# Strong interaction in dense matter

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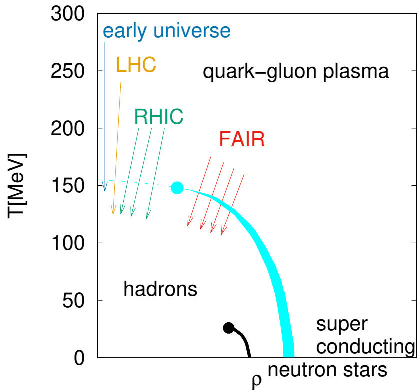
# Overview

# Dense strongly interacting matter

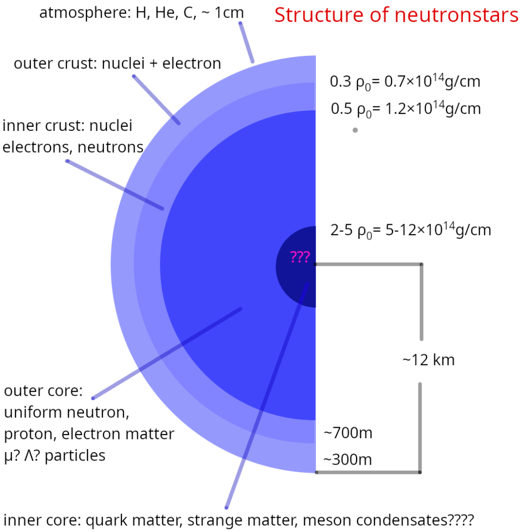



What is the phase diagram and EOS for dense strongly interacting matter?

At $\mu = 0$: lattice and experiments (STAR/PHENIX and ALICE).

For $\mu >> 0$ no precise theory and no heavy ion experiment.

# Neutron Stars a challenge and a possibility



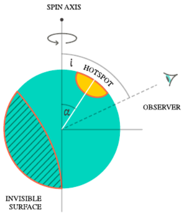

Neutron stars are contain cold, dense matter (T≈0, $\rho > 3\rho_0$) not available in terrestial experiments (Laboratory for strong interaction)

What is the structure of neutron stars (what are the constituents), hybrid stars? Superfluids?

YN, YNN interactions are important, three-body repulsion for $\Lambda, \Sigma$ (Weise)

# Neutron Star observations

- Discovering heavy neutron stars $M > 2M_{\odot}$ Demorest, et al., Nature. 467, 1081-1083 (2010). largest mass observed: $2.35 \pm 0.17\ M_{\odot}$ (2022) Black Widow (Shapiro-delay: pulsar+another star, at almost full covering the second member of the binary delays the radiation of the pulsar)
- Advanced gravitation wave detectors: Advanced Ligo, Virgo, Kagra: single neutron stars??, multichannel astronomy
  neutron star collision: GW170817 (130 million lightyears)
- Modern telescopes: NICER X-ray telescope: precise (<5%) mass and radius measurements (2020) “for nearby” neutron stars,
  5 stars yet, but more to come
  radiation bent by strong gravity: hotspots observation: M, R



# Observables for dense strongly interacting matter

1. Nuclear physics
   - $\rho = 0$
     nucleon-nucleon scattering, YN and YNN data from femtoscopy (ALICE), BB interaction and potential (lattice: HALQCD)
     femtoscopy data and HALQCD calculations are consistent
   - $\rho \approx \rho_0$
     masses of nuclei, isobaric analog states, hypernuclei, giant dipole and pigmy resonances, nuclear dipole polarizabilities, neutron skin thickness $\rightarrow$ normal nuclear density: $\rho_0$, binding energy, compressibility, symmetry energy (1st order in asymmetry expanded in density the 0th and 1st term)
2. Perturbative QCD: $\rho \approx 40\rho_0$
   $N^3LO$ calculation, hard thermal loops: $\mu$ =2.6 GeV, p = 3.8 GeV/$fm^3$.
   T. Gorda, A. Kurkela, et al., Phys. Rev. Lett. 127 (2021) 162003, arXiv:2103.05658.
3. Heavy ion collisions: $\rho : 1 - 8\rho_0$
   not very conclusive; there are many competing effects, like momentum dependent interaction, nonequilibrium, nonzero temperature
4. Neutron stars: $\rho : 1 - 8\rho_0$
   M, R, $\Lambda$. Quite strong constraints even with not yet very precise data

# Features of our quark meson model

Effective field theory: the same symmetry pattern as in QCD

- D.O.F's: scalar, pseudoscalar, vector, axial vector nonets,
- Polyakov loop variables, $\Phi, \bar{\Phi}$ with $U^{glue}$
- u,d,s constituent quarks, $(m_u = m_d)$
- mesonic fluctuations included in the grand canonical potential:

$$\Omega(T, \mu_q) = -\frac{1}{\beta V} ln(Z)$$

- Fermion vacuum and thermal fluctuations
- Five order parameters $(\phi_N, \phi_S, \Phi, \bar{\Phi}, v_0) \rightarrow$ five $T/\mu$-dependent equations

# Meson fields - pseudoscalar and scalar meson nonets

$$\Phi_{PS} = \sum_{i=0}^{8} \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_{S} = \sum_{i=0}^{8} \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} (\sim \bar{q}_i q_j)$$

Particle content:

Pseudoscalars: $\pi(138), K(495), \eta(548), \eta'(958)$
Scalars: $a_0(980$ or $1450), K_0^\star(800$ or $1430),$
$(\sigma_N, \sigma_S)$ : 2 of $f_0(500, 980, 1370, 1500, 1710)$

# Included fields - vector meson nonets

$$V^\mu = \sum_{i=0}^{8} \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star+} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star -} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu$$

$$A^\mu = \sum_{i=0}^{8} b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

Particle content:

Vector mesons: $\rho(770), K^\star(894), \omega_N = \omega(782), \omega_S = \phi(1020)$

Axial vectors: $a_1(1230), K_1(1270), f_{1N}(1280), f_{1S}(1426)$

# Lagrangian (2/1)

$$
\begin{aligned}
\mathcal{L}_{\text{Tot}} &= \mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2 \\
&- \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + \mathrm{Tr}[H(\Phi + \Phi^\dagger)] \\
&+ c_1(\det\Phi + \det\Phi^\dagger) + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
&+ \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2\mathrm{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger). \\
&+ \bar\Psi i\partial\!\!\!/\Psi - g_F\bar\Psi(\Phi_S + i\gamma_5\Phi_{PS})\Psi + g_V\bar\Psi\gamma^\mu\left(V_\mu + \frac{g_A}{g_V}\gamma_5 A_\mu\right)\Psi \\
&+ \text{Polyakov loops}
\end{aligned}
$$

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, D.H. Rischke, Phys. Rev. D87 (2013) 014011

# Lagrangian (2/2)

where

$$D^\mu\Phi = \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ieA_e^\mu[T_3,\Phi]$$

$$\Phi = \sum_{i=0}^{8}(\sigma_i + i\pi_i)T_i, \quad H = \sum_{i=0}^{8} h_i T_i \qquad T_i : U(3) \text{ generators}$$

$$R^\mu = \sum_{i=0}^{8}(\rho_i^\mu - b_i^\mu)T_i, \quad L^\mu = \sum_{i=0}^{8}(\rho_i^\mu + b_i^\mu)T_i$$

$$L^{\mu\nu} = \partial^\mu L^\nu - ieA_e^\mu[T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu[T_3, L^\mu]\}$$

$$R^{\mu\nu} = \partial^\mu R^\nu - ieA_e^\mu[T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu[T_3, R^\mu]\}$$

$$\bar\Psi = (\bar u, \bar d, \bar s)$$

non strange – strange base:

$$\varphi_N = \sqrt{2/3}\varphi_0 + \sqrt{1/3}\varphi_8,$$

$$\varphi_S = \sqrt{1/3}\varphi_0 - \sqrt{2/3}\varphi_8, \qquad \varphi \in (\sigma_i, \pi_i, \rho_i^\mu, b_i^\mu, h_i)$$

broken symmetry: non-zero condensates $\langle\sigma_N\rangle, \langle\sigma_S\rangle \longleftrightarrow \phi_N, \phi_S$

# Determination of the parameters of the Lagrangian

16 unknown parameters $(m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F, g_V, g_A) \longrightarrow$ Determined by the min. of $\chi^2$:

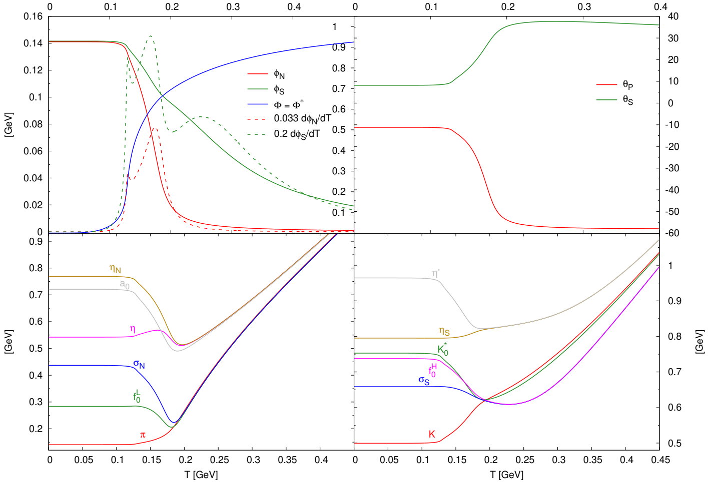
$$\chi^2(x_1,\dots,x_N) = \sum_{i=1}^{M} \left[\frac{Q_i(x_1,\dots,x_N) - Q_i^{\rm exp}}{\delta Q_i}\right]^2,$$

where $(x_1,\dots,x_N) = (m_0, \lambda_1, \lambda_2, \dots)$, $Q_i(x_1,\dots,x_N)$ calculated from the model, while $Q_i^{\rm exp}$ taken from the PDG

multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\rightarrow$ 2 physical quantities: $f_\pi, f_K$
- Tree-level masses $\rightarrow$ 15 physical quantities:
  $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\rightarrow$ 12 physical quantities:
  $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi},$
  $\Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$
- $T_c = 155$ MeV from lattice

# With low mass scalars, $m_{f_0^L} = 300$ MeV

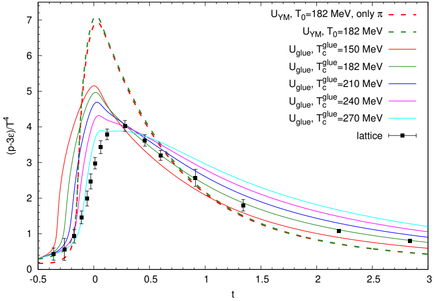



chiral symmetry is restored at high T as the chiral partners $(\pi, f_0^L)$, $(\eta, a_0)$ and $(K, K_0^\star)$, $(\eta', f_0^H)$ become degenerate

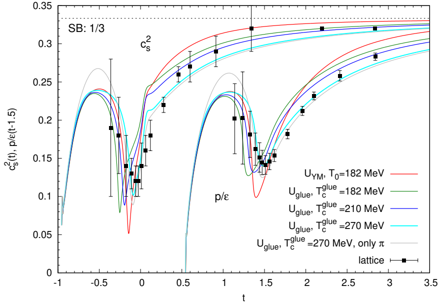
$U(1)_A$ symmetry is not restored, as the axial partners $(\pi, a_0)$ and $(\eta, f_0^L)$ do not become degenerate

# Observables

## interaction measure



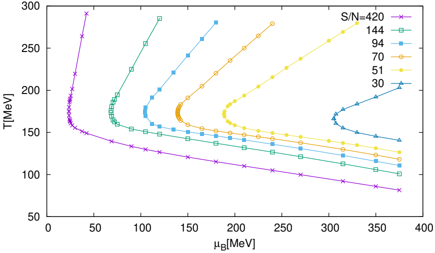

## speed of sound

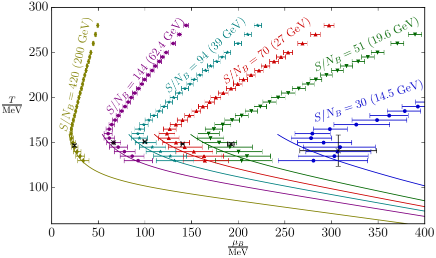


# Isentropic trajectories in the $T - \mu_{\rm B}$ plane

our model, where $\mu_B^{\rm CEP} > 850\rm MeV$

lattice (analytic continuation)
Günther *et al.*, arXiv:1607.02493




same qualitative behavior of the isentropic trajectories for $\mu_B \leq 400$ MeV
$\Longrightarrow$ indication that in the lattice result there is no CEP in this region of $\mu_B$

# EOS

1. $\rho \leq 2-4\rho_0$ ordinary nuclear potentials, CEFT, ...
2. $2-4\rho_0 \leq \rho \leq 6-8\rho_0$ quark matter model
3. $6-8\rho_0 \leq \rho$ extrapolation to the pQCD point

hadronic matter - soft: SFHo

(Steiner, A. W., Hempel, M., Fischer, T. Astrophys. J. 774 (2013) 17) and Hempel, M., Schaffner-Bielich, J. Nucl. Phys. A837 (2010) 210)

relativistic mean-field model (nucleons, $\sigma, \omega, \rho$ with quartic copulings), with K=245 MeV, L=47.1 MeV, $m^*/m_n$=0.76.

hadronic matter - stiff: DD2

(S. Typel, et al., Phys. Rev. C81 (2010) 015803

relativistic mean-field + light clusters, K = 243 MeV, L=58 MeV $m^*/m_n$=0.63

Quark matter: Quark-meson model - chiral $U(3)xU(3) \to SU(2)xU(1)$ model

degrees of freedom: 4 meson nonets, constituent quarks, Polyakov loops

condensates: 2 scalar (N,S), Polyakov loops (T>0), vector mesons ($\mu > 0$)

P. Kovács, Zs. Szép, Gy. Wolf, Phys. Rev. D93 (2016) 114014

# Concatenation

It seems that a strong first order phase transition is ruled out by astrophysical constraints: J.-E. Christian and J. Schaffner-Bielich, Phys. Rev. D 103, 063042 (2021), The allowed $p(\varepsilon)$ functions are in a rather narrow band, there can be no big jump

Hadron-quark crossover with polynomial interpolation ($\rho = \rho_B$):

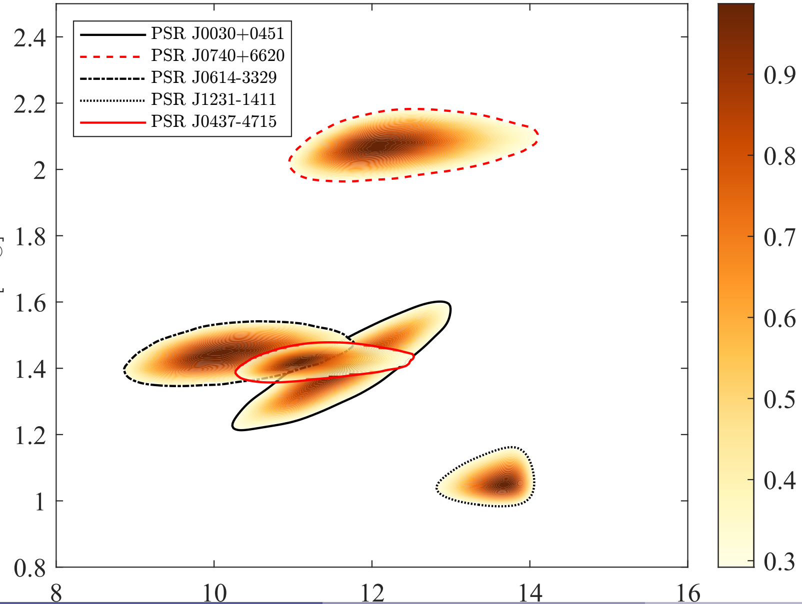
$$\begin{aligned} \varepsilon(\rho_B) &= \varepsilon_{hadronic}(\rho_B) \qquad \rho_B < \rho_{BL}, \\ \varepsilon(\rho_B) &= \sum_{k=0}^{5} C_k \rho_B^k \qquad \rho_{BL} \leq \rho_B \leq \rho_{BU} \\ \varepsilon(\rho_B) &= \varepsilon_{qm}(\rho_B) \qquad \rho_{BU} < \rho_B. \end{aligned}$$

$C_k$ is determined by the requirement that the energy density, $\varepsilon$ and its first two derivatives with respect to $\rho_B$, pressure and sound velocity is continuous at the boundaries.
2 parameters: $\Gamma = 0.5 * (\rho_{BU} - \rho_{BL})$ and $\overline{\rho_B} = 0.5 * (\rho_{BL} + \rho_{BU})$

# Data

- perturbative QCD EOS should converge to it keeping $c_s < 1$, $\mu_{QCD} = 2.6GeV, n_{QCD} = 6.471/fm^3, p_{QCD} = 3823MeV/fm^3$
- Minimal TOV mass constraint: $M^{min}_{TOV} = 2.22M_\odot$: PSR J0952–0607 a black widow pulsar, with a mass $2.35 \pm 0.17M_\odot$,
There are several pulsars mass above $2M_\odot$, using population analysis at the 1$\sigma$ confidence level $M^{min}_{TOV} = 2.22M_\odot$
- NICER: (M,R) values for PSR J0030+0451, PSR J0740+6620, PSR J0614–3329, PSR J1231–1411, and PSR J0437–4715
- tidal deformability GW170817: : $70 < \Lambda(1.4M_\odot) < 720$ Abbot (2019)
stronger constraint: $70 < \Lambda_{1.4} < 580$, $9.1$ km $< R_{1.4} < 12.8$ km
- Hess J1731-347 neutron star: mass= $0.77 \pm 0.19M_\odot$, $R = 10.4 \pm 0.8km$
- massgap neutron star: $2.59 \pm 0.09M_\odot$

# NICER data


PSR J0030+0451
PSR J0740+6620
PSR J0614-3329
PSR J1231-1411
PSR J0437-4715
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
8
10
12
14
16
0.9
0.8
0.7
0.6
0.5
0.4
0.3

# Bayesian inference

Unsetted parameters: $m_\sigma, g_v, \quad \overline{\rho_B} \equiv 0.5(\rho_{BL} + \rho_{BU}), \quad \Gamma \equiv 0.5(\rho_{BU} - \rho_{BL})$

290 MeV $\leq m_\sigma \leq$ 700 MeV
$0 \leq g_v \leq 10$
$2\rho_0 \leq \overline{\rho_B} \leq 5\rho_0$
$\rho_0 \leq \Gamma \leq 4\rho_0$ with the constraint: $\rho_{BL} = \overline{\rho_B} - \Gamma > \rho_0$
We created ∼ 18000 EOSs to be used in the Bayesian analysis
Bayes theorem:
$\theta$ is a parameter set, $p(\theta)$ is the prior probability for $\theta$, $p(data|\theta)$ is the probability that for given $\theta$, the data is measured. Then

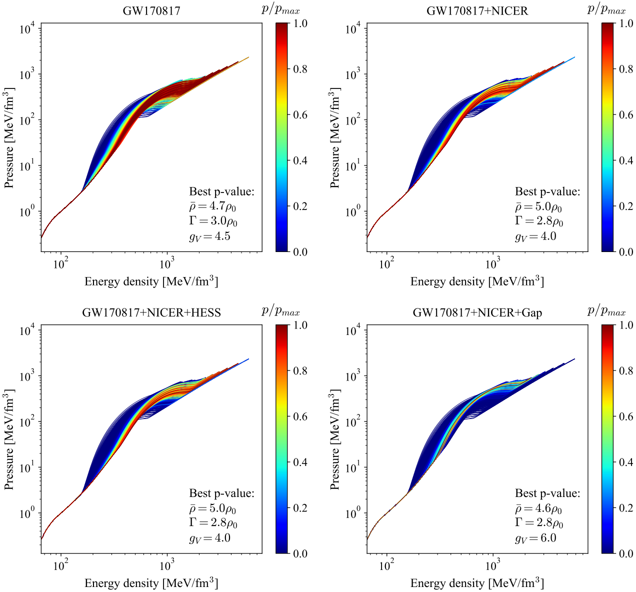
$$p(\theta|data) = \frac{p(data|\theta)p(\theta)}{p(data)}$$

$p(data)$ is a normalization constant. We assume $p(\theta)$ is uniform in the allowed hypersurface. For independent observations:

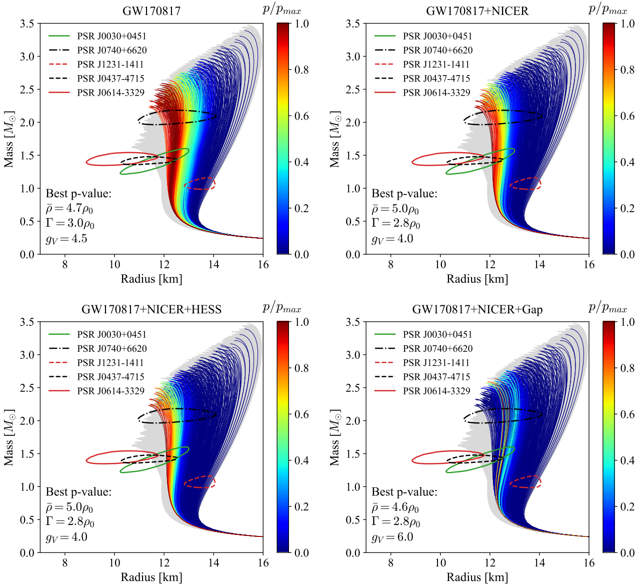
$$p(data|\theta) = p(M_{max}|\theta)p(NICER|\theta)p(\overline{\Lambda}|\theta)$$

Phys. Rev. D105 (2022) 103014, Phys. Rev. D108 (2023) 043002.

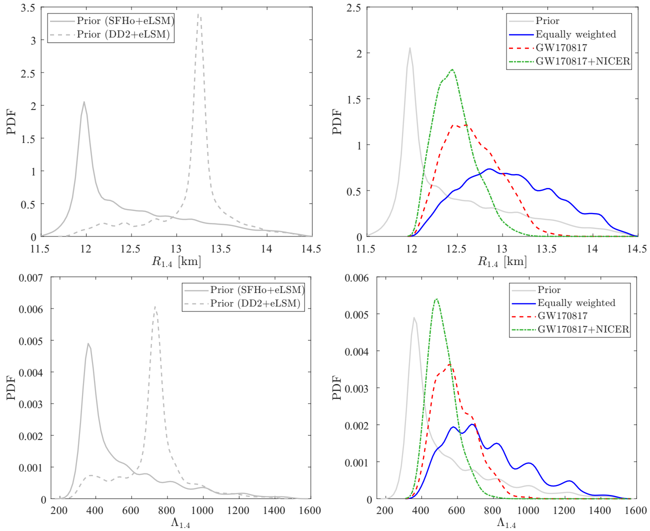
# EOS


GW170817
$p/p_{max}$
Pressure [MeV/fm$^3$]
Energy density [MeV/fm$^3$]
Best p-value:
$\bar{\rho} = 4.7\rho_0$
$\Gamma = 3.0\rho_0$
$g_V = 4.5$
GW170817+NICER
$p/p_{max}$
Pressure [MeV/fm$^3$]
Energy density [MeV/fm$^3$]
Best p-value:
$\bar{\rho} = 5.0\rho_0$
$\Gamma = 2.8\rho_0$
$g_V = 4.0$
GW170817+NICER+HESS
$p/p_{max}$
Pressure [MeV/fm$^3$]
Energy density [MeV/fm$^3$]
Best p-value:
$\bar{\rho} = 5.0\rho_0$
$\Gamma = 2.8\rho_0$
$g_V = 4.0$
GW170817+NICER+Gap
$p/p_{max}$
Pressure [MeV/fm$^3$]
Energy density [MeV/fm$^3$]
Best p-value:
$\bar{\rho} = 4.6\rho_0$
$\Gamma = 2.8\rho_0$
$g_V = 6.0$

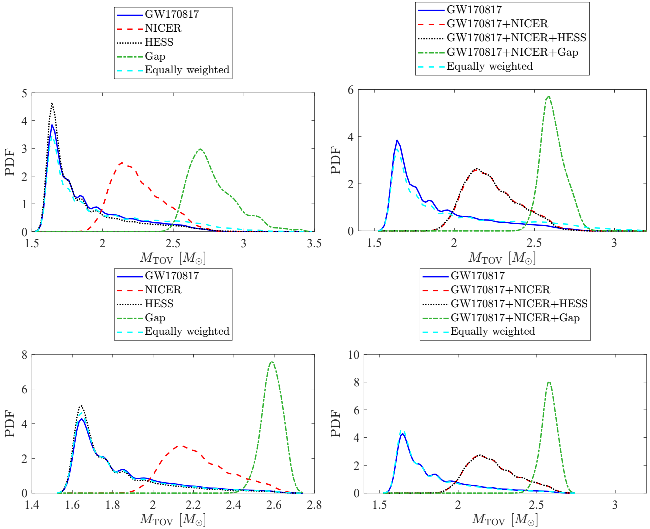
# M(R) probabilities



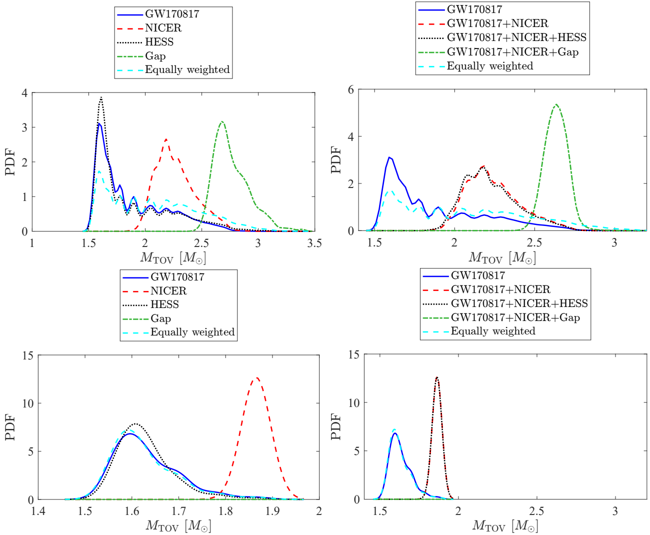
# PDF of the radii and tidal defomabilities



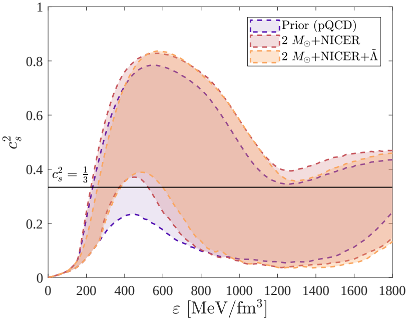
# Distribution of the maximal mass (SFHo and DD2)

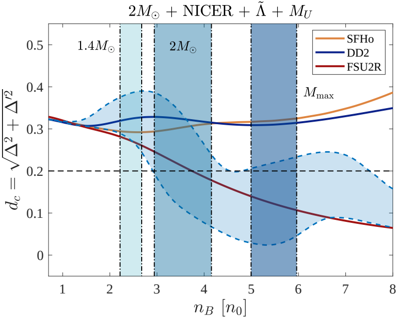


# Maximal mass (SFHo and DD2) with strong GW constraint

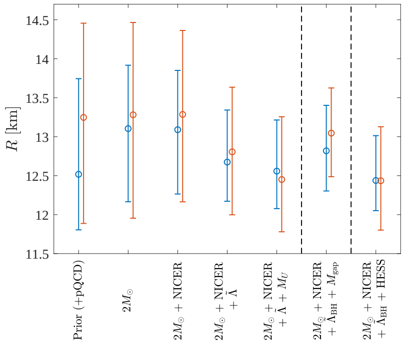


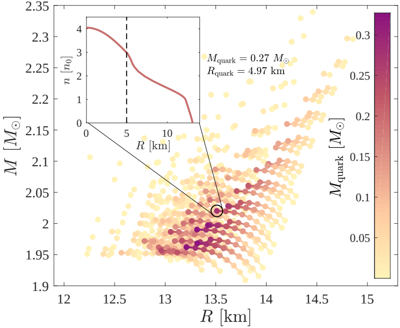
# Sound velocity



# Radius and Quark content



# Summary and Conclusions

- Our model can reproduce the lattice calculations at $\mu = 0$
- With our model we can fulfill the present astronomical constraints
- The central density do not go above $6\rho_0$.
- The radius of the neutron stars are $12.8 \pm 0.8$ km.
- strangeness should be included into the hadronic model
- hadronic and quark phase ought to be handled with the same model to drop ad-hoc parameters

# Thank you for your attention!