SOLUTIONS — PART I

1. BSE uranium mass

$$M_U = a_U M_{BSE} = (20.3 \times 10^{-9})(4.03 \times 10^{24}\ kg) = 8.18 \times 10^{16}\ kg$$

Expected result: $M_U \approx 8.2 \times 10^{16}$ kg.

2. Number of ^{238}U nuclei

$$n_U = (8.18 \times 10^{16}\ kg) / (0.238\ kg\ mol^{-1}) = 3.44 \times 10^{17}\ mol$$
$$N_U = n_U N_A = (3.44 \times 10^{17})(6.022 \times 10^{23}) = 2.07 \times 10^{41}\ nuclei$$

Expected result: $N_U \approx 2.07 \times 10^{41}$.

3. Decay constant and activity

$$T_1/_2 = (4.47 \times 10^9\ yr)(3.156 \times 10^7\ s\ yr^{-1}) = 1.41 \times 10^{17}\ s$$
$$\lambda = \ln 2\ /\ T_1/_2 = 4.91 \times 10^{-18}\ s^{-1}$$
$$A = \lambda N = (4.91 \times 10^{-18})(2.07 \times 10^{41}) = 1.02 \times 10^{24}\ s^{-1}$$

4. Total antineutrino luminosity

$$L\bar{\nu} = 6A = 6.10 \times 10^{24}\ s^{-1}$$

This uses secular equilibrium: the activity of each daughter in the undisturbed chain is taken to equal the ^{238}U parent activity.

5. Central-source surface flux

$$R\oplus = 6371\ km = 6.371 \times 10^8\ cm$$
$$\Phi\bar{\nu} = 6.10 \times 10^{24}\ /\ [4\pi(6.371 \times 10^8)^2] = 1.20 \times 10^6\ cm^{-2}\ s^{-1}$$

Expected result: $\Phi_{total} \approx 1.20 \times 10^6\ cm^{-2}\ s^{-1}$.

6. Flux above IBD threshold

$$\Phi_{>thr} = 0.065\ \Phi_{total} = 7.78 \times 10^4\ cm^{-2}\ s^{-1}$$

7. Averaged two-flavour survival probability

$$\langle P_{ee} \rangle = 1 - ½ \sin^2(66.8°) = 0.578 \approx 0.58$$

8. Oscillated electron-antineutrino flux above threshold

$$\Phi_e = (7.78 \times 10^4)(0.578) = 4.49 \times 10^4\ cm^{-2}\ s^{-1}$$

9. Signal in TNU

$$S = \Phi_e\ \sigma_{IBD}\ (10^{32})\ (1\ yr)$$
$$S = (4.49 \times 10^4\ cm^{-2}\ s^{-1})(1.3 \times 10^{-43}\ cm^2)(10^{32})(3.156 \times 10^7) = 18.4\ TNU$$

Note:

1) the exercise uses a single representative cross section rather than the full energy-dependent U spectrum, this number is intentionally a back-of-the-envelope estimate.

2) Neglects ^{235}U.

3) Assumes secular equilibrium of ^{238}U.

SOLUTIONS — PART II

1. Mantle signal in TNU

Direct scaling in TNU is valid because only the ^{238}U mass and its spatial distribution change. The threshold fraction, survival probability, representative IBD cross section and TNU exposure are already included in the Part I result.

From Part I: $S_{(\text{Part I})} = 18.4$ TNU

Mantle-to-BSE uranium-mass ratio: $(4.68 \times 10^{16}\ \text{kg})/(8.18 \times 10^{16}\ \text{kg}) = 0.572$

Central-source-equivalent mantle signal: (18.4 TNU)(0.572) = 10.5 TNU

Model A — homogeneous mantle: S_{hom} = (10.5 TNU)(1.55) = 16.3 TNU

Model B — CMB layer: S_{CMB} = (10.5 TNU)(1.12) = 11.8 TNU

2. Expected number of events

$N_p = 1.45 \times 10^{33}$ free protons, $\varepsilon = 0.80$, T = 1 year

Exposure factor = $(1.45 \times 10^{33}/10^{32})(0.80) = 11.6$

Homogeneous mantle: N_{hom} = (16.3 TNU)(11.6) = 189 ≈ 190 events yr^{-1}

CMB layer: N_{CMB} = (11.8 TNU)(11.6) = 137 ≈ 140 events yr^{-1}

3. Comparison

The homogeneous mantle gives a larger signal because some uranium is closer to the detector. The flux from each source element scales as $1/L^2$. In the CMB model, all uranium is far from the detector. Therefore, equal uranium masses and luminosities can produce different surface fluxes.

4. Realistic calculation

A realistic calculation integrates over antineutrino energy and the three-dimensional uranium distribution.

Energy dependent: ^{238}U spectrum, IBD cross section, survival probability and possibly also detection efficiency.

Position dependent: uranium abundance, rock density, source-to-detector distance, geometrical factor and oscillation baseline.

SOLUTIONS — PART III

A. IBD interaction recap

The IBD threshold follows from energy–momentum conservation and the neutron–proton mass difference. The exact threshold is ≈1.806 MeV. The positron deposits its kinetic energy and then annihilates, creating the prompt visible signal. To leading order, $E_{prompt} \approx E\nu - 0.78$ MeV. The neutron thermalizes and is captured after a characteristic delay, producing the delayed signal (for example, a 2.2 MeV γ after capture on hydrogen in a standard LS detector).

The coincidence is powerful because a prompt positron-like event followed after a characteristic time by a spatially nearby neutron-capture signal is uncommon for many single-event radioactive backgrounds. Useful selection observables include prompt energy, delayed energy, time separation Δt, spatial separation Δr, reconstructed vertex/fiducial volume, multiplicity, and relation to preceding muons.

B1. Accidentals

Accidentals are uncorrelated. Their rate and shapes can be obtained very directly from data by using off-time windows/time-shifted pairing. Because the prompt-like and delayed-like events are unrelated, moving the coincidence window far away in time preserves the accidental population while removing genuine IBD correlations. This is a classic fully data-driven background estimate.

B2. ^{9}Li / ^{8}He

These isotopes are produced by cosmic-ray muons and their β-n decays are correlated with a preceding muon. The time since the preceding muon is therefore a key observable. A control sample can be enriched using events following energetic or showering muons, and possibly using distance to the reconstructed muon track. Fits to the time-since-muon distribution can constrain the cosmogenic component.

B3. ^{13}C(α,n)^{16}O and ^{210}Po

In secular equilibrium, the activities of daughter isotopes in a long decay chain equal the activity of the long-lived parent after sufficient time and in the absence of chemical or physical separation. However, the lower ^{238}U-chain isotopes ^{210}Pb, ^{210}Bi and ^{210}Po can be introduced, removed, purified, plated onto surfaces, or transported independently. Their activities can therefore be far from equilibrium with the early ^{238}U chain.

For this reason, ^{210}Po activity should be measured directly from data rather than inferred from the uranium concentration. In liquid scintillator, the alpha energy is strongly quenched, but the ^{210}Po alpha population can still form a characteristic feature and can be identified statistically. Its measured activity can then be propagated, together with the ^{13}C(α,n) reaction yield and detector response, into an estimate of the (α,n) background.

Pedagogical link: Part I used secular equilibrium to associate one ^{238}U parent decay with a complete chain producing six antineutrinos. Part III shows why an equilibrium assumption that is reasonable for a geochemical source inventory cannot automatically be assumed for individual contaminant populations inside a detector.