Master(of)equations

In honor of Tamás S. Bíró on the occasion of his 70^{th} birthday

András Telcs

Wigner RCP

May 2026

When we met

When we met

Genesis 1:2:
“The earth was a formless chaos,
and darkness covered the face of the deep...”
and still it is ...

When we met

,

1984 Characterization paper: the prototype (Glanzel W. , T A , Schubert A)

A distribution can be characterized by relations by truncated moments,

$$e(x) = \mathbb{E}\left[X \mid X > x\right],$$

For suitable functions this identifies Pearson-type continuous distributions and their discrete analogues. In particular

$$e(x) = ax + b$$

characterizes the Warring distribution for $a + b - 1 \geq 2\sqrt{b(a-1)}$ and the uniform discrete distribution when $a = 0.5$ and $2b$ is a natural number.

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Based on that today the characterized distribution families are:

Pearson, beta, beta-prime/infinite beta, finite beta, gamma, exponential, Pareto, inverse-gamma/Pearson-V type, Irwin, hypergeometric, inverse Pólya–Eggenberger, Waring, binomial, Poisson, negative binomial, negative hypergeometric, discrete uniform, Cauchy, normal, uniform, power-function, modified Weibull, log-modified Weibull, Weibull-geometric, beta Weibull-geometric, generalized exponential geometric, gamma exponentiated Weibull, beta exponential, gamma exponentiated, gamma geometric, generalized exponential, exponentiated Pareto, exponentiated gamma, SKS, generalized gamma, Amoroso, New Weibull–Pareto, exponentiated transmuted generalized Rayleigh, transmuted Weibull Lomax, McDonald generalized-K, beta generalized, Kumaraswamy generalized, McDonald normal, Kumaraswamy-inverse Weibull, beta extended Weibull, gamma-G, gamma extended Weibull, generalized beta extended Pareto, Poisson Birnbaum–Saunders, skew normal, ratio of Rayleigh, ratio of gamma/exponential, ratio of Weibull, Zografos–Balakrishnan log-logistic, Lindley variants.

2020 Bíró line: replace truncated moments by tail cumulatives

For a nonnegative variable X with density P,

$$C(x) = \int_x^\infty P(y)\,\mathrm{d}y, \qquad F(x) = \int_x^\infty \frac{y}{\langle X \rangle} P(y)\,\mathrm{d}y.$$

These are the population tail and the value-weighted tail. In scientometrics, X becomes citations or productivity.

The common technical object: tail-integrated structure

Characterization view

- Condition on the tail $X > x$.
- Compare different truncated expectations.
- Distribution class follows from simple tail relations.

$$\mathbb{E}[g(X) \mid X > x] = \lambda(x)\mathbb{E}[h(X) \mid X > x].$$

Gintropic view

- Keep two canonical tail summaries.
- Plot F against C as a Lorenz map.
- Study the gap from equality.

$$\sigma(x) = F(x) - C(x), \qquad G = 2\int_0^1 \sigma(C)\,\mathrm{d}C.$$

The continuity is methodological: infer global distributional structure from tail-conditioned quantities.

Dynamics: growth, reset, and emergence of Pareto tails

Matthew (25:29), "For to everyone who has, more will be given, and he will have an abundance; but from the one who has not, even what he has will be taken away. "

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2018,19 Bíró TS. Néda Z growth-reset master equation supplies a dynamical route to the Tsallis–Pareto family.

Linear growth with constant reset

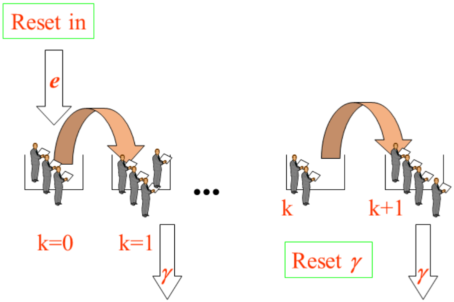
$$\dot{P}(k,t) = \mu(k-1)P(k-1,t) - [\mu(k) + \gamma(k)]P(k,t),$$

with a reset boundary equation at $k = 0$.

$$\mu(k) = ak + b, \qquad \gamma(k) = \gamma.$$

This mechanism produces a stationary Tsallis–Pareto / cut power-law distribution.

Features



- Preferential growth: cumulative advantage, "rich get richer" – Matthew effect.
- Reset: retirement, failure, dropout, field exit, loss of visibility.
- Gini dynamics tracks the emergence of inequality in the evolving distribution.

Continuous-state master equation

The population density $\rho(x, t)$ evolves through transition rates $W(y \mid x, \rho)$.

$$\frac{\partial \rho(x,t)}{\partial t} = \int W(x \mid y, \rho)\rho(y,t)\ \mathrm{d}y - \rho(x,t) \int W(y \mid x, \rho)\ \mathrm{d}y.$$

- The first term is inflow into state x from other states y.
- The second term is outflow from state x to other states y.

Let X be citation count per publication for a researcher or institution.

$$\langle X \rangle = \frac{N_{\mathrm{cit}}}{N_{\mathrm{pub}}}, \qquad h = N_{\mathrm{pub}} C(h).$$

For scaling distributions,

$$P(x) = \frac{1}{\langle X \rangle} p\left(\frac{x}{\langle X \rangle}\right),$$

so C, F, σ depend on $z = x/\langle X \rangle$. Do a little calculation...

$$\frac{h^2}{N_{\mathrm{cit}}} = z\left(1 + \frac{z}{b-1}\right)^{-b}.$$

At $z = 1$ when $b = 2$

$$\frac{h^2}{N_{\mathrm{cit}}} \simeq \frac{1}{4}.$$

Consequence

The Hirsch index is not independent: it is constrained by N_{pub}, N_{cit}, and the tail parameter; the rule $\sqrt{N_{\mathrm{cit}}} \simeq 2h$ is a bulk scaling trend, not an identity.

All who take the citation will judged by the citation.

Tamás S. Biró

Wigner Research Centre for Physics
Field: Physical Sciences
Discipline: Physics
Total publications: 236
Total predicted citations: 6,738
Overall (All Fields) predicted h-index: 45

	Scholars ranked	Productivity			Impact			Quality			ScholarGPS® Ranks		
		Publications	Rank	Top Percentage Rank	Citations	Rank	Top Percentage Rank	h-index	Rank	Top Percentage Rank	Rank	Z Score	Top Percentage Rank
Overall (All Fields)	29,161,103	96.08	45,743	0.16%	1,358.12	180,491	0.62%	20	107,399	0.4%	82,985	1.86	0.28%
Physical Sciences	3,705,091	96.08	9,632	0.26%	1,358.12	38,041	1.03%	20	22,069	0.65%	17,638	1.81	0.48%
Physics	998,186	96.08	3,153	0.32%	1,358.12	11,535	1.16%	20	6,745	0.73%	5,598	1.79	0.56%
Specialties													
Nuclear physics	54,086	10.95	1,174	2.17%	248.75	1,502	2.78%	8	959	2.07%	1,136	1.78	2.1%
Quark	16,088	10.78	331	2.05%	189.98	570	3.54%	7	345	2.64%	378	1.8	2.35%
Gauge theory	12,224	9.42	189	1.55%	44.67	1,645	13.46%	4	567	6.06%	521	1.74	4.26%
Collision	108,844	7.27	948	0.87%	352.65	483	0.44%	10	204	0.21%	353	1.95	0.32%
QCD matter	1,653	6.87	20	1.18%	94.15	78	4.69%	4	48	4.02%	43	1.8	2.57%
Particle physics	44,978	5.75	2,431	5.4%	57.3	4,931	10.96%	4	3,558	9.41%	3,430	1.57	7.62%
Energy	1,669,067	5.7	27,224	1.63%	82.96	70,795	4.24%	6	22,774	1.71%	31,262	1.85	1.87%
Statistical mechanics	38,762	5.15	1,015	2.62%	68.77	1,957	5.05%	5	600	2.1%	961	1.84	2.48%
Thermodynamics	190,811	4	3,524	1.89%	109.42	3,743	1.96%	4	3,252	2.37%	3,098	1.89	1.62%
Lattice gauge theory	1,636	3.42	112	6.82%	12.5	542	33.1%	3	122	10.45%	204	1.41	12.44%
Plasma (physics)	229,182	2.67	13,295	5.83%	63.83	8,794	3.84%	3	10,444	6.6%	10,525	1.71	4.59%
Power (physics)	119,741	2.45	1,654	1.39%	71.7	1,463	1.22%	4	567	0.76%	794	1.94	0.66%
Radiation	653,917	2.33	34,148	5.27%	6.33	168,572	25.86%	2	41,237	10.74%	60,442	1.49	9.24%

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Gintropy: inequality kernel on the Lorenz map

For nonnegative X with finite mean,

$$C(x) = \Pr\{X \geq x\}, \qquad F(x) = \frac{\mathbb{E}[X\mathbf{1}_{X \geq x}]}{\mathbb{E}X}, \qquad \sigma(x) = F(x) - C(x).$$

- $F \geq C$: the value-share tail is above the population-share tail.
- σ is nonnegative and reaches its maximum at $x = \langle X \rangle$.
- The Gini index is the area under the gintropy curve:

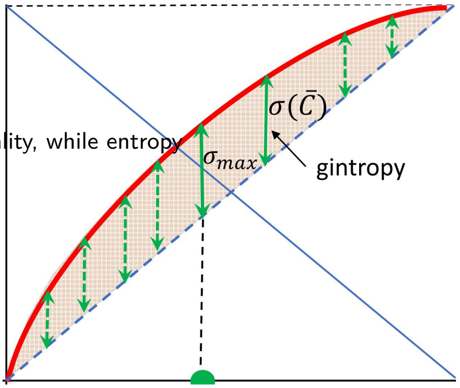
$$G = 2\int_0^1 \sigma(C)\,\mathrm{d}C.$$

Interpretation

Gintropy is not merely another index. It is the local Lorenz-gap density whose integral is the Gini index.

2023 Biro, T, Jozsa Néda - The Gini index measures inequality, while entropy measures disorder or uncertainty.

The idea of *gintropy* is to connect these two viewpoints through the Lorenz curve.



Gintropy as the density under the Gini index

2020 Bíró Néda The Gini index can be written as the average of the gintropy density:

$$G = 2\int_0^1 \sigma(C)\,\mathrm{d}C = 2\int_0^\infty \sigma(x)\rho(x)\,\mathrm{d}x.$$

Thus σ plays the role of an "entropy-like density", but its integral gives an inequality measure rather than a thermodynamic entropy.

Interpretation

$$\sigma(C) = 0 \quad \Longleftrightarrow \quad \text{perfect equality at that cumulative level.}$$

Large $\sigma(C)$ means that the rich-end population share and the rich-end output share strongly separate.

The maximum of σ occurs at the mean output level:

$$\sigma_{\text{max}} = \sigma(\langle x \rangle).$$

Pareto / Tsallis bridge

For a Tsallis–Pareto tail

$$C(x) = (1 + ax)^{-b}, \qquad P(x) = ab(1 + ax)^{-b-1}, \qquad b > 1,$$

with $q = 1 - 1/b$,

$$\sigma(C) = \frac{C^q - C}{1 - q}, \qquad \lim_{q \to 1} \sigma(C) = -C \log C, \qquad G = \frac{1}{q + 1}.$$

- Pareto tails generate Tsallis-type gintropy.
- The exponential limit returns the Shannon kernel.
- The same formulas apply to wealth, citations, and other valued outputs if the tail law is comparable.

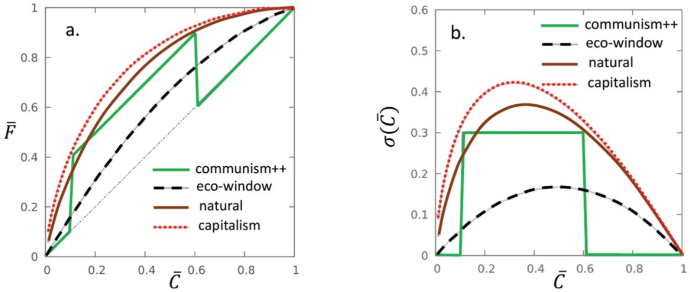
Social caricatures: block, natural, capitalism

Caricature	Distribution type	Gintropy shape
Communism	Single value	$\sigma(C) = 0$
Communism++	Two classes	Step-like plateau
Eco-window	Uniform window	$\sigma(C) = 3G\,C(1-C)$
Natural	Exponential	$\sigma(C) = -C \ln C$
Capitalism	Pareto / Tsallis–Pareto	$\sigma(C) = \dfrac{C^q - C}{1-q}$

Message

Different social caricatures have different entropy-like densities. The Gini index is not only a number; through $\sigma(C)$ it has a whole "spectral profile" over cumulative social rank.

Reading the caricatures



The labels are intentionally schematic.

- **Communism:** everyone has the same output; inequality vanishes.
- **Communism++:** a two-class society; inequality appears as a discrete jump.
- **Eco-window:** outputs are uniformly spread over an interval.
- **Natural:** exponential output distribution; the Shannon form appears.
- **Capitalism:** heavy-tailed output distribution; the Tsallis form appears.

Bis Hundert und Zwanzig Tamás!