Thermodynamics and relativistic dissipation

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In honor of Tamás Biró

BTSLXX, Budapest, 11.06.2026

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Common projects I

Entrance examination to Institute of Particle and Nuclear Physics: NEXT 2005.

Quark mass distribution: with J. Zimányi, P. Lévai and A. László

- Biro, TS; LP; VP; Zimanyi, J: Equation of state for distributed mass quark matter, JoP G, 32/12, S205-S212, (2006).
- Biro, TS; LA; P: Mass gap from pressure inequalities, hep-ph/0612085, (2006)
- Biro, TS; LP; VP; Zimányi, J.: Mass distribution from a quark matter equation of state, PRC 75/3:034910 (2007).

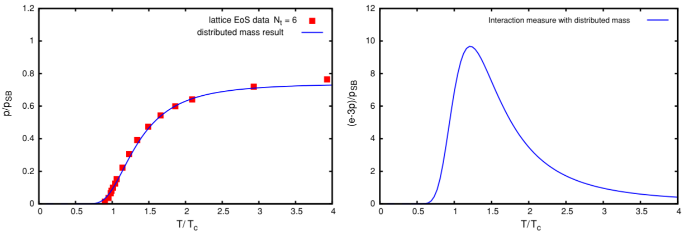


Figure 3. The normalized pressure (left) and the interaction measure $(\epsilon - 3p)/T^4$ (right) obtained by using the mass distribution (7) with adjusted mass scale $M(T)$.

Common projects II

Nonadditive thermodynamics: with GG Barnaföldi and K Ürmössy

- Biro, TS; VP: Zeroth law compatibility of nonadditive thermodynamics, PRE 83:061147, (2011).
- Biró, TS; Barnaföldi, GG; VP: Quark-gluon plasma connected to finite heat bath EPJA 49:110, (2013).
- Biró, TS; Barnaföldi, GG; VP; Ürmössy, K: Statistical Power Law due to Reservoir Fluctuations and the Universal Thermostat Independence Principle, Entropy 16:6497-6514, (2014).
- Biró, TS; Barnaföldi, GG; VP: New entropy formula with fluctuating reservoir, Physica A 417:215-220, (2015).

Origin of Tsallis-like thermostatistics:

- Physical temperature: Rényi.
- Finite size reservoir effect: calculable universal q.
- Fluctuating reservoirs: new entropy formula.

Common projects III

Thermodynamics and fluids: with VG Czinner and H. Iguchi

- VP; Biró, TS: Relativistic hydrodynamics - causality and stability, EPJ-ST 155/1:201-212 (2008).
- Biró, TS; Molnár, E; VP: Thermodynamic approach to the relaxation of viscosity and thermal conductivity, PRC 78:014909, (2008).
- Biró, TS; VP: About the temperature of moving bodies, EPL 89:30001 (2010).
- VP; Biró, TS: First order and stable relativistic dissipative hydrodynamics, PLB 709/1-2:106-110 (2012).
- VP; Biró, TS: Thermodynamics and flow-frames for dissipative relativistic fluids, AIP CONF PROC 1578:114-121, (2014).
- Biró, TS; VP: Splitting the Source Term for the Einstein Equation to Classical and Quantum Parts, Foundations of Physics 45/11:1465-1482, (2015).
- Biró, TS; Czinner, VG; Iguchi, H; VP: Black hole horizons can hide positive heat capacity, PLB 782:228-231, (2018) and Volume dependent extension of Kerr-Newman black hole thermodynamics, PLB 803:135344, (2020).

Thermodynamics is a theory of stability: physical conditions of Lyapunov's theorem

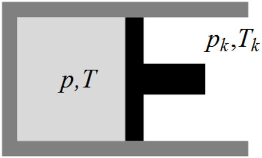
Matolcsi T. Ordinary Thermodynamics (Akadémiai 2005)
VP (PTRSA, 2023)

Ordinary thermodynamics

System, body, environment??

$$dE = TdS - pdV \Rightarrow \dot{E} = T\dot{S} - p\dot{V} \neq Q + W$$

- State space (E, V). Entropy is a potential of the vector field $\left(\frac{1}{T}, \frac{p}{T}\right)(E, V)$.

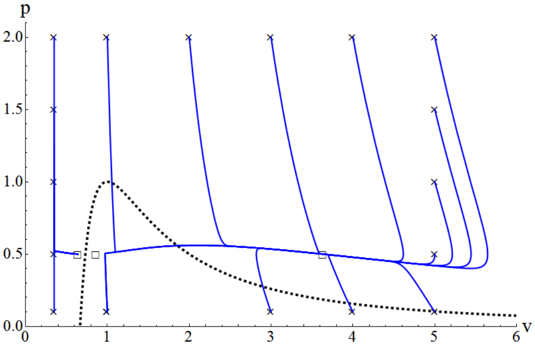


$$S(E, V), \quad \left.\frac{\partial S}{\partial E}\right|_V = \frac{1}{T}, \quad \left.\frac{\partial S}{\partial V}\right|_E = \frac{p}{T}$$

- $S(E, V)$ is concave (thermodynamic stability)
- Insulation constraints: $E + E_k = const.$, $V + V_k = cons.$
- Evolution equation:

$$\boxed{\frac{dE}{dt} = -\alpha(T(E, V) - T_k) - p(E, V)\dot{V}, \quad \frac{dV}{dt} = \beta(p(E, V) - p_k).}$$

$$\dot{E} = Q - pF, \quad \dot{V} = F.$$



- Critical point units.
- Slow manifold: isotherm.
- Bifurcation parameters: T_k, p_k.

Total entropy = exergy $\times$ T_k:

Entropy procustion (entropy rate, along the evolution equations):

$$S_T^{\blacksquare} = Q\left(\frac{1}{T} - \frac{1}{T_k}\right) + \frac{F}{T_k}(p - p_k) \geq 0 \quad (\longrightarrow \quad \alpha \geq 0, \beta \geq 0)$$

conditions of Lyapunov's theorem.

Is stability fundamental?

Some instability challenges:

(In)famous hydrodynamic instabilities:

- **Jeans** (gravity-hydro). About the hoomogeneous equilibrium of selfgravitating fluid. Equilibrium?
- **Bobylev** (kinetic-hydro). Absent stable closure of moment series expansion. Solved (?) with Second Law constraints by Paolucci?
- **Eckart** (relativistic-hydro). Homogneous equilibrium of first order relativistic hydro is unstable. Solved (?) by BDNK.
- turbulence, ...
- **Korteweg** (gradient-hydro). Grandient expansion of Navier-Stokes is unstable Solved with Second Law constraints by Dunn and Serrin.
- **Lorentz-Abraham-Dirac** (electromagnetic-mechanical – hydro(?)), NOT solved. What about thermodynamics?

Eckart problem

📖 Eckart, C.: The thermodynamics of irreversible processes, III. Relativistic theory of the simple fluid, Physical Review 58:919-924 (1940).

📖 Hiscock, WA and Lindblom, L.: Generic instabilities in first-order dissipative relativistic fluid theories, PRD 31/4:725-733 (1985).

📖 VP; Biró, TS: Relativistic hydrodynamics - causality and stability, EPJ-ST 155/1:201-212 (2008).

📖 VP; Generic stability of dissipative non-relativistic and relativistic fluids, JSM:Theory and Experiment, p02054 (2009).

Relativistic fluid theory

$$T^{ab} = e\,u^a u^b + q^a u^b + q^b u^a + P^{ab},$$
$$N^a = n\,u^a + j^a.$$

energy-momentum density
particle number density

$$q^a u_a = j^a u_a = 0, \quad P^{ba} u_a = P^{ab} u_a = 0^b$$

$$T^{ab} = \begin{pmatrix} e & q^i \\ q^j & P^{ij} \end{pmatrix}, \qquad N^a = \begin{pmatrix} n \\ j^i \end{pmatrix}$$

$a, b \in \{0,1,2,3\}$; $i, j \in \{1,2,3\}$;
$\mathrm{diag}(1,-1,-1,-1)$, $\dot{e} = u^a \partial_a e$

- u^a – velocity field
- e – energy density
- q^a – **momentum density** **or energy flux??**
- P^{ab} – pressure
- n – particle num. density
- j^a – particle current

General, expressed by comoving splitting:

$$u_a \partial_b T^{ab} = \dot{e} + e\,\partial_a u^a + \partial_a q^a + u_a \dot{q}^a + u_a \partial_b P^{ab} = 0^a \quad \text{energy balance}$$
$$\partial_b N^b = \dot{n} + n\,\partial_a u^a + \partial_a j^a = 0 \quad \text{pnumber balance}$$

Dissipative or ideal? Pressure splitting:

$$P^{ab} = -p\,\Delta^{ab} + \Pi^{ab} = (-p + \Pi)\Delta^{ab} + \pi^{ab}$$

Method of Eckart: entropy production

Background. Ideal fluid: $j^a = 0$, $q^a = 0$, $\Pi^{ab} = 0^{ab}$.
Entropy flux and Gibbs relation:

$$J^a = \beta\, q^a, \quad \mathrm{d}s = \beta\, \mathrm{d}e - \alpha\, \mathrm{d}n$$

Nonrelativistic ideas: local equilibrium in rest frame.

$$\Sigma = -j^a \partial_a \alpha - \beta\, \Pi^{ab} \partial_b u_a + q^a (\partial_a \beta + \beta\, \dot{u}_a) \geq 0$$

Closure by linear relations:

$$\begin{aligned} j^a &= \eta\, \Delta^{ab} \partial_b \alpha \\ \Pi^{ab} &= \eta_v\, \partial_c u^c + \eta\, \Delta^{ac} \Delta^{bd} \tfrac{1}{2} (\partial_c u_d + \partial_d u_c) \\ q^a &= \lambda\, \Delta^{ab} (\partial_a \beta + \beta\, \dot{u}_a) \end{aligned}$$

Heat flux or momentum density?

Thermodynamic ideas

$$\partial_a S^a = \dot{s} + s\,\partial_a u^a + \partial_a J^a \geq 0$$

Eckart (1940), theory and flow-frame:

$$S^a(T^{ab}, N^a) = s(e,n)\,u^a + \frac{q^a}{T}$$

(Müller)-Israel-Stewart (1969–72) theory: suppression in higher order:

$$S^a(T^{ab}, N^a) = \left(s(e,n) - \frac{\beta_0}{2T}\Pi^2 - \frac{\beta_1}{2T}q_b q^b - \frac{\beta_2}{2T}\pi^{bc}\pi_{bc}\right)u^a + \frac{1}{T}(q^a + \alpha_0\,\Pi q^a$$

$$+\alpha_1\,\pi^{ab}q_b)$$

Divergence type theories (Müller-Ruggeri-Geroch-Lindblom):
hyperbolic by construction

$$\partial_a N^a = 0; \qquad \partial_a T^{ab} = 0^b; \qquad \partial_a A^{abc} = I^{bc}$$

$$\partial_a S^a + \chi\,\partial_a N^a + \chi_b\,\partial_a T^{ab} + \chi_{bc}\,\partial_a A^{abc} = 0,$$

$$\partial_a S^a = -\chi_{bc}\,I^{bc} \geq 0.$$

Our thermodynamic ideas

$$\partial_a S^a = \dot{s} + s\,\partial_a u^a + \partial_a J^a \geq 0$$

Thermal assymetry (2008):

$$S^a(T^{ab}, N^a) = s(E, n)\,u^a + \frac{q^a}{T}, \qquad E = \sqrt{e^2 - q^a q_a}.$$

Analysis of kinetic equilibrium → **thermal frame (2012)**:

$$\beta^a = \frac{u^a + w^a}{T}, \qquad \mathrm{d}e + \frac{q^a}{h}\mathrm{d}q_a = T\mathrm{d}s + \mu\mathrm{d}n$$

Bypoduct: **relativistic temperature** with thermal frame. Equilibrium with four-temperatures:

$$\beta_1^a = \frac{g_1^a}{T_1} = \frac{g_2^a}{T_2} = \beta_2^a, \qquad g^a = u^a + w^a.$$

Compatibility with hydrodynamics.

Stability vs dissipation in relativistic fluids

Problem set

- Nonrelativistic local equilibrium cannot be transferred to special relativity. First order and second order fluids: unexpected **violent dissipative instability**.
- The enigma of covariant thermodynamics vs local equilibrium.
- Temperature of moving bodies: a requirement.
- Kinetic theory: not a big help.

Paradox solved?

- First order hyperbolicity: several theories. Main idea of Kovtun (2017): flow-frame is fixed by stability.
- Parabolic or hyperbolic? Causality and stability?
- What is flowing? Particles, the energy or something else?
- What is ideal fluid? Thermostatic pressure without thermodynamics?

Closing remarks

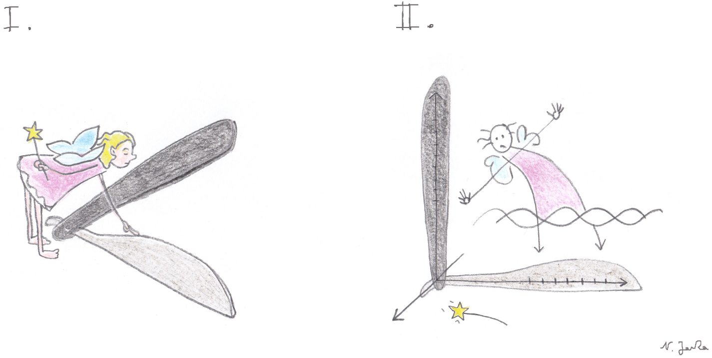
Tamás as my colleague, my teacher and my boss

- Characteristic working style: virtuous blackboard calculations, group discussion.
- Fast preparation of manuscripts: a coworker should work hard to contribute in time.
- Regular group lunch with scientific-historical entertraining conversations.
- Problem focused but publication centered working style.
- New and new ideas for attacking and simplifying the problem and for the sake of elegant presentation.
- Adminstrative tasks are invisible.

Thank you Tamás!

VP: about 📖 Biró Tamás Sándor: Gintropy (A fiction on inequality), FIZIKAI SZEMLE 75:36-36 (2025).

Thank you for your attention !



What is ideal?

Landau–Lifshitz: $N^a = \hat{n}\,\hat{u}^a + \hat{j}^a$,
$T^{ab} = \hat{e}\,\hat{u}^b\hat{u}^a + \hat{P}^{ab} = \hat{e}\,\hat{u}^b\hat{u}^a - \hat{p}\,\hat{\Delta}^{ab} + \hat{\Pi}^{ab}$

Eckart: $N^a = n\,u^a, \quad T^{ab} = e\,u^b u^a + q^b u^a + q^a u^b - p\,\Delta^{ab} + \Pi^{ab}$

Transformation:

$$\hat{u}^a = \frac{u^a + w^a}{\zeta}$$

What is ideal?

$$\begin{aligned} N_0^a &= n\,u^a \\ T_0^{ab} &= e\,u^b u^a - p\,\Delta^{ab} \end{aligned} \quad \Longrightarrow \quad \begin{aligned} N_0^a &= \hat{n}\,\hat{u}^a + j^a \\ T_0^{ab} &= \hat{e}\,\hat{u}^b\hat{u}^a + q^b\hat{u}^a + q^a\hat{u}^b - p\,\hat{\Delta}^{ab} + \Pi^{ab} \end{aligned}$$

$$\hat{n} = \frac{n}{\zeta}, \qquad j^a = n\,\frac{\hat{w}^a}{\zeta}, \qquad \hat{e} = \frac{h}{\zeta^2} - p, \qquad q^a = h\,\hat{w}^a, \qquad \Pi^{ab} = \frac{\hat{w}^a\hat{w}^b}{h}$$

Ideal fluid is a class of N^a, T^{ab}.
Dissipation leads to homogenization? Equilibrium is a submanifold?

Fields and equations

Fields:

N^a	4		j^a	3
T^{ba}	10		q^a	3
u^a	3		Π^{ab}	6
Σ	17		Σ	12

$$q^a u_a = j^a u_a = 0,\ \Pi^{ba} u_a = \Pi^{ab} u_a = 0^b$$

Equations:

$$\partial_a N^a = 0 \quad 1$$

$$\partial_b T^{ab} = 0^a \quad 4$$

N^a – particle number vector
T^{ab} – energy-momentum density
u^a – velocity field
j^a – particle flux/current
q^a – energy flux and momentum density
Π^{ab} – viscous pressure
n, e, u^a – basic fields

Flow-frames, non-equilibrium thermodynamics, second law