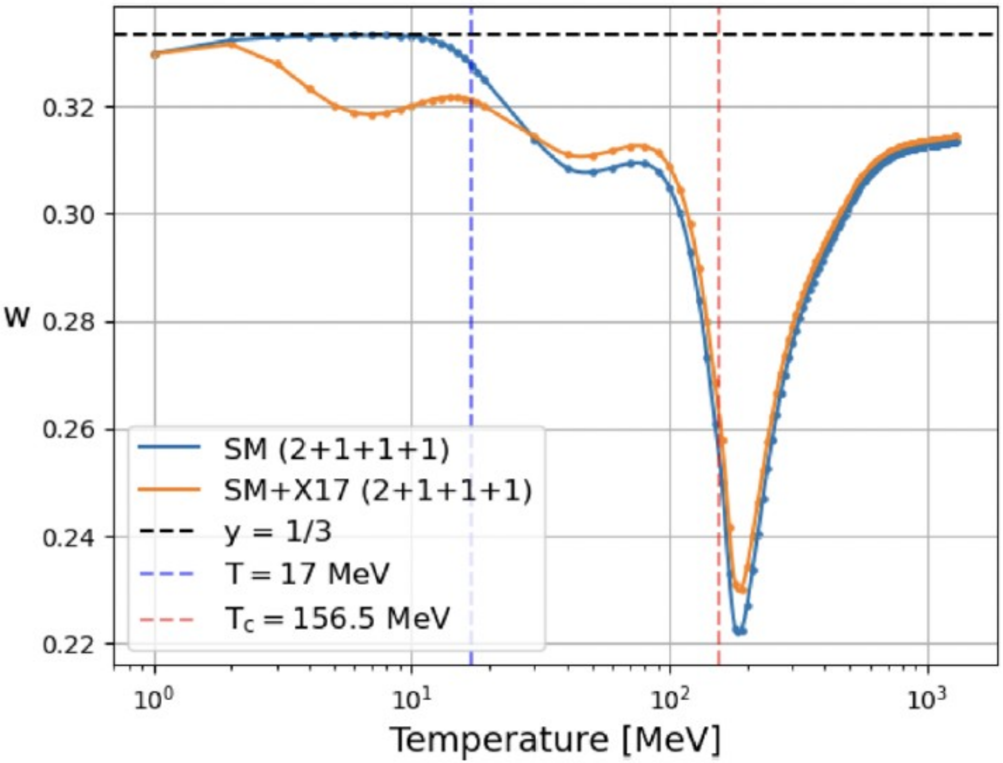
arXiv:2507.10497; PoS (QCHS2024) 247

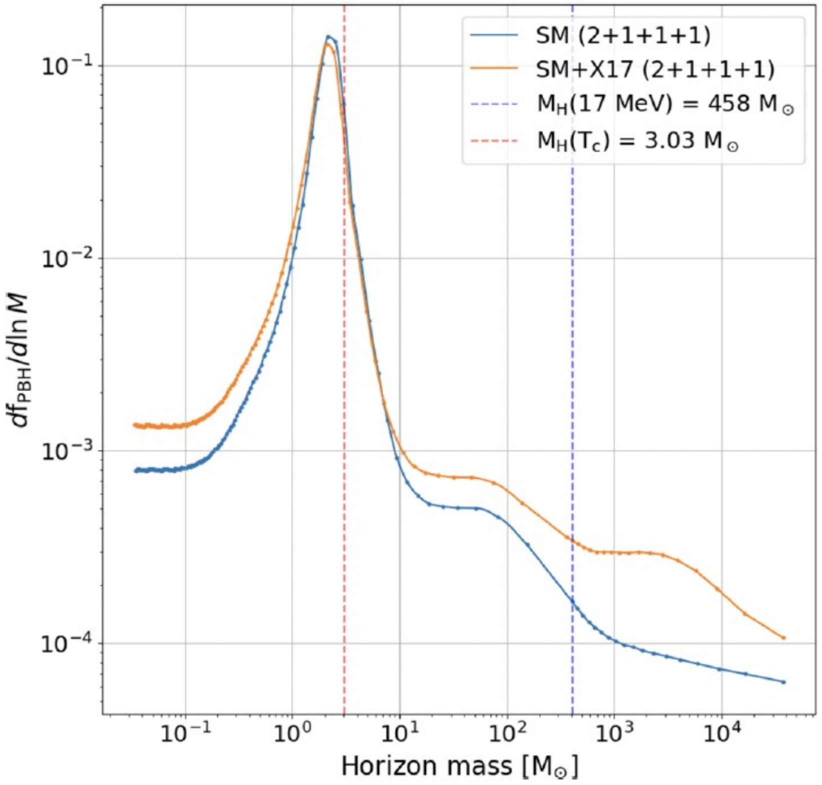
PBH Formation and Nuclear Droplets?



Two paths @ QCD hadronization: 1) PBH formation & 2) Nuclear droplets !

PBH Formation and CFO of heavy elements?

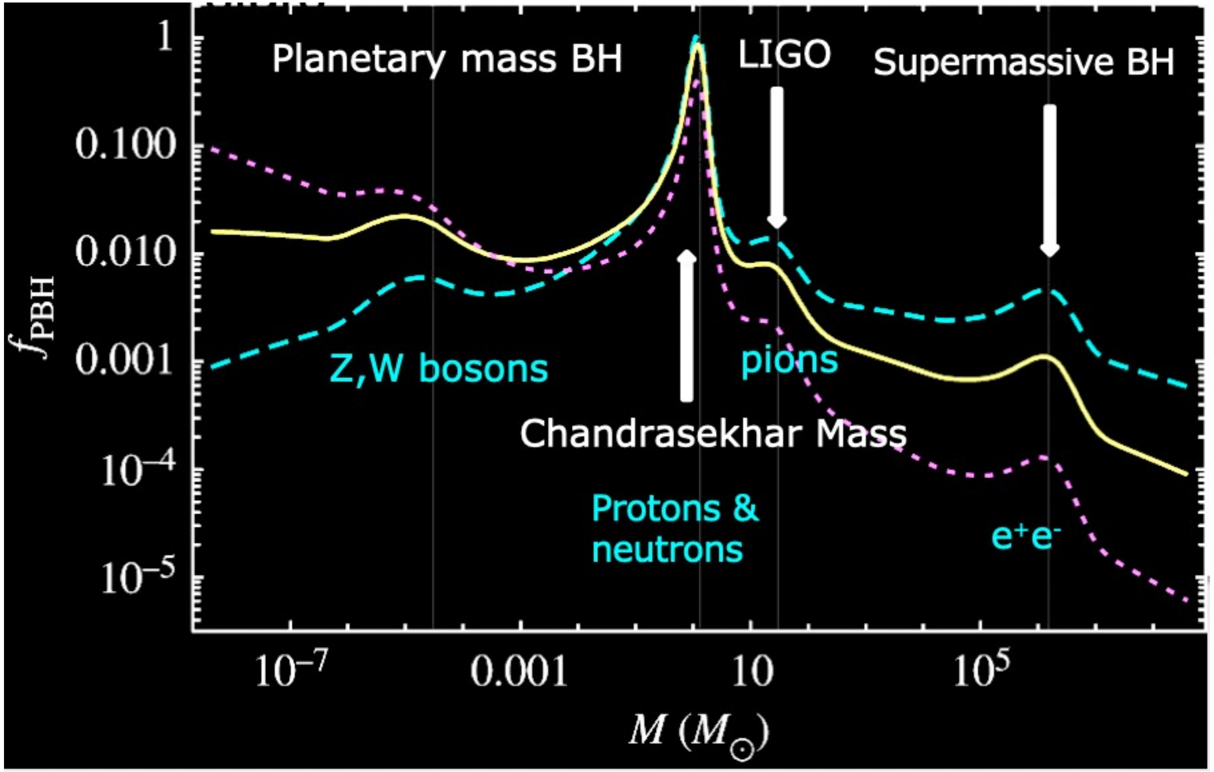




QCD hadronization transition plays key role plays for PBH formation !

PBH Formation and CFO of heavy elements?

JWST results – primordial black holes !



Different peaks correspond to different particles created at the early universe phase transitions and the corresponding reduction in the sound velocity.

BH mass corresponds to the horizon size at each time.

Only requirement is enough fluctuation power in a volume fraction of 10^{-9} of the early Universe.

Carr, Clesse, García-Bellido 2019

QCD hadronization transition plays key role plays for PBH formation !

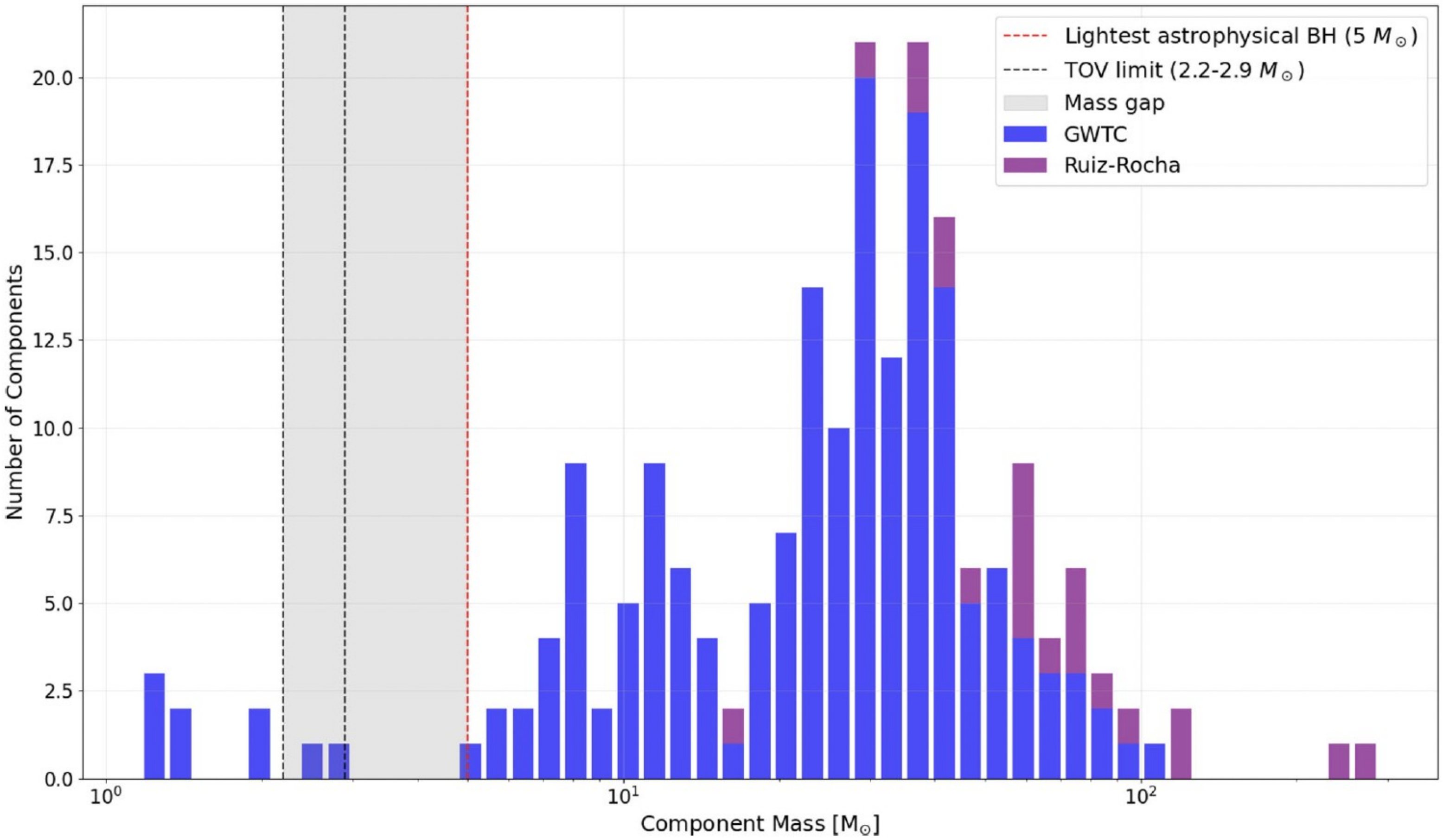
Surprised: Dirk Rischke with Nu Xu and Ramona Vogt

PBH Formation and CFO of heavy elements?

LIGO BBH Merger Mass Distribution

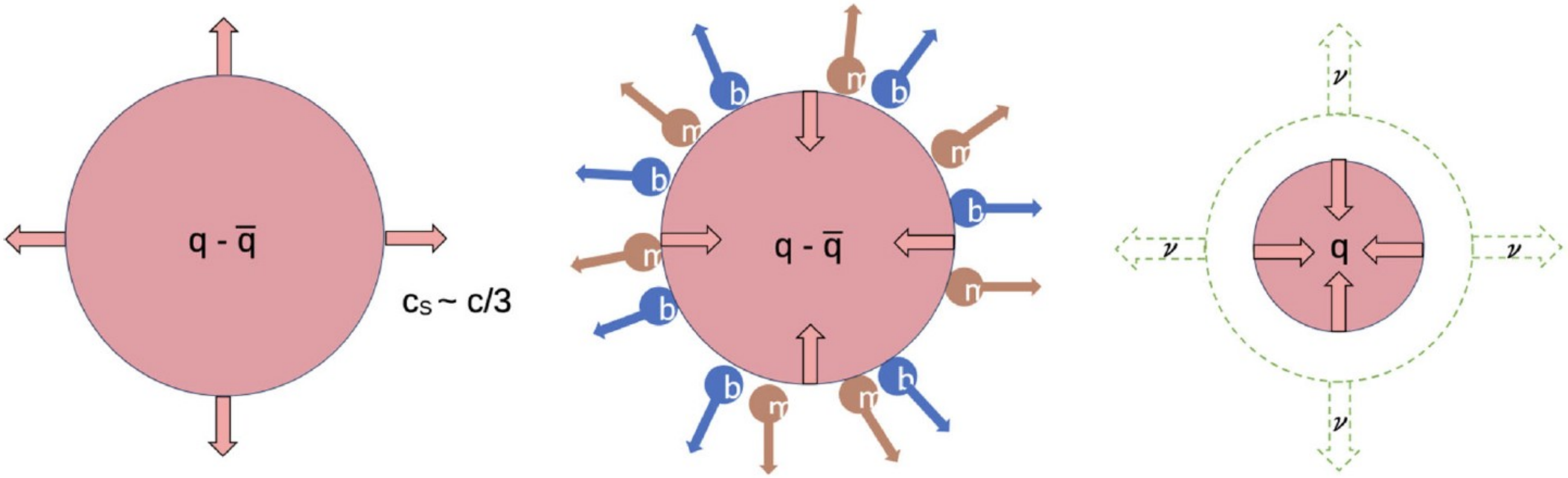
Crazy conjecture:

Peak mass “measures“ Cosmic QCD Transition TemperatureT_H~50 MeV ?



PBH Formation and CFO of heavy elements?

In this scenario the shrinking stops when the energy density gets to protons energy density.
This scenario requires $T > T_{QCD}$. It can be applied for pre-QCD transition failed collapse.

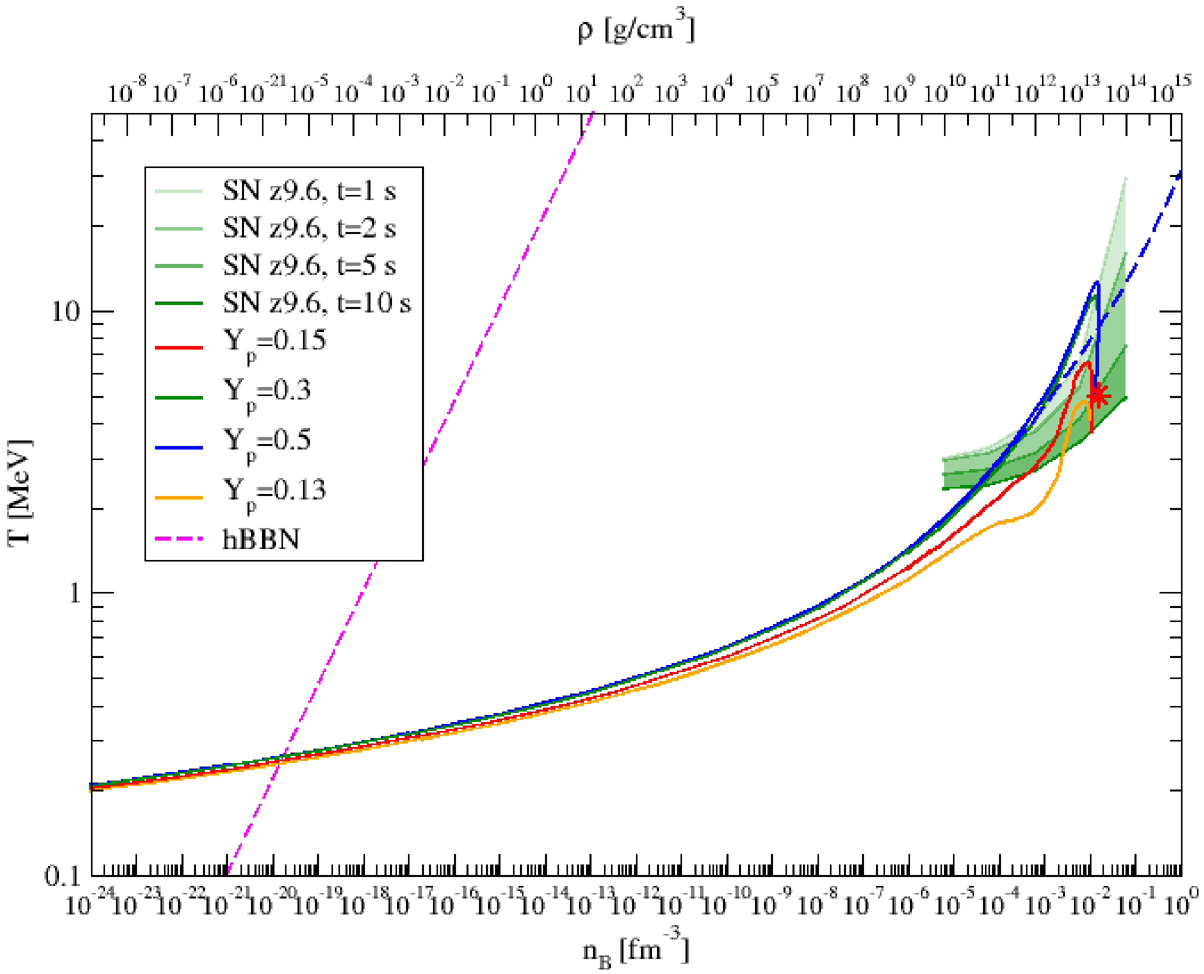


At first the overdensity expands at the speed of sound

Following Bjerrum-Bohr et al. 2012 and 2014. The quark plasma droplet shrinks by emitting baryons marked with b and mesons marked with m.

Following Witten 1984, the surplus quarks could be distilled by quark-antiquark annihilation to neutrinos being radiated off the glob.

PBH Formation and Nuclear Droplet Formation?



Can the primordial evolution of the Universe lead to these freeze-out Parameters (red star):

T=5 MeV,
μ_n=940.317 MeV,
μ_p= 845.069 MeV

Maybe inhomogeneous Big Bang?

The freeze-out point lies in the domain of supernova explosions and
binary neutron star mergers

G. Röpke, D. Blaschke, F. Röpke, arXiv:2411.00535

QCD phase transition → Quark Matter nuggets

PHYSICAL REVIEW D VOLUME 30, NUMBER 2 15 JULY 1984

Cosmic separation of phases

Edward Witten*
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 9 April 1984)

A first-order QCD phase transition that occurred reversibly in the early universe would lead to a surprisingly rich cosmological scenario. Although observable consequences would not necessarily survive, it is at least conceivable that the phase transition would concentrate most of the quark excess in dense, invisible quark nuggets, providing an explanation for the dark matter in terms of QCD effects only. This possibility is viable only if quark matter has energy per baryon less than 938 MeV. Two related issues are considered in appendices: the possibility that neutron stars generate a quark-matter component of cosmic rays, and the possibility that the QCD phase transition may have produced a detectable gravitational signal.

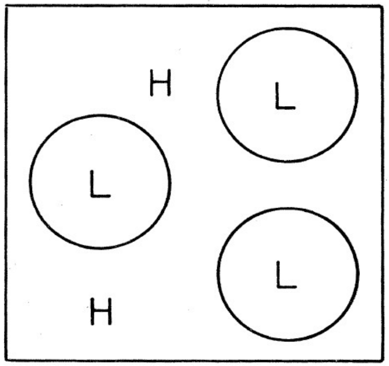


FIG. 1. Isolated expanding bubbles of low-temperature phase in the high-temperature phase.

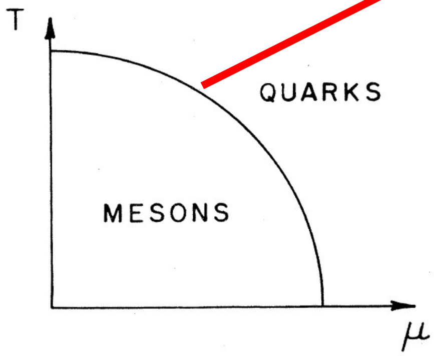
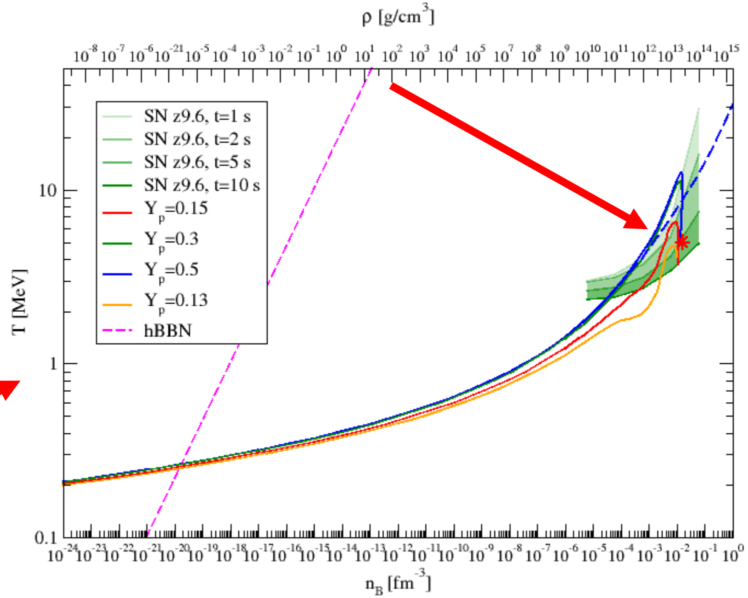


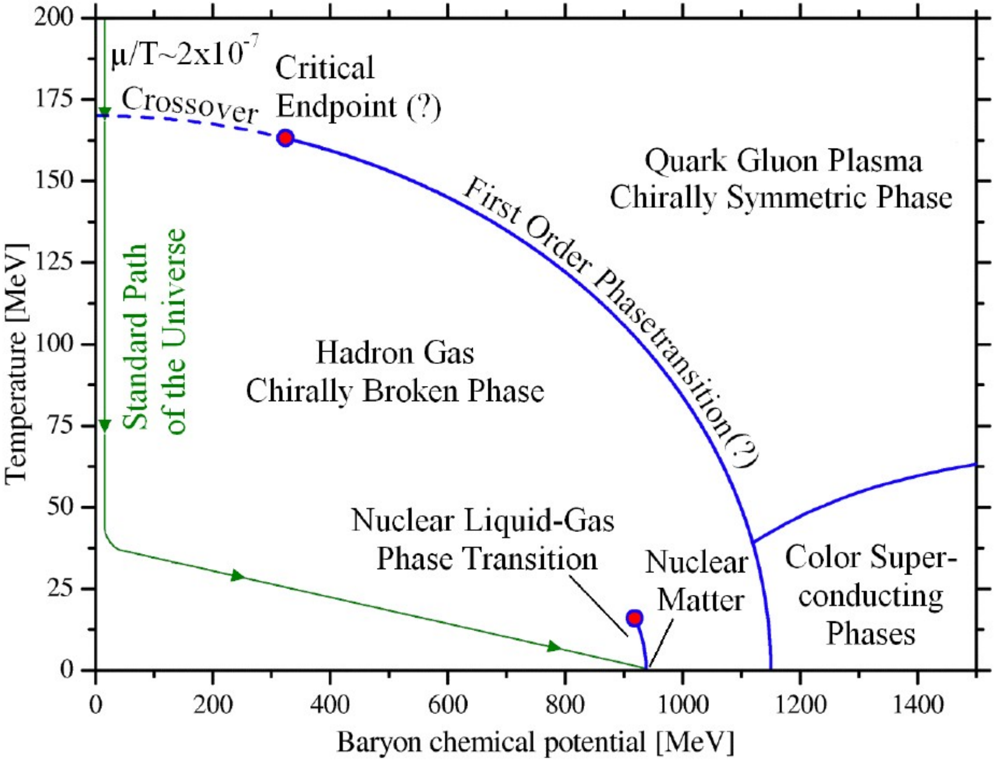
FIG. 4. A sketch of the coexistence temperature for quark matter of chemical potential μ coexisting with the meson-baryon phase of $\mu=0$. What is shown is the temperature, as a function of μ, at which the two phases exert equal pressure.

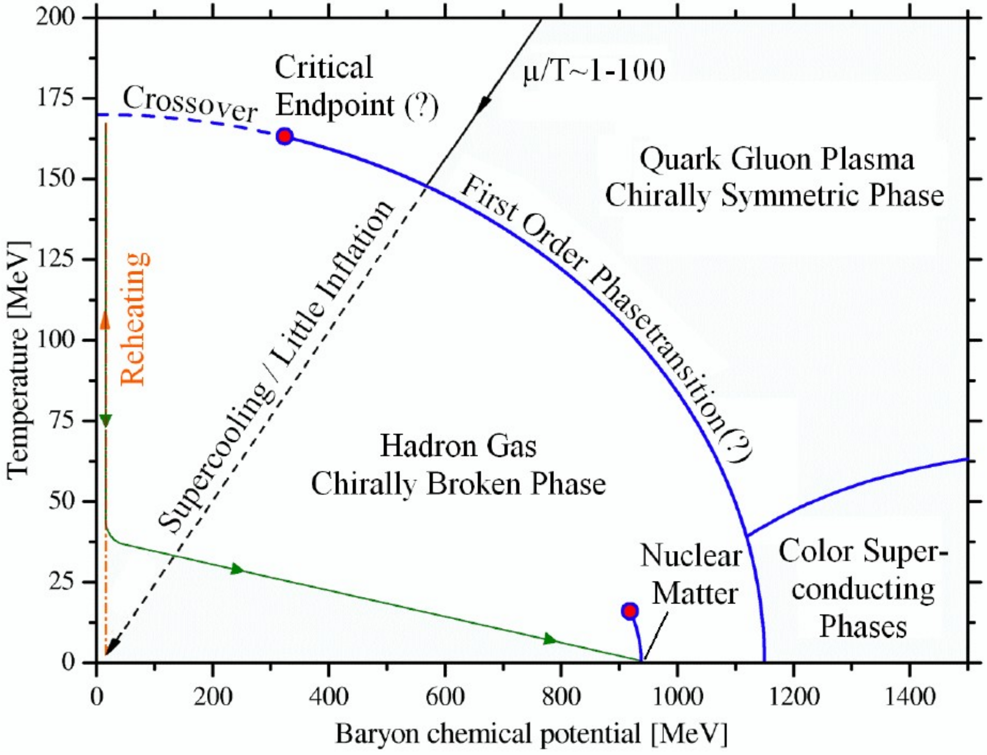


Accumulated mass fraction vs. mass number Â for solar element abundances compared with freeze-out model after neutron evaporation for T=5 MeV, μ_n=940.317 MeV, μ_p= 845.069 MeV

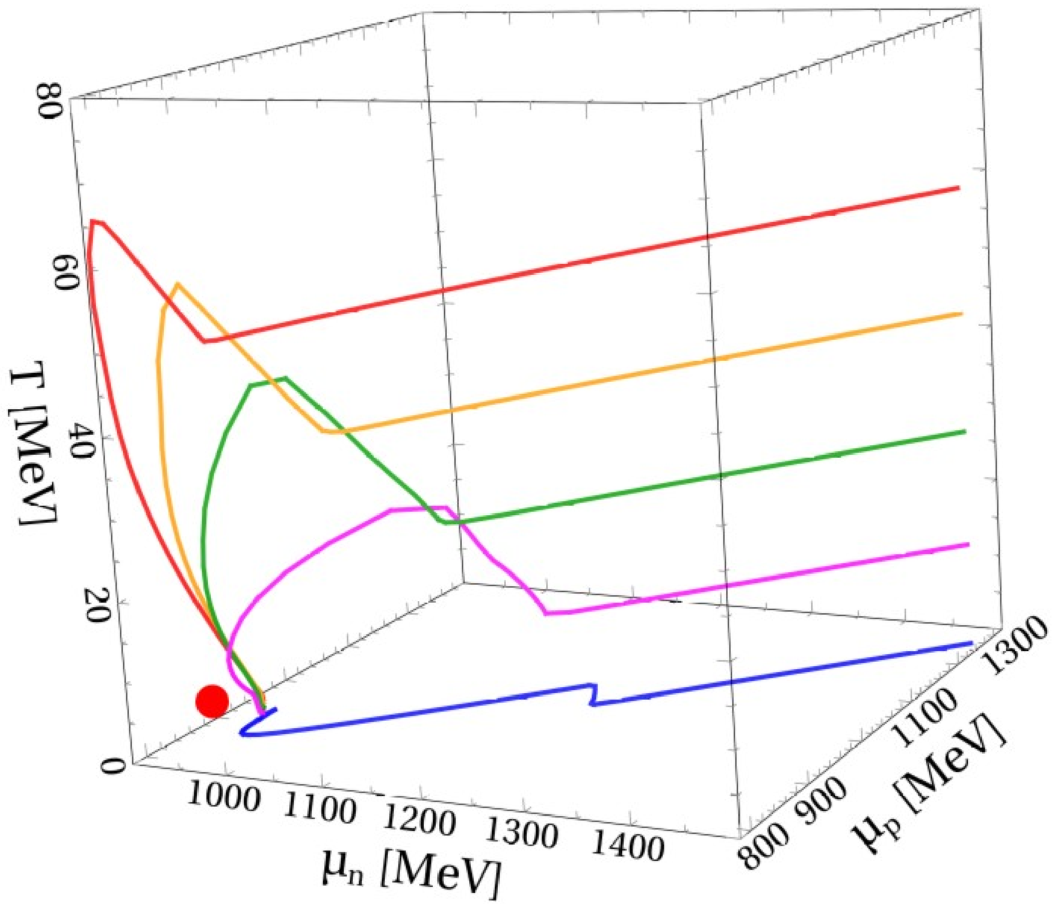
G. Röpke, D. Blaschke, F. Röpke, arXiv:2411.00535

Task: Explain how primordial evolution leads to Lagrange parameters T=5 MeV and n_B=n_sat/10





Simon Schettler, Tillmann Boeckel, Jürgen Schaffner-Bielich, arXiv:1010.4857
Tillmann Boeckel, Jürgen Schaffner-Bielich, arXiv:1105.0832



Preliminary result

for isentropic evolution of a quark nugget with high mu_b/T ~ 100 in the QCD phase diagram under cosmic plasma conditions with photons, leptons, charge neutrality and beta equilibrium.

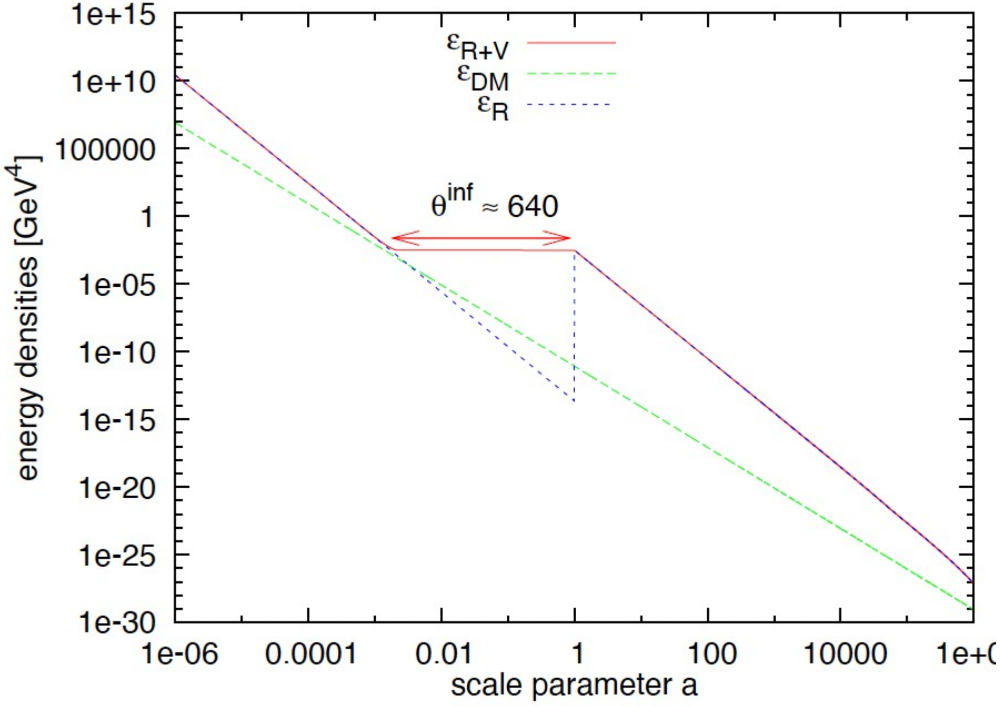
S/N = const

0.1 (blue)
1.0 (magenta)
2.0 (green)
3.0 (yellow)
4.0 (red)

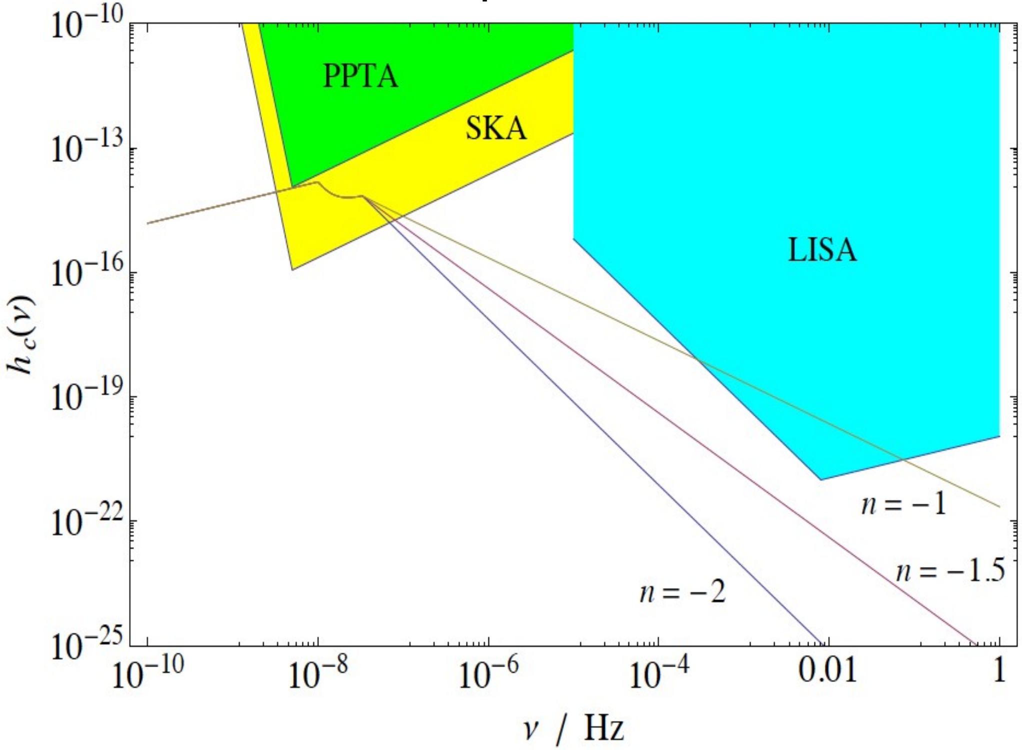
Red dot = freeze-out parameter set by Oleksii Ivanytskyi (05.09.2025)

Task: Explain how primordial evolution leads to Lagrange parameters T=5 MeV and n_B=n_sat/10

Supercooling, Mini-Inflation, Reheating



Gravitational wave spectrum: PTA, SKA, ...



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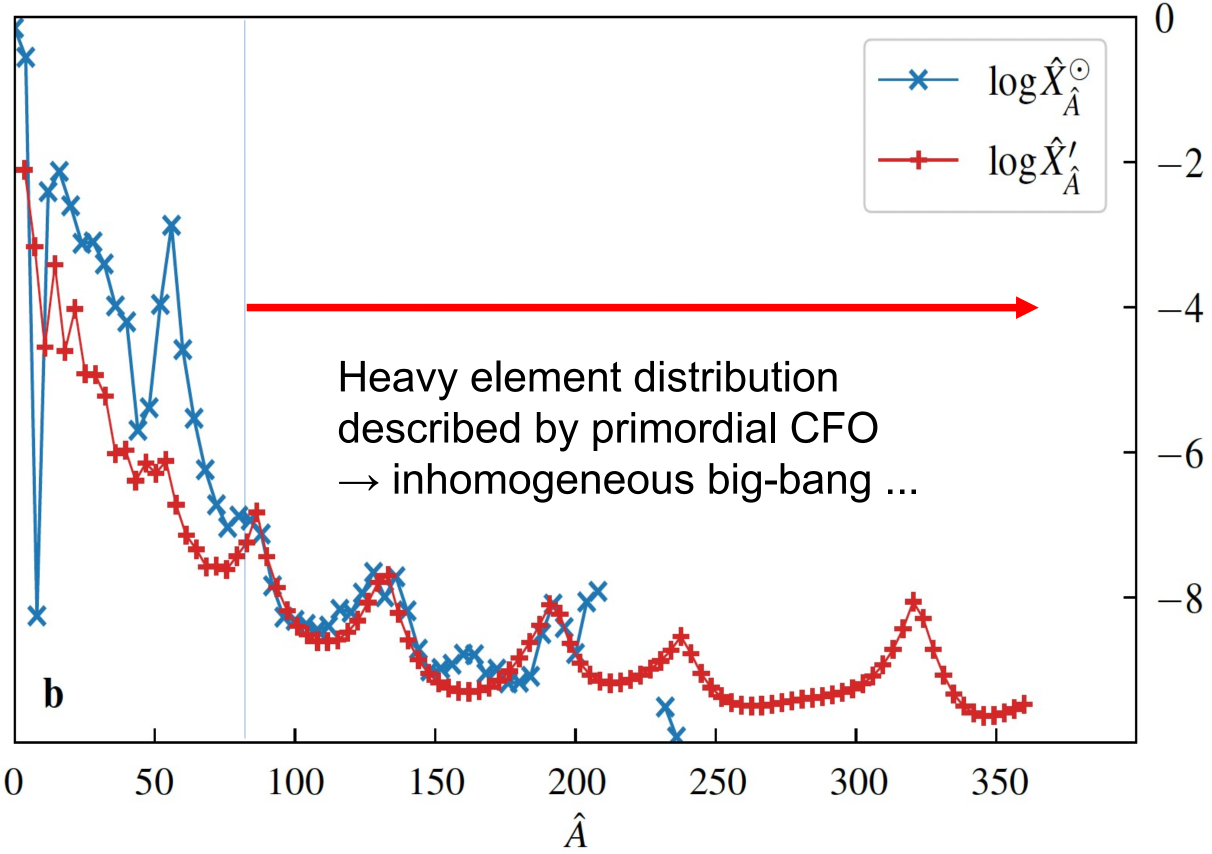

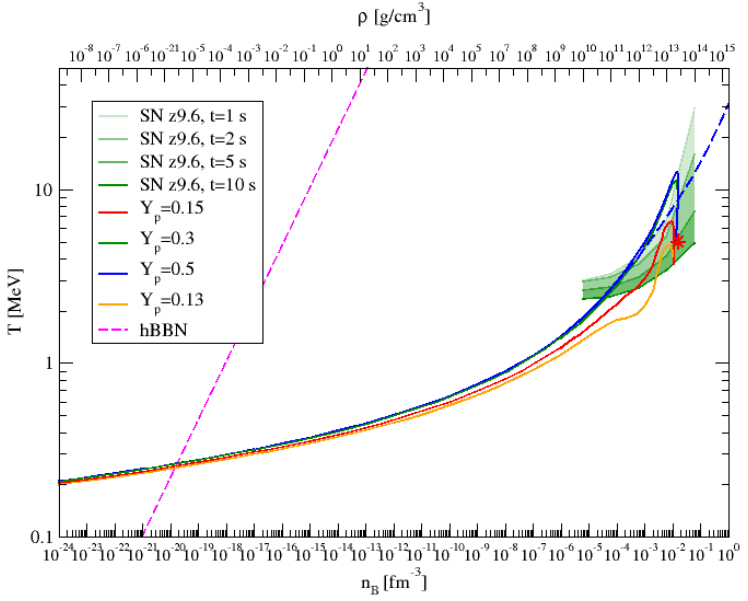

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G. Röpke, D. Blaschke, F. Röpke, arXiv:2411.00535

International Conference on Quark Confinement and the Hadron Spectrum (QCHS 2026)
29.6. - 4.7.2026, Wroclaw, Poland

David Blaschke (Uni Wroclaw & HZDR/CASUS),
Nora Brambilla (TU Munich),
Pior Surowka (Wroclaw Univ. Science & Technology)

https://indico.cern.ch/event/1531304

BP 2002

INTERNATIONAL WORKSHOP ON QUARK AND HADRON DYNAMICS IN RELATIVISTIC HEAVY ION COLLISIONS

March 3-7, 2002
Budapest, Hungary

Boromisza, Tibor: "*Early morning light*"

ZIMÁNYI SCHOOL 2025

I. Csók: Lightning over Balaton

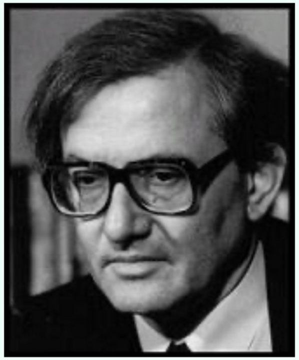
25th ZIMÁNYI SCHOOL

WINTER WORKSHOP

ON HEAVY ION PHYSICS

December 1-5, 2025

Budapest, Hungary



József Zimányi (1931 - 2006)

BP 2002:

Ivan Vitev

Jozef Zimanyi

Magdolna Zimanyi

Istvan Lovas

His wife

Husband of Judit

Judit Nemeth

Peter Levai