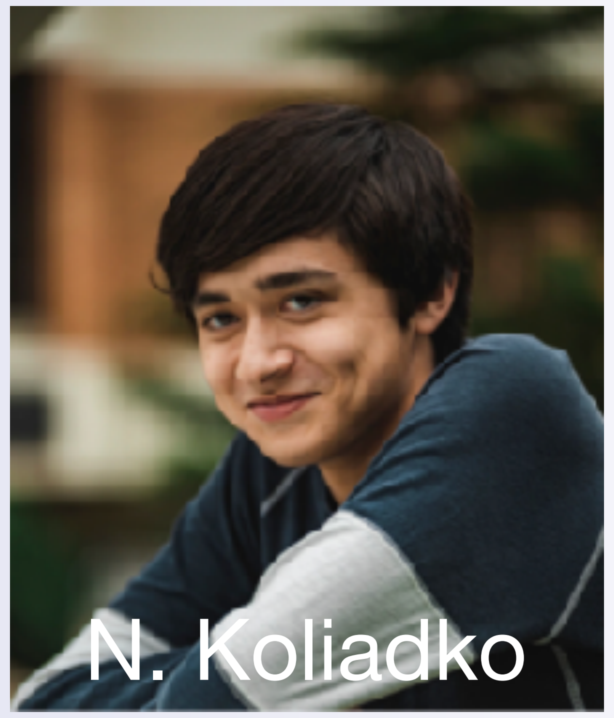
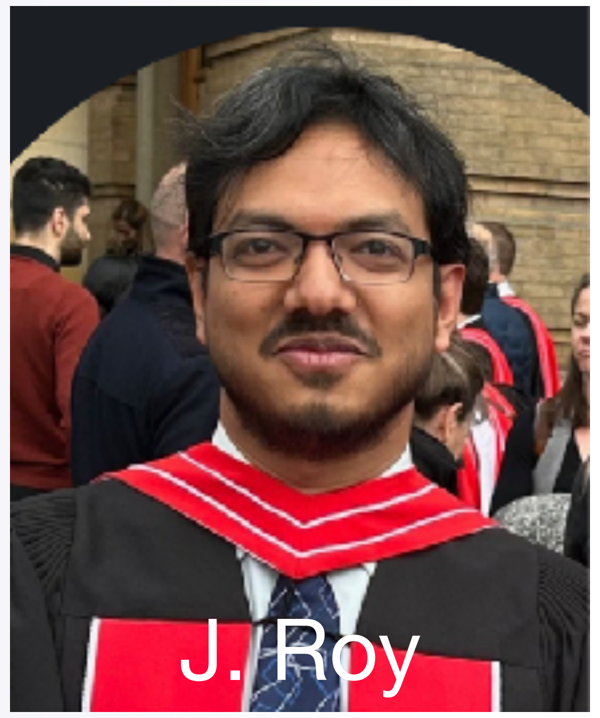
Heavy Quarks Floating in the QGP Fluid

Berndt Mueller

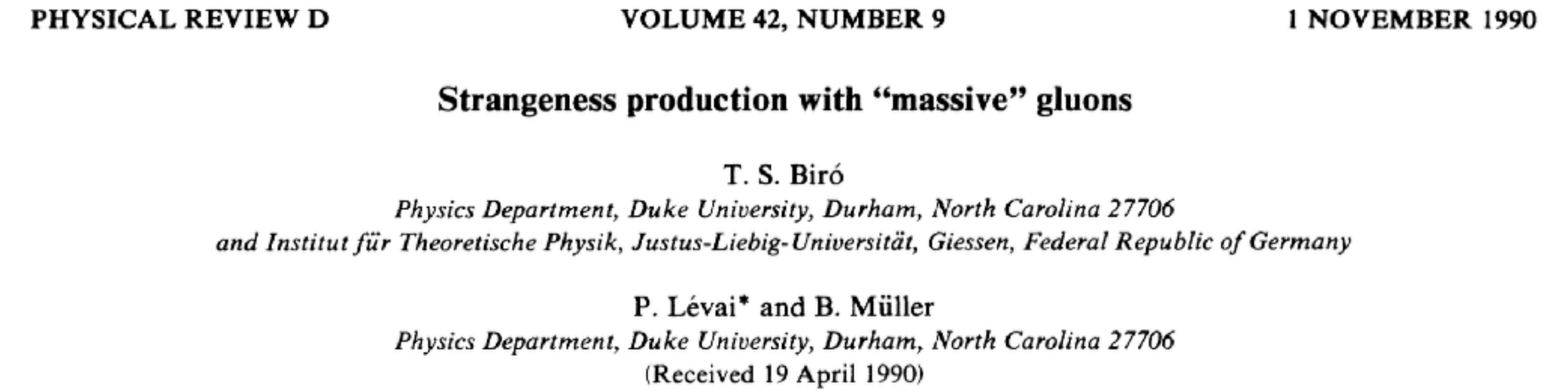

A. Kirchner


N. Koliadko


J. Roy


C. Sirimanna

12 June 2026
Budapest, Hungary



PHYSICAL REVIEW D VOLUME 42, NUMBER 9 1 NOVEMBER 1990

Strangeness production with "massive" gluons

T. S. Biró
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and Institut für Theoretische Physik, Justus-Liebig-Universität, Giessen, Federal Republic of Germany

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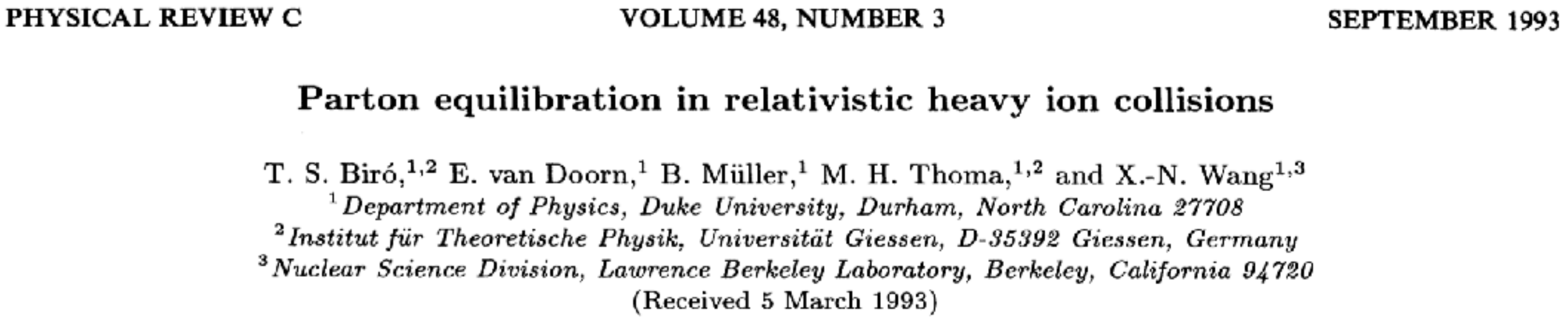

PHYSICAL REVIEW C VOLUME 48, NUMBER 3 SEPTEMBER 1993

Parton equilibration in relativistic heavy ion collisions

T. S. Biró,[1,2] E. van Doorn,[1] B. Müller,[1] M. H. Thoma,[1,2] and X.-N. Wang[1,3]
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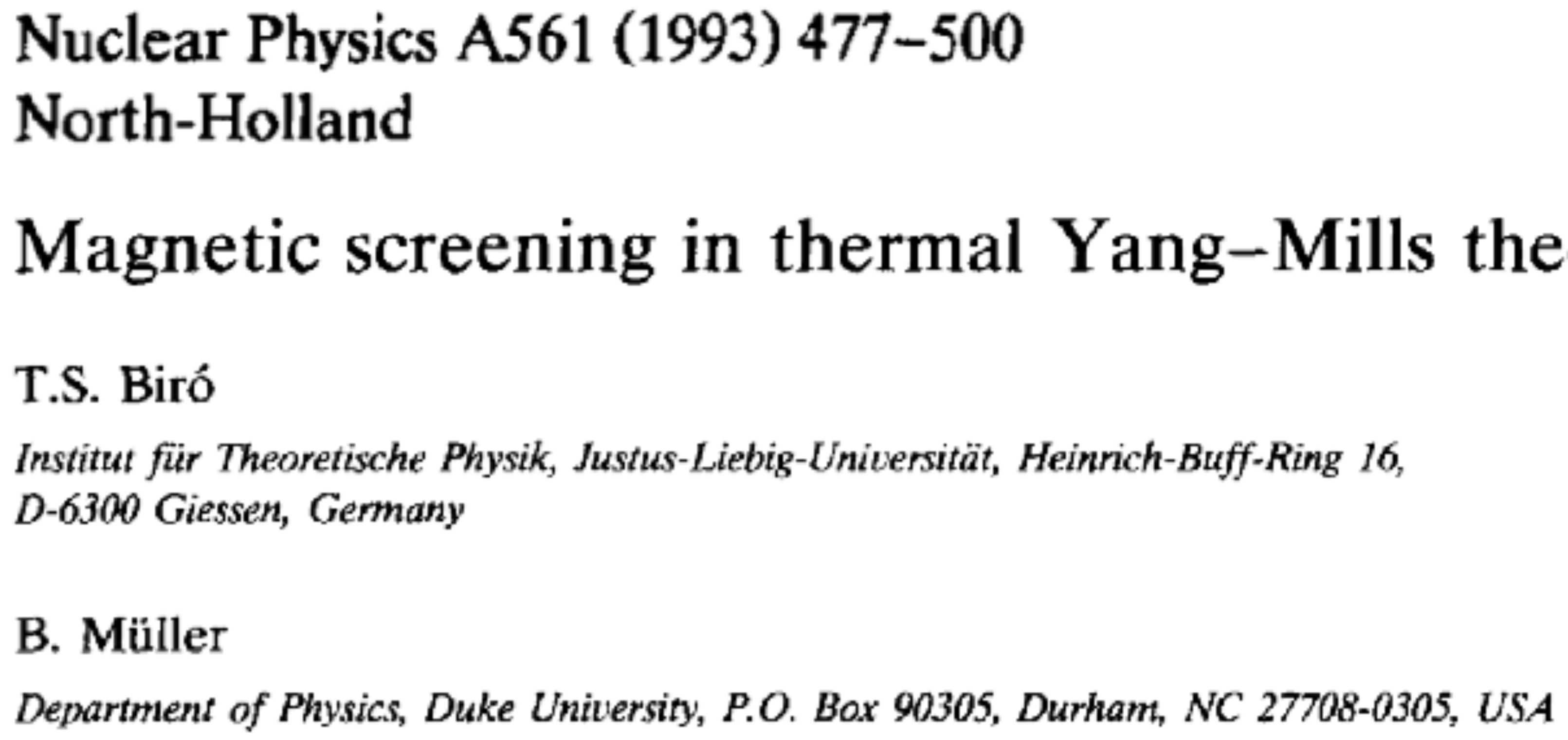

Nuclear Physics A561 (1993) 477–500
North-Holland

Magnetic screening in thermal Yang–Mills theories

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D-6300 Giessen, Germany

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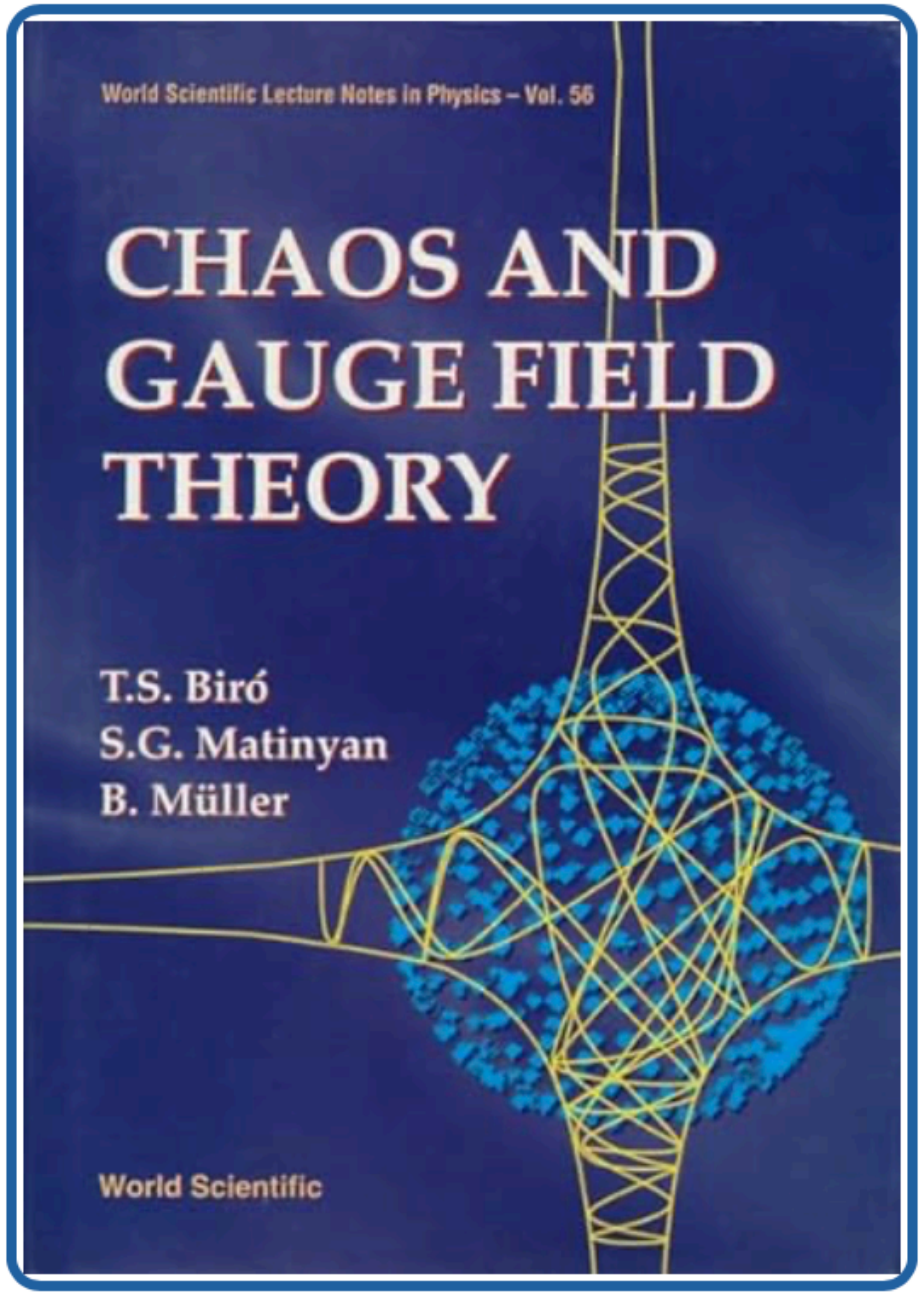
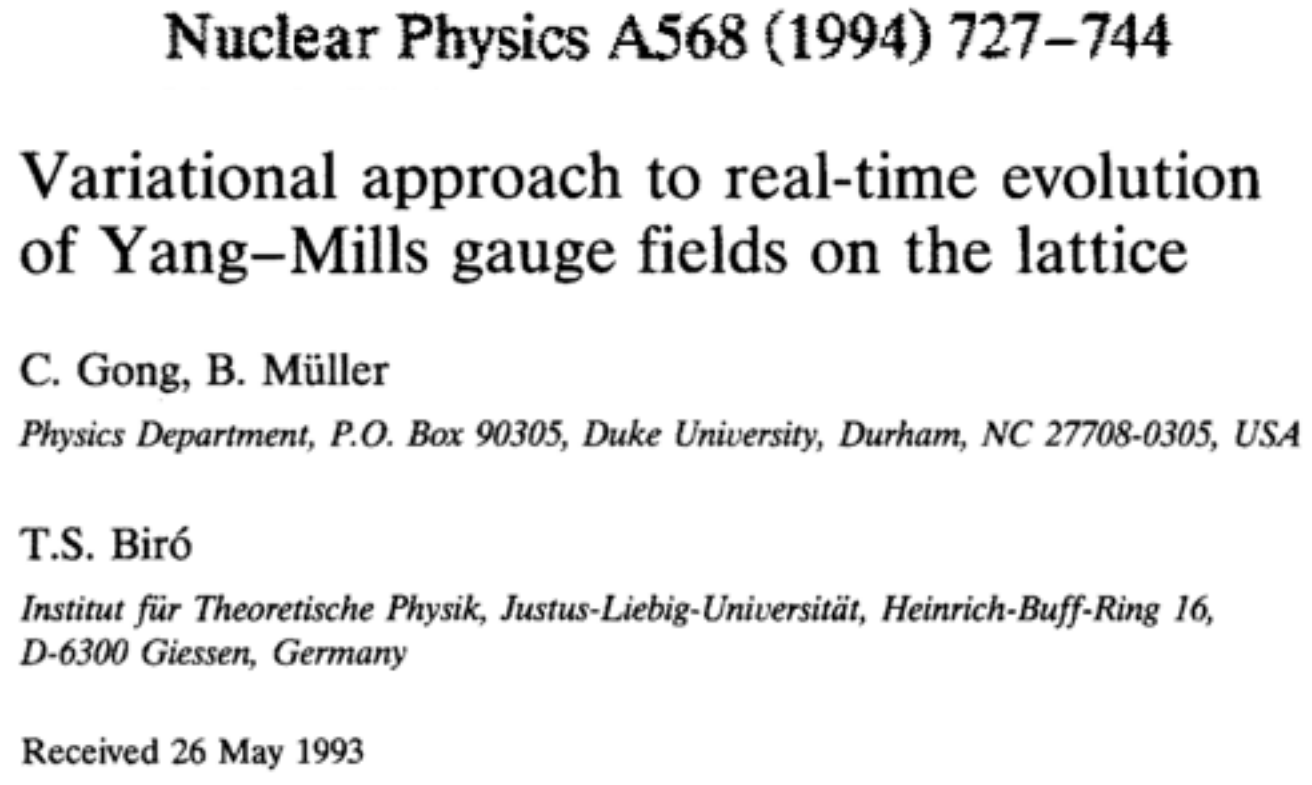


World Scientific Lecture Notes in Physics – Vol. 56

CHAOS AND GAUGE FIELD THEORY

T.S. Biró
S.G. Matinyan
B. Müller

World Scientific



Nuclear Physics A568 (1994) 727–744

Variational approach to real-time evolution of Yang–Mills gauge fields on the lattice

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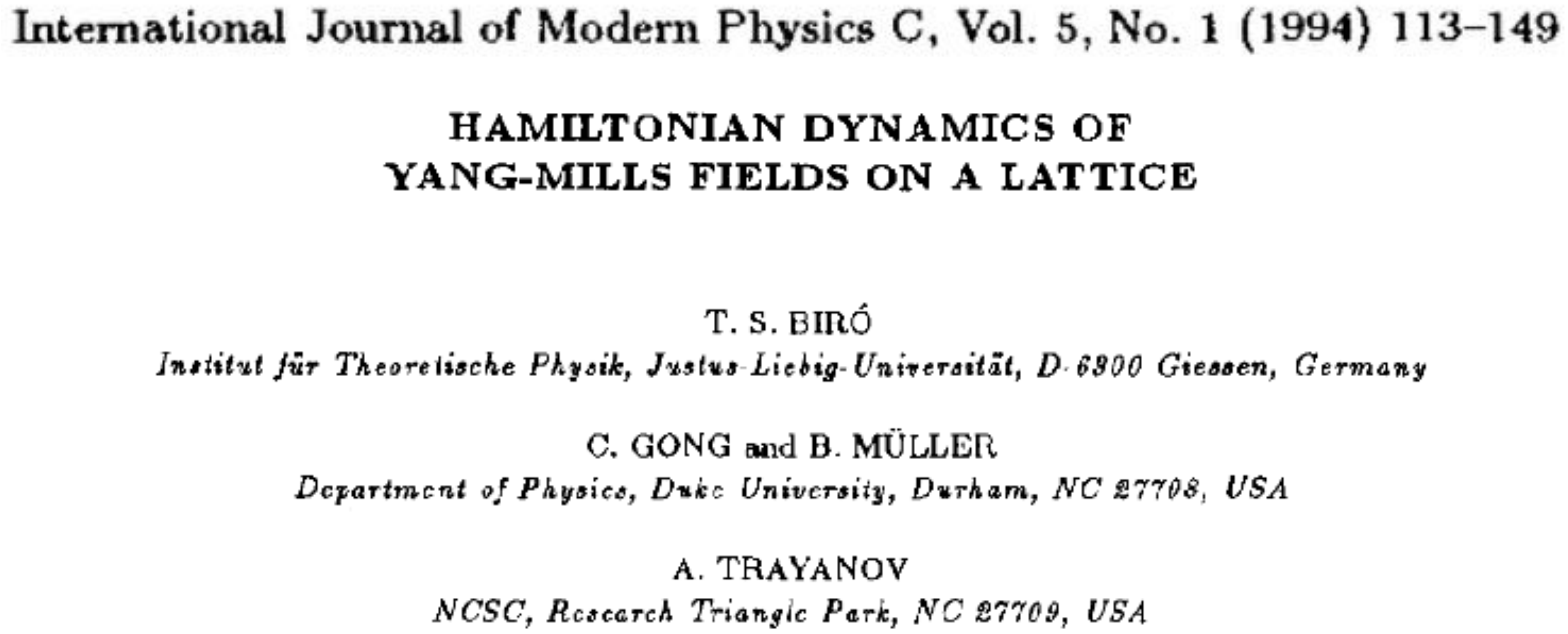

International Journal of Modern Physics C, Vol. 5, No. 1 (1994) 113–149

HAMILTONIAN DYNAMICS OF YANG-MILLS FIELDS ON A LATTICE

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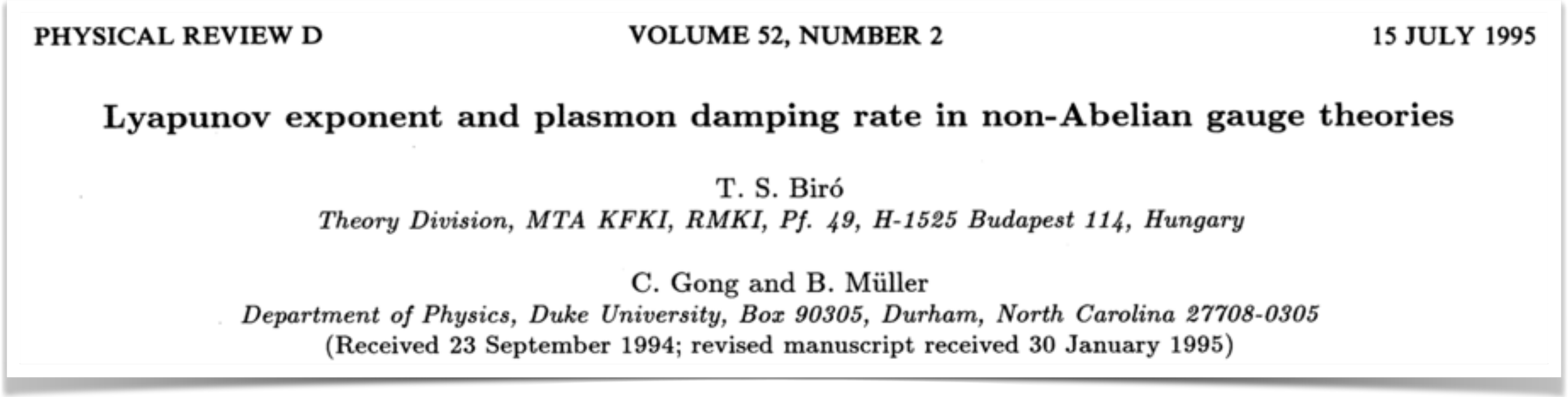

PHYSICAL REVIEW D VOLUME 52, NUMBER 2 15 JULY 1995

Lyapunov exponent and plasmon damping rate in non-Abelian gauge theories

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(Received 23 September 1994; revised manuscript received 30 January 1995)

Foundations of Physics Letters, Vol. 14, No. 5, October 2001 (©2001)

CHAOTIC QUANTIZATION OF CLASSICAL GAUGE FIELDS

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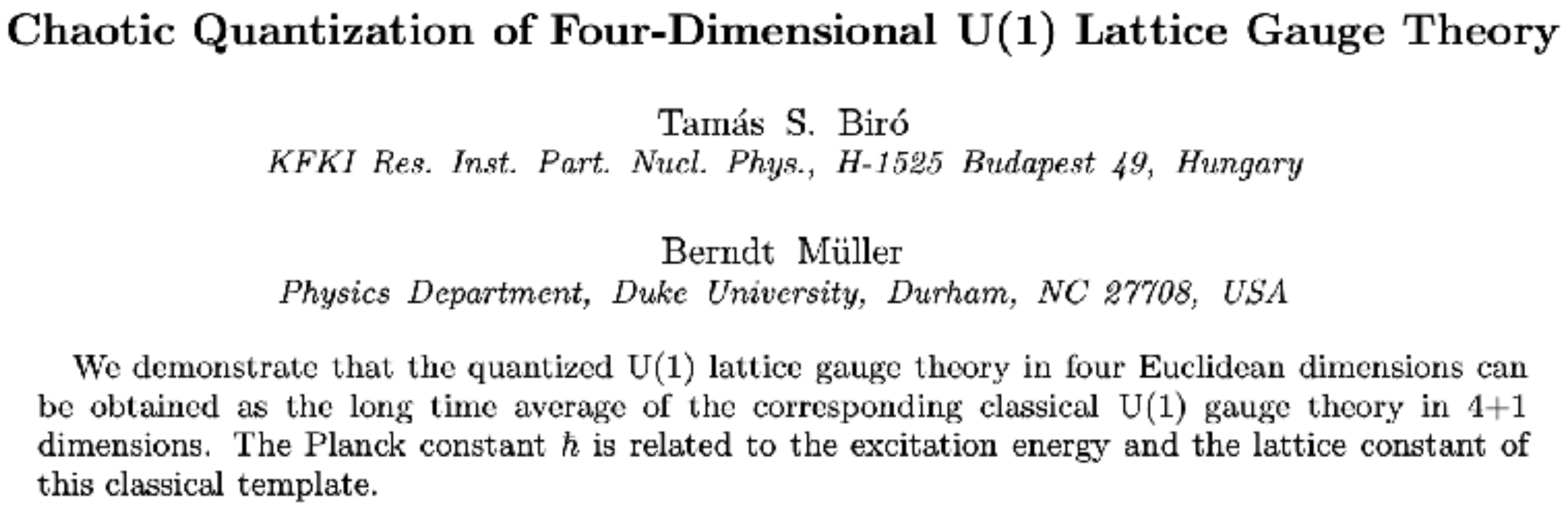
e-Print: hep-lat/0307028 [hep-lat]



Chaotic Quantization of Four-Dimensional U(1) Lattice Gauge Theory

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Berndt Müller
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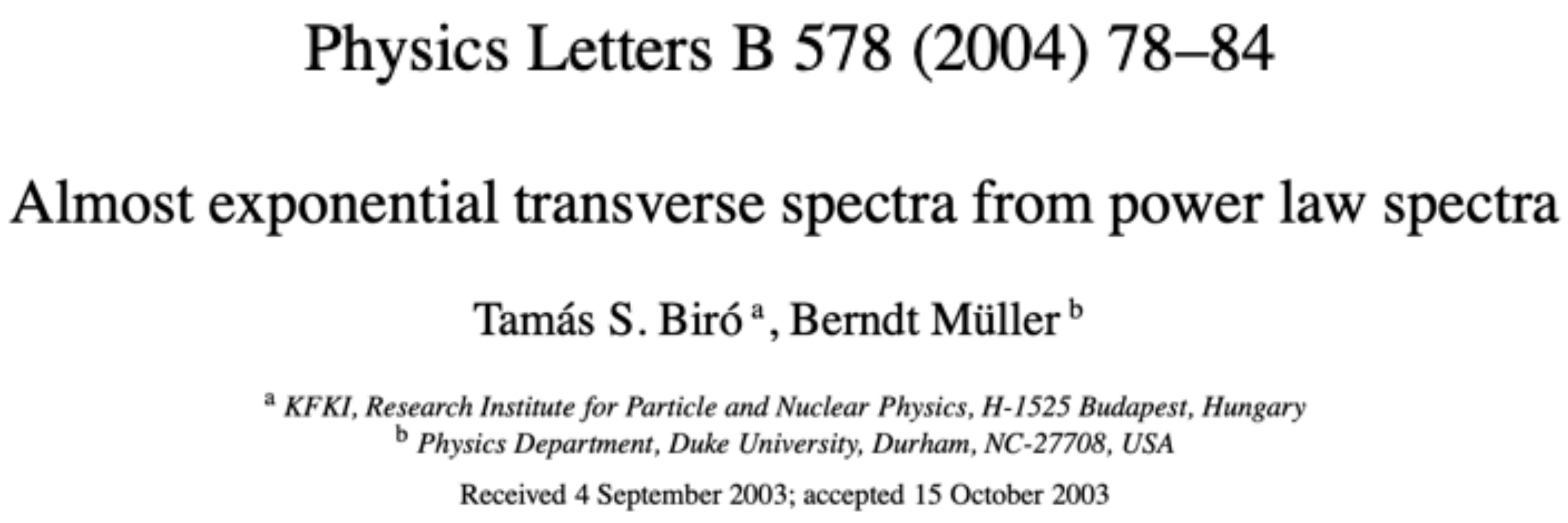
We demonstrate that the quantized U(1) lattice gauge theory in four Euclidean dimensions can be obtained as the long time average of the corresponding classical U(1) gauge theory in 4+1 dimensions. The Planck constant $\hbar$ is related to the excitation energy and the lattice constant of this classical template.


Physics Letters B 578 (2004) 78–84

Almost exponential transverse spectra from power law spectra

Tamás S. Biró [a], Berndt Müller [b]

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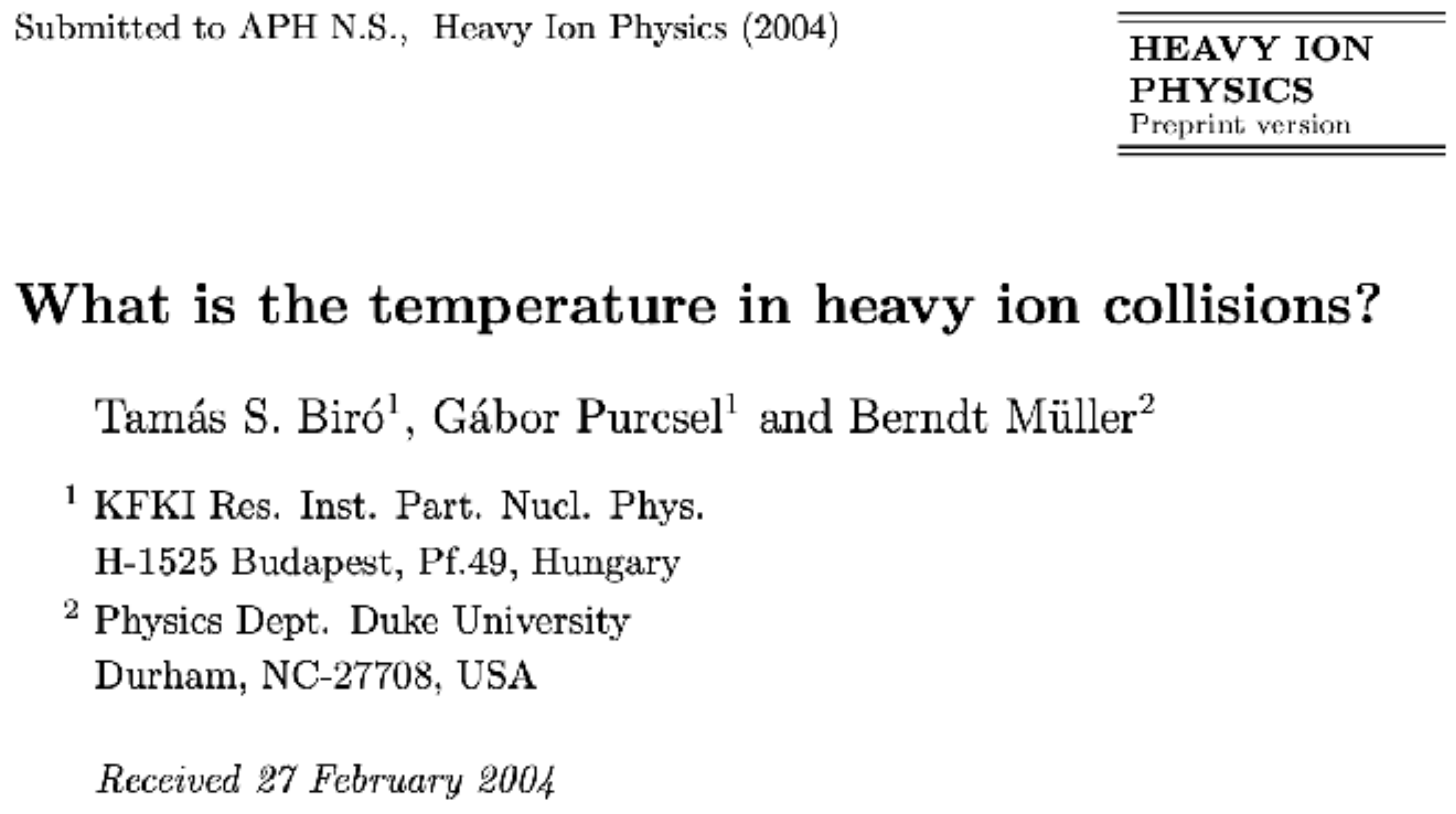
Submitted to APH N.S., Heavy Ion Physics (2004)

HEAVY ION PHYSICS
Preprint version

What is the temperature in heavy ion collisions?

Tamás S. Biró[1], Gábor Purcsel[1] and Berndt Müller[2]

[1] KFKI Res. Inst. Part. Nucl. Phys.
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Received 27 February 2004

Dear Tamás:

Thank you for decades of inspired and inspiring scientific collaboration.

I learned great deal from you that helped me understand physics much better.

You taught me how to make Palacsinta.

I wish you continued insights, discoveries and enjoyment in your “retirement”.

Happy seventieth birthday - and many even happier returns!

Berndt

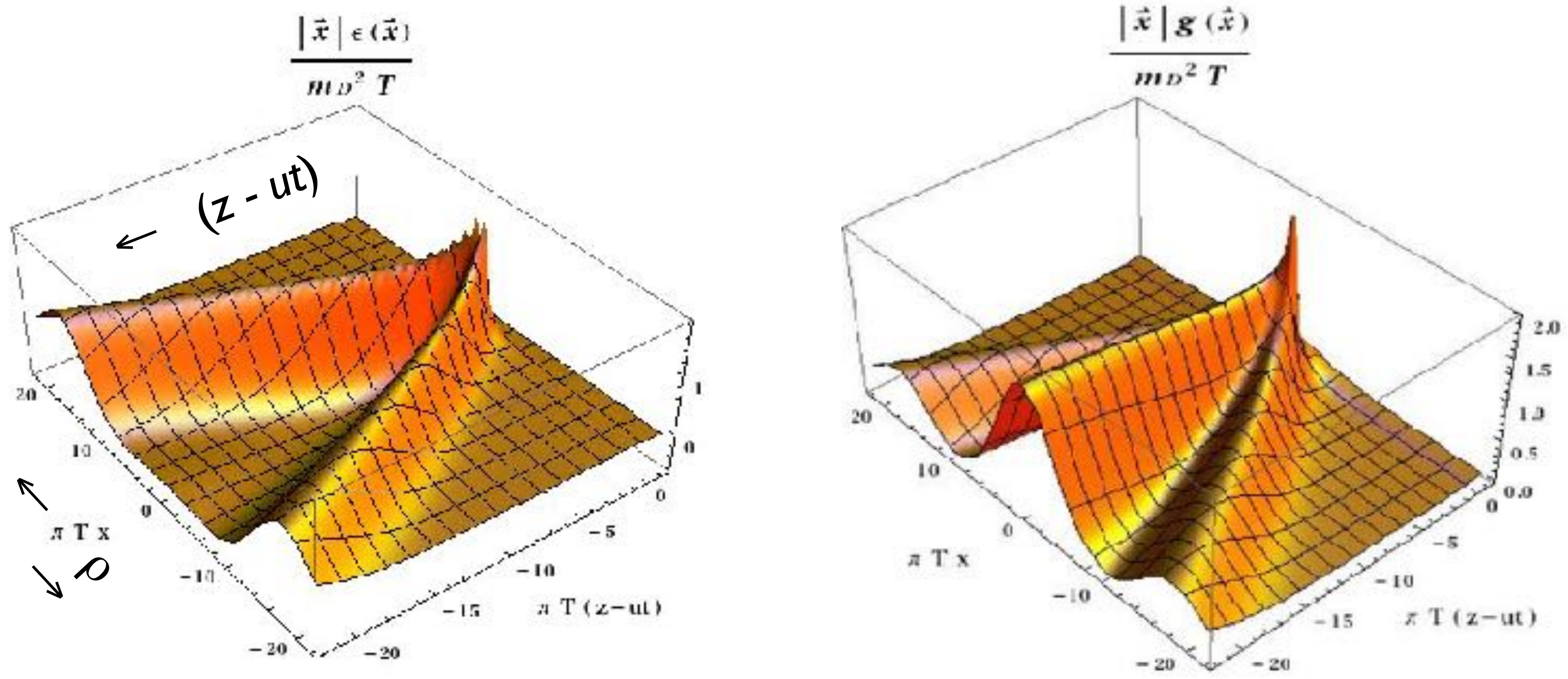
Mach cone & diffusion wake

An energetic (“heavy”) colored object moving through the QGP deposits energy-momentum, which creates a comoving perturbation.

The leading part is mostly phonons; the trailing part is diffusive, as low-frequency shear does not propagate ballistically in the QGP liquid.

Energy response

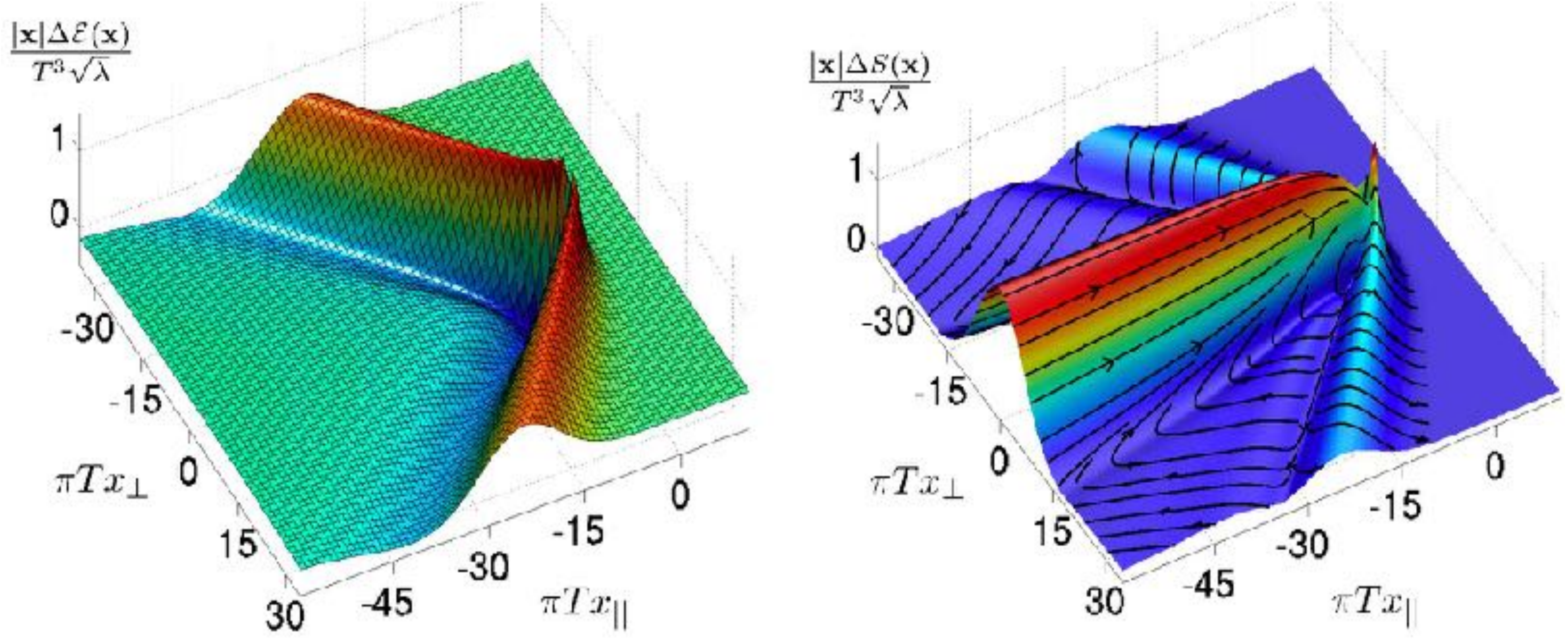
Momentum response



u = 0.99955 *c*

Neufeld et al.
arXiv:0802.2254

QCD—HTL



Chesler & Yaffe
arXiv:0712.0050

N=4 SYM

u = 0.75 *c*

Theoretical approaches

Diffusion and drag of heavy quarks:

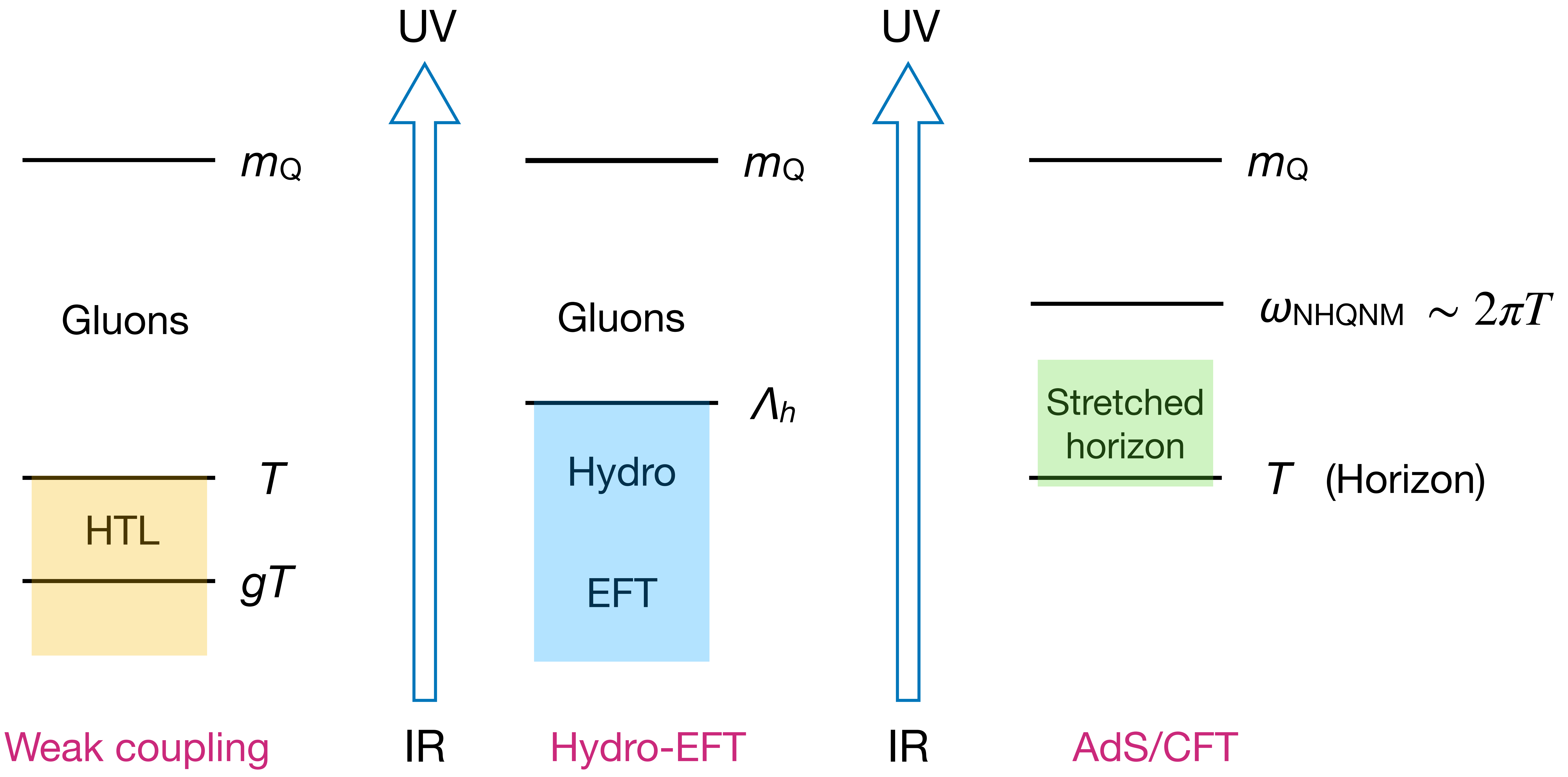
- HTL / pQCD [Svetitsky PRD 37 (1988) 2484; Caron-Huot & Moore, hep-ph/0801.2173]
- Lattice QCD [Petreczky & Teaney, hep-ph/0507318; HotQCD - 2311.01525, 2505.11958]
- T-matrix [Huggins & Rapp - 1206.6537; Tang, Mukherjee, Petreczky & Rapp - 2310.18864]
- AdS/CFT [Herzog et al. - hep-th/0605158; Casalderrey-Solana & Teaney - hep-ph/ 0605199; Vazquez-Poritz - 0803.2890]

Transport formalisms:

- Langevin, Fokker-Planck, Linearized Boltzmann, Hybrid approaches

New approach: Treat QGP as a fluid and couple it to heavy quark probes

Relevant scales



Hydro EFT

[Endlich et al. - 1011.6396; Dubovsky et al. - 1107.0731]

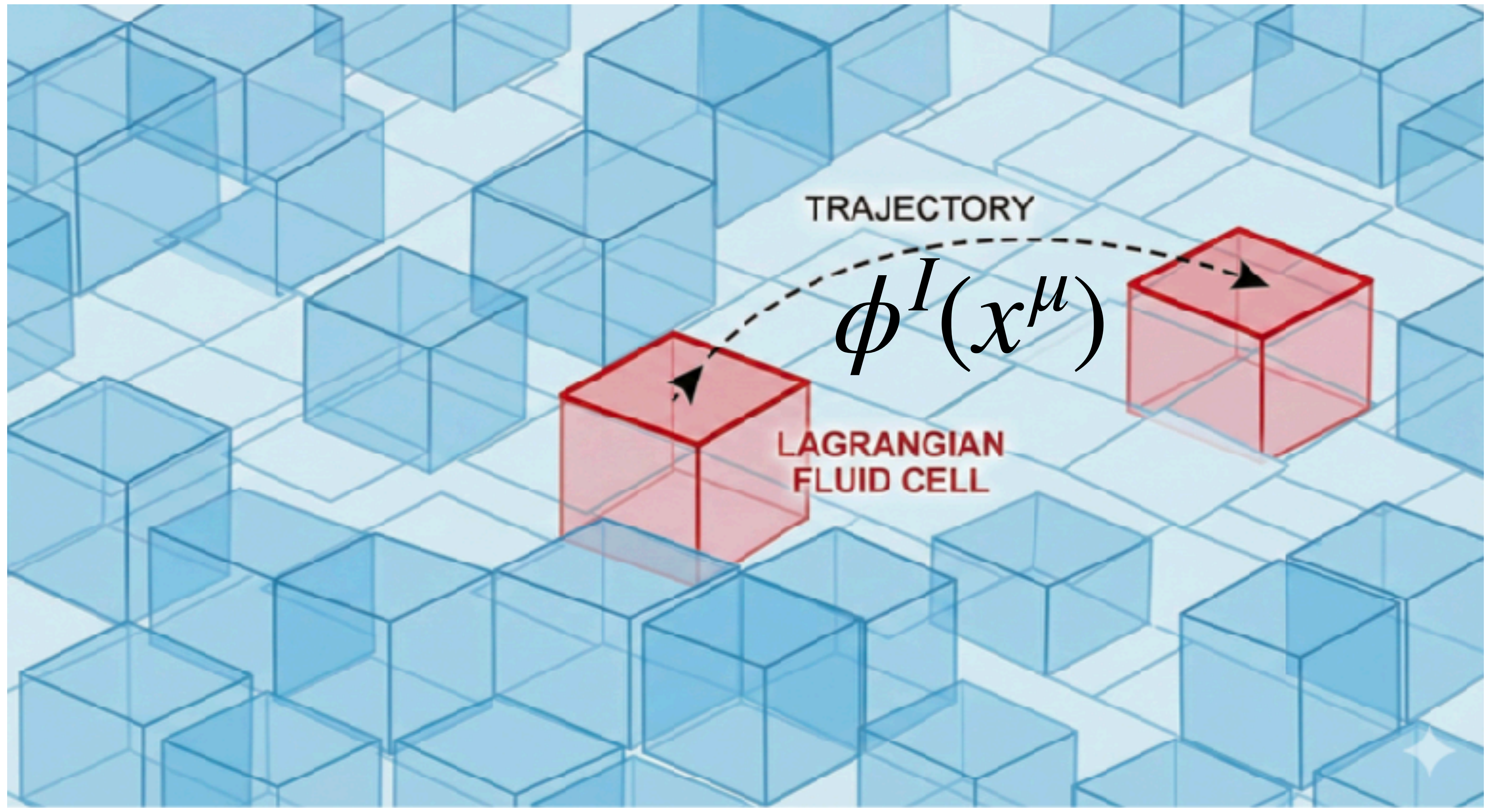
Fluid cells are described by comoving labels ϕ^I.

Fluid field $\phi^I(x^\mu)$ signifies which fluid cell is present at space-time point $x^\mu = (\vec{x}, t)$.

Fluid at rest everywhere: $\phi^I(x^\mu) = x^I$ for all *t.*

Ideal fluid: Action must be invariant under

- Translations: $\phi^I \to \phi^I + a^I$
- Rotations: $\phi^I \to R^I_K \phi^K$
- Volume preserving diffeomorphisms:
  $\phi^I \to \xi^I(\phi) \quad [\det(\partial \xi^I / \partial \phi^K) = 1].$
  (fluid can assume any shape!)



Simplest invariant: $X = J^\mu J_\mu$ with

$$J^\mu = \frac{1}{3!}\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{IJK}\partial_\rho\phi^J\partial_\nu\phi^K\partial_\sigma\phi^I$$

Ideal fluid Lagrangian: $\mathscr{L} = F(X)$

Properties of the Lagrangian

Relation to standard fluid properties:

$$\varepsilon = -F(X), \quad p = F(X) - 2XF'(X), \quad \varepsilon + p = -2XF'(X)\,.$$

Equation of state:

Conformal fluid: $F(X) \propto X^{2/3}$

Constant speed of sound: $F(X) \propto X^{(1+c_s^2)/2}$

Equilibrium state (fluid at rest): $\phi^I(x^\mu) = x^I \;\rightarrow\; X = X_0 = 1$

Goldstone's theorem: fluctuation modes are derivatively coupled.

For later: $\dfrac{\varepsilon_0 + p_0}{2} = -F'(1)$ $\quad c_s^2 = \left(\dfrac{\partial p}{\partial \varepsilon}\right)_0 = \dfrac{F'(1) + 2F''(1)}{F'(1)}$

Fluctuations

Small fluctuations: $\phi^I = x^I + \lambda \pi^I(t, \vec{x}) \quad [\lambda \ll 1]$ are governed by

$$\mathscr{L}[\pi^I] = F^{\mu\nu}_{IJ} \partial_\mu \pi^I \partial_\nu \pi^J + F^{\mu\nu\rho}_{IJK} \partial_\mu \pi^I \partial_\nu \pi^J \partial_\rho \pi^K + \mathcal{O}((\partial\pi)^4)$$

Quadratic part:

$$\mathscr{L}^{(2)}[\vec{\pi}] = -F'(1) \left[\frac{1}{2} (\partial_t \vec{\pi})^2 - \frac{1}{2} \frac{F'(1) + 2F''(1)}{F'(1)} (\nabla \cdot \vec{\pi})^2 \right]$$

Field redefinition: $\vec{\pi}' = \Lambda_h^2 \vec{\pi} \equiv \sqrt{-F'(1)}\, \vec{\pi} = \sqrt{(\varepsilon_0 + p_0)/2}\, \vec{\pi}$ $\qquad \mathrm{dim}[\pi'] = 1$

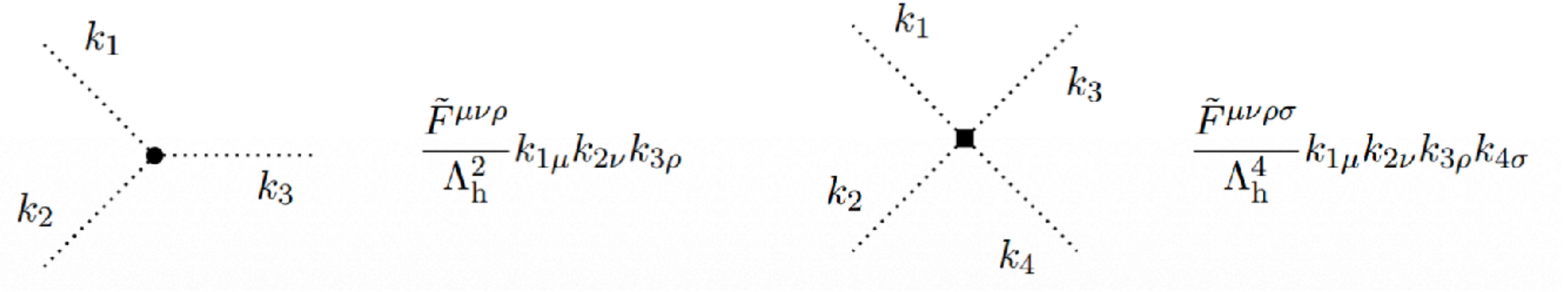
Canonical Lagrangian: $\mathscr{L}^{(2)}[\vec{\pi}'] = \frac{1}{2} (\partial_t \vec{\pi}')^2 - \frac{1}{2} c_s^2 (\nabla \cdot \vec{\pi}')^2$

Only longitudinal modes (phonons) are dynamical — transverse modes (shear) decouple

Phonon interactions

Fluid Lagrangian $\mathscr{L} = -F(J^\mu J_\mu)$ contains infinitely many elementary *N*-phonon vertices.

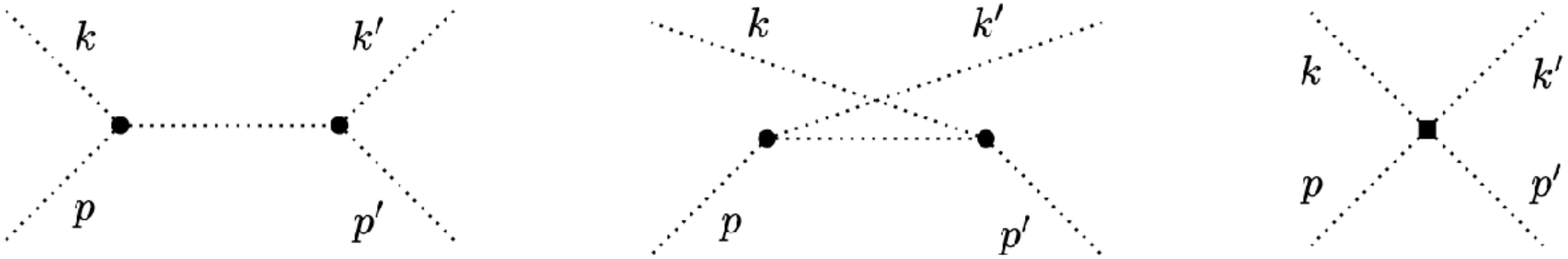
Field redefinition introduces scale Λ_h into interaction terms:



$$\frac{\tilde{F}^{\mu\nu\rho}}{\Lambda_{\rm h}^2} k_{1\mu} k_{2\nu} k_{3\rho} \qquad \frac{\tilde{F}^{\mu\nu\rho\sigma}}{\Lambda_{\rm h}^4} k_{1\mu} k_{2\nu} k_{3\rho} k_{4\sigma}$$

…….

Higher vertices are suppressed by increasing powers of Λ_h^{-2}. Phonon-phonon scattering:

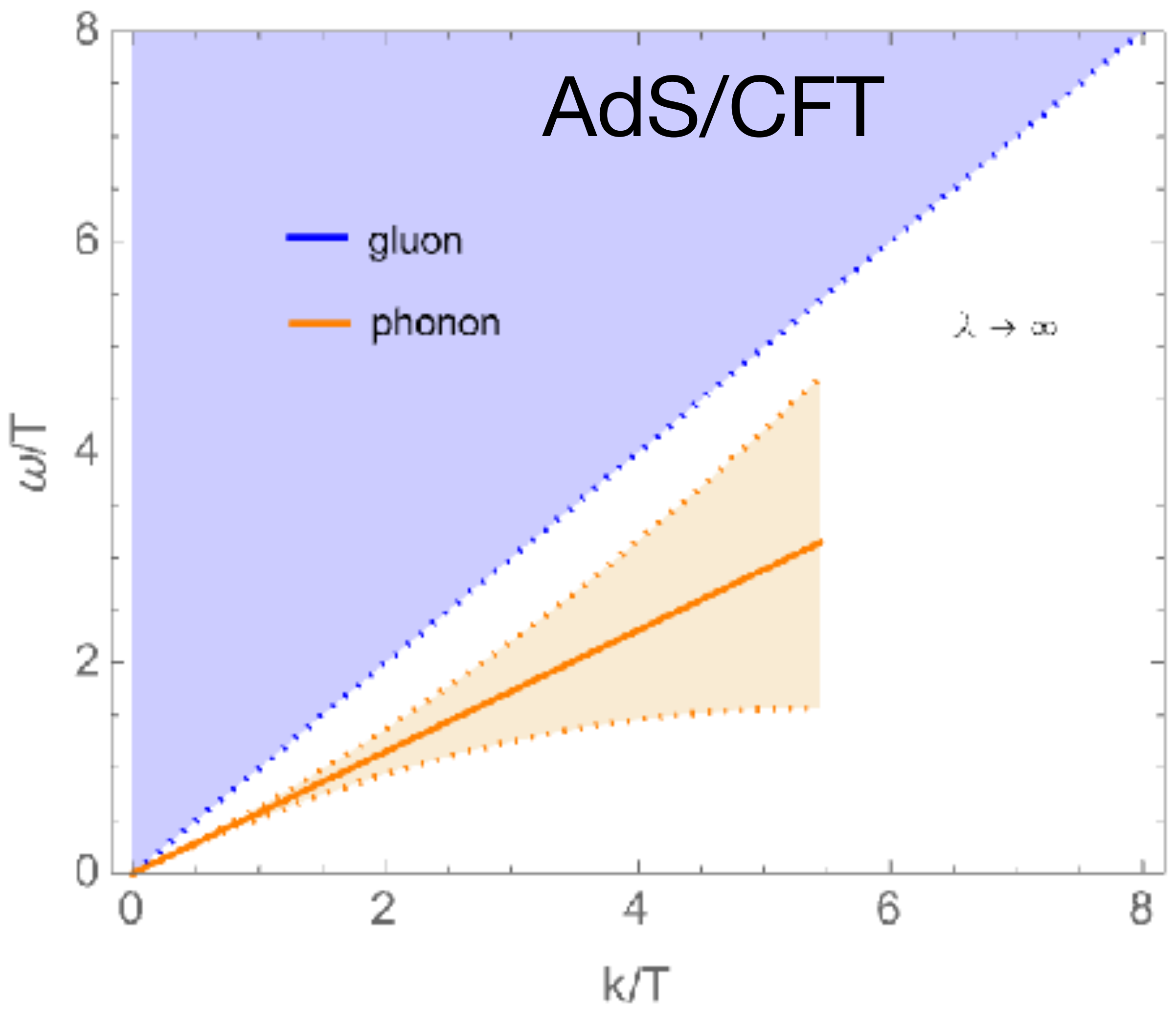


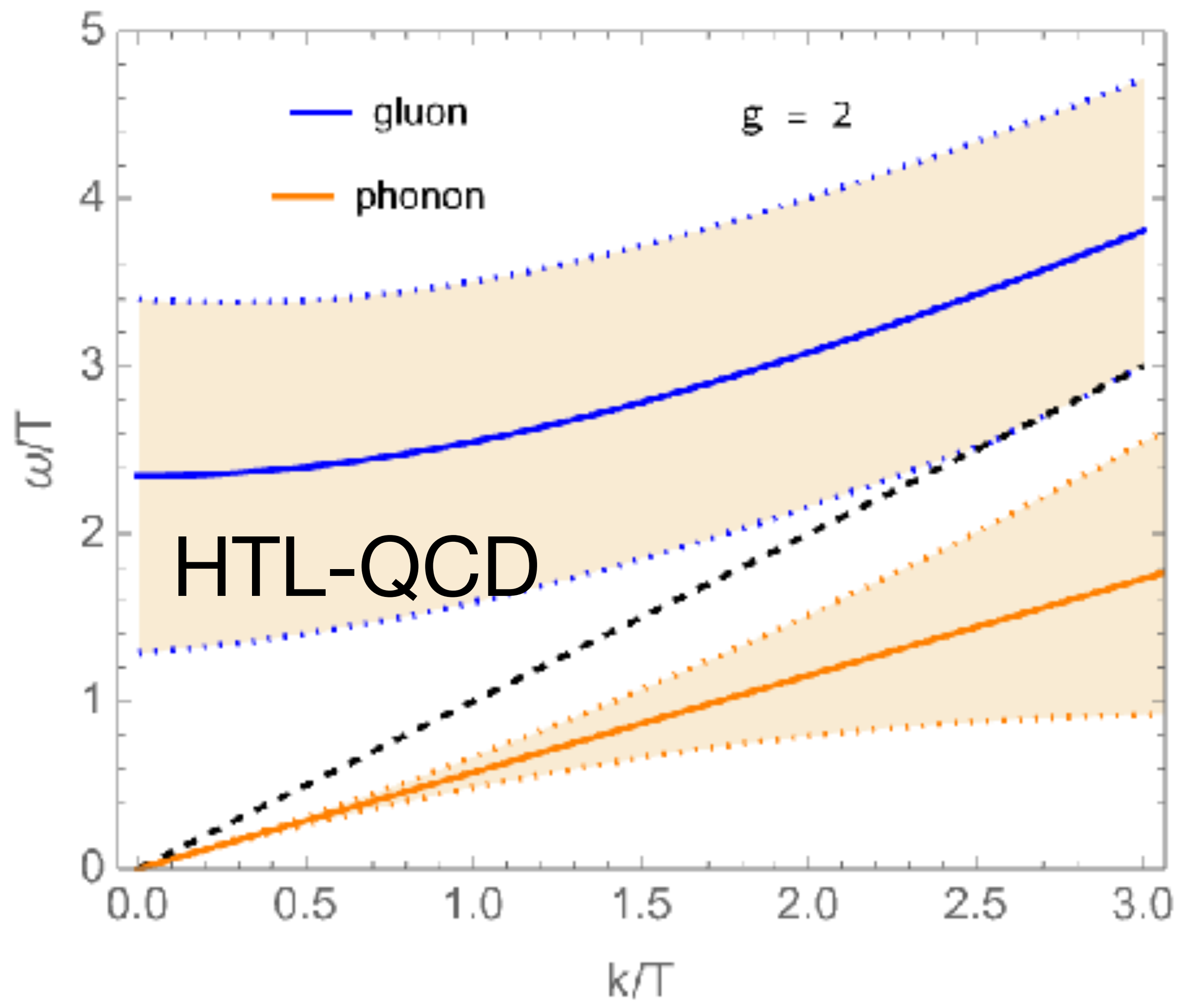
Scale setting

Rescaling definition: $\Lambda_h = \left(\frac{s_0 T}{2}\right)^{1/4} \approx 1.8\,T$ (QCD) $\Lambda_h \approx 1.25\,N_c^{1/2}\,T$ (AdS/CFT)

Kinematic considerations:

- Sound attenuation length (pQCD at $\alpha_s \approx 0.3$): $k \ll \frac{3c_s T}{2\eta/s} \approx 5\,T$
- Sound attenuation length (AdS/CFT at $\lambda \to \infty$): $k \ll 2\sqrt{3}\pi T \approx 11\,T$
- First non-hydro quasinormal mode: $k \ll \omega_1/c_s \approx 17\,T$





Conservative estimates:

$$k_{\max} \approx 3\,T \quad \text{(QCD)}$$

$$k_{\max} \approx 5\,T \quad \text{(AdS/CFT)}$$

Quark-Hydro-EFT (QHEFT)

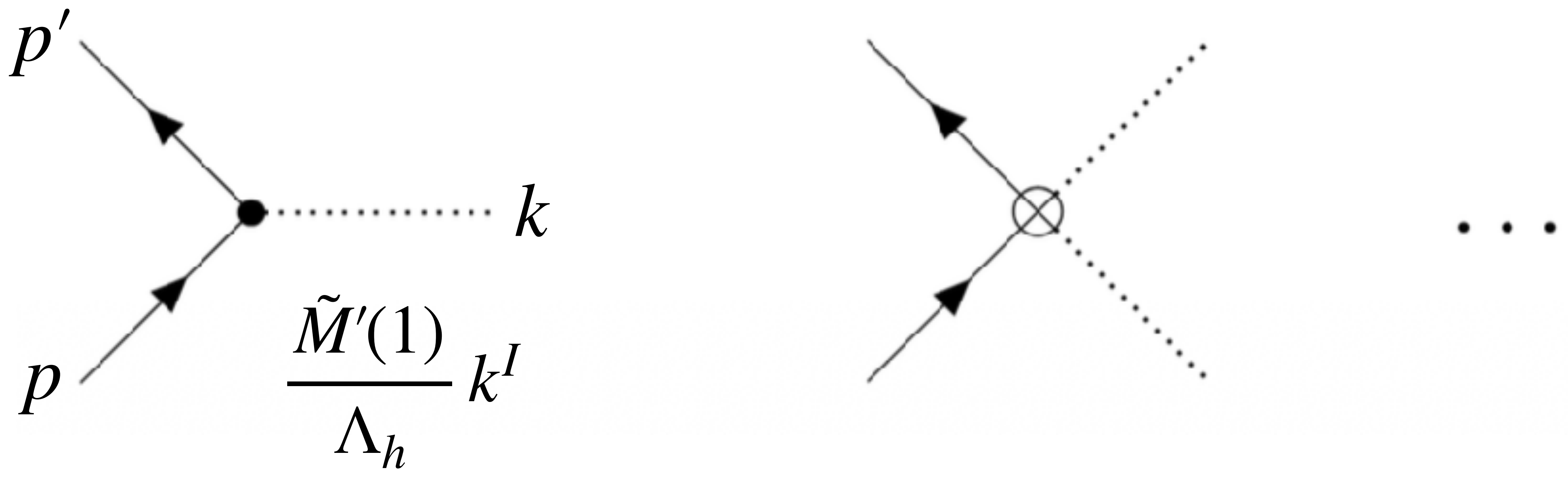
Kirchner, BM, Roy, Sirimanna - 2510.13942

Adding a quark field - Lagrangian up to dimension 4:

$$\mathcal{L} = F(X) + Z_\psi(X)\bar{\psi}i\not{D}\psi - M(X)\bar{\psi}\psi + Z_A(X)G^a_{\mu\nu}G_a^{\mu\nu}$$
$$+ P(X)\bar{\psi}\gamma_\mu\psi J^\mu + Q(X)\bar{\psi}i\overleftrightarrow{D}_\mu\psi J^\mu + R(X)\bar{\psi}\sigma_{\mu\nu}\psi\omega^{\mu\nu}$$

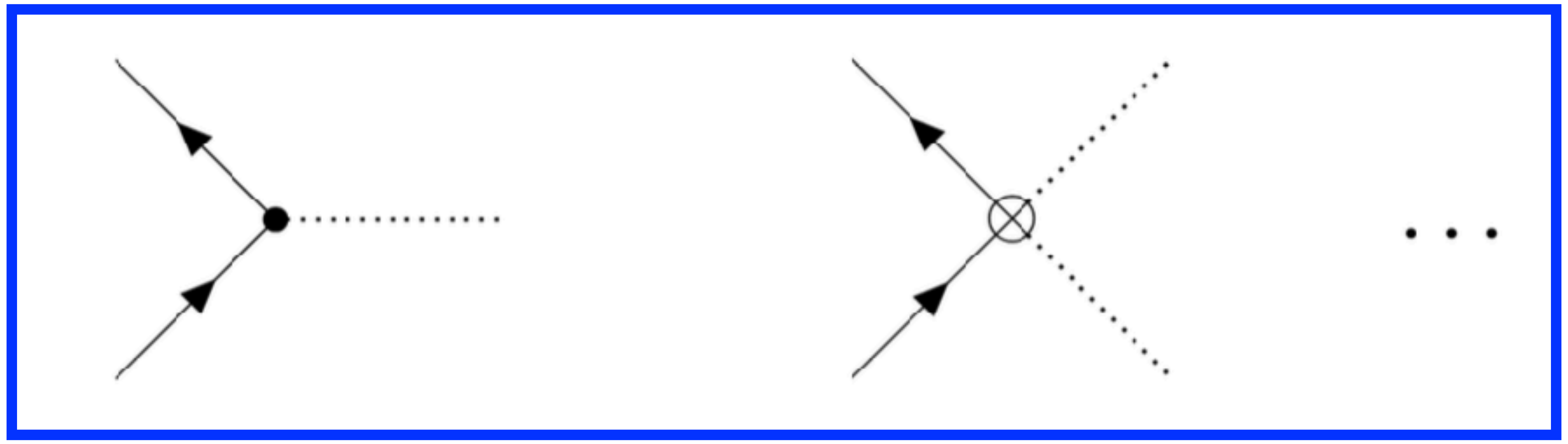
Consider the leading (dimension 3) term: $M(X)\bar{\psi}\psi$, which describes medium modifications to the quark mass. Expand in powers of phonon fields to obtain quark-phonon vertices:

$$M(X)\bar{\psi}\psi = \bar{\psi}\psi\left[M(1) + \frac{\tilde{M}^\mu_I}{\Lambda_h}\partial_\mu\pi^I + \frac{\tilde{M}^{\mu\nu}_{IJ}}{\Lambda_h^3}\partial_\mu\pi^I\partial_\nu\pi^J + \mathcal{O}\left(\frac{1}{\Lambda_h^5}\right)\right]$$

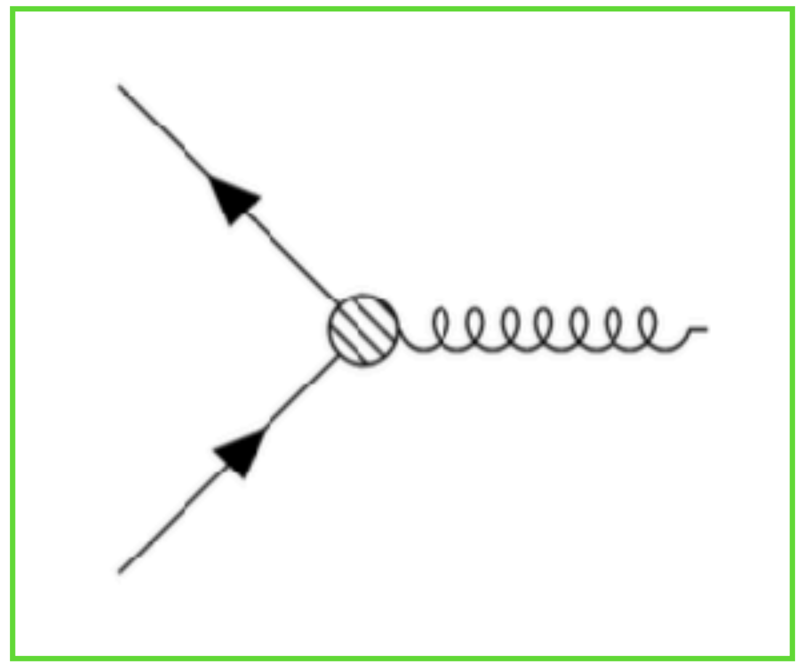


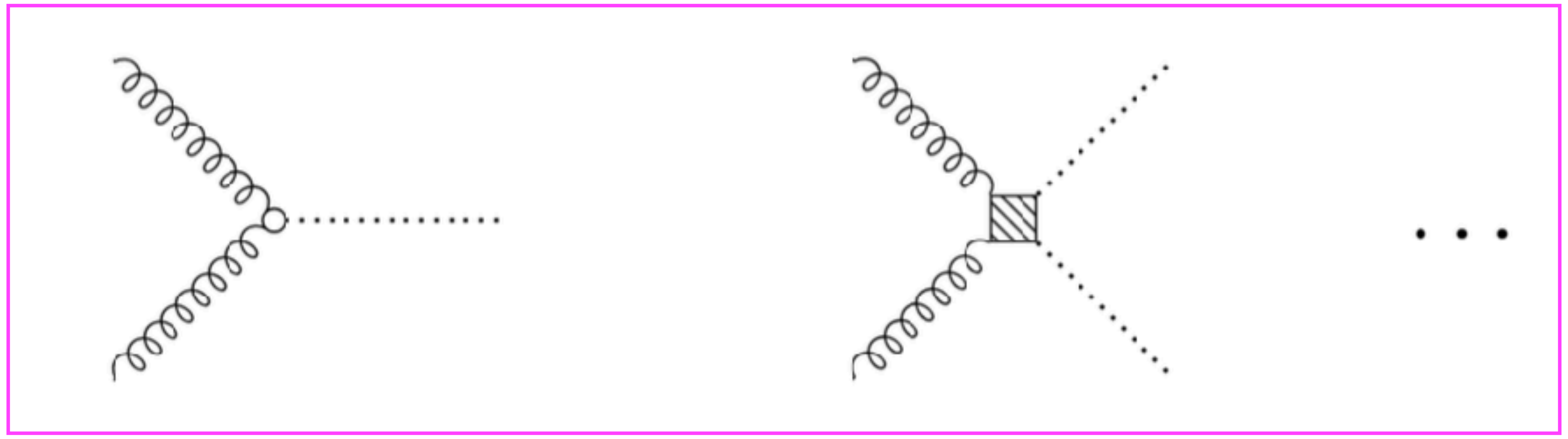
In-medium interactions

Quark-phonon int’s

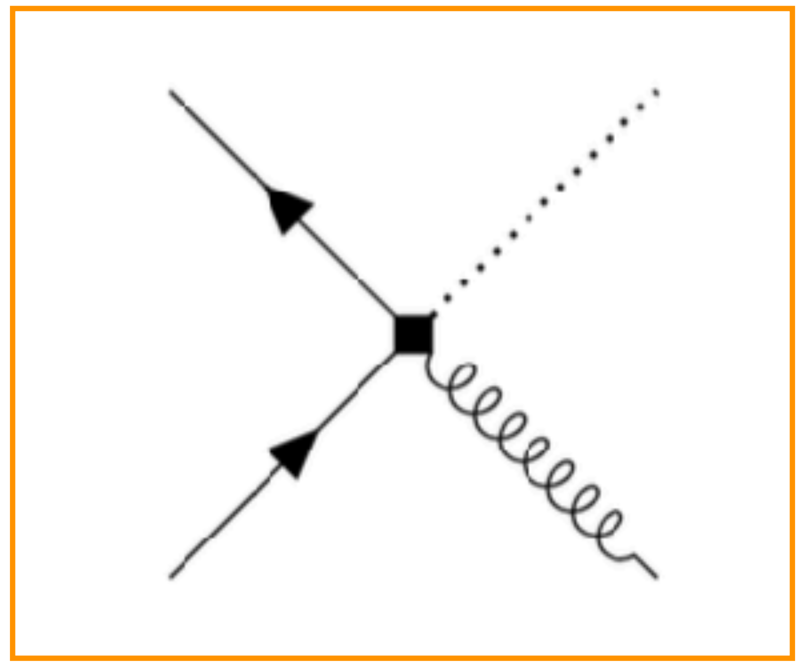


Quark-gluon int

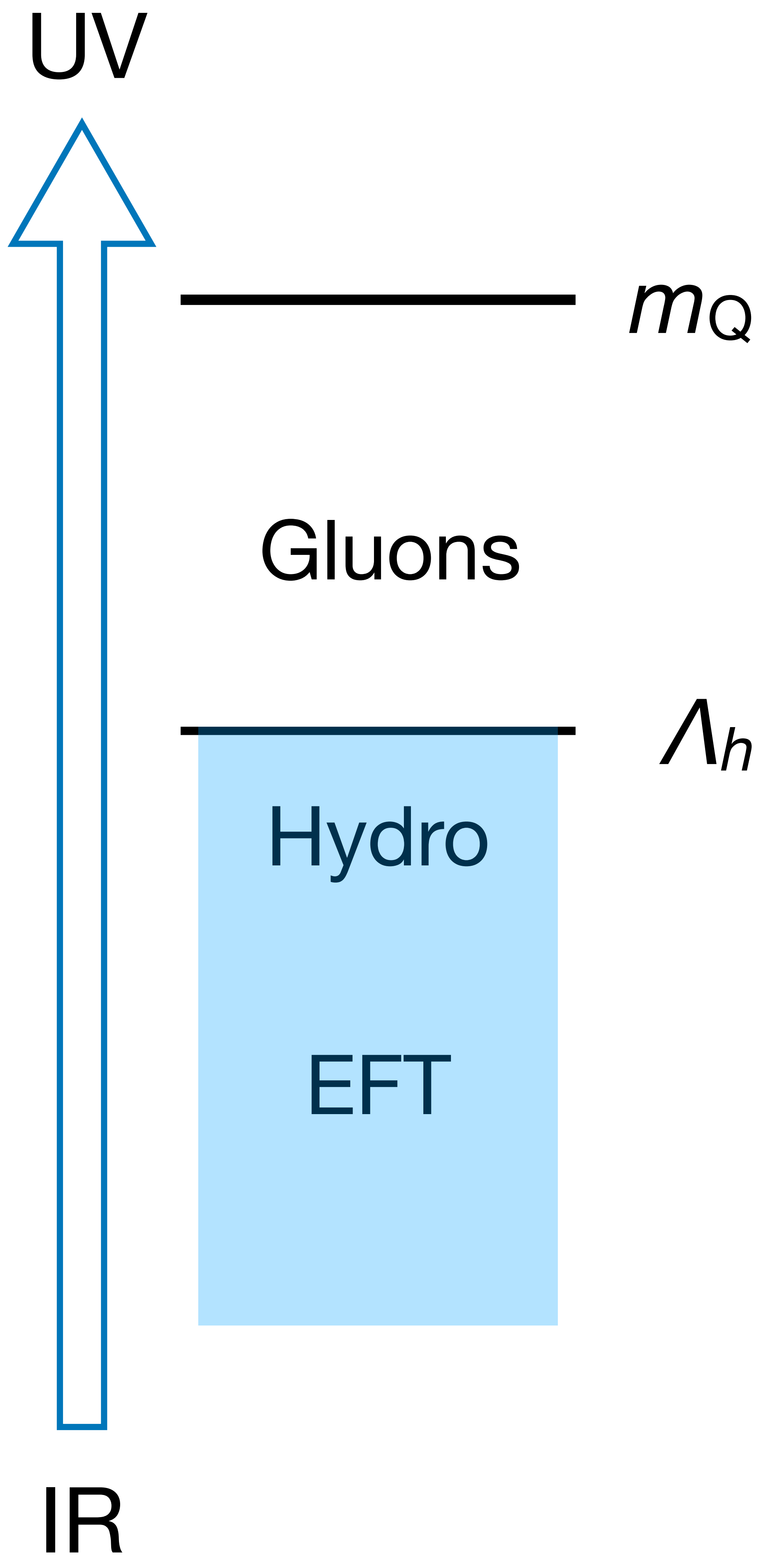




Gluon-phonon int’s



Gluon-phonon Compton



UV

m_Q

Gluons

Λ_h

Hydro

EFT

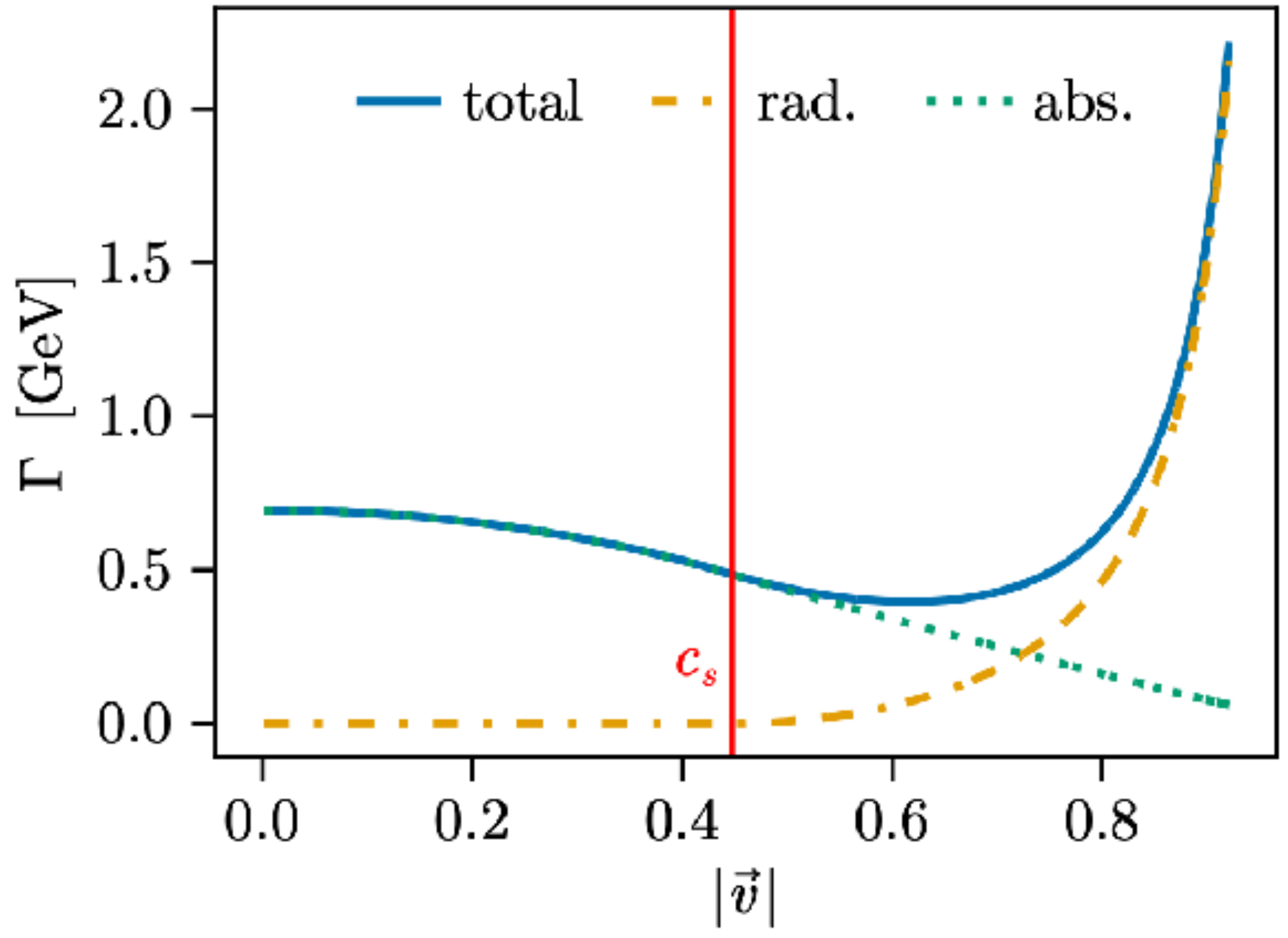
IR

Quark-phonon scattering

Leading terms occur at order Λ_h^{-2}

[Note:4-point interaction $O(\Lambda_h^{-3})$]

Because the phonon is a space-like mode ($\omega < |k|$), phonons can also be emitted or absorbed by a quark:

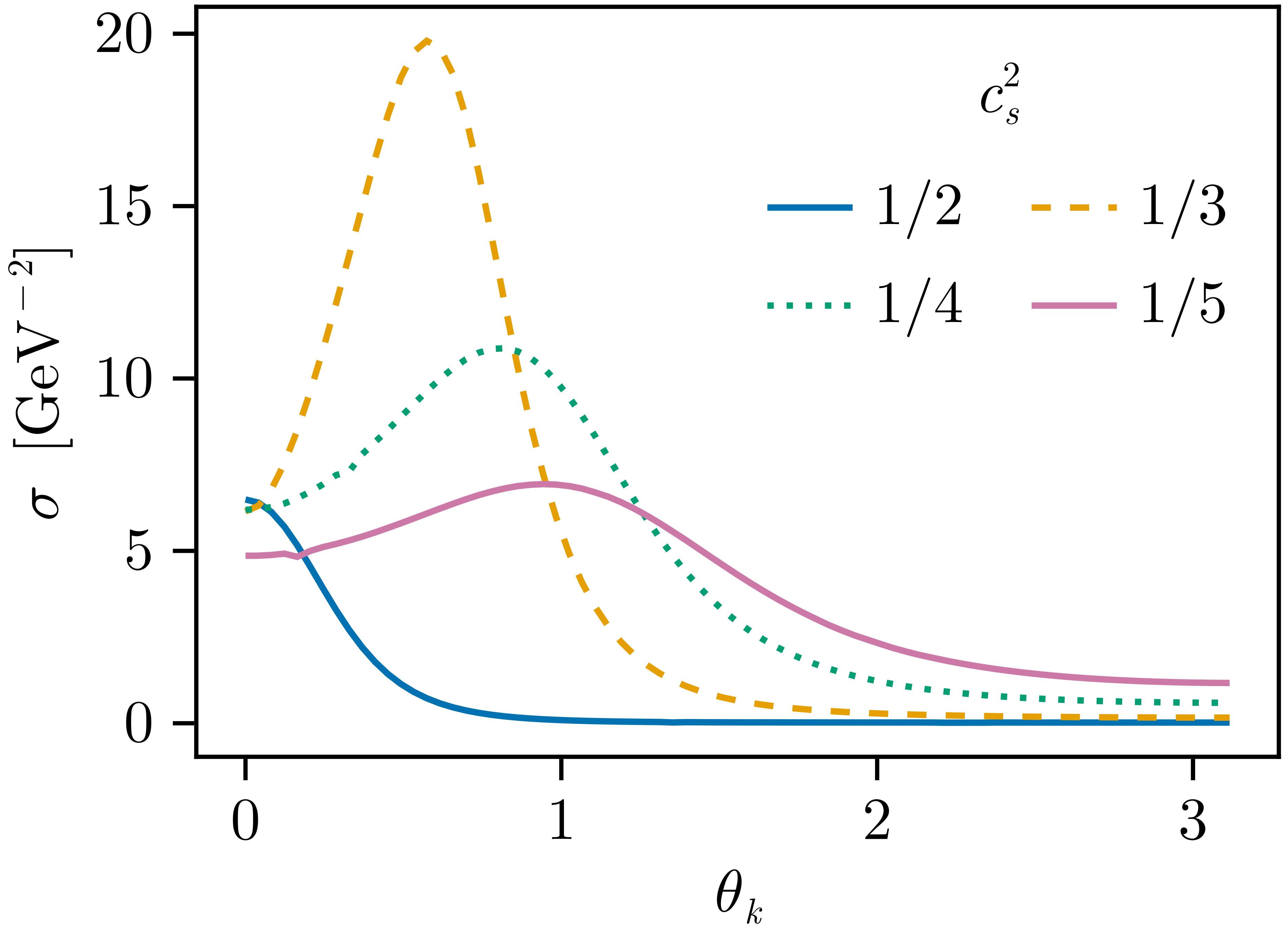


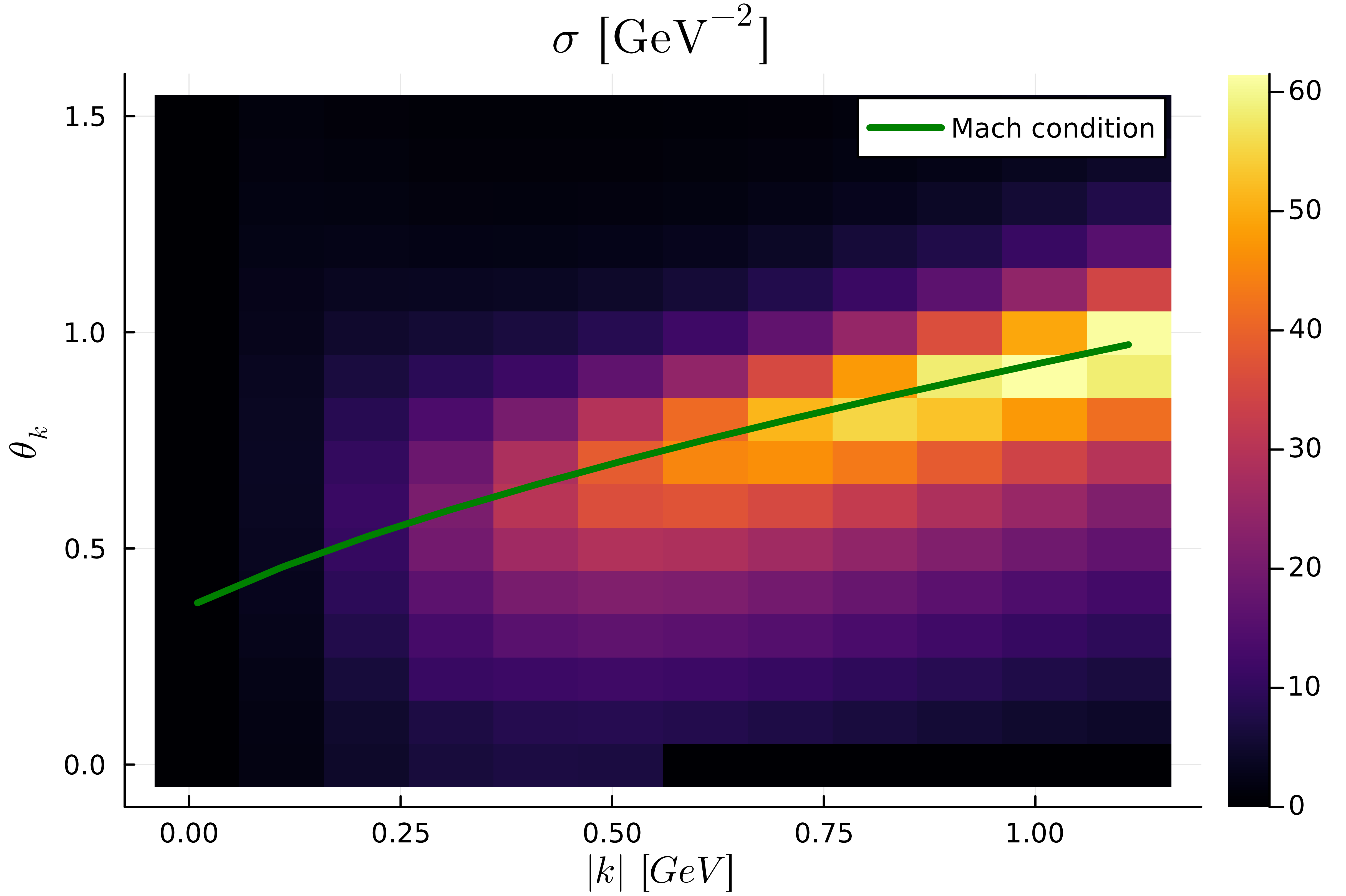
Heavy quarks inside the fluid acquire a quasiparticle width Γ: A quark can always absorb a phonon, but the emission rate of a phonon by a moving quark (dashed red line) vanishes for subsonic motion:

$$\Gamma_{\text{decay}}(\text{v}) = \gamma(\text{v})\theta(|\text{v}| - c_s)$$

Quark-phonon scattering

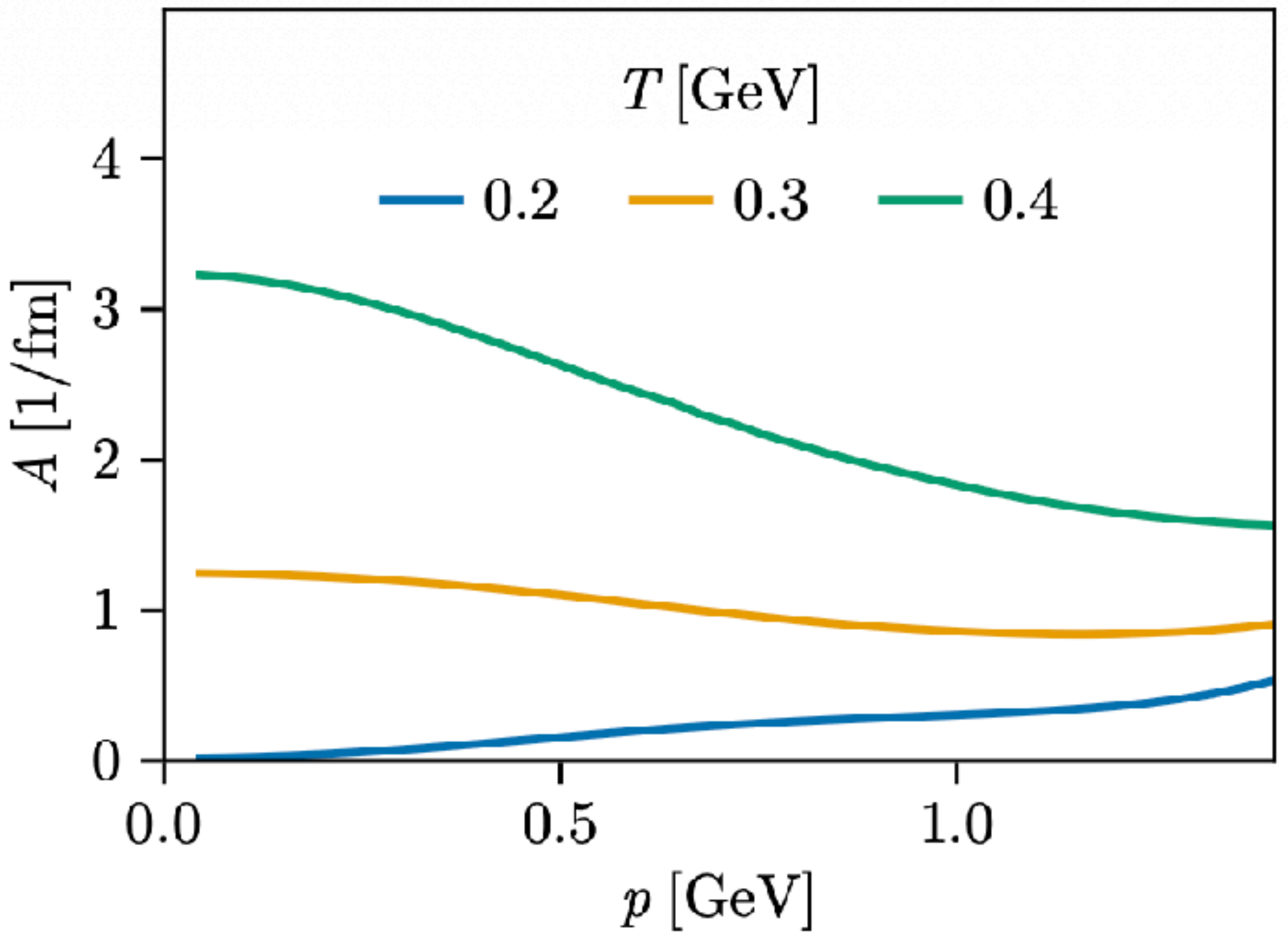
- Quark in-medium width needs to be included in the quark propagator
- Phonon emission appears as Bose-enhanced phonon scattering
- Resonance position depends on c_s
- $\sigma(\theta = 0)$ is finite; $\sigma(\theta = \pi)$ is small

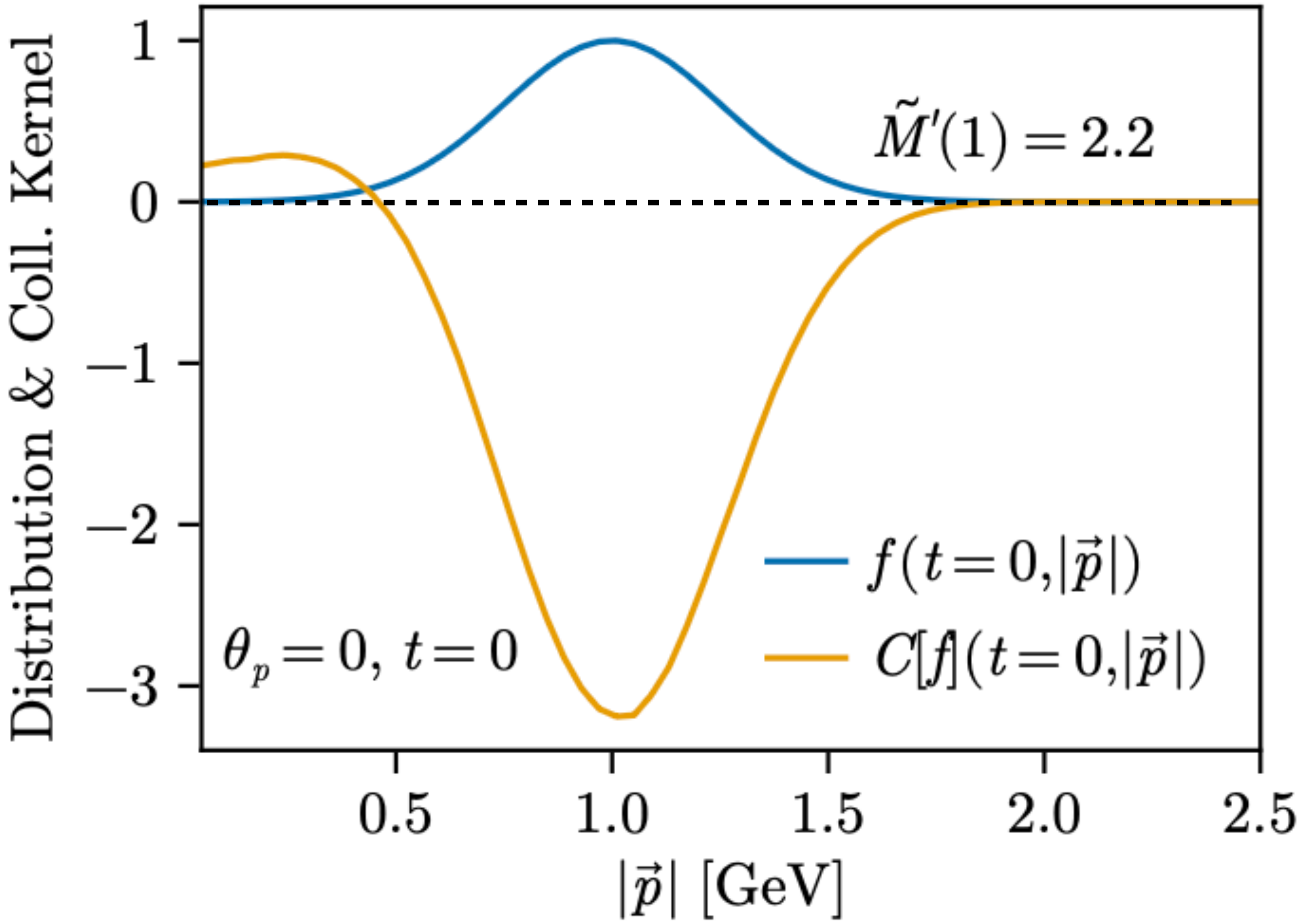




Drag coefficient & collision kernel

A. Kirchner & N. Koliadko (in progress)





Drag coefficient $A(p)$ in Langevin equation

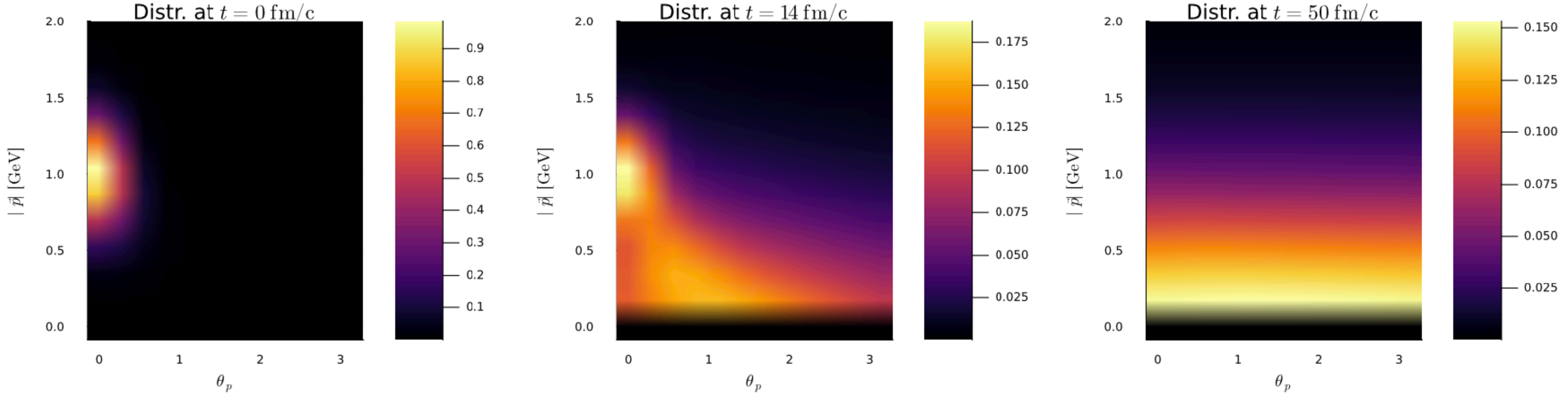
$$\frac{d\vec{p}}{dt} = -A(p)\vec{p} + \vec{\xi}$$

Collision kernel $C(p)$ of Boltzmann equation in thermal medium

$$\partial_t f(\vec{p}, t) = C[f, T](\vec{p}, t)$$

Thermalization

A. Kirchner & N. Koliadko (in progress)



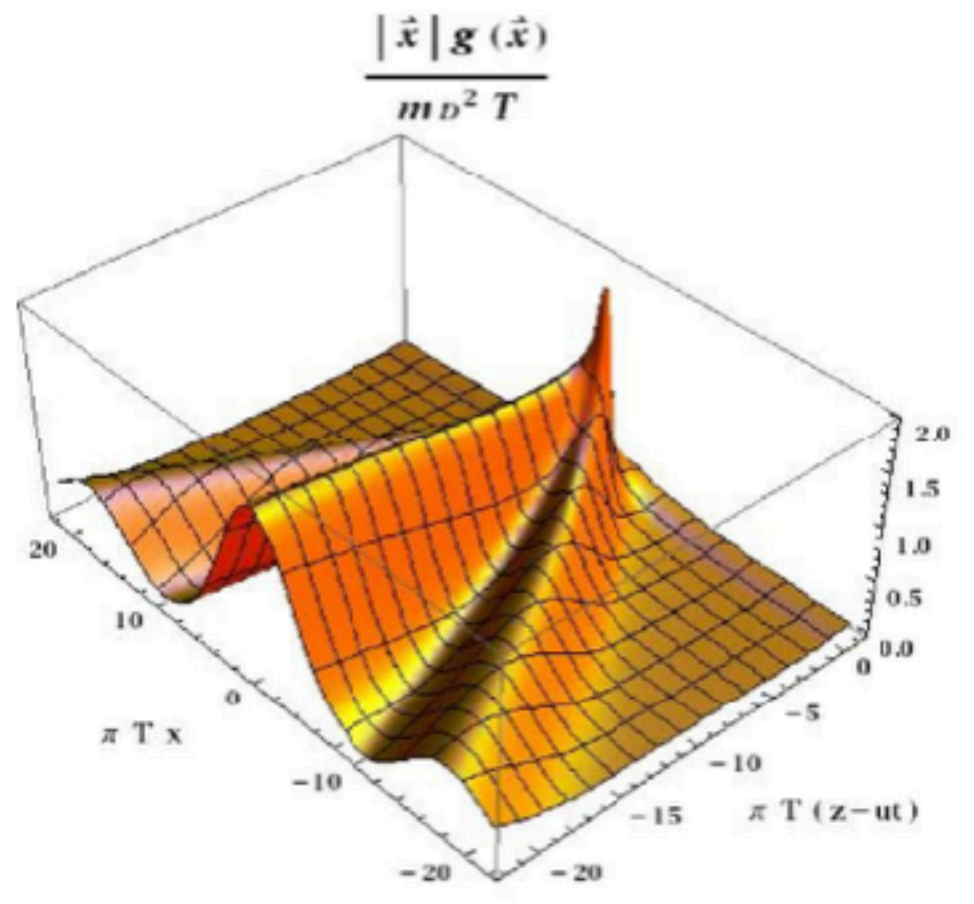
- Initial distribution centered around $\vec{p} = (0,0,1)$ GeV/c; medium at rest
- Only thermal phonon-quark scattering
- Distribution isotropies and thermalizes due to interactions with the medium
- Final distribution is isotropic (quark loses directional memory)

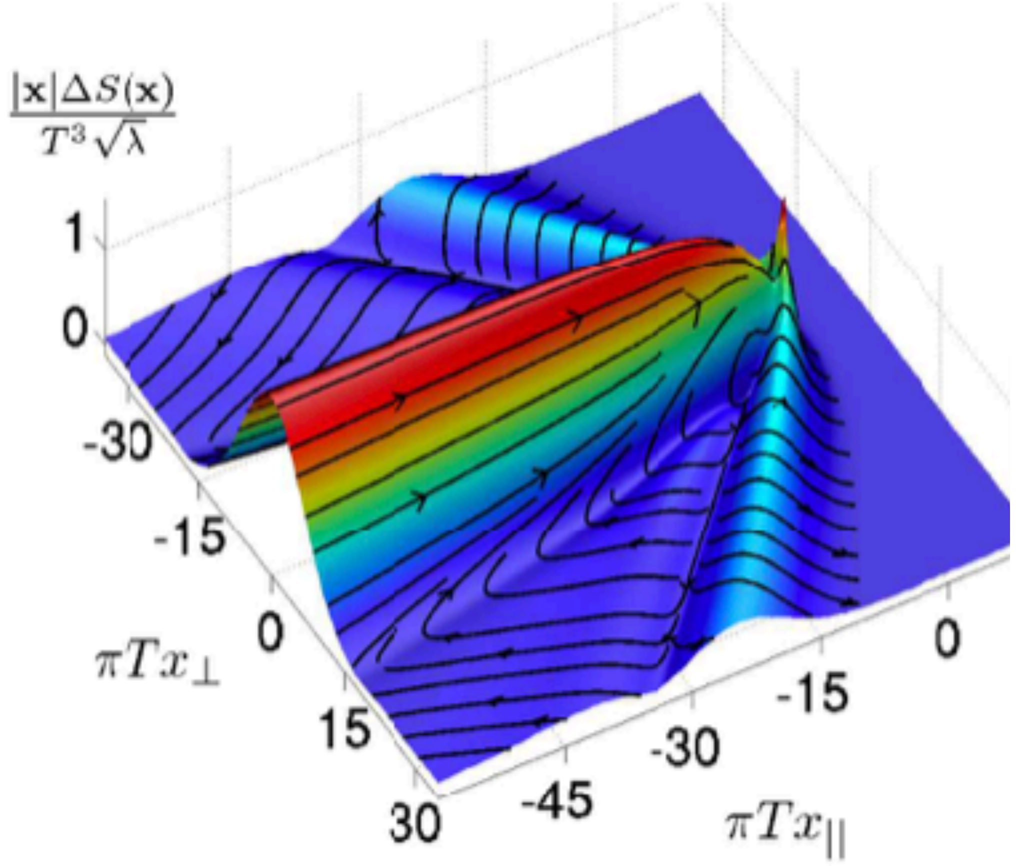
Diffusion wake

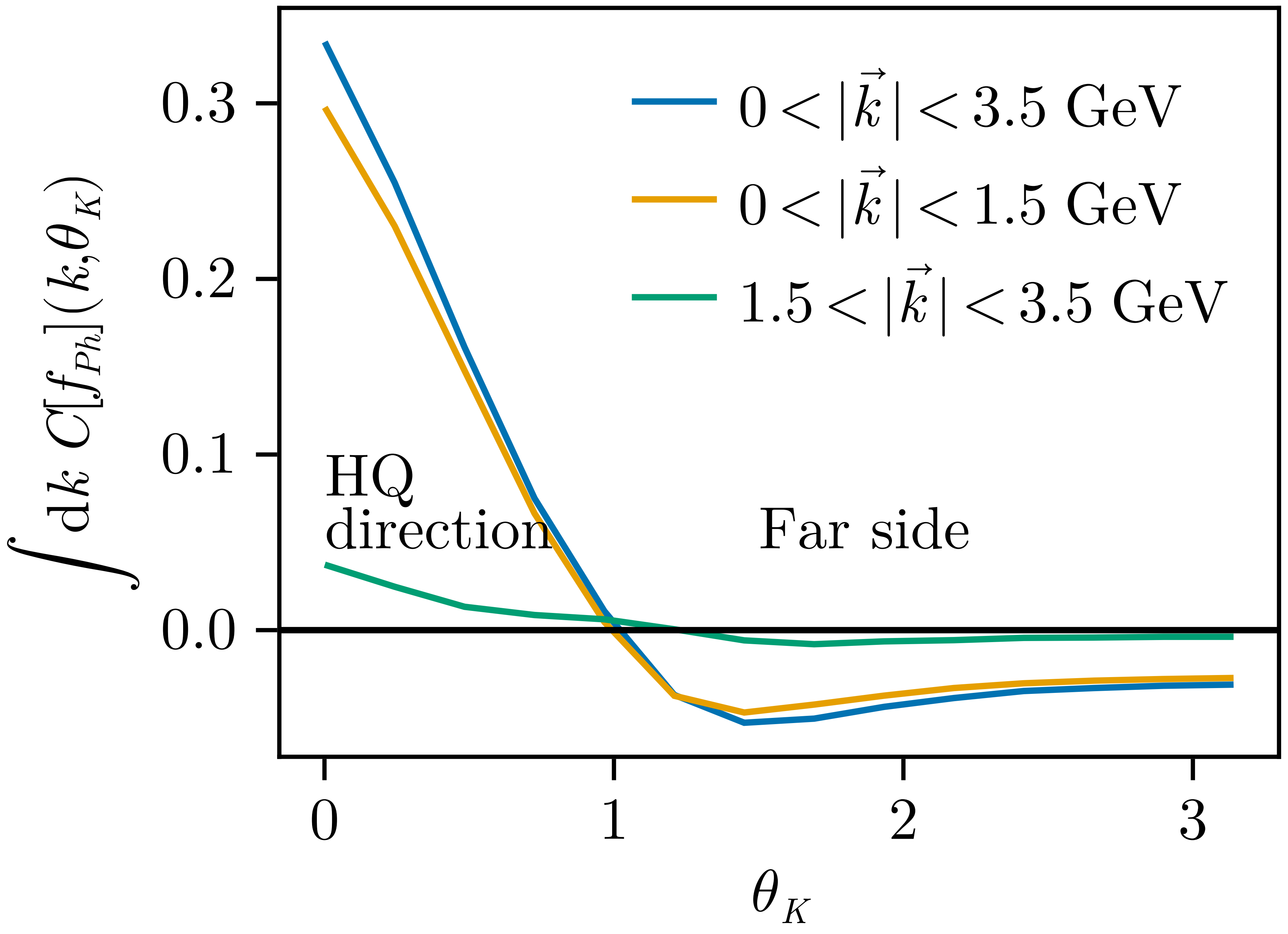
The diffusion wake follows the quark through the medium

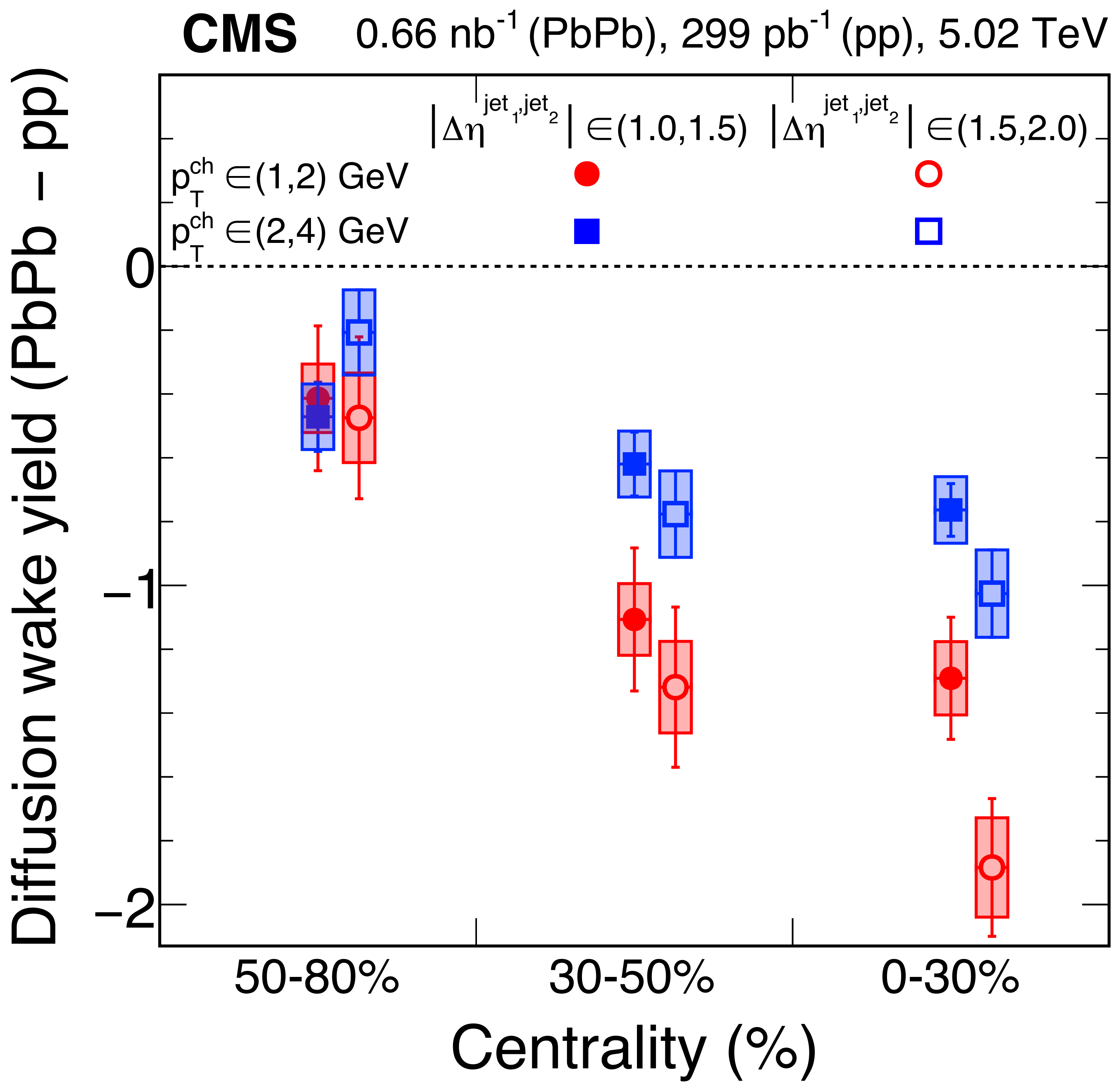
It reveals itself by a depleting of medium particles emitted opposite to the quark. First seen by CMS in Z-jet events.

Momentum response









Summary

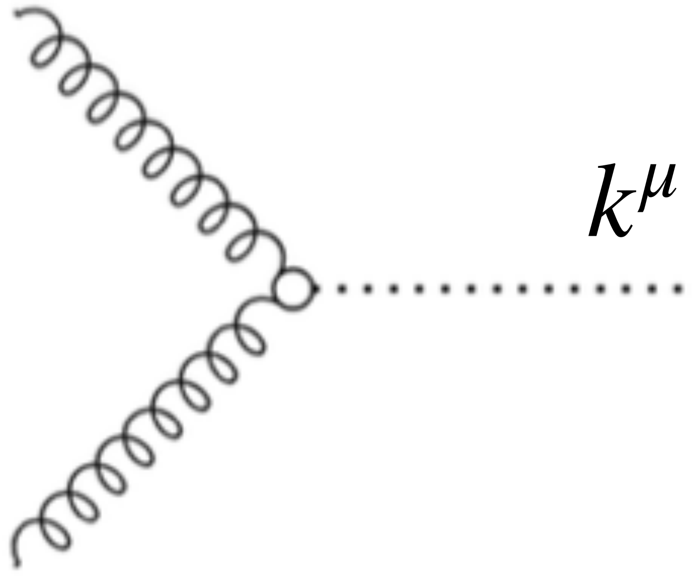
- The combination of heavy quark EFT and hydro EFT provide a consistent framework of power counting of quark-fluid interactions that has the potential to interpolate between moderately coupled pQCD and strongly coupled AdS/CFT
- We have evaluated the in-medium quark quasiparticle width, drag force and quark-phonon collision kernel and identified the diffusion wake.
- We showed how a heavy quark thermalizes in the linearized Boltzmann formalism
- Open issues:
    - Matching of Wilson coefficients in QCD and AdS/CFT
    - Inclusion of non-fluid gluons in the quark collision kernel: Consistent hybrid model of heavy quark energy loss
    - Are there tree-level dissipative terms that contribute to the drag force [see Modrekiladze et al - 2412.06747 for EFT description of drag force on point particles]?
    - Extension of the formalism to high-momentum light quarks
    - Extension to viscous hydro EFT

Viscous Hydro-EFT

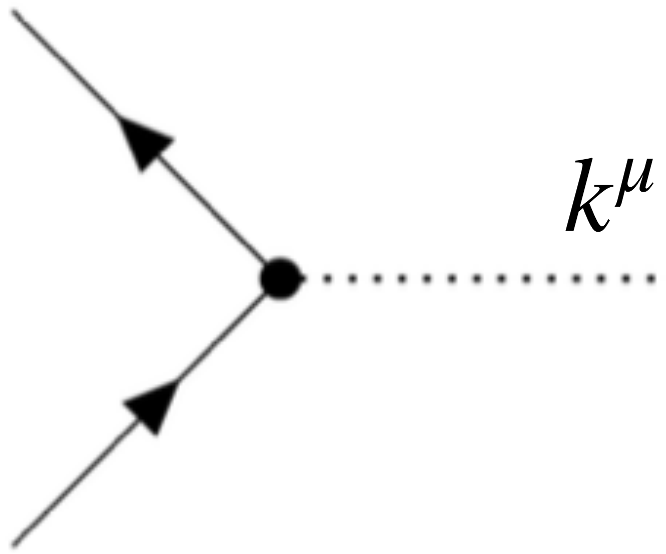
Endlich, Nicolas, Porto, Wang - 1211.6461
Grozdanov & Polonyi - 1305.3670

Consider QGP fluid ϕ in interaction with non-fluid sector of QGP $(\omega, k) > k_{\max}$

$$\mathscr{L}[\phi, A, q] = -F[\phi_0^I + \pi^I] + \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} - \partial_\mu \phi^I \left(T^{\mu I}[A] + T^{\mu I}[q] \right)$$



Following standard CTP formalism one then finds the effective equation for π^I:



$$(\varepsilon_0 + p_0)\left[\omega^2 \pi^I - c_s^2 k^I k^J \pi^J\right] + i G_{\mathrm{R}}^{IJ}(\omega, \vec{k}) \pi^J = 0$$

where

$$G_{\mathrm{R}}^{IJ}(\omega, \vec{k}) = k_\mu k_n u \left\langle T^{\mu I}[A, q] T^{\nu J}[A, q] \right\rangle$$

The analytic structure at low (ω, k) is:

$$\mathrm{Re}\, G_{\mathrm{R}}^{IJ} = \omega k^2 \left[(\tilde{\zeta} + 4\tilde{\eta}/3) P_{\mathrm{L}}^{IJ} + \tilde{\eta} P_{\mathrm{T}}^{IJ} \right] + \cdots$$

where $\tilde{\eta}, \tilde{\zeta}$ are viscosity contribution from non-hydro gluons and light quarks.