Geoneutrinos: From Bulk Silicate Earth Uranium to Experimental Measurement

PART I — First estimate: all uranium at the Earth's center

Student task: Consider only the ^{238}U decay chain. For this first estimate, make the deliberately simplified assumption that all uranium in the Bulk Silicate Earth (BSE) is concentrated at the center of the Earth.

Quantity	Value
BSE mass, M_{BSE}	4.03×10^{24} kg
U abundance in BSE, a_U	20.3 ppb
Molar mass of ^{238}U	238 g mol^{-1}
Avogadro constant, N_A	6.022×10^{23} mol^{-1}
Half-life of ^{238}U	4.47×10^9 yr
Earth radius, $R_\oplus$	6371 km
Antineutrinos per complete ^{238}U chain	6
Fraction of summed U-chain spectrum above IBD threshold	$f(E\nu > 1.806 \text{ MeV}) = 0.065$
Solar mixing angle	$\theta_{12} = 33.4°$
Representative IBD cross section	$\sigma_{IBD} = 1.3 \times 10^{-43}$ cm^2
One year	3.156×10^7 s
TNU definition	1 event per 10^{32} free protons per year and 100% detection efficiency

1. Calculate the total mass of uranium in the Bulk Silicate Earth. For simplicity, treat natural uranium as ^{238}U.
2. Calculate the total number of ^{238}U nuclei in the BSE.
3. Calculate the decay constant λ and the total ^{238}U activity A.

$$\lambda = \ln 2 / T_{1/2}, \qquad A = \lambda N$$

4. Assuming secular equilibrium in the complete ^{238}U decay chain, calculate the total antineutrino luminosity $L\bar{\nu}$. Each complete chain produces six electron antineutrinos.
5. Because all uranium is assumed to be at the Earth's center, calculate the antineutrino flux at the Earth's surface. Express the result in cm^{-2} s^{-1}.
6. Only part of the summed ^{238}U-chain antineutrino spectrum lies above the inverse beta decay threshold of 1.806 MeV. Using f = 0.065, calculate the flux above threshold.
7. Include neutrino oscillations. Assuming the oscillatory term is completely averaged, calculate the two-flavour electron-antineutrino survival probability using only $\theta_{12} = 33.4°$.

$$\langle P_{ee} \rangle = 1 - ½ \sin^2(2\theta_{12})$$

8. Calculate the surviving electron-antineutrino flux above the IBD threshold.
9. Using the representative IBD cross section $\sigma_{IBD} = 1.3 \times 10^{-43}$ cm^2, calculate the expected signal in TNU.

PART II — Uranium distribution and the mantle signal

Student task: The central-source approximation is unrealistic. A source element at distance L from the detector contributes a flux proportional to $1/L^2$. Therefore the spatial distribution of uranium matters.

$$d\Phi \propto 1 / L^2$$

Use the BSE uranium inventory obtained in Part I. A global crust model is assumed to contain

$$M_U^{crust} = 3.5 \times 10^{16}\ \mathrm{kg}$$

1. Using your ^{238}U signal in TNU from Part I, estimate the mantle signal for the following distributions. Account for the smaller uranium mass in the mantle and apply the stated geometrical factor.

   - **Model A — homogeneous mantle:** $g_{hom} = 1.55$
   - **Model B — all mantle uranium in a thin layer at the core–mantle boundary:** $g_{CMB} = 1.12$

A full spatial integration is not required.

2. Now consider a 20 kt JUNO-like LAB-based liquid-scintillator detector with

$$N_p = 1.45 \times 10^{33}\ \text{free protons}, \qquad \varepsilon = 0.80$$

   For both mantle models, calculate the expected number of detected mantle geoneutrino events in 1 year.
3. Compare the two results. Why do the two models give different detector signals although the total mantle uranium mass and antineutrino luminosity are the same?
4. How would a realistic calculation replace the simplified single-energy and geometrical-factor treatment? Identify the quantities that must be energy dependent and position dependent.

PART III — From prediction to experimental measurement

Discussion section: No extended numerical calculation is intended here. Use the questions to connect the geophysical prediction to an actual liquid-scintillator measurement.

A. Inverse beta decay signature

$$\bar{\nu}_e + p \rightarrow e^+ + n$$

- Where does the 1.806 MeV IBD threshold come from?
- What produces the prompt signal?
- Why is the approximate relation $E_{prompt} \approx E\nu - 0.78$ MeV useful?
- What happens to the neutron after the interaction, and what produces the delayed signal?
- Why is the prompt–delayed coincidence such a powerful signature?
- Which reconstructed observables can be used to define an IBD candidate?

B. Data-driven evaluation of backgrounds

A key experimental principle is to determine background rates and shapes from control data whenever possible. Discuss the following three examples.

1. Accidentals

Accidental coincidences are unrelated prompt-like and delayed-like events that happen to satisfy the IBD coincidence cuts.

- How can their rate be measured directly from data?
- How can their prompt-energy and other distributions be obtained with minimal reliance on simulation?

2. Cosmogenic ^{9}Li / ^{8}He

Cosmogenic ^{9}Li and ^{8}He can undergo β^--n decays that imitate an IBD coincidence.

- Which observable connects these events to their production mechanism?
- How can a muon-correlated control sample be constructed?
- Which additional information could improve the separation from genuine IBD events?

3. ^{13}C(α,n)^{16}O and ^{210}Po

Alpha particles in the scintillator can interact with ^{13}C and produce neutrons. ^{210}Po can be an important alpha source.

- What is meant by secular equilibrium in a radioactive decay chain?
- Why should the ^{210}Po activity not simply be inferred from the measured uranium concentration?
- Why can ^{210}Pb, ^{210}Bi and ^{210}Po be out of equilibrium with the early ^{238}U chain in a real detector?
- How can the ^{210}Po activity be measured directly in liquid-scintillator data and used to constrain the (α,n) background?