

Neutrino Earth Tomography

Sort of...

Andrea Donini

Instituto de Física Corpuscular

Consejo Superior de Investigaciones Científicas - Universitat de València



MINISTERIO
DE CIENCIA, INNOVACIÓN
Y UNIVERSIDADES



CSIC

CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS



VNIVERSITAT
DE VALÈNCIA



INSTITUT DE
FÍSICA
CORPUSCULAR

General view

- 1. Neutrinos in particle physics**
- 2. Neutrino Earth Tomography**
- 3. Hands-on: two simple exercises**

Class 1: outline

1. Neutrinos in the Standard Model

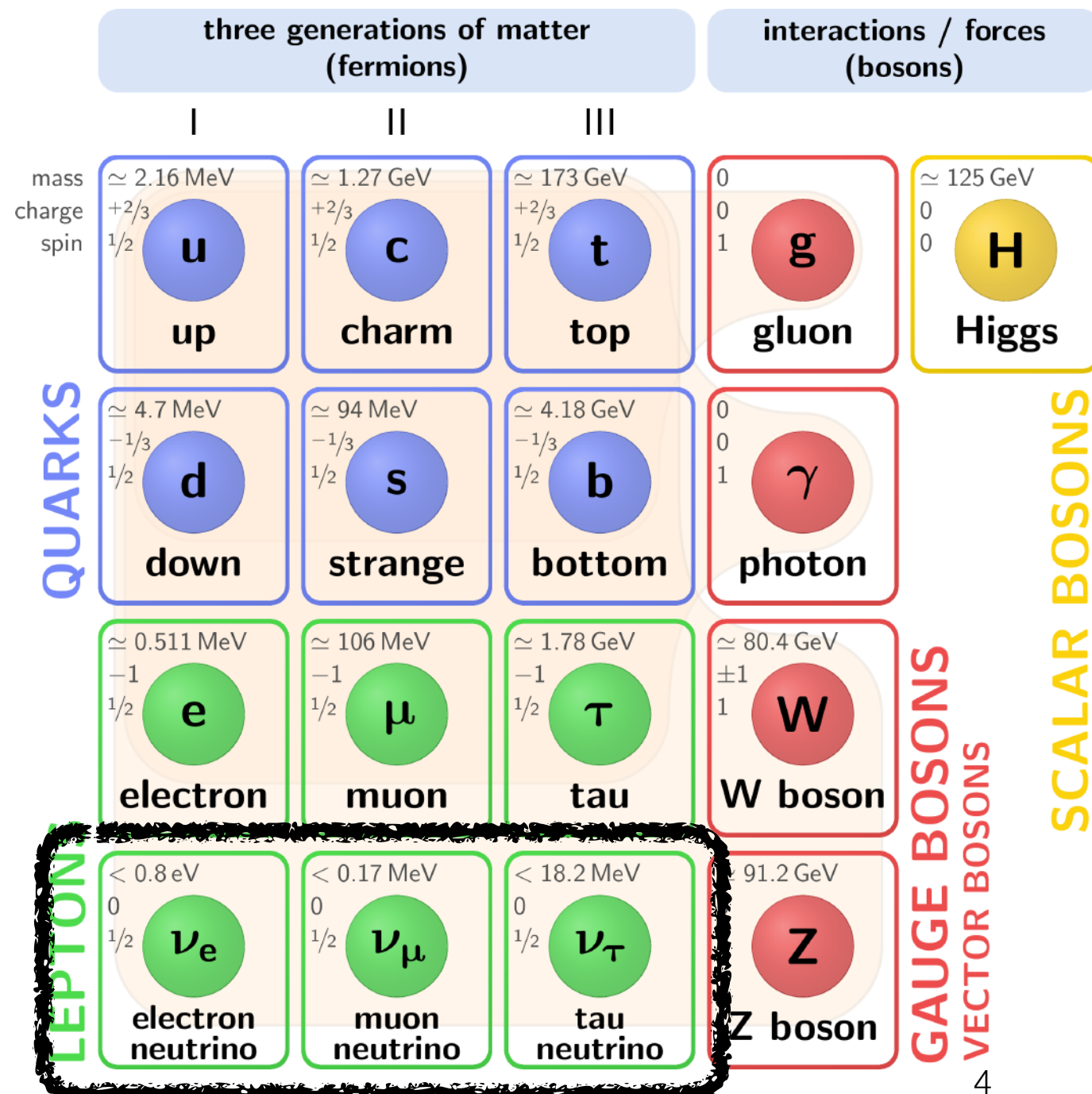
2. Neutrino Masses

3. Neutrino Oscillations

4. Neutrino Sources

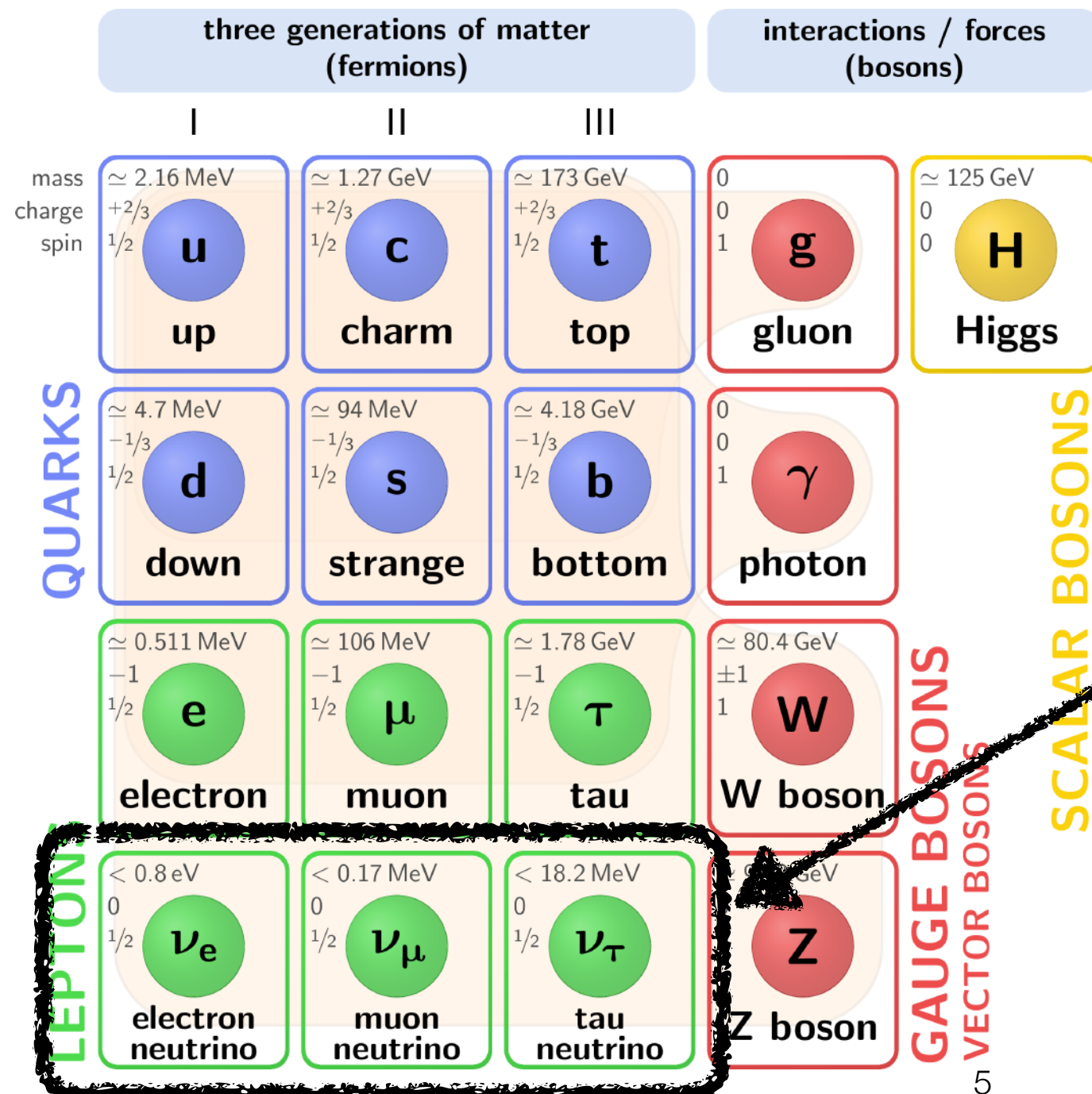
Neutrinos in the Standard Model

(of fundamental interactions)



Neutrinos in the Standard Model

(of fundamental interactions)

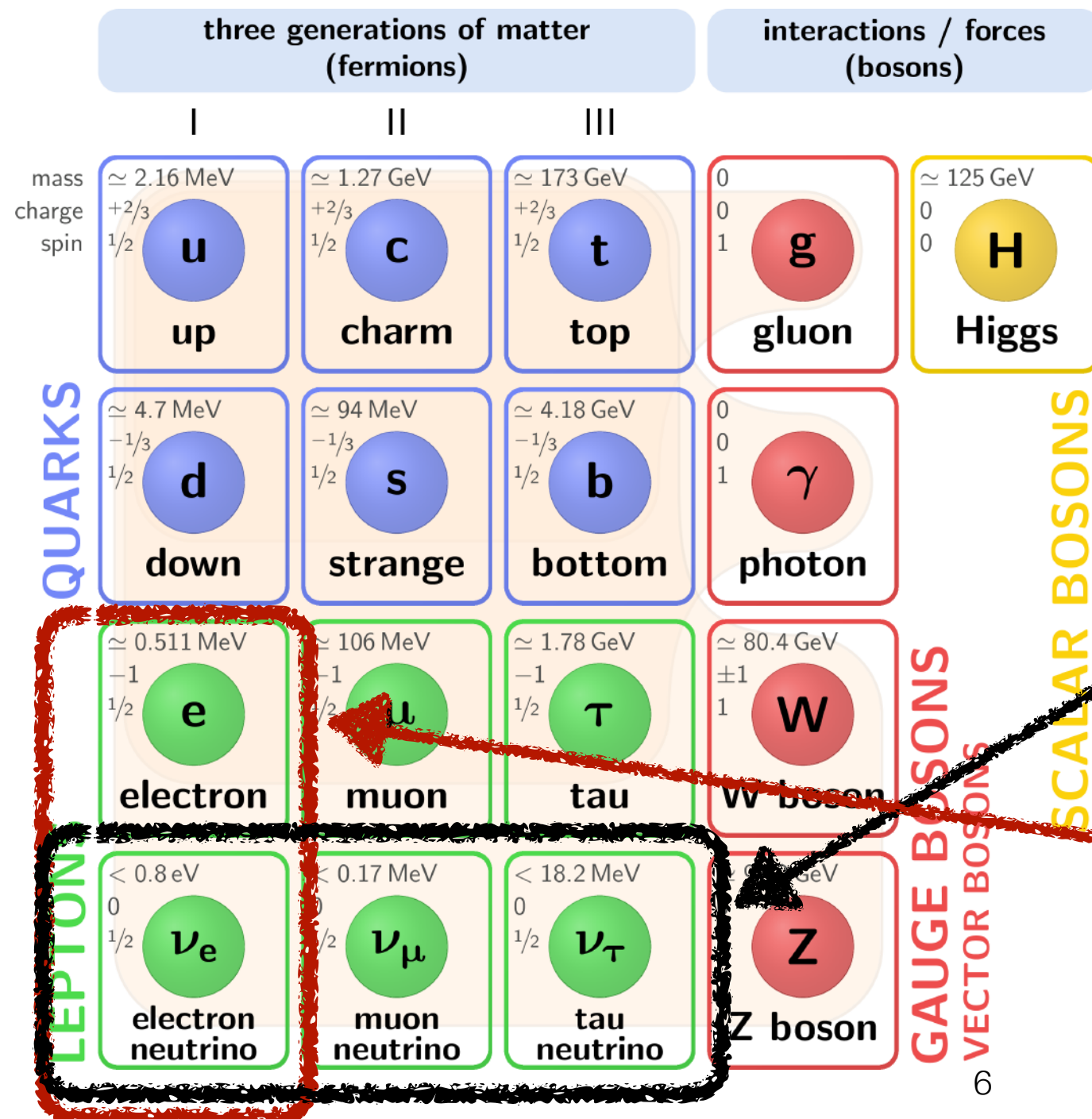


Three fermions

Chargeless
Colourless

Neutrinos in the Standard Model

(of fundamental interactions)

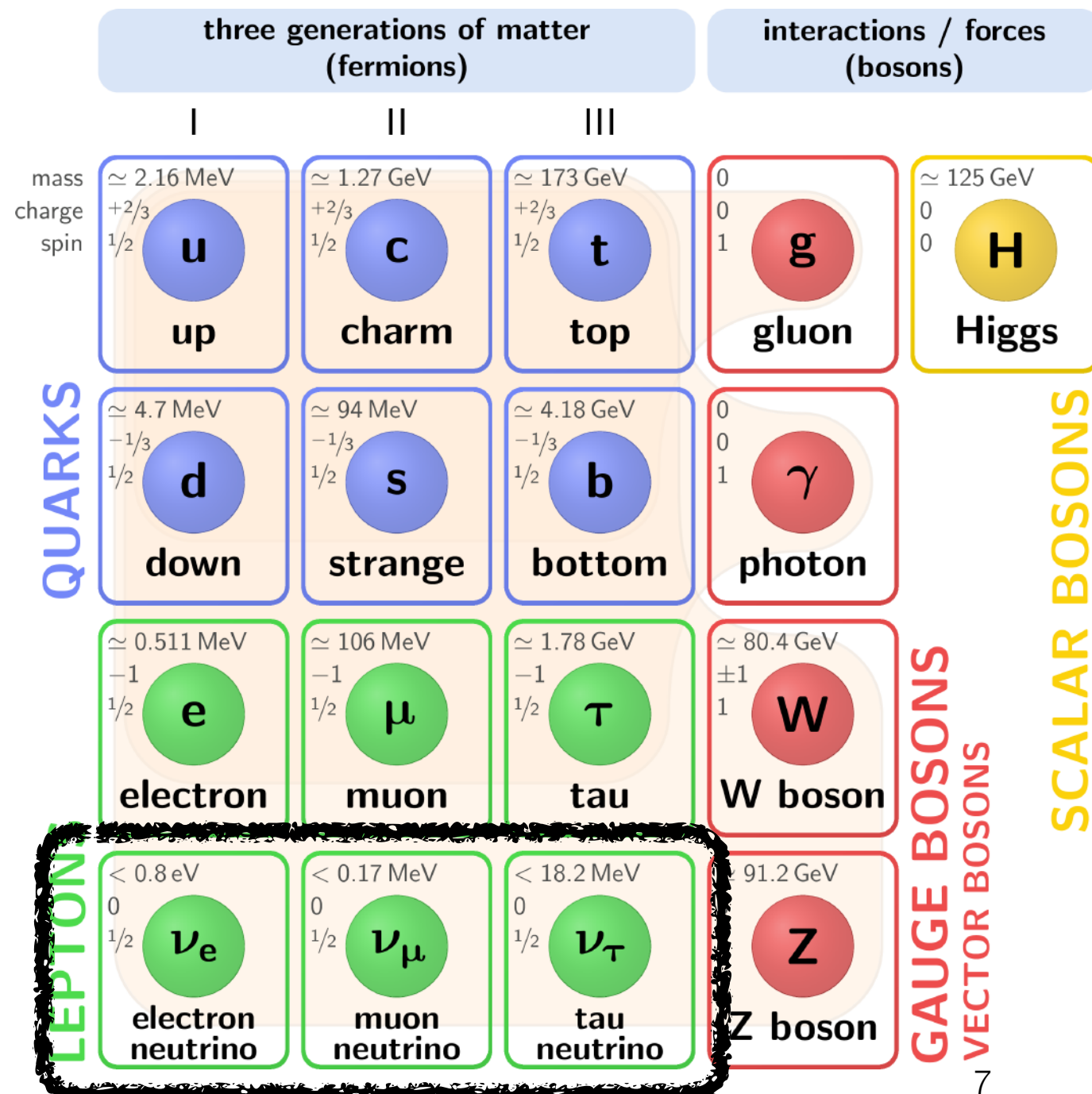


Three fermions

Chargeless
Colourless

SU(2) doublets
(weakly interacting)

Are neutrinos massive?



Not in the
Standard Model

They are massive,
though...

Neutrino oscillations, 1

(two families)

1. Flavour basis (interaction eigenstates)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \equiv (\nu_\alpha)_{\text{flavour}}$$

$$\mathcal{L}_{\text{CC}} \sim \bar{\ell}_\alpha \gamma^\mu P_L \nu_\alpha W_\mu + \text{h.c.}$$

2. Mass basis (propagation eigenstates)

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \equiv (\nu_i)_{\text{mass}}$$

$$\mathcal{L}_{\text{mass}} = - \sum_{i=1}^2 m_i \bar{\nu}_i \nu_i$$

Neutrino oscillations, 1

(two families)

1. Flavour basis (interaction eigenstates)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \equiv (\nu_\alpha)_{\text{flavour}}$$

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(...just as for quarks, 1)

1. Charged–current interactions (gauge basis)

$$\mathcal{L}_{\text{CC}} = \frac{g}{\sqrt{2}} \bar{u}_\alpha \gamma^\mu P_L d_\alpha W_\mu^+ + \frac{g}{\sqrt{2}} \bar{\ell}_\alpha \gamma^\mu P_L \nu_\alpha W_\mu^+ + \text{h.c.}$$

Here α is a *flavour* index: (u, d) for quarks, (ℓ, ν) for leptons.

2. Mass diagonalization

$$u_L = U_u u_L^{\text{mass}}, \quad d_L = U_d d_L^{\text{mass}},$$

$$\ell_L = U_\ell \ell_L^{\text{mass}}, \quad \nu_L = U_\nu \nu_L^{\text{mass}}.$$

(...just as for quarks, 2)

3. Mixing matrices appear in the CC term After rotating to the mass basis:

$$\mathcal{L}_{\text{CC}} = \frac{g}{\sqrt{2}} \bar{u}^{\text{mass}} U_u^\dagger U_d \gamma^\mu P_L d^{\text{mass}} W_\mu^+ + \frac{g}{\sqrt{2}} \bar{\ell}^{\text{mass}} U_\ell^\dagger U_\nu \gamma^\mu P_L \nu^{\text{mass}} W_\mu^+$$

Quark sector: CKM matrix $V_{\text{CKM}} = U_u^\dagger U_d$

$$V_{\text{CKM}} \simeq \begin{pmatrix} 0.974 & 0.226 & 0.0036 \\ -0.226 & 0.973 & 0.042 \\ 0.0087 & -0.041 & 0.999 \end{pmatrix}$$

Mixing angles

$$\theta_{12}^q \simeq 13^\circ, \quad \theta_{23}^q \simeq 2.4^\circ, \quad \theta_{13}^q \simeq 0.2^\circ.$$

Hierarchical, close to identity.

Neutrino oscillations, 2a

(Three families)

Lepton sector: PMNS matrix

$$U_{\text{PMNS}} = U_{\ell}^{\dagger} U_{\nu}.$$

$$U_{\text{PMNS}} \simeq \begin{pmatrix} 0.82 & 0.55 & 0.15 \\ -0.36 & 0.70 & 0.62 \\ 0.44 & -0.45 & 0.78 \end{pmatrix}$$

Mixing angles

$$\theta_{12}^{\ell} \simeq 33^{\circ}, \quad \theta_{23}^{\ell} \simeq 49^{\circ}, \quad \theta_{13}^{\ell} \simeq 8.6^{\circ}.$$

For massless neutrinos...

...there would be no misalignment

Neutrino oscillations, 2b

(Three families)

$$\nu_\alpha = \sum_{i=1}^3 U_{\alpha i} \nu_i$$
$$P_{\alpha \rightarrow \beta}(L) = \left| \sum_{i=1}^3 U_{\beta i} U_{\alpha i}^* e^{-i \frac{m_i^2 L}{2E}} \right|^2$$

$$P_{\alpha \rightarrow \beta} = -4 \sum_{i>j} \Re \left(U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j} \right) \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right) + 2 \sum_{i>j} \Im \left(U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j} \right) \sin \left(\frac{\Delta m_{ij}^2 L}{2E} \right)$$

Neutrino oscillations, 2b

(Three families)

$$\nu_\alpha = \sum_{i=1}^3 U_{\alpha i} \nu_i \qquad P_{\alpha \rightarrow \beta}(L) = \left| \sum_{i=1}^3 U_{\beta i} U_{\alpha i}^* e^{-i \frac{m_i^2 L}{2E}} \right|^2$$

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CP-even
(disappearance and appearance)

Neutrino oscillations, 2b

(Three families)

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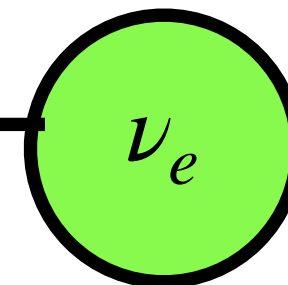
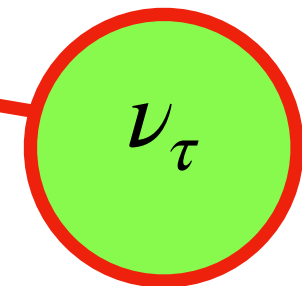
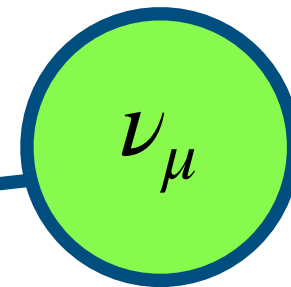
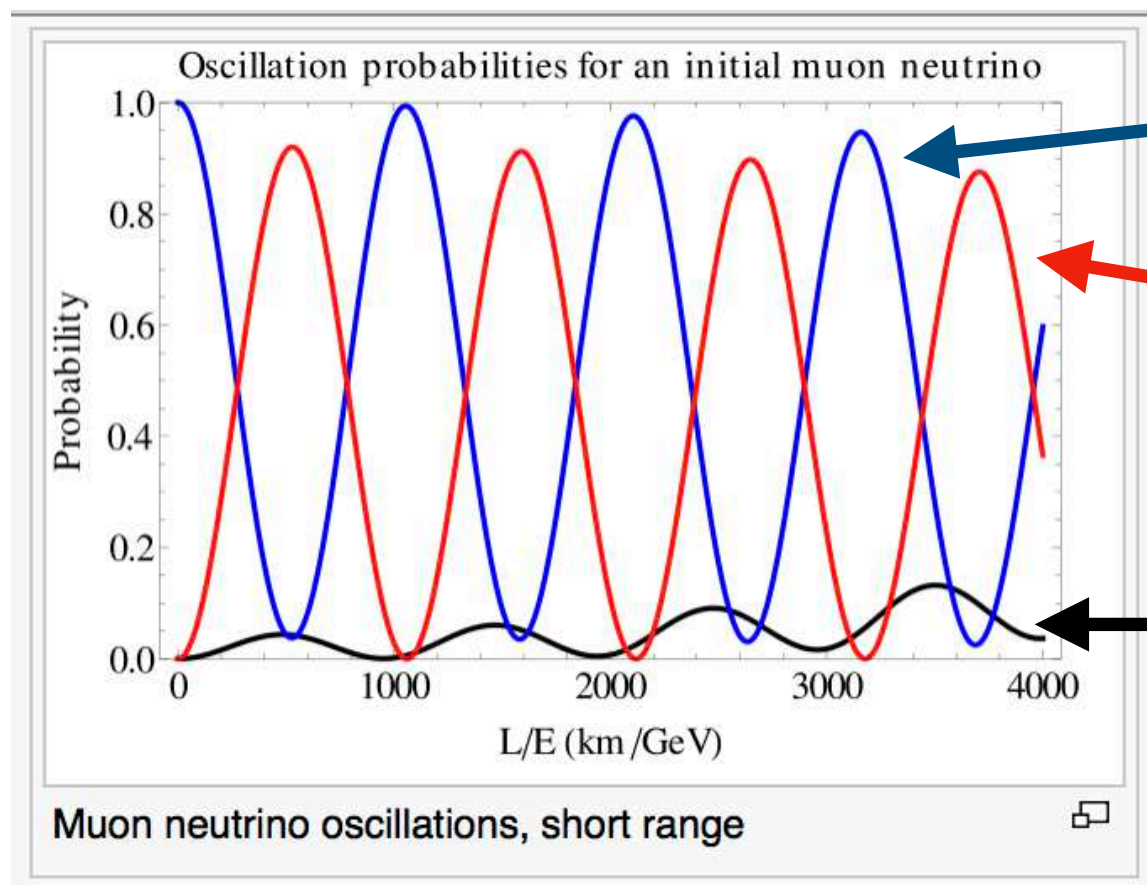
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CP-even
(disappearance and appearance)

CP-odd
(appearance only)

Neutrino oscillations, 3a

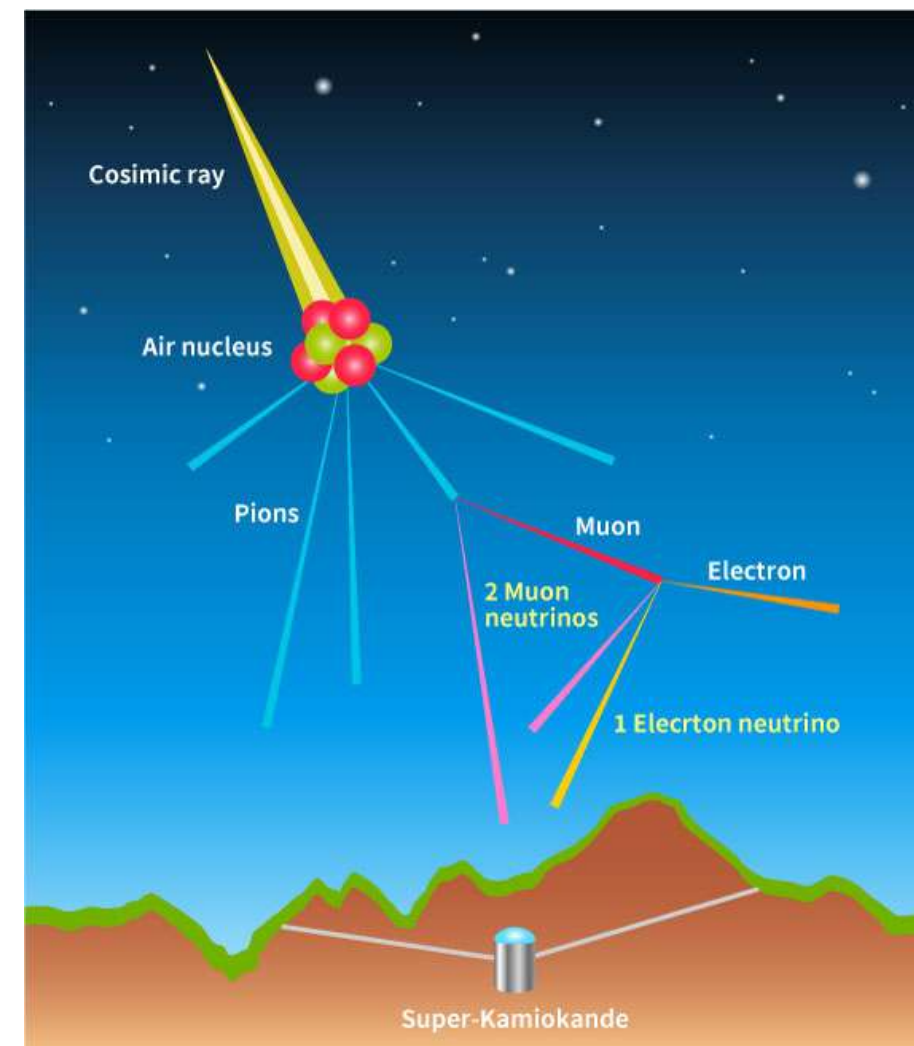
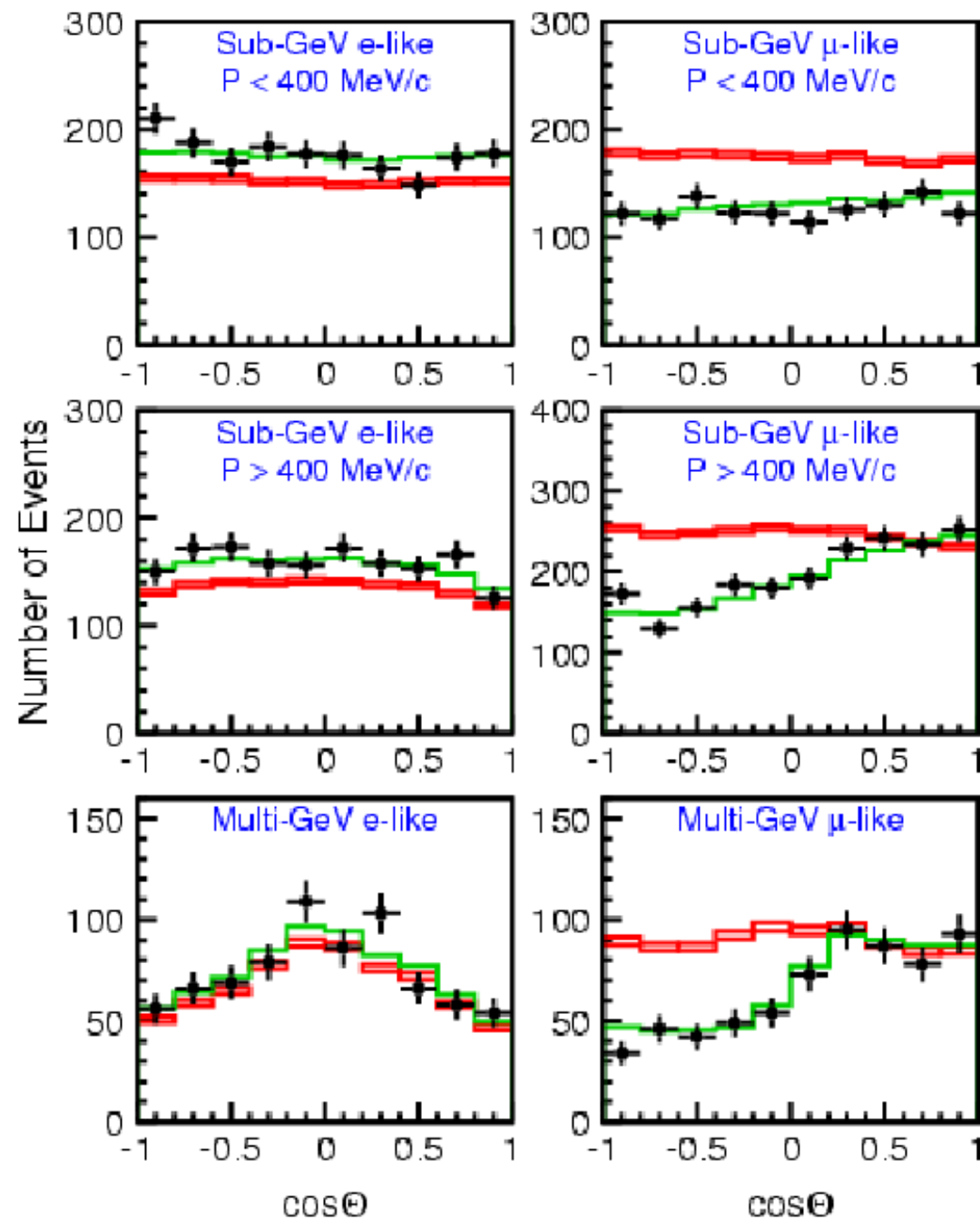
(Probabilities in vacuum)



$$i \frac{d}{dx} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}.$$

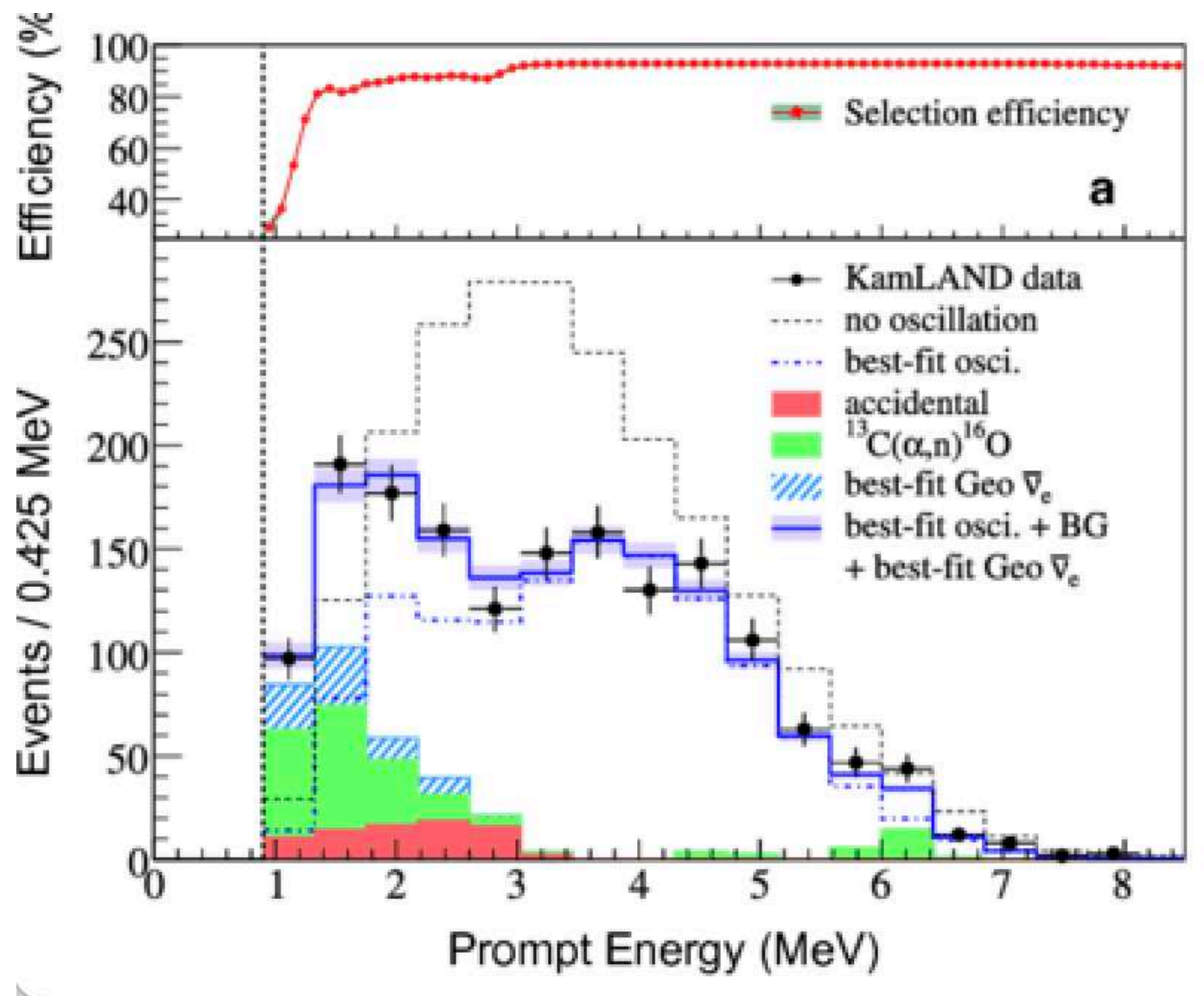
Neutrino oscillations, 3b

(SuperKamiokaNDE data: atmospheric neutrinos)

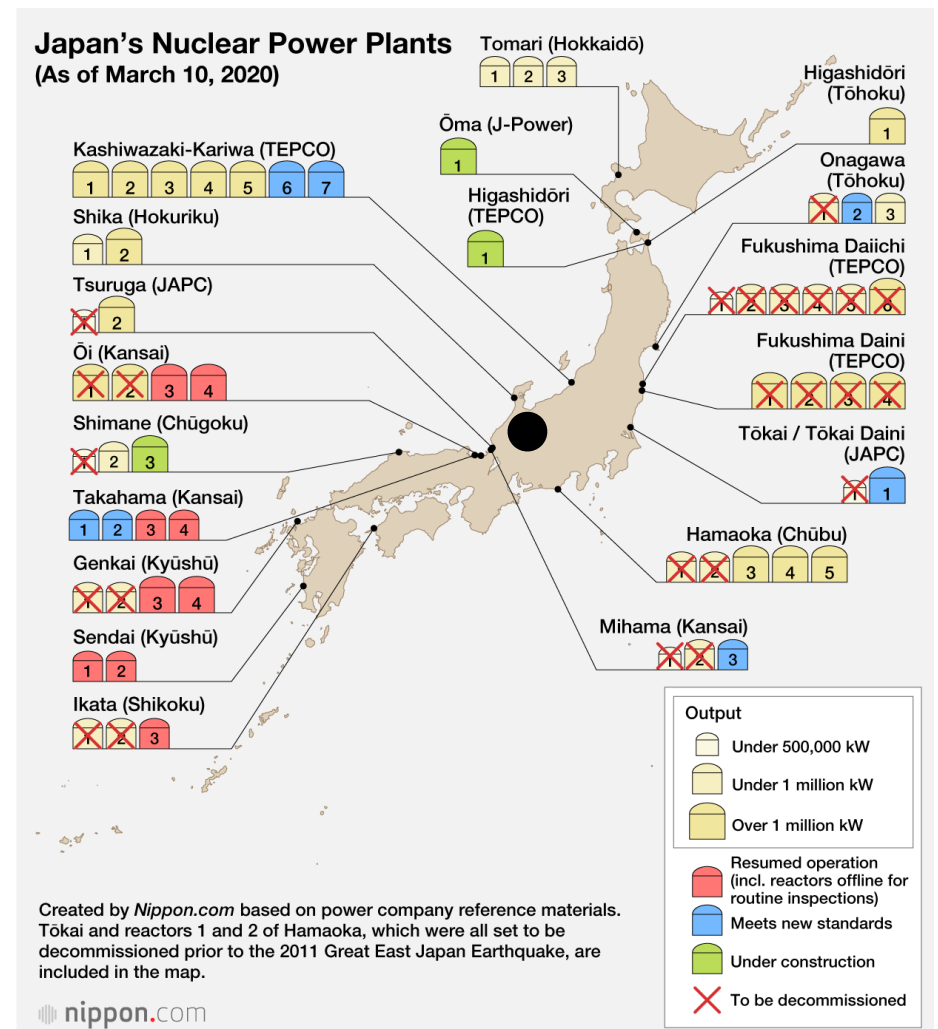


Neutrino oscillations, 3c

(KamLAND data: reactor neutrinos)

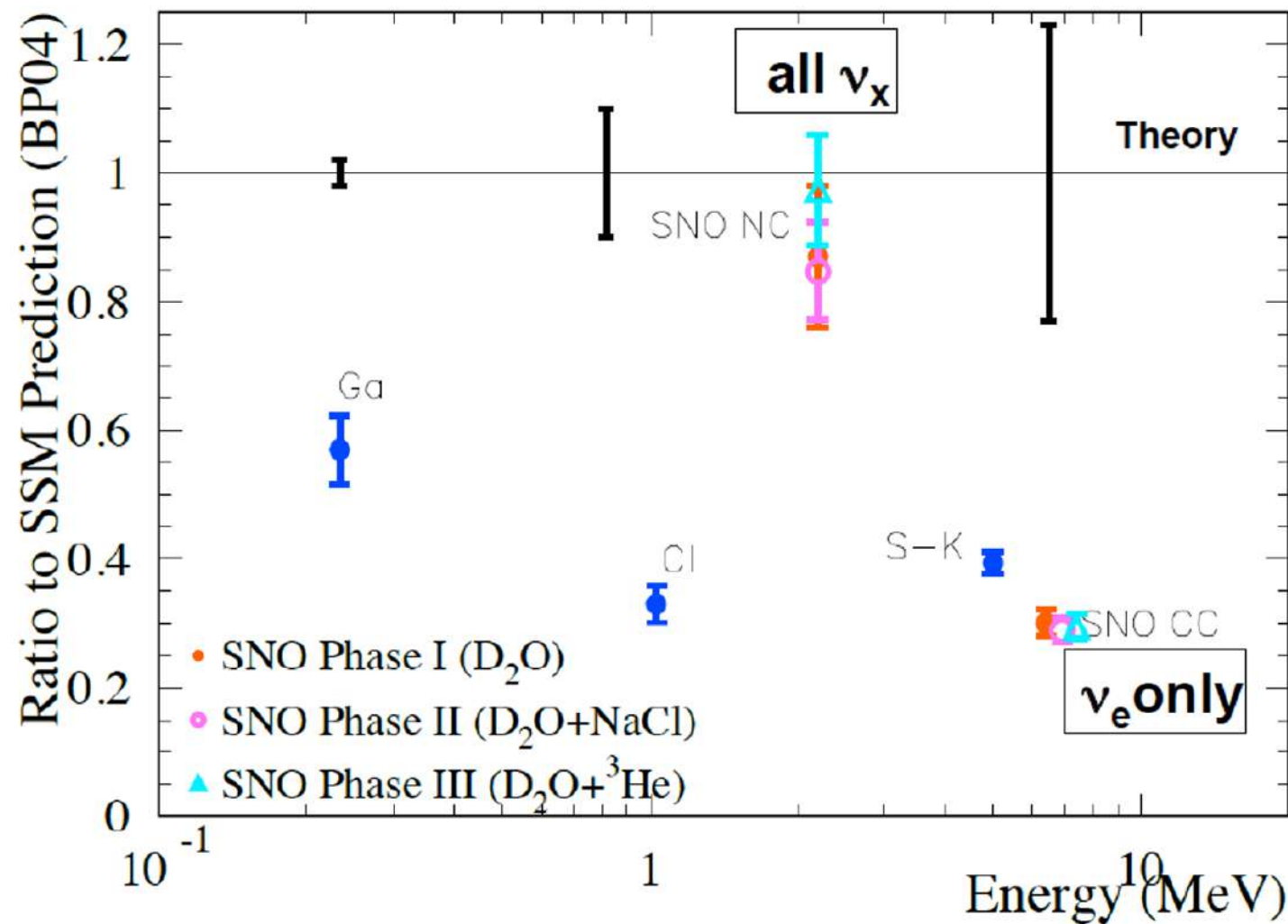


Expected
Observed



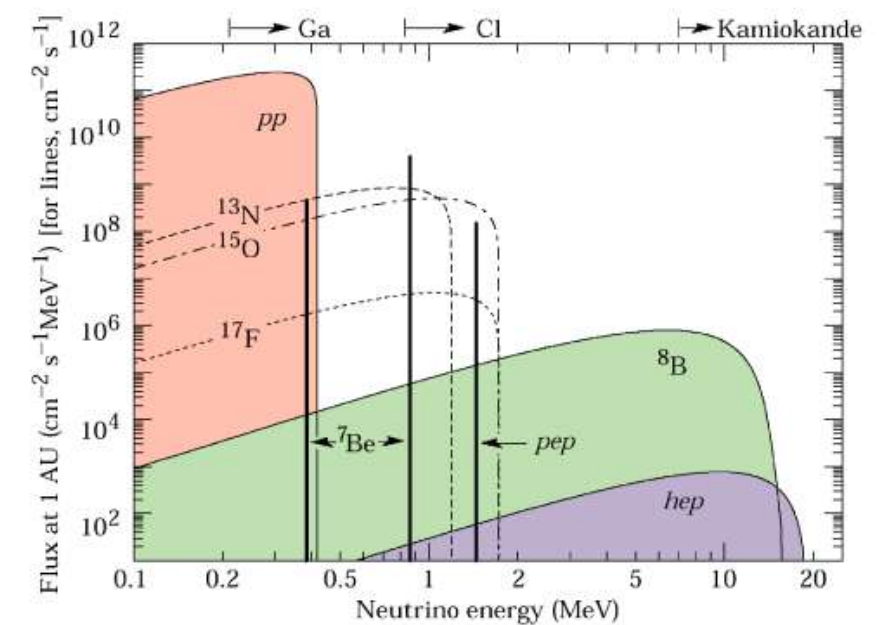
Neutrino oscillations, 3d

(SNO data: solar neutrinos NC vs CC)



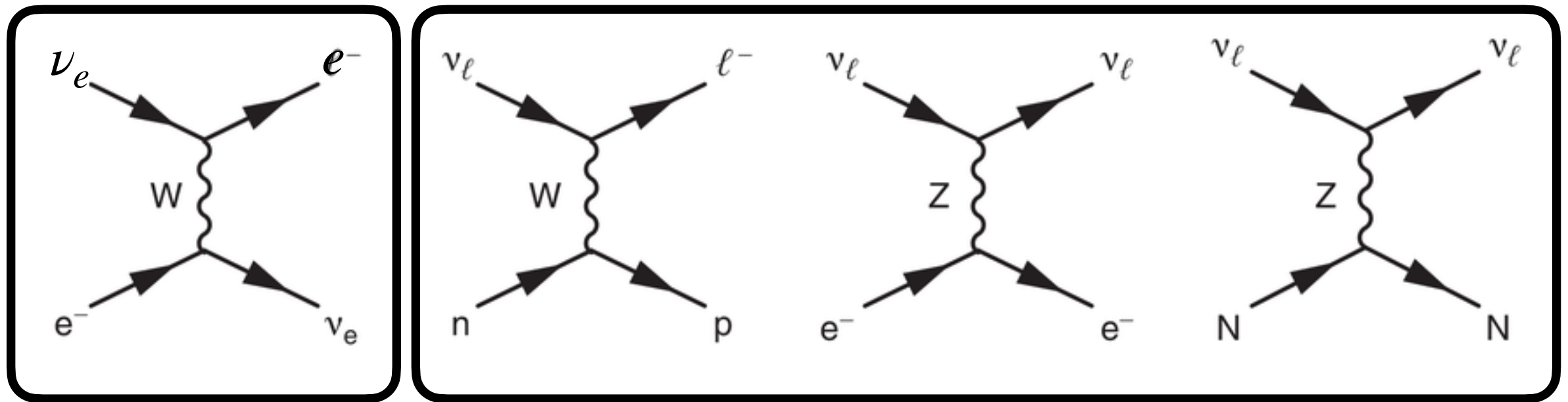
Observed all neutrinos

Observed ν_e



Neutrino oscillations, 4

(Probabilities in matter)



Only ν_e

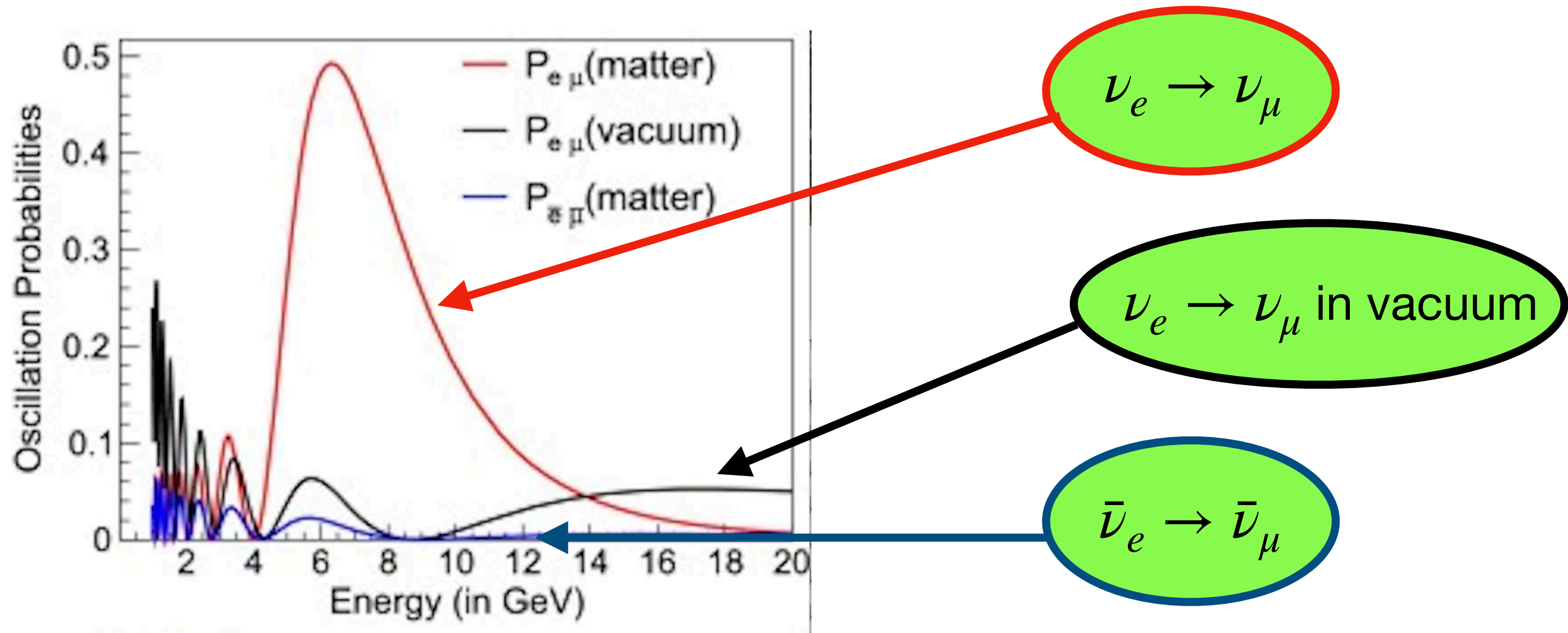
All neutrino species

$$i \frac{d}{dx} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \left[\frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \begin{pmatrix} \sqrt{2} G_F n_e(x) & 0 \\ 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \nu_e(x) \\ \nu_\mu(x) \end{pmatrix}$$

Hands-on session, exercise 1: diagonalize and find the value of the angle in vacuum for which there is resonant oscillation

Neutrino oscillations, 5a

(Probabilities in matter)



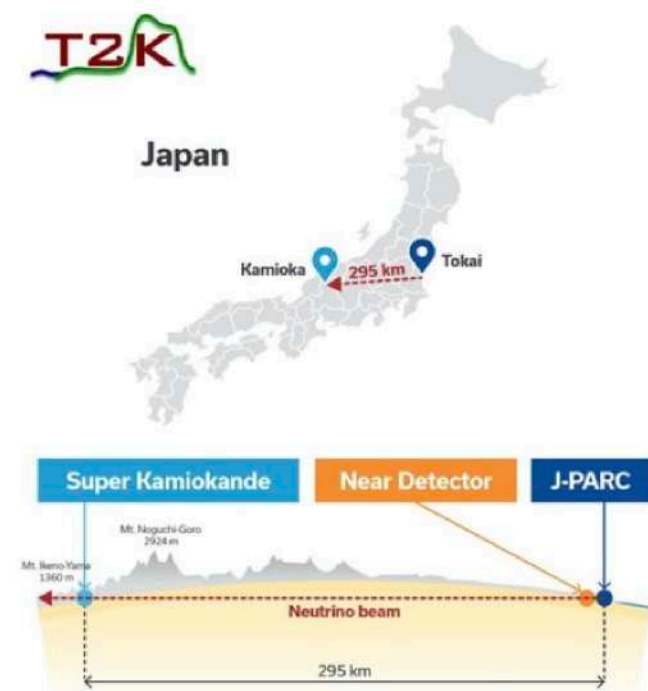
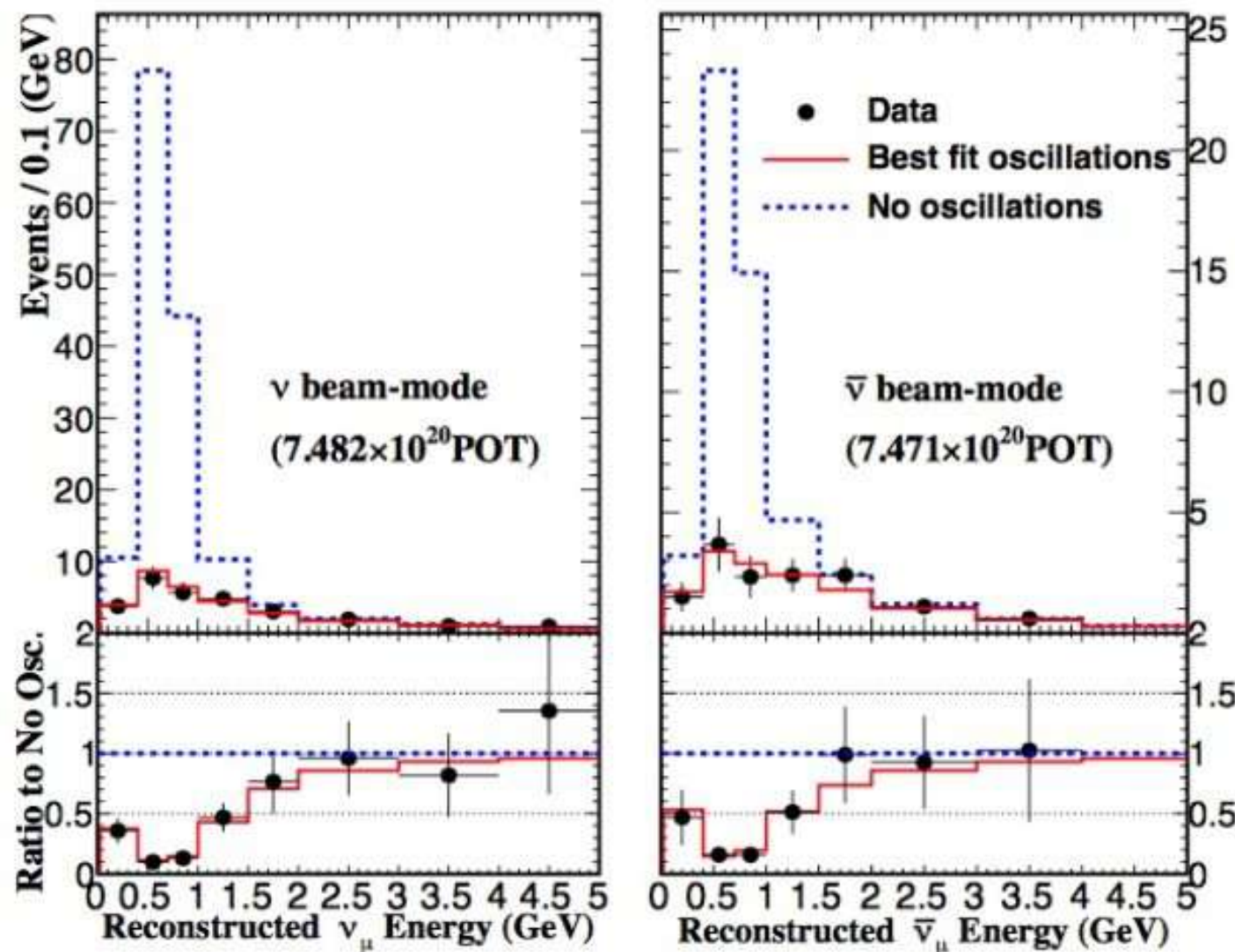
Matter matters!

Neutrino oscillations, 5b

(Probabilities in matter)

Matter is CP-odd!

(if you are looking for
leptonic CP-violation,
better you know this...)



Neutrino sources

1. Solar neutrinos: ν_e

2. Atmospheric neutrinos: $\nu_e, \nu_\mu, \bar{\nu}_e, \bar{\nu}_\mu$

3. Reactor neutrinos: $\bar{\nu}_e$

4. Accelerator-produced: $\nu_\mu, \bar{\nu}_\mu$ with some $\nu_e, \bar{\nu}_e$ component

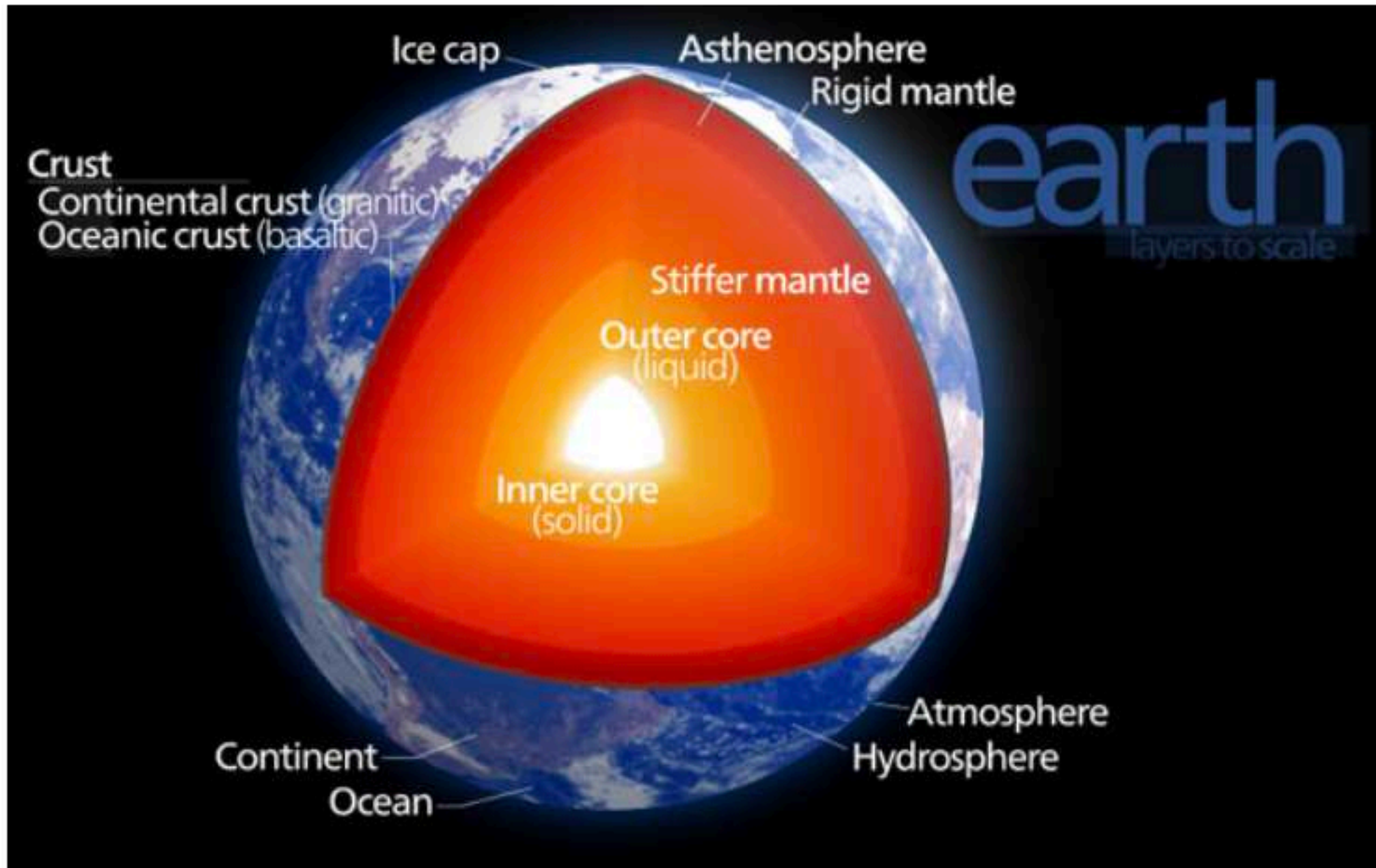
5. Geo-neutrinos: see L. Ludhova course

6. Astrophysical neutrinos (neutrino telescopes)

Class 2: outline

- 1. Why a neutrino Earth tomography**
- 2. Neutrino telescopes**
- 3. From the source to the detector**
- 4. 1st Neutrino Earth Tomography** (2019)
- 5. 2nd Neutrino Earth Tomography** (2026)

The Earth interior



Tomography

noun to·mog·ra·phy \ tō-'mä-grə-fē \

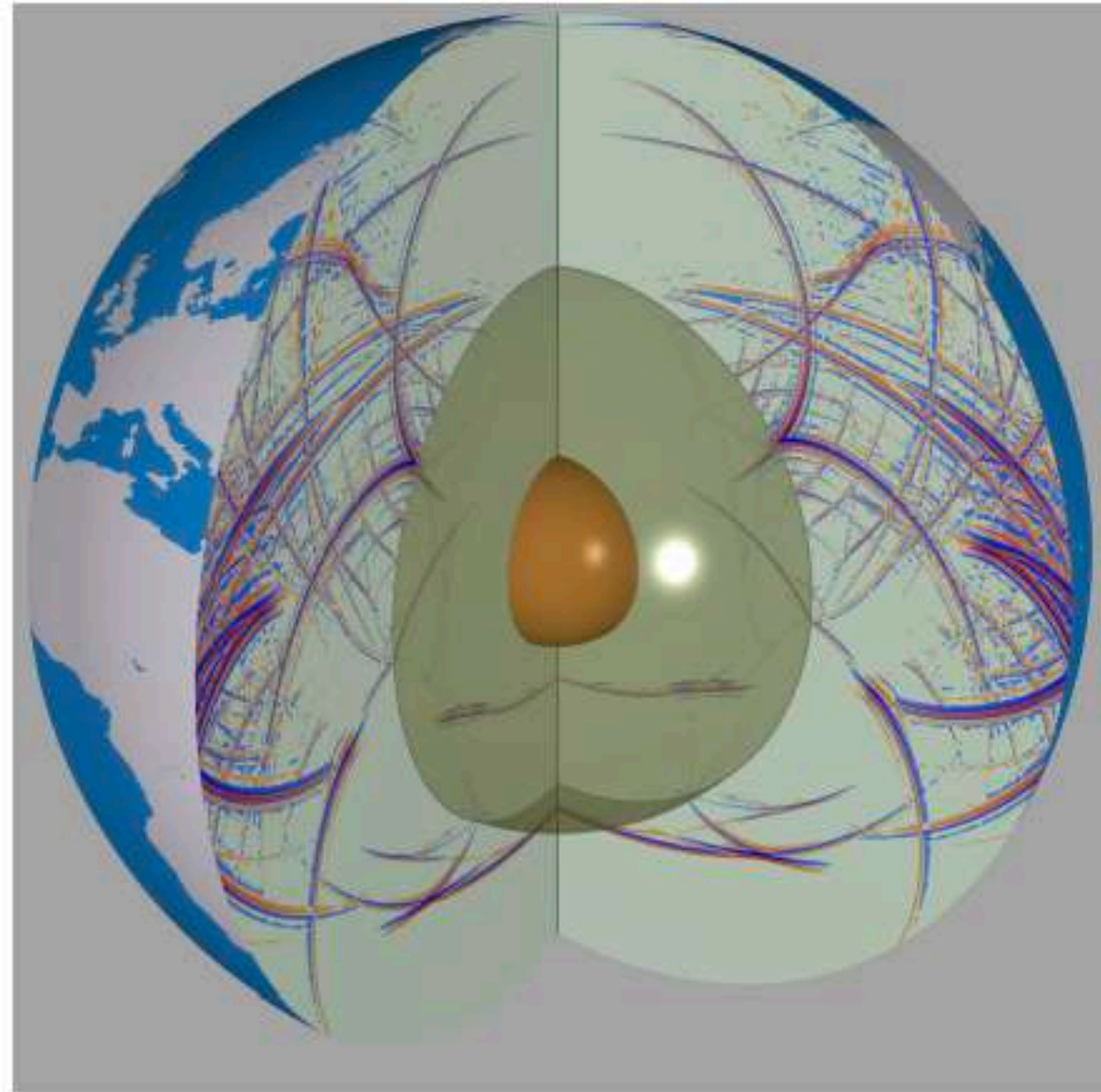
imaging by sections or sectioning, through the use of any kind of penetrating wave ([Wikipedia](#))

a method of producing a three-dimensional image of the internal structures of a solid object (such as the [human body](#) or the [earth](#)) by the observation and recording of the differences in the effects on the passage of waves of energy impinging on those structures ([Merriam-Webster](#))

How densities are measured?

seismology

$O(100)$ /year earthquakes
with magnitude
larger than 6



How densities are measured?

seismology

propagation of earthquake waves
through the Earth: p-waves and s-waves
(v_p and v_s)

Adams-Williamson equation (1924)

$$\frac{d\rho}{dr} = -\rho(r) \frac{g(r)}{\Phi(r)}$$

$$\Phi(r) = v_p^2 - \frac{4}{3}v_s^2$$

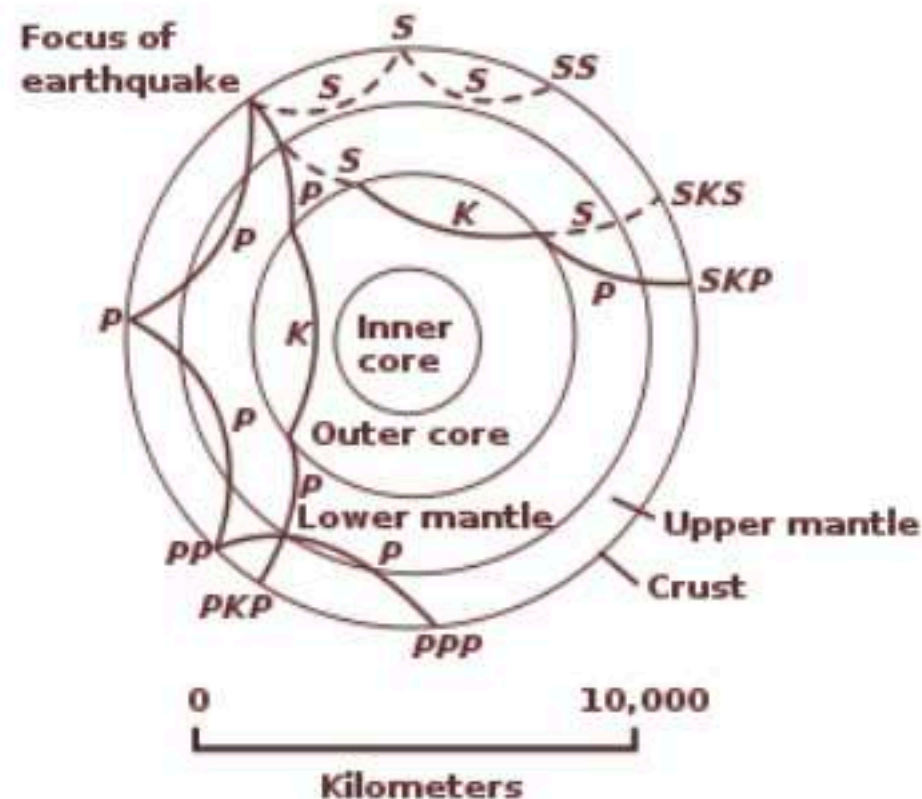
Composition dependence! Gravitational profile dependence!

The Earth's core

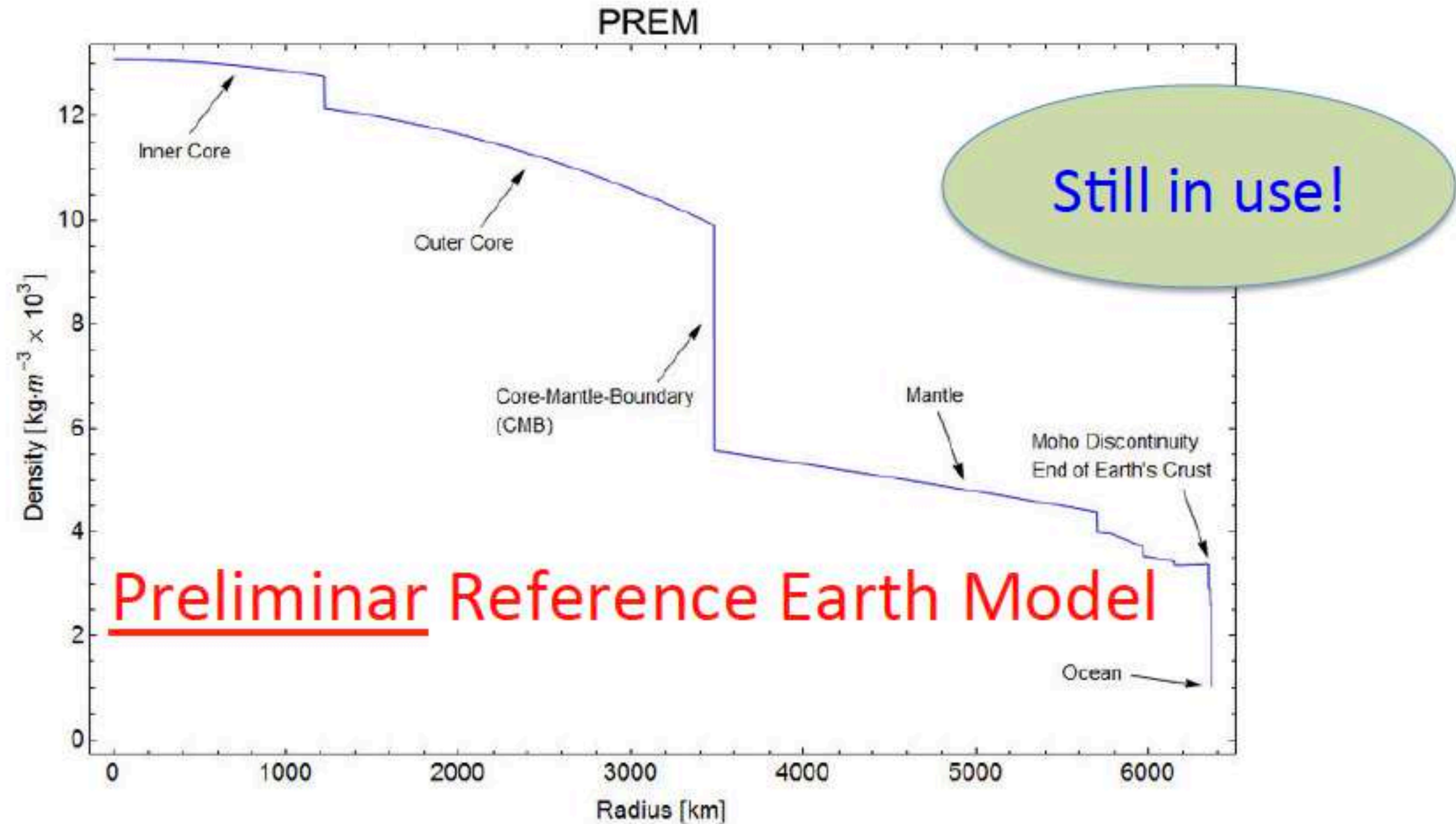
The **OUTER CORE IS LIQUID**,
whereas
the **INNER CORE IS SOLID**
(source of the geodynamo)

IT IS VERY DIFFICULT TO
HAVE DIRECT
INFORMATIONS from the
INNER CORE

Mostly through **global constraints**
and **extrapolations**

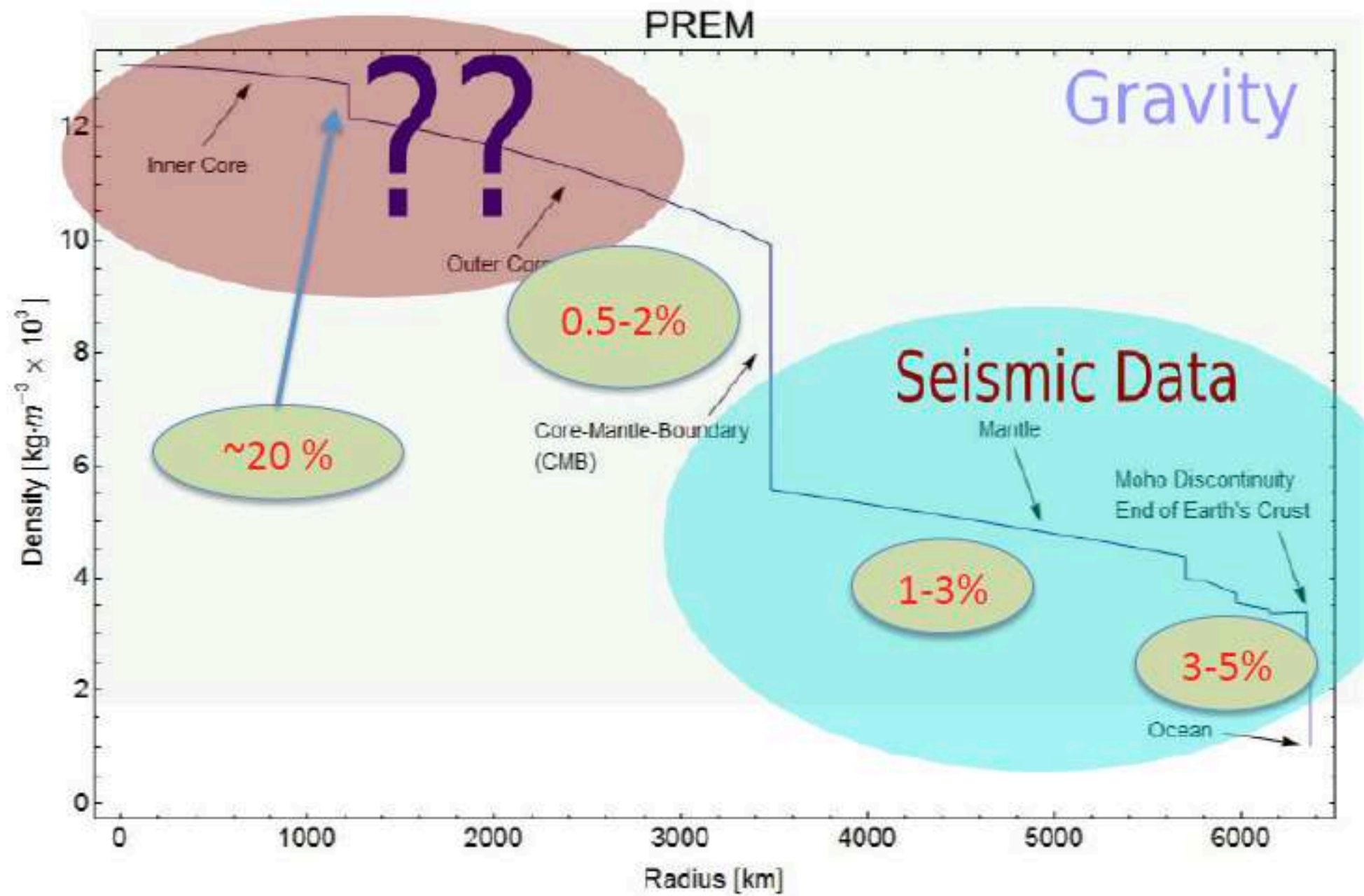


1-dimensional density profile



[Dziewonski and Anderson, Physics of the Earth and Planetary Interiors, 25 (1981)]

Uncertainties from seismology



Neutrinos to study the Earth's interior

An old idea: first mentioned in an unpublished CERN preprint,

A.Placci and E. Zavattini, submitted in Oct 1973 to Nuovo Cimento;
rejected?... never received?....

and in a talk

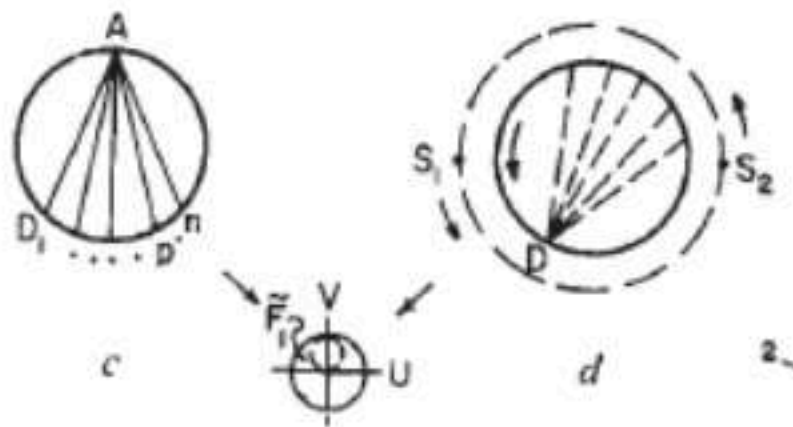
L. V. Volkova and G. T. Zatsepin, Izv. Akad. Nauk. Ser. Fiz. 38N5 (1974)

In modern language, a long-baseline experiment

The idea was premature!

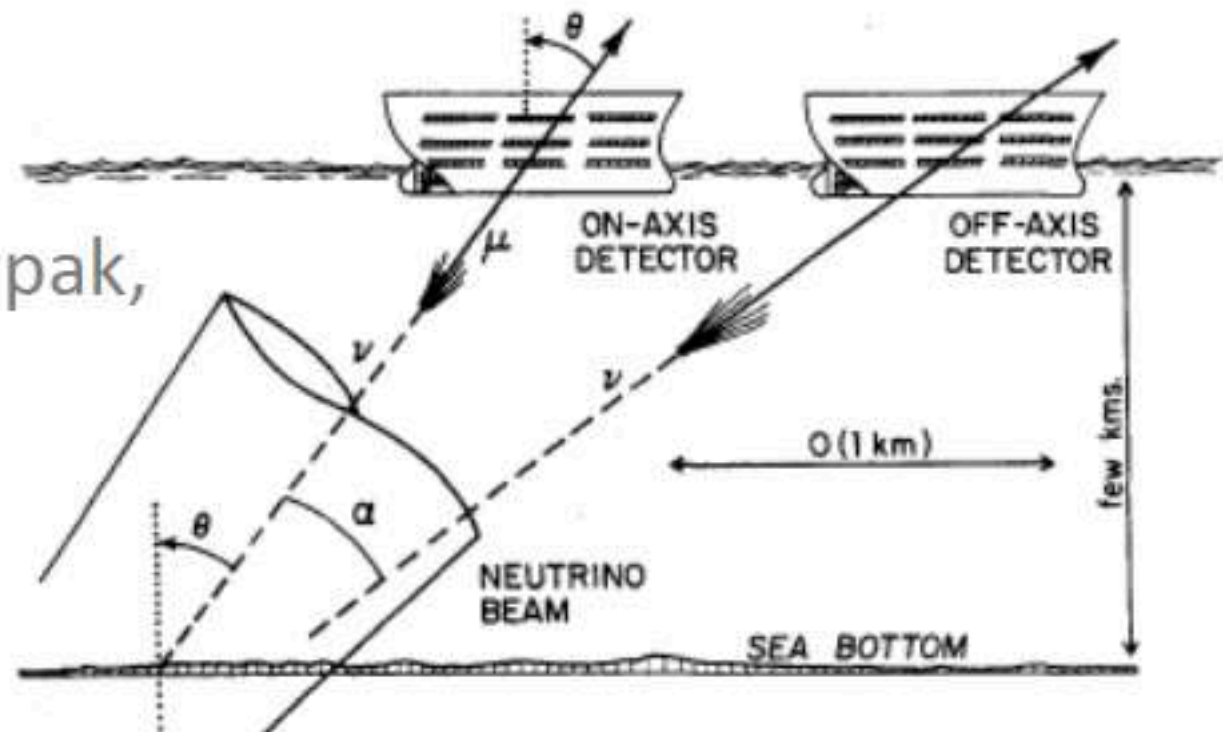
Neutrino tomography

Even more premature...



T. Wilson, Nature 309 (1984)

De Rújula, Glashow, Wilson, Charpak,
Phys. Rept. 99 (1983)



Two ways to scan the Earth

- Neutrino oscillations (< 1 TeV)

$$P_{ee}^{\pm} = 1 - \left(\frac{\Delta_{23}}{B_{\mp}} \right)^2 \sin^2(2\theta_{13}) \sin^2 \left(\frac{B_{\mp} L}{2} \right) - \left(\frac{\Delta_{12}}{A} \right)^2 \sin^2(2\theta_{12}) \sin^2 \left(\frac{A L}{2} \right)$$

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W. Winter, Nucl. Phys. B 908 (2016) 250; Km3Net, PoS ICRC2017 (2018) 1020

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- Neutrino flux attenuation (> 1 TeV)

$$\frac{d\phi_{\nu}(E, \tau)}{d\tau} = -\sigma_{tot}(E)\phi_{\nu}(E, \tau)$$

Two ways to scan the Earth

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$$\sigma_{tot}(E) = \rho(E) \times \sigma_{\nu N}(E)$$

Gonzalez-García, Halzen, Maltoni, Tanaka, Phys. Rev. Lett. 100 (2008)

Two ways to scan the Earth

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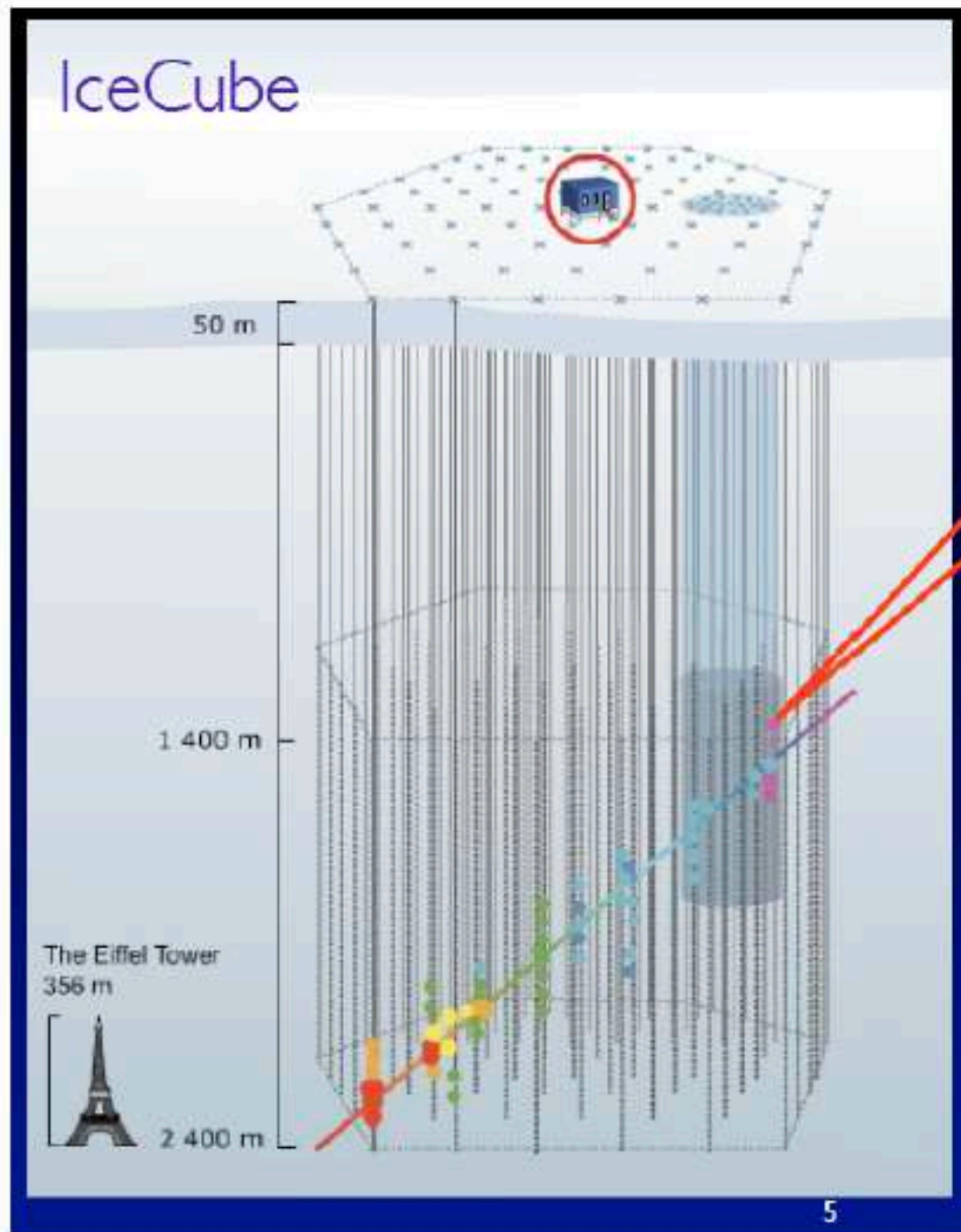
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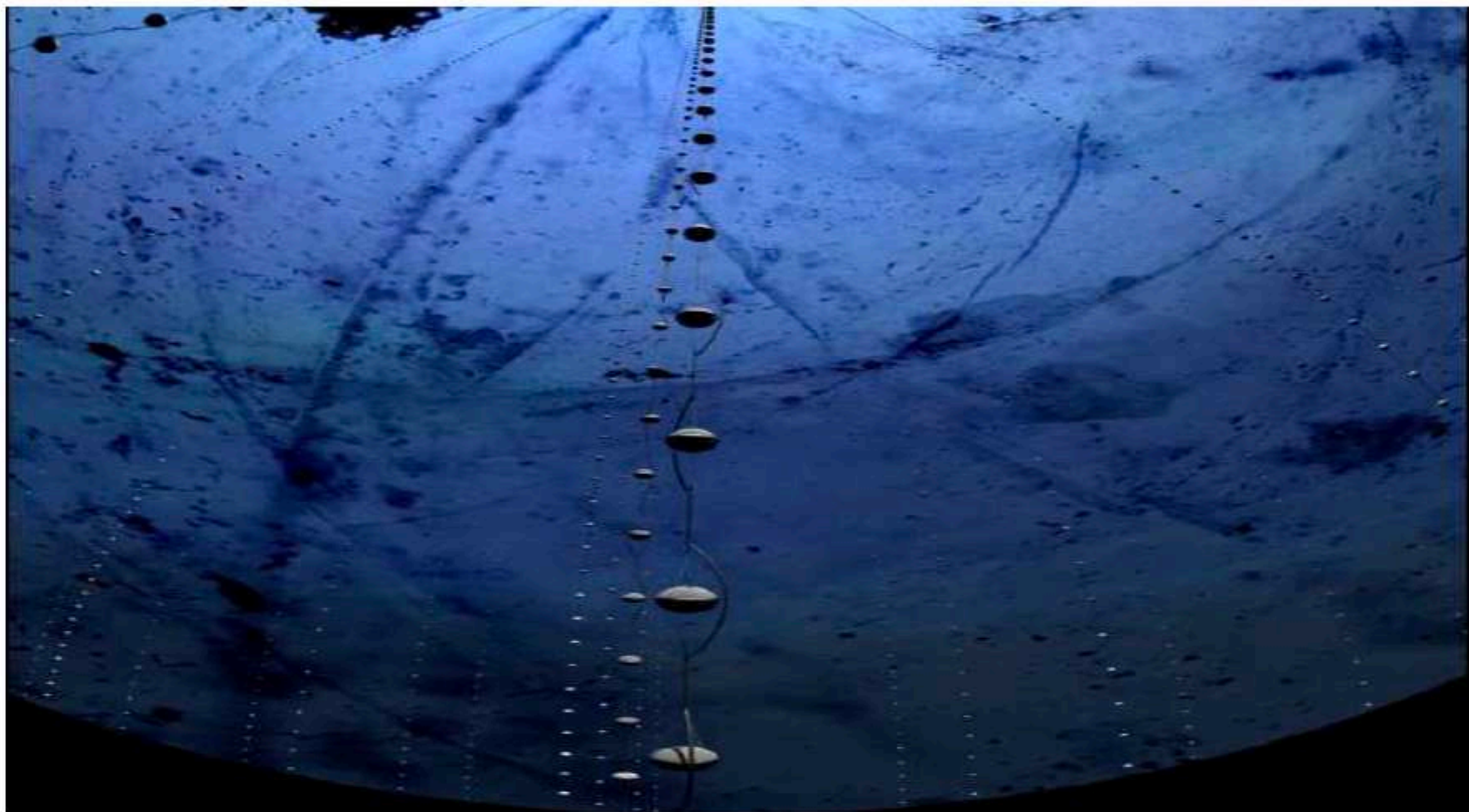
IceCube: a 1 km³ neutrino telescope



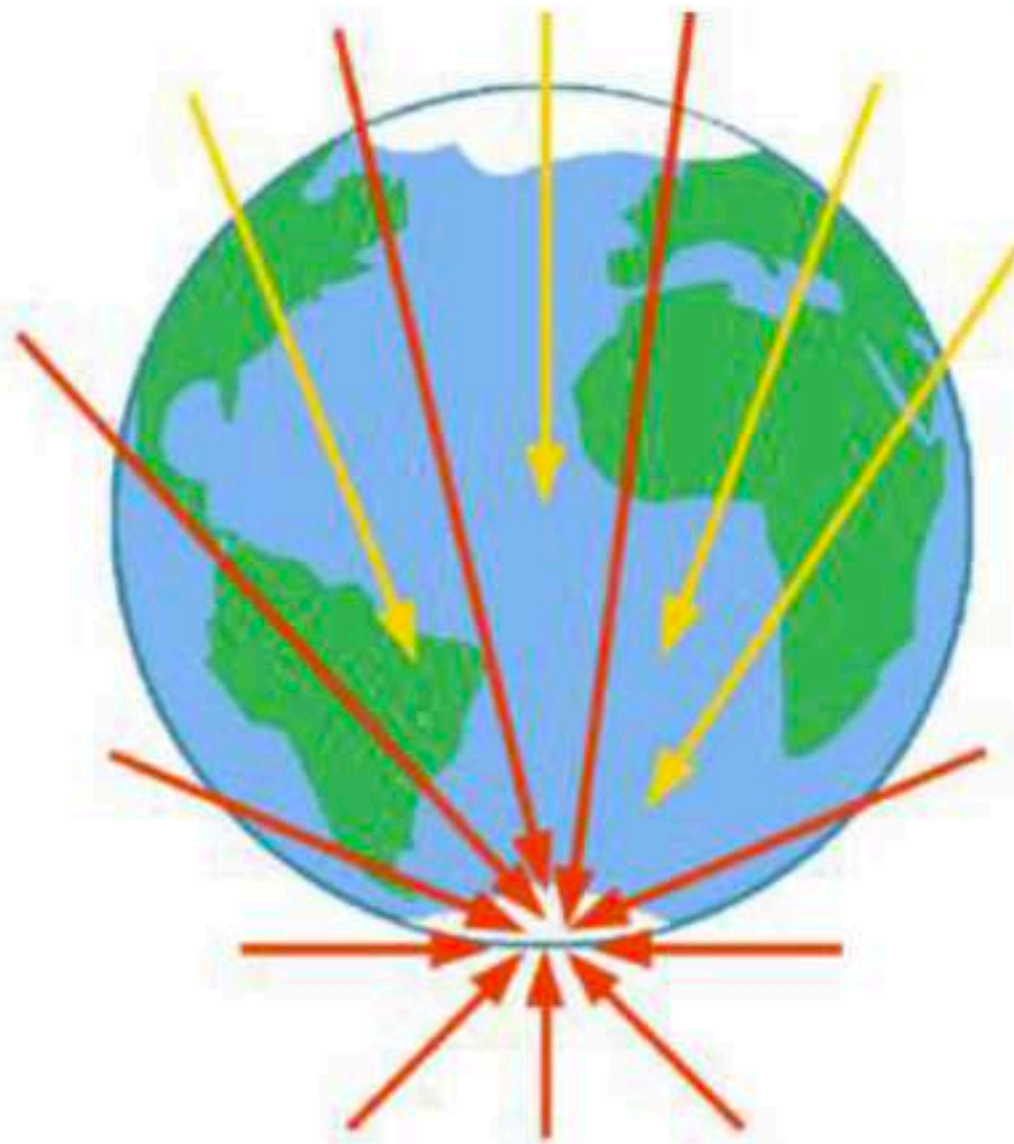
The IceCube Experiment



- Deployed in glacial ice at the South Pole
- Array size 1 km^3 , 86 strings, 60 optical sensors (DOMs) per string

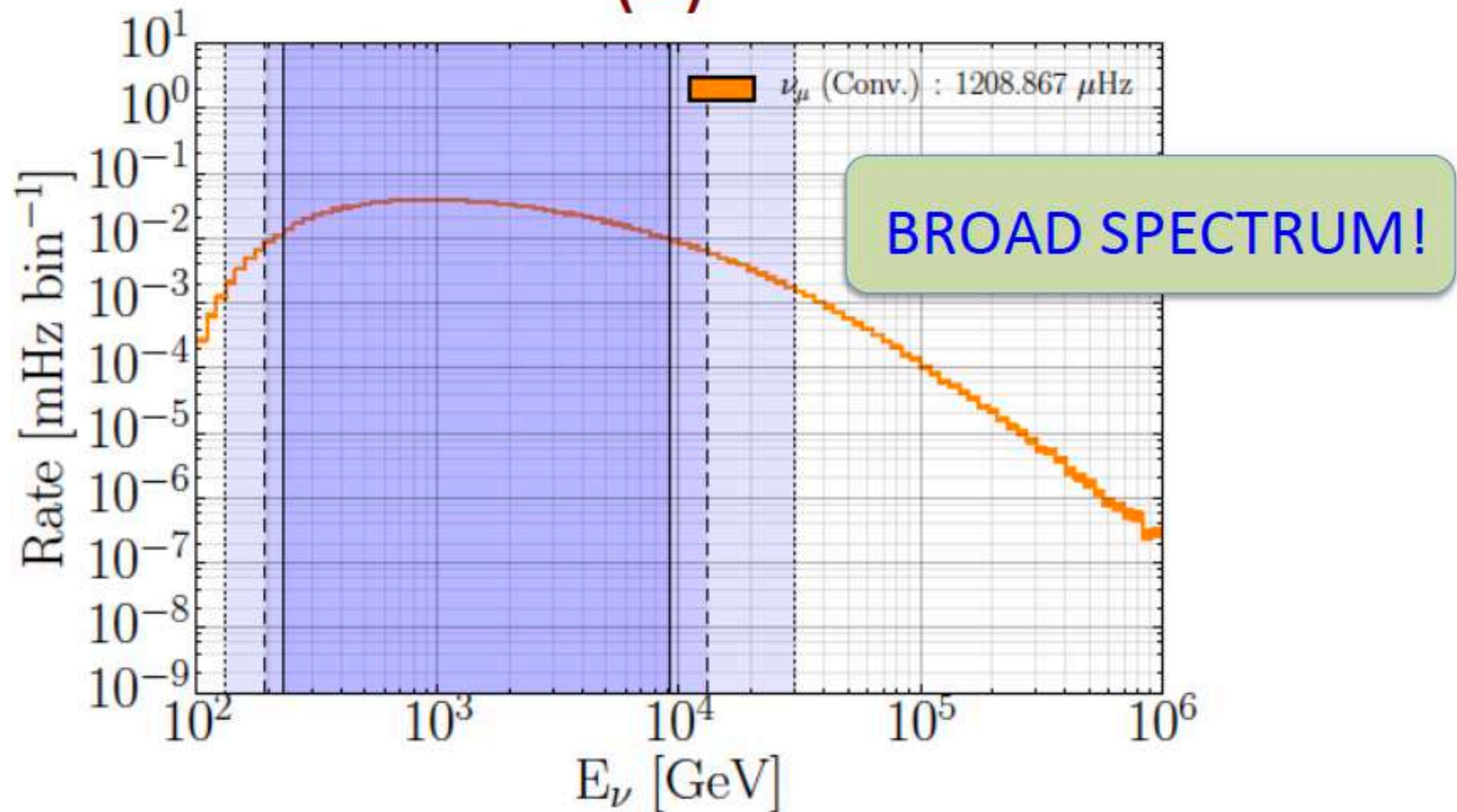


Atmo-neutrinos are an optimal source (1)



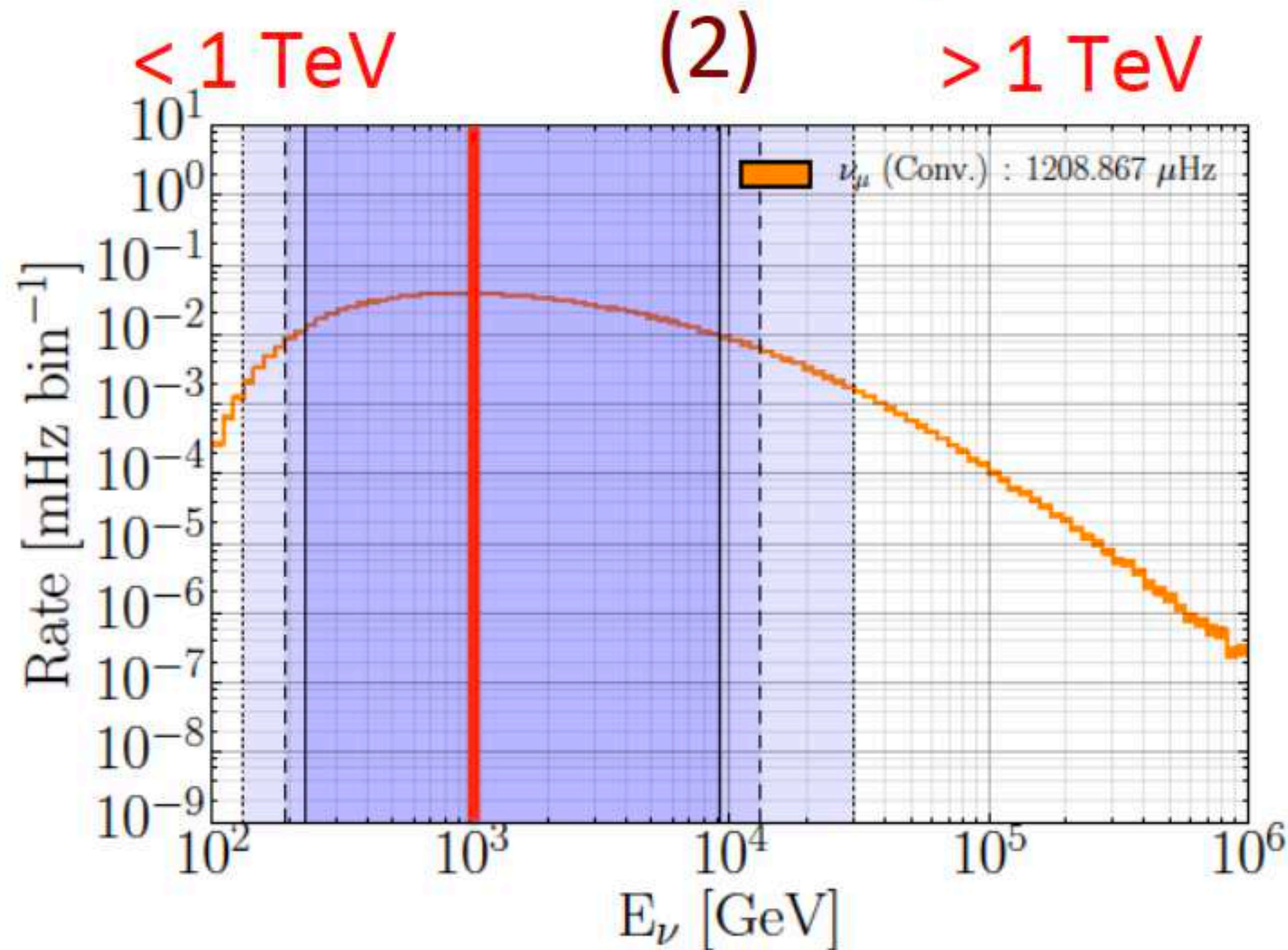
They reach a
detector
from all
directions

Atmo-neutrinos are an optimal source (2)



IceCube Collaboration, Aartsen et al , Phys. Rev. D102 (2020)

Atmo-neutrinos are an optimal source



IceCube Collaboration, Aartsen et al , Phys. Rev. D102 (2020)

The IceCube IC86 data sample

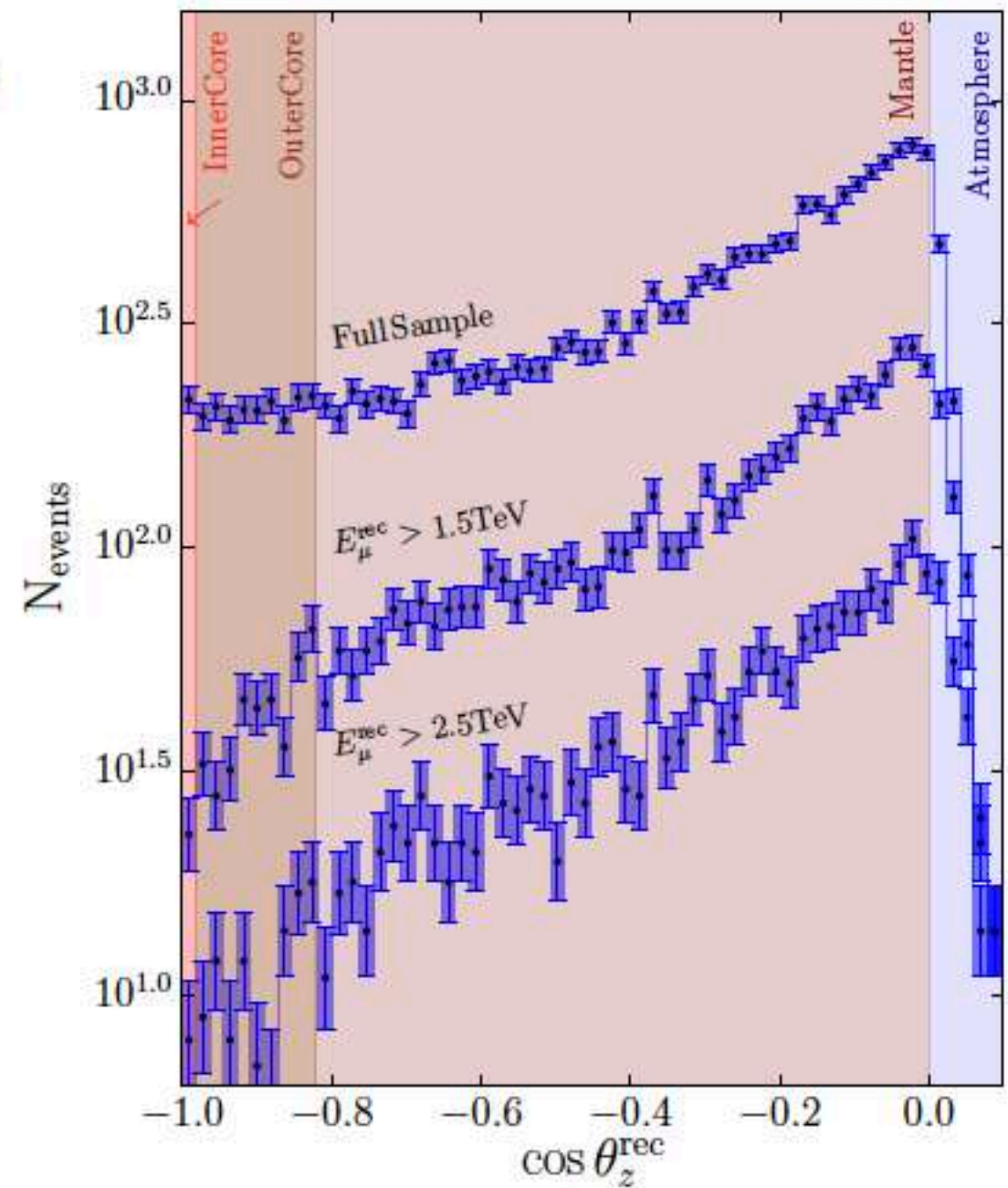
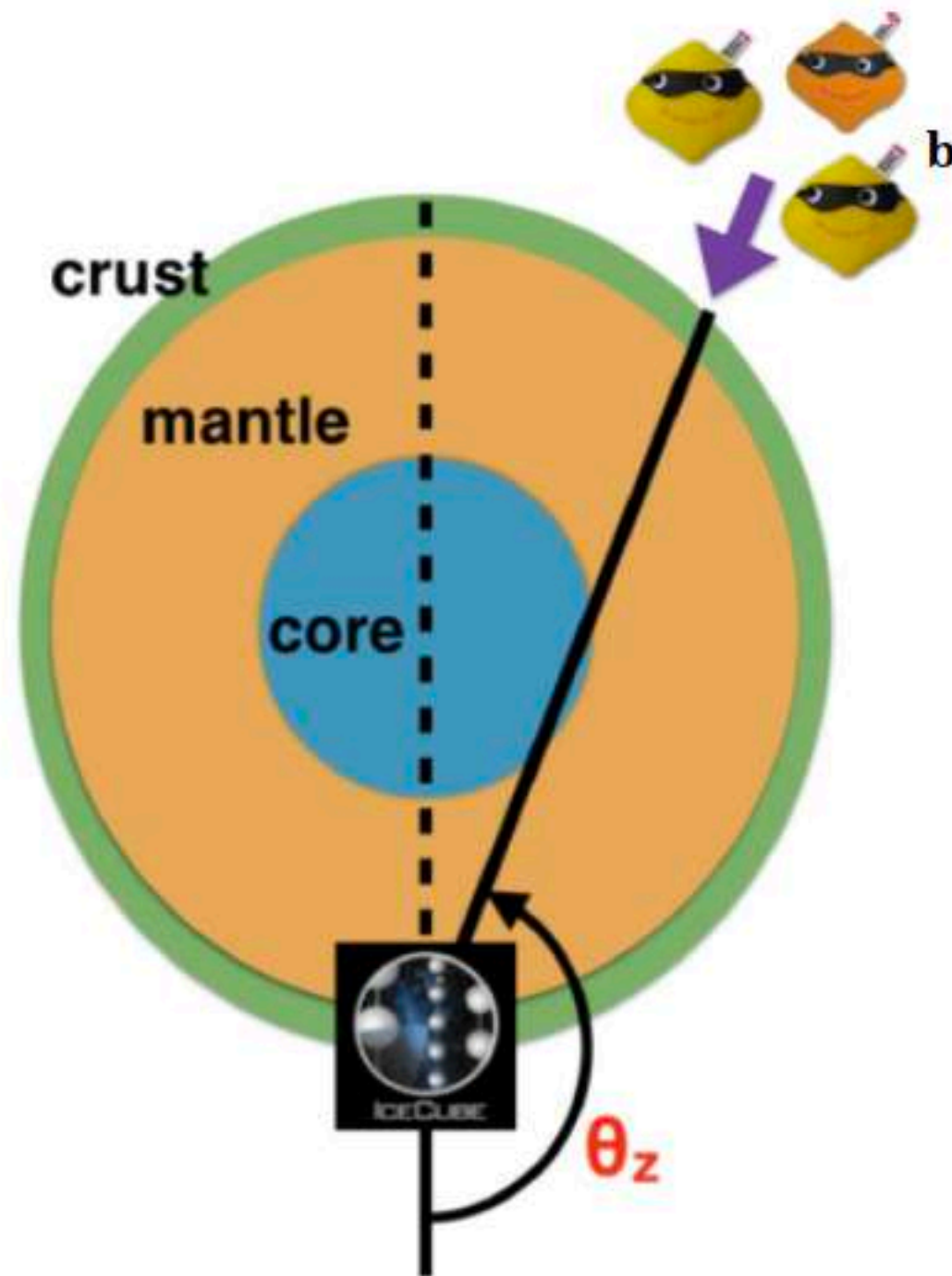
- 1 year of data taking (2011-2012)
- 20145 muon events over 343.7 days
- $E_{\mu} = [400 \text{ GeV} \div 20 \text{ TeV}]$
- The muon direction is a very good proxy of the neutrino direction, with $\Delta\cos\theta < 0.01$
- PUBLICLY AVAILABLE!

The IceCube IC86 data sample

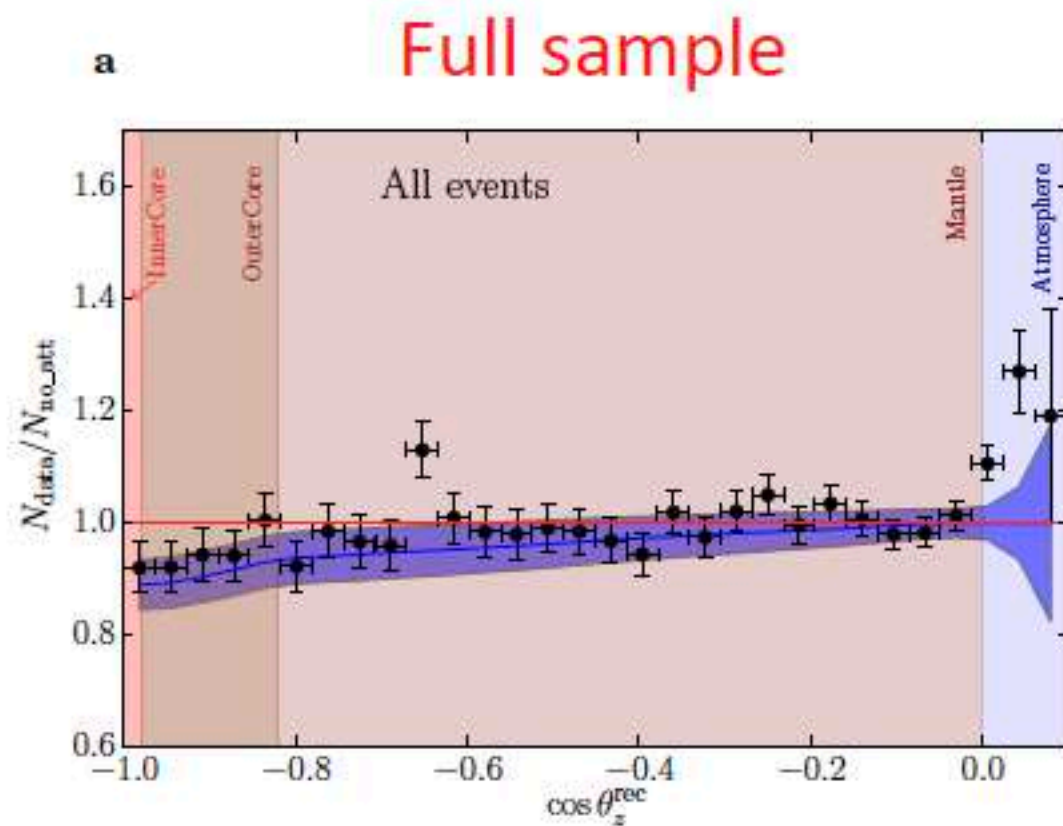
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WE USED THIS SAMPLE IN 2018 TO PERFORM THE FIRST NEUTRINO EARTH TOMOGRAPHY. SEVEN MORE YEARS OF DATA WERE NOT AVAILABLE AT THAT TIME

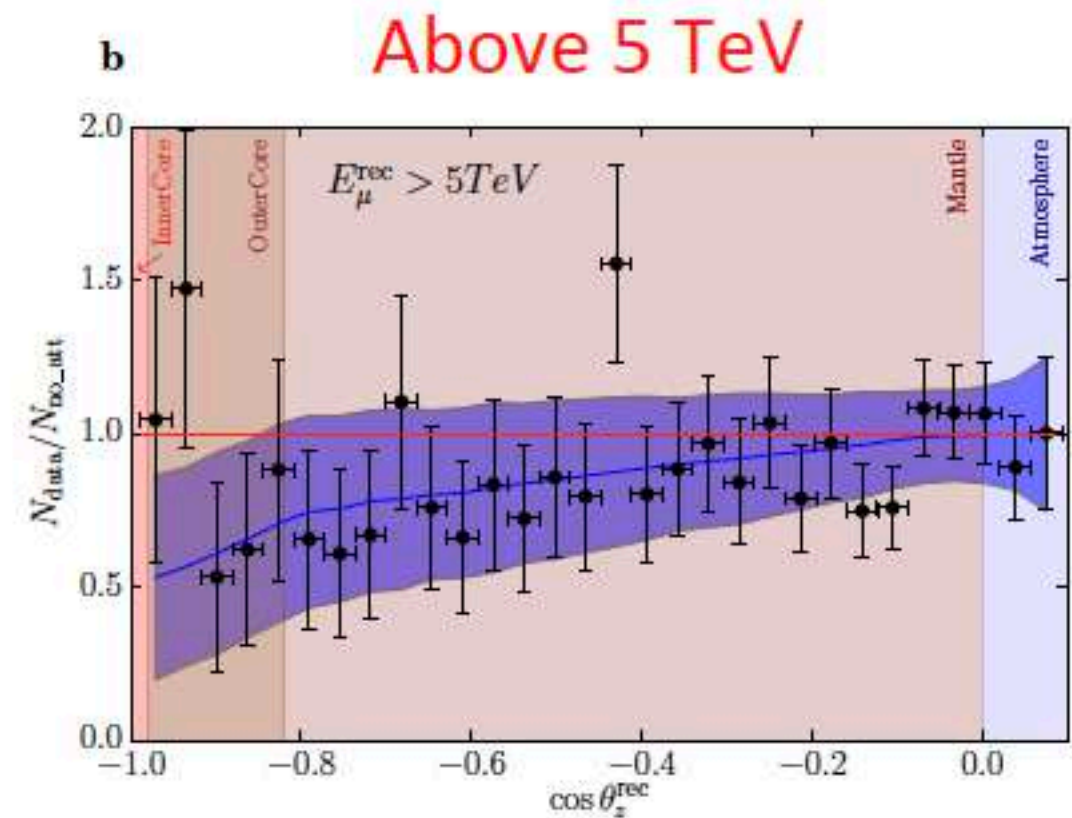
Raw data as a function of E_μ and θ



Including energy cuts: $N_{\text{data}}/N_{\text{noatt}}$

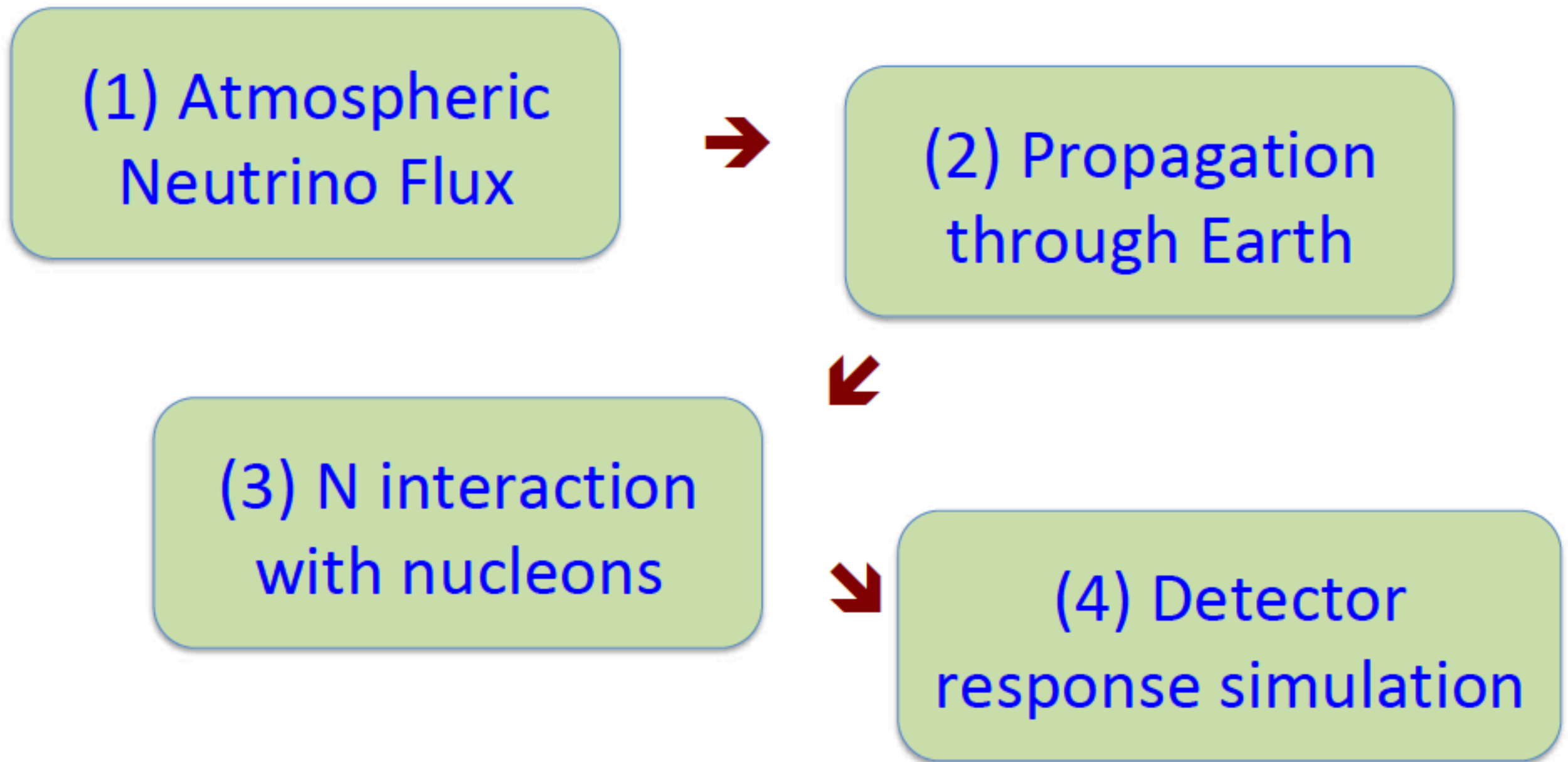


Full sample
useful for
normalization

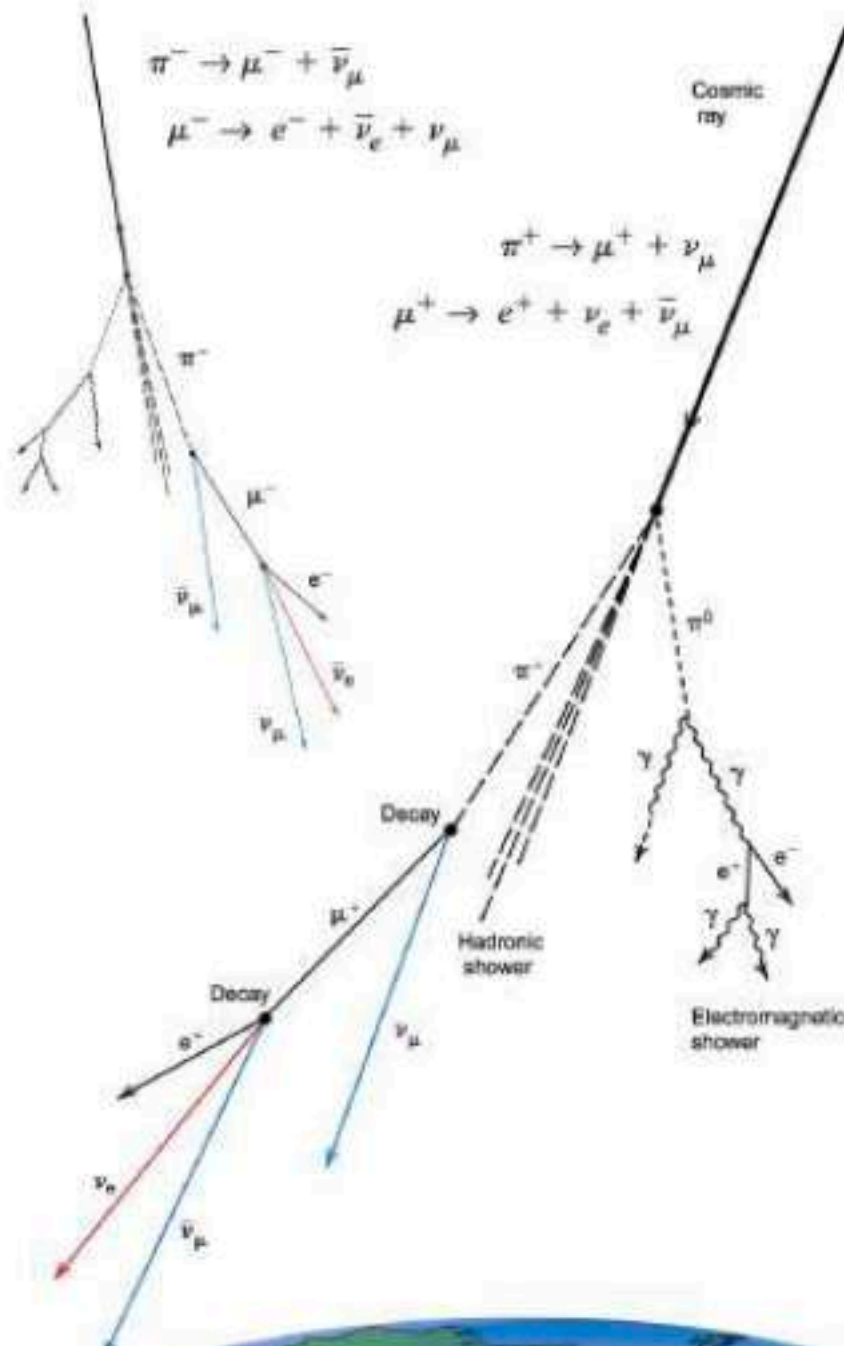


For $\cos \theta > -0.6$,
attenuation can be
as large as 50%

From raw data to measurements



N_{exp} : (1) atmo- ν flux model



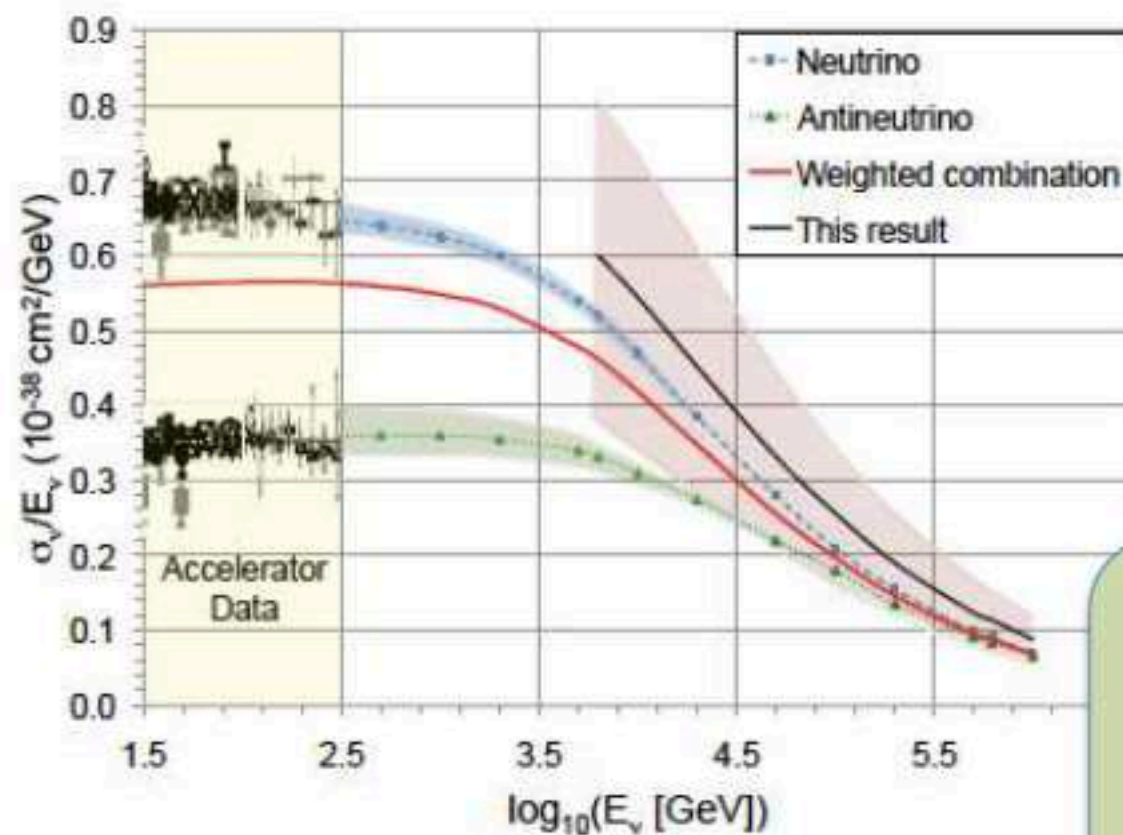
Primary cosmic ray flux:
Honda-Gaisser model +
Gaisser-Hillas corrections
(HG-GH-H3a)

Hadronic model: QGSJET-II-04

We tried other options →
“discrete” systematics

$N_{\text{exp}}: (3)$ neutrino-nucleon interaction

Remember: $\sigma_{\text{tot}} = \sigma_{\nu N} \times \rho_E$



Aarsten et al, Nature 551 (2017)

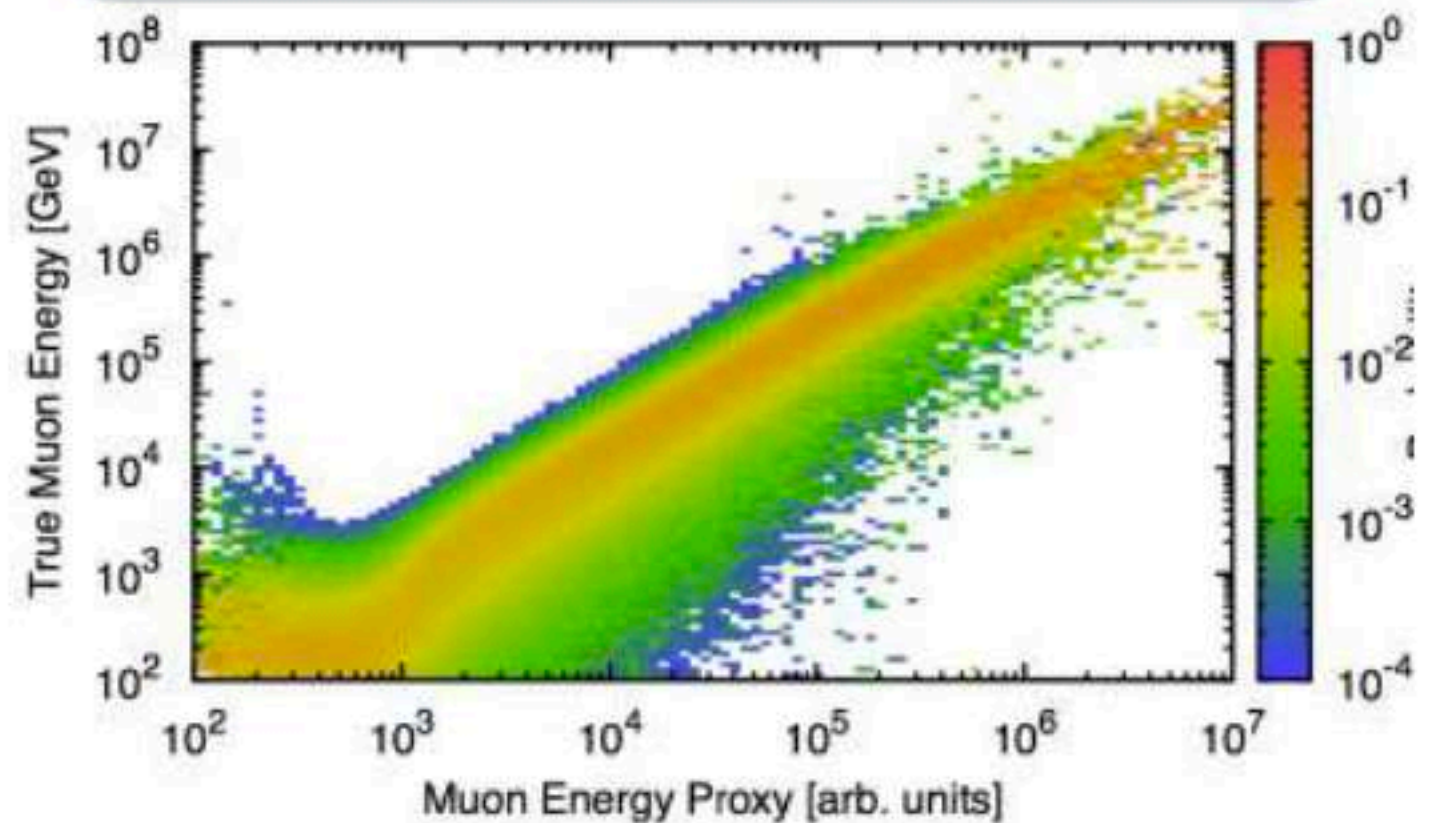
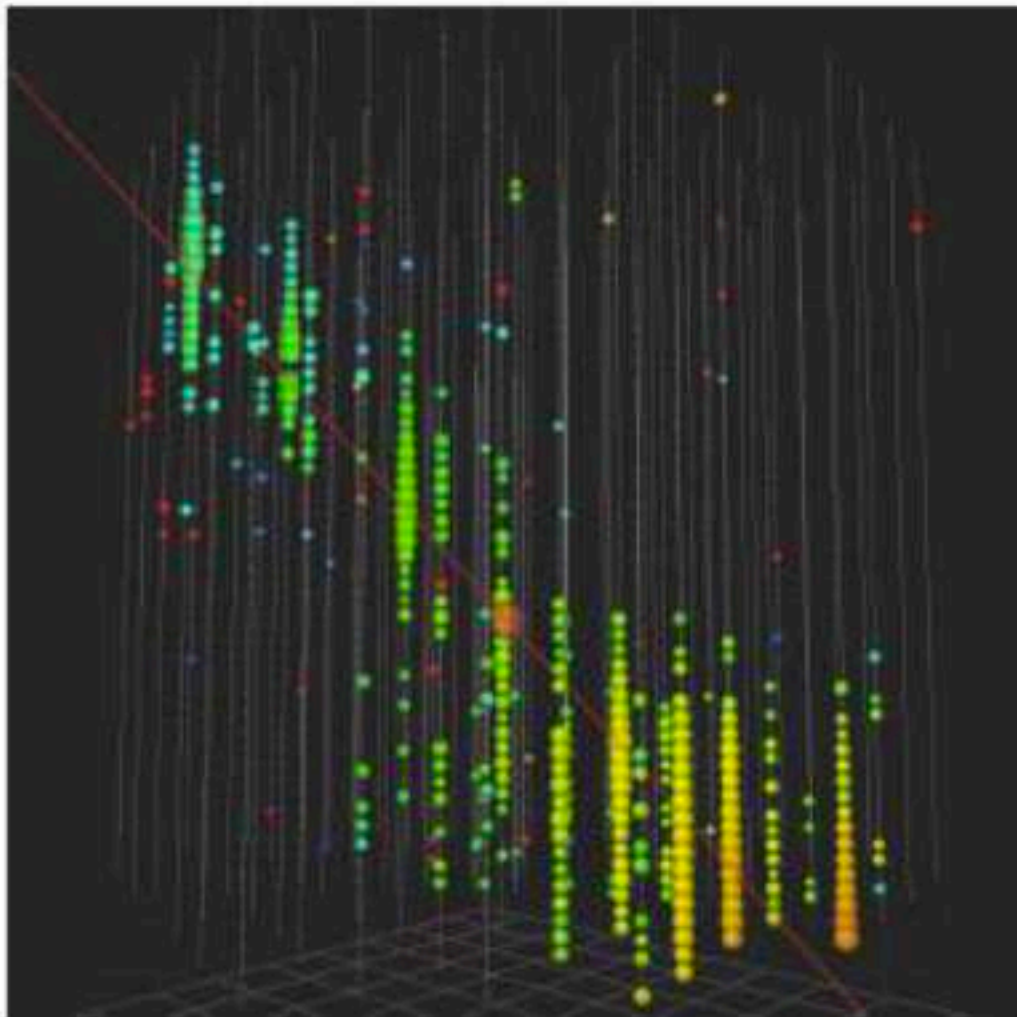
νN ($\bar{\nu} N$) cross-sections
at 2-3% (4-10%) errors

ICECUBE MEASUREMENT
 $1.30^{+0.21}_{-0.19} \text{ (stat)}^{+0.39}_{-0.43} \text{ (syst)} \times \sigma_{\text{SM}}$

N_{exp} : (4) detector simulation

We use the official IceCube MC to map

$E_{\text{obs}}^{\mu}, \theta_{\text{obs}}^{\mu}$ into $E_{\text{rec}}^{\nu}, \theta_{\text{rec}}^{\nu}$



<https://icecube.wisc.edu/science/data/IC86-sterile-neutrino>

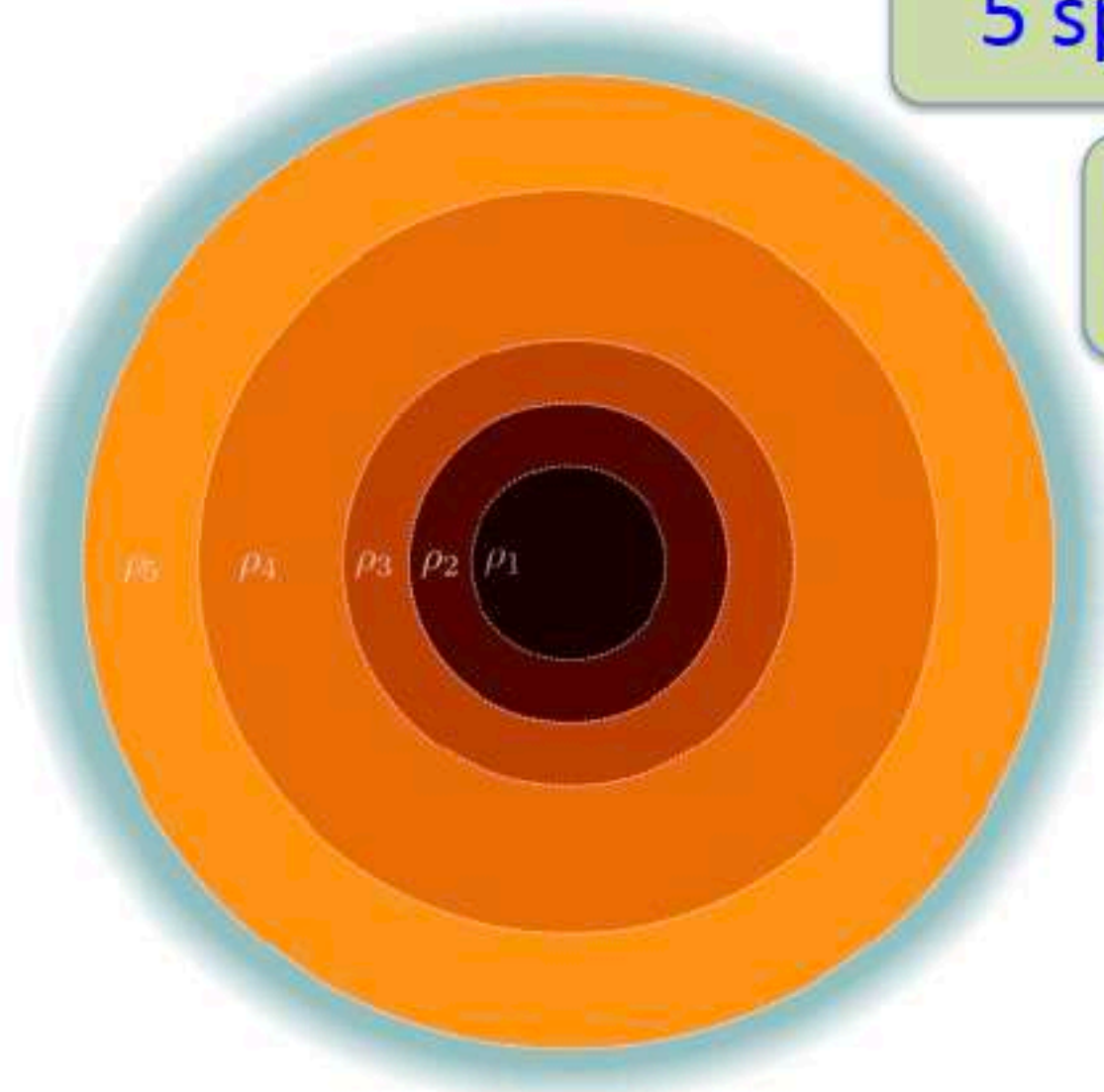
N_{exp} : (2) neutrino propagation

Propagation through the Earth with ν -SQulDs

Argüelles Delgado, Salvado & Weaver, Comput. Phys. Commun. 196 (2015)

- **Neutrino Oscillations:** evolution Hamiltonian in matter
(dominant below 1 TeV)
- **Neutrino Attenuation:** inelastic CC and NC interactions with matter
(dominant above 1 TeV)
- **Neutrino Regeneration:** $\nu_e, \nu_\mu \rightarrow \nu_\tau \rightarrow \text{CC } \tau \rightarrow \nu_e, \nu_\mu$
through leptonic decays
- **Migration to lower energy bins:** due to NC interactions

Our Earth's model



5 spherical layers

Inner Core, one layer

$L_1 = 1242$ km

Outer Core, two layers

$L_2 = 2373$ km,

$L_3 = 3504$ km

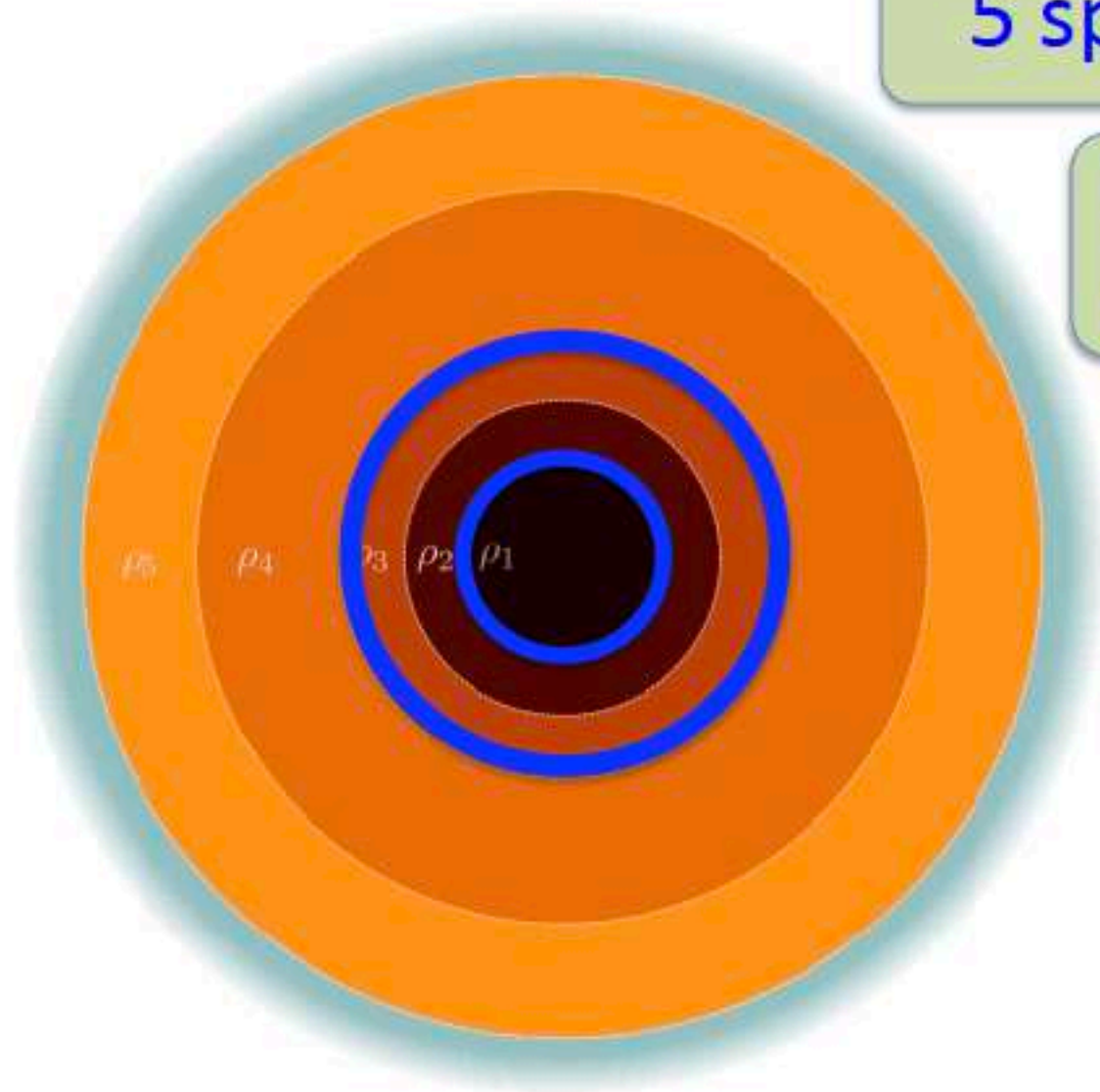
Mantle, two layers

$L_4 = 4938$ km,

$L_5 = 6371$ km

No crust!

Our Earth's model



ICB and CMB fixed!

5 spherical layers

Inner Core, one layer

$L_1 = 1242$ km

Outer Core, two layers

$L_2 = 2373$ km,

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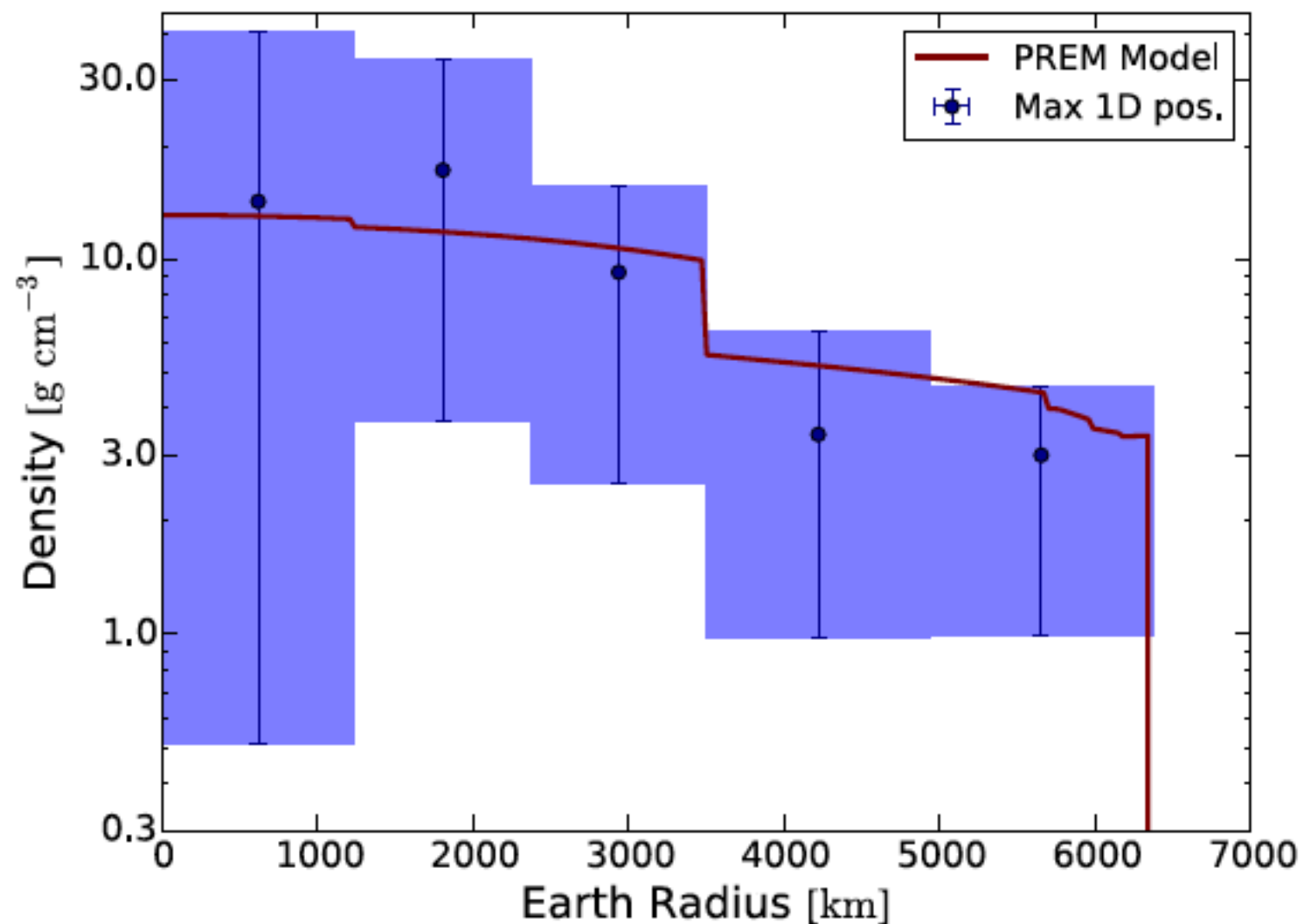
Mantle, two layers

$L_4 = 4938$ km,

$L_5 = 6371$ km

No crust!

First 1-d density profile with neutrinos



Analysis performed
with MultiNest

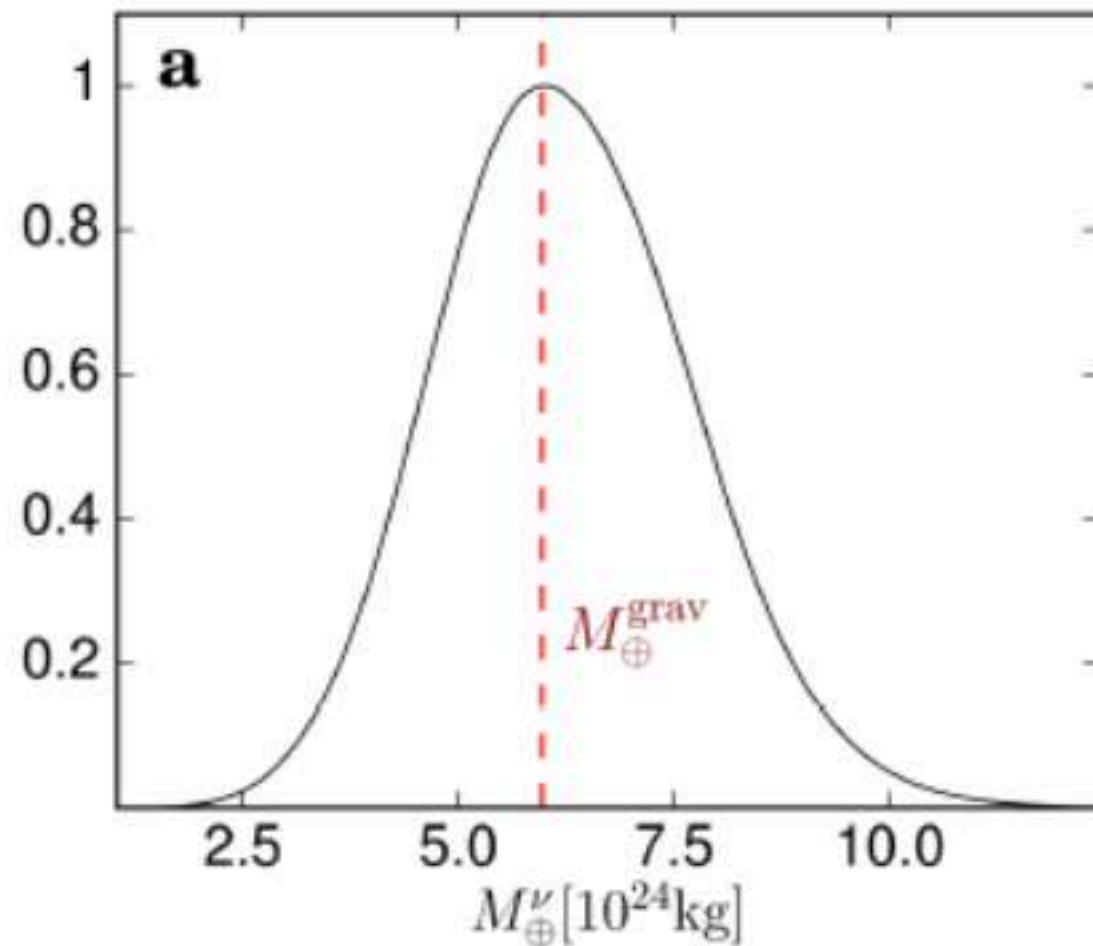
5 Earth layers densities

and

4 systematic errors:

- Flux normalization
- Pion-to-kaon ratio
- Spectral shape
- DOM Efficiency

The Earth's mass



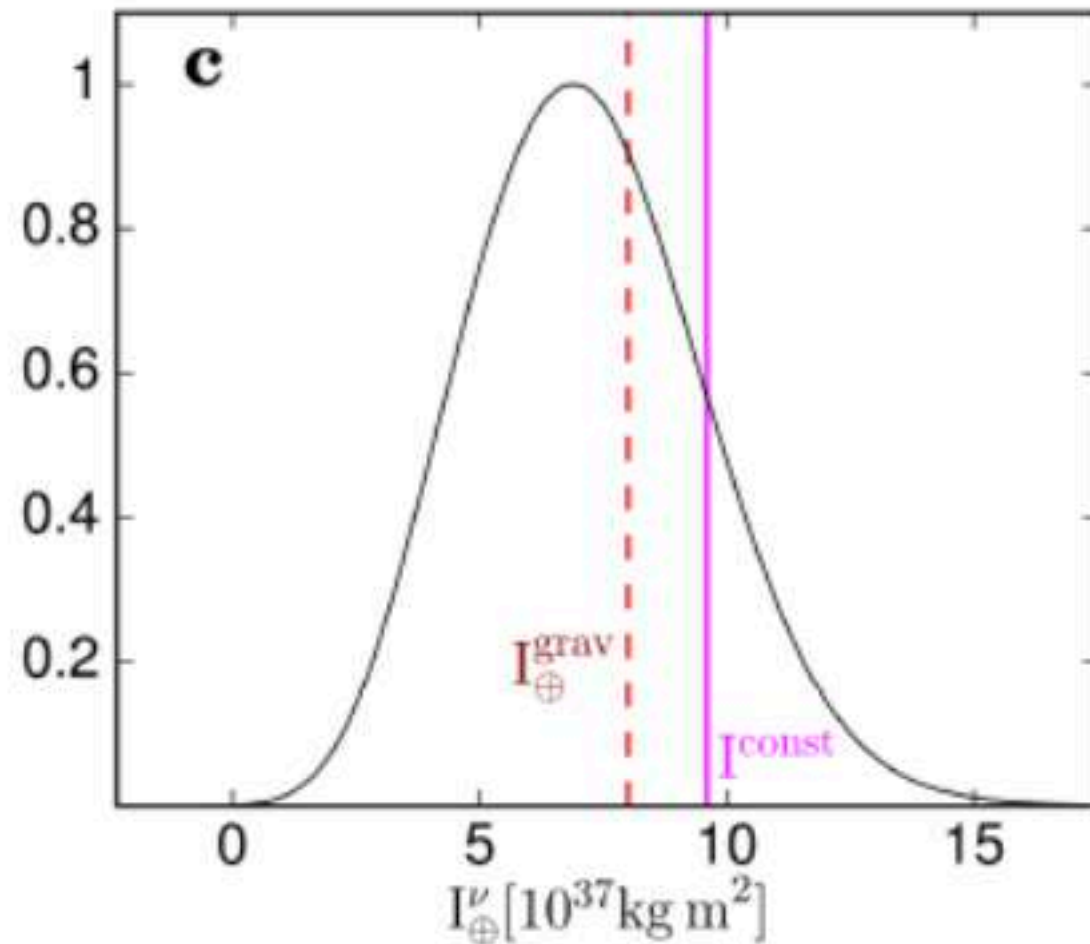
First measurement of the Earth's mass using the weak force!

$$M_\nu = (6.0^{+1.6}_{-1.3}) \times 10^{24} \text{ kg}$$

Gravitational measurement of the Earth's mass

$$M_{\text{earth-grav}} = (5.9722 \pm 0.0006) \times 10^{24} \text{ kg}$$

The Earth's moment of inertia



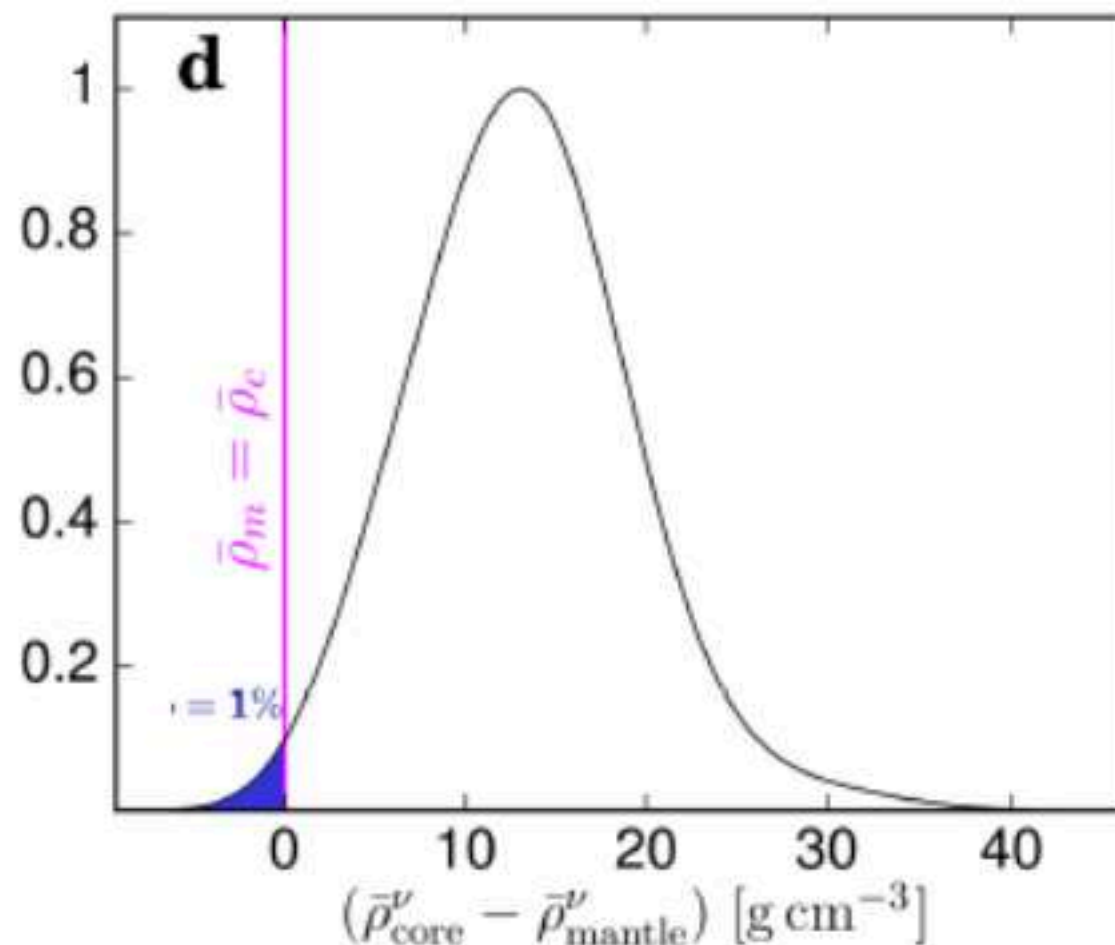
First measurement of the Earth's
moment of inertia using
the weak force!

$$I_\nu = (6.9 \pm 2.4) \times 10^{37} \text{ kg m}^2$$

Gravitational measurement of the Earth's moment of inertia

$$I_{\text{earth-grav}} = (8.01736 \pm 0.00097) \times 10^{37} \text{ kg m}^2$$

Earth's non-homogeneity



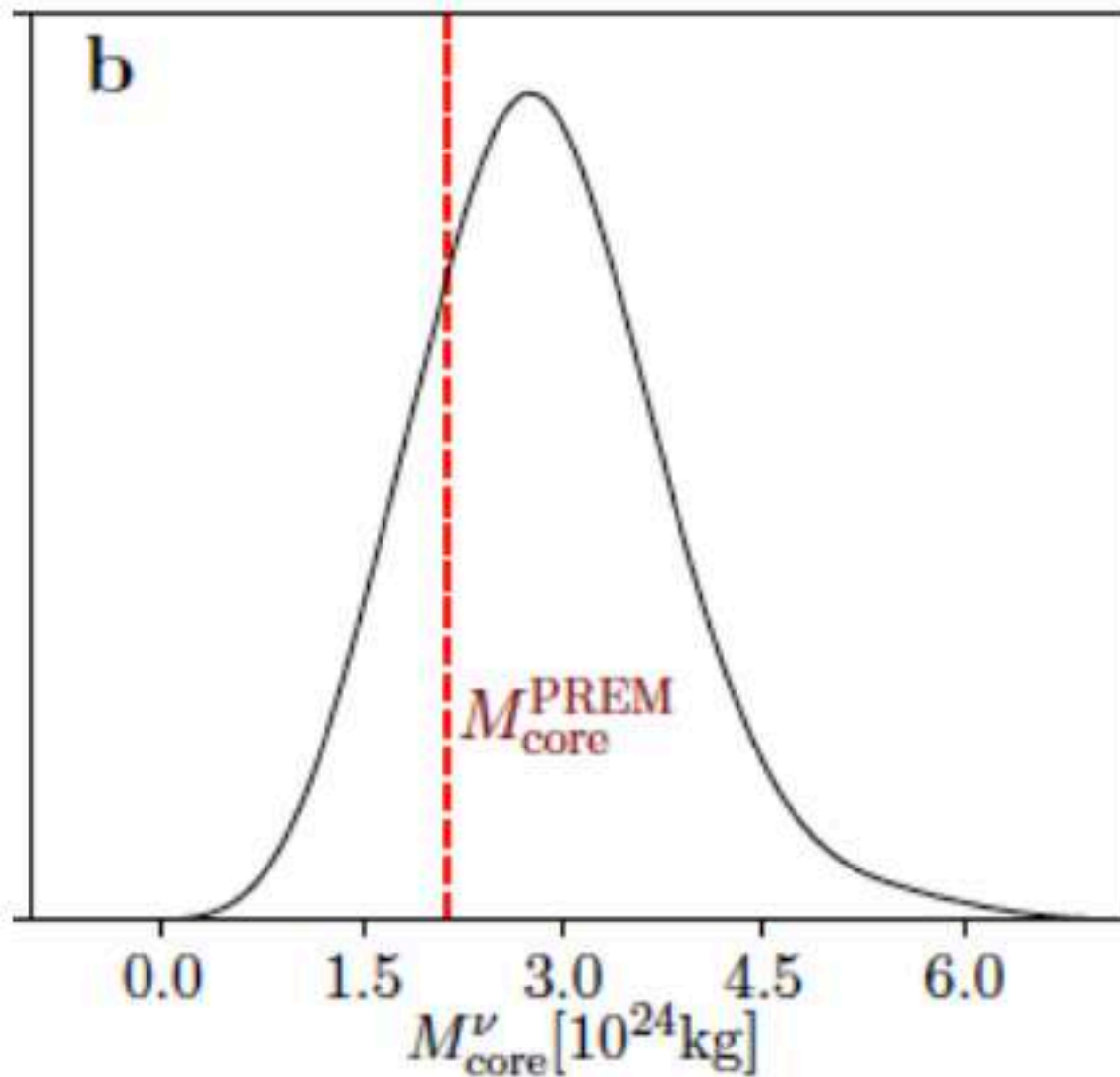
First measurement of the Earth's core-mantle discontinuity using the weak force!

$$\left(\bar{\rho}_{core}^{\nu} - \bar{\rho}_{mantle}^{\nu} \right) = \left(13.1 \begin{smallmatrix} +5.8 \\ -6.3 \end{smallmatrix} \right) \text{ g / cm}^3$$

A homogenous Earth has a p-value $p = 0.01$!!!

2008 Claim: IceCube could reject a homogeneous Earth at 5σ in ten years

The Earth's core mass



First measurement of the Earth's core mass using the weak force!

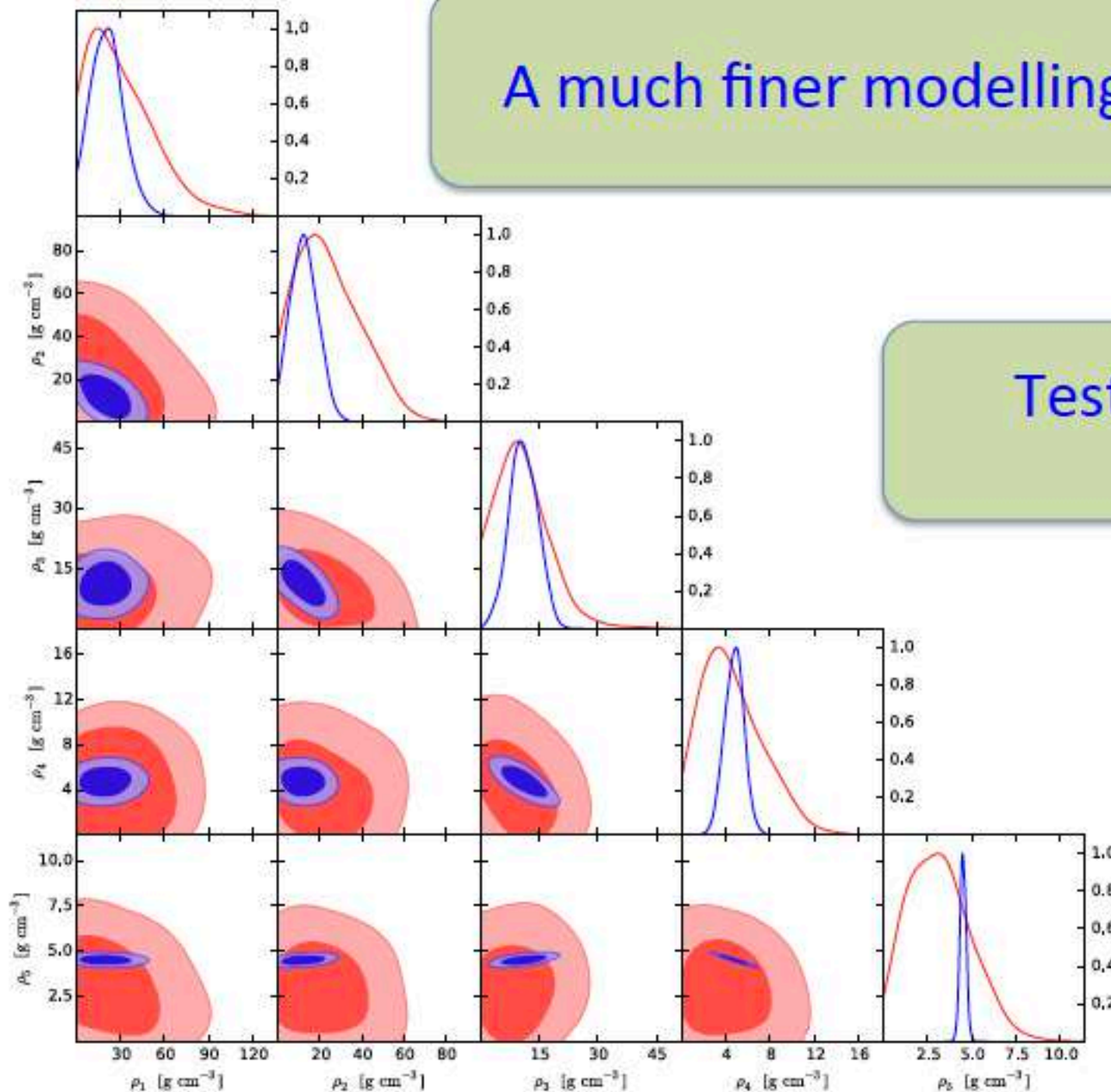
$$M_{\text{core-}\nu} = \left(2.7^{+1.0}_{-0.9}\right) \times 10^{24} \text{ kg}$$

Forecast with 10 years of data

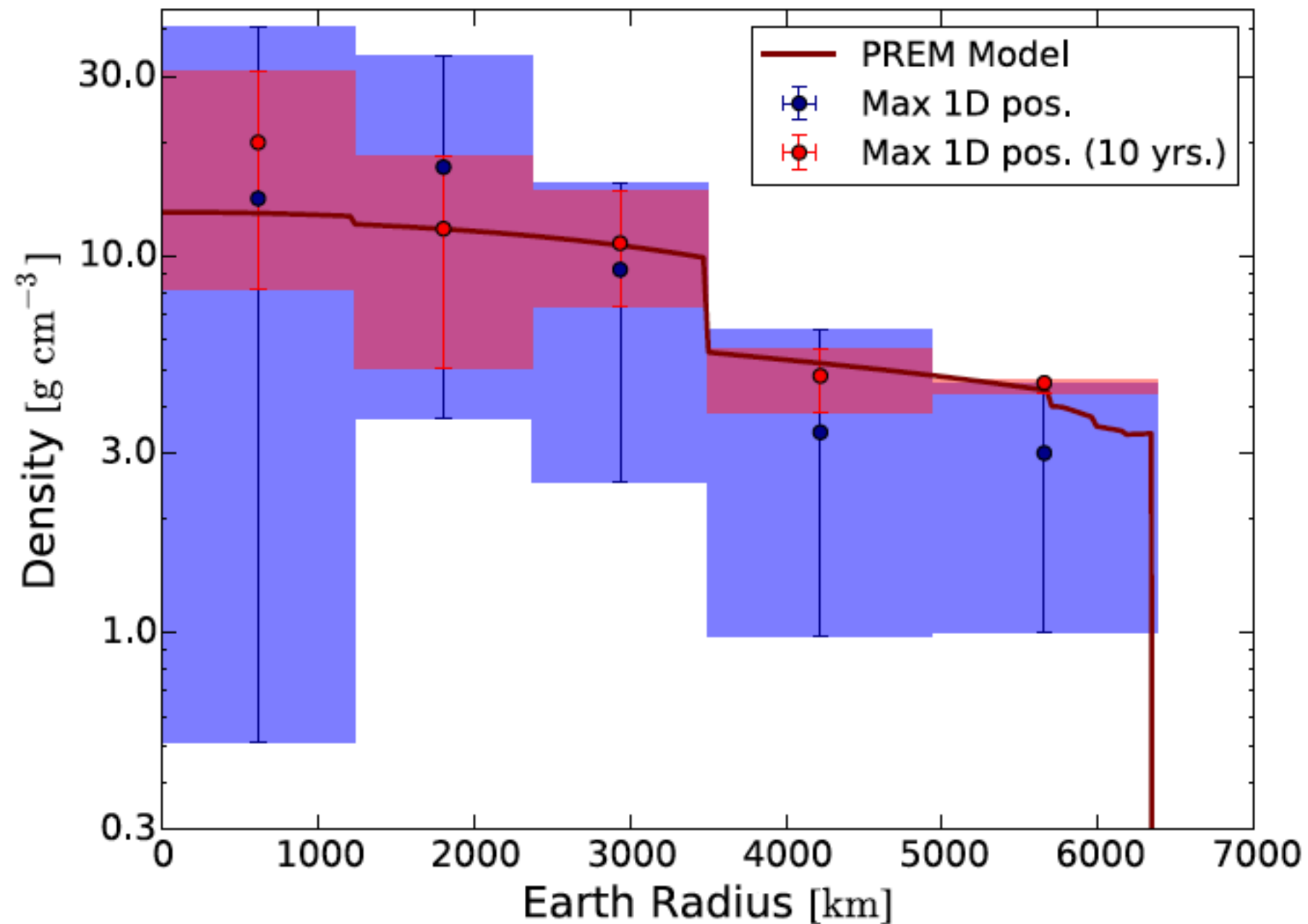
A much finer modelling of the Earth could be done

Test of the Inner-Outer Core discontinuity

Independent localization of the Core-Mantle Boundary

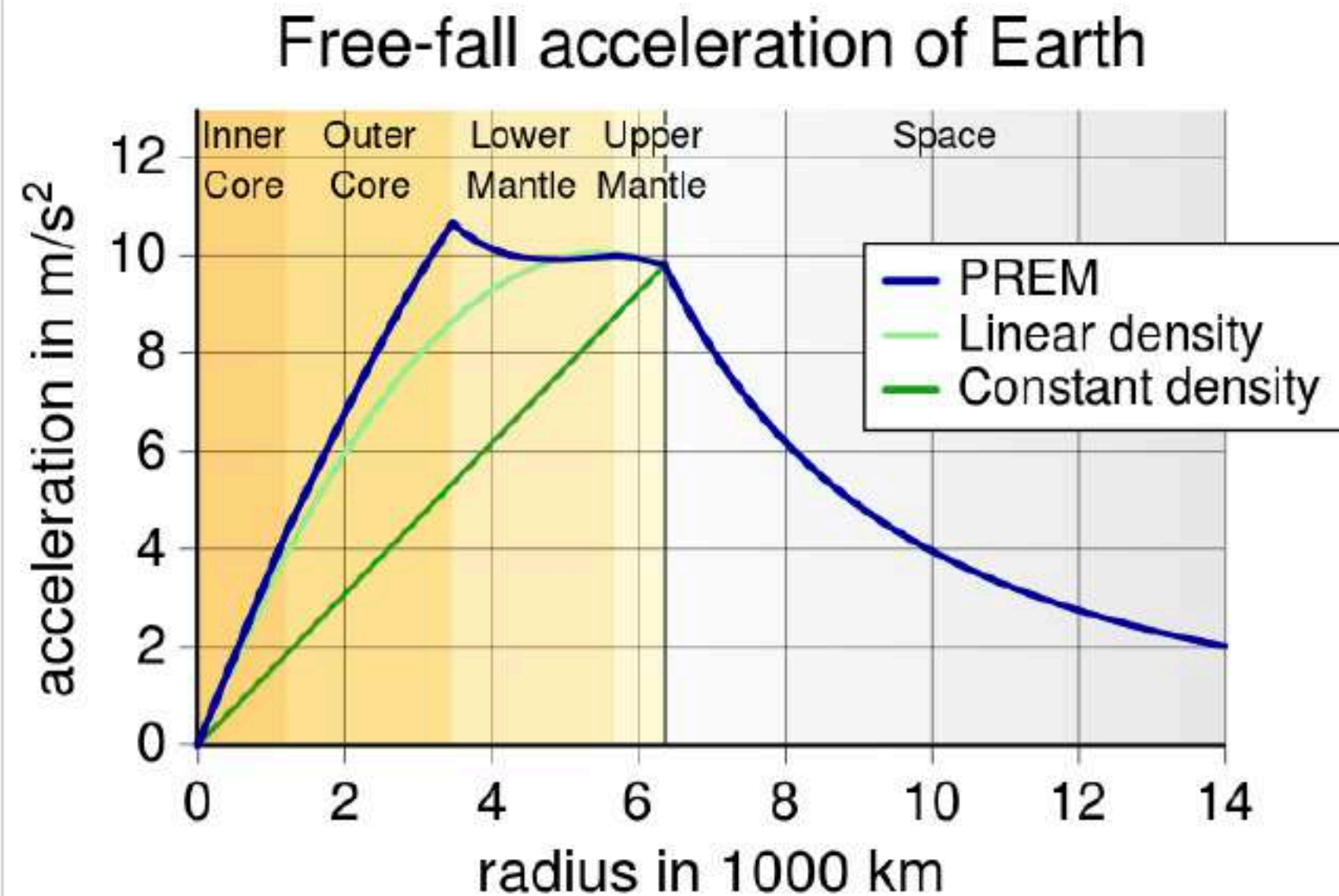


1-d density profile with 10 years



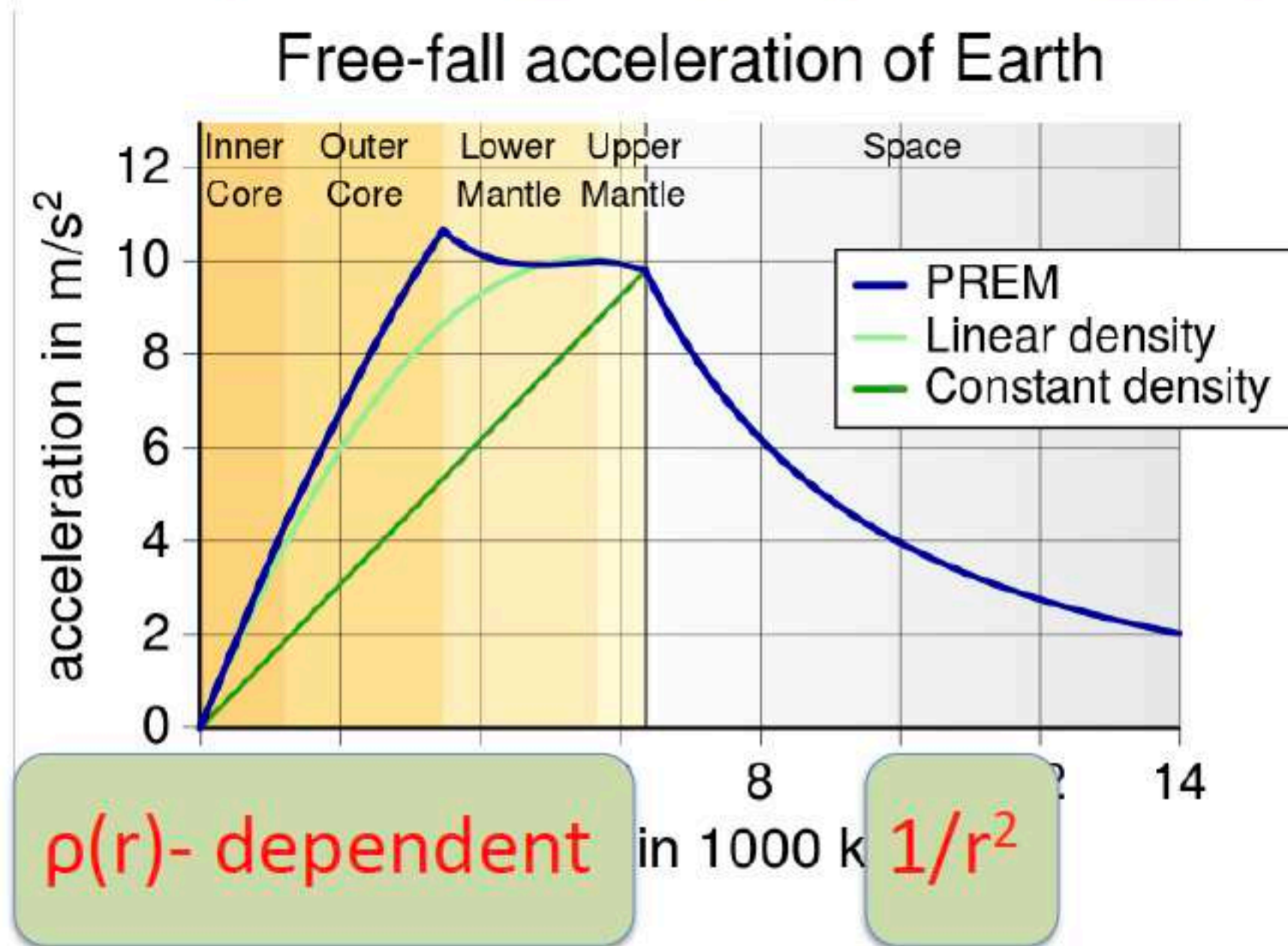
A. Donini, S. Palomares-Ruiz, J. Salvado, Nature Phys. (2019)

An input to geophysics: $g(r)$



The Earth's gravitational profile is needed to compute $\rho(r)$ from earthquake waves velocities

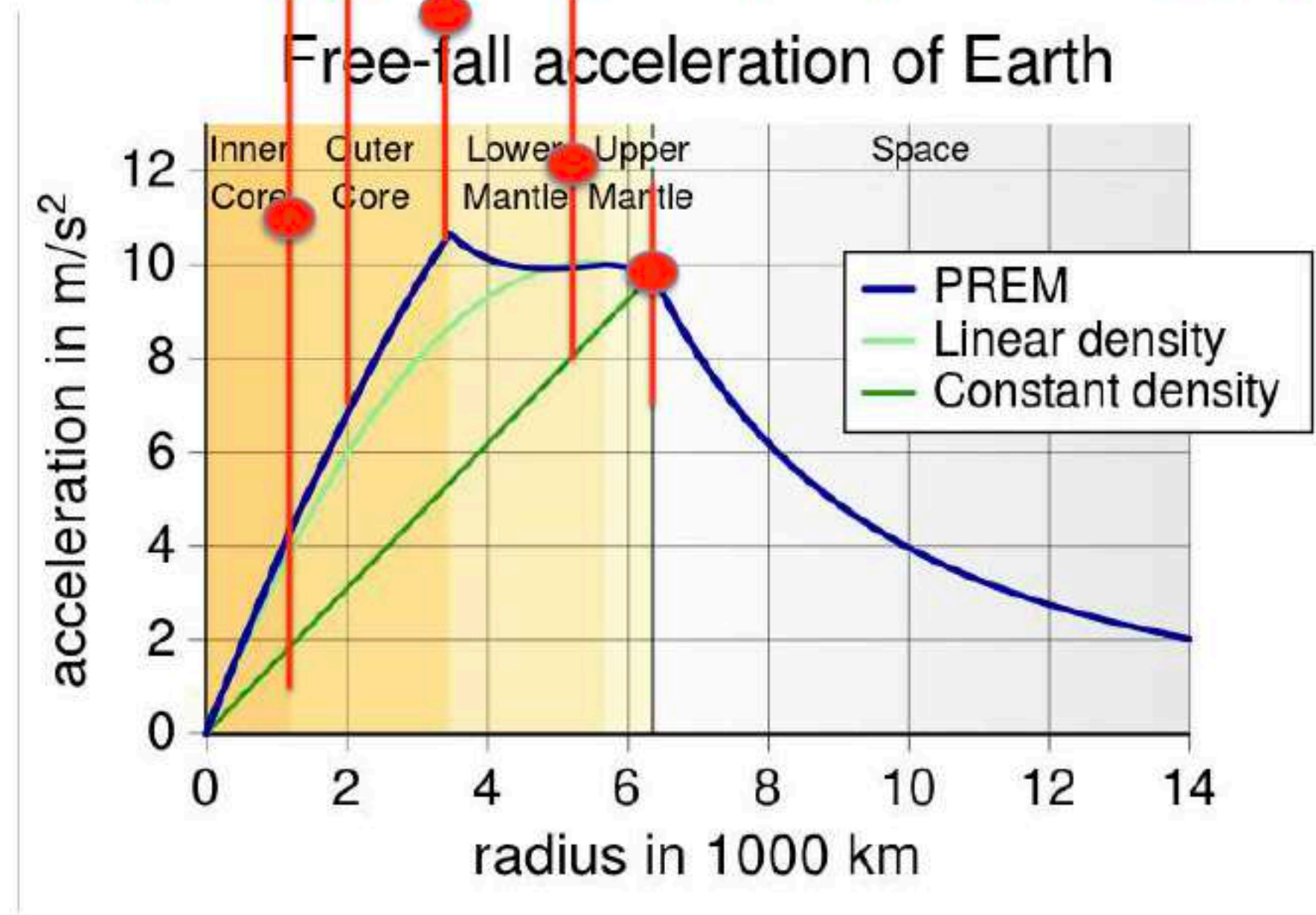
An input to geophysics: $g(r)$



The Earth's gravitational profile is needed to compute $\rho(r)$ from earthquake waves velocities

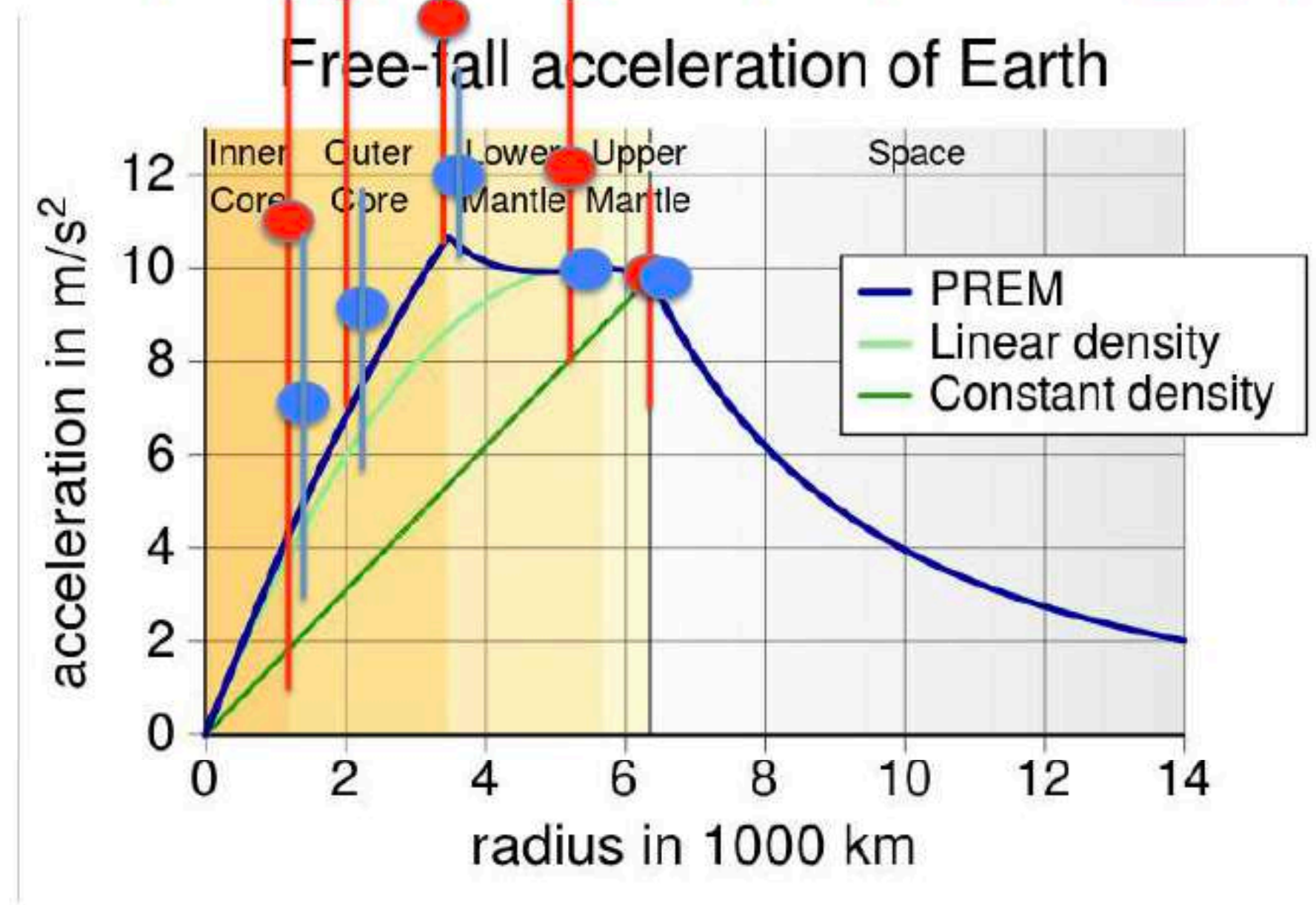
1 year measurement

Complement geophysics: $g(r)$



The Earth's gravitational profile is needed to compute $p(r)$ from earthquake waves velocities

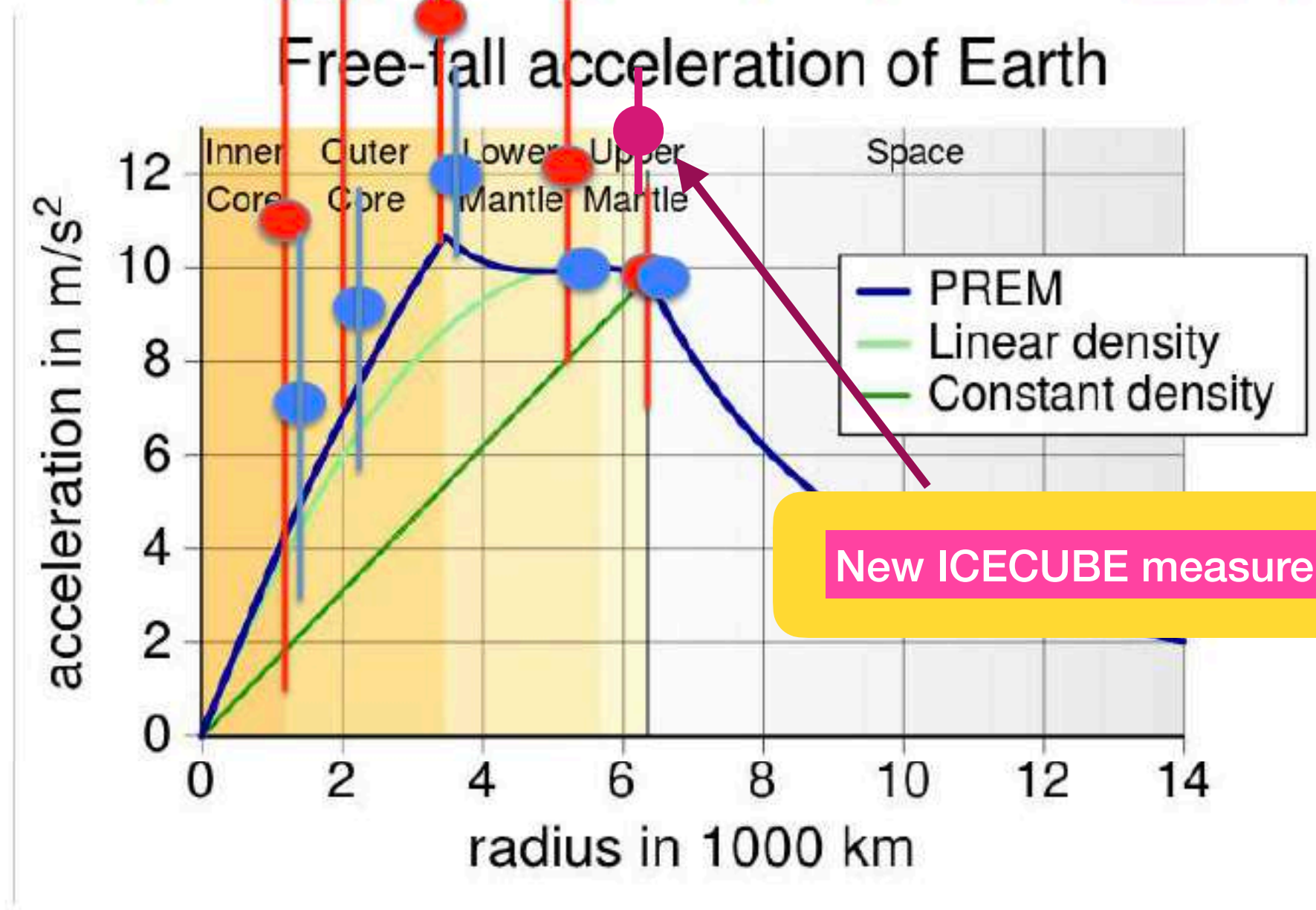
Complement geophysics: $g(r)$



The Earth's gravitational profile is needed to compute $p(r)$ from earthquake waves velocities

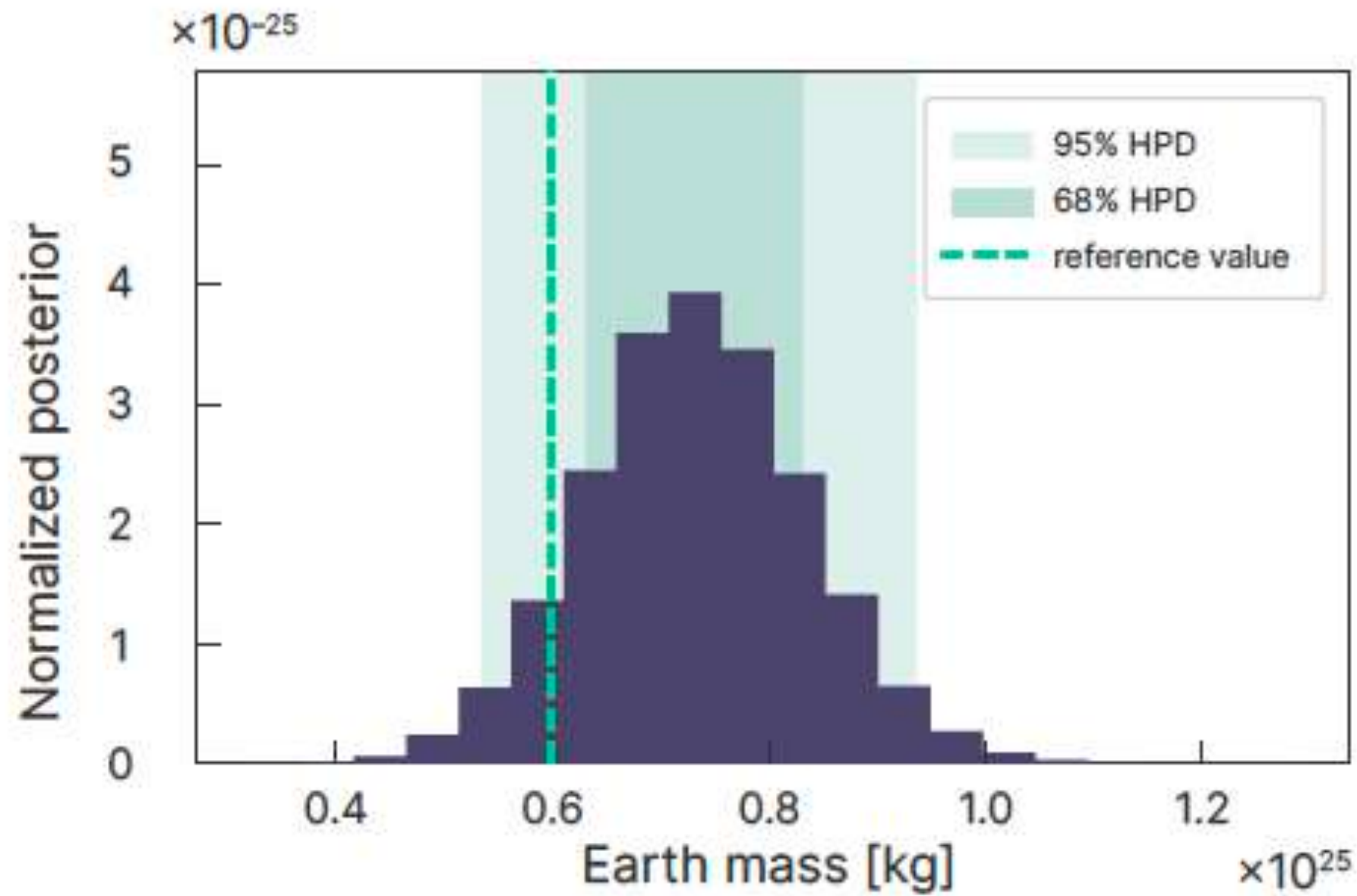
15 years measurement !!!!!!!

Complement geophysics: $g(r)$

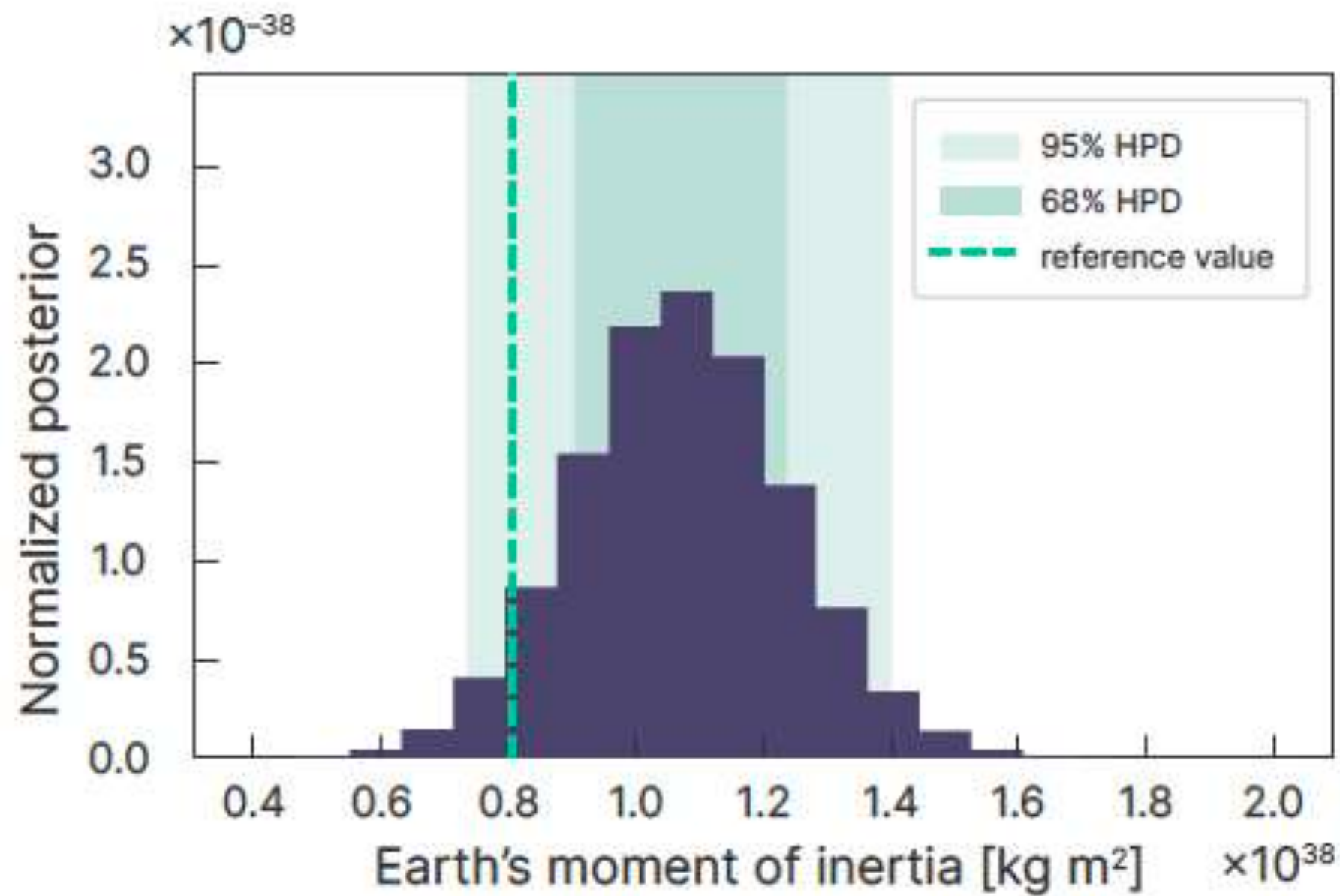


The Earth's gravitational profile is needed to compute $p(r)$ from earthquake waves velocities

NEW measurement of Earth's mass: two sigma off



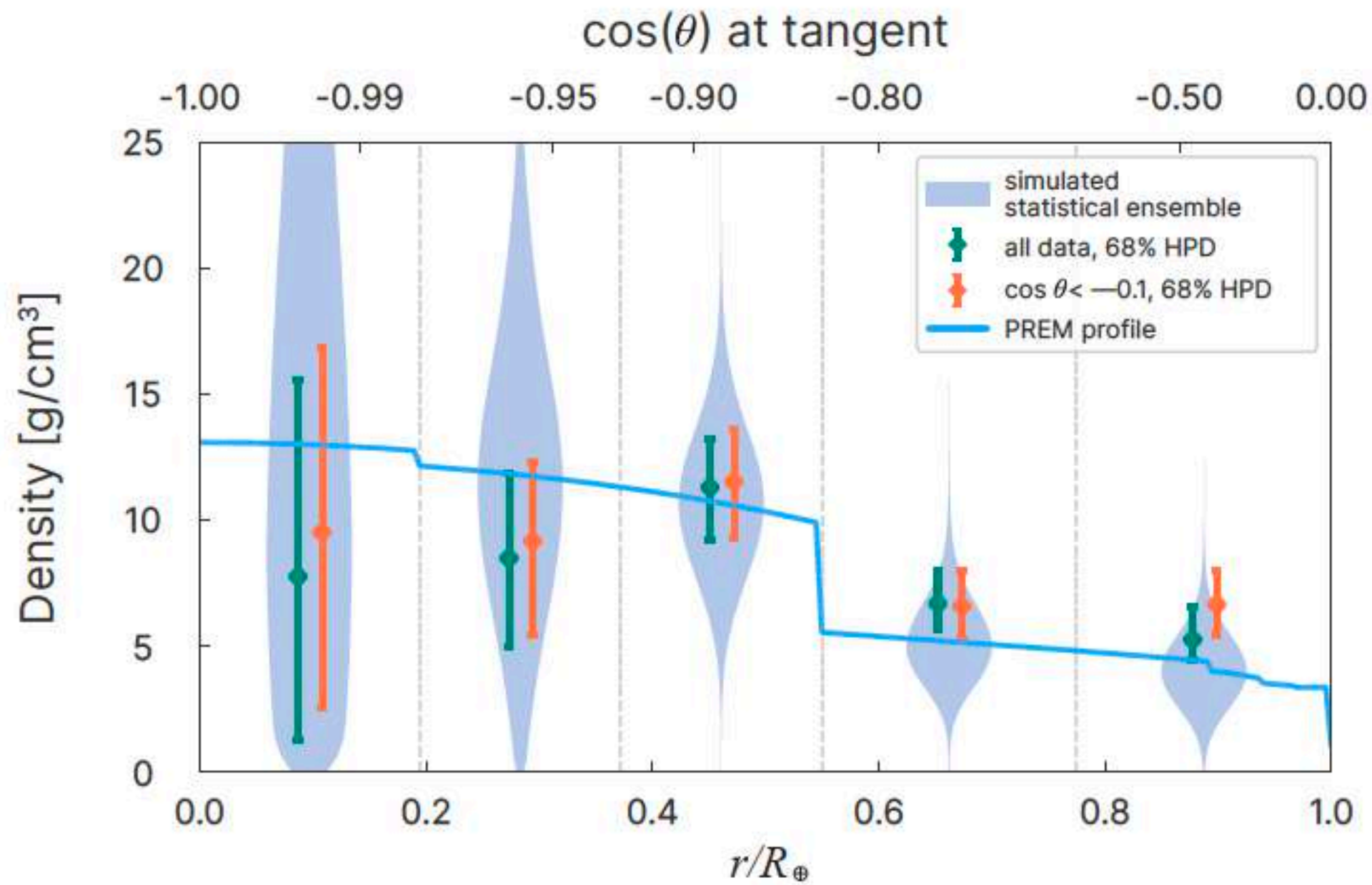
NEW measurement of Earth's moment of inertia: two sigma off



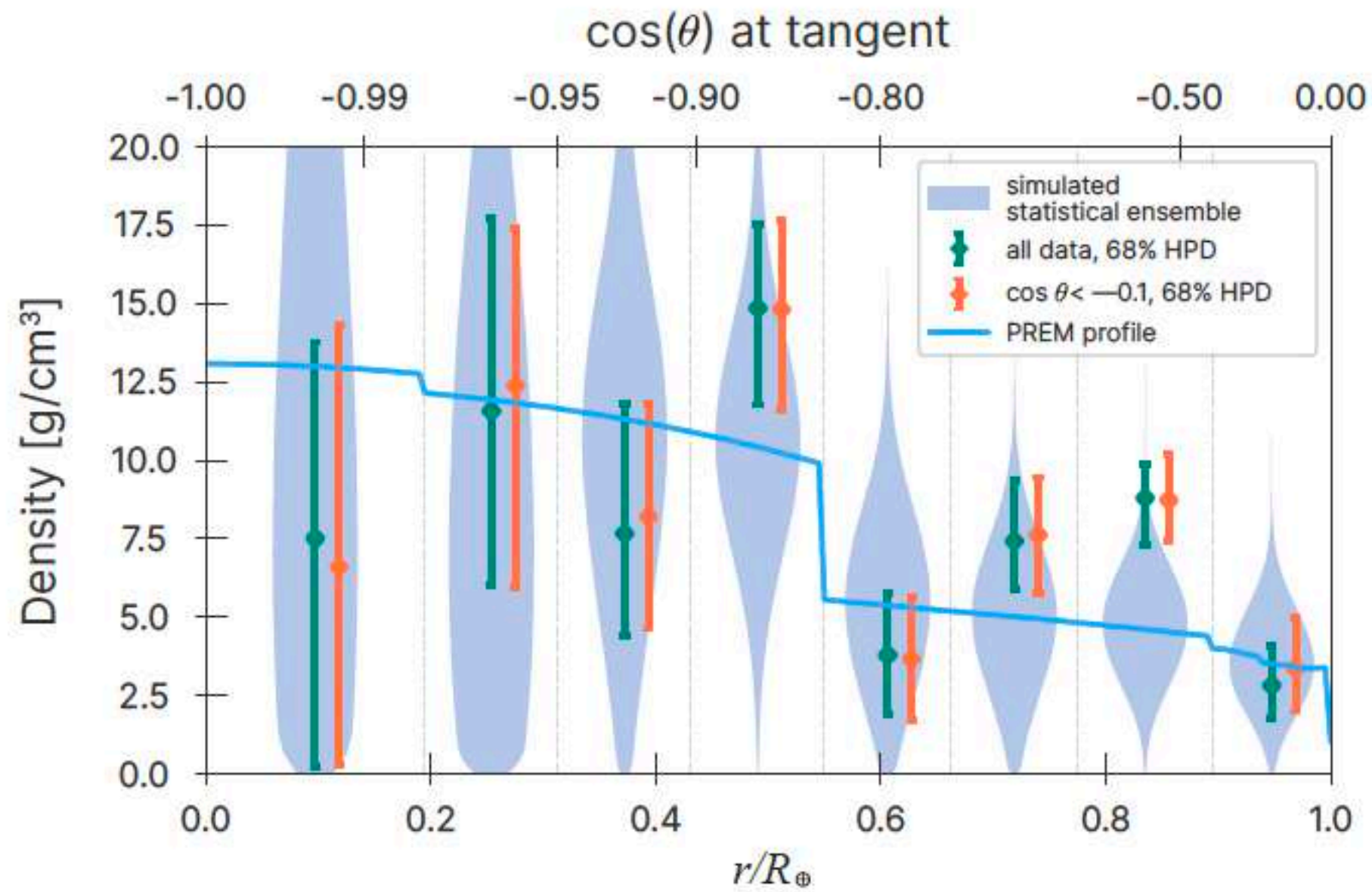
Understanding the new measurement

1. Which data push the Earth's mass to large values?
2. What if we fix the Earth's mass?
3. New vs Old analyses

FIVE LAYERS



EIGHT LAYERS



Constraints and derived quantities

- Gravitational measurement of the Earth's mass

$$M_{\oplus} = \frac{4\pi}{3} \int_0^{R_{\oplus}} dr r^2 \rho(r) = 5.972 \times 10^{24} \text{kg}$$

- A derived quantity: Earth's mean moment of inertia

$$I_{\oplus} = \frac{8\pi}{3} \int_0^{R_{\oplus}} dr r^4 \rho(r) = 0.3307144 M_{\oplus} R_{\oplus}^2$$

A constant density would give $I_{\oplus}(\rho(r) = \rho_0) = 0.4 M_{\oplus} R^2$.

Impact of constraints

OLD DATA

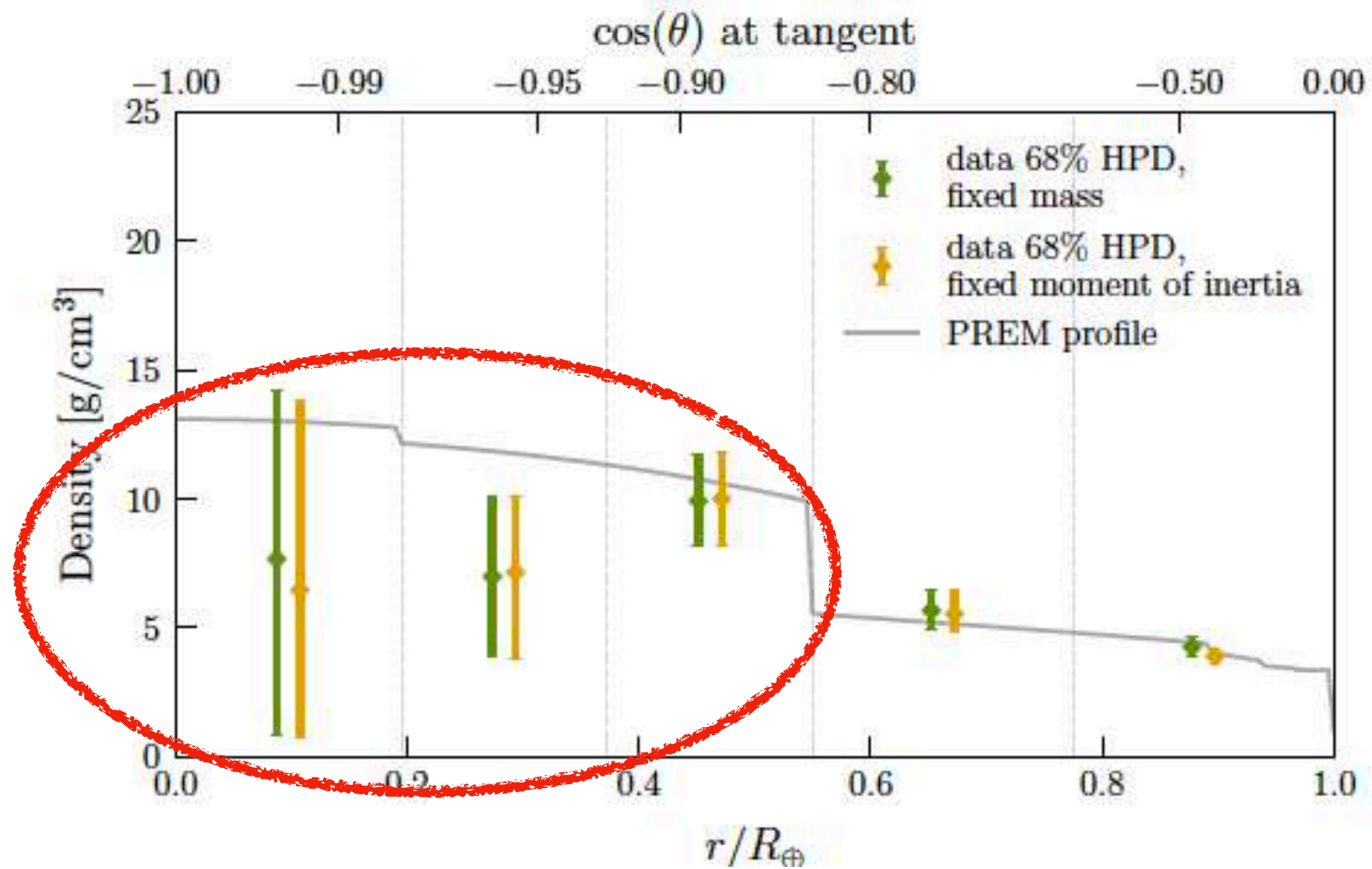
Adding the gravitational Earth's mass as an external constraint, results in fixing the mantle density:

$$\rho_5 = [1.22-4.78] \text{ g/cm}^3 \rightarrow [4.43-4.79] \text{ g/cm}^3$$

Rather small impact on the core density, instead:

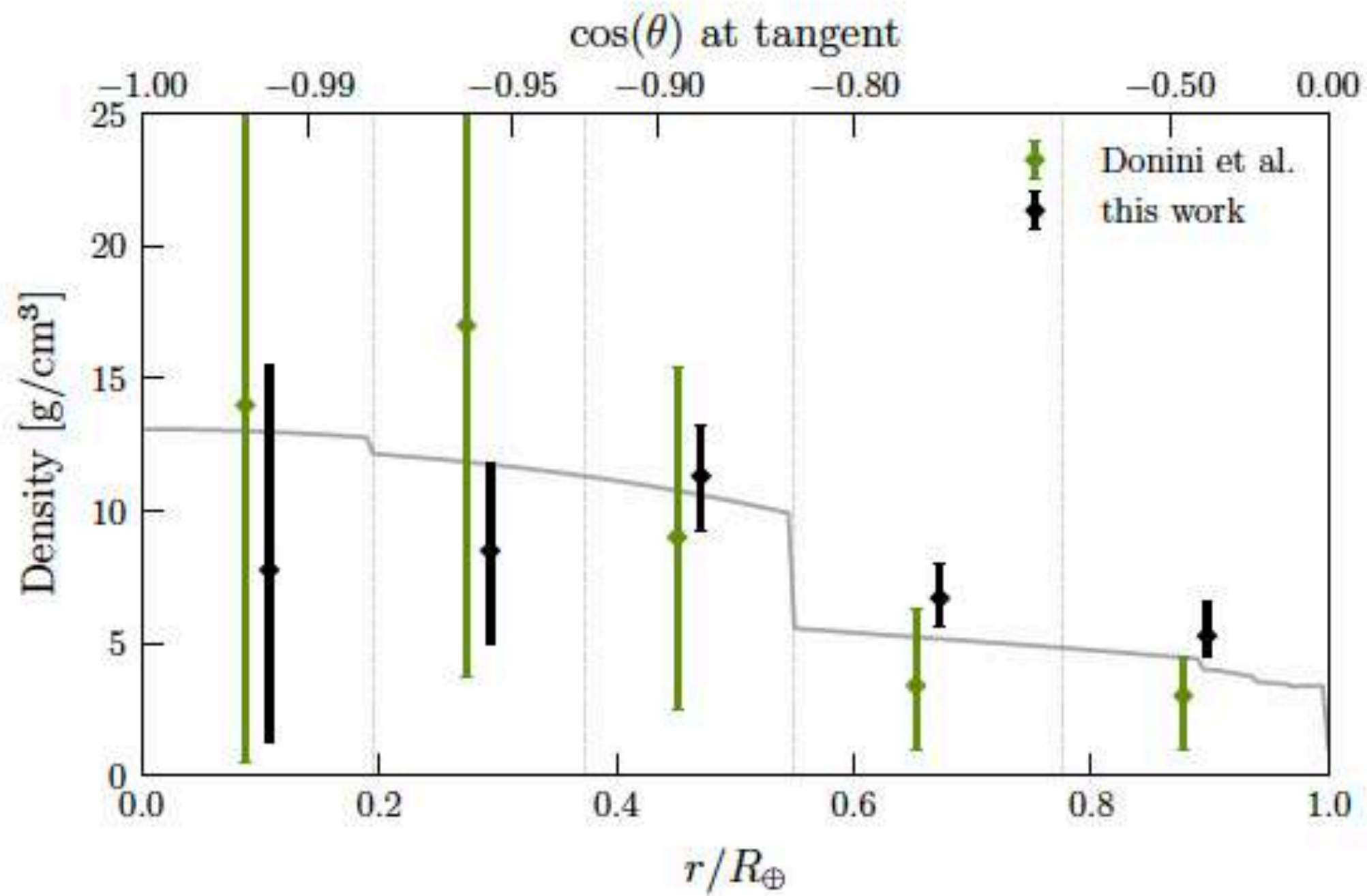
$$\rho_{\text{core}} = [10.2-20.8] \text{ g/cm}^3 \rightarrow [9.7-18.6] \text{ g/cm}^3$$

NEW DATA



YOU NEED LESS MATTER IN THE CORE

OLD vs NEW DATA



My personal guess

1. Missing systematics: best choice could be related to ice opacity
Dust accumulates on mostly horizontal planes,
a wrong reconstruction may affect neutrino detection with low angle

My personal guess

1. Missing systematics: best choice could be related to ice opacity

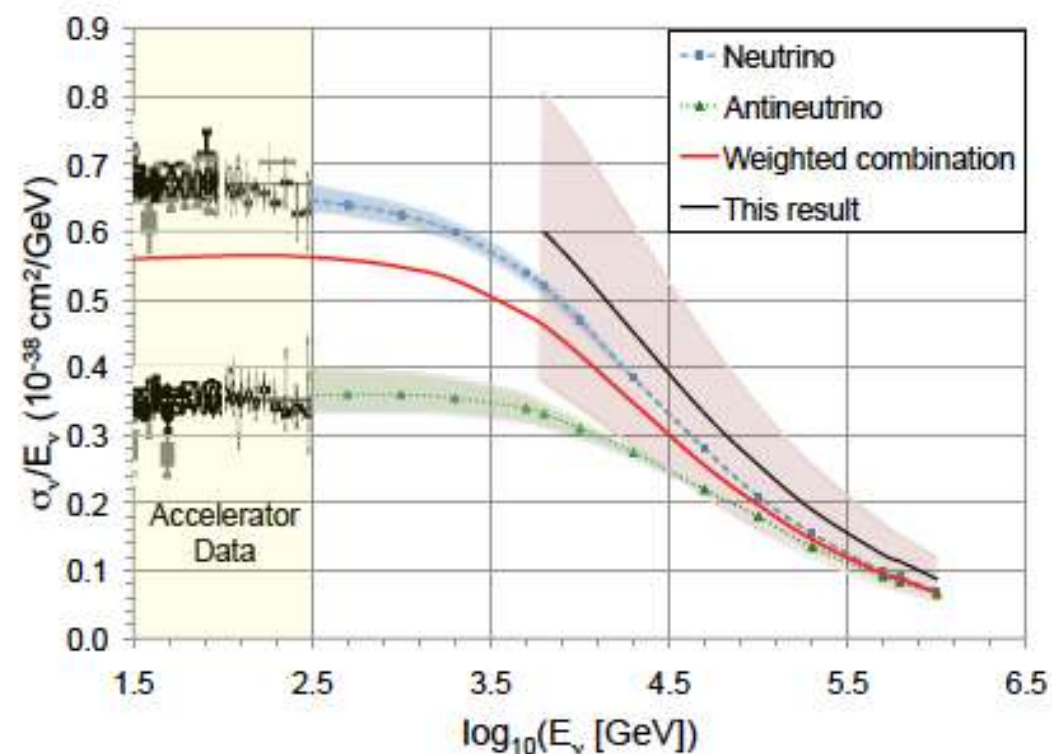
Dust accumulates on mostly horizontal planes,

a wrong reconstruction may affect neutrino detection with low angle

2. Do you remember this?

1 year sample, 2017

Could you please check again?



How to use neutrinos?

What if we had a gravitational model different from Newtonian gravity?

1) Adams-Williamson equation ('20s)

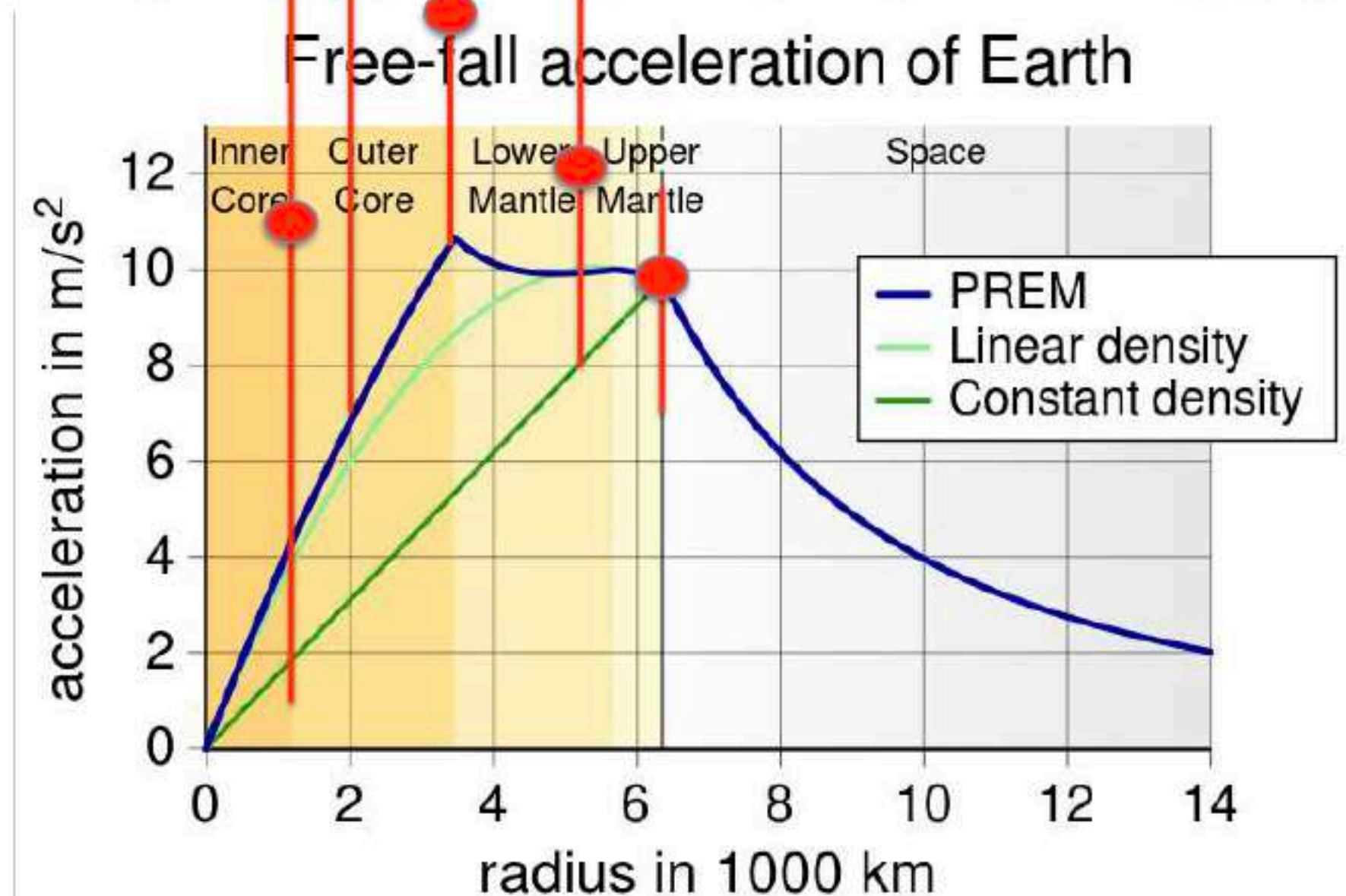
$$\frac{d\rho}{dr} = -\rho(r) \frac{g(r)}{\Phi(r)}$$

$$\Phi(r) = v_p^2 - \frac{4}{3}v_s^2$$

The density profile depends on the gravitational model!

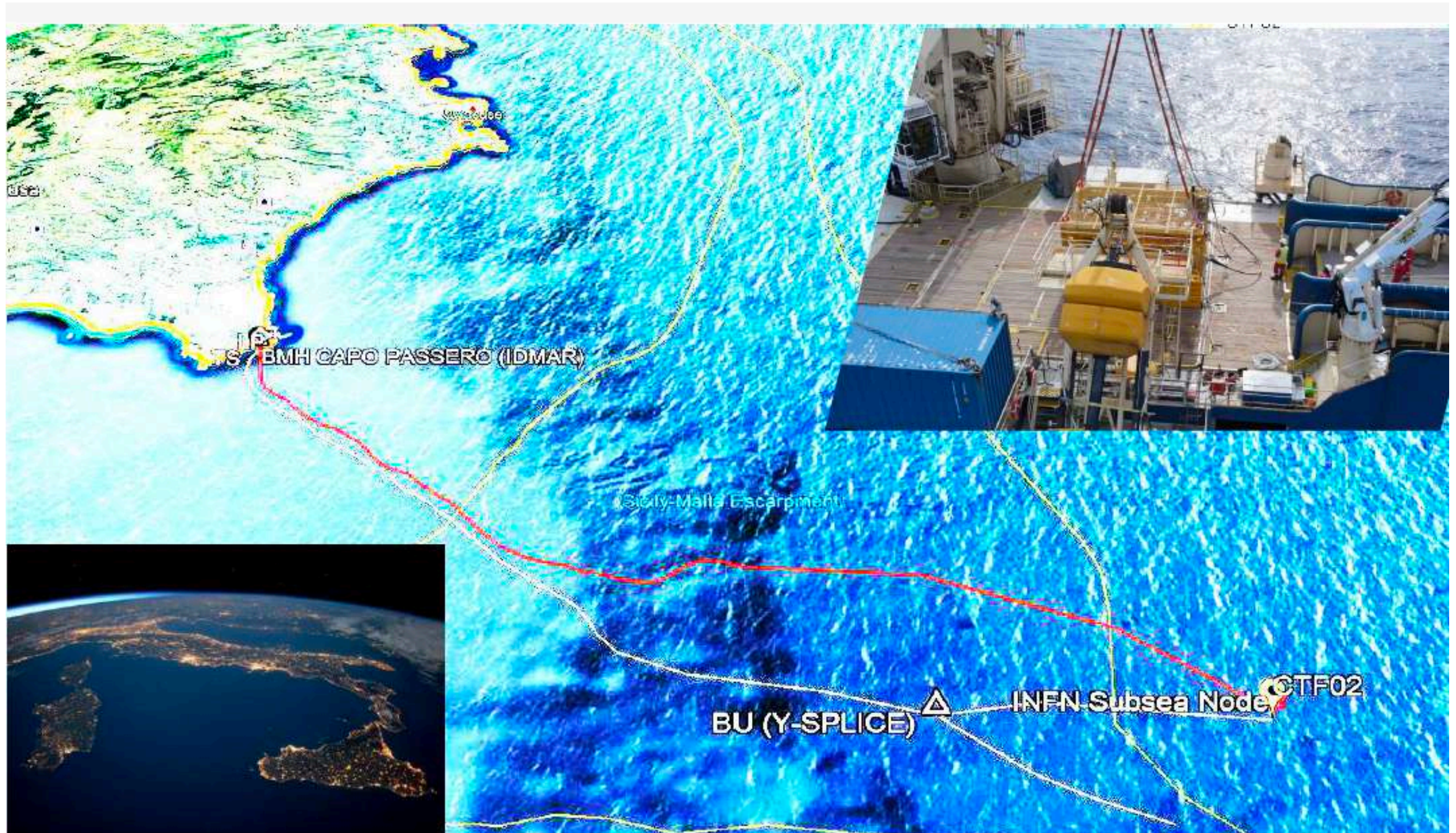
1 year measurement

Complement geophysics: $g(r)$

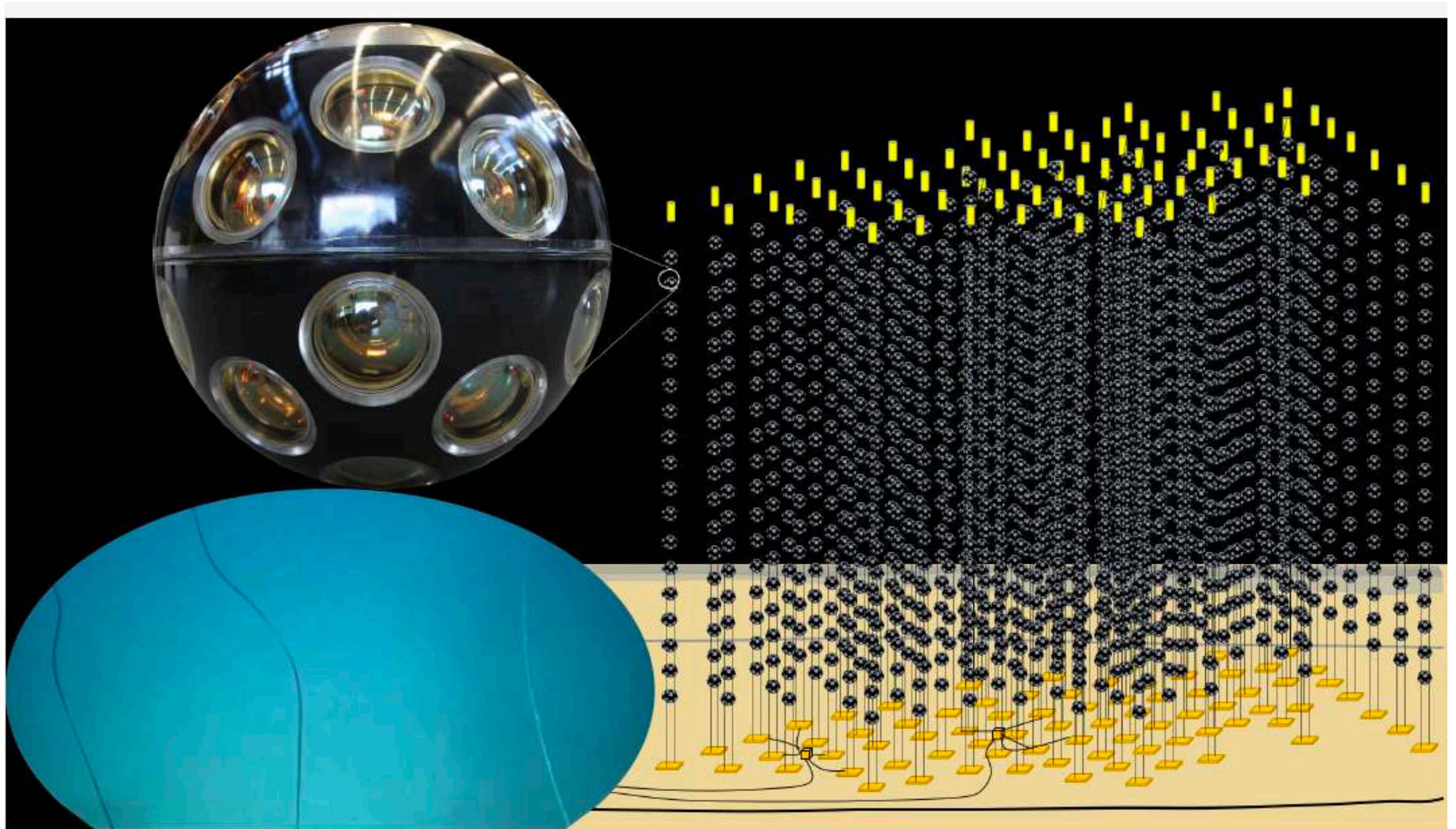


SO WE ARE BACK TO THIS!

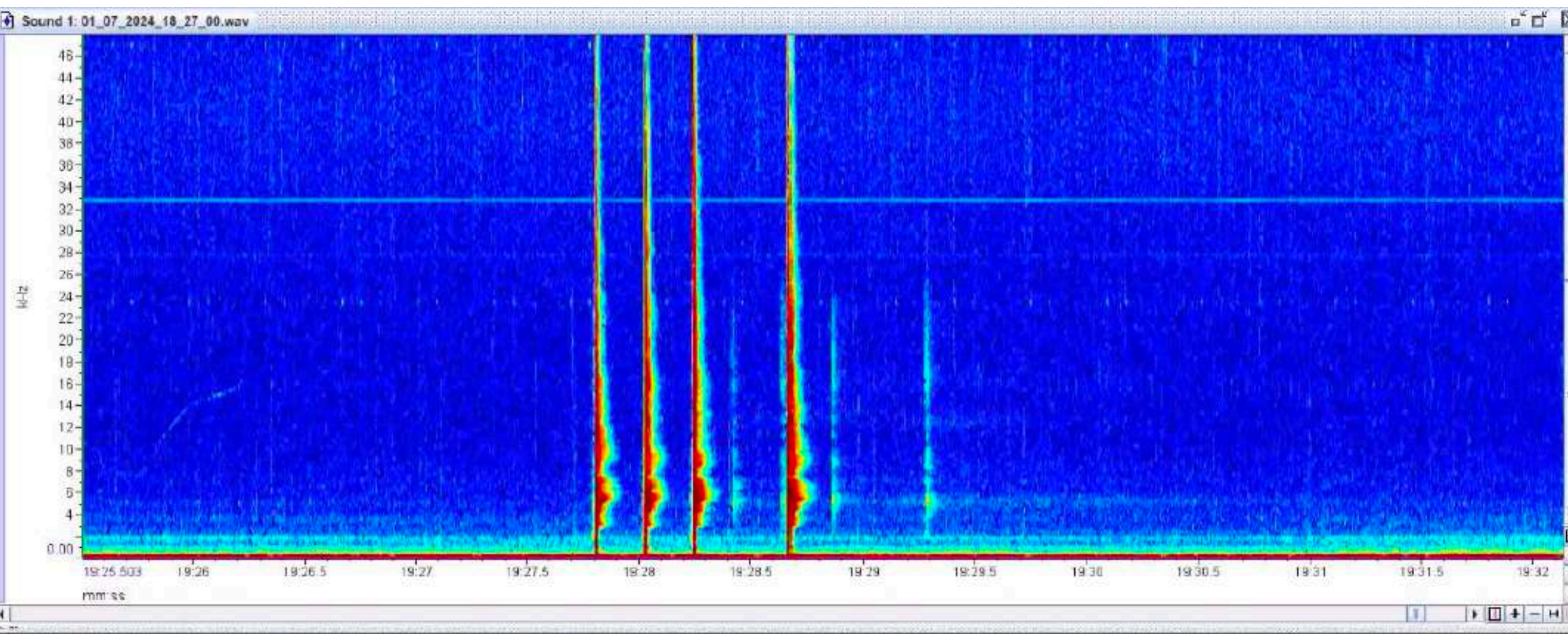
New telescopes: Km3Net



New telescopes: Km3Net

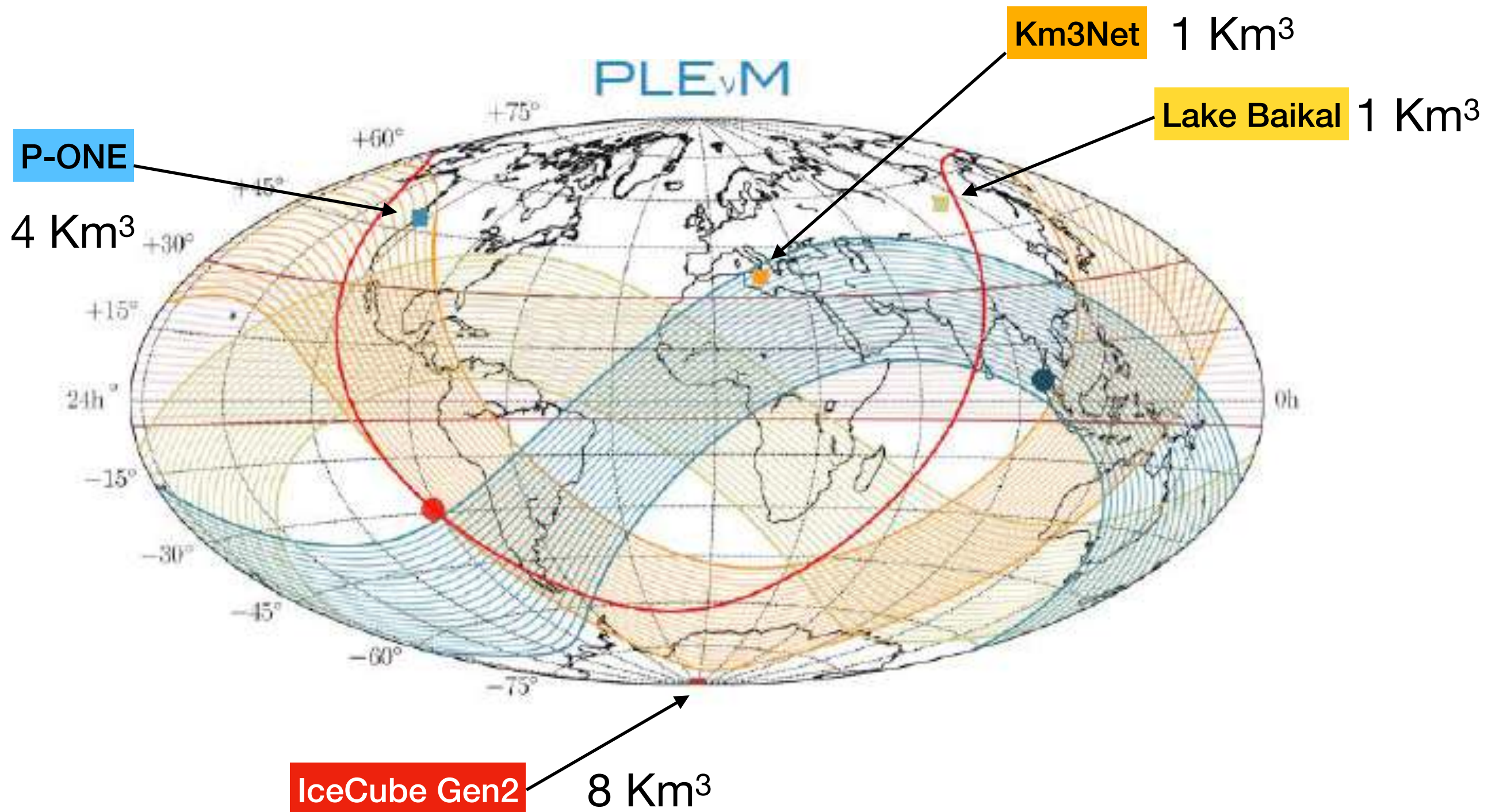


New telescopes: Km3Net



Mediterranean Sperm Whale-singing recorded by Km3Net

More telescopes



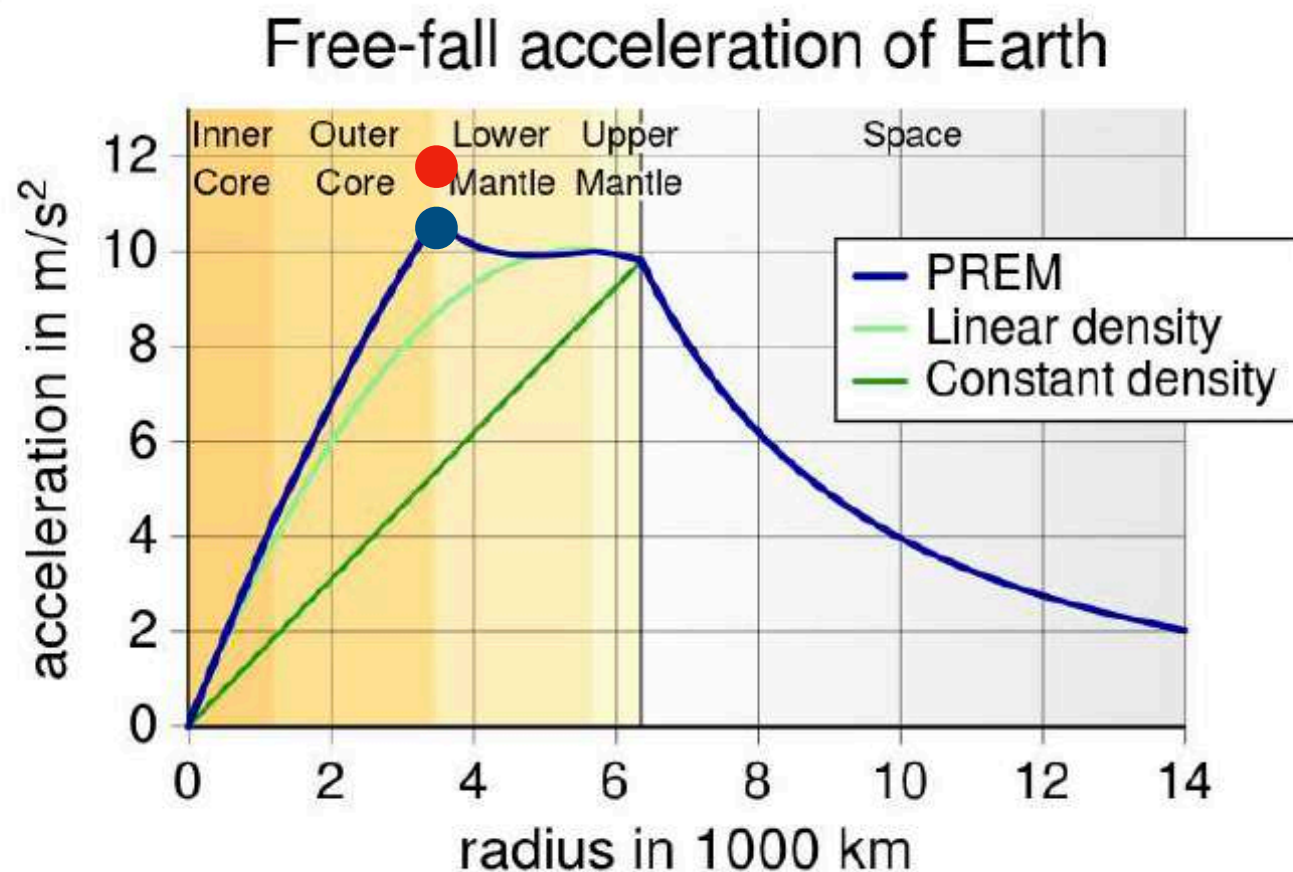
HANDS-ON

MSW resonance

Hands-on session, exercise 1: diagonalize and find the value of the angle in vacuum for which there is resonant oscillation

Neutrino usefulness

Hands-on session, exercise 2: estimate the exposure you need to test a 10% difference between the PREM (Newton-driven) and a mod-gravity model Earth's core mass?



Assume that errors from seismology are negligible

SOLUTIONS

MSW resonance

Hands-on session, exercise 1: diagonalize and find the value of the angle in vacuum for which there is resonant oscillation

$$i \frac{d}{dx} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \left[\frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \begin{pmatrix} \sqrt{2} G_F n_e(x) & 0 \\ 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \nu_e(x) \\ \nu_\mu(x) \end{pmatrix}$$

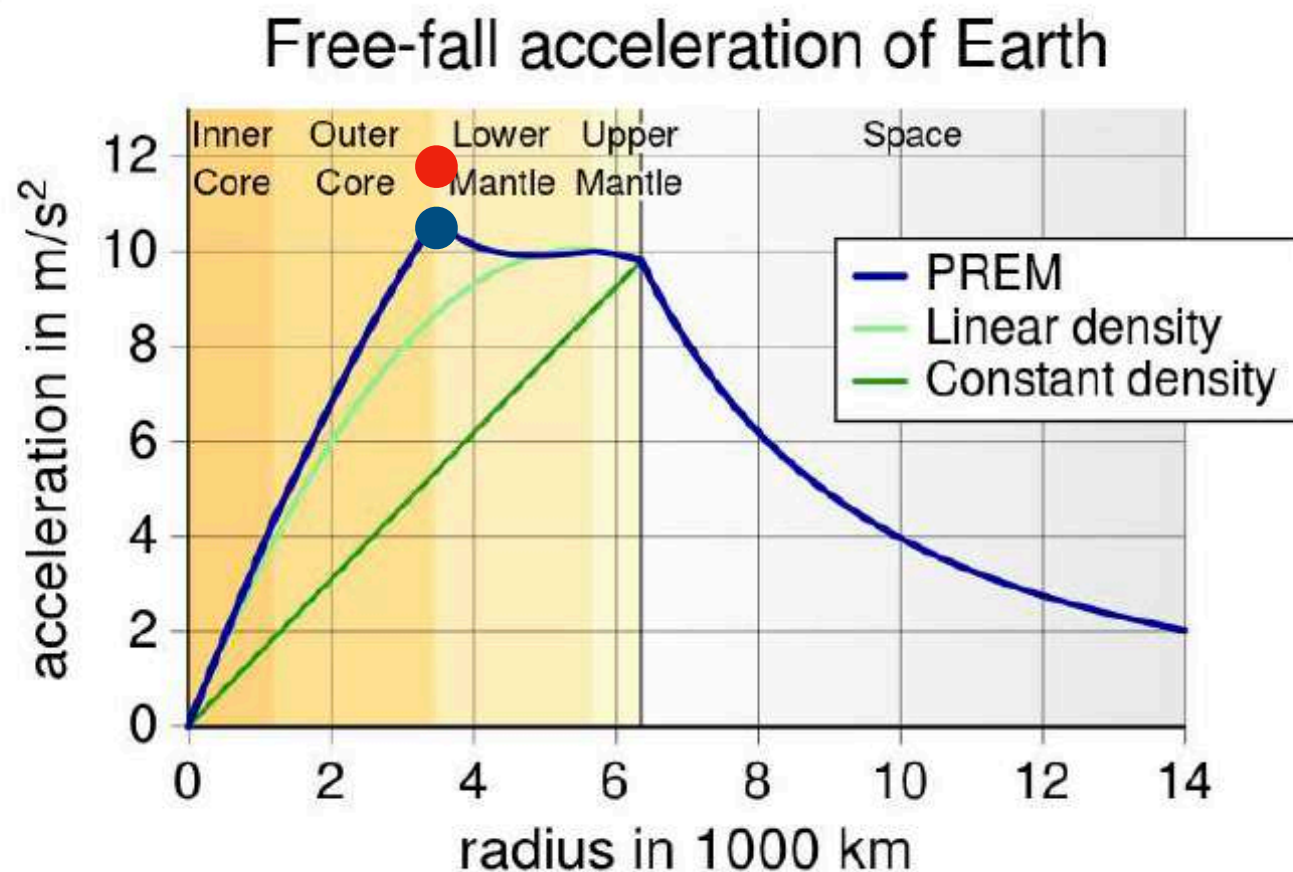
$$\sin 2\theta_m = \frac{\sin 2\theta}{\sqrt{(\cos 2\theta - A)^2 + \sin^2 2\theta}}, \quad A = \frac{2\sqrt{2} G_F n_e E}{\Delta m^2}.$$

$$A_{\text{res}} = \cos 2\theta \quad \Rightarrow \quad n_e^{\text{res}} = \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2} G_F E}.$$

MSW condition

Neutrino usefulness

Hands-on session, exercise 2: estimate the exposure you need to test a 10% difference between the PREM (Newton-driven) and a mod-gravity model Earth's core mass?



Assume that errors from seismology are negligible

Neutrino usefulness

1-year IceCube analysis

$$m_{\text{core}}^{(1 \text{ yr})} = (2.7 \pm 1.0) \times 10^{24} \text{ kg} \quad \Rightarrow \quad \frac{\sigma_1}{m_{\text{core}}} \approx \frac{1.0}{2.7} \approx 37\%.$$

15-year IceCube analysis (present)

$$m_{\text{core}}^{(15 \text{ yr})} \simeq (2.7 \pm 0.27) \times 10^{24} \text{ kg} \quad \Rightarrow \quad \frac{\sigma_{15}}{m_{\text{core}}} \approx 10\%.$$

Empirical scaling of the error Pure statistics would give:

$$\sigma(T) \propto \frac{1}{\sqrt{T}} \quad \Rightarrow \quad \frac{\sigma_{15}}{\sigma_1} \approx \frac{1}{\sqrt{15}} \approx 0.26.$$

Neutrino usefulness

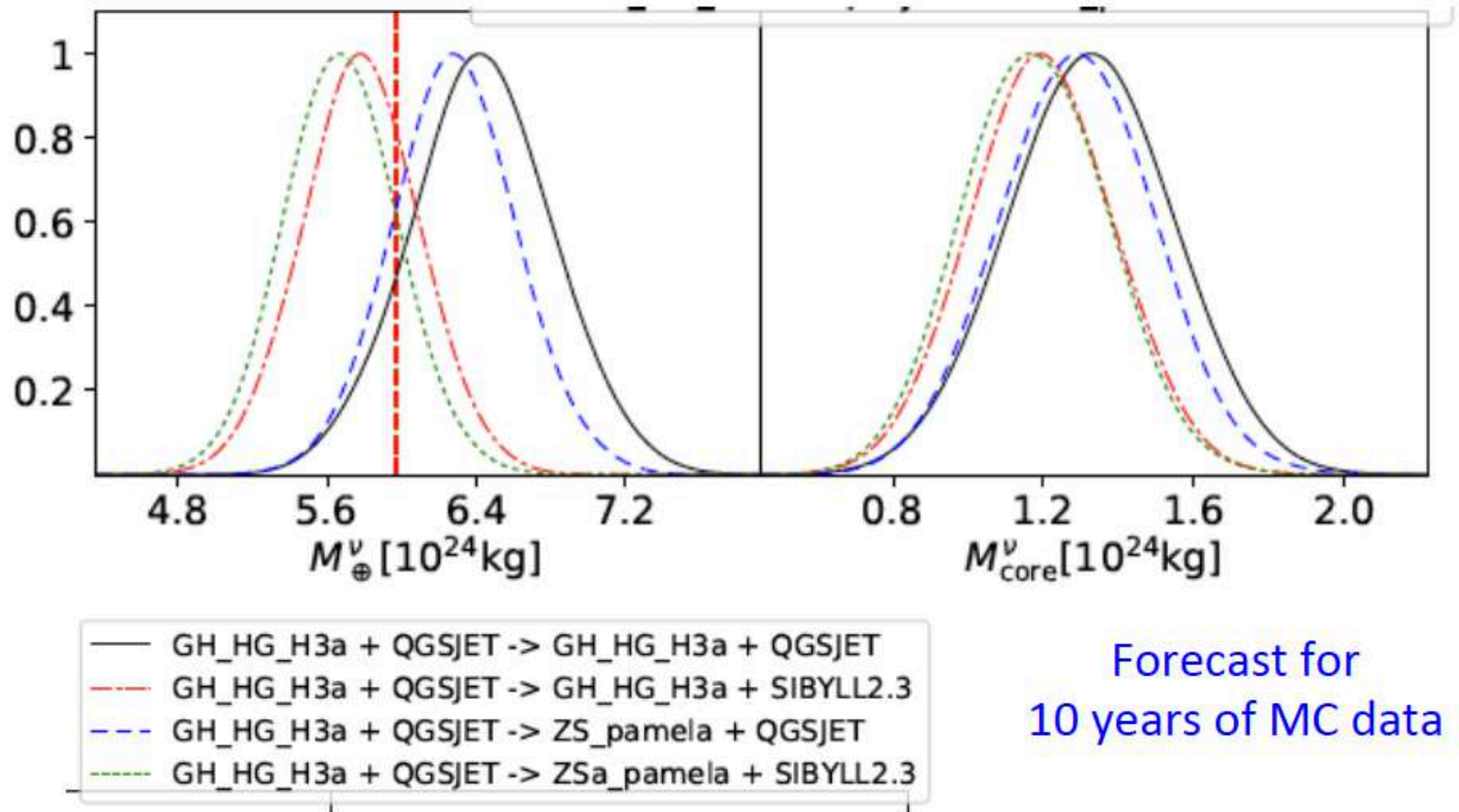
You need the error to go down to 2-3% in order to distinguish
between two models 10% apart...

Extra factor 10: $\frac{\sigma_{150}}{\sigma_{15}} \frac{\sigma_{15}}{\sigma_1} \approx \frac{1}{\sqrt{15}} \frac{1}{\sqrt{10}} \sim 0.1$

If you do not want to wait 150 years, we need at least 10 Km³

BACKUPS

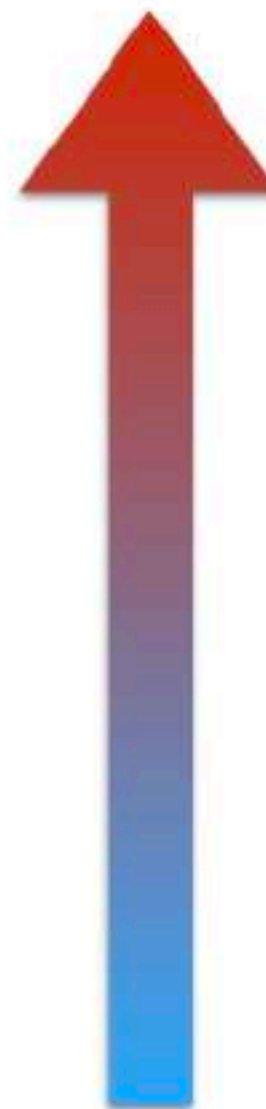
Flux and hadronic model dependence, 2



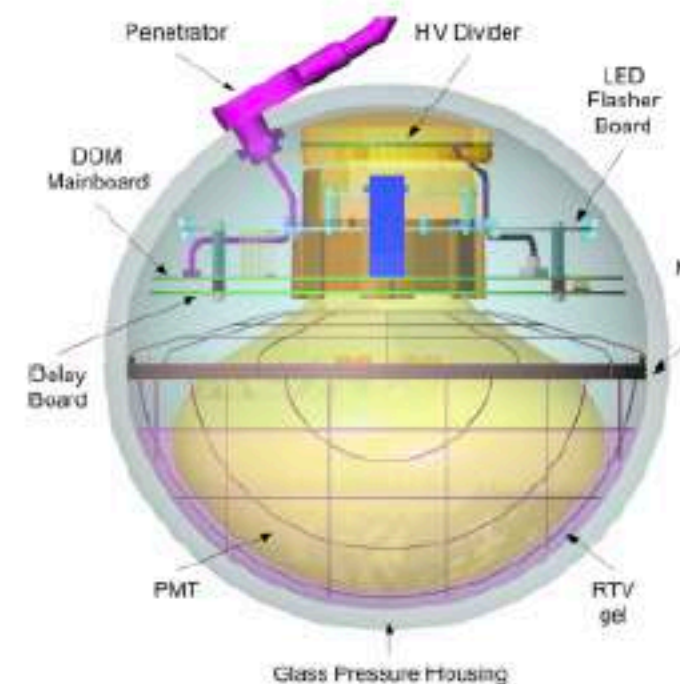
Systematics importance

- ▶ DOM efficiency
- ▶ Flux continuous parameters
 - ▶ spectral index
 - ▶ π/K ratio
 - ▶ $\nu/\bar{\nu}$ ratio Full Implementation
- ▶ Air shower hadronic models Marginally irrelevant precise check
- ▶ Primary cosmic ray fluxes Marginally irrelevant precise check
- ▶ Hole Ice Irrelevant
- ▶ Neutrino cross sections Irrelevant
- ▶ Bulk ice scattering/absorption Irrelevant

continuous systematics
discrete systematic



Important



D.O.M.

Not important

Systematics importance

- ▶ DOM efficiency
- ▶ Flux c
 - ▶ S
 - ▶ τ
 - ▶ λ
- ▶ Air sh
irrelev
- ▶ Primar
irrelev
- ▶ Hole I
- ▶ Neutric
- ▶ Bulk ice scattering/absorption
Irrelevant

Not taken into account:
OPTICAL PROPERTIES OF THE ICE

continuous systematics
discrete systematic

Not important

