From black holes at the edge of the Universe to observatories underground: the gravitational wave detection tale

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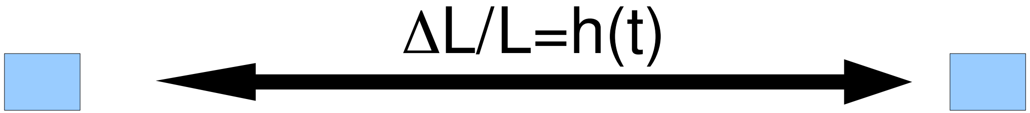
Basics



- The waves affect free floating bodies
- Space is curved
- There is no global flat coordinate system

Physical basics

- Spacetime oscillations
- Measurement with light

- Curvature – tensor of 4^{th} order: R

Physical basics

- Transverse wave
- Distortions of perpendicular geometry
- Symmetry to rotation by 180 deg
- For the EM case – 360 deg
- Photon spin =1, graviton =2
- Two polarizations: EM 90 deg; GW 45deg

Multipole expansion

- Waves - amplitude goes as 1/r
- Oscillating masses
- Multipole moments – mass, momentum
- Dimensional analysis
- Mass quadrupole moment
- Momentum quadrupole moment

The waves

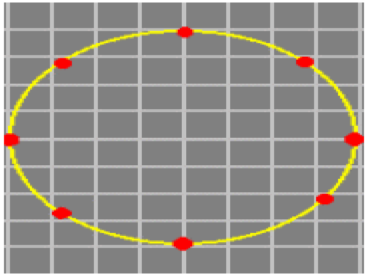
- Source: mass M, size L, period P
- Quadrupole moment ML^2

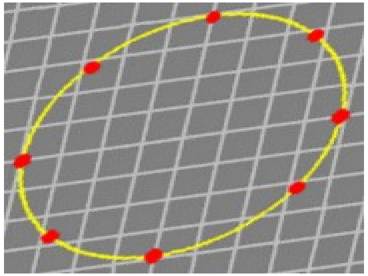
$$h=(G/c^4)(ML^2/P^2)/r$$

- Newtonian potential of the kinetic energy
- Higher moments – factors of (v/c)
- Maximally: $$h \approx \frac{R_g}{D}\left(\frac{v}{c}\right)^2$$

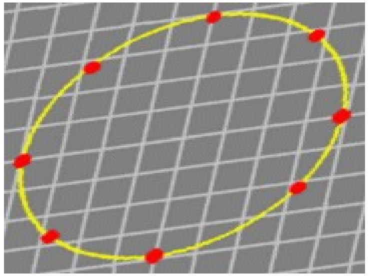
Gravitational wave

Linear polarization



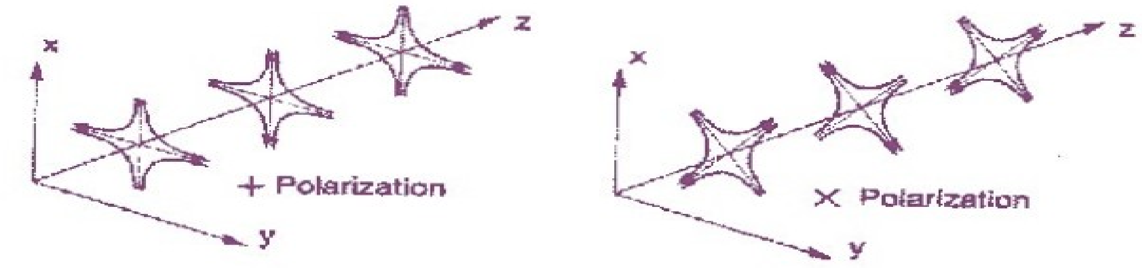


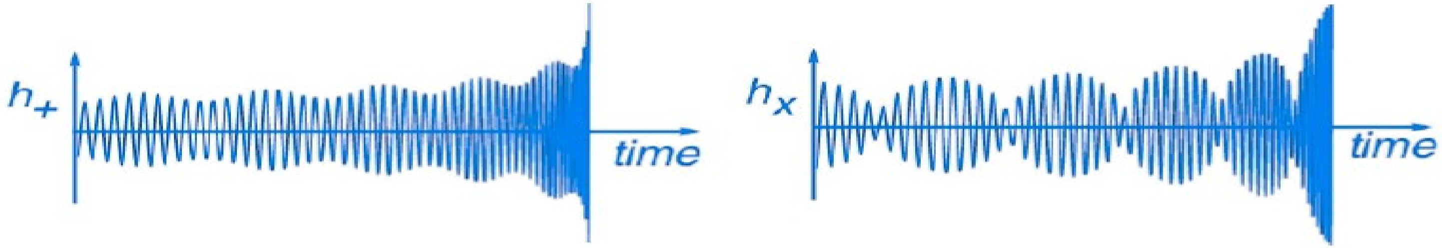
Circular polarization



Basics

- Two independent polarisations



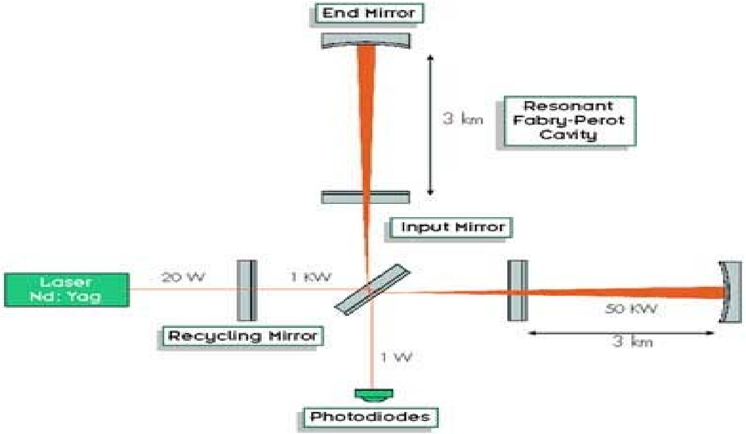


Wave propagation

- High frequency waves (wavelength smaller than curvature radius move along straight (geodetic) lines –geometrical optics
- Zero mass of graviton
- Redshift
- If wavelength comparable to curvature - scattering
- Dispersion: Negliglible- less than one wavelength in propagation through the Universe

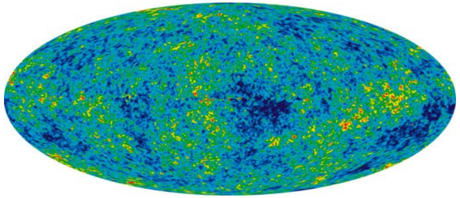
Spectrum

- 20 orders of magnitude
- Basics 4 ranges:
  - ELF log f = -16 do -10
  - VLF log f= -10 do -6
  - LF log f = -6 do 0
  - HF log f = 0 do 6

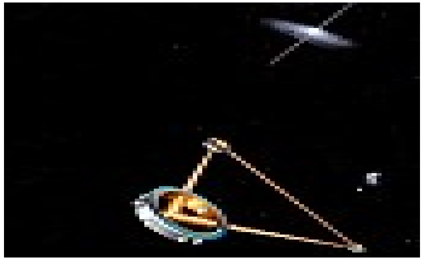


Cosmic observatories
LISA

Backgroud
radiation



Pulsar timing

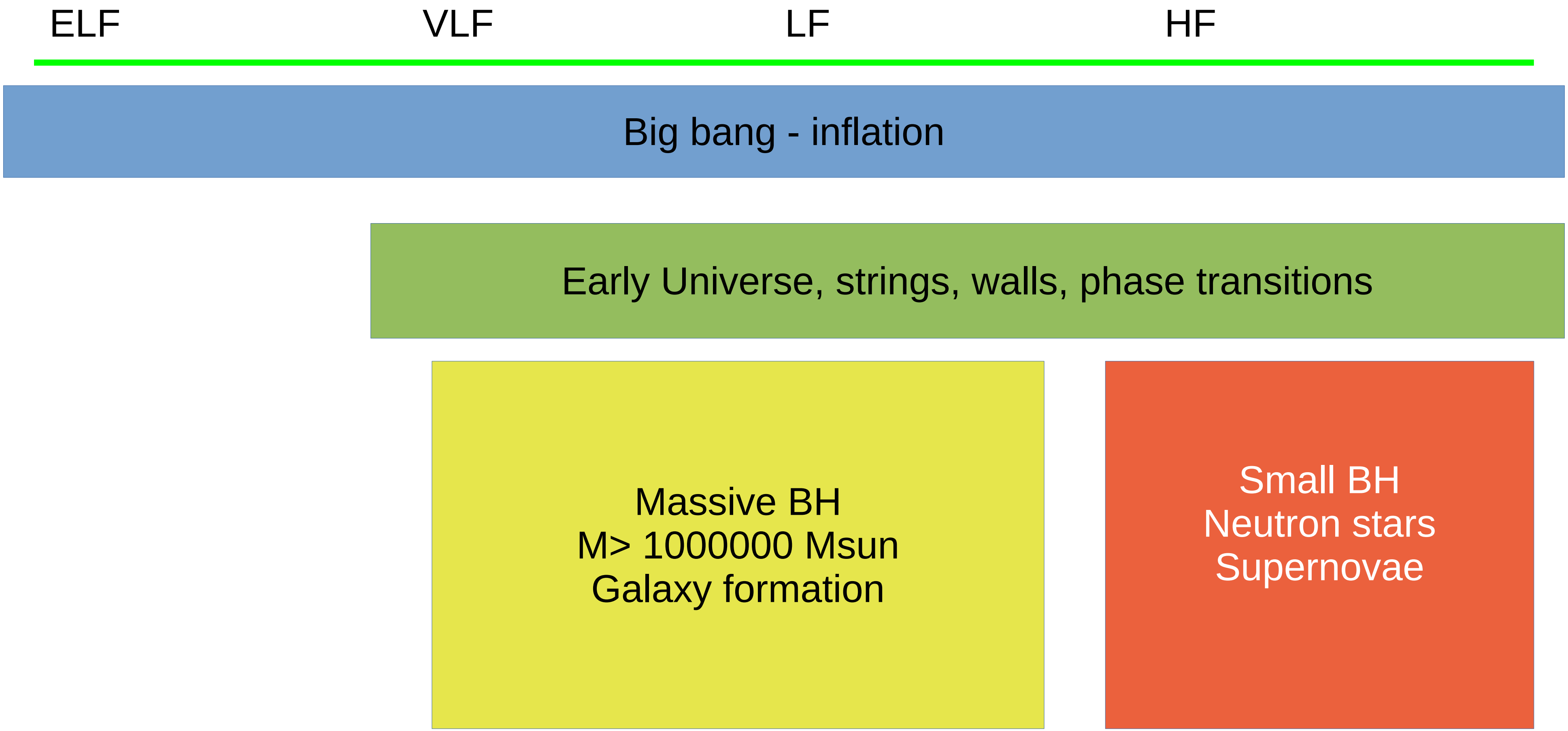


Interferometers

Resonsnt
detectors

-16 -8 0 8 log f

Sources



ELF
VLF
LF
HF
Big bang - inflation
Early Universe, strings, walls, phase transitions
Massive BH
M> 1000000 Msun
Galaxy formation
Small BH
Neutron stars
Supernovae

Basic wave generation

Look for perpendicular part

$$I_{jk} = \sum_A m_A (x_j^A x_k^A - \frac{1}{3}\delta_{jk} x_m^A x_m^A)$$

$$P_{ij} = \delta_{ij} - n_i n_j$$

$$I_{jk}^{TT} = P_{jk} P_{km} I_{lm} - \frac{1}{2} P_{jk} (P_{lm} I_{lm})$$

$$h_{jk}^{TT}(t, \vec{x}) = \frac{2}{r} \frac{G}{c^4} \ddot{I}_{jk}^{TT}(t - \frac{r}{c})$$

Expected amplitude

$$R_g = 2\frac{GM}{c^2}$$

Gravitational radius

$$h \approx \frac{R_g}{D}\frac{v^2}{c^2}$$

For a fiducial source: two 30 solar mass black holes at 100Mpc

$$h \approx \frac{R_g}{D}\frac{v^2}{c^2} \approx 2 \times 10^{-21}$$

Energy and angular momentum

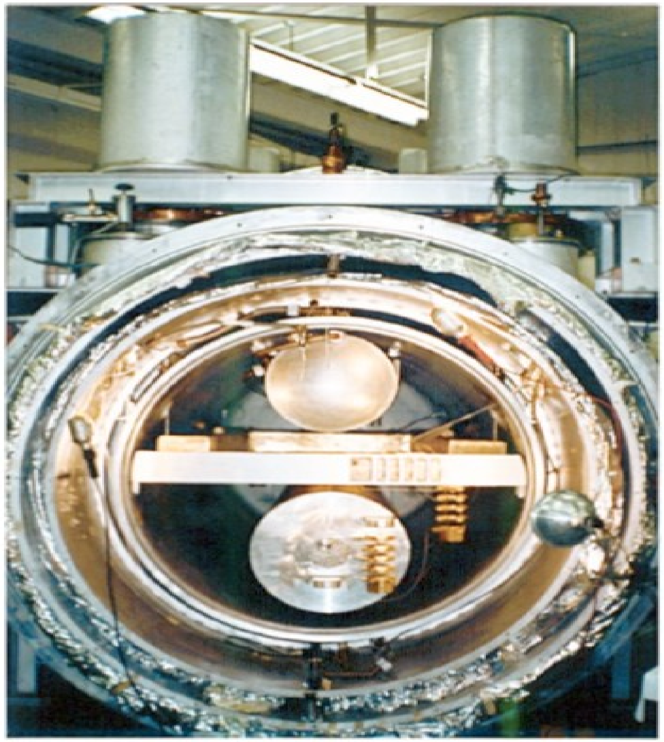
$$L_{GW} = \frac{1}{5}\,\frac{G}{c^5}\,\frac{d^3\,I_{jk}}{dt^3}\,\frac{d^3\,I_{jk}}{dt^3}$$

$$\frac{dJ_i}{dt} = -\frac{2}{5}\,\frac{G}{c^5}\epsilon_{ijk}\,\frac{d^2\,I_{jm}}{dt^2}\,\frac{d^3\,I_{km}}{dt^3}$$

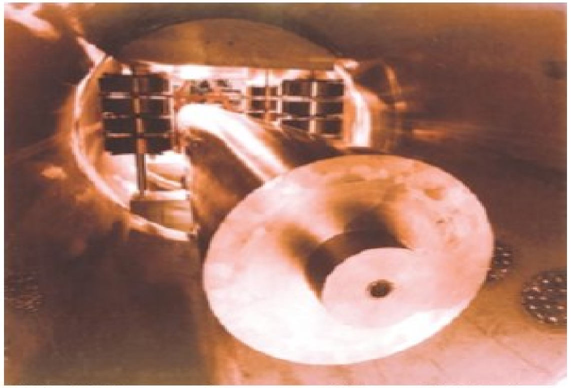
Detection

- Test masses
- Measuring distances
- Need a network of detectors

First attempts: resonant detectors



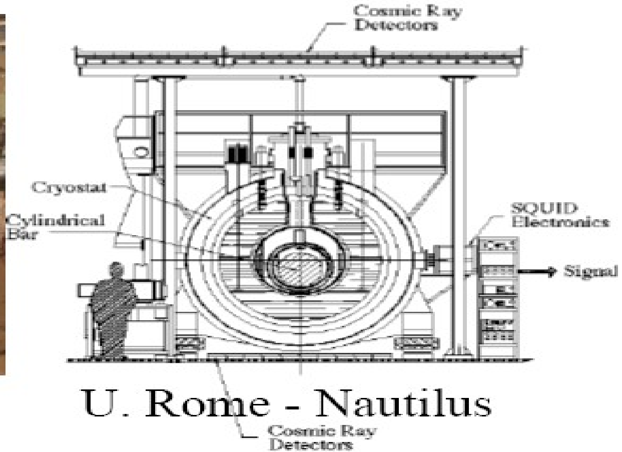
Louisiana State U. - Allegro



U. West Australia - Niobe

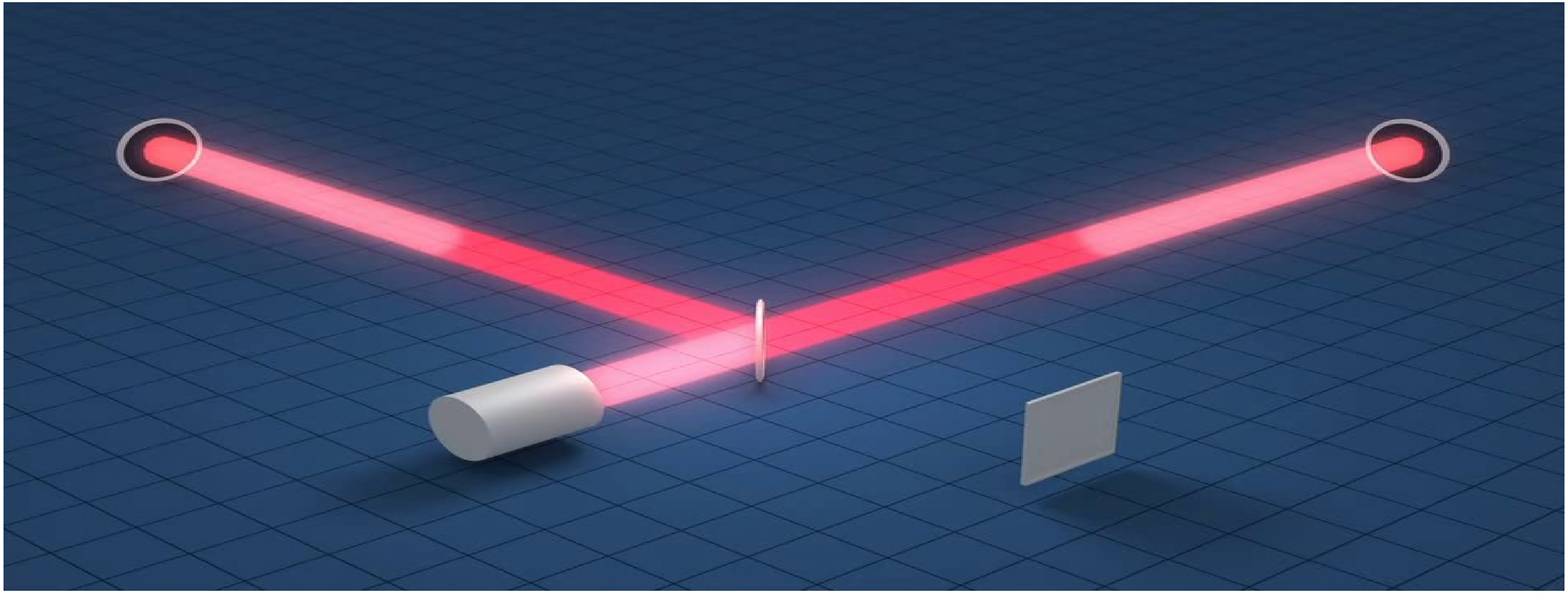
U. Padova - Auriga

CERN - Explorer

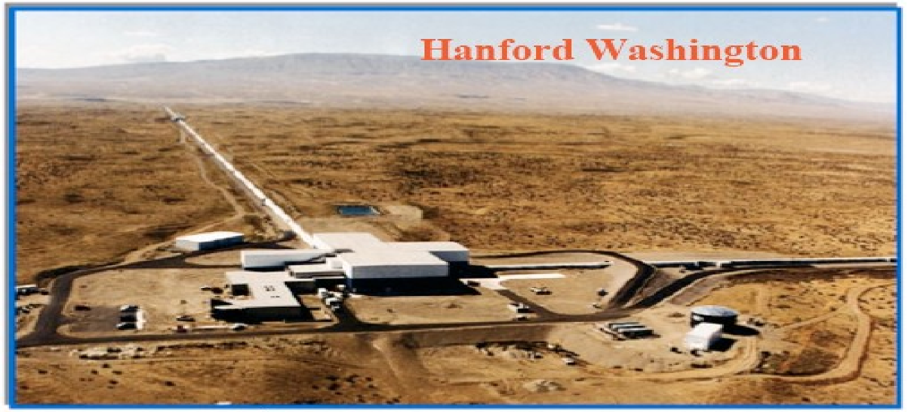


U. Rome - Nautilus

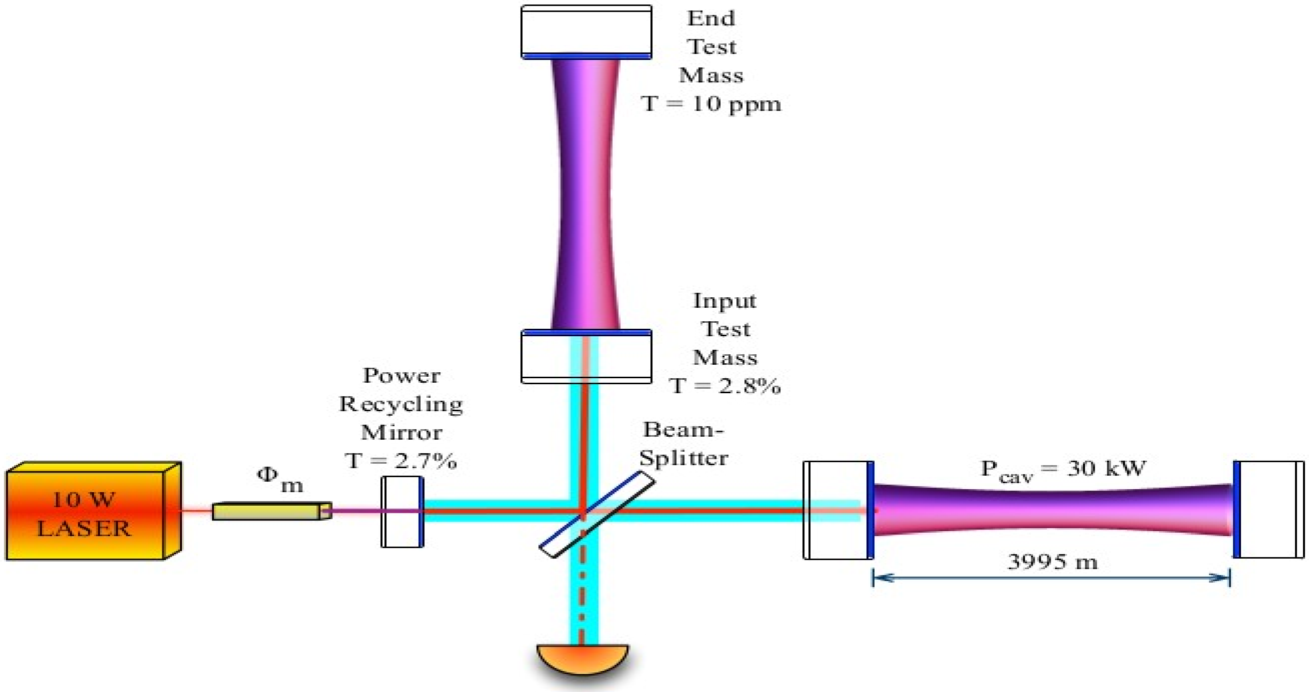
Interferometers



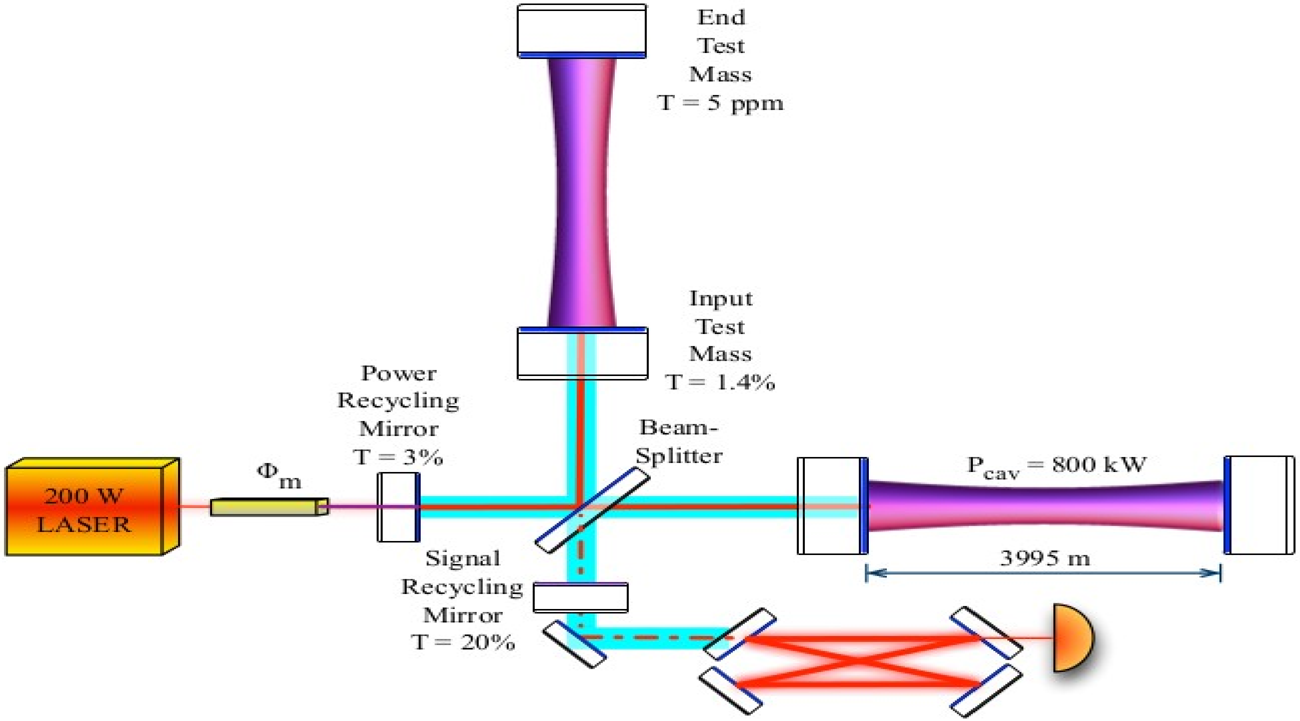
Interferometers - HF



Interferometers: initial



Second generation interferometers



Noise reduction

- Seismic noise quenched by 10 orders of magnitude
- Thermal noise – heavy mass, smart coating
- Quantum shot noise - laser power increase – about 100kW in the cavity
- But – note – fundamental quantum limit:
    - Low vs. high frequency

Order of magnitude estimates

$$\Delta L \approx 10^{-18} m$$

500 round trips, expected delay:

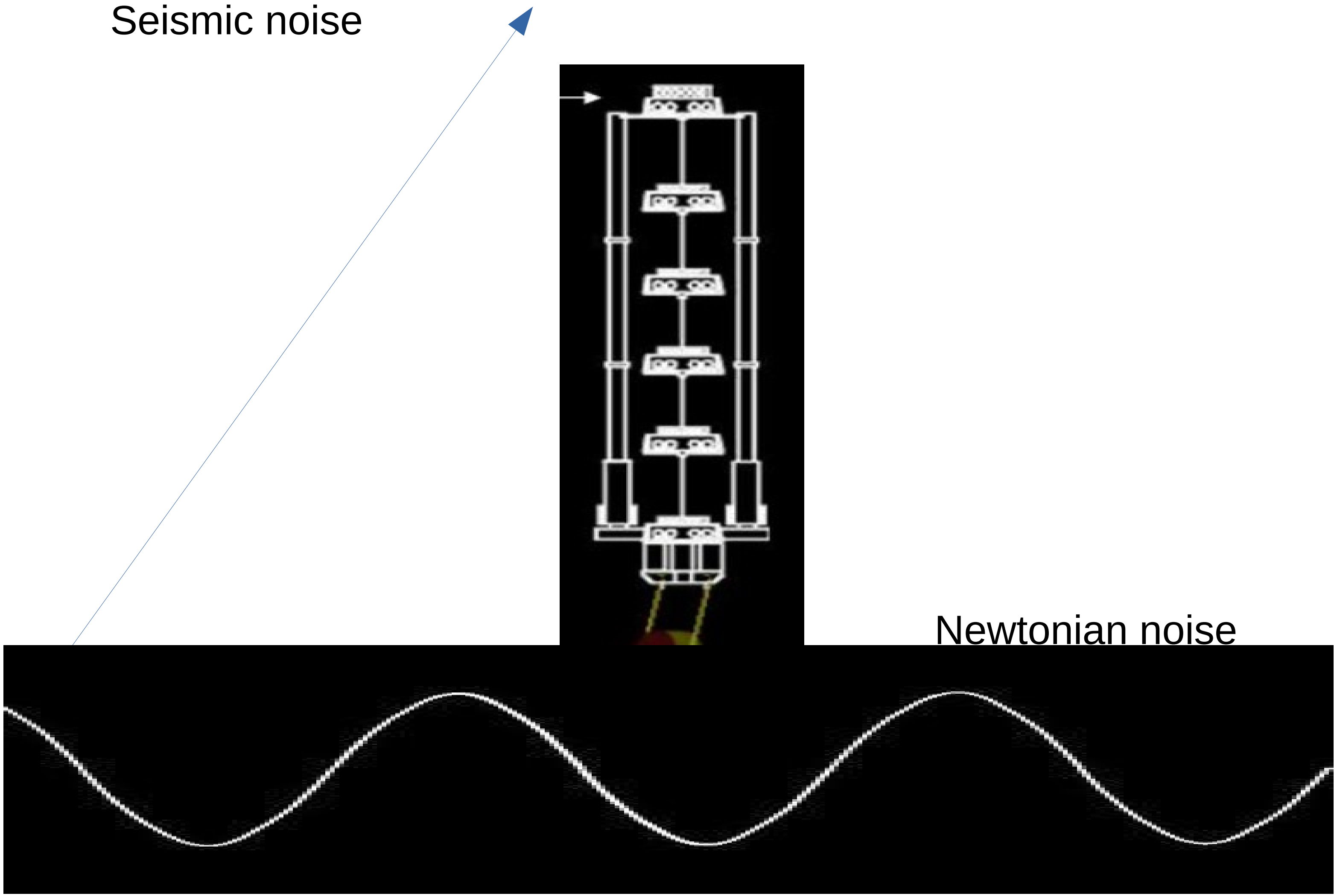
$$\Delta L_{eff} = 500 \times 10^{-18}\mathrm{m} \approx 5 \times 10^{-16}\mathrm{m}$$

The laser wavelength is 1064nm. Expected phase amplitude:

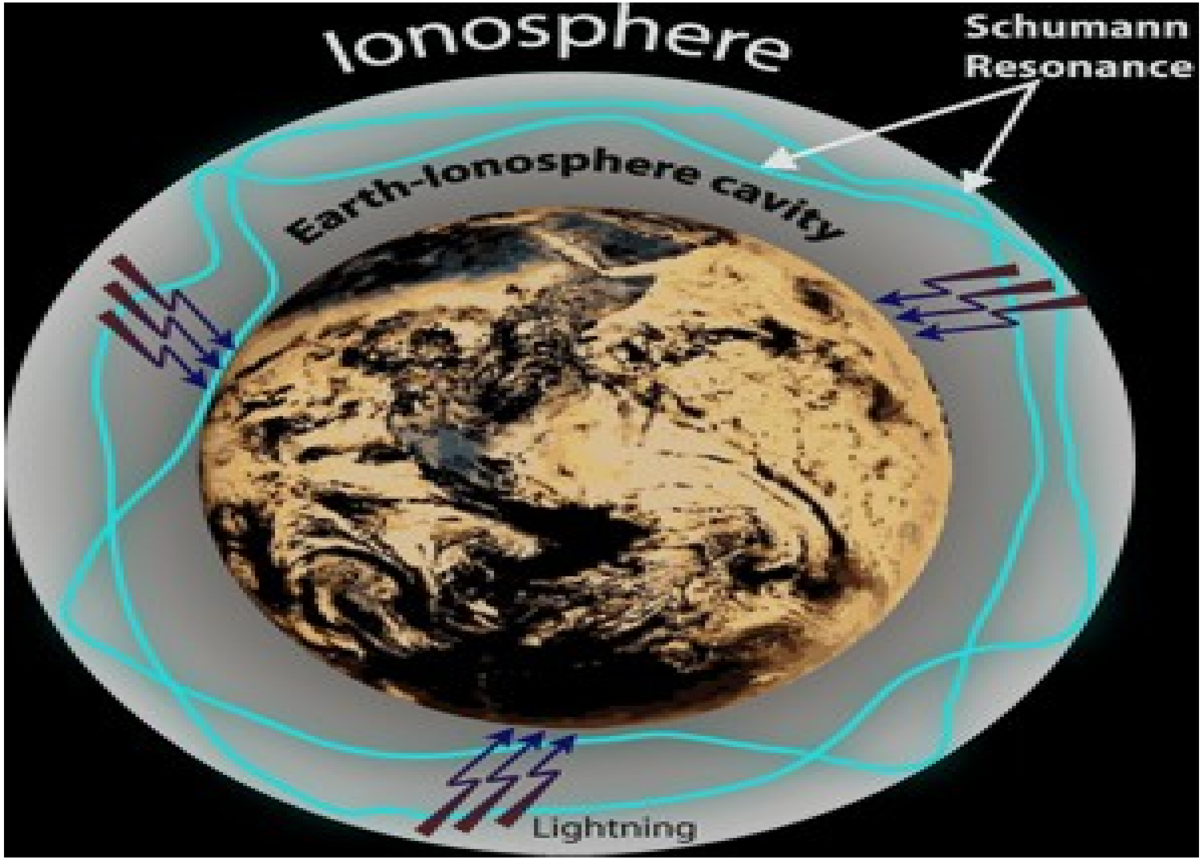
$$\Delta\phi \approx 5 \times 10^{-16}/10^{-6} = 5 \times 10^{-10}$$

With 100kW power the Poisson noise fluctuations are

$$\delta\phi = N^{-1/2} = \sqrt{\frac{hc}{\lambda P \Delta T}} = \sqrt{\frac{10^{-19}\mathrm{J}}{10^{5}\mathrm{W}10^{-2}\mathrm{s}}} \approx 10^{-11}$$

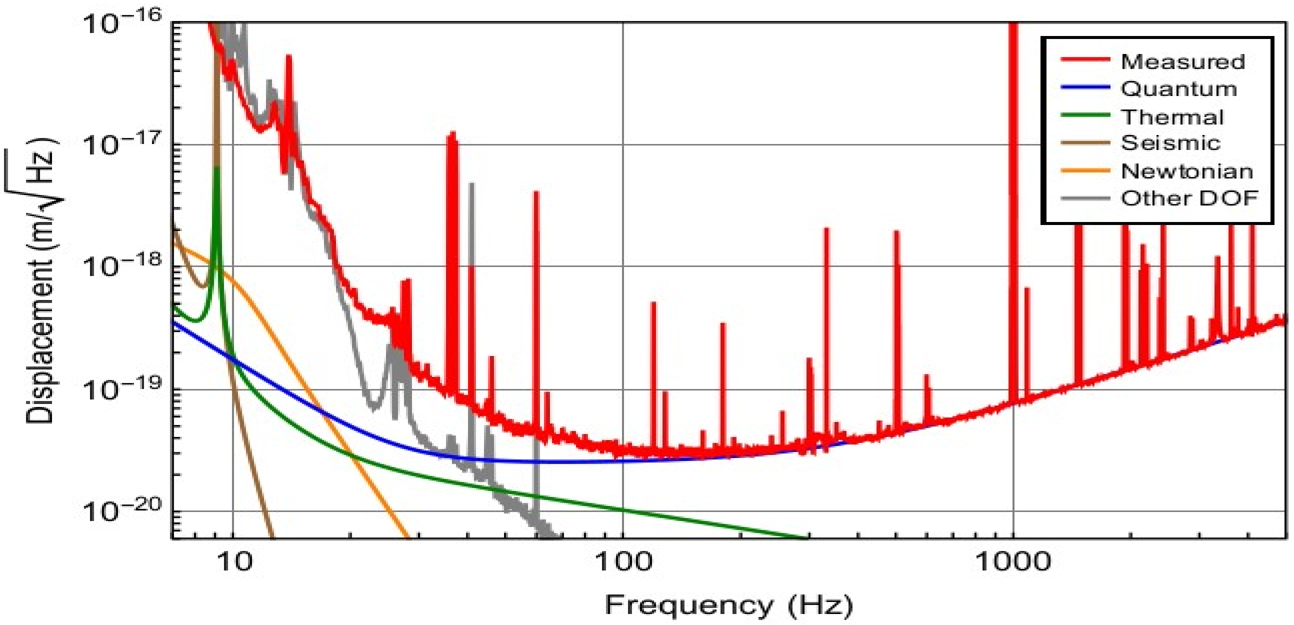

Seismic noise
Newtonian noise

Magnetic noise



Technical challenges

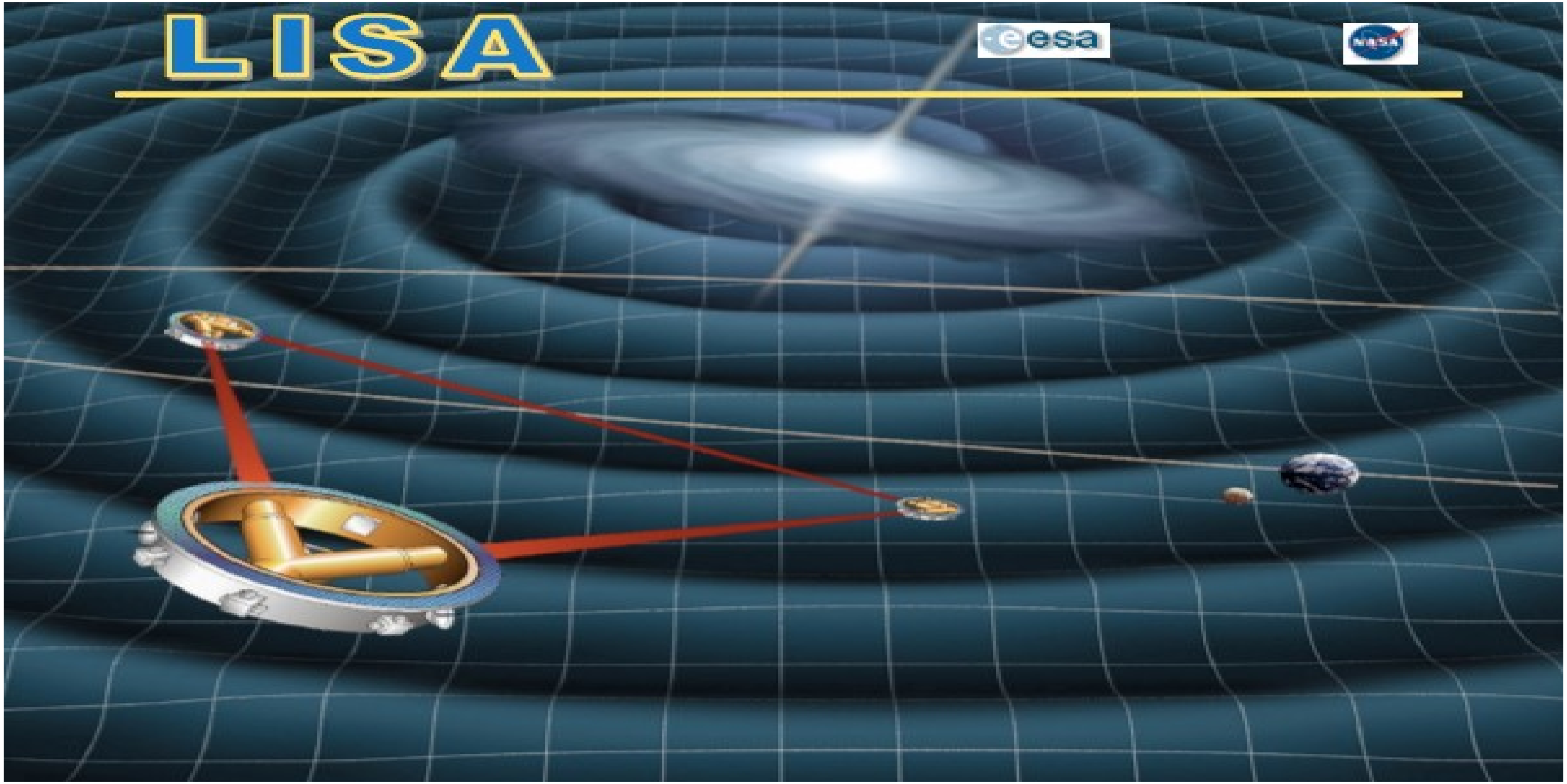
- Seismic isolation
- Vacuum system
- High power lasers
- Thermal noise
- Quantum fluctuations



Abbott et al 2016

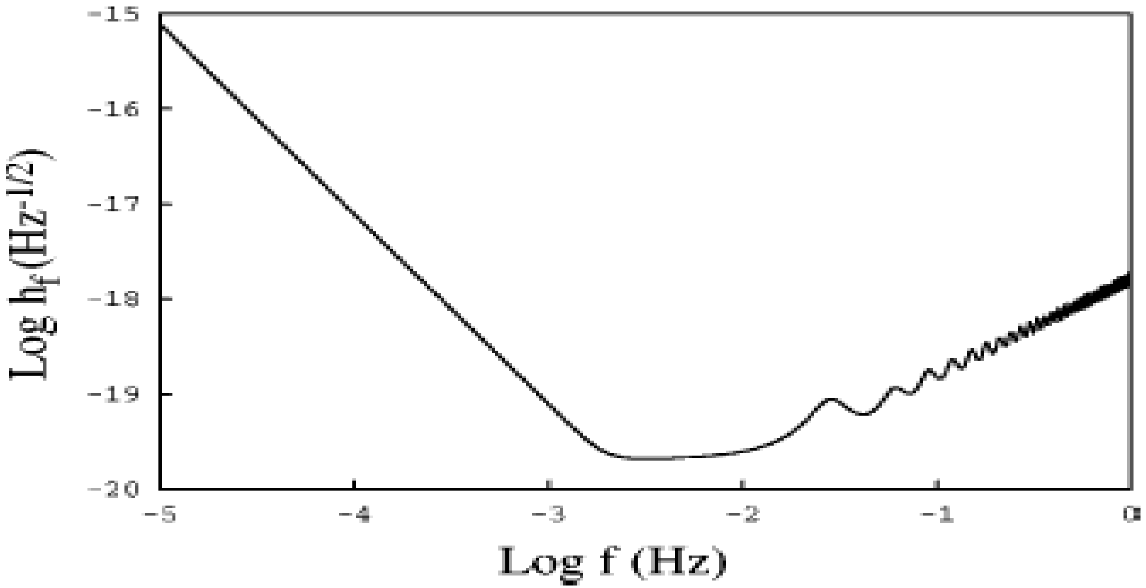
Also – newtonian noise, magnetic noise

LF instruments



LISA

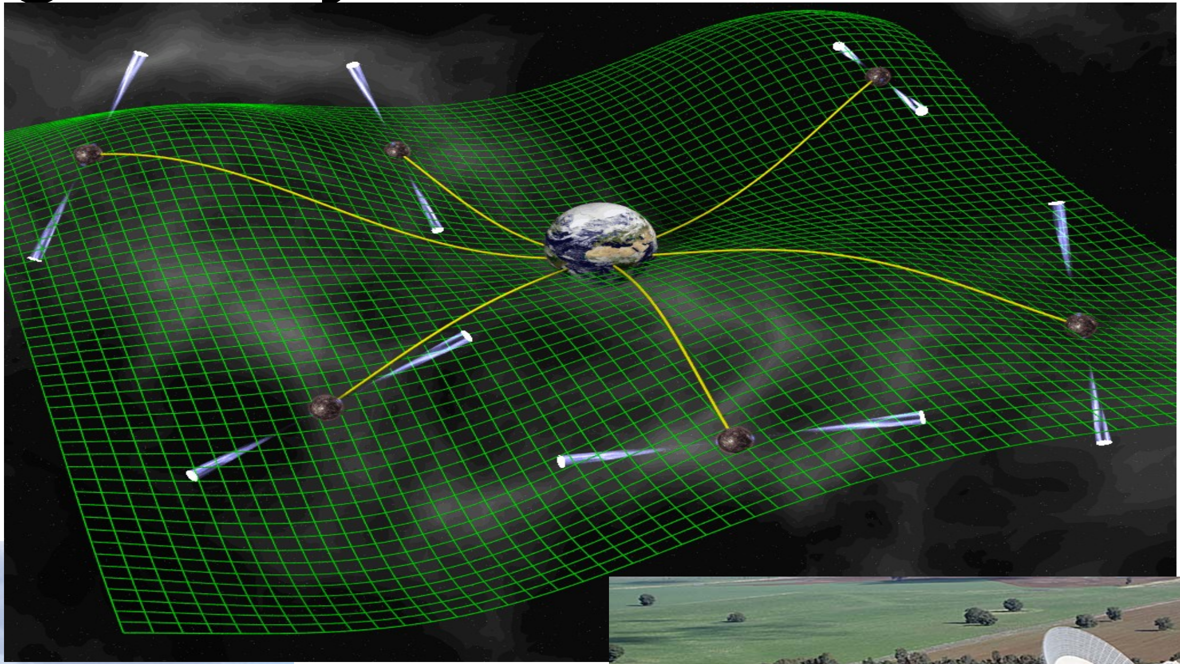
- Three satellites 5 Gm apart
- Lasers 1W
- Telescopes 30 cm
- Satellite motion



- Launch 2037
- DECIGO, BBO, ALIA

Pulsar Timing Arrays

- PPTA
- EPTA
- SKA

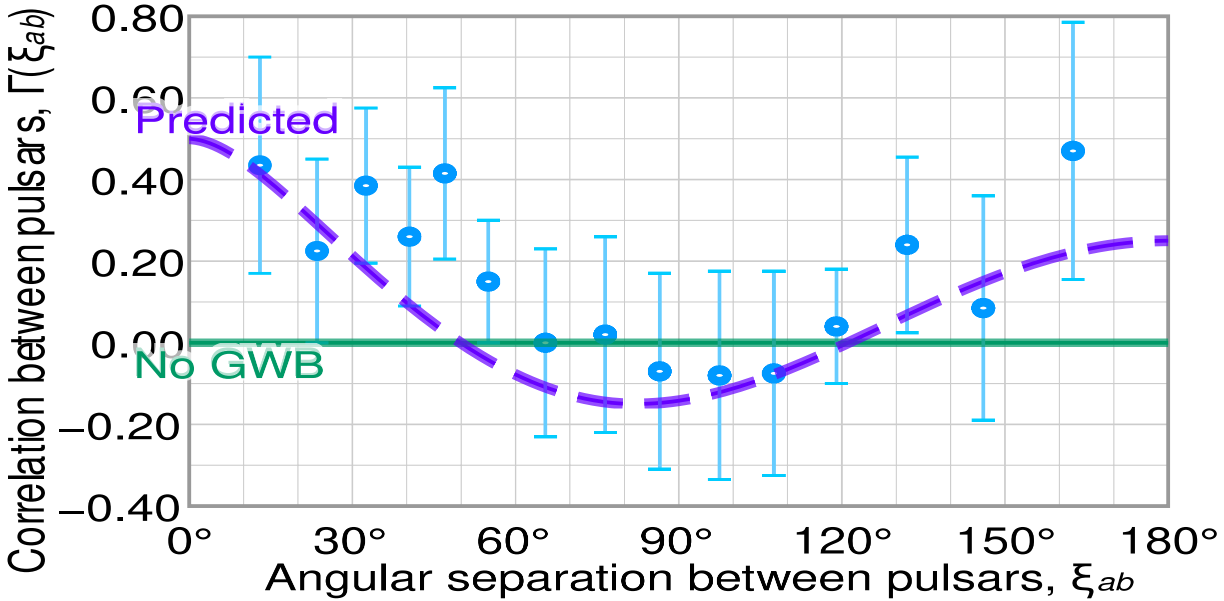


PTA search and detection

Correlation between timing residuals of distant pulsars

Helling Downs curve.

Expect more in the coming months!



Detector sensitivities and readout

Detector provides a digitized output – a string of numbers.

h1 h2 h3 ... hn

$$\tilde{h}(f) = \mathcal{F}(x(t))(f) = \int_{-\infty}^{\infty} h(t) \exp(2\pi i f t) dt$$

$$h(t) = \mathcal{F}^{-1}(\tilde{h}(f))(t) = \int_{-\infty}^{\infty} \tilde{h}(f) \exp(-2\pi i f t) df$$

Power spectral density

$$\left\langle \tilde{h}(f) \tilde{h}^*(f') \right\rangle = \frac{1}{2} \delta(f - f') S_c(f)$$

Units!

Detector noise curve

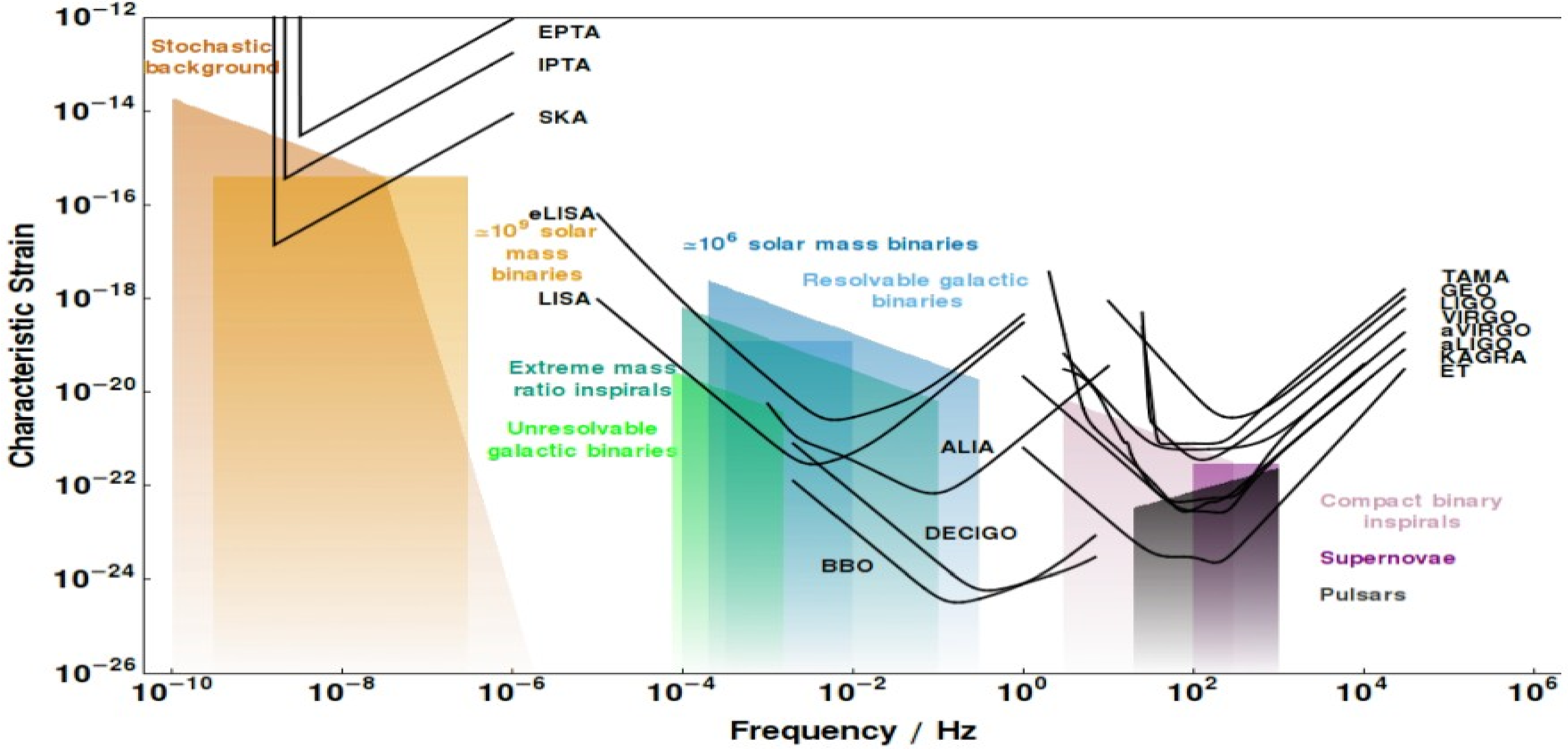
Characteristic strain

$$h_c(f) = \sqrt{f S_c(f)}$$

Signal to noise

$$\rho^2 = \int d(\log f) \frac{|h_c(f)|^2}{|h_n(f)|^2}$$

+ Easy to read the S/N off graphs
- does not correspond to signal amplitude
+ shows the rms in a given band



Characteristic Strain
Frequency / Hz
Stochastic background
EPTA
IPTA
SKA
eLISA
LISA
≃10⁹ solar mass binaries
≃10⁶ solar mass binaries
Resolvable galactic binaries
Extreme mass ratio inspirals
Unresolvable galactic binaries
ALIA
DECIGO
BBO
TAMA
GEO
LIGO
VIRGO
aVIRGO
aLIGO
KAGRA
ET
Compact binary inspirals
Supernovae
Pulsars

Detector noise curves

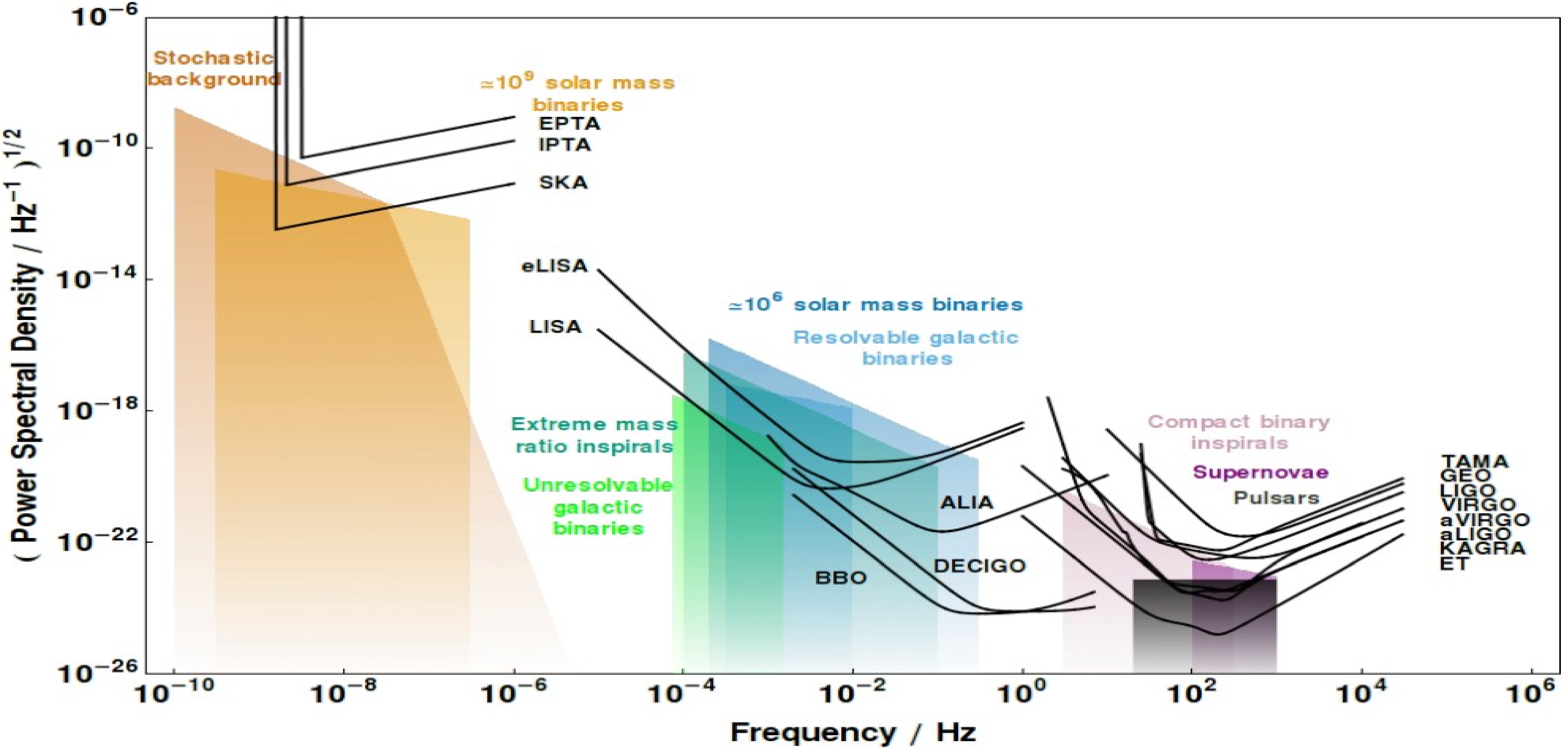
Amplitude spectral density (or root PSD)

$$\mathrm{ASD}(f) = \sqrt{S_h(f)} = h(f) f^{-1/2}$$

- units

- mean square amplitude of the signal from PSD

- most commonly used



Stochastic background
≃10^9 solar mass binaries
EPTA
IPTA
SKA
eLISA
LISA
≃10^6 solar mass binaries
Resolvable galactic binaries
Extreme mass ratio inspirals
Unresolvable galactic binaries
ALIA
BBO
DECIGO
Compact binary inspirals
Supernovae
Pulsars
TAMA
GEO
LIGO
VIRGO
aVIRGO
aLIGO
KAGRA
ET
(Power Spectral Density / Hz^-1)^1/2
Frequency / Hz

Detector noise curves

Energy density

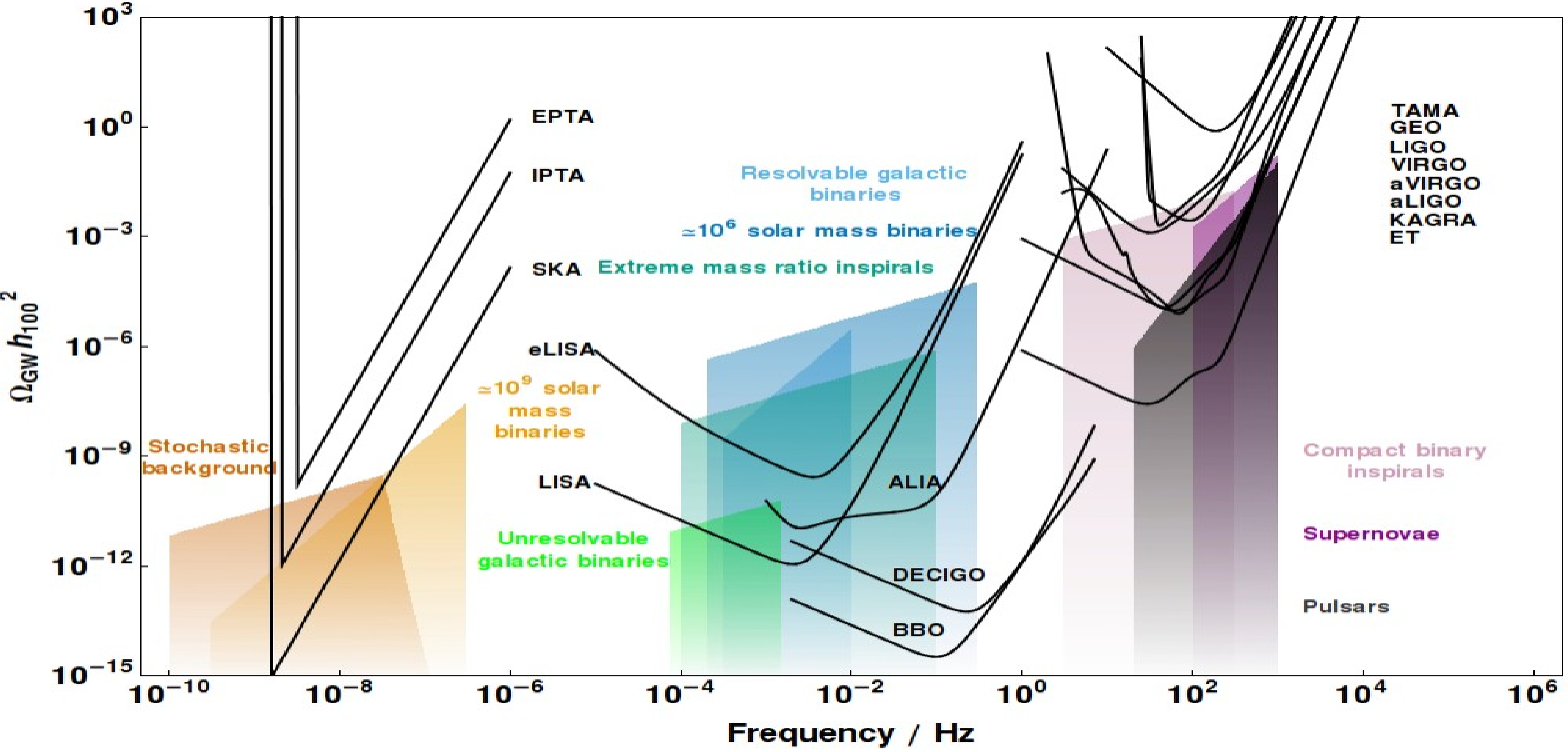
$$T_{\mu\nu} = \frac{c^4}{32\pi G} \langle \partial_\mu h_{\alpha\beta} \partial_\nu h_{\alpha\beta} \rangle$$

$$\rho c^2 = \frac{c^2}{16\pi G} \int_0^\infty df (2\pi f)^2 \tilde{h}(f) \tilde{h}^*(f) = \int_0^\infty df \frac{\pi c^2}{4G} f^2 S_h(f)$$

$$S_E(f) = \frac{\pi c^2}{4G} f^2 S_h(f)$$

Energy density per logarithmic frequency interval

$$\Omega_{GW}(f) = \frac{f S_E(f)}{\rho_c c^2} = \frac{8\pi G}{3H_0^2 c^2} f S_E(f)$$



EPTA
IPTA
SKA
eLISA
LISA
ALIA
DECIGO
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ET
Resolvable galactic binaries
≃10^6 solar mass binaries
Extreme mass ratio inspirals
≃10^9 solar mass binaries
Stochastic background
Unresolvable galactic binaries
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Supernovae
Pulsars
$\Omega_{GW} h_{100}^2$
Frequency / Hz
10^3
10^0
10^{-3}
10^{-6}
10^{-9}
10^{-12}
10^{-15}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^0
10^2
10^4
10^6

Let us come down to Earth!

Conversions

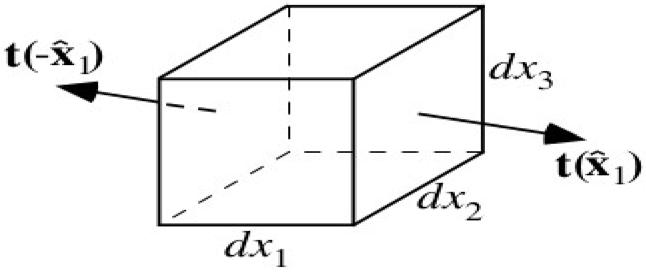
$$\Omega_{GW}(f) = \frac{8\pi G}{3H_0^2 c^2} f S_E(f) = \frac{2\pi^2}{3H_0^2} f^3 S_h(f) = \frac{2\pi^2}{3H_0^2} f^2 [h_c(f)]^2 = \frac{8\pi^2}{3H_0^2} f^4 \left|\tilde{h}(f)\right|^2$$

$$\frac{3H_0^2 c^2}{8\pi G} \frac{\Omega_{GW}(f)}{f} = S_E(f) = \frac{c^2\pi}{4G} f^2 S_h(f) = \frac{c^2\pi}{4G} f [h_c(f)]^2 = \frac{\pi c^2}{G} f^3 \left|\tilde{h}(f)\right|^2$$

$$ASD(f) = S_h(f)^{1/2} = \frac{|h_c(f)|}{f} = 2f|\tilde{h}(f)| = \sqrt{\frac{4GS_E(f)}{\pi c^2 f^2}} = \sqrt{\frac{3H_0^2 \Omega_{GW}}{2\pi^2 f^3}}$$

$$h_c(f) = f\,ASD(f)$$

Seismic waves 1



Stress tensor τ_{ij}

coordinates u_i

Force on a volume element:

$$F_i = \partial_i \tau_{ij} dx_1 dx_2 dx_3$$

Seismic waves 2

Mass of the volume element: $$m = \rho dx_1 dx_2 dx_3$$

Newton equation of motion:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \partial_j \tau_{ij} + f_i$$

stress

Body (external) forces

Seismic waves 3

We want to express stress tensor as a function of strain!

$$e_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i)$$

The stress tensor of an isotropic medium can be expressed as

$$\tau_{ij} = \lambda \delta_{ij} e_{kk} + 2\mu e_{ij}$$

$\lambda \text{ and } \mu$ are Lame coeficients. So

$$\tau_{ij} = \lambda \delta ij \partial_k u_k + \mu(\partial_i u_j + \partial_j u_i)$$

Seismic waves 4

Insert this into the equation of motion

$$\begin{aligned}
\rho\frac{\partial^2 u_i}{\partial t^2} &= \partial_j[\lambda\delta_{ij}\partial_k u_k + \mu(\partial_i u_j + \partial_j u_i)] \\
&= \partial_i\lambda\partial_k u_k + \lambda\partial_i\partial_k u_k + \partial_j\mu(\partial_i u_j + \partial_j u_i) + \mu\partial_j\partial_i u_j + \mu\partial_j\partial_j u_i \\
&= \partial_i\lambda\partial_k u_k + \partial_j\mu(\partial_i u_j + \partial_j u_i) + \lambda\partial_i\partial_k u_k + \mu\partial_i\partial_j u_j + \mu\partial_j\partial_j u_i.
\end{aligned}$$

Or a little simpler:

$$\rho\ddot{\mathbf{u}} = \nabla\lambda(\nabla\cdot\mathbf{u}) + \nabla\mu\cdot[\nabla\mathbf{u} + (\nabla\mathbf{u})^T] + (\lambda+\mu)\nabla\nabla\cdot\mathbf{u} + \mu\nabla^2\mathbf{u}.$$

Seismic waves 5

$$\nabla \times \nabla \times \mathbf{u} = \nabla\nabla \cdot \mathbf{u} - \nabla^2 \mathbf{u}$$

$$\nabla^2 \mathbf{u} = \nabla\nabla \cdot \mathbf{u} - \nabla \times \nabla \times \mathbf{u}.$$

The seismic wave equation:

$$\rho \ddot{\mathbf{u}} = \nabla\lambda(\nabla \cdot \mathbf{u}) + \nabla\mu \cdot [\nabla\mathbf{u} + (\nabla\mathbf{u})^T] + (\lambda + 2\mu)\nabla\nabla \cdot \mathbf{u} - \mu\nabla \times \nabla \times \mathbf{u}.$$

For an isotropic medium

$$\boxed{\rho \ddot{\mathbf{u}} = (\lambda + 2\mu)\nabla\nabla \cdot \mathbf{u} - \mu\nabla \times \nabla \times \mathbf{u}.}$$

Seismic waves 6

Simple solutions:

Take div of the seismic equation:

$$\frac{\partial^2(\nabla \cdot \mathbf{u})}{\partial t^2} = \frac{\lambda + 2\mu}{\rho} \nabla^2(\nabla \cdot \mathbf{u})$$

$$\nabla^2(\nabla \cdot \mathbf{u}) - \frac{1}{\alpha^2} \frac{\partial^2(\nabla \cdot \mathbf{u})}{\partial t^2} = 0,$$

Wave equation with velocity

$$\alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$

P - wave

Seismic waves 7

Take curl of the seismic equation:

$$\frac{\partial^2(\nabla \times \mathbf{u})}{\partial t^2} = -\frac{\mu}{\rho}\nabla \times \nabla \times (\nabla \times \mathbf{u}).$$

$$\frac{\partial^2(\nabla \times \mathbf{u})}{\partial t^2} = \frac{\mu}{\rho}\nabla^2(\nabla \times \mathbf{u})$$

Wave equation: S wave

$$\nabla^2(\nabla \times \mathbf{u}) - \frac{1}{\beta^2}\frac{\partial^2(\nabla \times \mathbf{u})}{\partial t^2} = 0,$$

velocity

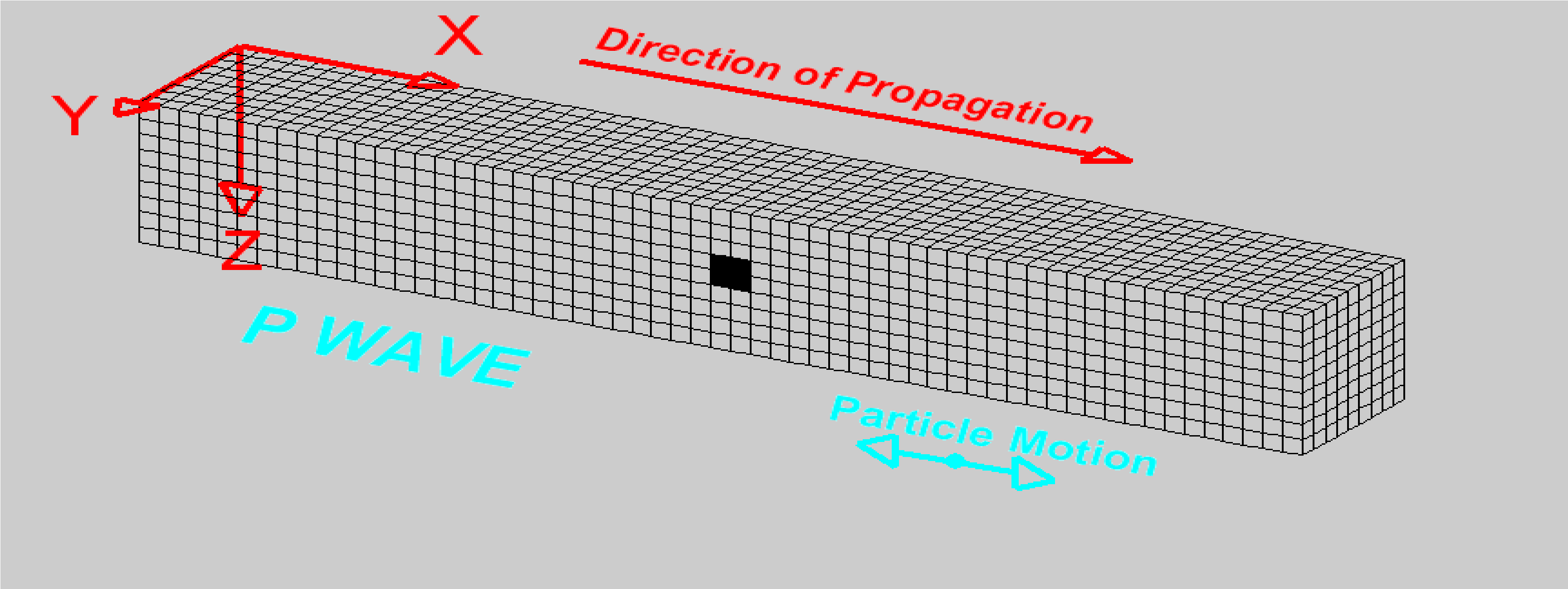
$$\beta = \sqrt{\frac{\mu}{\rho}}$$

Body waves

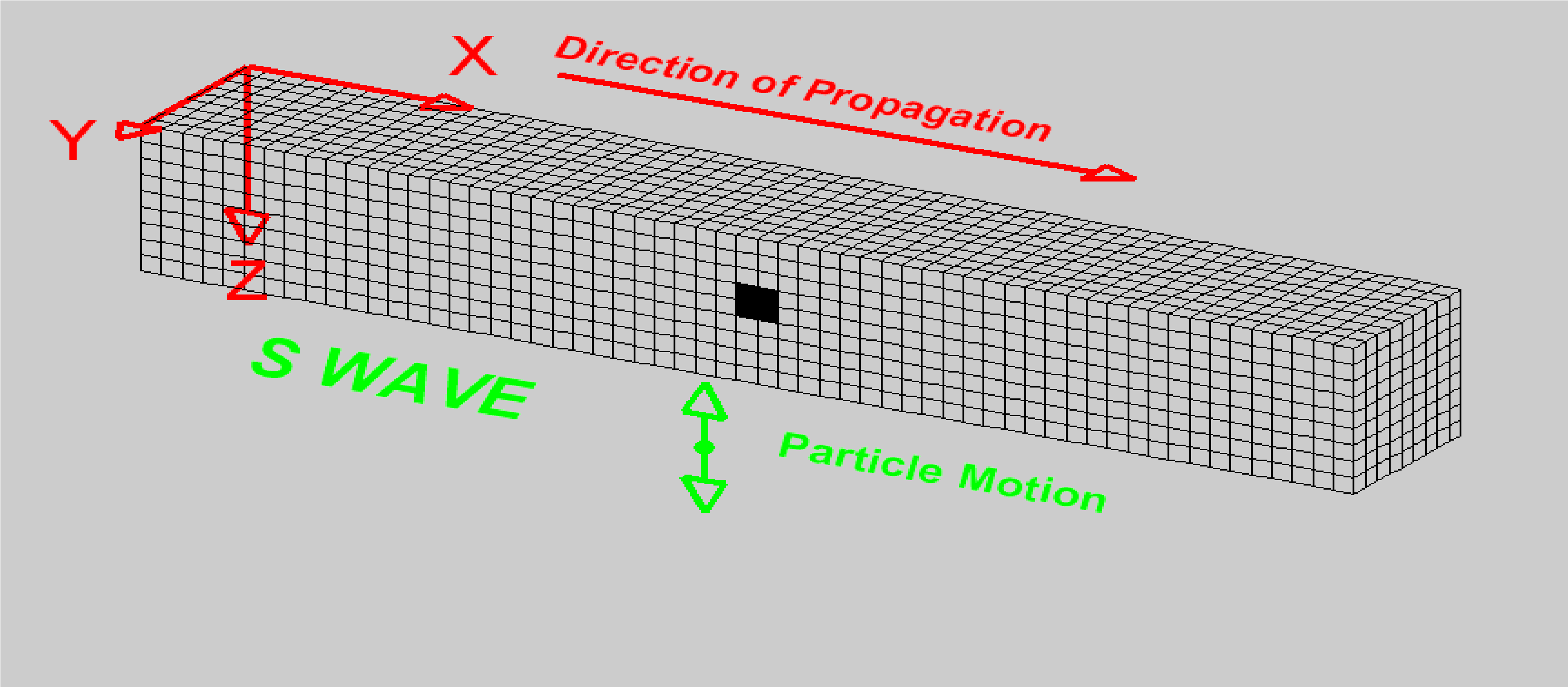
- P-wave (primary wave)

- S-wave (secondary wave)

P-wave



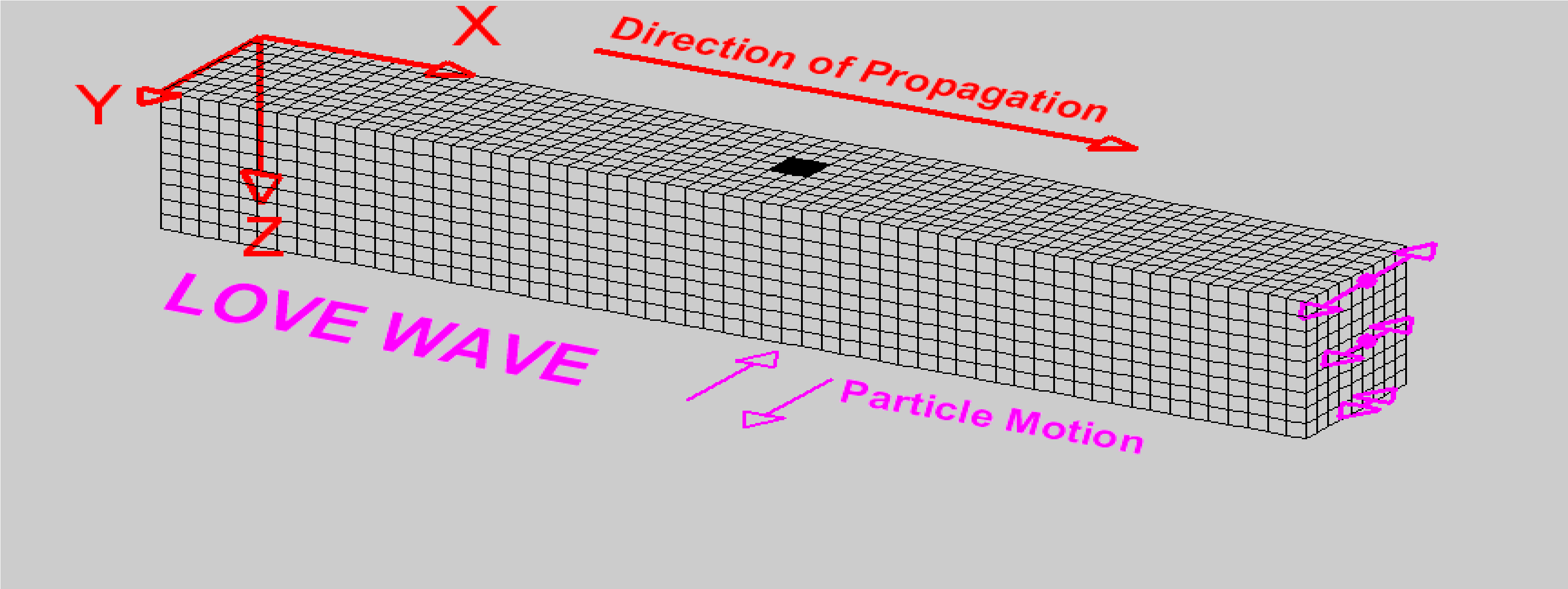
S-wave



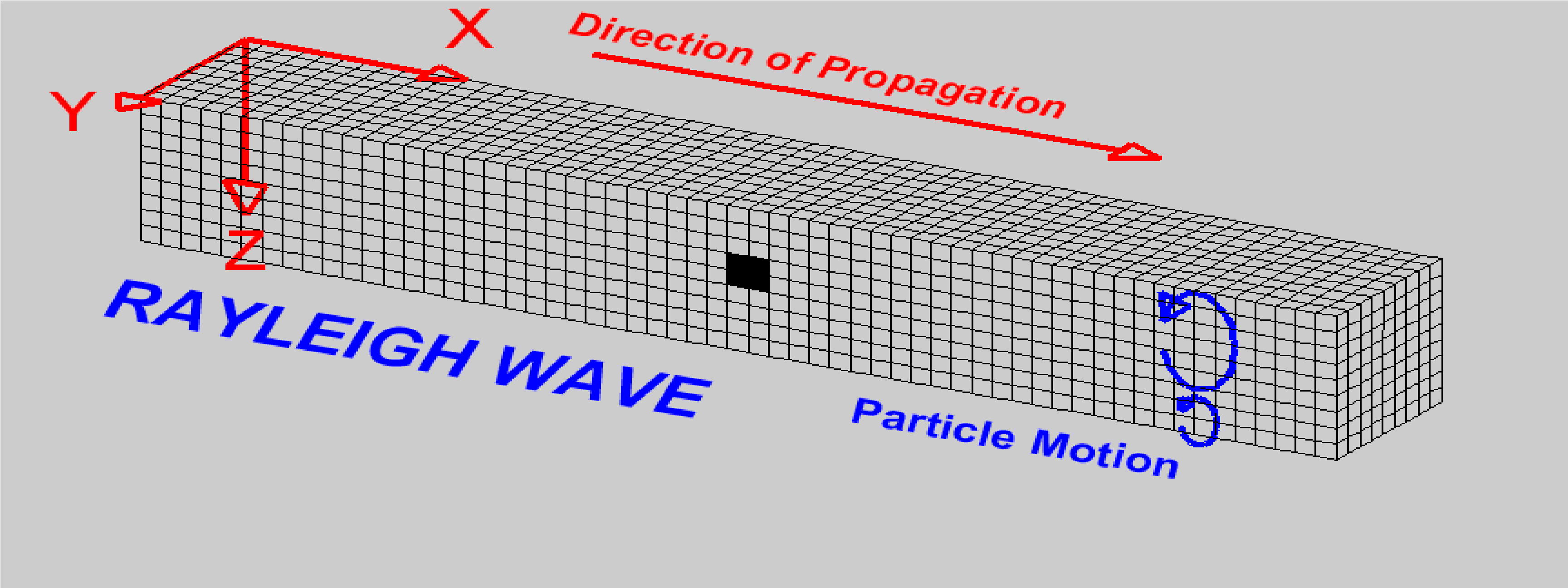
Surface waves

- Need to solve the equation with specific boundary conditions
- Space- half uniform density, half zero density
- Displacement vanished at -infinity
- Solutions can be thought as linear combinations of S and P wave that are reflected off the boundary (surface)

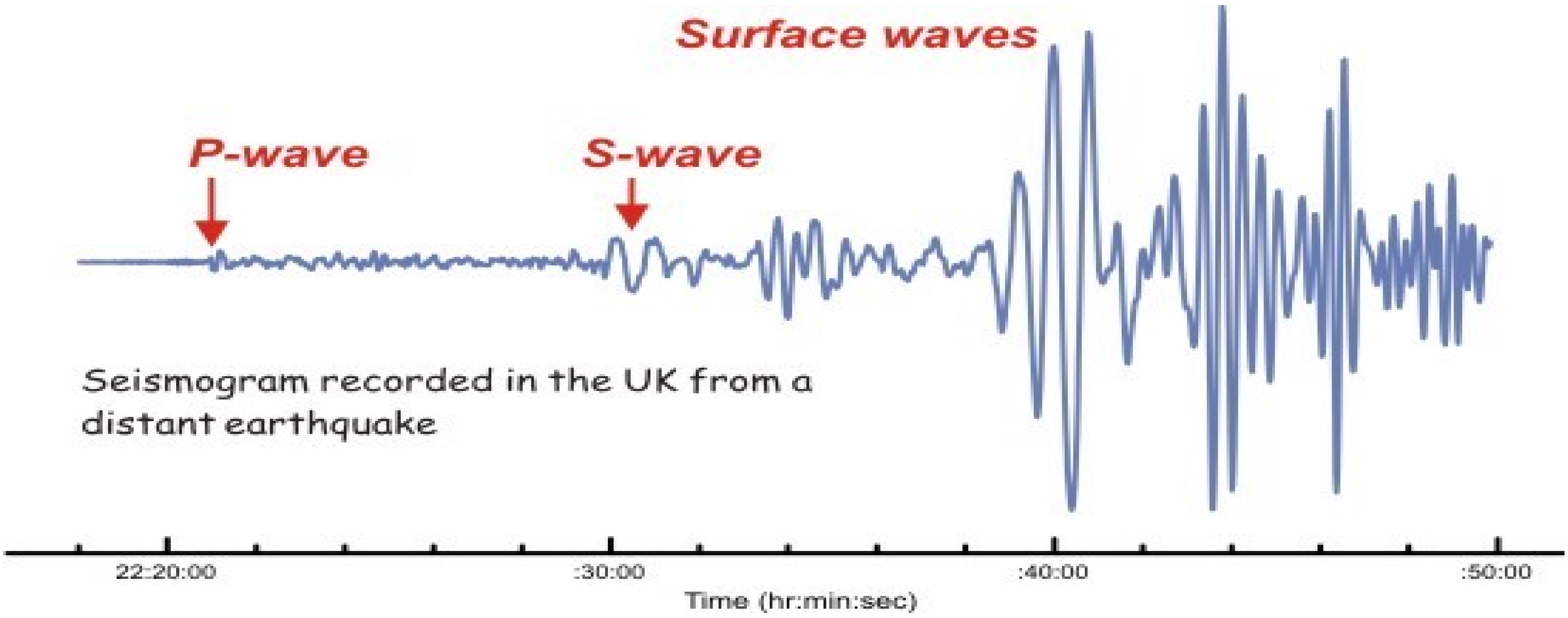
Love wave



Rayleigh wave



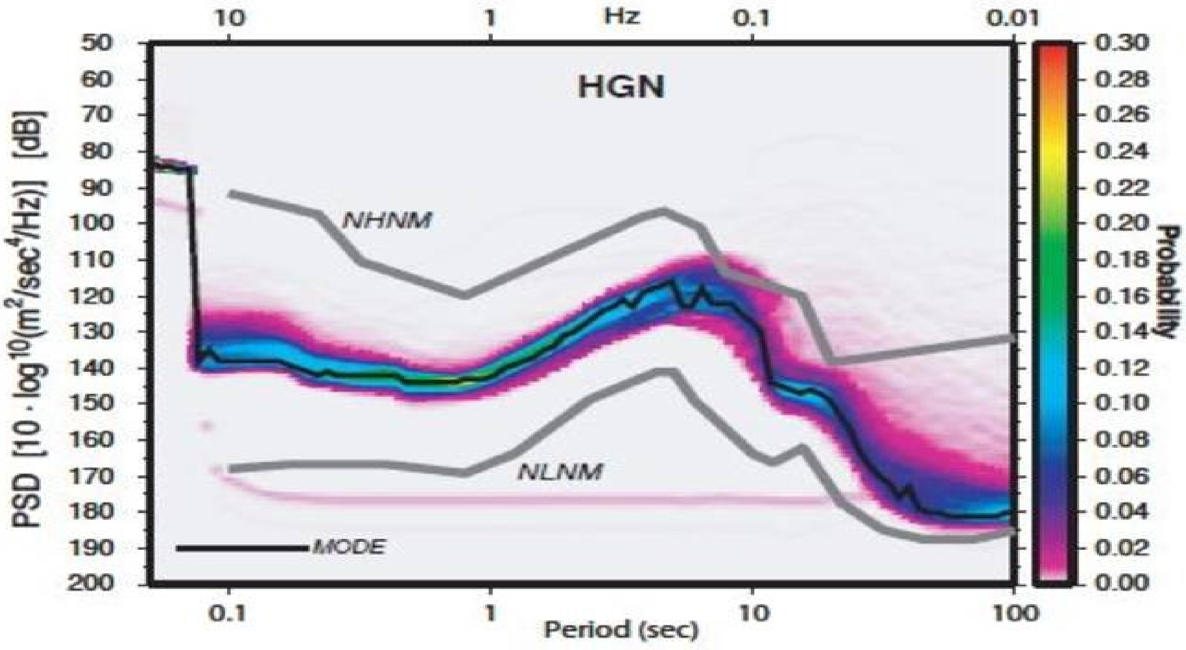
Earthquakes



Seismic noise

- Originates in
  - Wind
  - Waves
  - Seismicity
  - Human activity

Seismic noise



A note on units

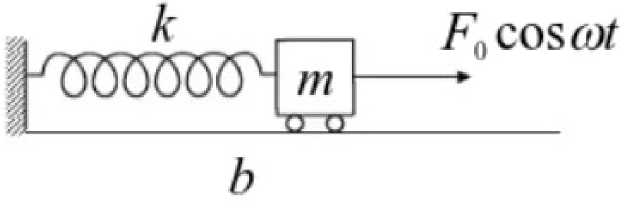
- Displacement, velocity, acceleration
- Time series $u(t)$, $v(t)$, $a(t)$
- Power spectral density, e.g. m^2/Hz
- Amplitude spectral density e.g. m/sqrt(Hz)

Influence of seismic noise on gravitational interferometers

- Mechanical coupling
- Gravitational coupling

<u>Forced Harmonic Oscillator</u>

$$\gamma = \frac{b}{2m}; \quad \omega_0 = \sqrt{\frac{k}{m}}$$



$$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega_0^2 x = \frac{F_0}{m}\cos\omega t$$

Solution: $$x(t) = x^H(t) + \frac{F_0/m}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4\gamma^2\omega^2}}\cos\left(\omega t - \varphi\right)$$

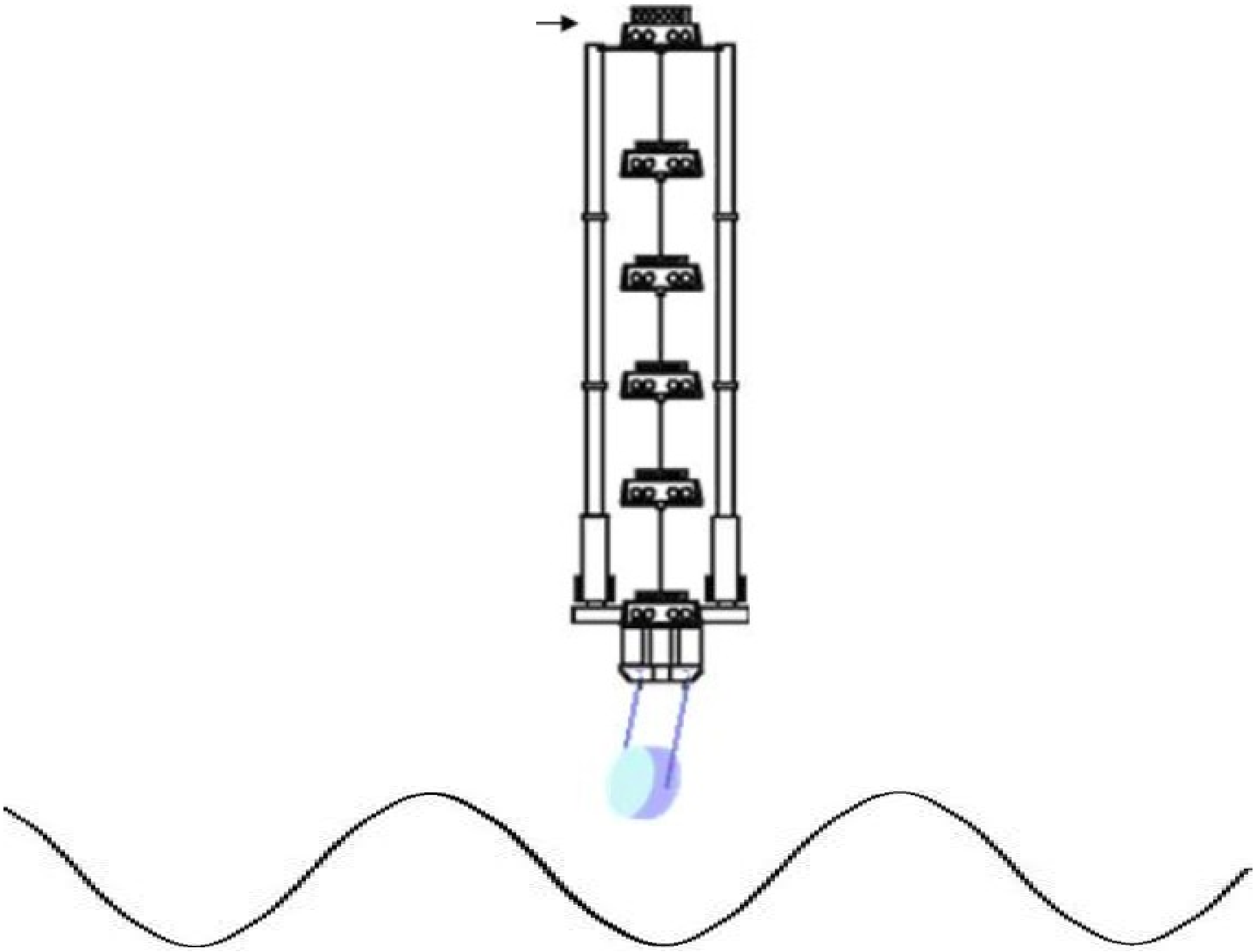
General solution of associated homogeneous equation
(**Transient Term**)
(**Decays to zero as $t \to \infty$**)

Particular solution of inhomogeneous equation
(**Steady State Term**)

$$\tan\varphi = \frac{2\gamma\omega}{\omega_0^2 - \omega^2}$$

Mechanical filtering

- Forced oscillators are mechanical low pass filters
- A series of mechanical filters can quench the mechanical noise sufficiently



Gravity perturbation from seismic waves

Density fluctuation

$$\delta\rho = -\nabla(\rho(\vec{r})\vec{\zeta}(\vec{r},t))$$

Potential fluctuation

$$\delta\phi = -G\int dV\,\frac{\delta\rho(\vec{r},t)}{|\vec{r}-\vec{r}_0|}$$

A little algebra

$$\delta\phi = -G\int dV\,\frac{\nabla(\rho(\vec{r},t)\vec{u}(r,t))}{|\vec{r}-\vec{r}_0|}$$

Integration by parts

$$\delta\phi = -G\int dV\,\rho(\vec{r},t)\vec{u}(r,t))\nabla\frac{1}{|\vec{r}-\vec{r}_0|}$$

Change of volume to surface integral (Gauss theorem)

So finally

$$\delta\phi = -G\int dV\,\rho(\vec{r},t)\vec{u}(r,t))\,\frac{\vec{r}-\vec{r}_0}{|\vec{r}-\vec{r}_0|^3}$$

And now acceleration

$$\vec{a} = \nabla\phi$$

$$\delta\vec{a}(\vec{r}_0, t) = G\int dV\,\rho(\vec{r})\vec{\zeta}(\vec{r}, t)\nabla_0\frac{\vec{r}-\vec{r}_0}{|\vec{r}-\vec{r}_0|^3}$$

In the case of surface waves

Half space:

$$\rho = \rho_0 \Theta(-\sigma(\vec{r}))$$

$$\delta\phi = G\rho \int dV \, \frac{\nabla(\Theta(-\sigma(\vec{r})\vec{u}(\vec{r},t))}{|\vec{r}-\vec{r}_0|}$$

$$\delta\phi = G\rho \int \frac{dV}{|\vec{r}-\vec{r}_0|} \left[\Theta(-\sigma(\vec{r})\nabla\vec{u}(\vec{r},t)) - \delta(-r)\vec{n}(\vec{r})\vec{u}(\vec{r},t)\right]$$

$$\delta\phi = G\rho \int dV \, \frac{\nabla\vec{u}(\vec{r},t)}{|\vec{r}-\vec{r}_0|} + G\rho \int dS \, \frac{n(\vec{r})\vec{u}(\vec{r},t)}{|\vec{r}-\vec{r}_0|}$$

Bulk surface

Which waves contribute to gravity fluctuations?

An example

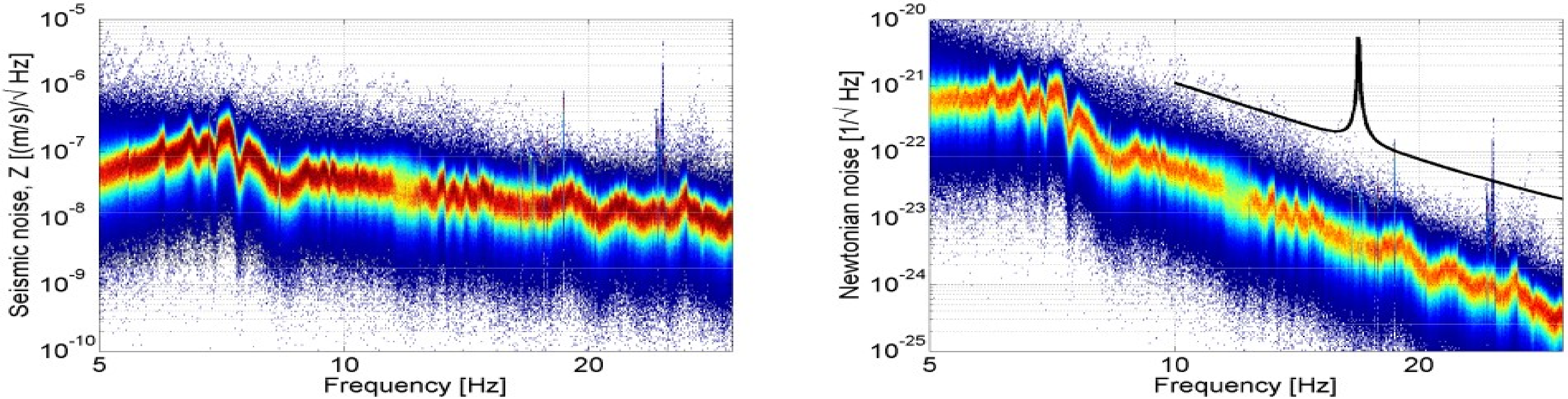
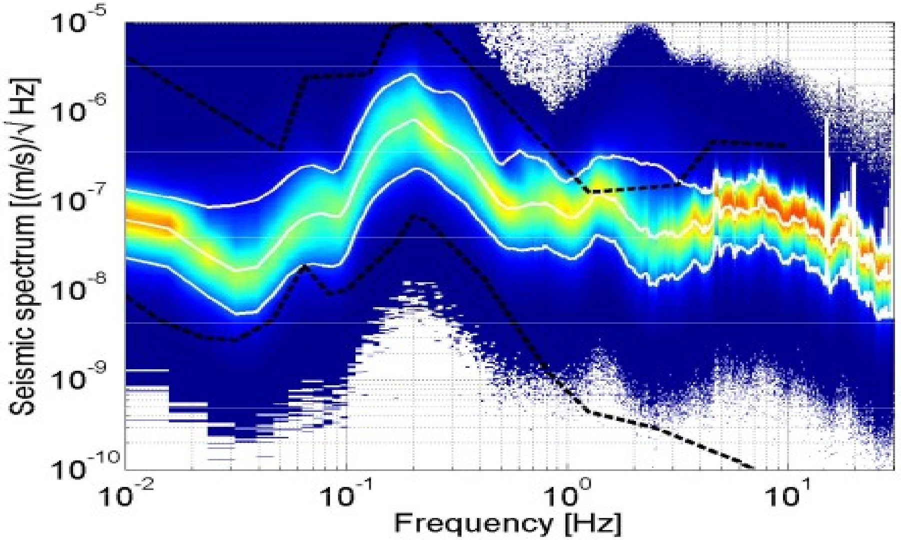


Figure 14: Histograms of seismic spectra at the central station of the Virgo detector and modelled Rayleigh-wave Newtonian noise. A sensitivity model of the Advanced Virgo detector is plotted for comparison. (Seismic data stem from channel `SEBDCE06` between June 4, 2011, UTC 00:00 and September 3, 2011, UTC 00:00.)

Harms, LRR 2020

And another



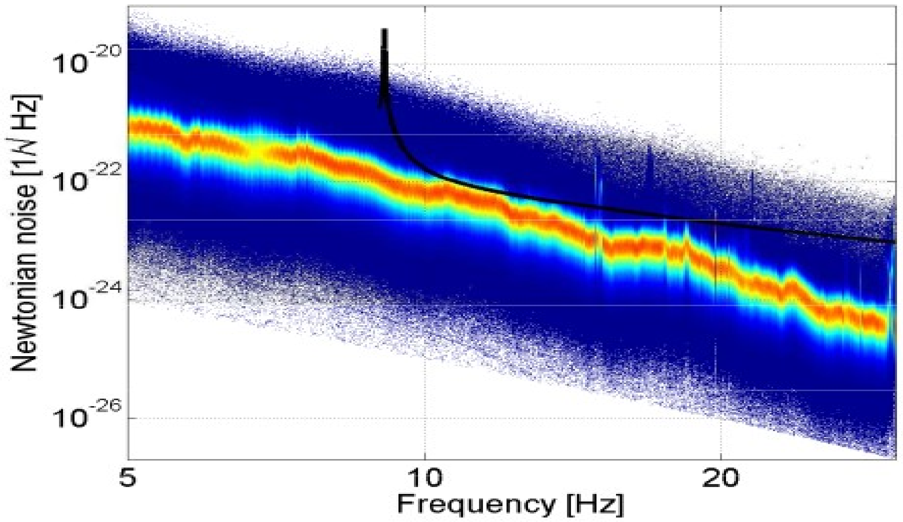
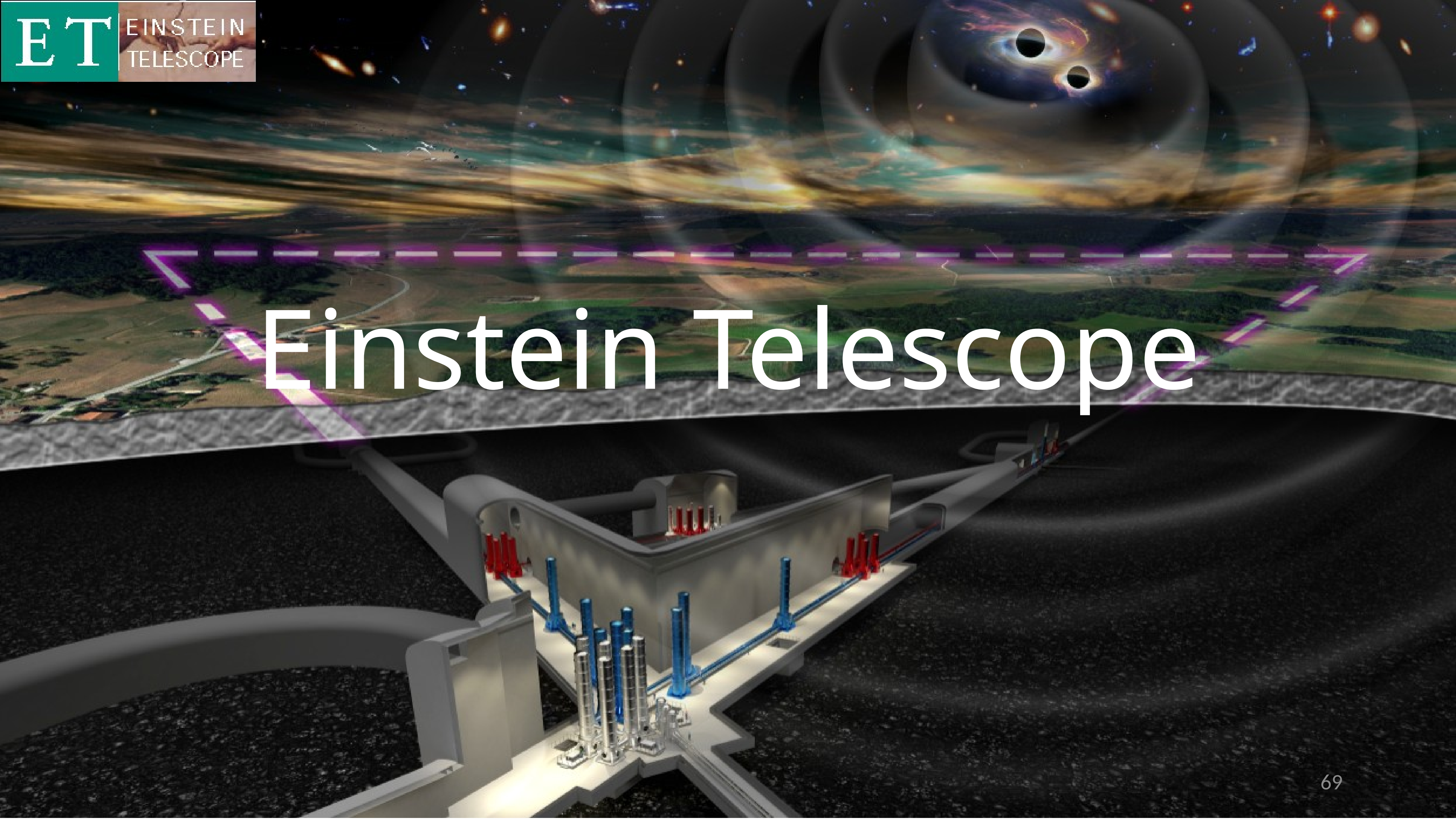


Figure 15: Histograms of seismic spectra at the central station of the LIGO Livingston detector and modelled Rayleigh-wave Newtonian noise. In the left plot, the dashed black curves are the global seismic high-noise and low-noise models. The white curves are the 10th, 50th, and 90th percentiles of the histogram. In the right plot, a sensitivity model of the Advanced LIGO detector is plotted for comparison. (Seismic data stem from channel `L0:PEM-LVEA_SEISZ` between August 1, 2009, UTC 00:00 and August 1, 2010, UTC 00:00.

Gravity gradients mitigation strategy

- Measure and subtract
- Use networks of sensors
- Find the best combinations
- Use Wiener filter techniques
- Best subtraction – a factor of a few
- Move detector underground!

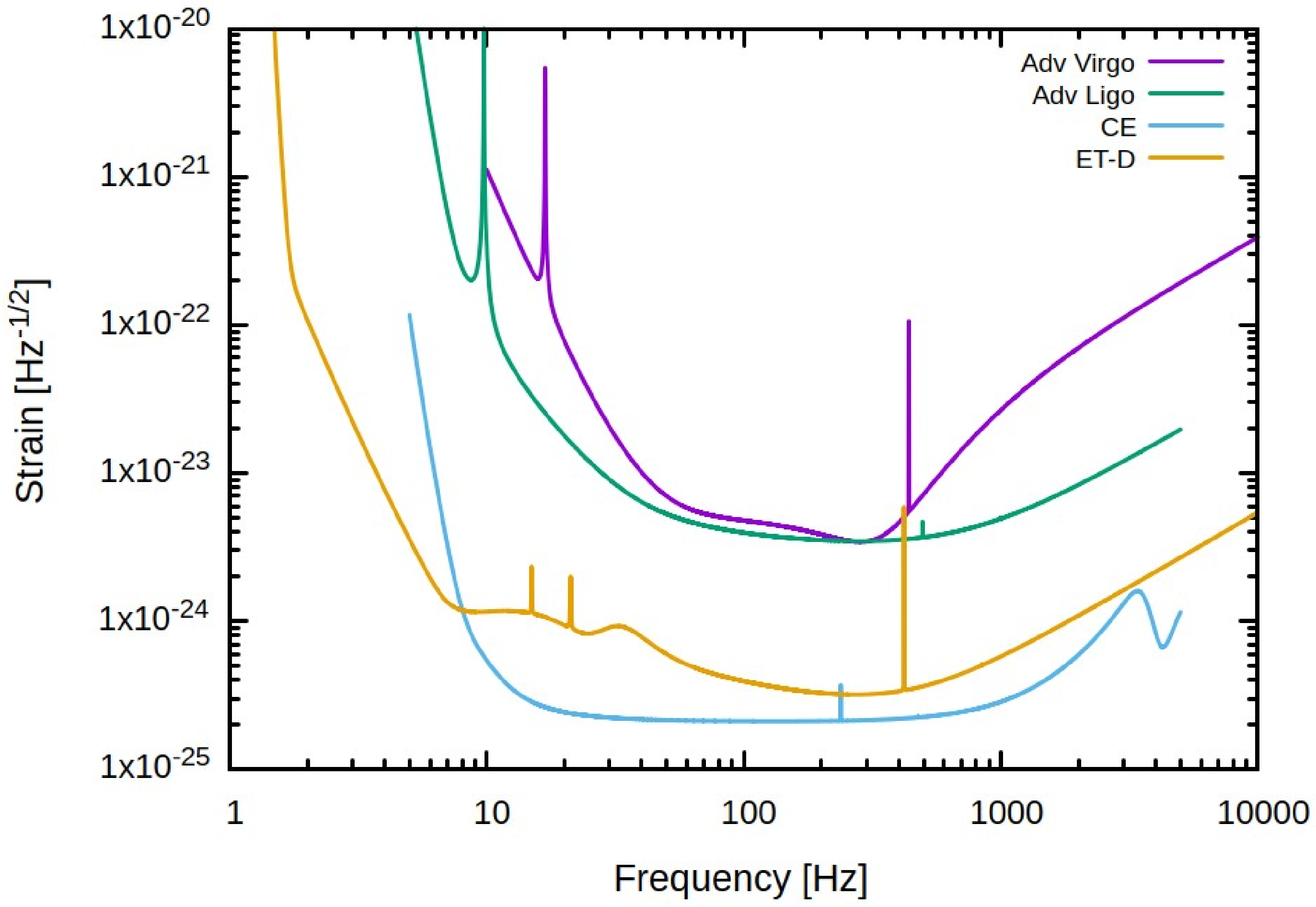
Einstein Telescope



What is Einstein Telescope (ET)?

- ET pioneered the concept of a 3rd generation GW observatory, evolved in the following key requirements
  - **ET must be a 3rd generation GW observatory**, and thus its sensitivity aims to be at least one order of magnitude better with respect to the nominal sensitivity of advanced detectors in all the detection frequency band
  - **ET must be both a precision measurement and a new discovery project**, and thus it aims to be a wide frequency band observatory,
  - **ET science has a special focus on massive (or intermediate mass) black holes**, and thus it aims to have an extraordinary sensitivity at low frequency (few Hz)
  - **ET must have a high reliability**, and thus it aims to obtain a high observation duty cycle
  - **ET must be able to make science in a standalone configuration**; ET aims to have localisation and GW polarisation disentanglement capabilities and to show a more uniform sky coverage. Nevertheless, it is worth to note that the ET full science potential will be achieved in a 3G network, and it is important that other projects, like the Cosmic Explorer (CE) project in US, are developed in parallel to ET. In case of a full 3G global network, this requirement can be revised.
  - **ET must have a lifetime of several decades,** (50 years in the ET proposal), being capable to host the evolution of the detectors, without limiting their sensitivity.
- ET path in the last decade:
  - A conceptual design
  - Few enabling technologies
  - A scientific collaboration (currently under definition)
  - An official ET project (recently entered in the ESFRI roadmap and whose structure is under definition by the funding agencies engaged on ET)

ET planned sensitivity



ET Science in a nutshell

ASTROPHYSICS

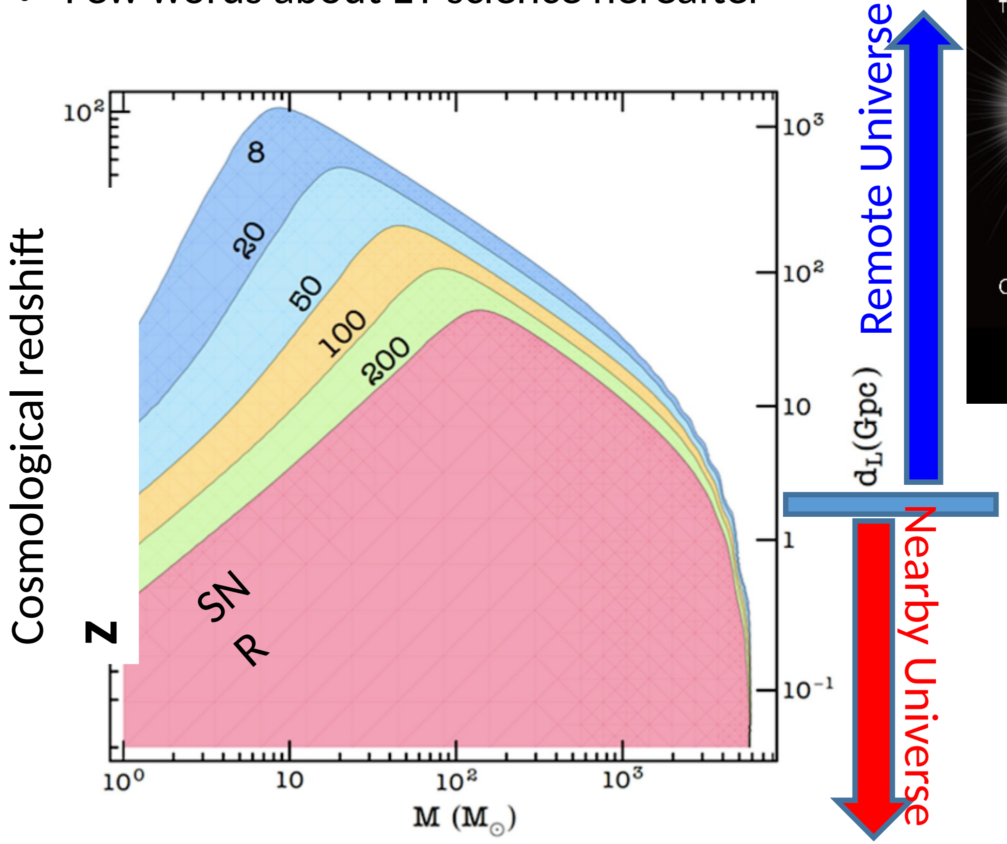
- **Black hole properties**
  - origin (stellar vs. primordial)
  - evolution, demography
- **Neutron star properties**
  - interior structure (QCD at ultra-high densities, exotic states of matter)
  - demography
- **Multi-band and -messenger astronomy**
  - joint GW/EM observations (GRB, kilonova,...)
  - multiband GW detection (LISA)
  - neutrinos
- **Detection of new astrophysical sources**
  - core collapse supernovae

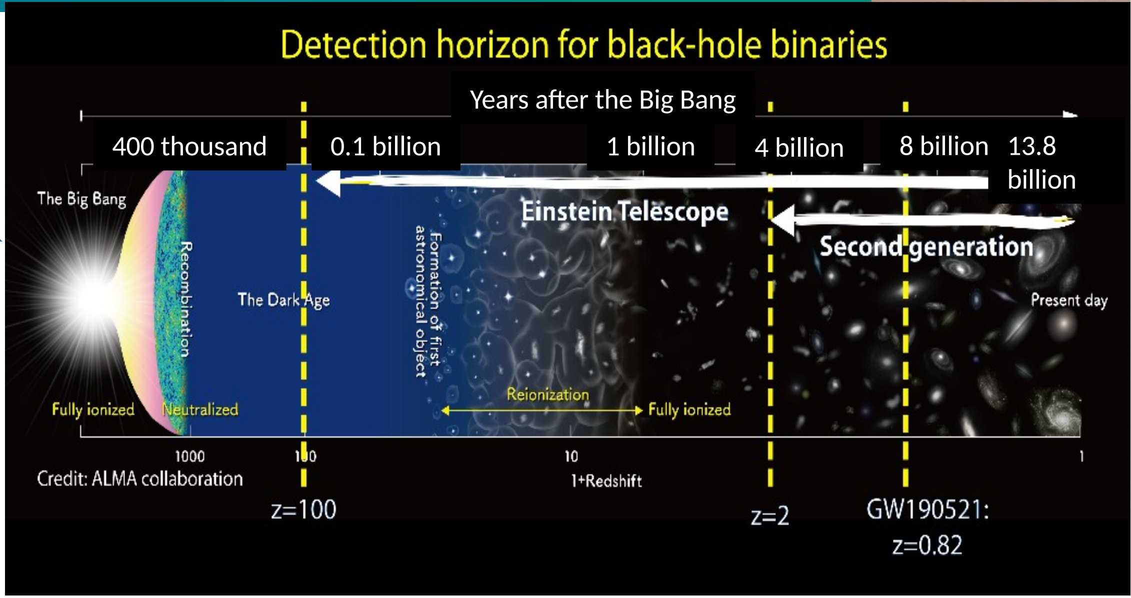
FUNDAMENTAL PHYSICS AND COSMOLOGY

- **The nature of compact objects**
  - near-horizon physics
  - tests of no-hair theorem
  - exotic compact objects
- **Tests of General Relativity**
  - post-Newtonian expansion
  - strong field regime
- **Dark matter**
  - primordial BHs
  - axion clouds, dark matter accreting on compact objects
- **Dark energy and modifications of gravity on cosmological scales**
  - dark energy equation of state

ET Science in a nutshell

- ET will be both a discovery machine and a precision measurement instrument
- Few words about ET science hereafter





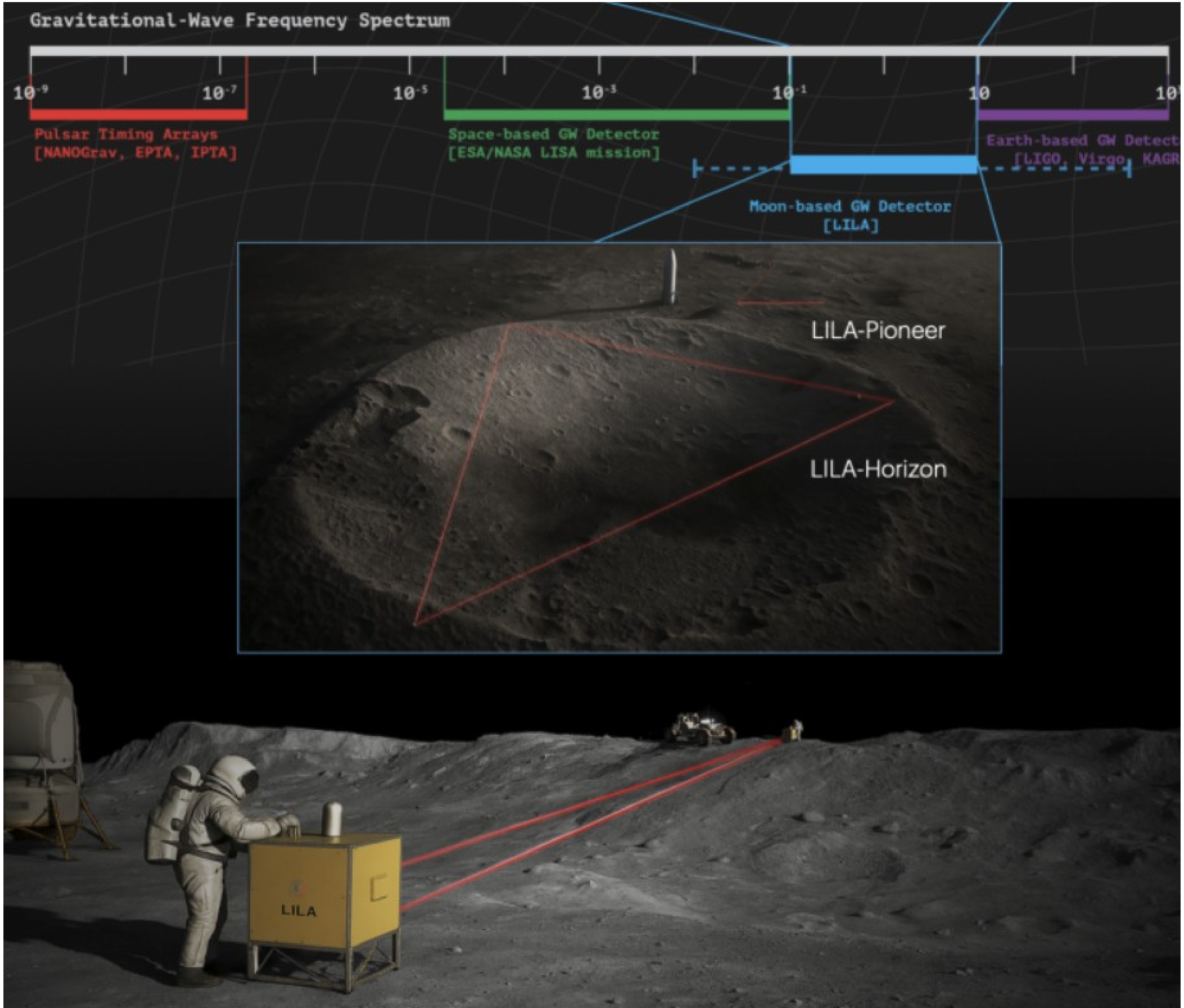
The combination of

- distances and masses explored
- number of detections
- detections with very high SNR

will provide a wealth of data expected to generate **revolutions in astrophysics, cosmology**

Going extraterrestial: the Moon

- Lunar Gravitational Wave Antenna concept
- Laser Interferometer Lunar Antenna


Gravitational-Wave Frequency Spectrum
10-9
10-7
10-5
10-3
10-1
10
Pulsar Timing Arrays
[NANOGrav, EPTA, IPTA]
Space-based GW Detector
[ESA/NASA LISA mission]
Earth-based GW Detect
[LIGO, Virgo, KAGR
Moon-based GW Detector
[LILA]
LILA-Pioneer
LILA-Horizon
LILA

Further reading

Jan Harms, “Terrestrial gravity fluctuations”Living Reviews of Relativity.

Peter Shearer, “Introduction to seismology”, Cambridge University Press

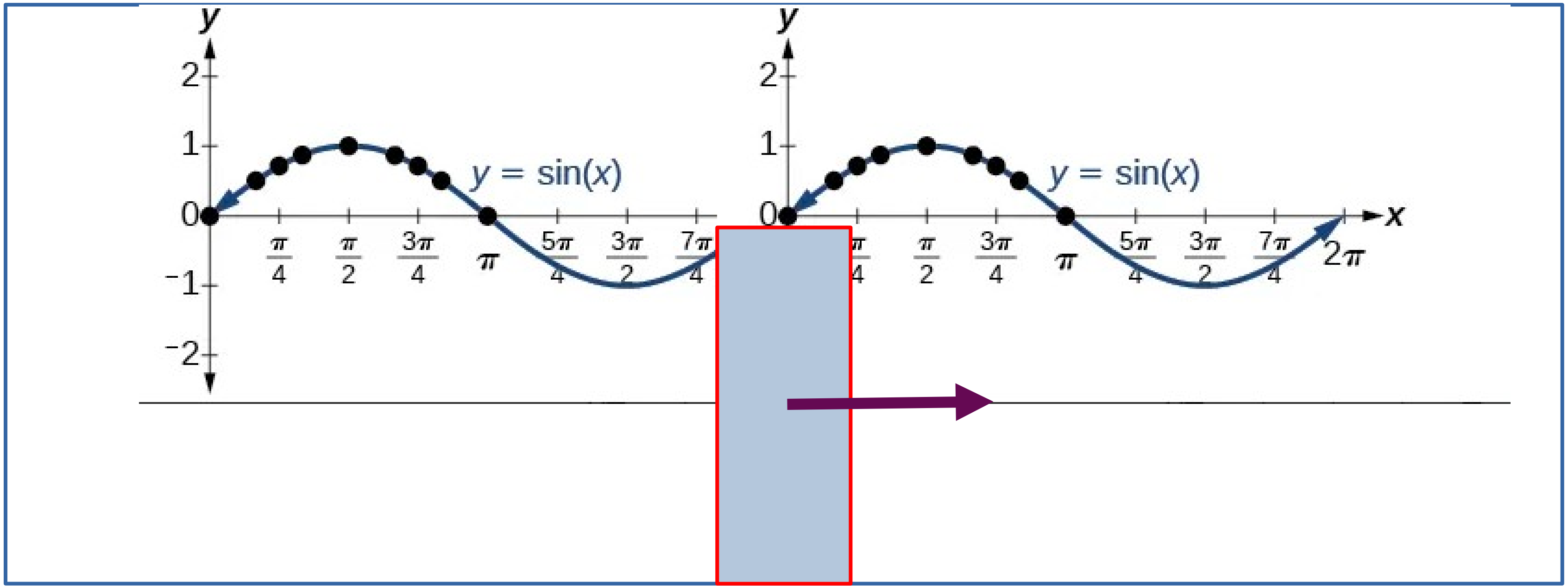
Hands on

Infrasound induced Newtonian Noise

$$
\begin{aligned}
S_{\mathrm{tot}}(\vec{r}_0,\omega) =& S_{\mathrm{int}}(\vec{r}_0,\omega) + S_{\mathrm{ext}}(\vec{r}_0,\omega) = \\
& \frac{1}{\left(L\omega^2\right)^2} S_{\delta p}^{\mathrm{int}}(\omega)\langle \mid a_{\mathrm{int}}(z_0,\theta,\phi,\omega) \mid^2\rangle + \\
& \frac{1}{\left(L\omega^2\right)^2} S_{\delta p}^{\mathrm{ext}}(\omega)\langle \mid a_{\mathrm{ext}}(z_0,\theta,\phi,\omega) \mid^2\rangle, \quad (15)
\end{aligned}
$$

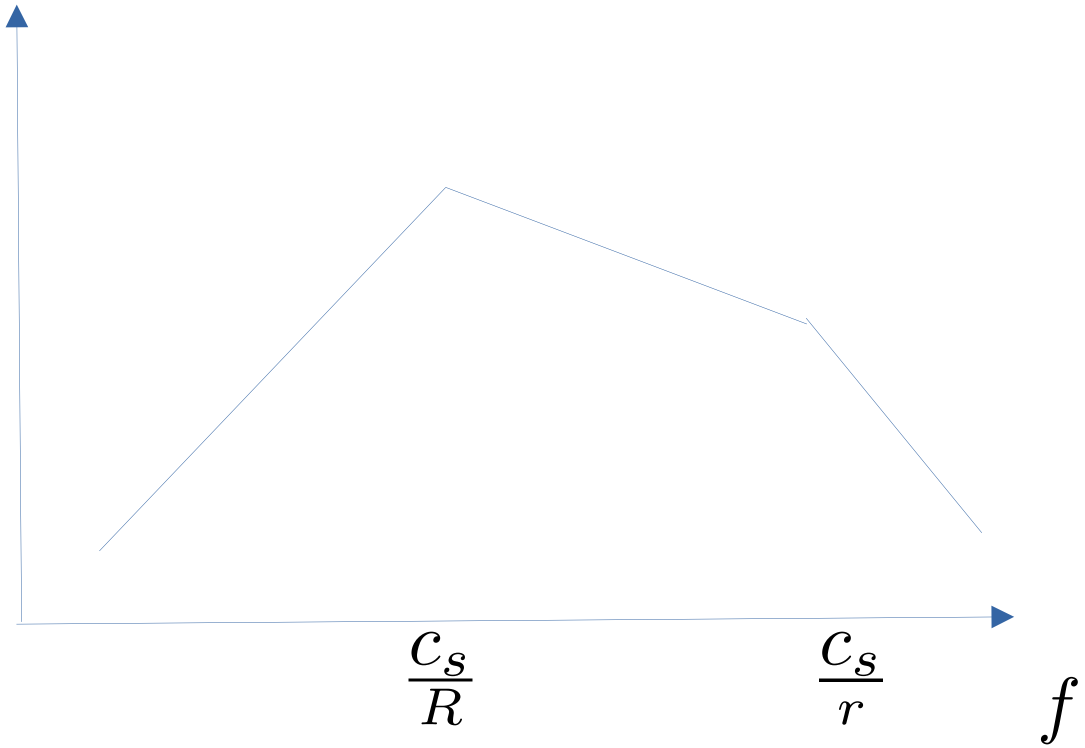
Imprtant scales

Cavern size R, wavelength λ,



Vacuum tower
width r, height h

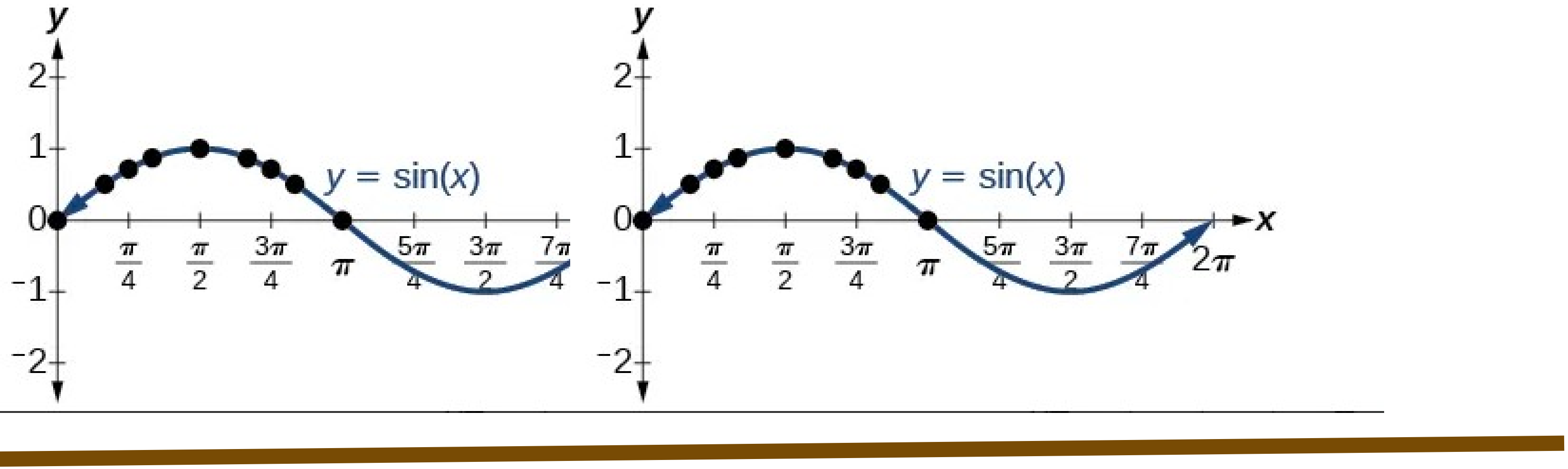
Transfer function



Mitigation

- Make R small - divide caverns or make them small
- Make caverns quiet – what is the ultimate silence limit with all machinery and ventilation?
- Total fantiasy: decrease pressure in caverns :-)

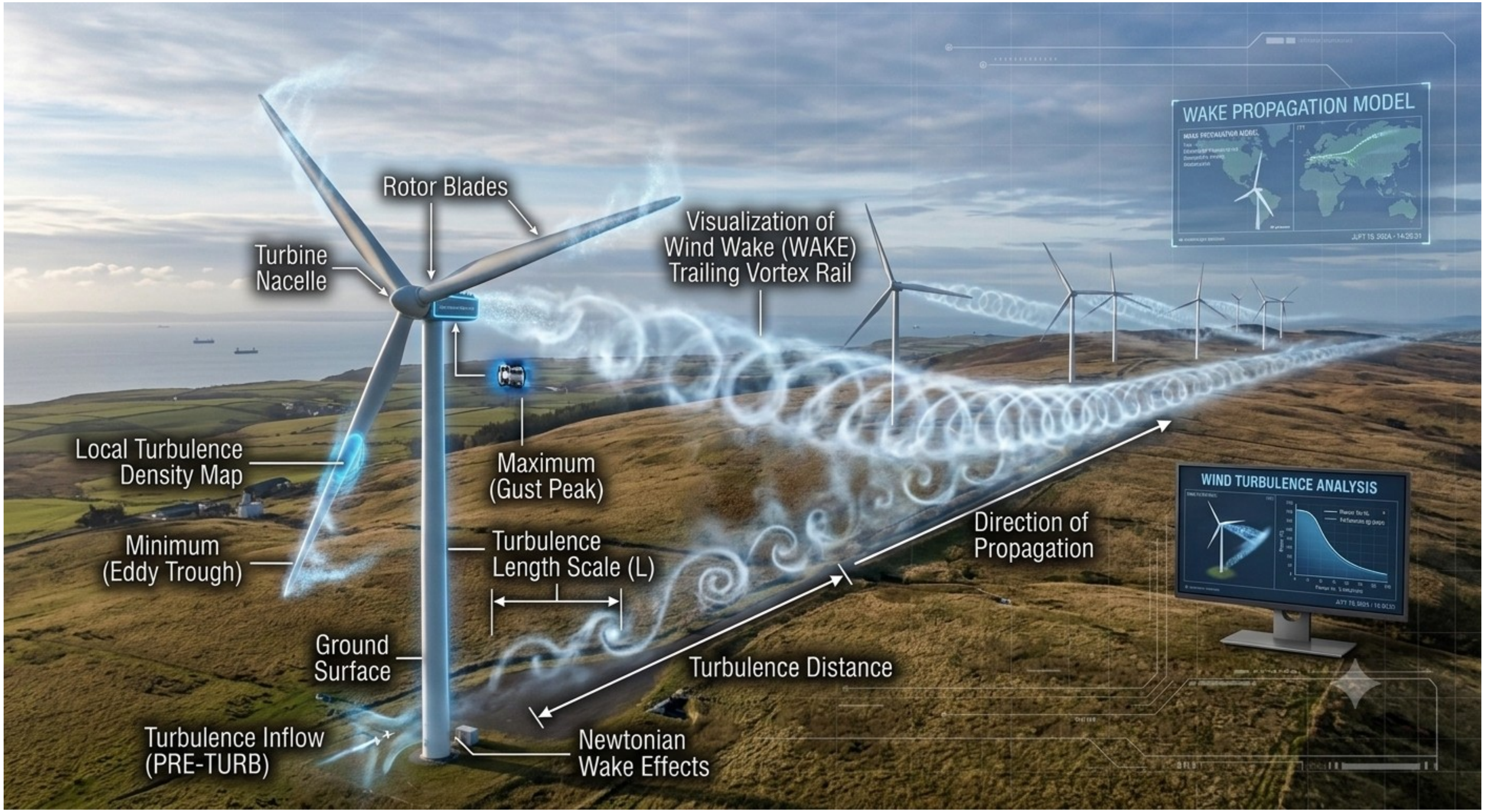
The case of the surface waves



Depth D
wavelength λ,

But hat are the integration limits?

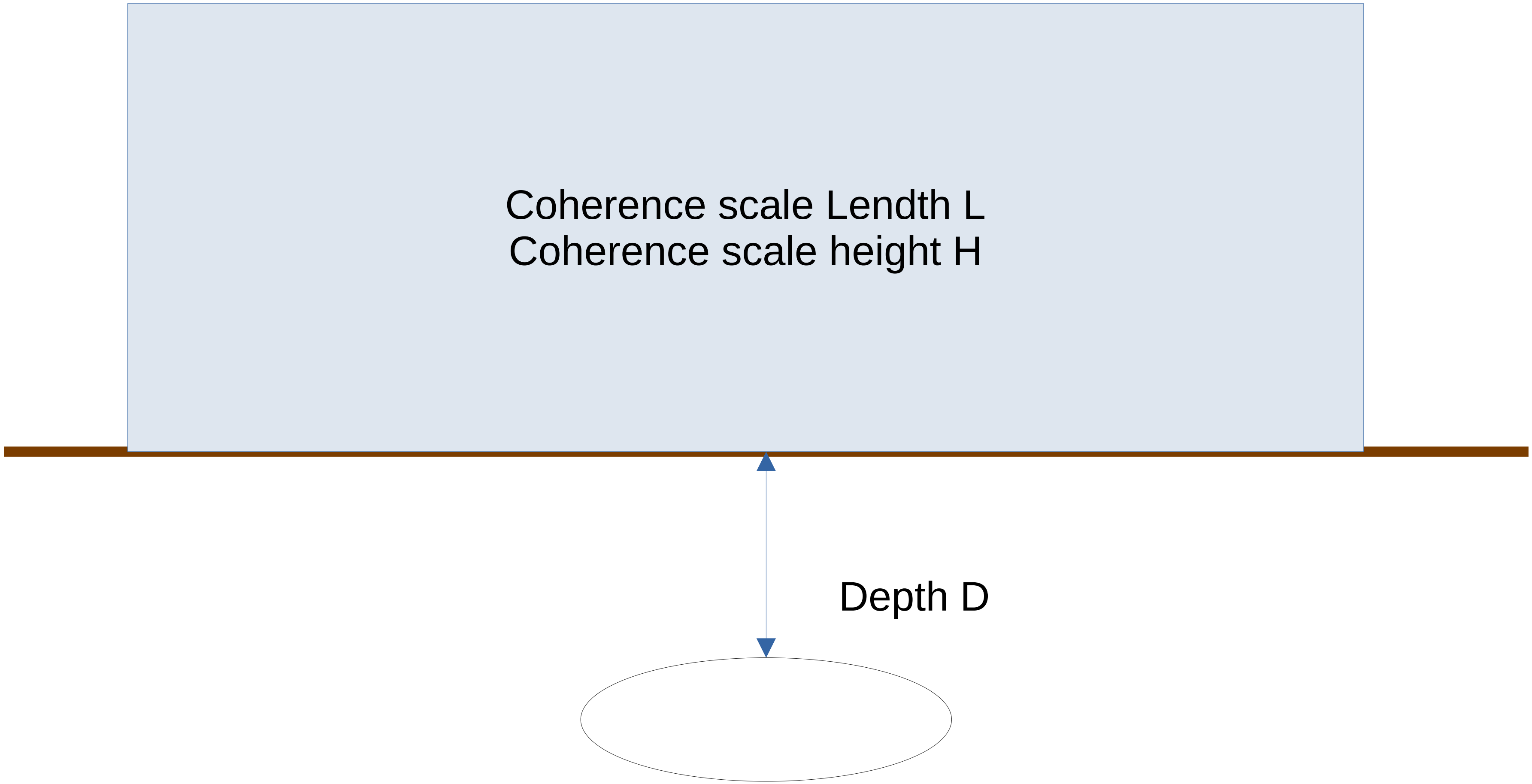
How far a wave is wave?



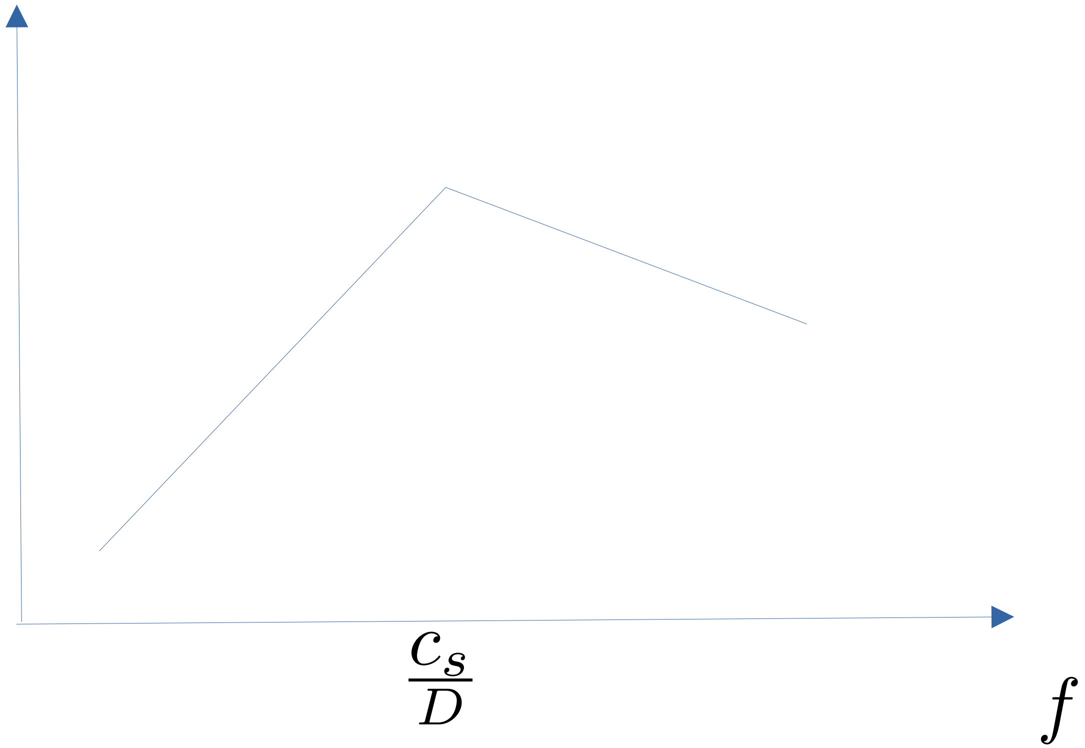
What we need to measure

- Infrasound wave spectrum, and variability
- Coherence length scale of the wave
- Coherence height scale of the waves

Integration region



Transfer function



Level depends on
volume of integration
H*L*L

Real case - cavern

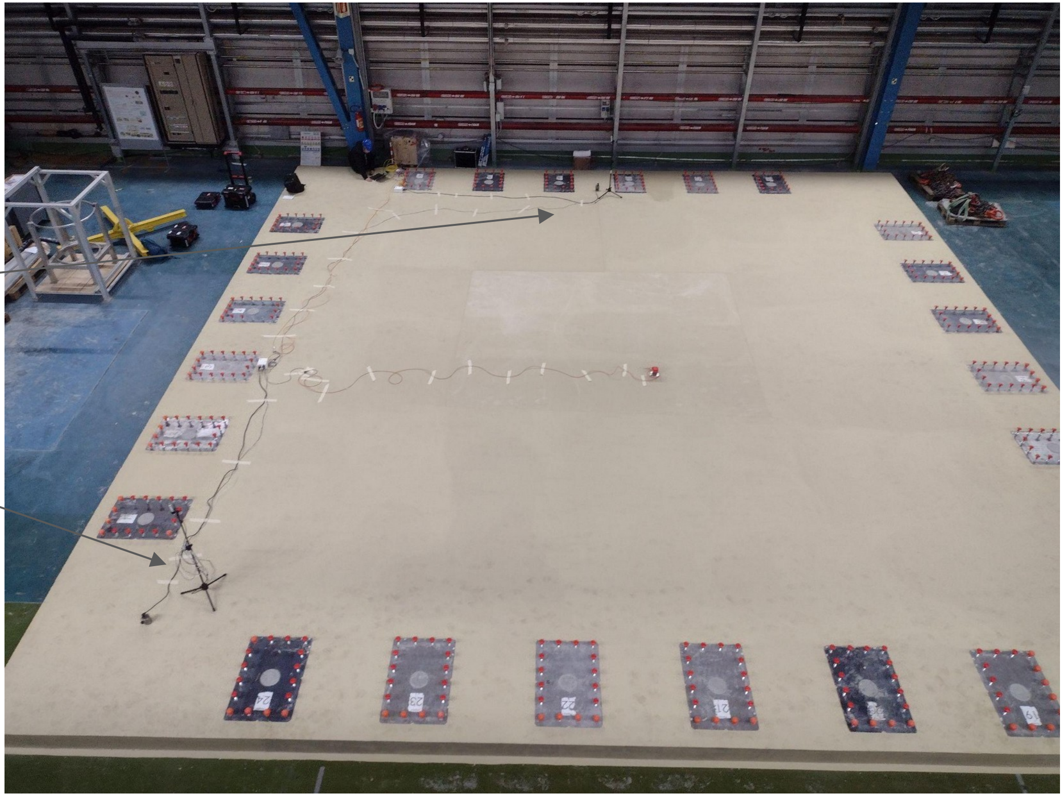
DarkSide campaign

Measure the seismic and infrasound noise at the DarkSide location

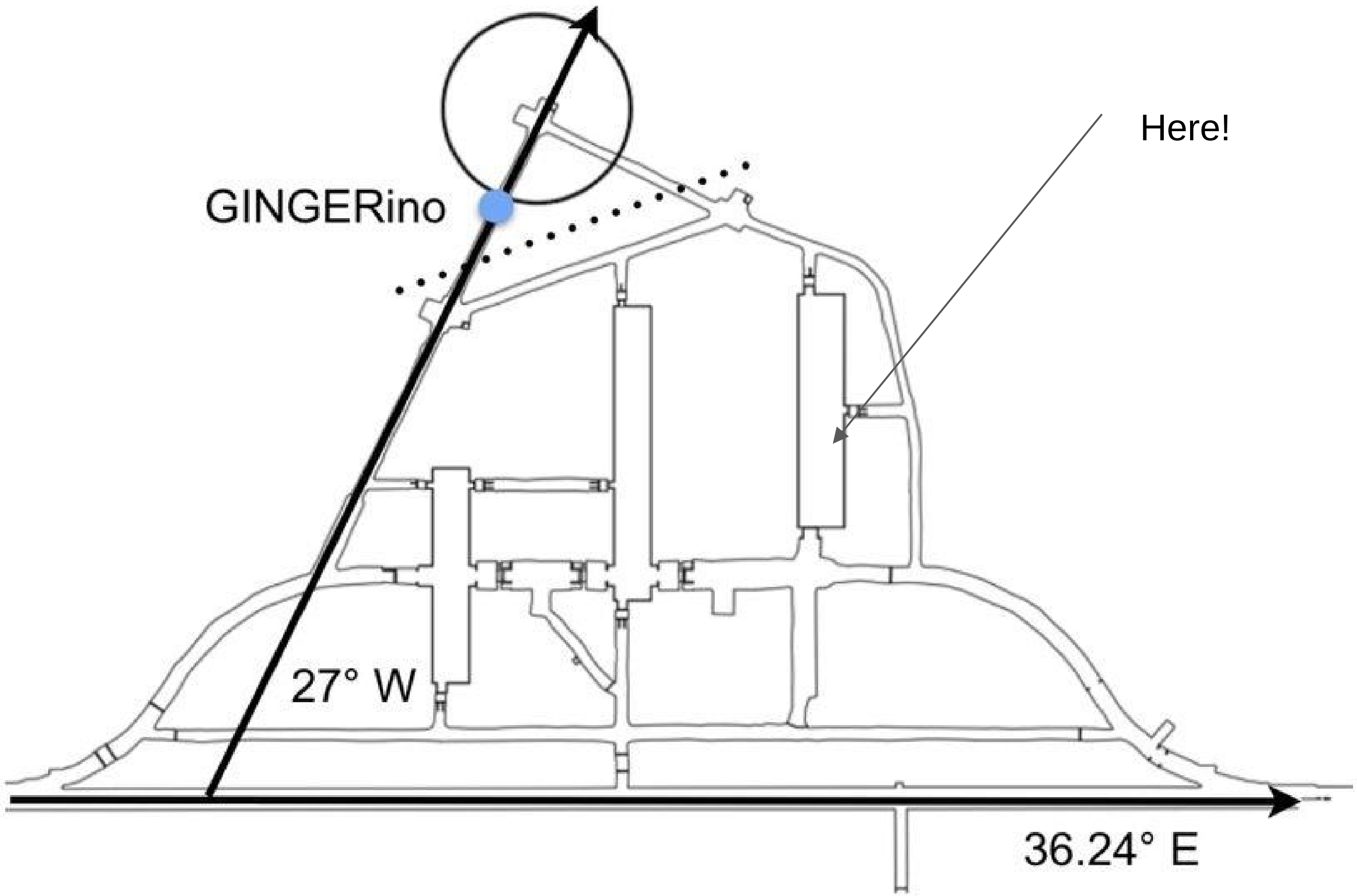
3 week campaign

2 microphones

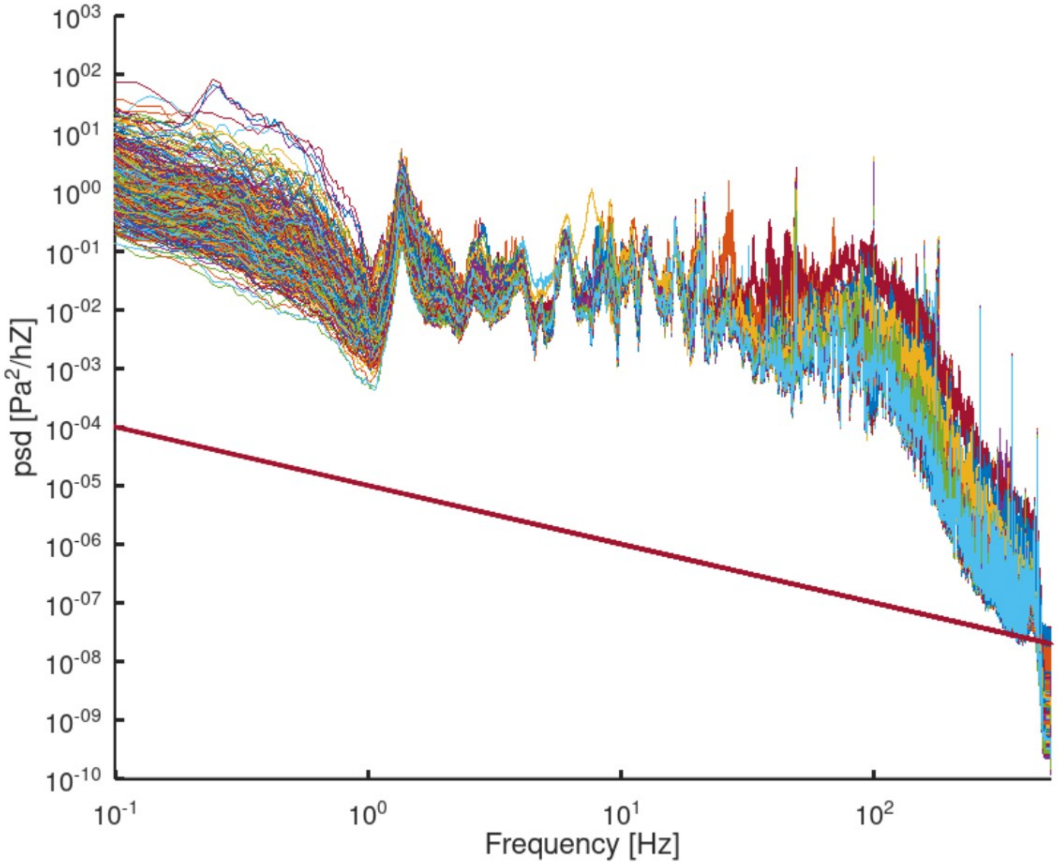
Hall 1 with existing experiments



Where were the measurements taken?



Sound spectra



Ambient noise model
Bowman et al. 2005

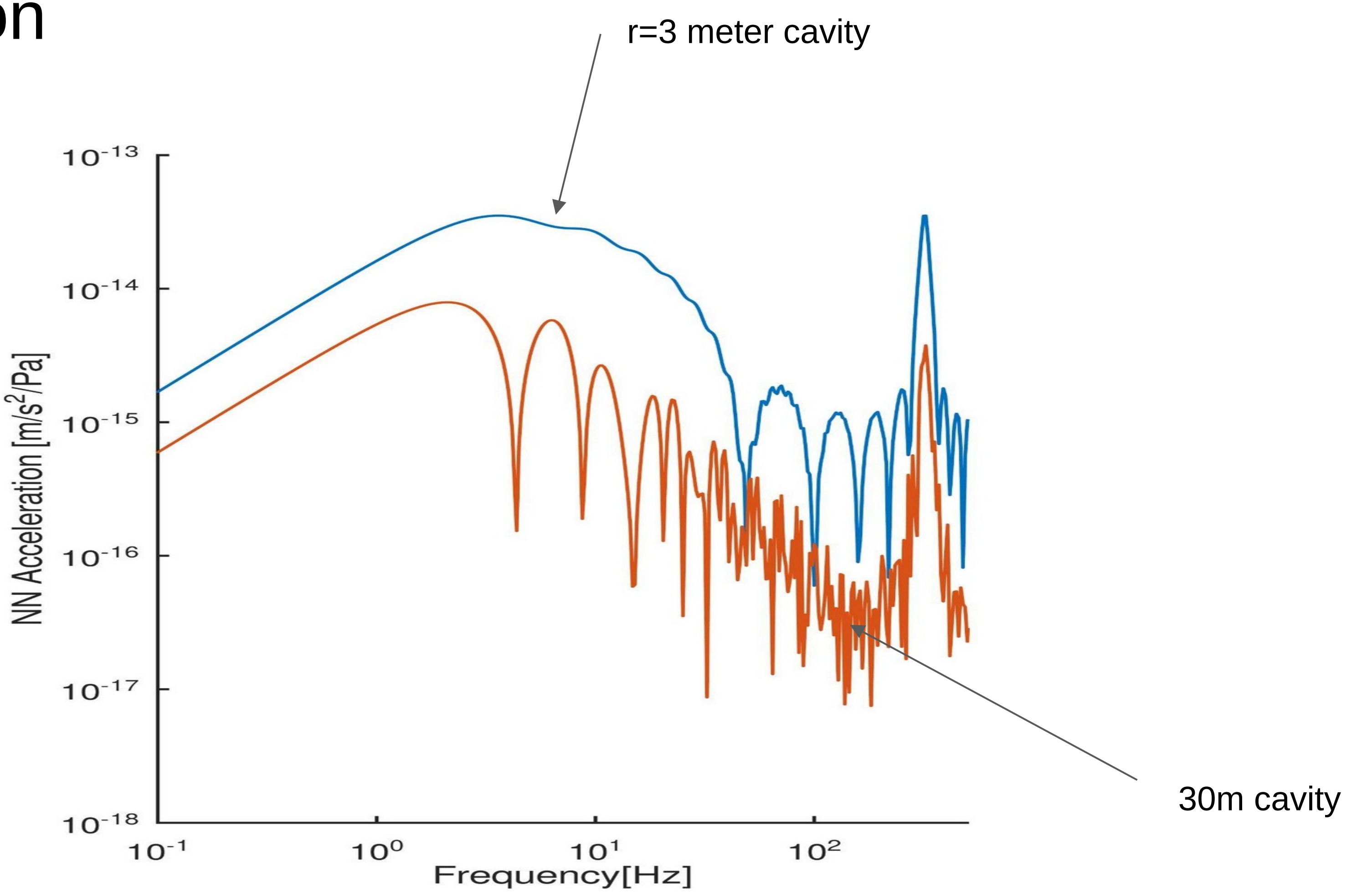
Calculating the Newtonian noise

1. Assume a geometry of detector: hall 100m x 20m x 25 m
2. Assume position of test mass in the center 3 m above the ground
3. Calculate NN acceleration per unit pressure fluctuation as function of frequency - straightforward numerical calculation
4. Assume four test masses have similar uncorrelated noise
5. Follow the calculations in Fiorucci etal. 2018

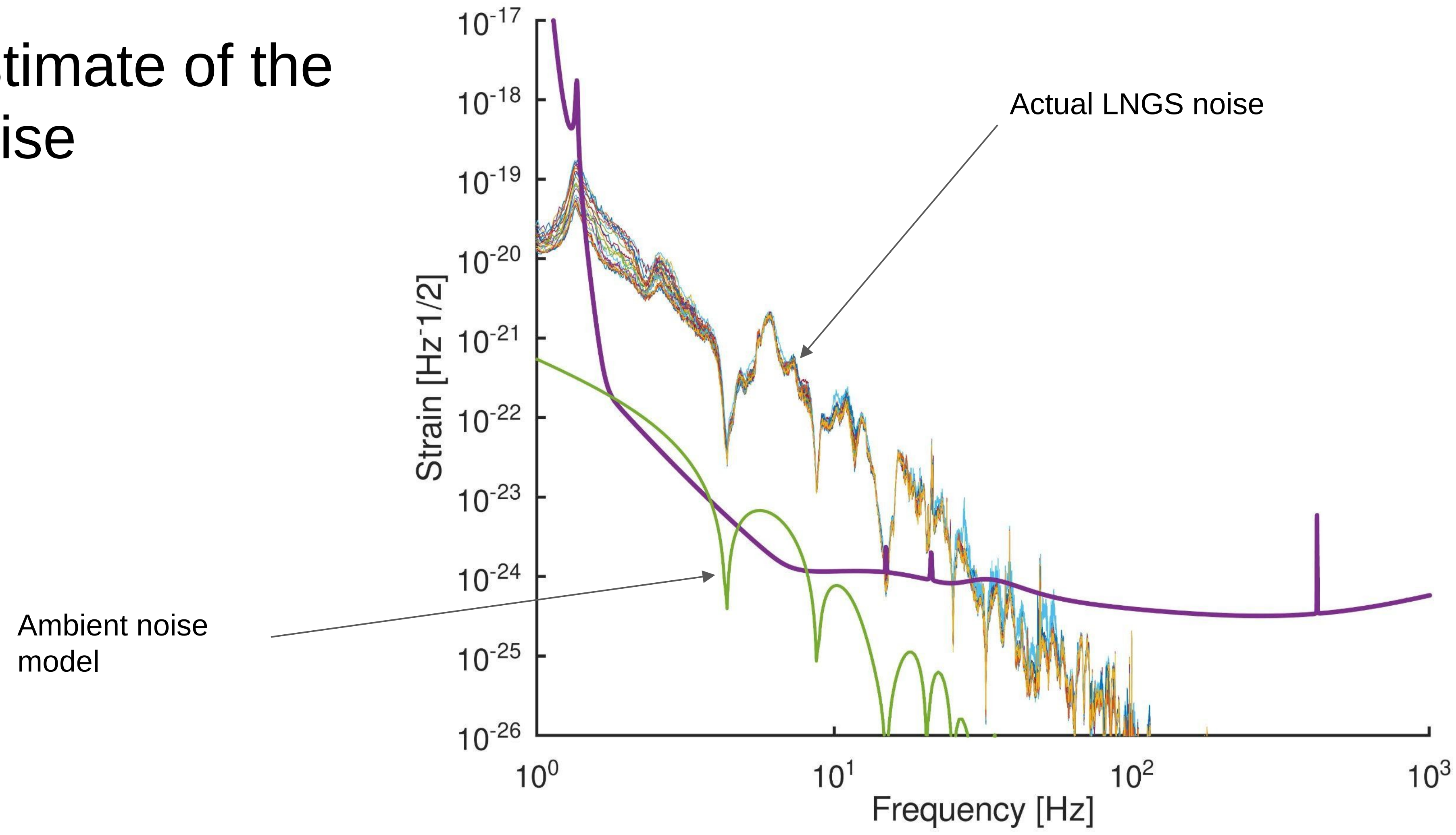
$$\begin{aligned} S_{\text{tot}}(\vec{r}_0,\omega) =& S_{\text{int}}(\vec{r}_0,\omega) + S_{\text{ext}}(\vec{r}_0,\omega) = \\ & \frac{1}{(L\omega^2)^2} S_{\delta p}^{\text{int}}(\omega) \langle \mid a_{\text{int}}(z_0,\theta,\phi,\omega) \mid^2 \rangle + \\ & \frac{1}{(L\omega^2)^2} S_{\delta p}^{\text{ext}}(\omega) \langle \mid a_{\text{ext}}(z_0,\theta,\phi,\omega) \mid^2 \rangle, \quad (15) \end{aligned}$$

$$\begin{aligned} S_{\text{tot}}^{AdV}(\omega) =& 2 \times S_{\text{tot,CEB}}(\omega) + \\ & S_{\text{tot,NEB}}(\omega) + S_{\text{tot,WEB}}(\omega) \end{aligned}$$

NN acceleration



Estimate of the noise



External noise