

Cosmic Muontomography



FuSe

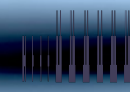
TESTING FUNDAMENTAL
PHYSICS WITH SEISMOLOGY

TIHANY, HUNGARY
7-11 September 2026

FuSe School 2026
Gergely Gábor Banraföldi
Gergely Surányi



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the European Union



MTA
Kiváló
Kutatók

**HUN
REN**



- The electroscope loses its charge
 - Monk Wulf (1909): experiments at school and in nearby sandstone quarries.
 - The rock itself was originally suspected, but the loss was slower down in the quarry.
 - University of Paris + Eiffel Tower: the radiation is stronger at altitude
 - Viktor Hess (1912): balloon measurements up to 6 km. (Nobel Prize: 1936)
 - Carl Anderson: positron (1932)
 - Powell: muons are heavy electrons (1947)
 - Pierre Auger, Lajos Jánossy

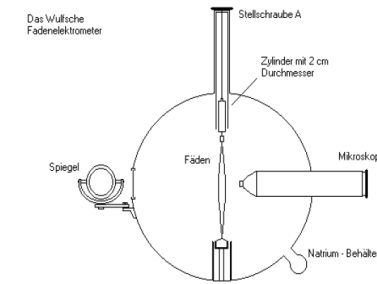
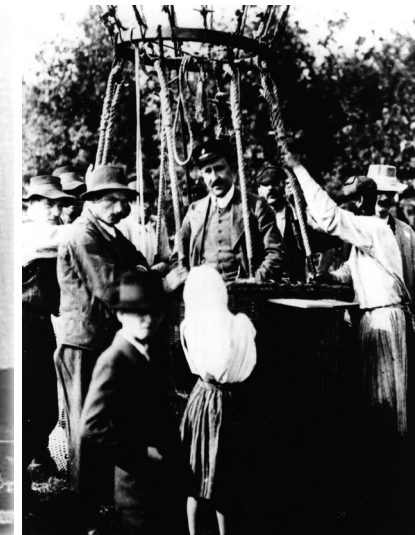
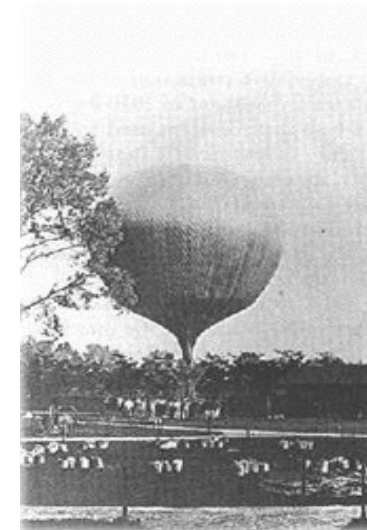


Fig. 2: Wulf electrometer.



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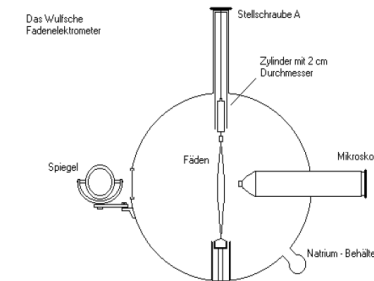
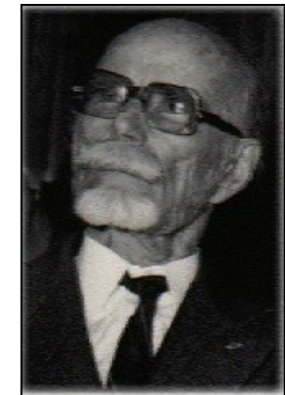
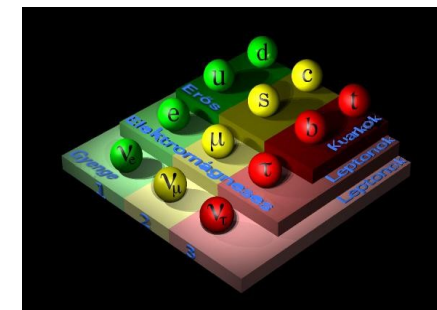
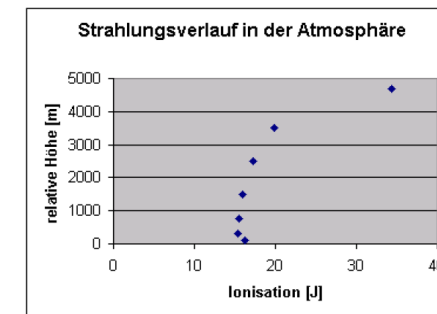
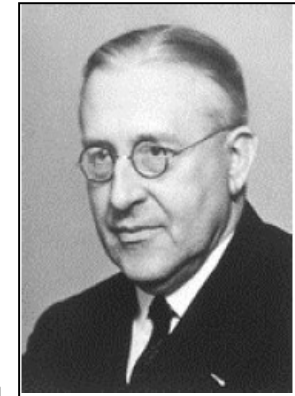
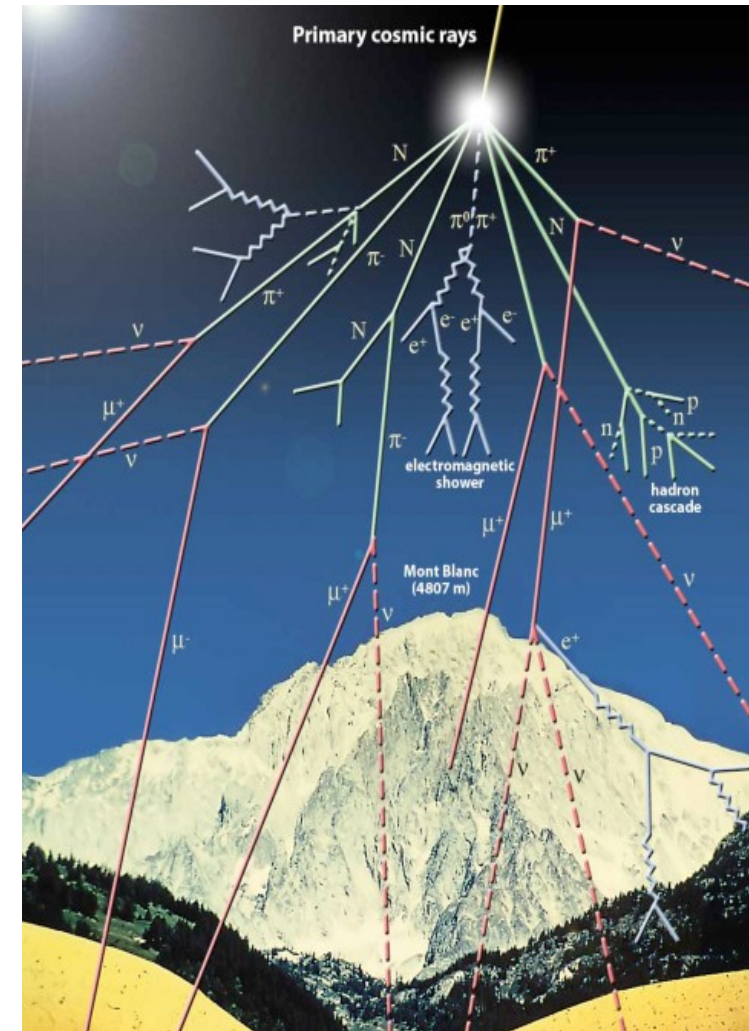


Fig. 2: Wulf electrometer.



- Primary cosmic radiation:
 - High-energy particles arriving from outer space
 - Mainly p (89%), He^{2+} (10%), e^- , other nuclei (1%)
 - Up to 10^{20} eV in energy.
- Secondary particles
 - Produce pions and kaons
 - Decay into electrons and muons
 - **Muon: negatively charged, 200 times the electron mass: μ^-**



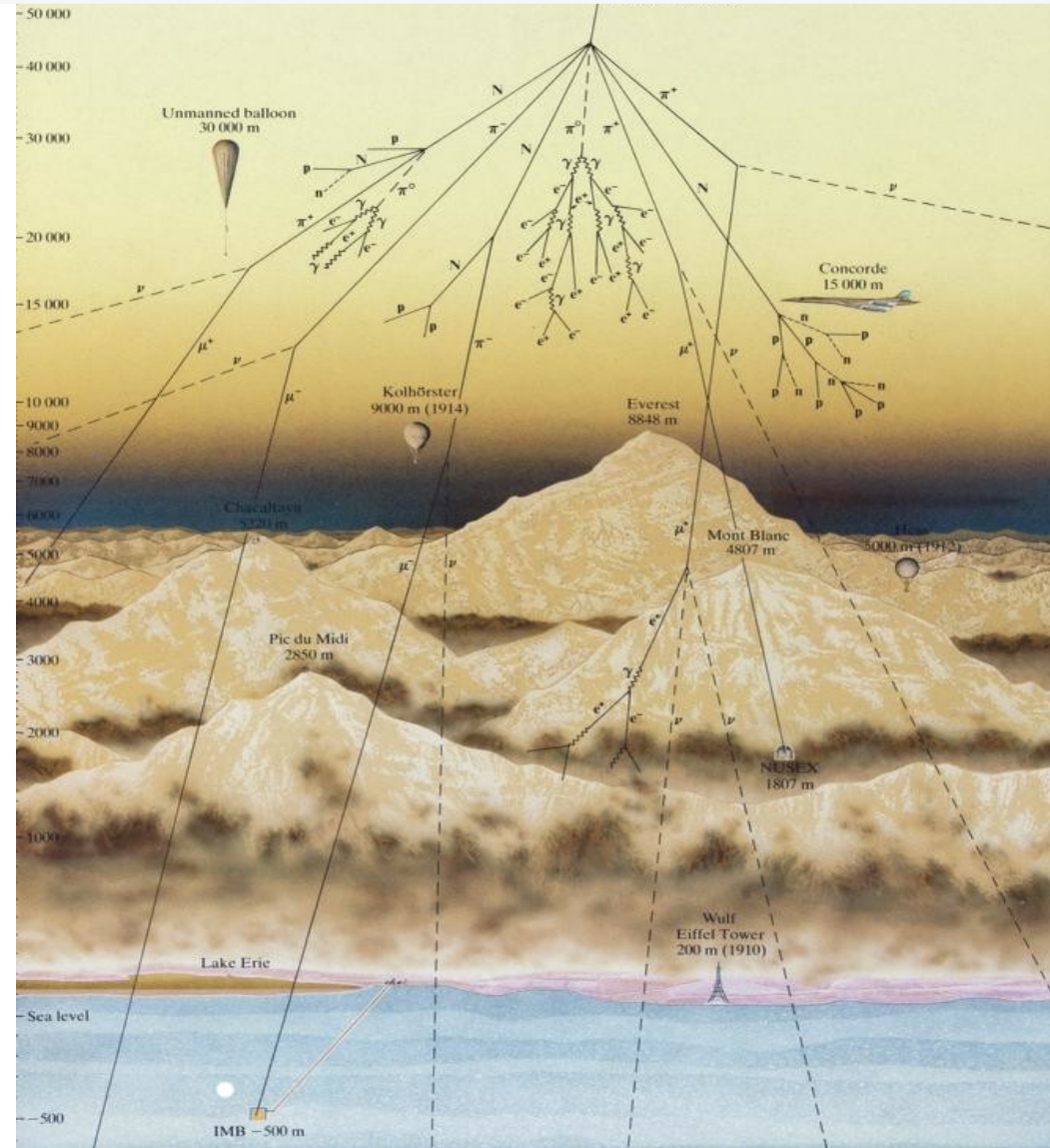
Detecting Cosmic Particles

Large extent!

Over an area larger than Paris
Or larger ($\sim \text{km}^3$) in volume,
even

On the surface, underground,
in mines, underwater

Even the Earth itself can be a
detector



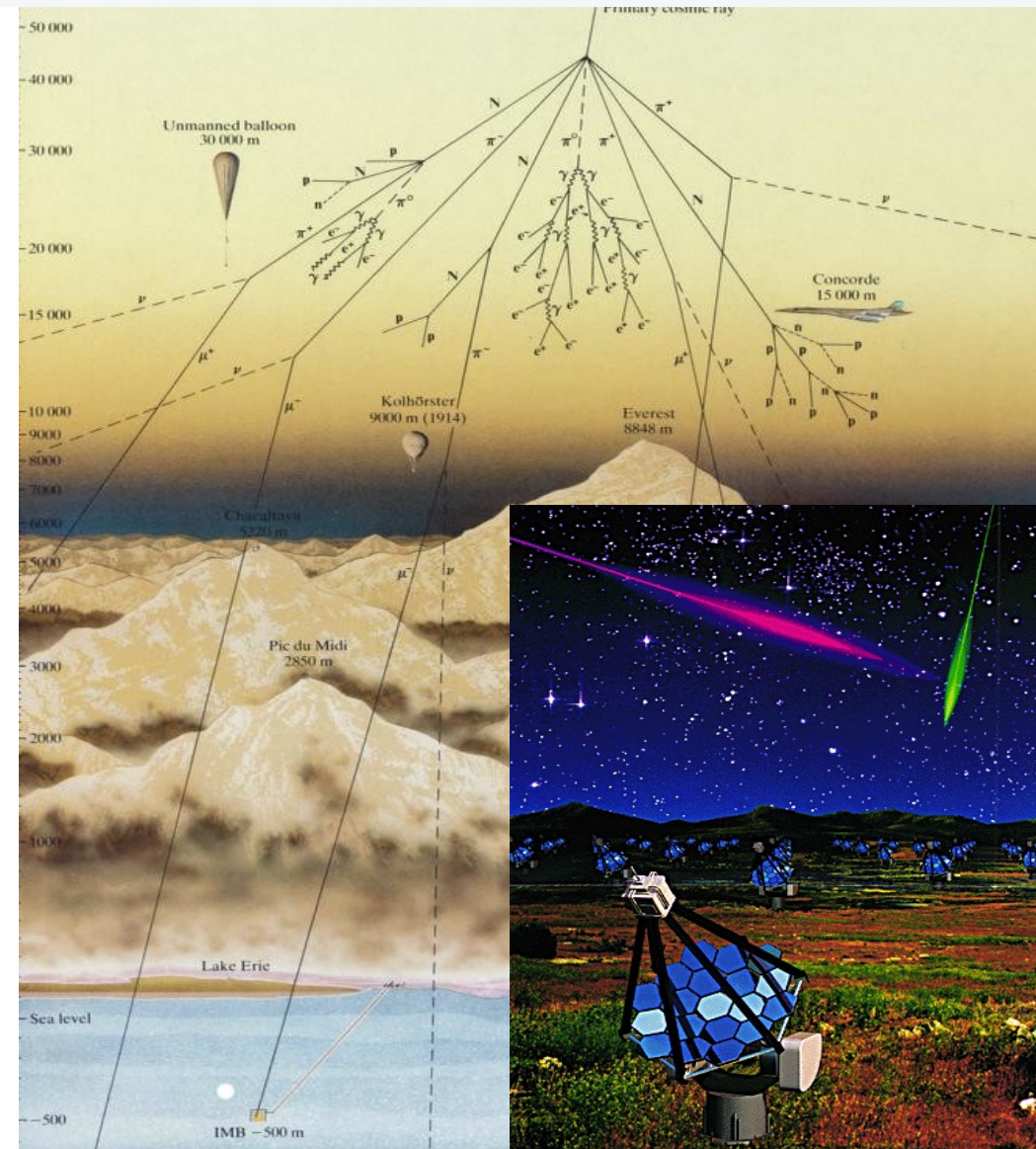
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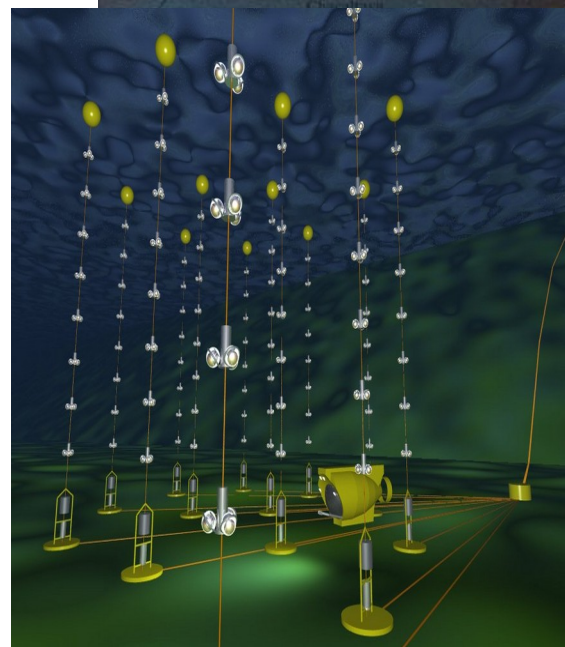
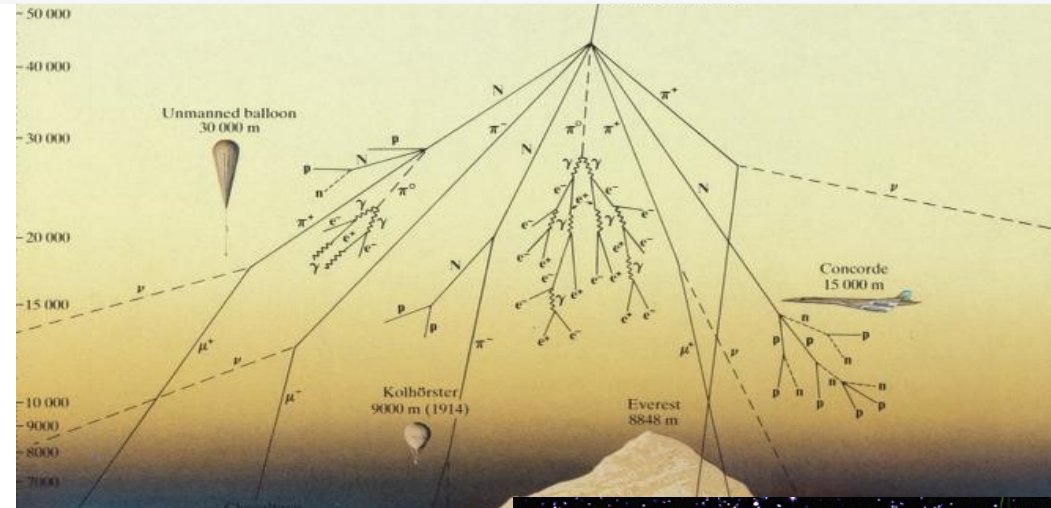
The Origin of Cosmic Rays

Large extent!

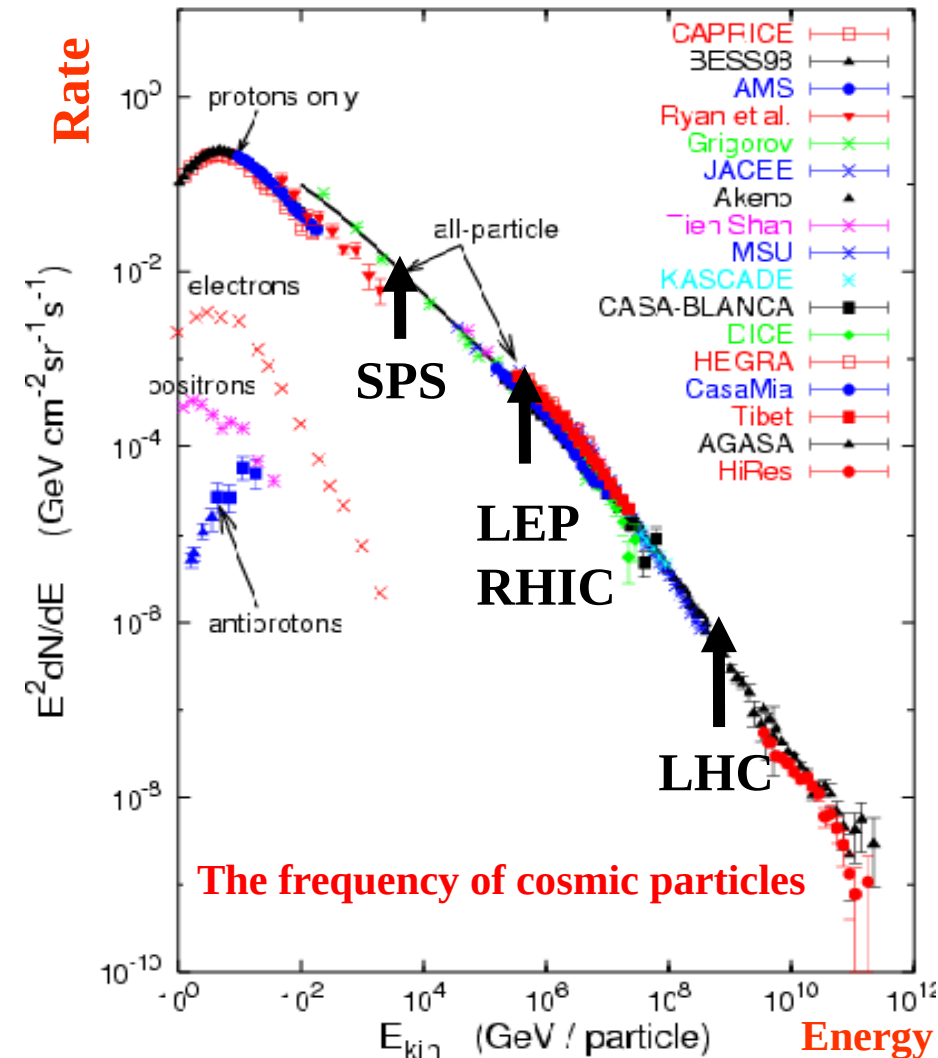
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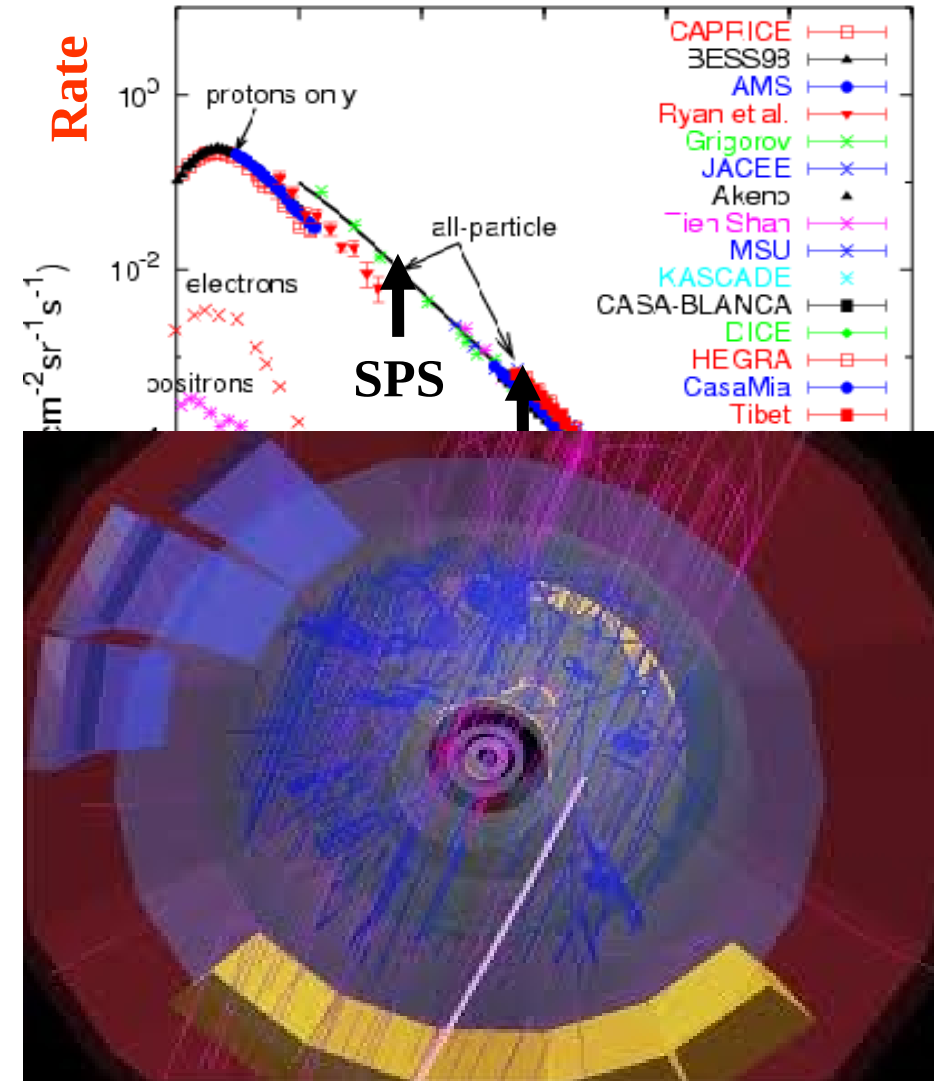


- A particle accelerator: 10–15 years
- Is there any experimental possibility above LHC energies?
- Of course, yes:
 CERN SPS ~20 GeV
 RHIC BNL, LEP ~100 GeV
 LHC ~10 TeV
 Cosmic ~1 EeV
- Of course, we are already planning the next one: eLHC, sLHC, FCC
- We calibrate the LHC detectors using cosmic radiation:



Higher Energies than Man-made Accelerators

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- Energy distribution:

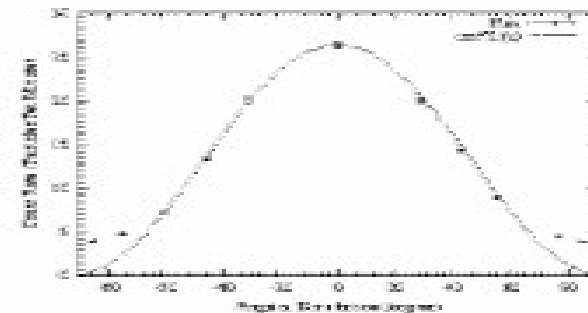
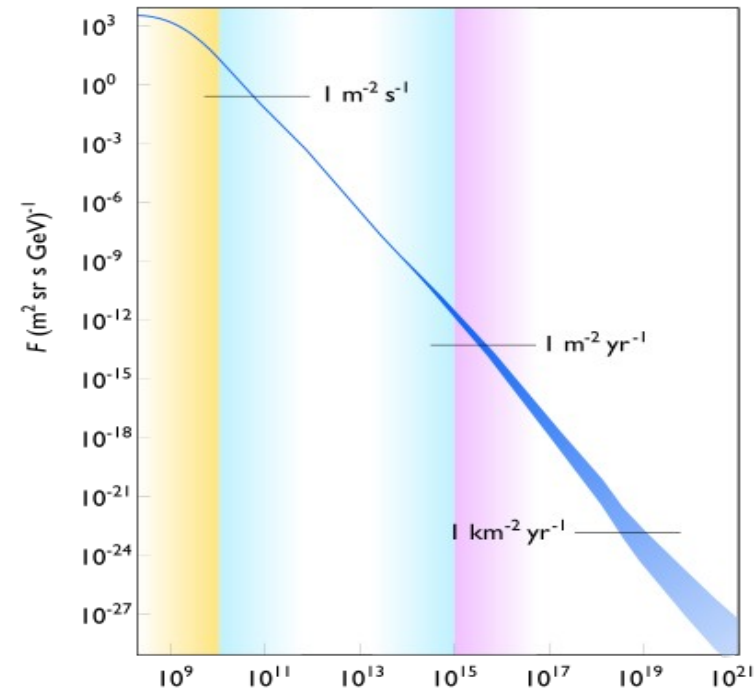
$$N(>E) = k \cdot (E+1)^{-a}$$

- E is the particle energy [GeV];
- $N(>E)$: the number of particles with energy greater than E;
- $k \approx 5000$ particles/m²;
- $a \approx 1.6$

- Angular distribution:

$$\Phi(\theta) \sim \cos^m(\theta)$$

- θ is the so-called zenith angle, measured from vertical
- $m=2$, but there is also a dependence on latitude and altitude.



- Principle: a cosmic “X-ray image”
 - First application by L.W. Alvarez: searching for hidden chambers in the Pyramid of Khafre. (1957)
 - Physical result:
There are no secret chambers in the Pyramid of Khafre
- The real result:

No need to drill through the pyramid!

L. Alvarez et al.: Search for Hidden Chambers in the Pyramids, Science, 167, 832-839, 1970.



Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720
Telephone 415/843-2740

August 2, 1978

Senorita Linda Manzanilla
Cerro Del Agua No. 106
Mexico 21, D. F.
Mexico

Dear Senorita Manzanilla:

I have not been working in Egypt for the last several years, so it won't be possible for you to be with me in Giza, in the near future.

I am enclosing some printed material on the work that my colleagues and I did at Giza, and I can say that since those things were published, we completed a survey of the Second Pyramid, pointing our cosmic ray telescope in six or seven different compass directions, tilted each time at 45 degrees above the horizon. We also repeated the vertical scan that is described in the material I am sending you. The results of all this is that we found the pyramid to be quite solid, with no chambers comparable in size to those found above the plateau level, in the Great Pyramid.

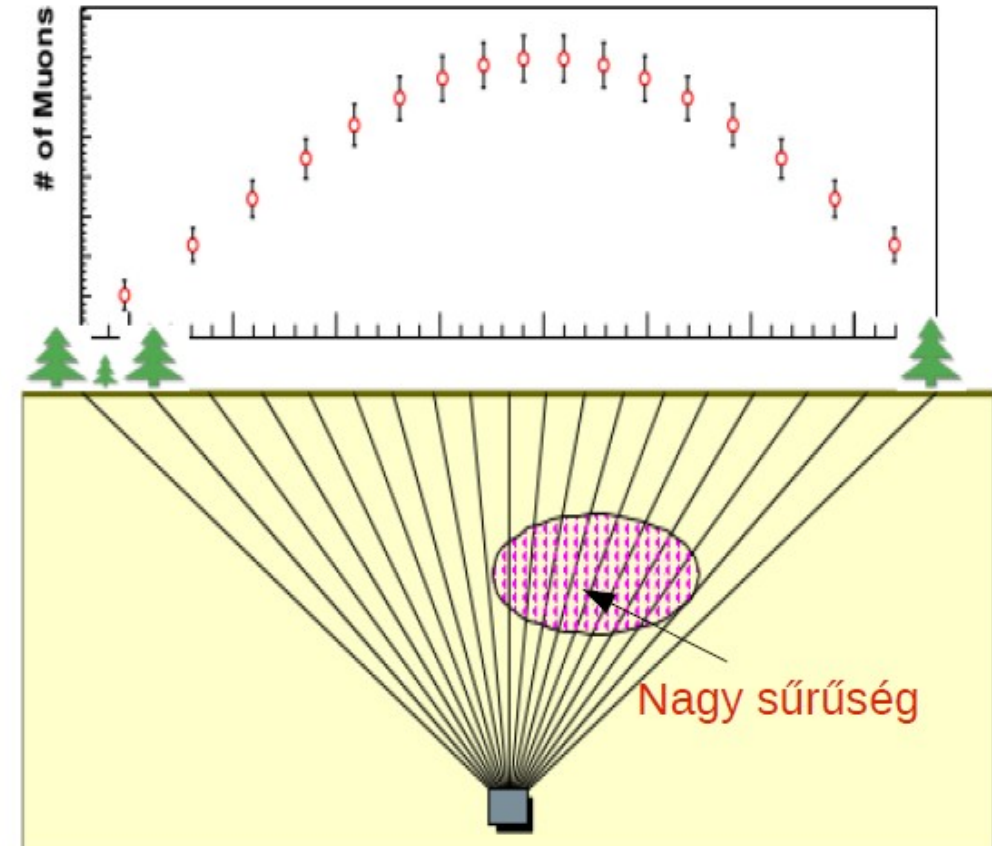
Very sincerely,

Luis W. Alvarez
Luis W. Alvarez

LWA/lb
Encls.

Goal:

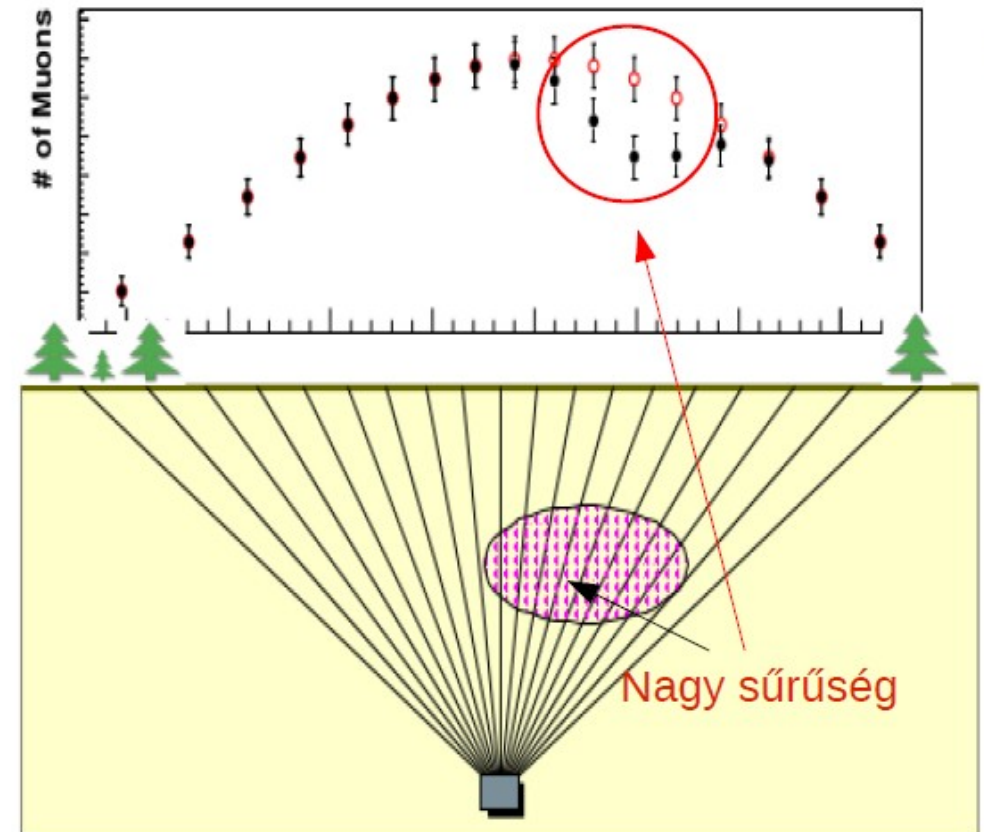
Finding & locating : large-scale inhomogenities: ores, caverns, by using cosmic muons.



Goal:

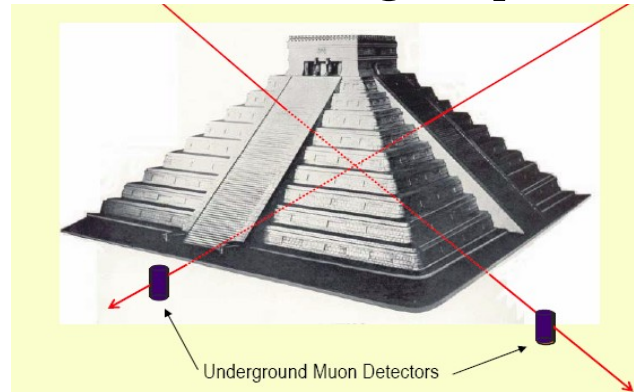
Finding & locating : large-scale inhomogenities: ores, caverns, by using cosmic muons.

Having a lower (higher) flux in the distribution, caused by higher (lower) density volume.

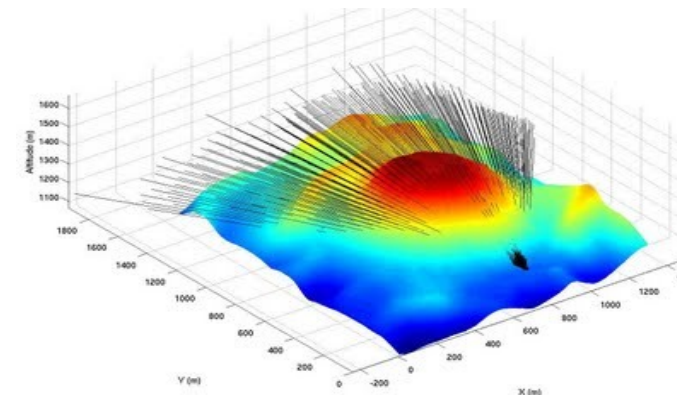
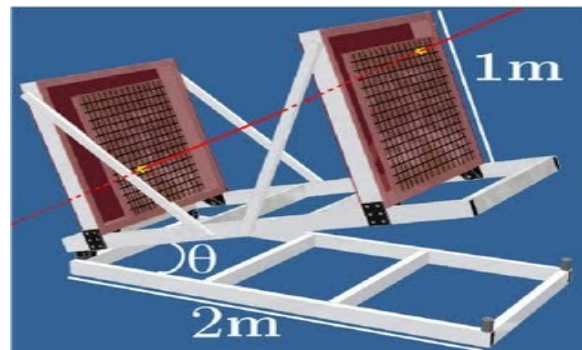


- Pyramid of the Sun (Mexico)

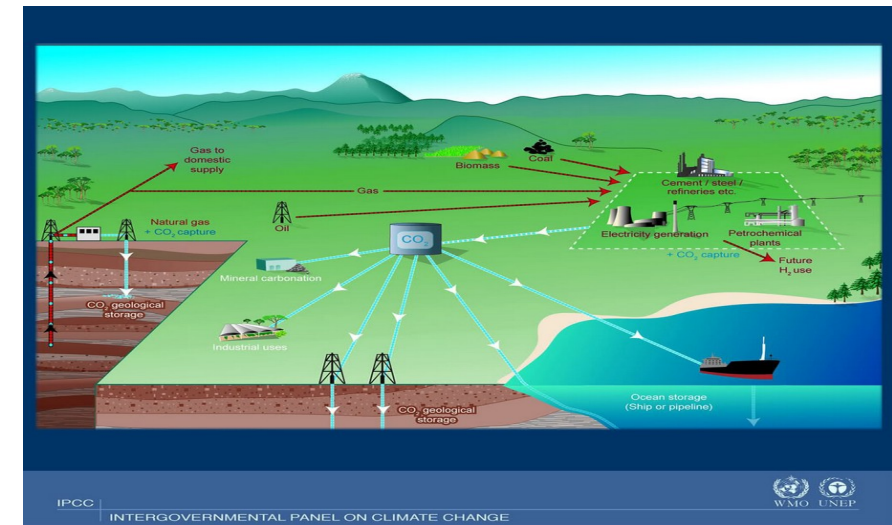
R. Alfaro, V. Grabski: Searching for possible hidden chambers in the Pyramid of the Sun.



- Volcano investigation (Japan)

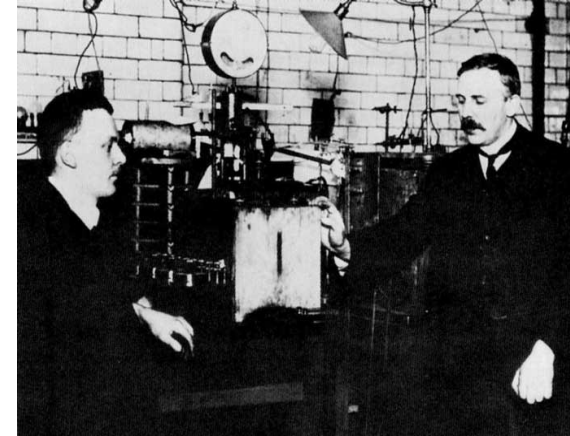


- National-security application
- Measuring the volume of a CO₂ trap



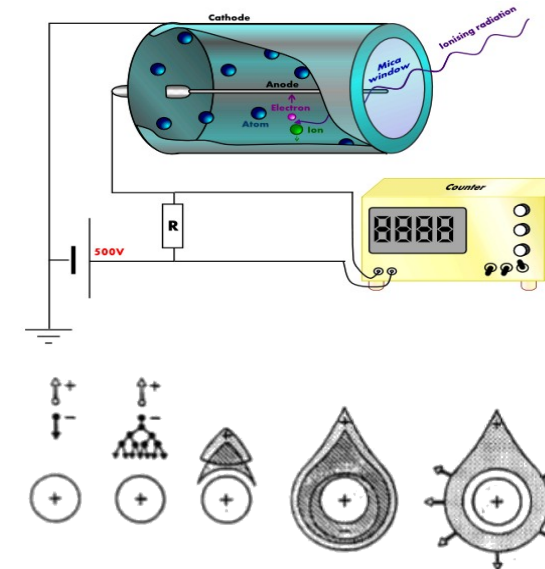
- Geiger–Müller counter

- H. Geiger, E. Rutherford W. Müller
- 1908 H. Geiger tube
- 1928 perfected by W. Müller

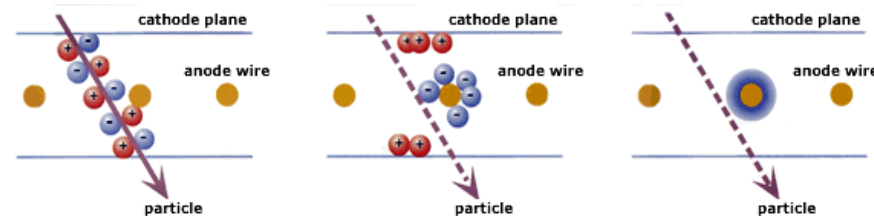
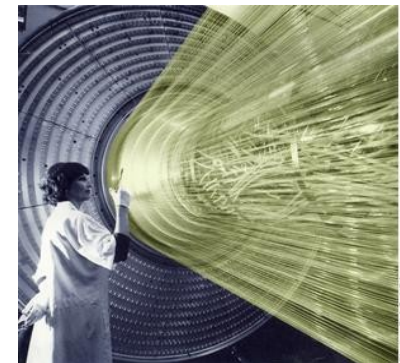
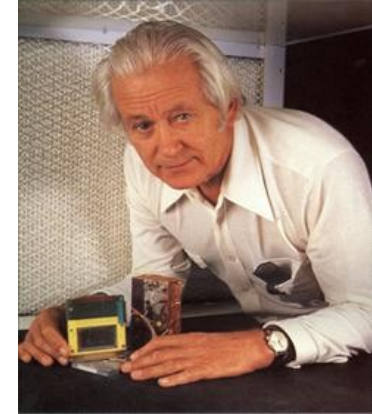


- Operation

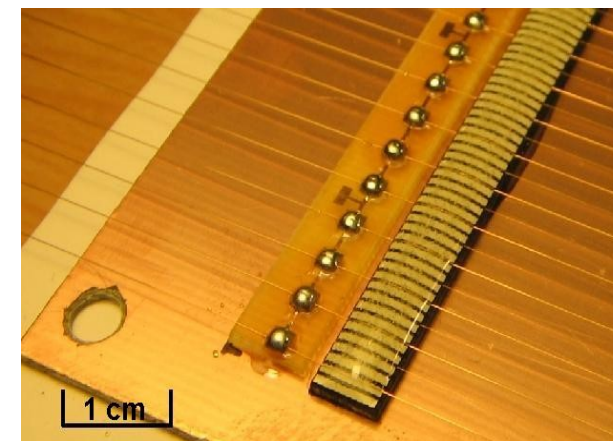
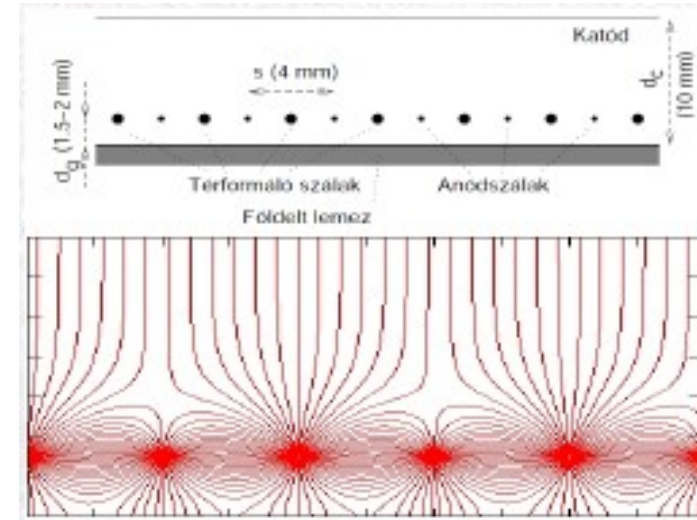
- A charged particle entering a strong electric field collides with the gas molecules and triggers an electron avalanche.
- The electron avalanche is already measurable, e.g., 10^6 electrons



- Multi-Wire Proportional Chamber (MWPC)
 - Georges Charpak (1924 -)
 - 1968 multi-wire chamber
 - Measures charged particles
 - 1992 Nobel Prize
- Operation
 - Same principle as the GM tube, but with far more anode wires.
 - Larger volume.

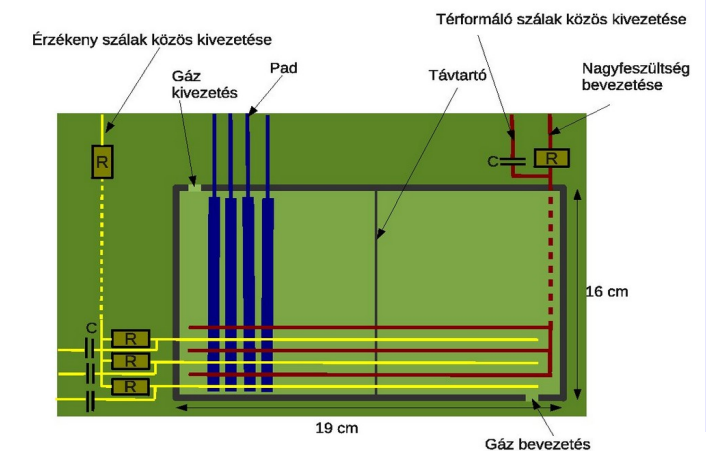
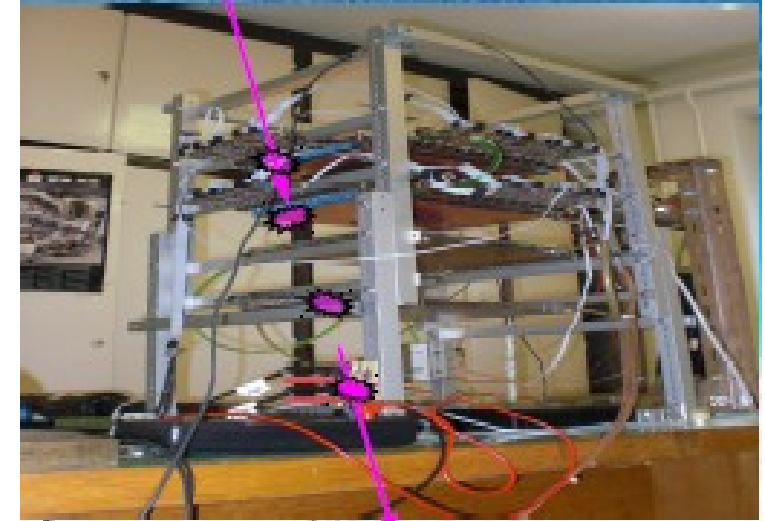


- Detector development
 - Micro-pattern detectors
 - Gas Electron Multiplier, GEM
 - Multi-wire chambers (MWPC)
- In-house development: CCC
 - Close Cathode Chamber (CCC)
 - Not sensitive
 - even to extreme conditions
 - Inexpensive to produce



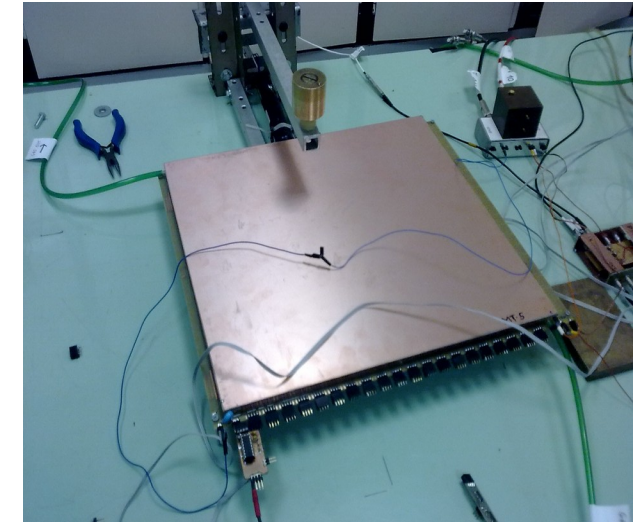
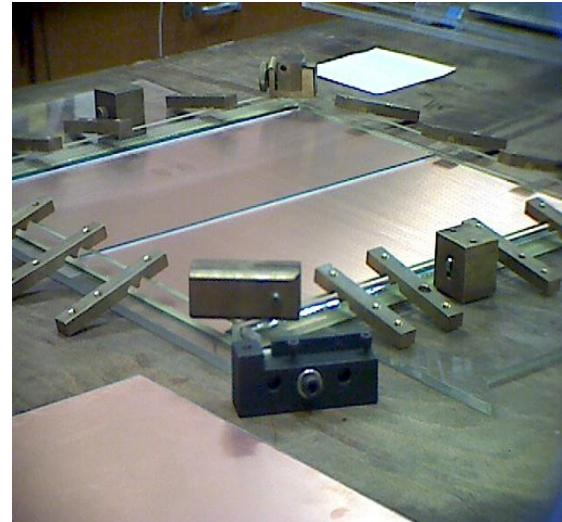
The Detector: MWPC & CCC chambers

- Muon tomography:
 - Measuring the angular distribution of high-energy muons
 - Studying 1D or 2D distributions in coincidence.
 - This requires multi-wire chambers, but those are cumbersome.
- The 2D CCC chamber
 - Direct 2D measurement
 - ArCO₂-filled, asymmetric-cathode chambers
 - Not (very) sensitive to deformation or temperature

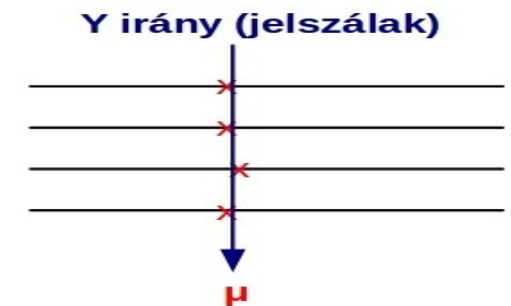
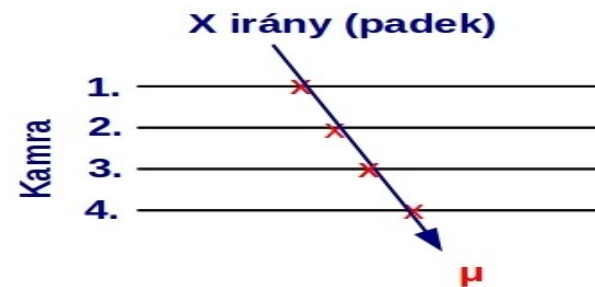


The Detector: MWPC & CCC chambers

- Making a CCC chamber



- The muon tomograph in operation



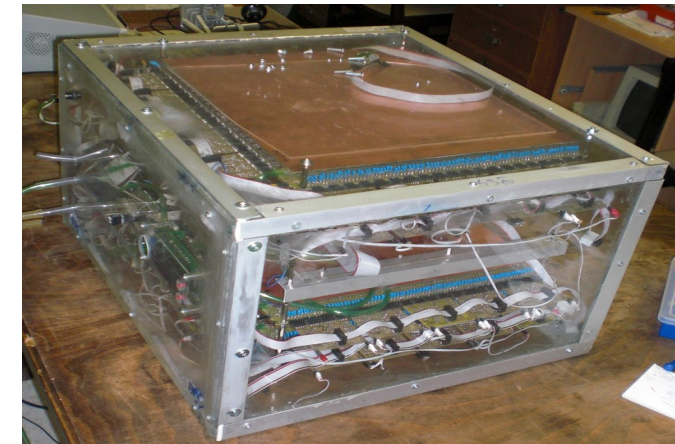
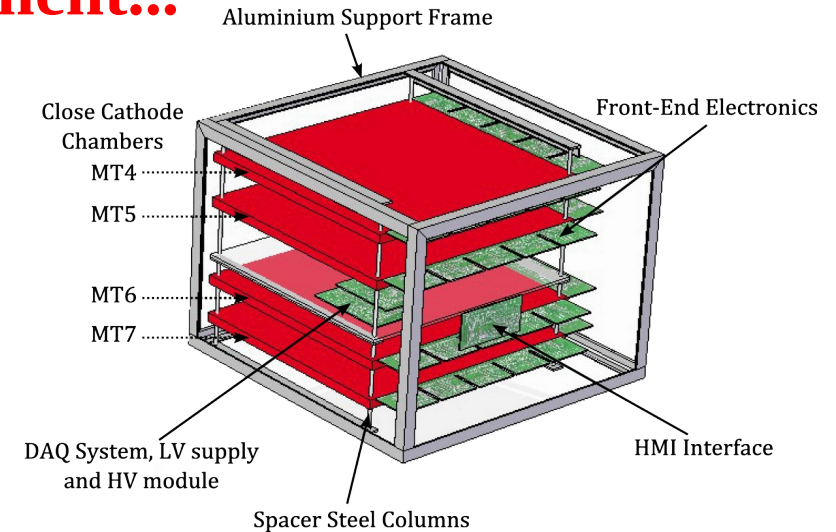
A by-product of the ALICE VHMPID development...

- 4 close-cathode chambers (CCC)
- Active area: 32 cm × 32 cm
- In a hermetically sealed acrylic box
- Easy to handle:

volume: 51 × 46 × 32 cm³

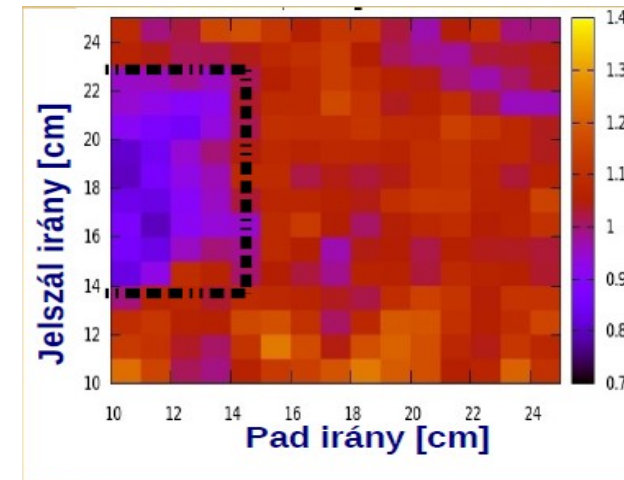
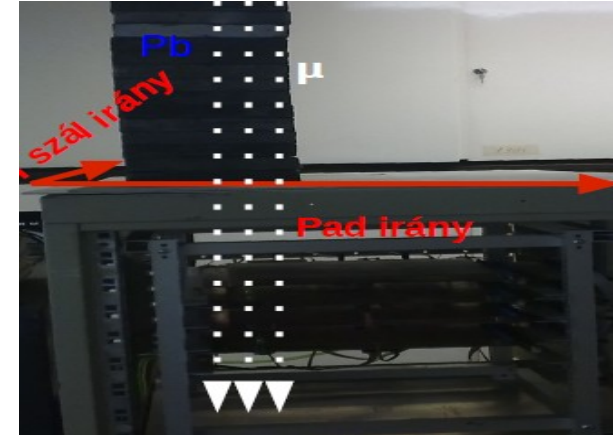
Total weight: 13 kg

- DAQ system
- Simple: LCD display, SD card



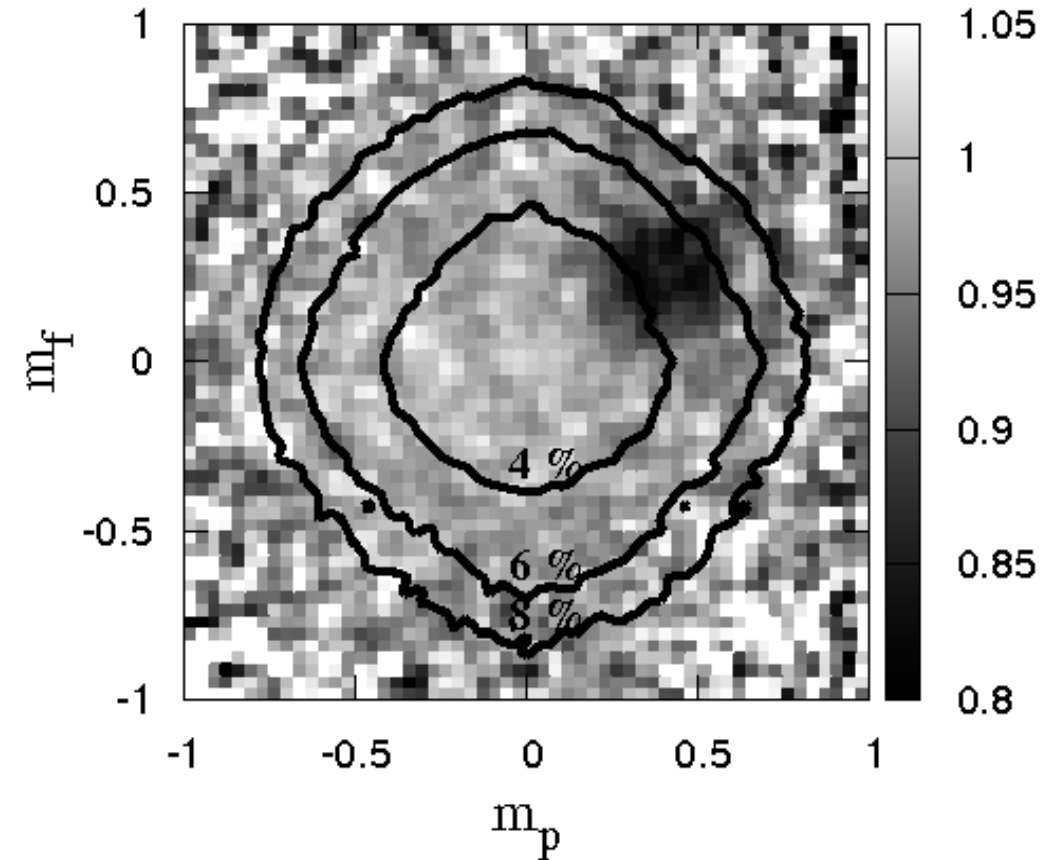
Simple test: muon imaging of a lead brick

- Muon tomography:
 - Muons are absorbed in very dense materials (e.g., lead)
 - Can we see the “muon shadow” of the lead block?
- YES: with the muon tomograph
 - **Purple** (few muons arrive)
 - **Red** (many muons arrive)



Simple test: muon imaging of a lead brick

- Laboratory conditions
- Absorber tower:
 - mass: 150 kg
 - volume: $20 \times 20 \times 50 \text{ cm}^3$
- Data taken at the same position:
with tower / without tower
- 7 M events, 10-day measurement
- 20% decrease in muon rate (Figure: ratio of fluxes measured with and without the tower)



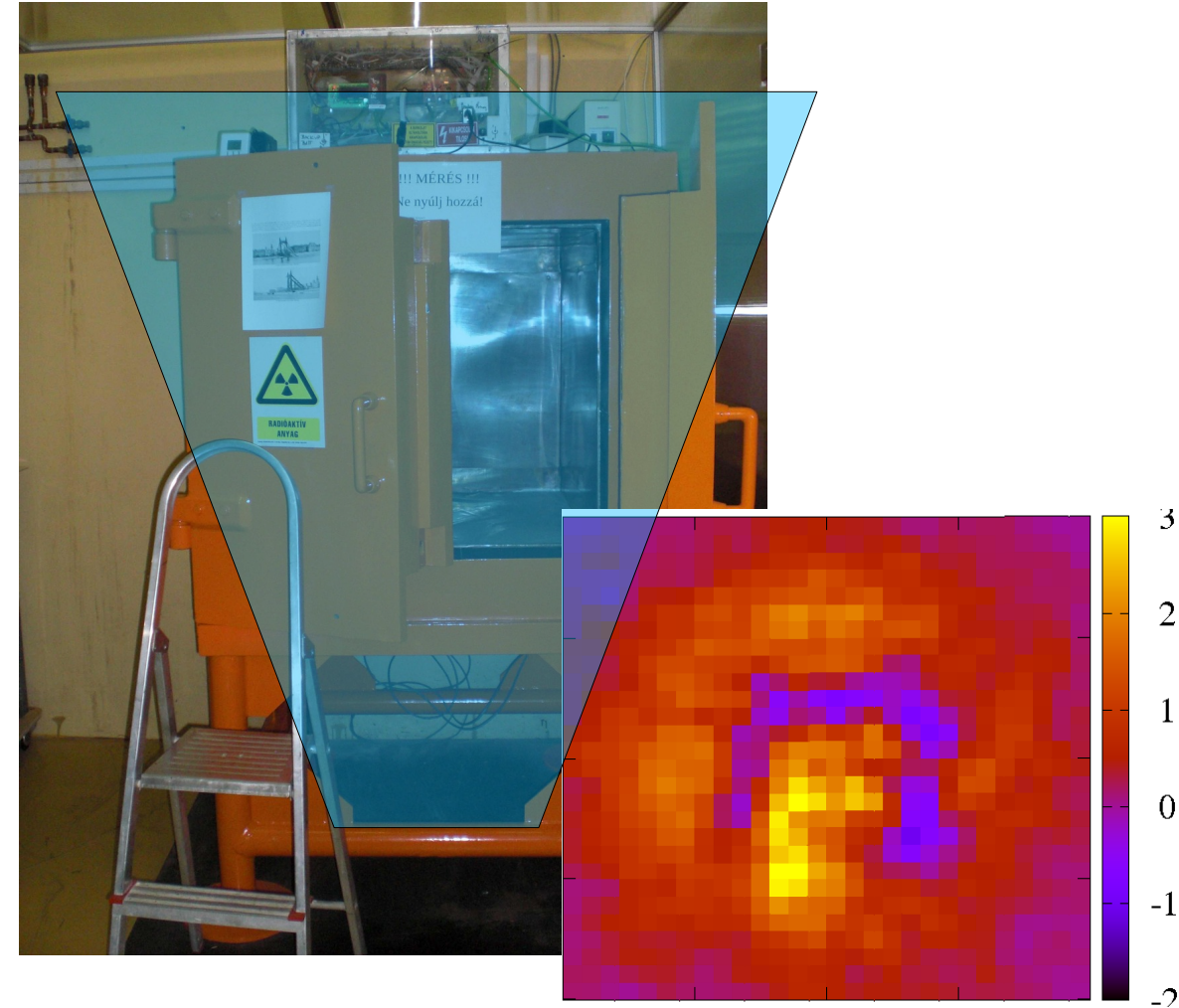
Simple test: muon imaging of safe (DÖME)

- DÖME – "Ridiculously Heavy Measuring Equipment" (Hungarian acronym)
- 8.5 t of iron from the old Erzsébet Bridge
- Pre-WWII iron (no atomic-bomb contamination)

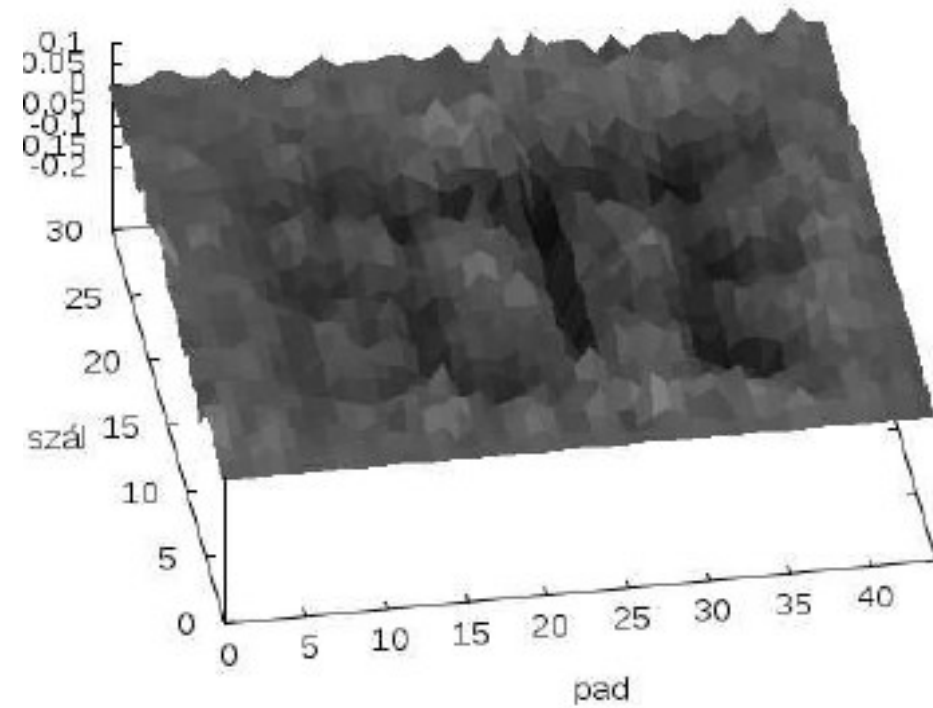


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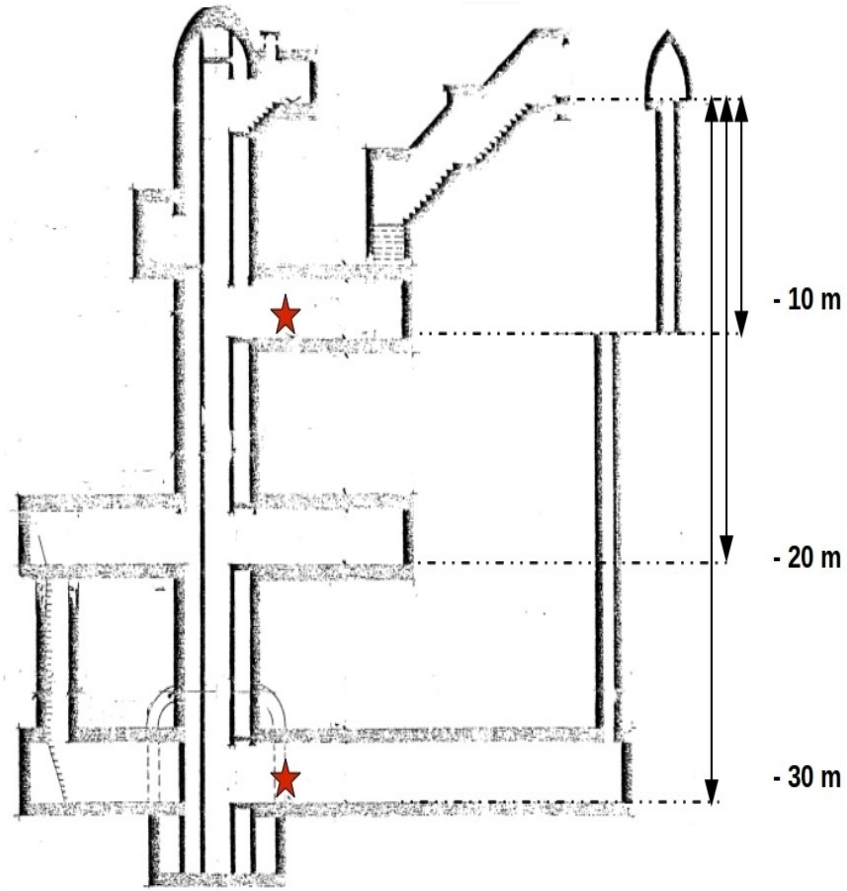
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Simple test: imaging 'ELTE' with radionuclei

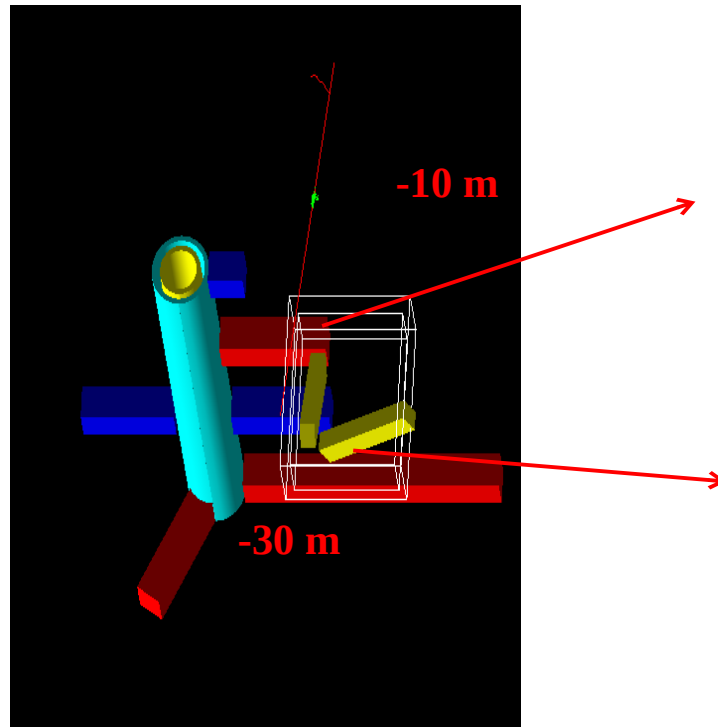


Simple test: In the JURLAB

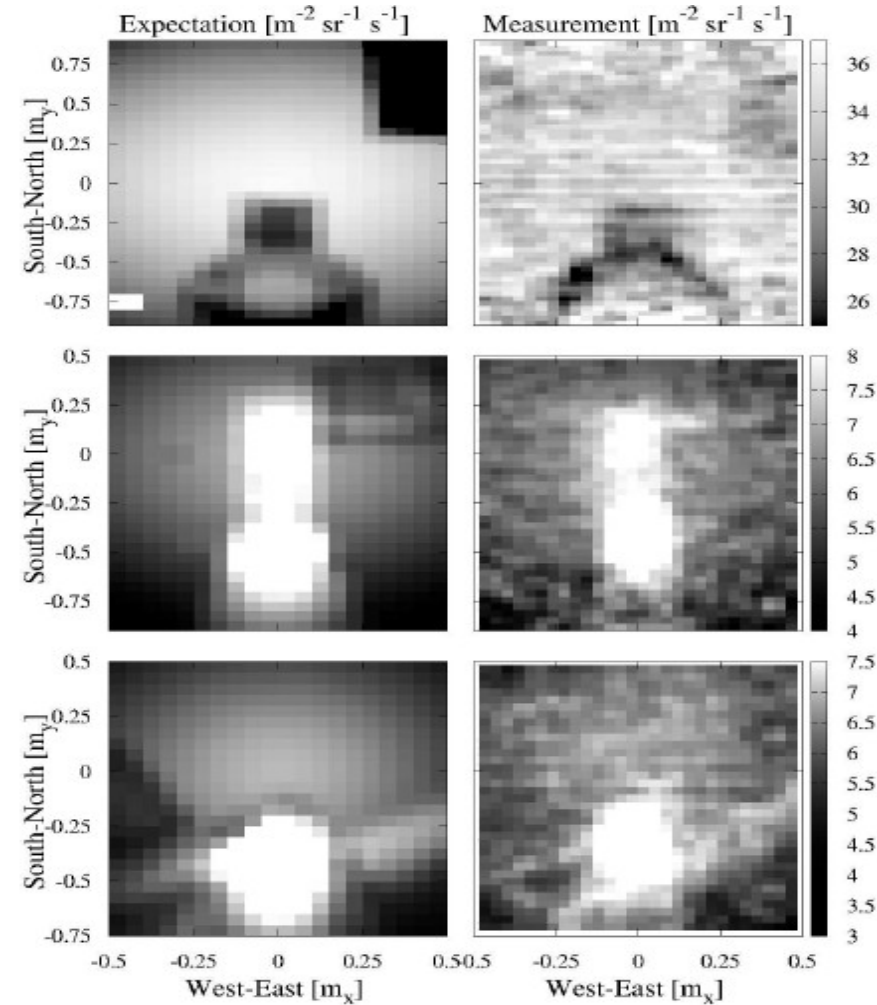


- 3 levels, 32 m deep underground structure (since 1951)
- Suitable for testing:
 - high humidity
 - several galleries stacked one below another
- Earlier results:
 - Angular-distribution measurements:
P. Király: Investigation of cosmic radiation at 40 m water-equivalent depth (1963)
 - Forbush decrease (> 10 GeV)

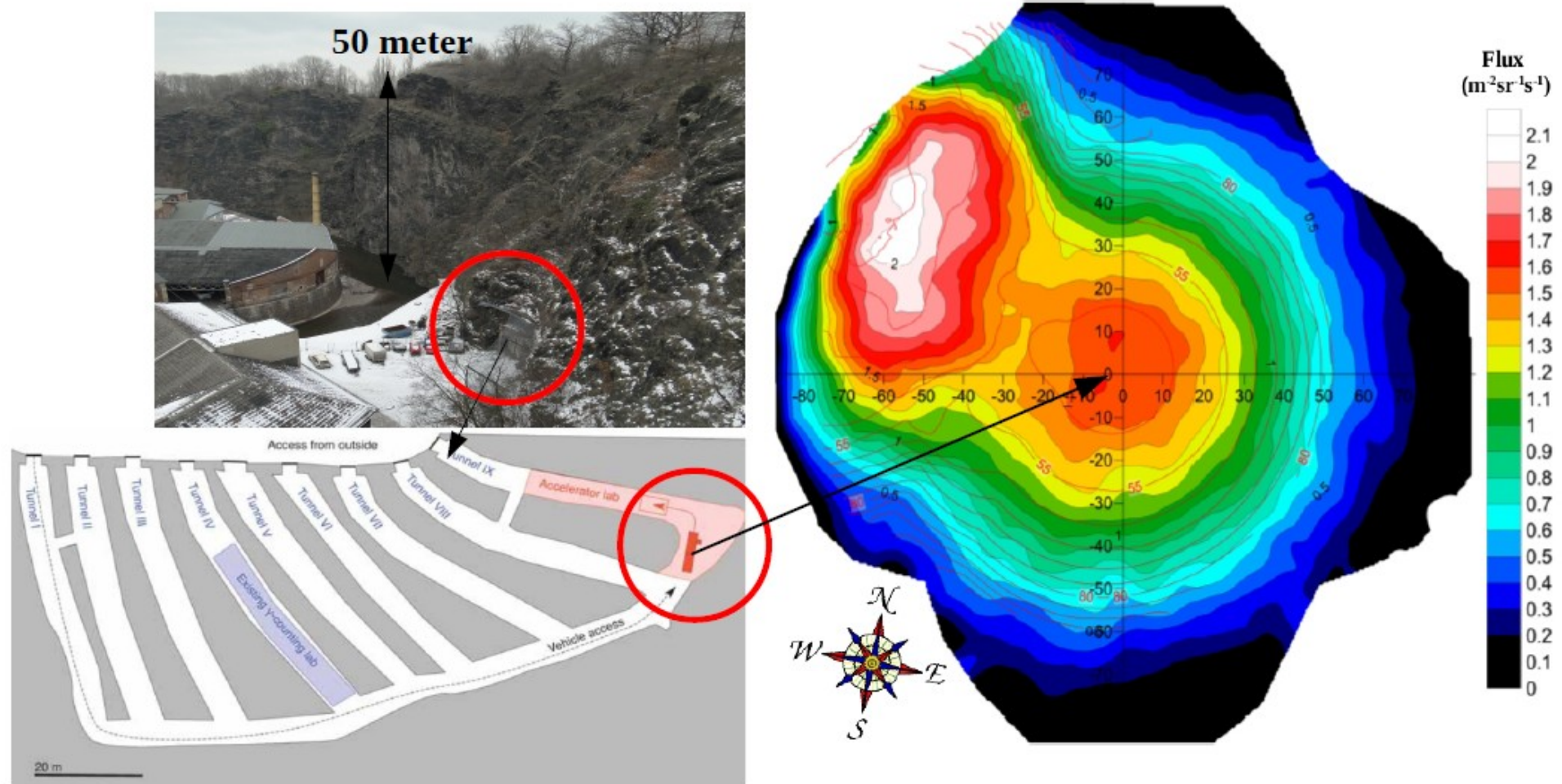
Simple test: In the JURLAB



- Creating a geometry model
- Carrying out measurements
- Can be reconstructed and compared using Monte Carlo simulation



Measurements: Flesenkeller, Dresden



Measurements: in Hungarian caves



- **Lab (0 m):**
> 100 days, > 100 M muon events
- **Jánossy shaft (-10, -20, -30 m):**
15 days, 2 M muon events
- **Molnár János Cave (-45 m):**
77 days, 110 k muon events
- **Ajándék Cave (-60 m):**
50 days, 170 k muon events
- **Pilis Mountains (0 m):**
1 day, 300 k muon events
- **Kőbánya, Dreher (-20 m):**
40 days, 500 k muon events
- **Hármashatárhegy Gas Storage (-40 m):**
200 days, 10 M muon events
- **Sátorkőpuszta Cave (-60 m):**
90 days, 500 k muon events
- **Felsenkeller, Dresden (-20 m)**
150 days, 3 M muon events

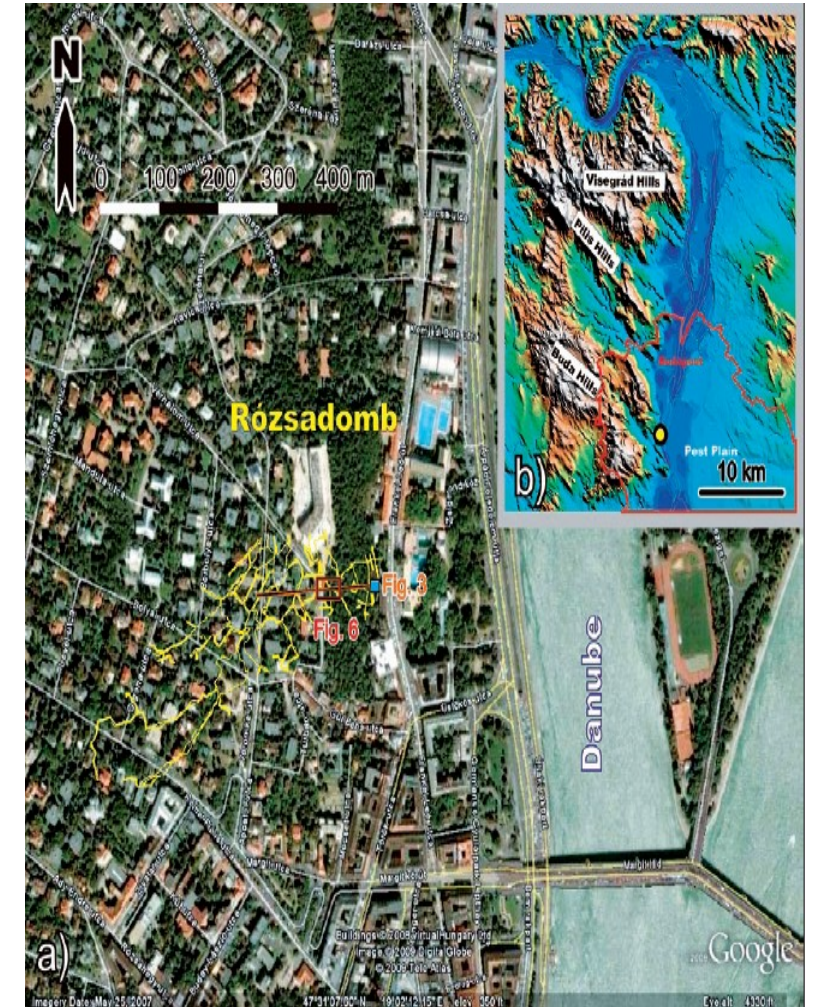
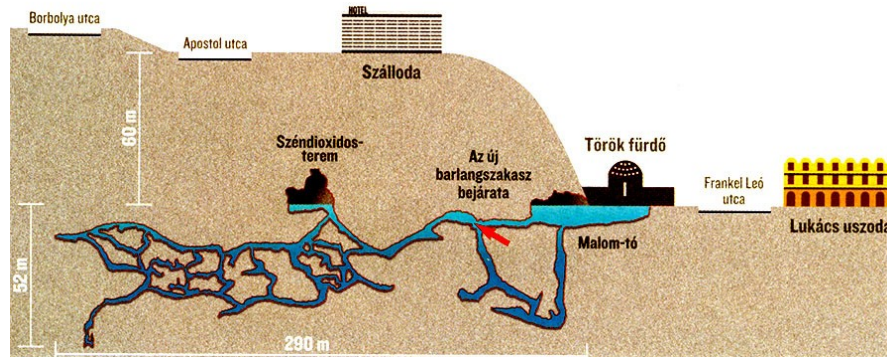


Measurements: in János Molnár cave

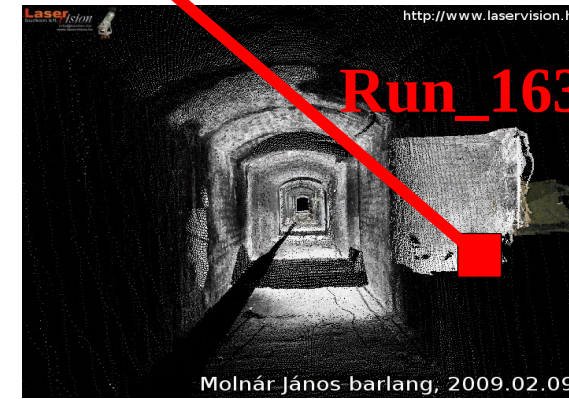
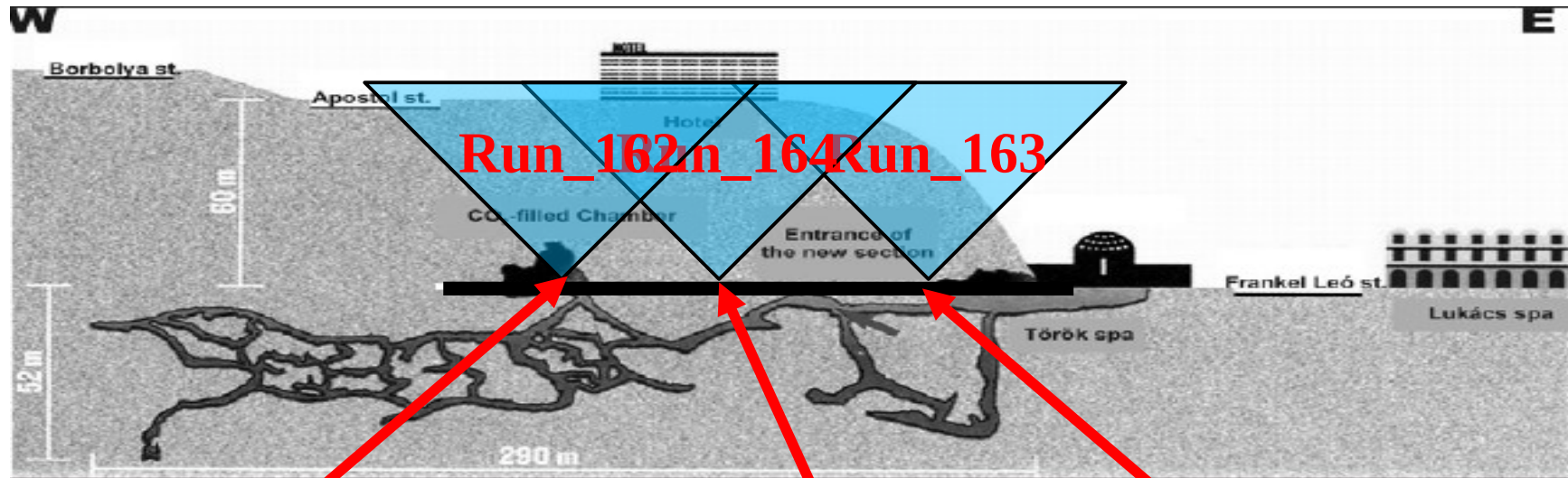


Measurements: in János Molnár cave

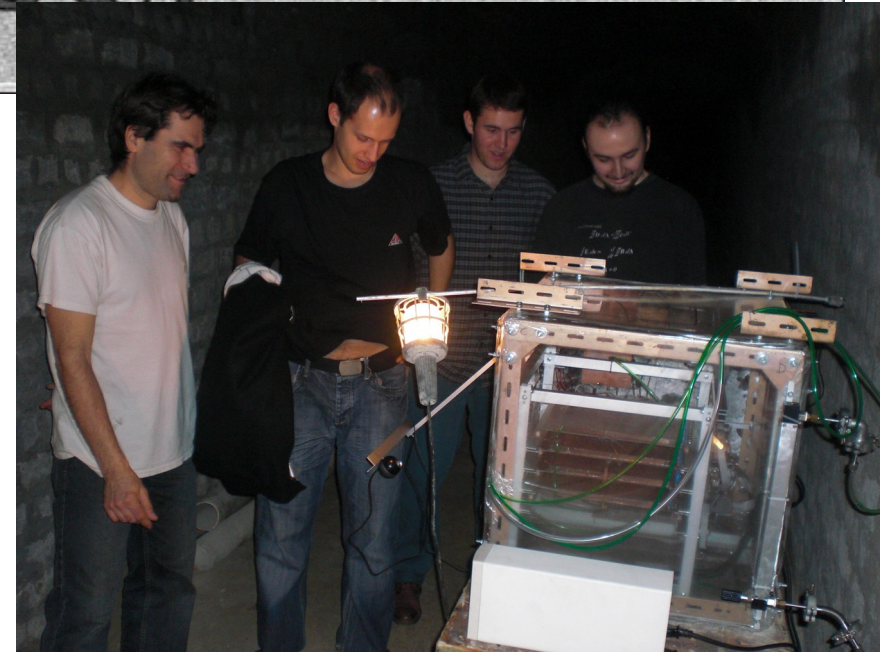
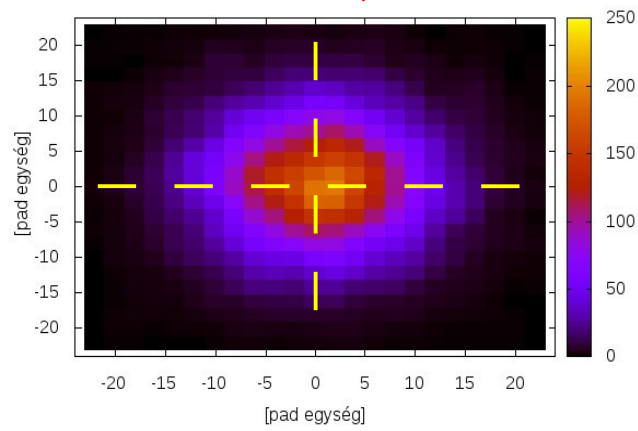
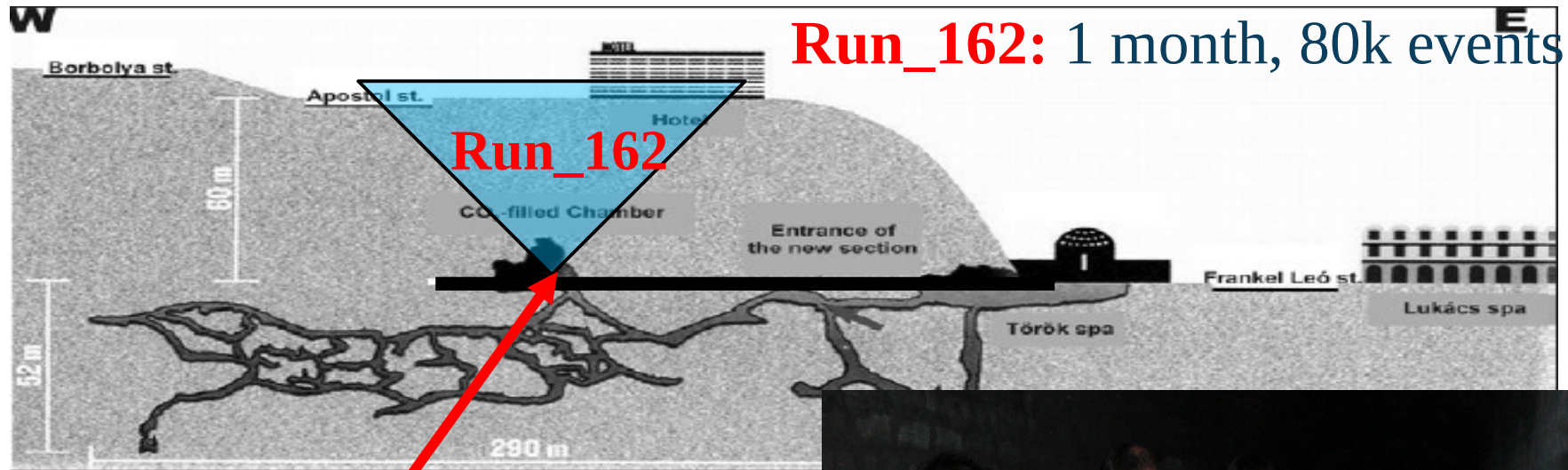
- A thermal cave in the city
 - A young cave: 10,000–100,000 years old
 - 99% of it is underwater
 - fed by several springs.
 - 1860: János Molnár, pharmacist
 - 1977: Hubert Kessler, 180 m
 - Currently: 6,000–7,000 m



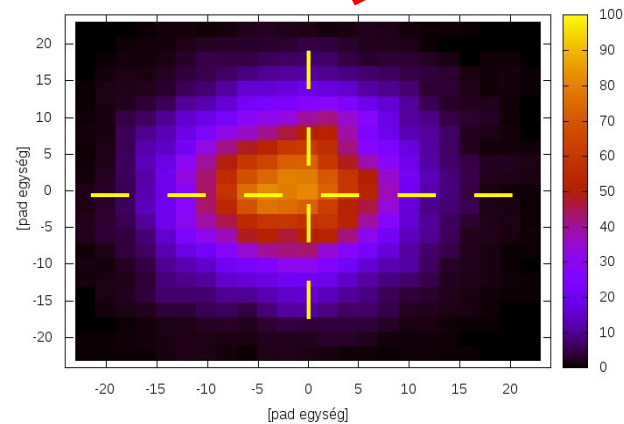
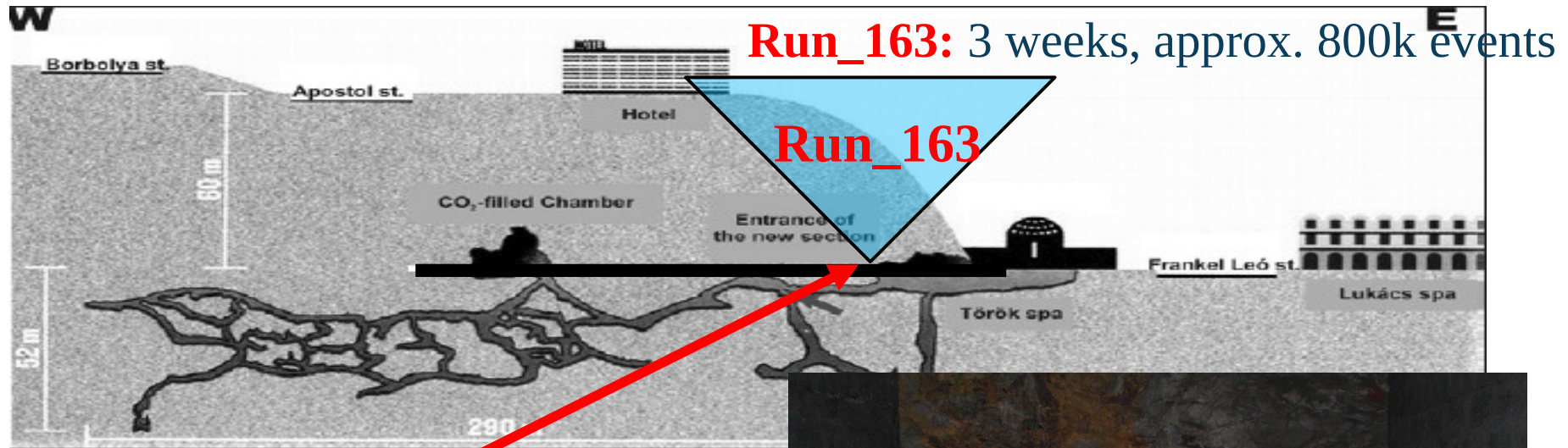
Measurements: in János Molnár cave



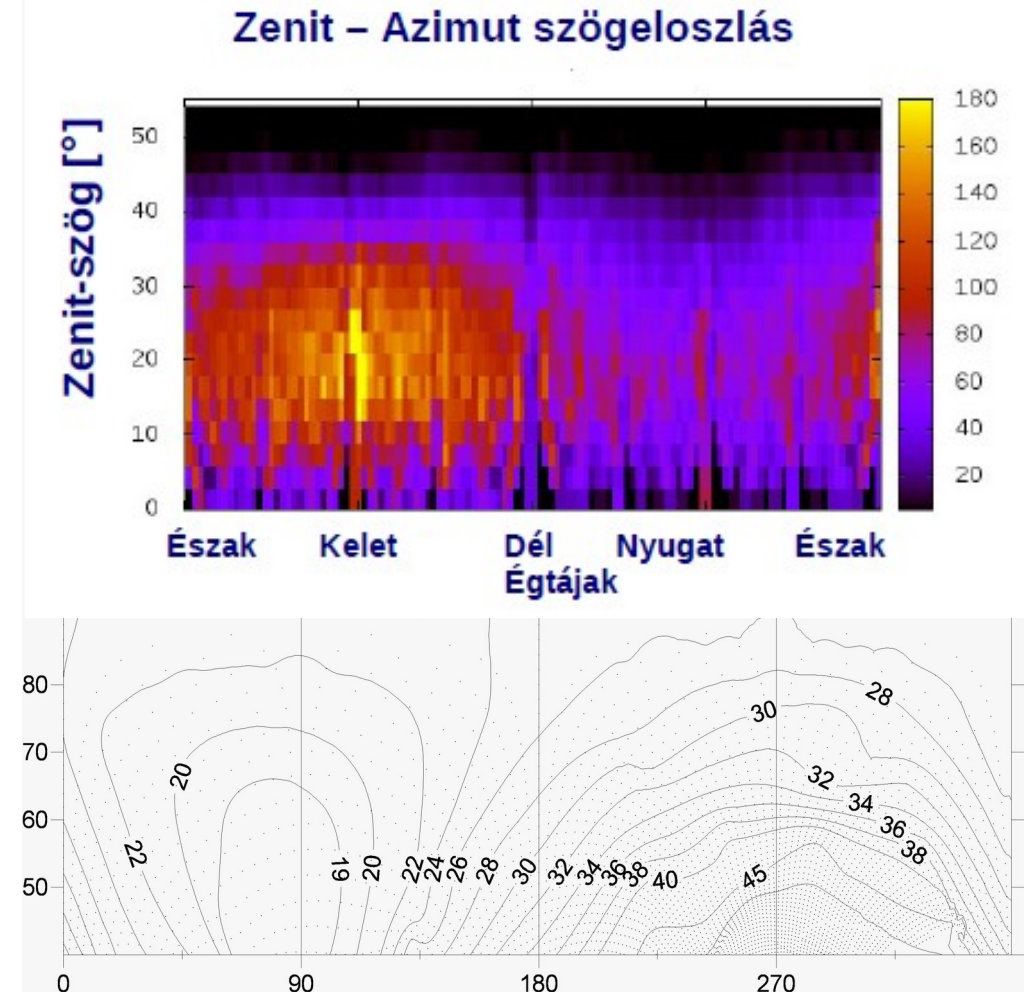
Measurements: in János Molnár cave



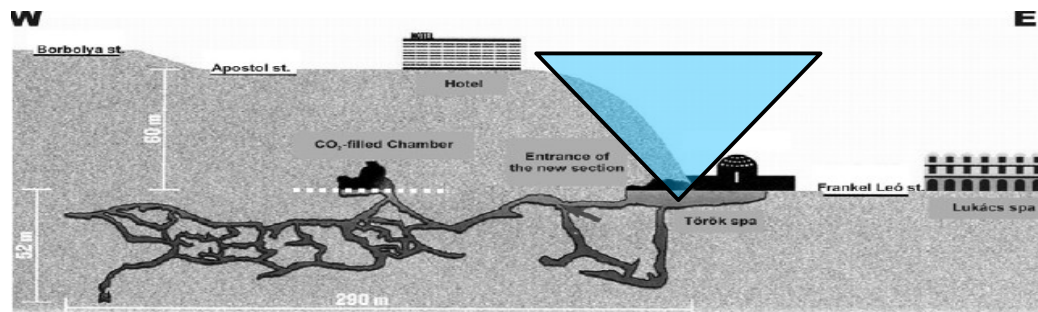
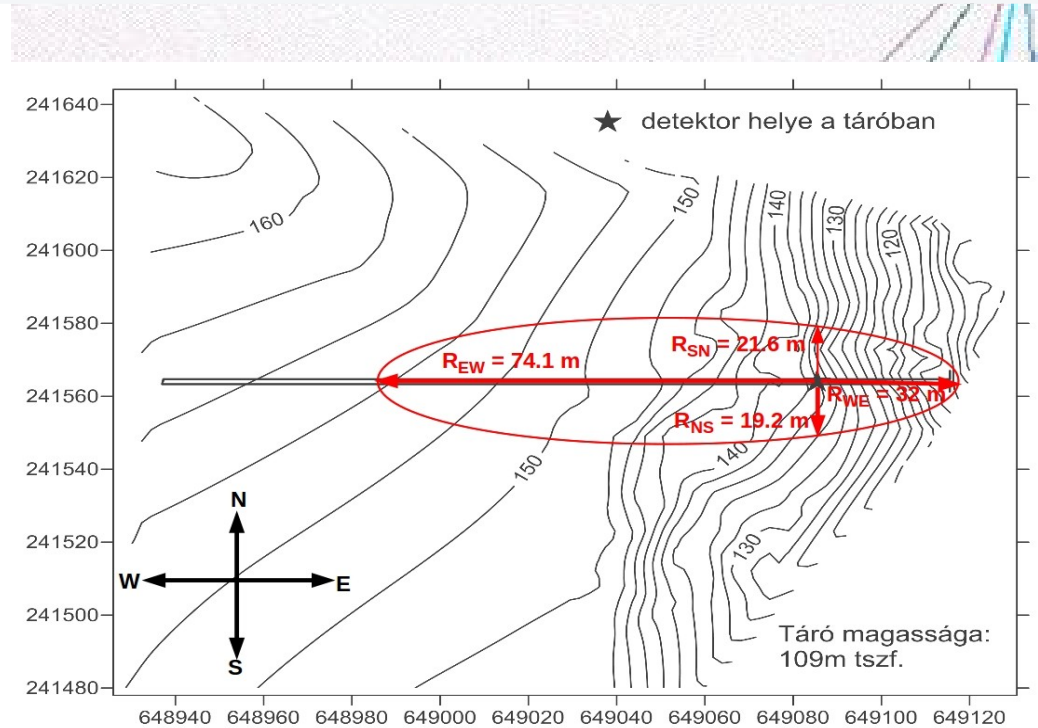
Measurements: in János Molnár cave



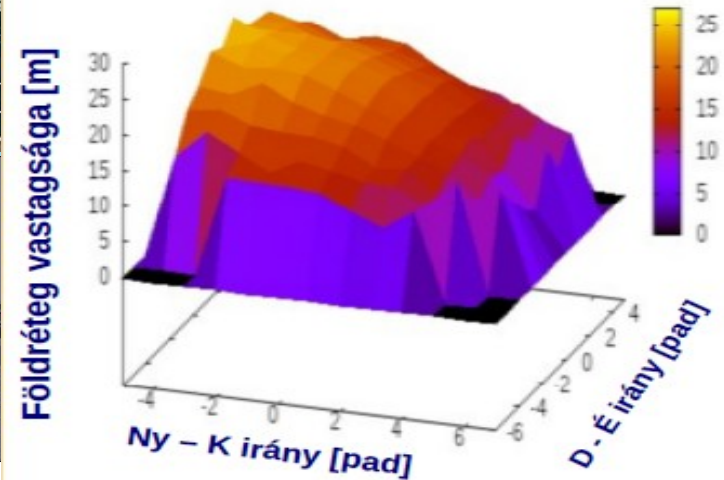
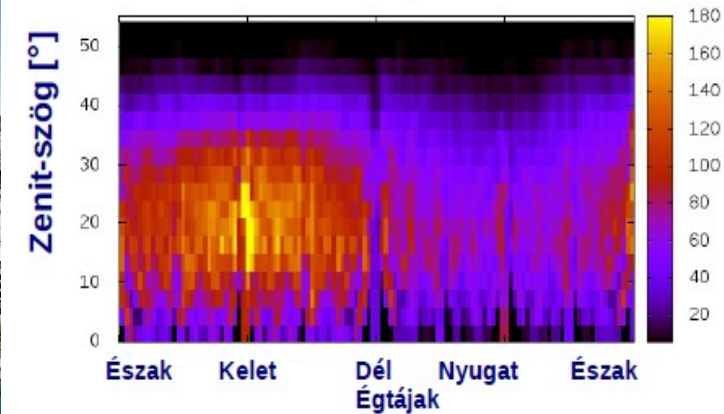
- Mapping (GPS):
 - 3 cm accuracy(!)
 - Our own contour map
 - Rock thickness determined to an accuracy of 0.5–1 m
- Corrected angular distribution
 - Rock thickness is proportional to the attenuation along the given direction
- The similarity to the terrain is visible!



Measurements: in János Molnár cave



Zenit – Azimut szögeloszlás



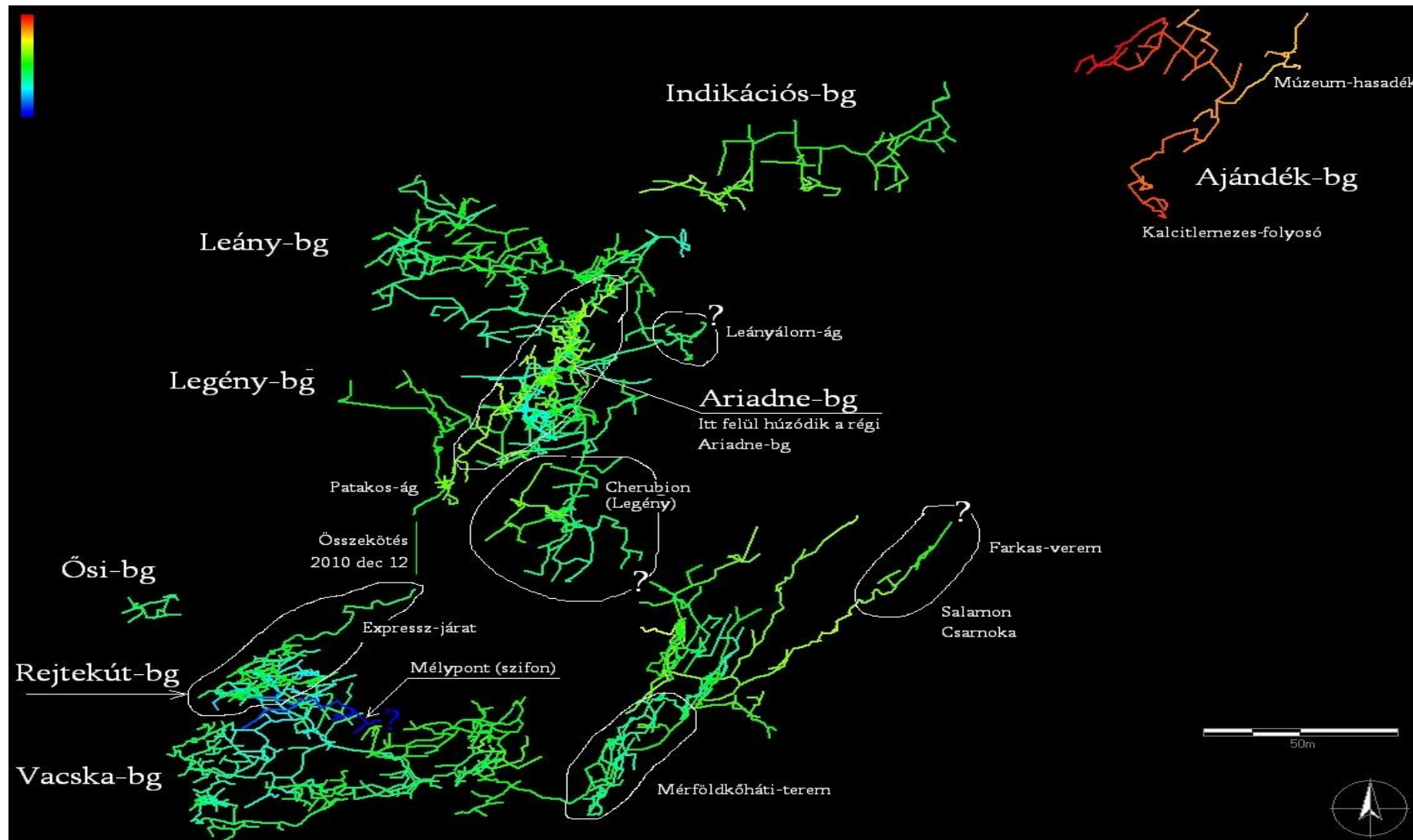
Measurements: in Ajándék cave



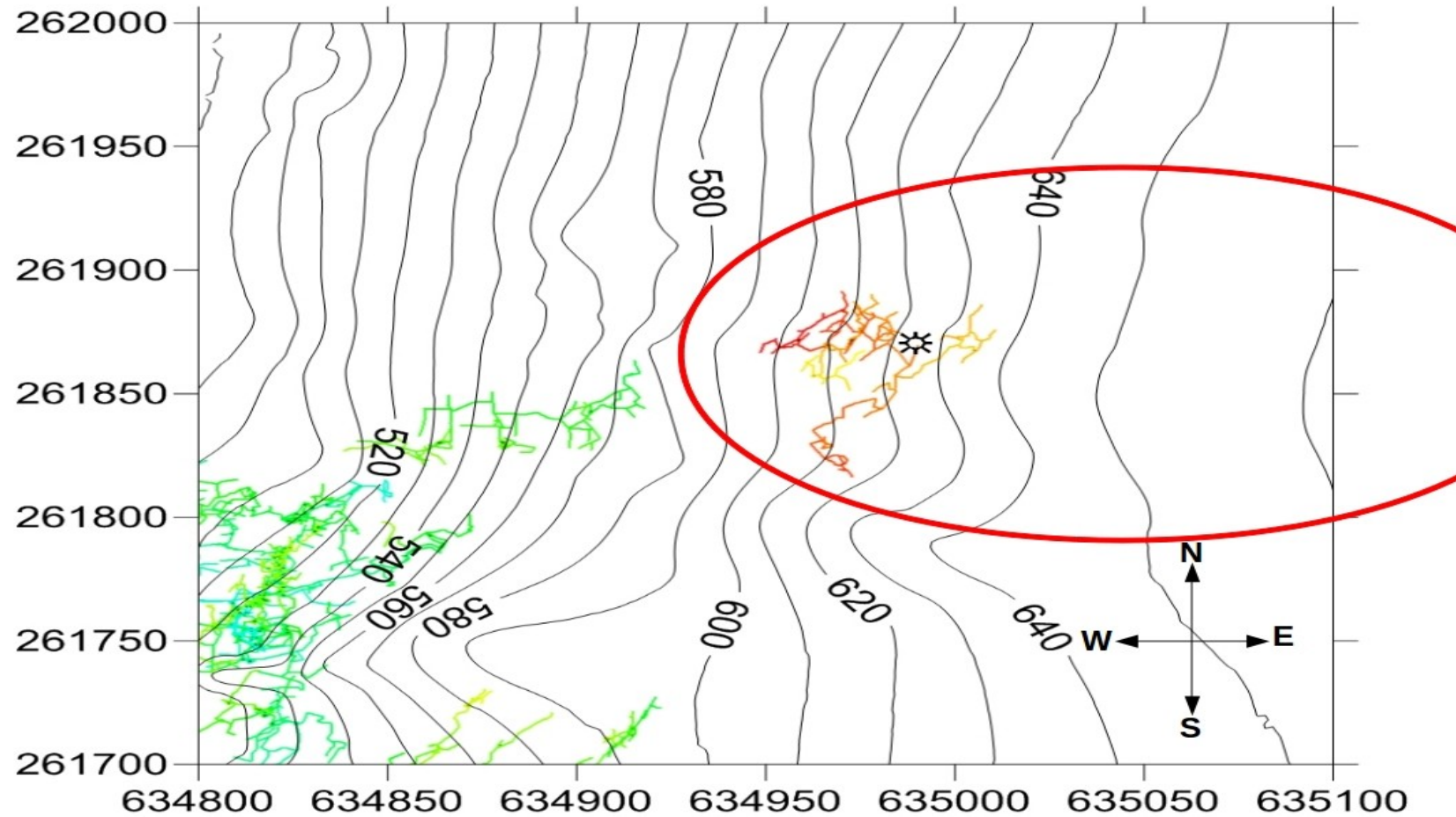
Measurements: in Ajándék cave



Measurements: in Ajándék cave



Measurements: in Ajándék cave

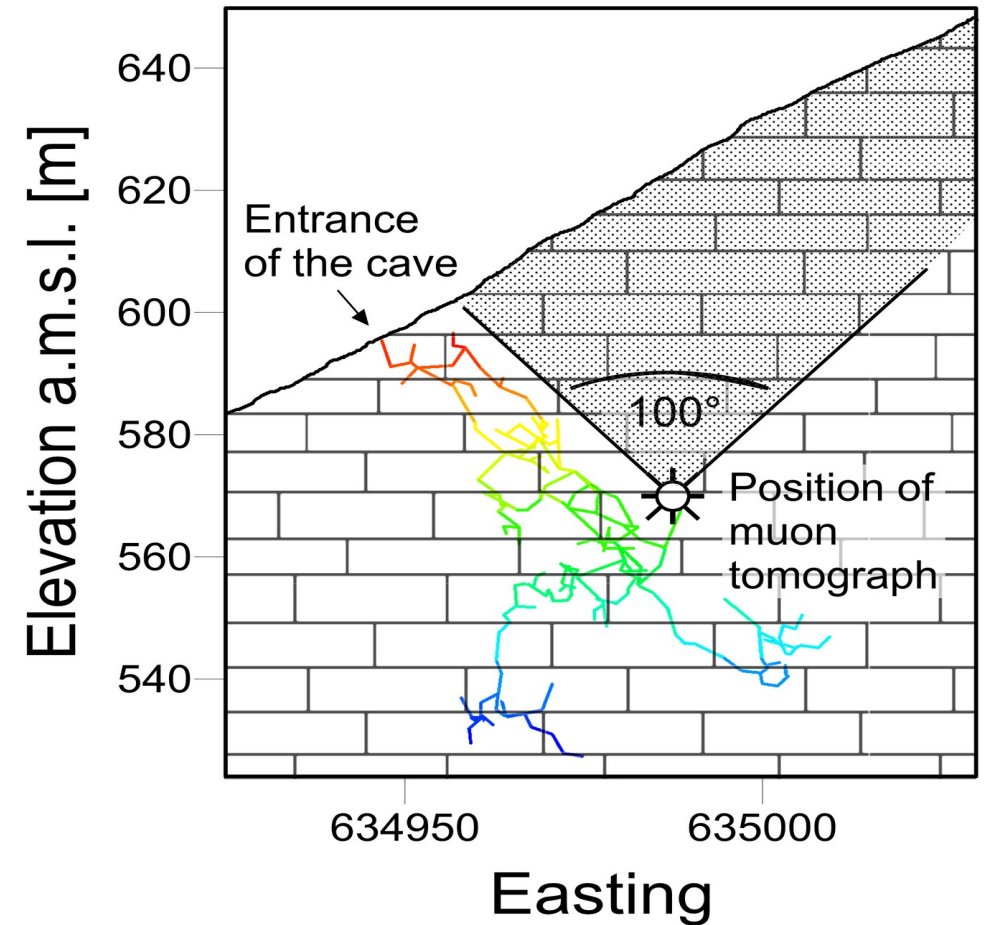


- Ajándék Cave (Ariadné):**

a natural cave system in the Pilis Mountains

Explorations: 1973–2012

Part of one of Hungary's largest cave systems.



Measurements: in Ajándék cave



- Ajándék Cave (Ariadné):**

a natural cave system in the Pilis Mountains

Explorations: 1973–2012

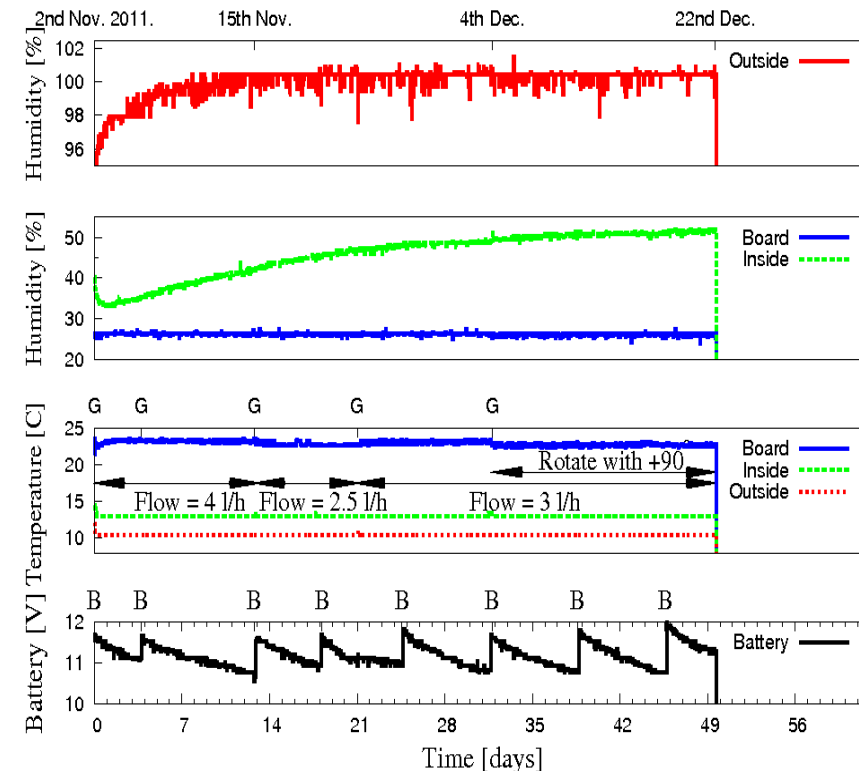
Part of one of Hungary's largest cave systems.

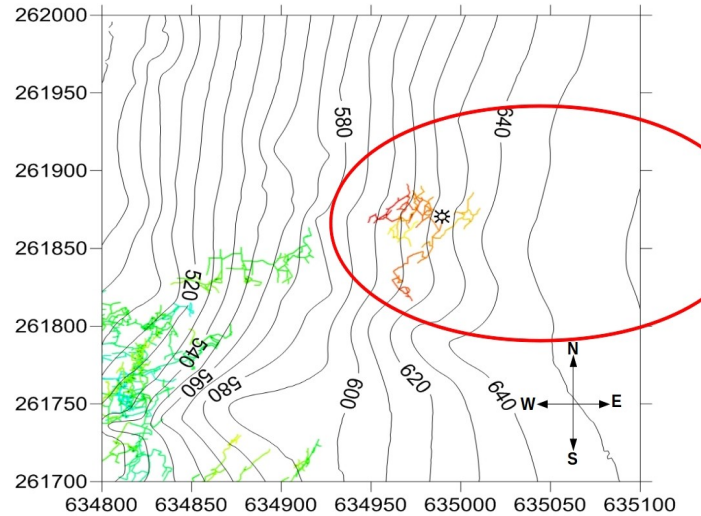
- Measurement:**

We are searching for hidden passages 2–4 m in size

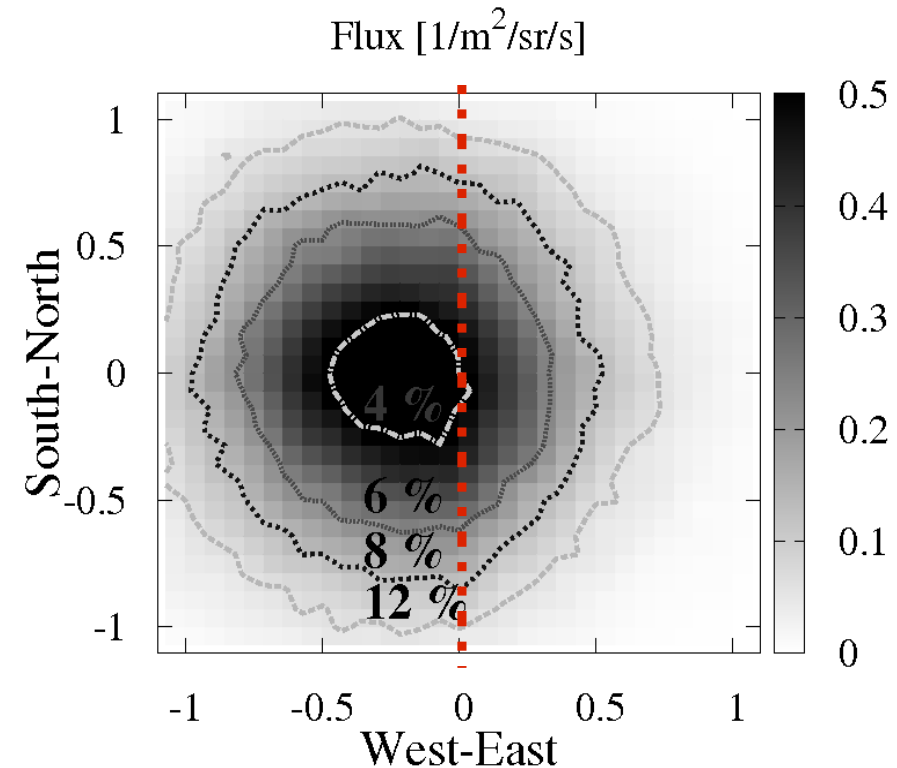
50 days of measurement data (!)

Gas system and batteries at the cave entrance, 100 m of cable

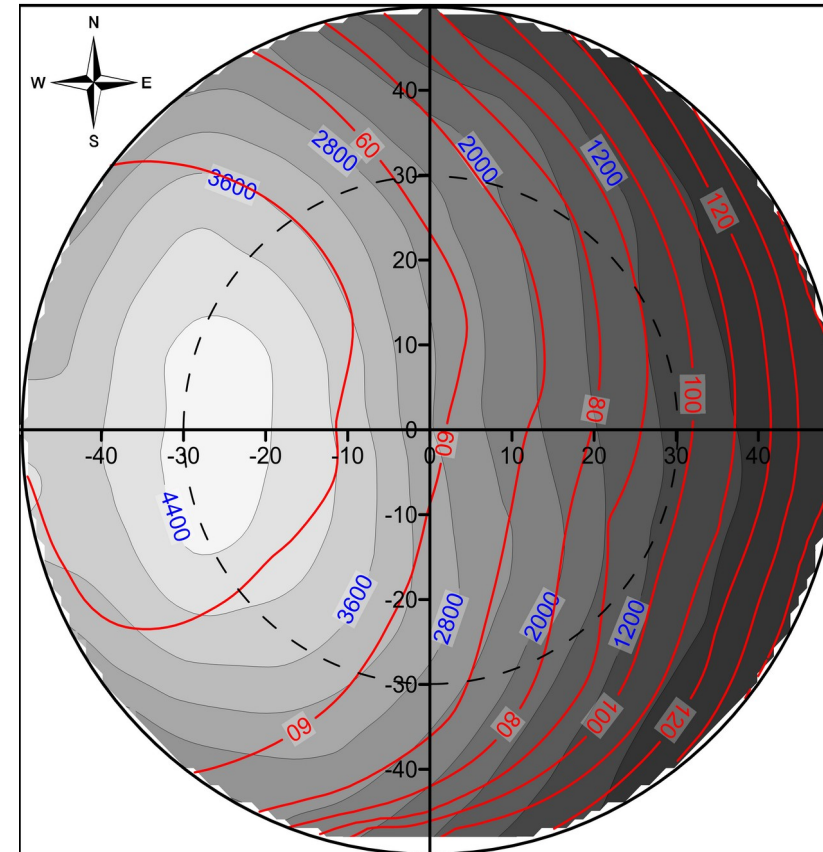




- Muon flux with statistical error per bin
- Muon rate shifts towards the west



- The Pilis hillside from below
 - Red lines:
rock thickness calculated
from GPS
 - Shadowing:
number of detected muons,
indicative of depth



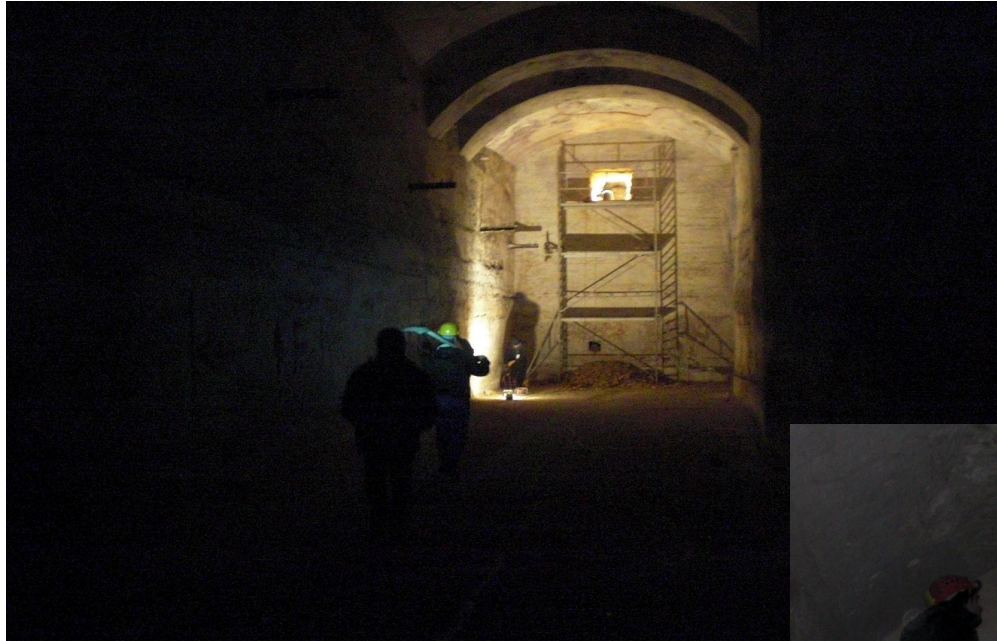
Measurements: in Kőbánya beer cellar



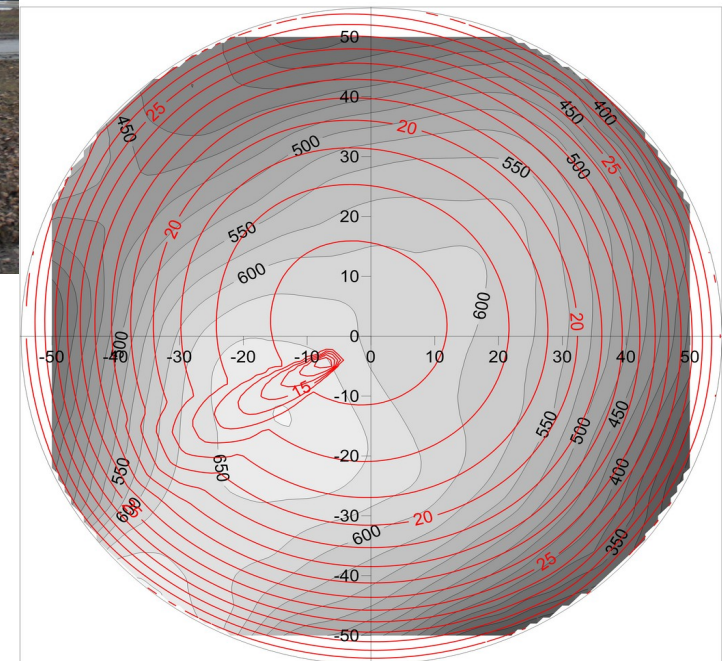
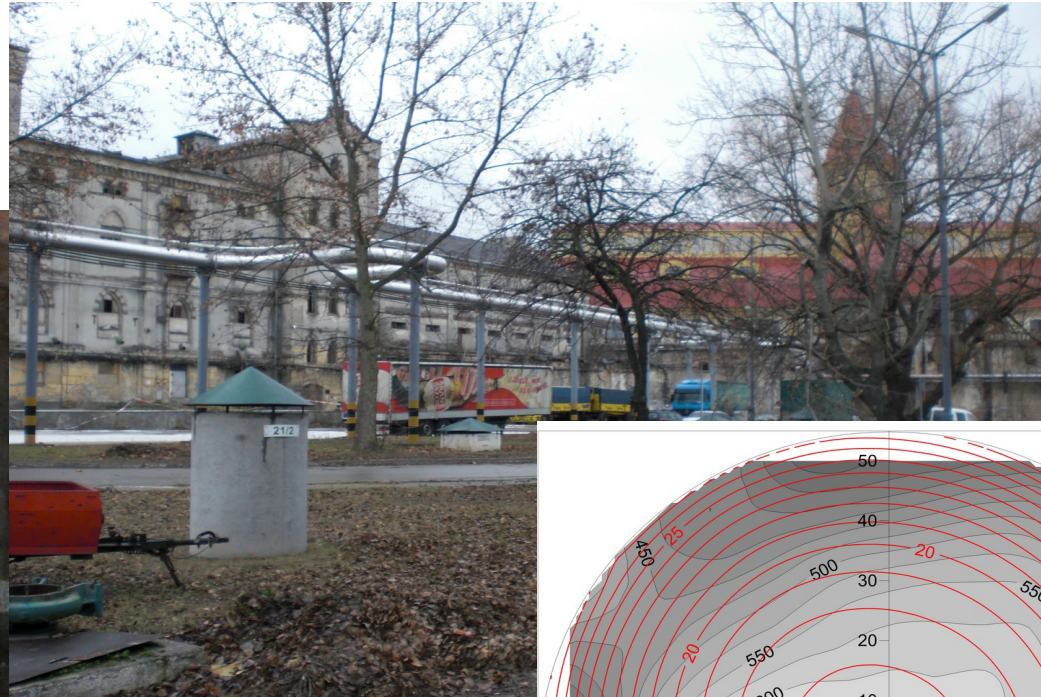
Measurements: in Kőbánya beer cellar



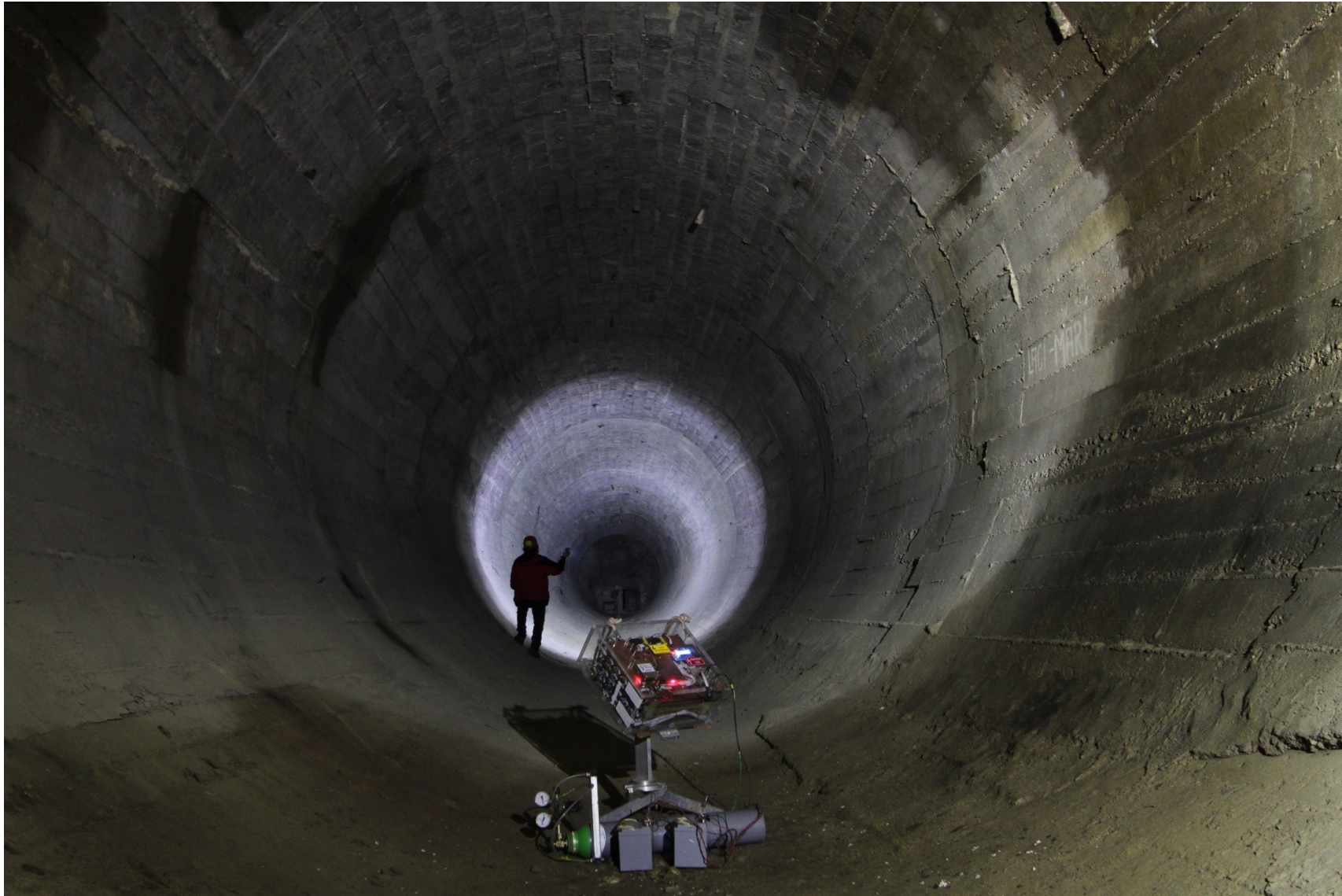
Measurements: in Kőbánya beer cellar



Measurements: in Kőbánya beer cellar



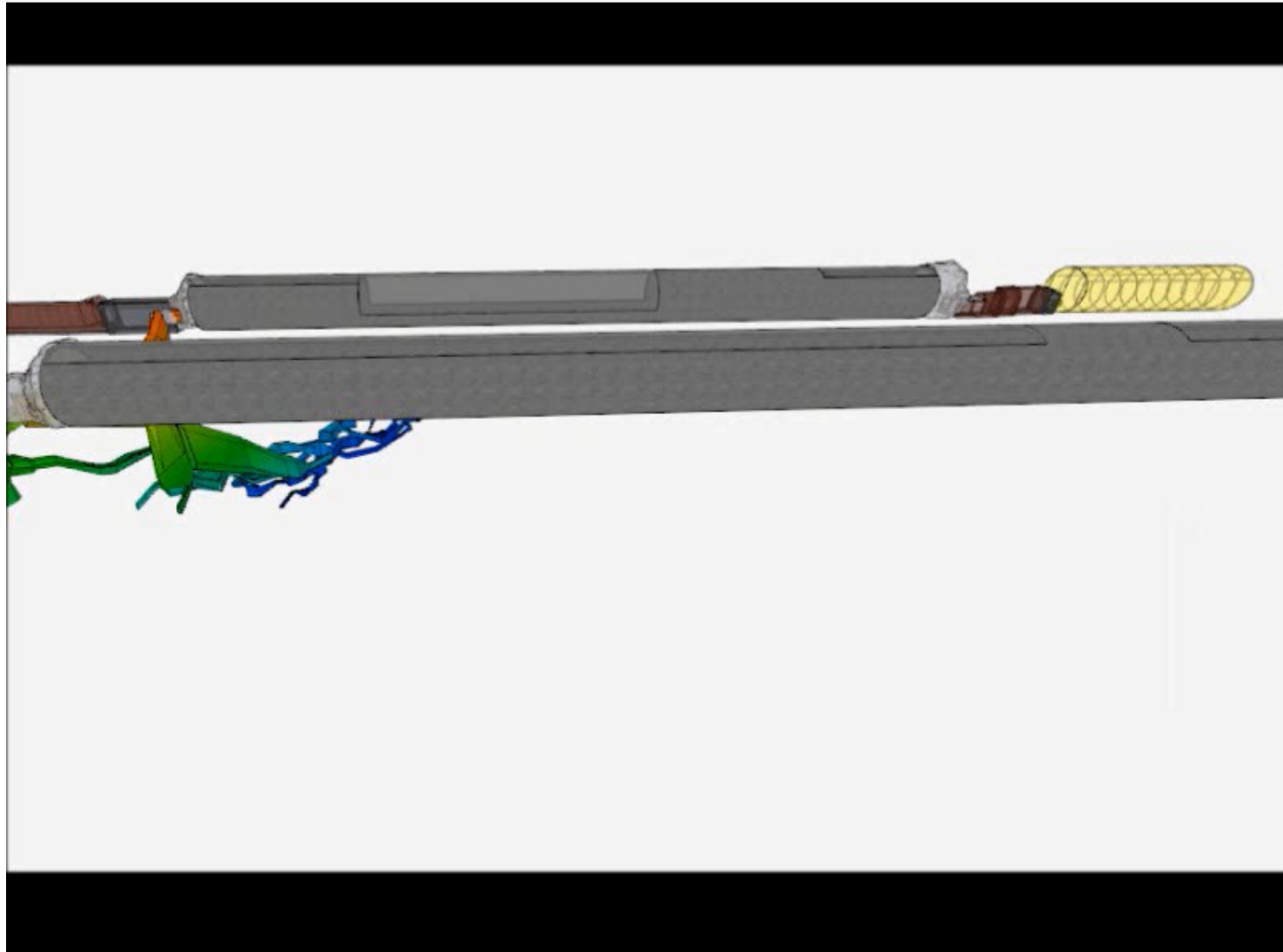
Measurements: Királylaki gas storage



Measurements: Királylaki gas storage



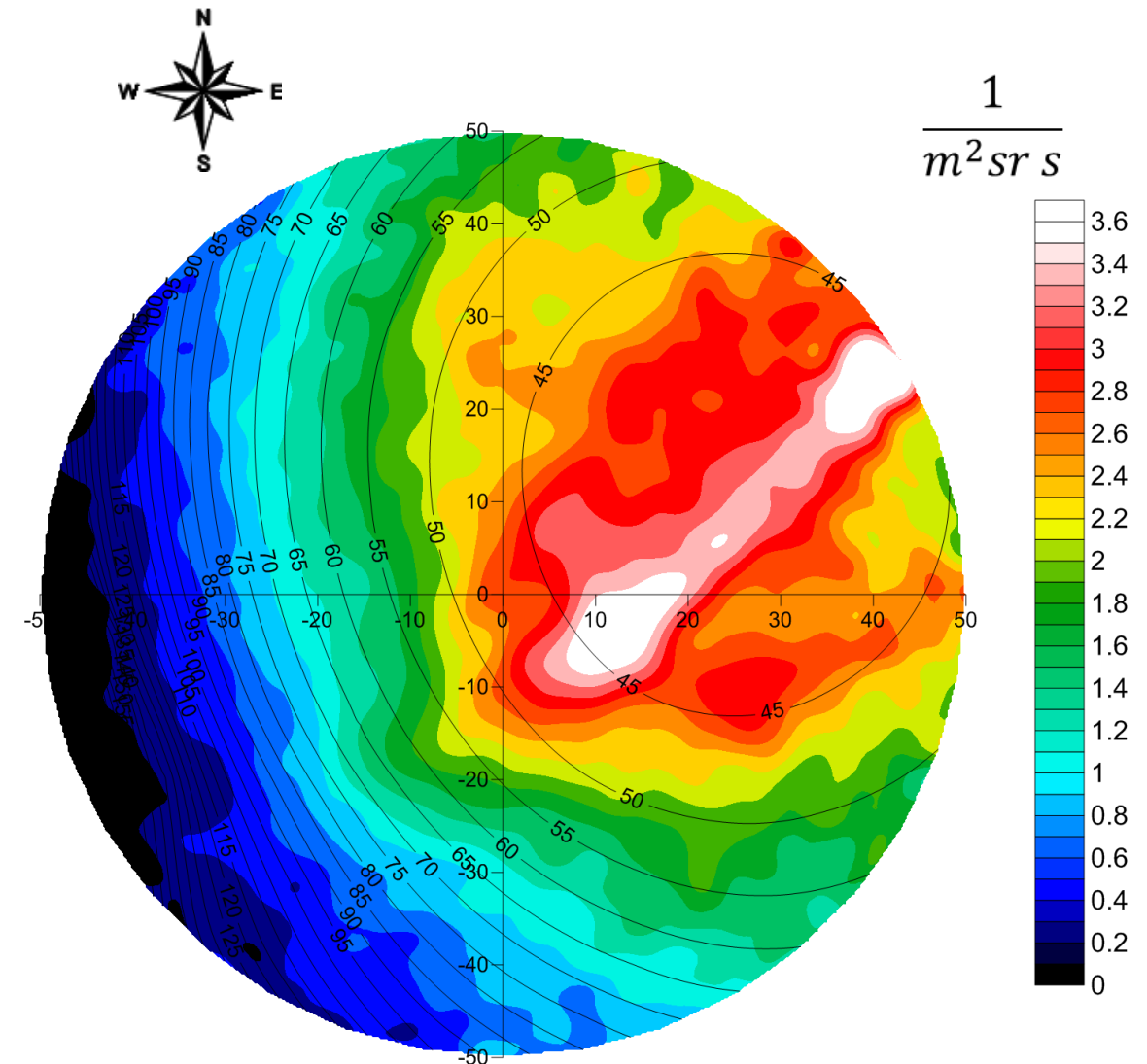
Measurements: Királylaki gas storage



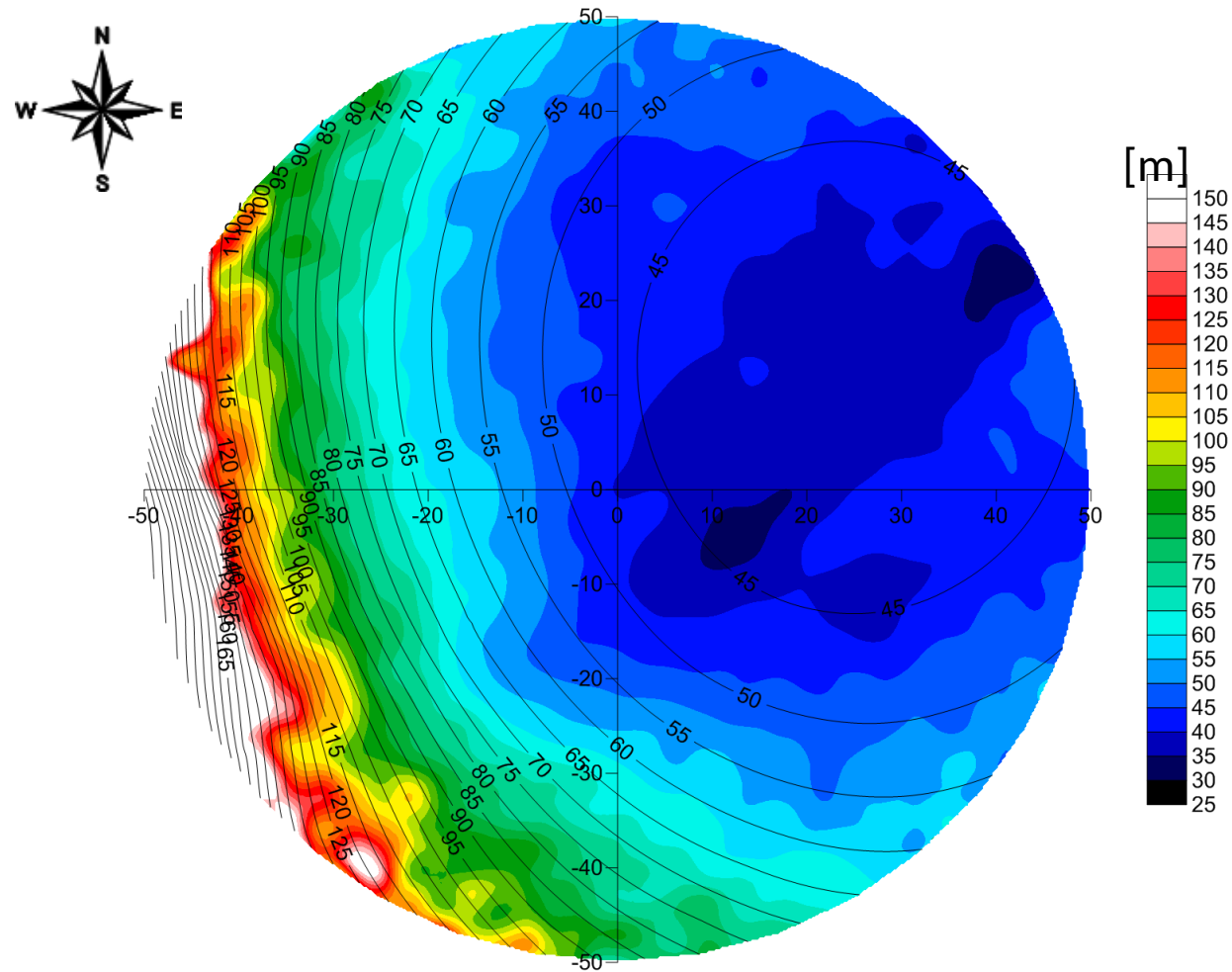
Measurements: Királylaki gas storage



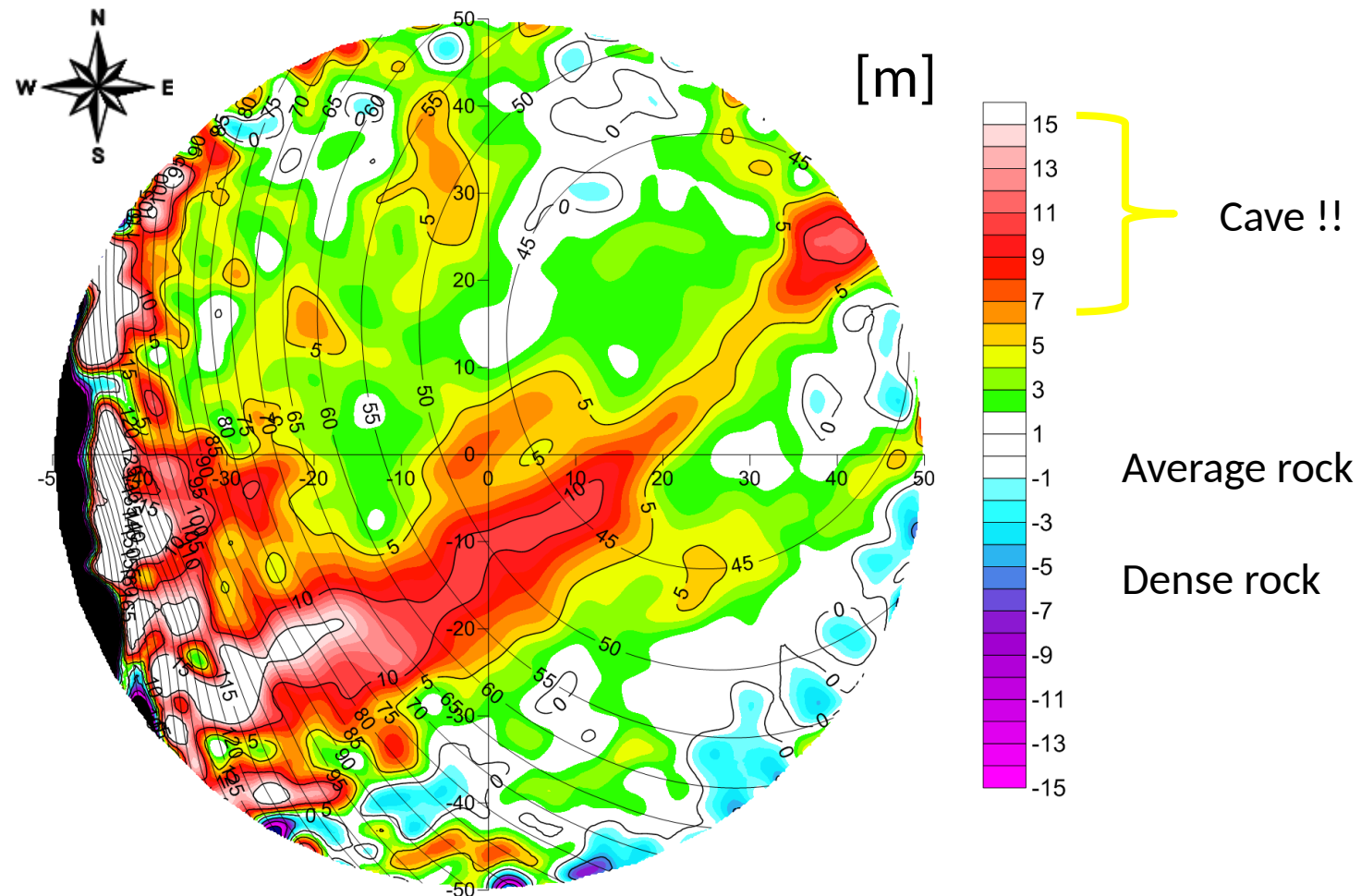
Measurements: Királylaki gas storage



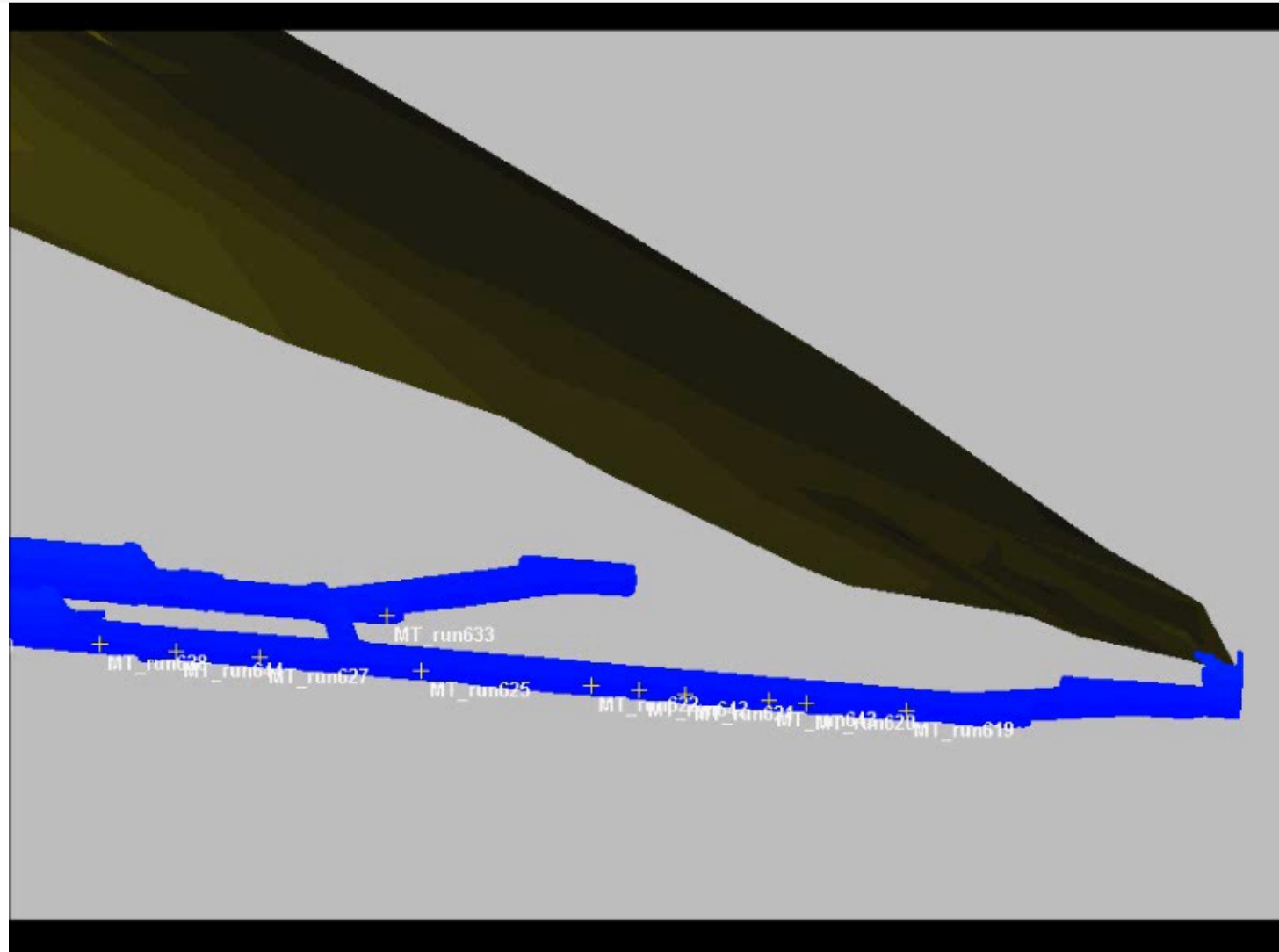
Measurements: Királylaki gas storage



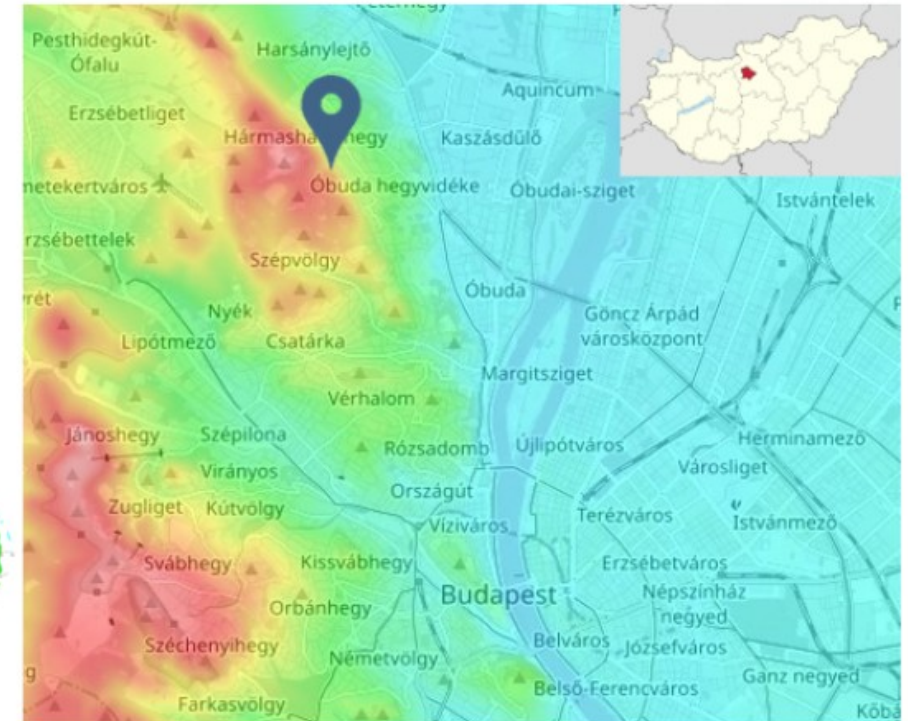
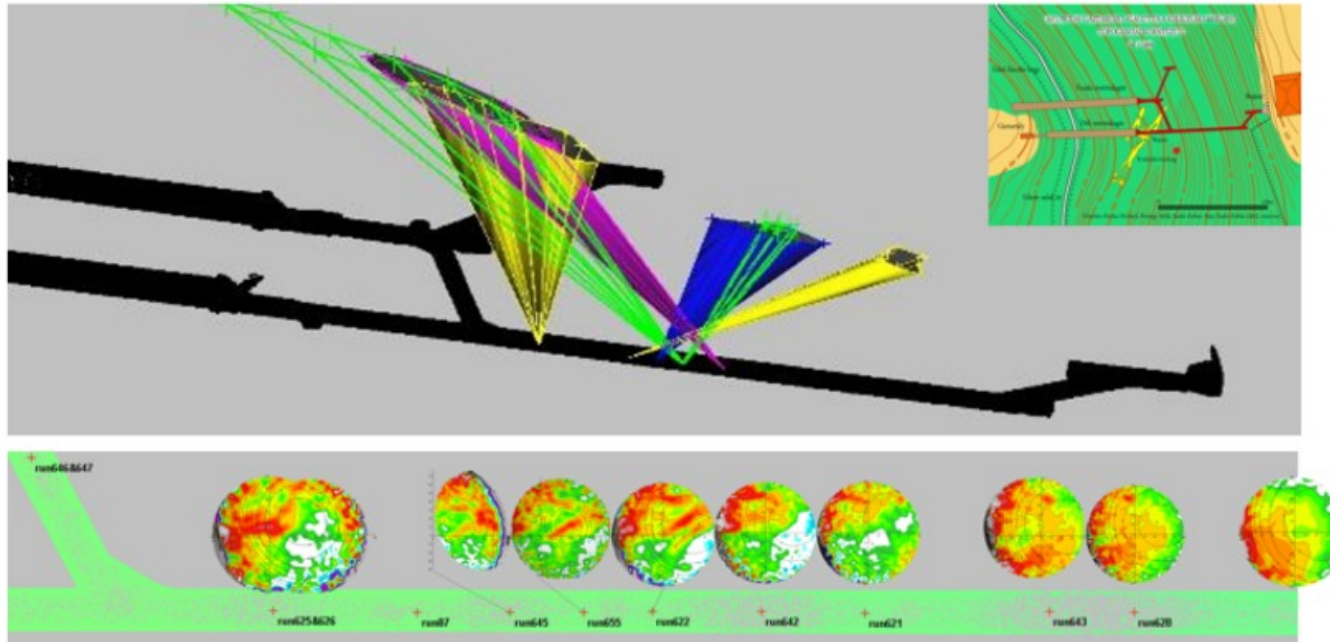
Measurements: Királylaki gas storage



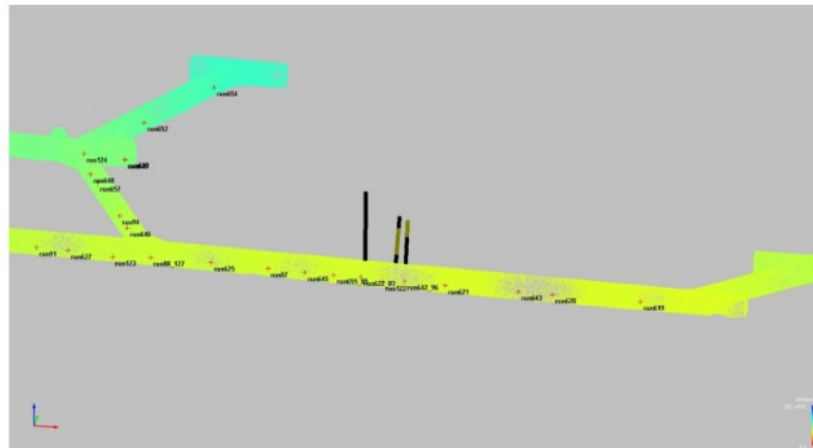
Measurements: Királylaki gas storage



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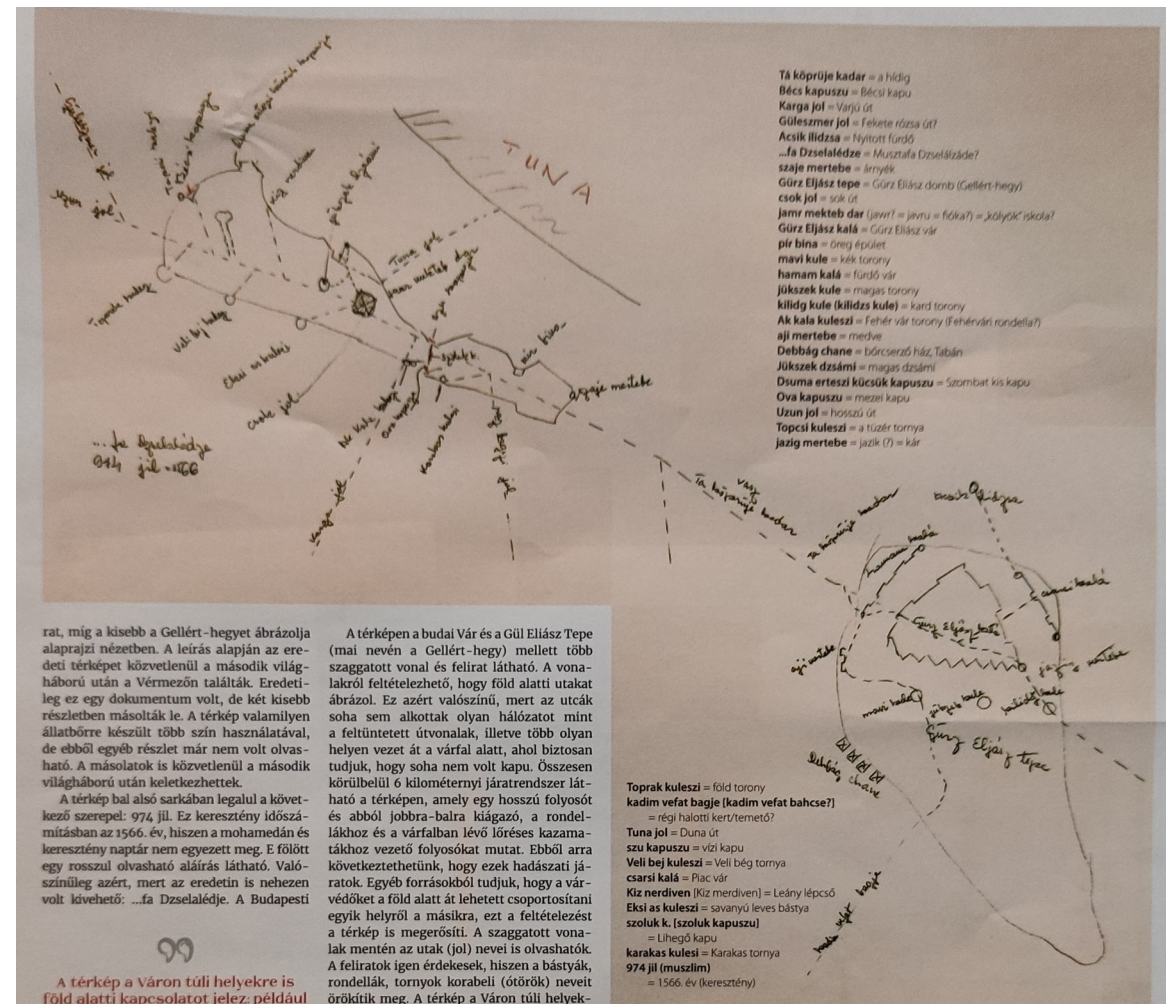
Measurements: Királylaki gas storage



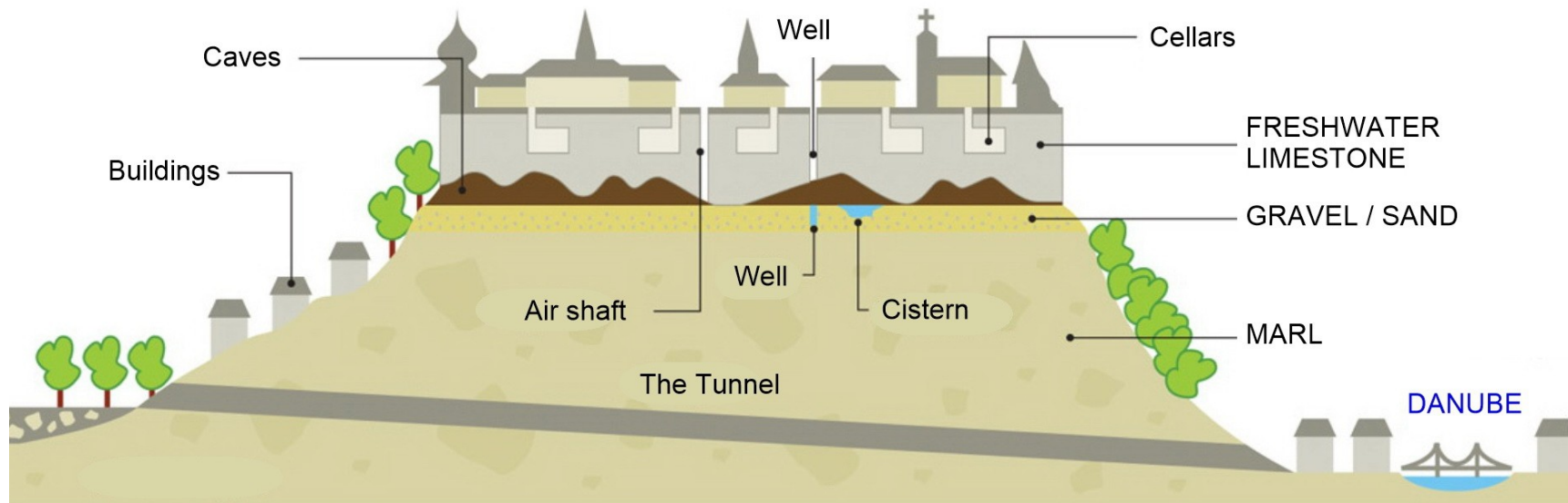
Measurements: Castle of Buda



Measurements: Castle of Buda



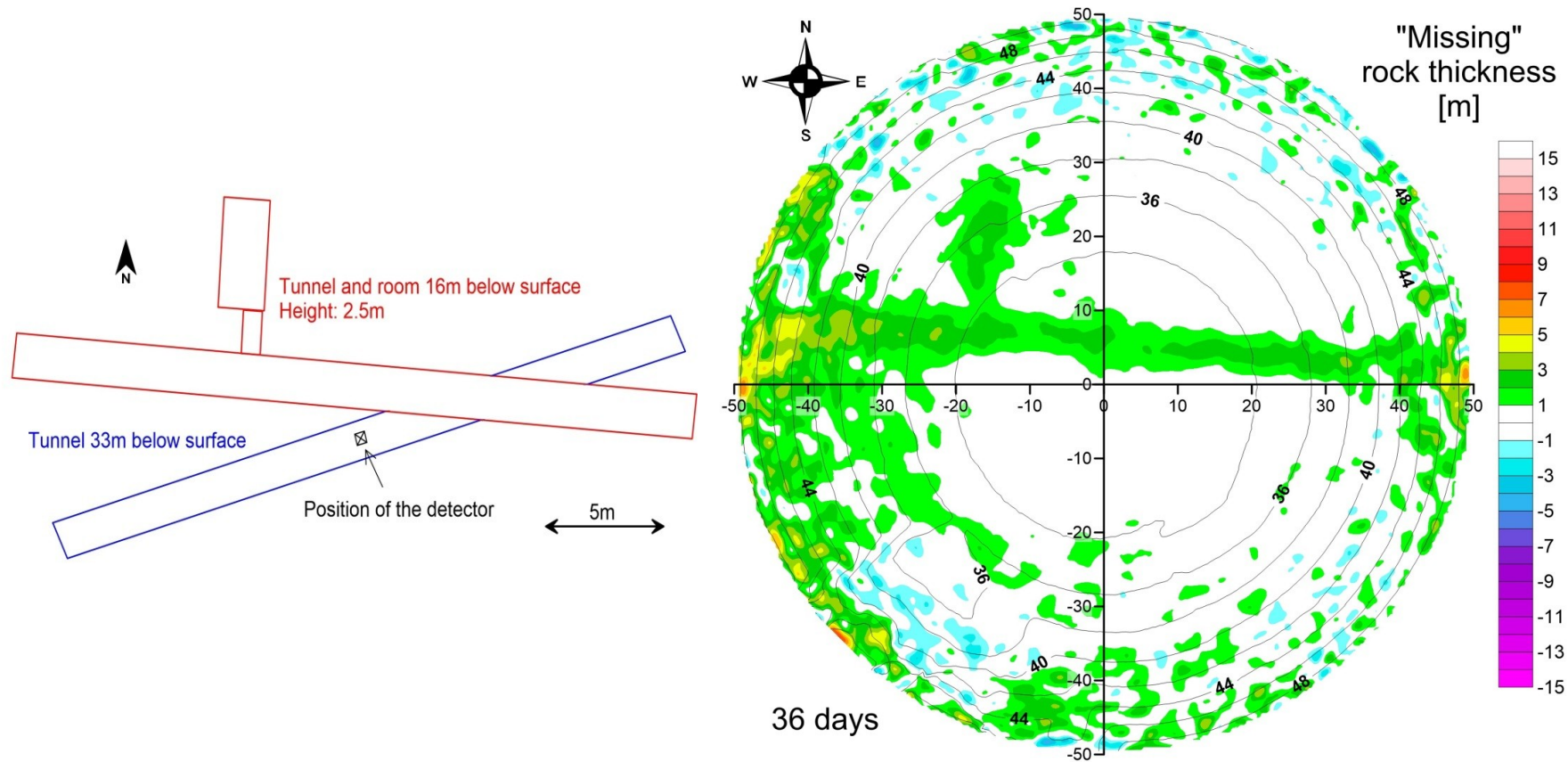
Measurements: Castle of Buda



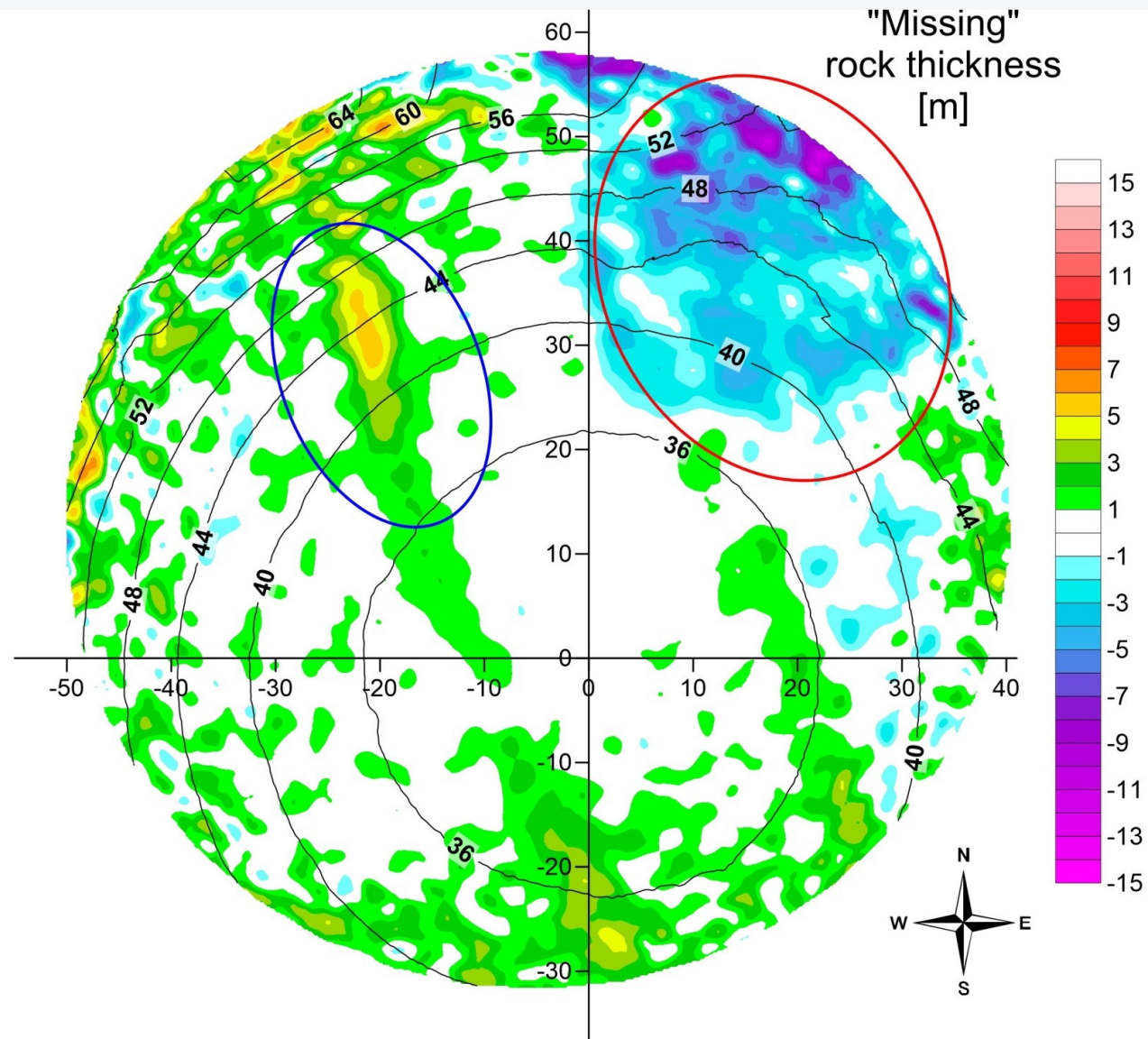
Measurements: Castle of Buda



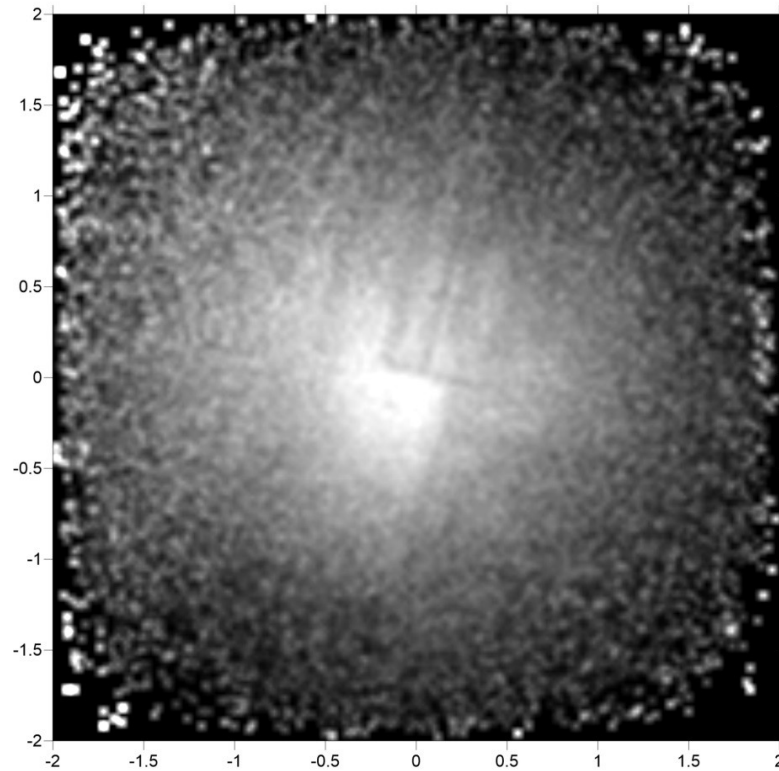
Measurements: Castle of Buda



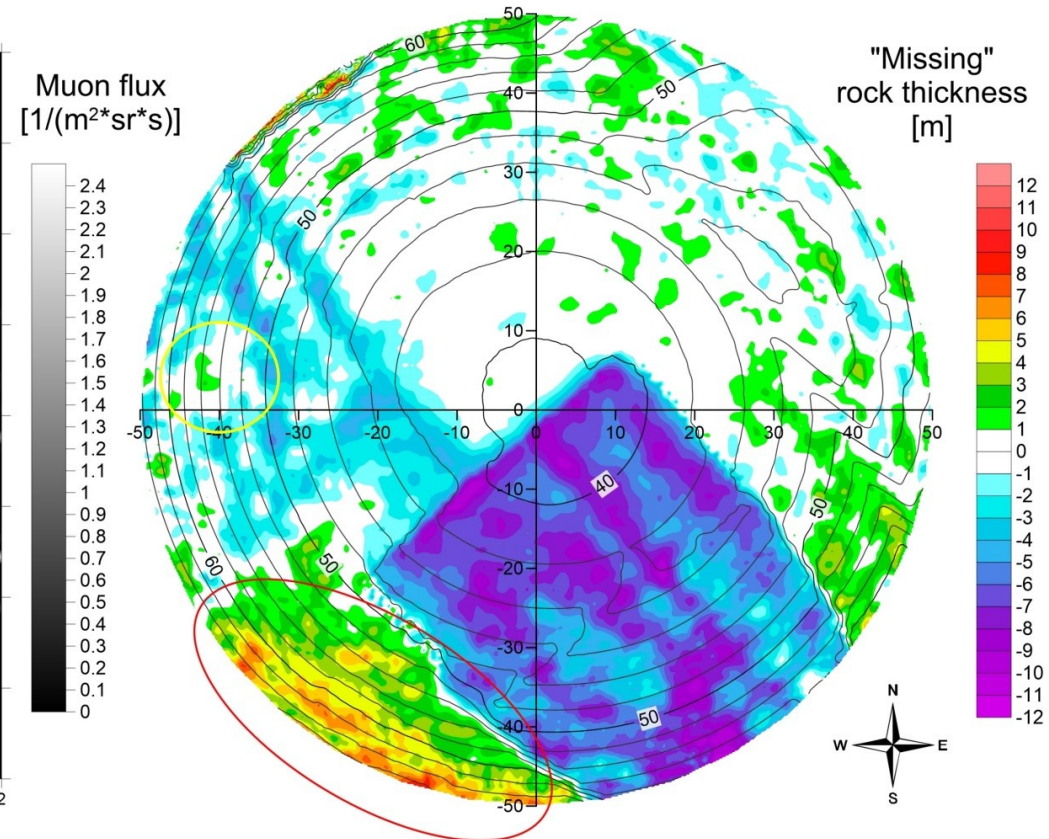
Measurements: Castle of Buda



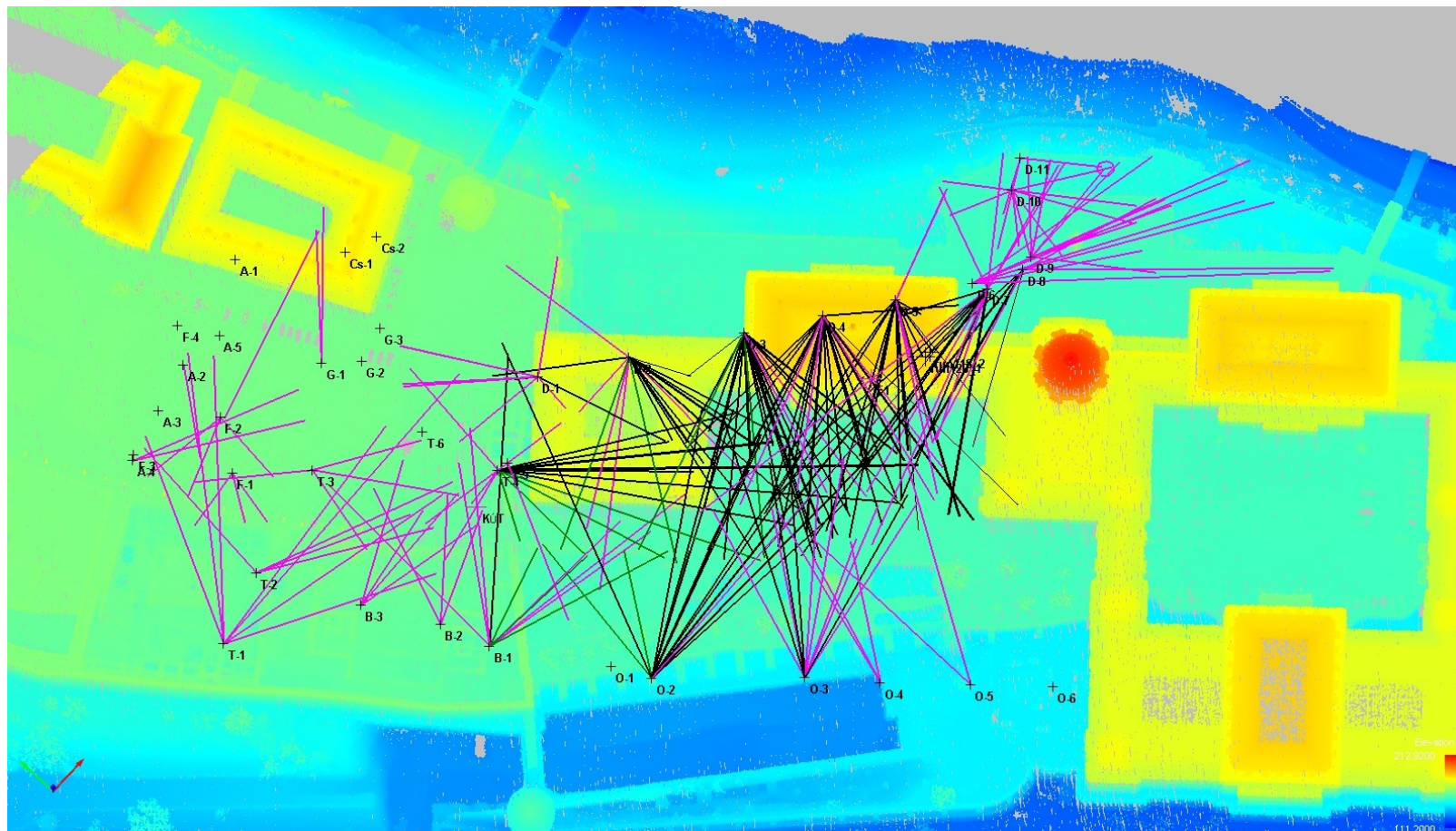
Detector coordinate system



Geographical polar coordinate system

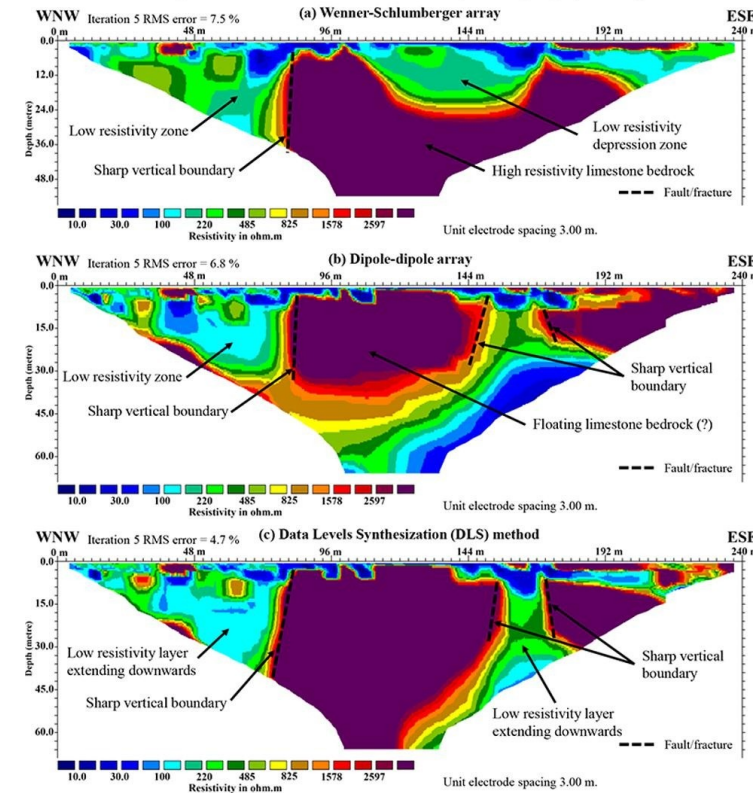


Measurements: Castle of Buda → 3D tomography



Tomography in Geophysics

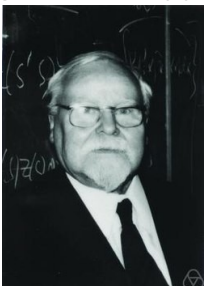
High-resolution electrical resistivity tomography survey



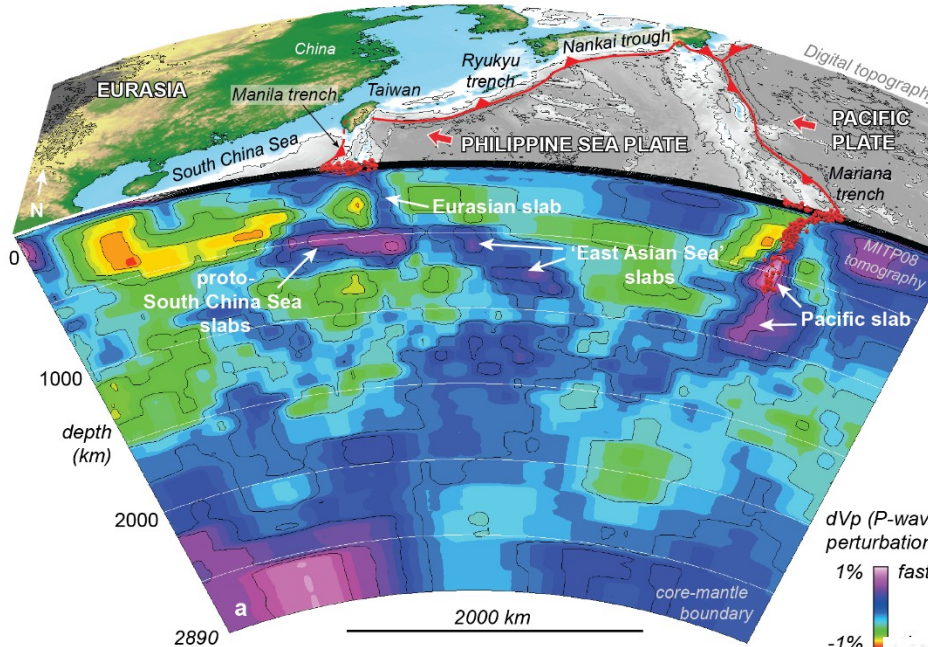
(Lee et al., 2021)

<https://doi.org/10.1016/j.jseas.2021.104880>

ERT



One-sided, underdetermined tomographic problems

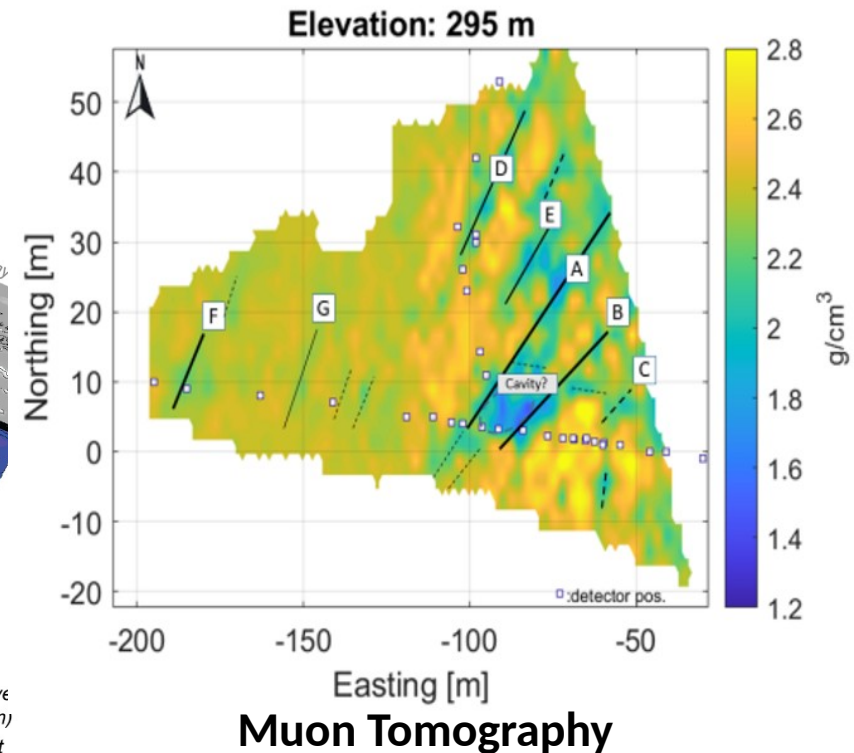


(Li et al., 2008)

<https://doi.org/10.1029/2007GC001806>

Seismic Tomography

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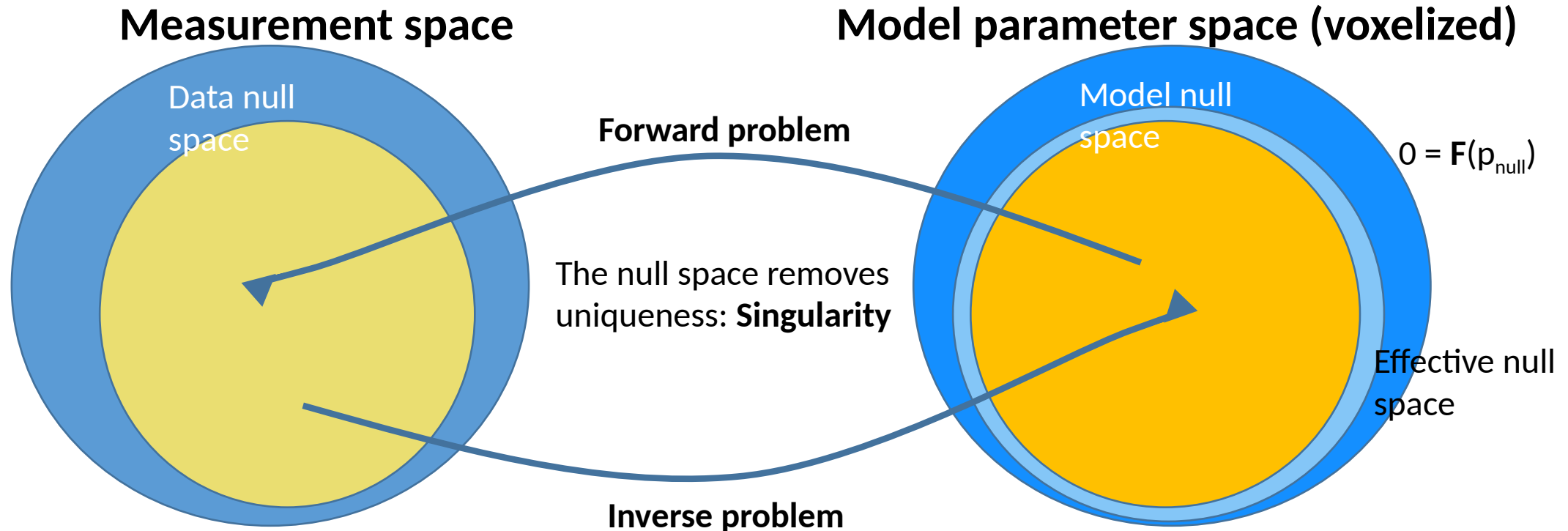


Muon Tomography



Strongly underdetermined inverse problem...

REGULARIZATION , SVD, EM, stopped Landweber iteration, etc.



The regularized inversion's efficiency and bias are determined by the **noise** and the **structure and symmetries of the null space** (see below).

The symmetries of the null space arise from the nature of the mapping (Jacobian matrix symmetries)

3D: Limited-Angle Tomographic Inversion

Regularized inversion works....

Estimator derived from an L2-L2 Maximum Likelihood + Bayesian-type functional

$$\hat{\rho} = \left(\mathbf{R} + \mathbf{C}_B^{-1} \right)^{-1} \left(\mathbf{F}^T \mathbf{C}_y^{-1} \mathbf{y} + \mathbf{C}_B^{-1} \rho_B \right)$$

Weight of
measurement
information

Weight of prior
information

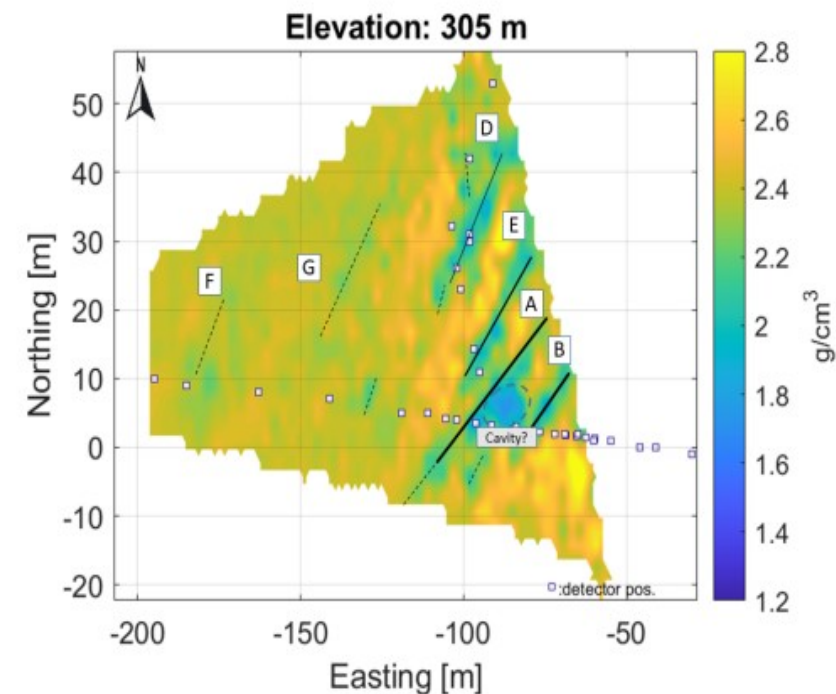
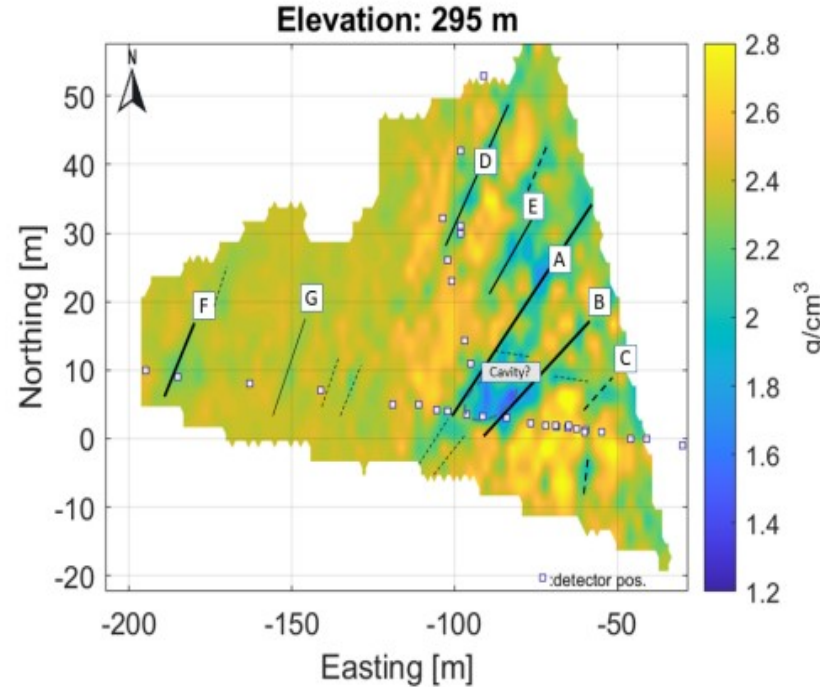
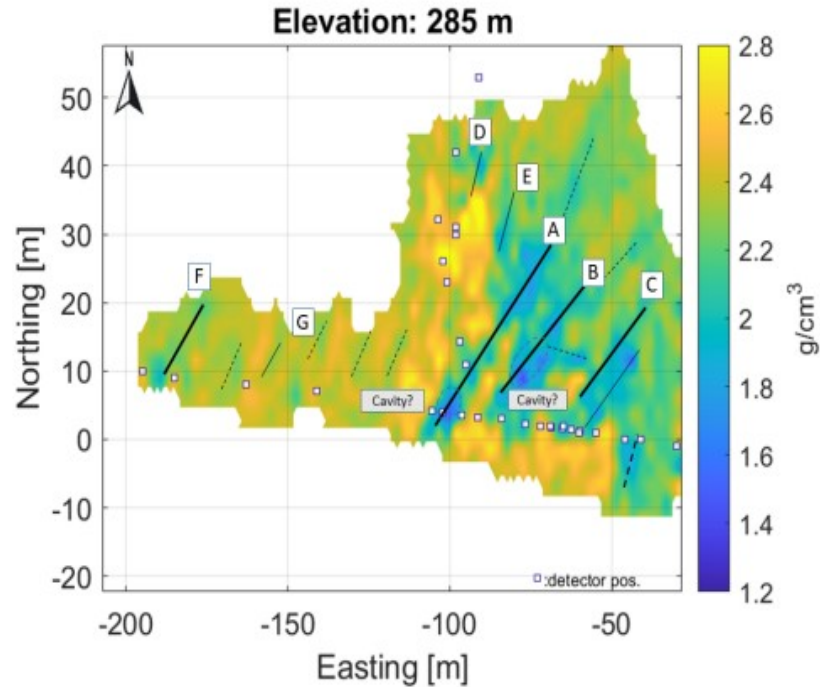
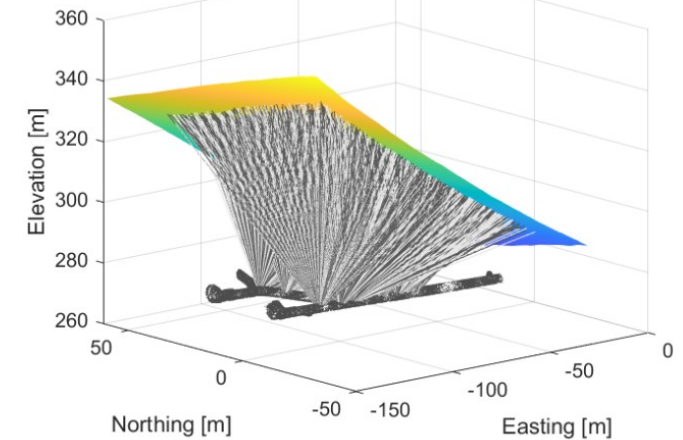
measurement

Prior
distribution

$$\mathbf{R} = \mathbf{F}^T \mathbf{C}_y^{-1} \mathbf{F}$$

Fisher matrix

15,000,000 muon events, 46 measurement points



3D: Limited-Angle Tomographic Inversion

Inversion in a built environment:

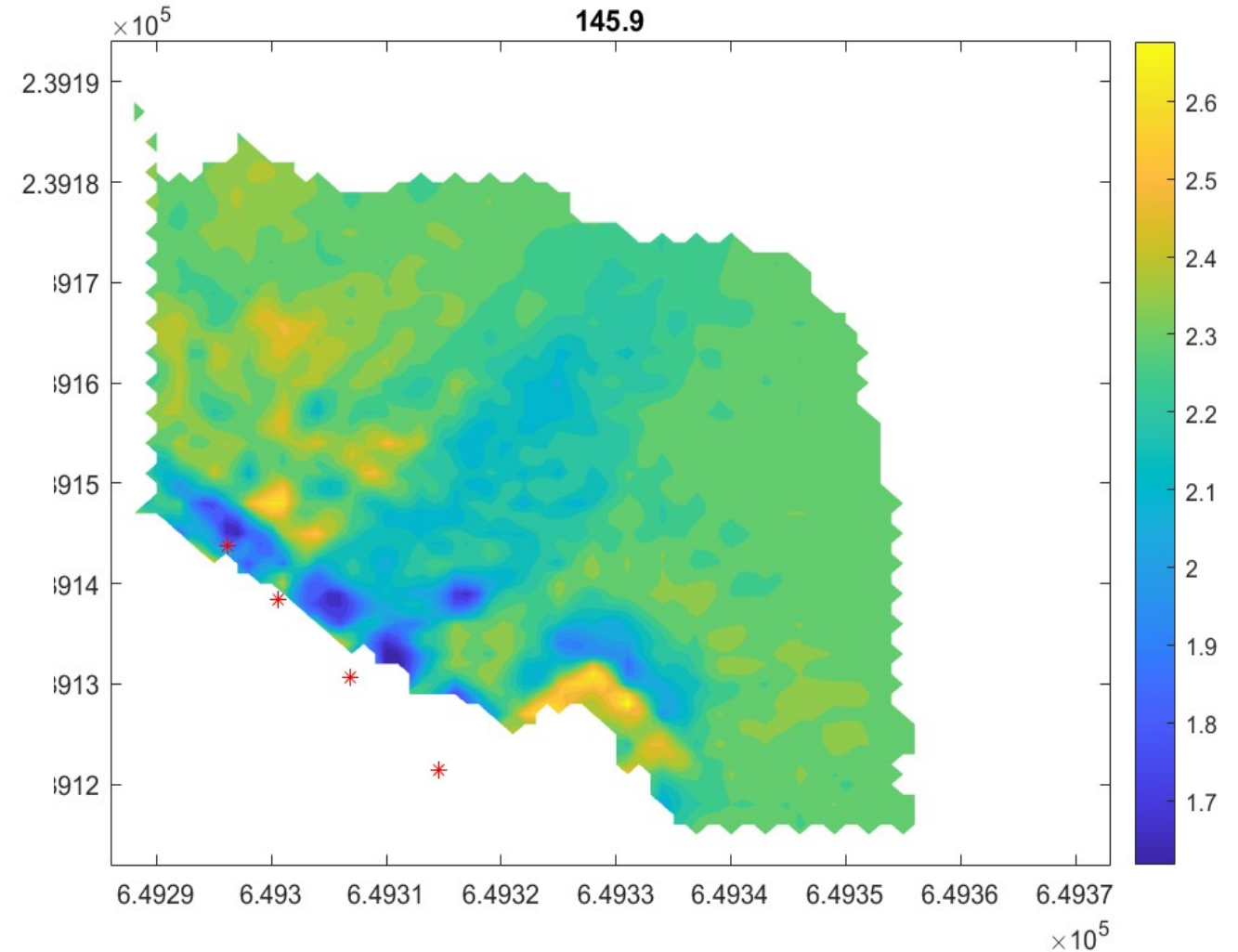
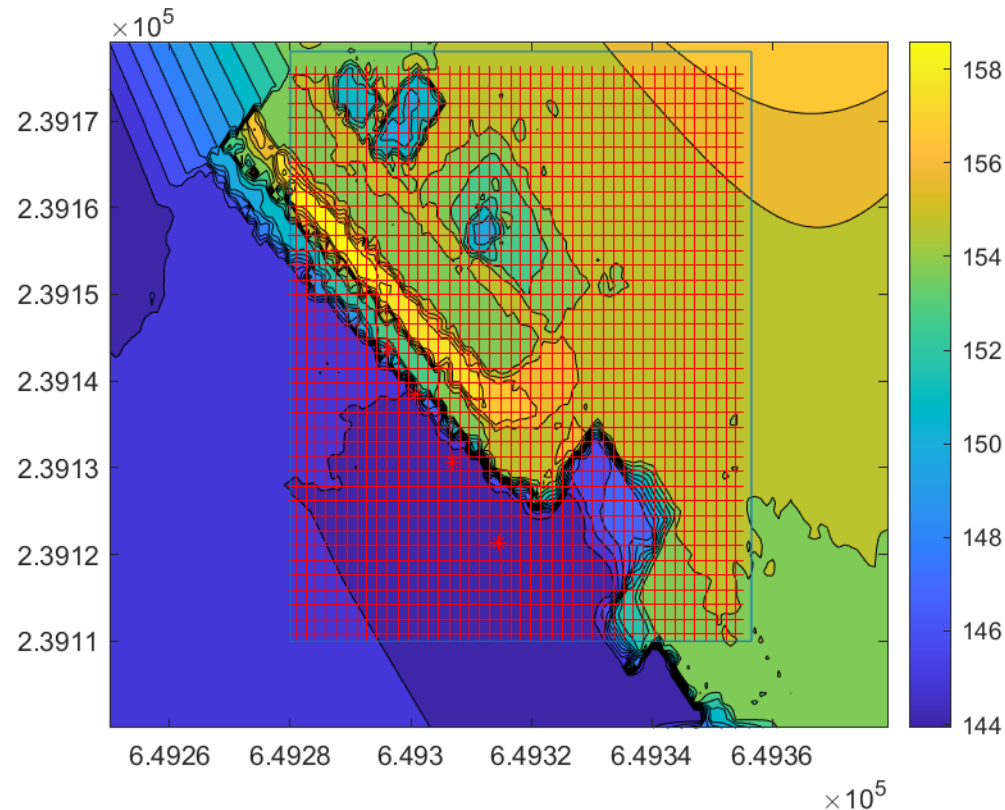
3D reconstruction from only 4 (!) detector positions

Scanned thickness approx. 40 m

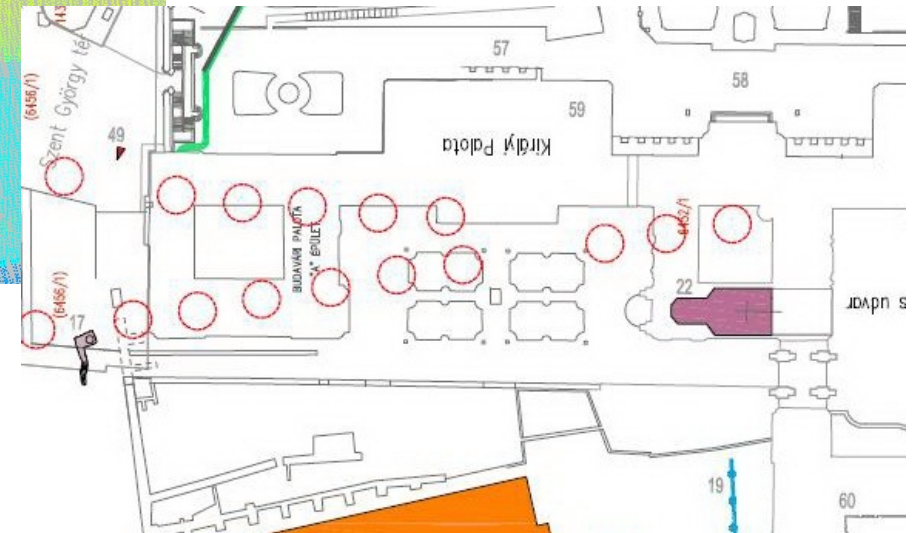
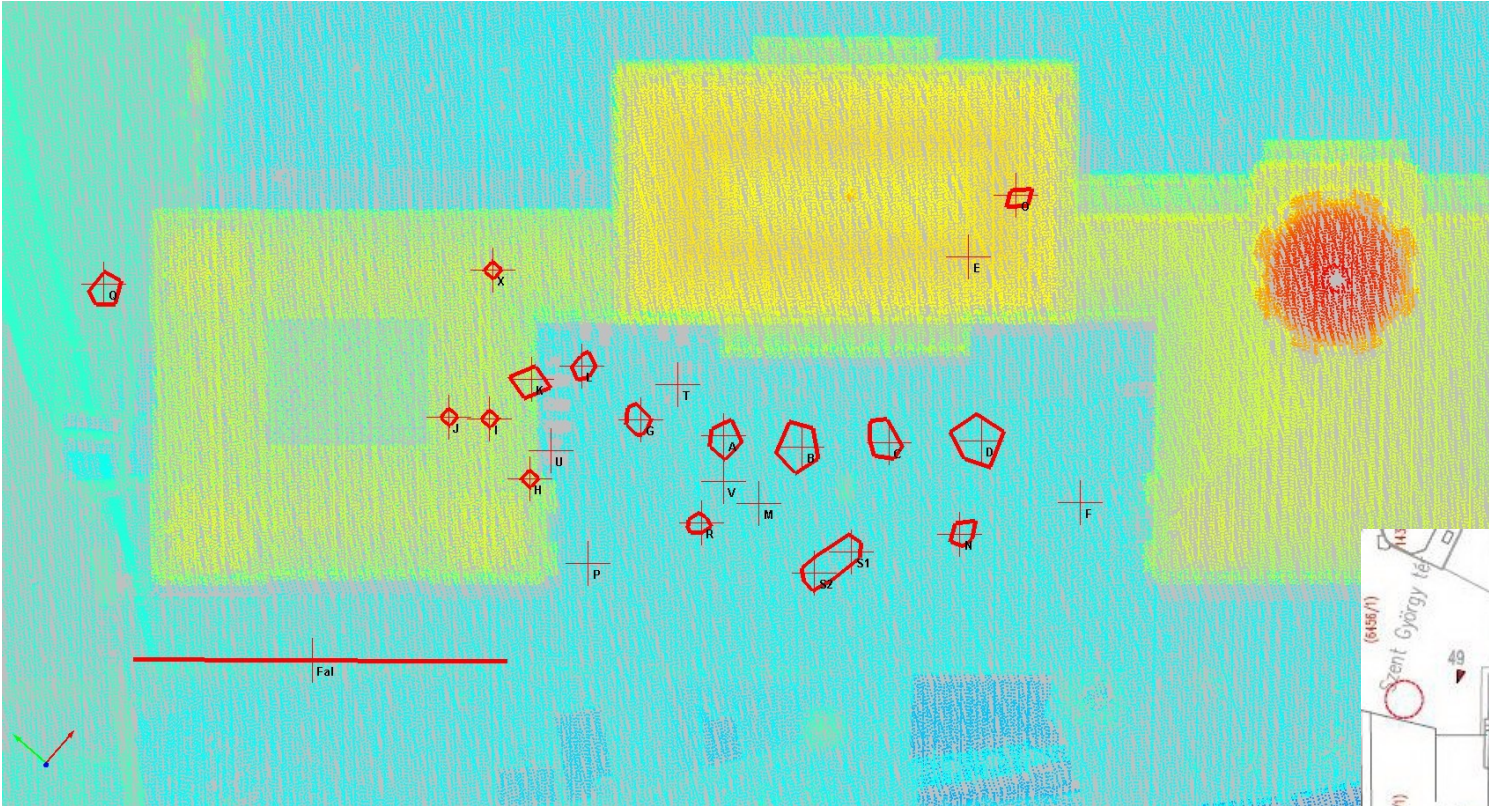
Spatial resolution: 0.8-1 m

Voxel count: $45 \times 40 \times 40$

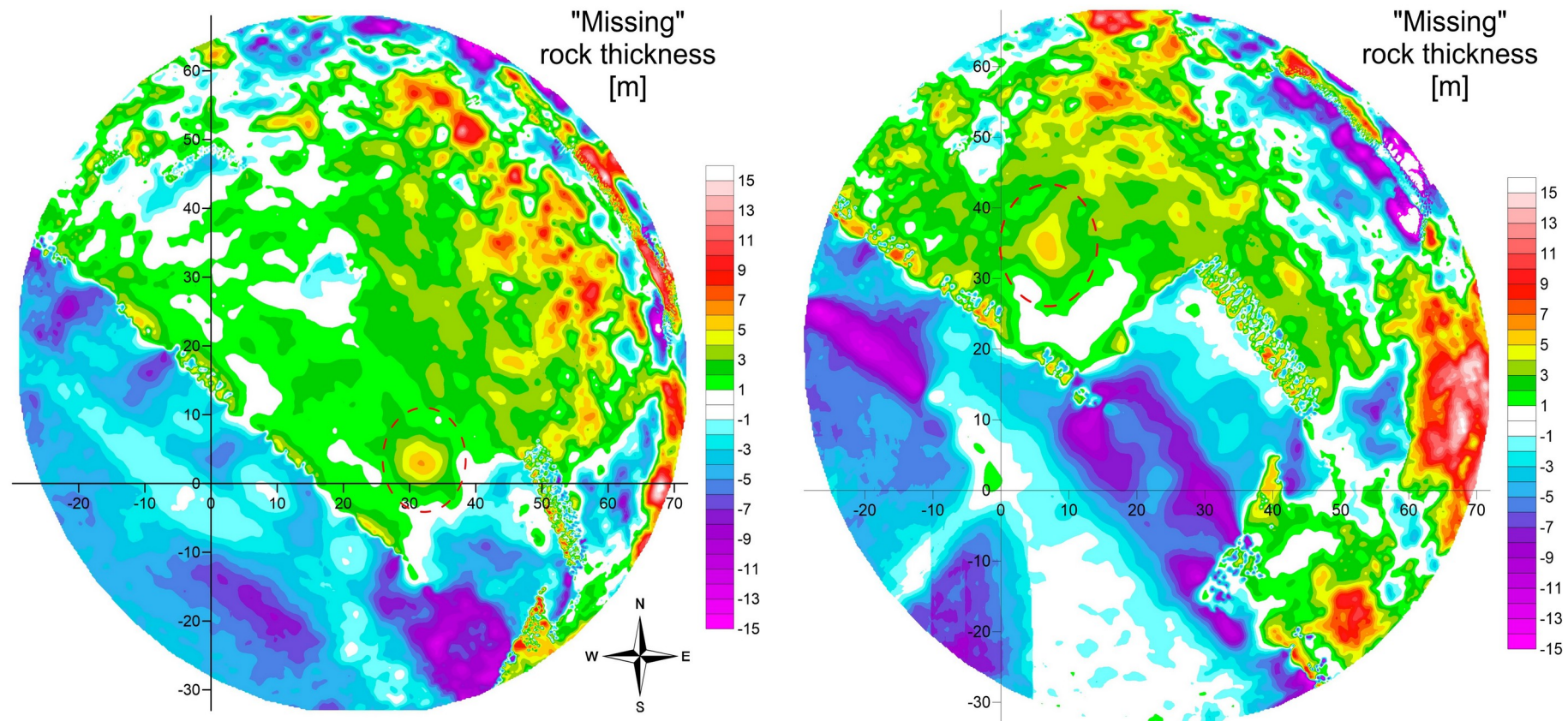
Bayesian inversion



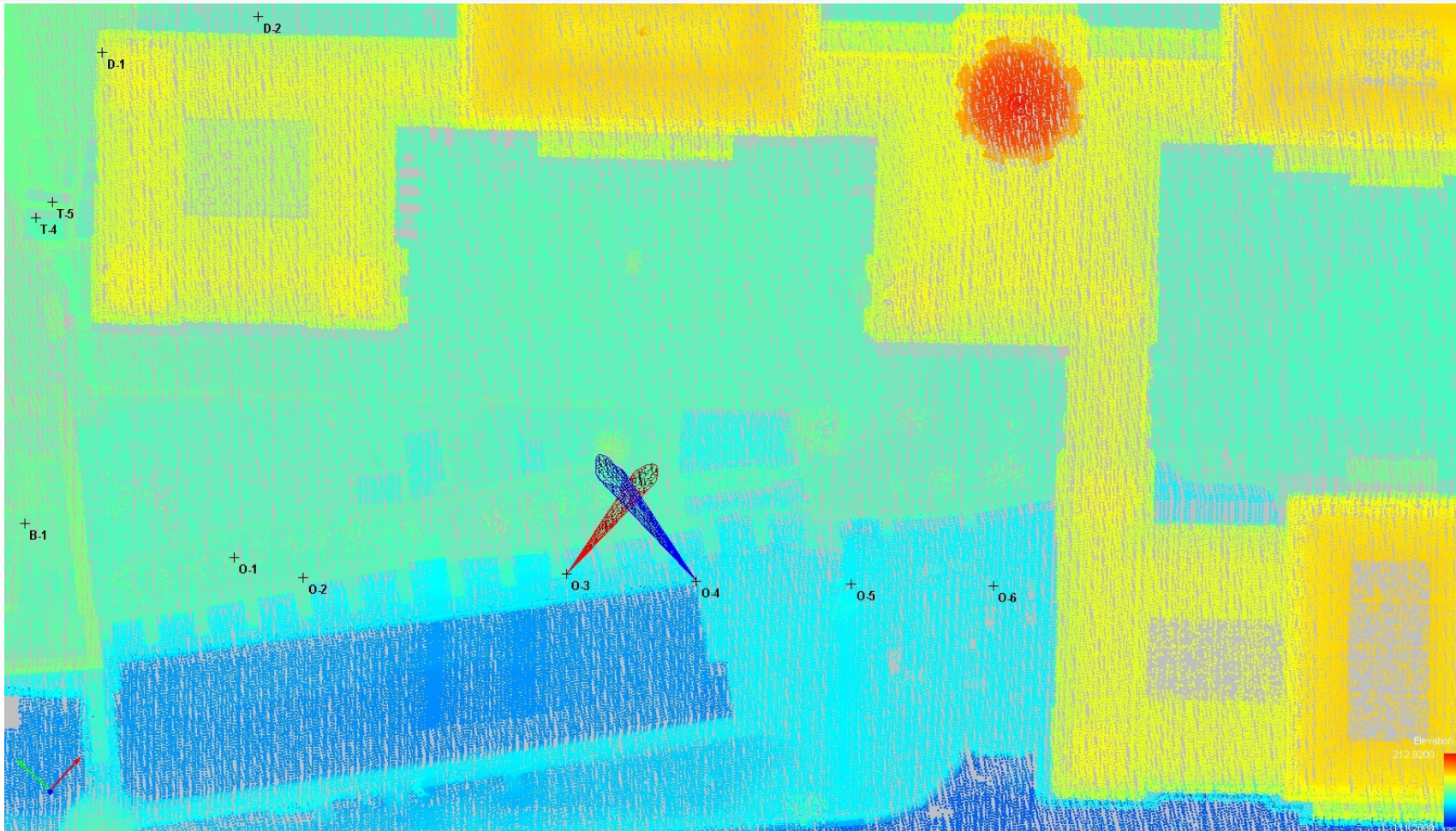
Measurements: Castle of Buda → 3D tomography

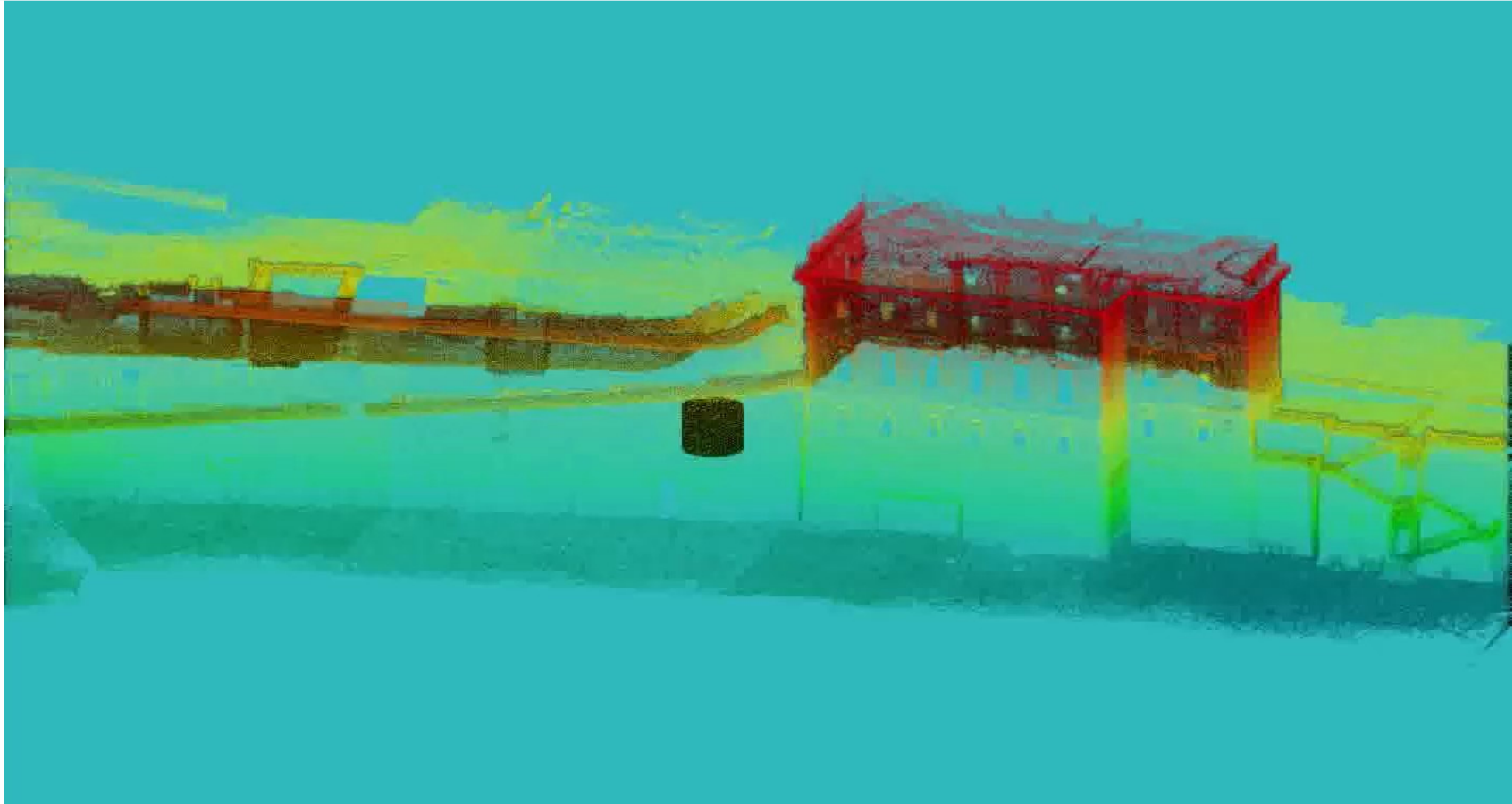


Measurements: Castle of Buda → 3D tomography



Measurements: Castle of Buda → 3D tomography





Summary: REGARD Muontomograph

- **Large-scale structural investigation using cosmic particles.**
- **Implementation:**

Detector:

- for detecting cosmic muons;
- for imaging high-density objects with muons;
- for detecting soil inhomogeneities and underground cavities.
- Precision:
 - 1–2 mm spatial resolution;
 - 2-3° angular resolution.

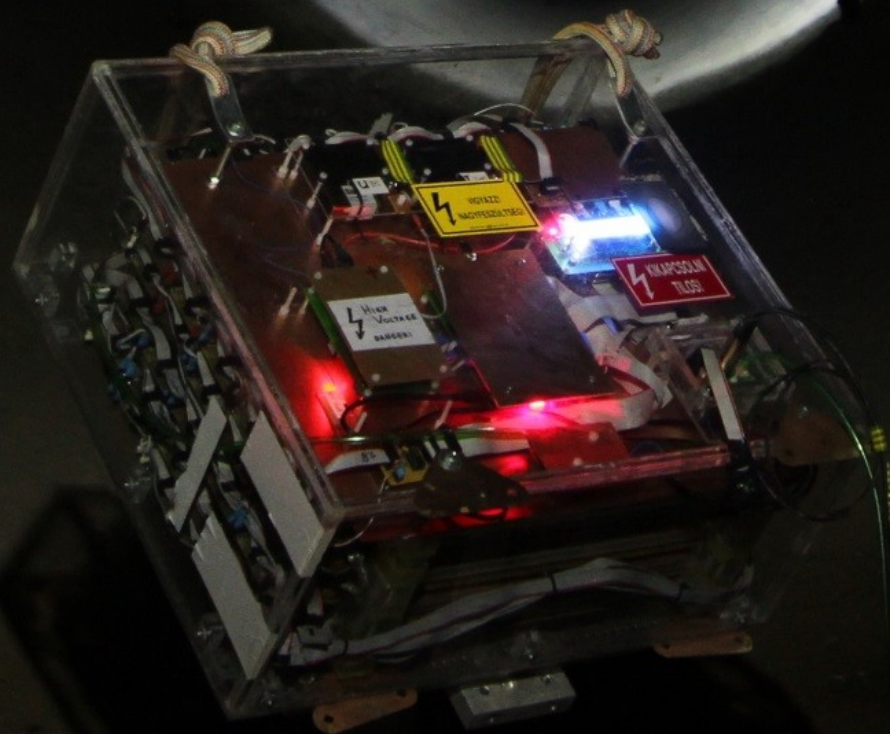
Mobile (< 50-80 kg).

Cost-effective.

Low energy consumption.



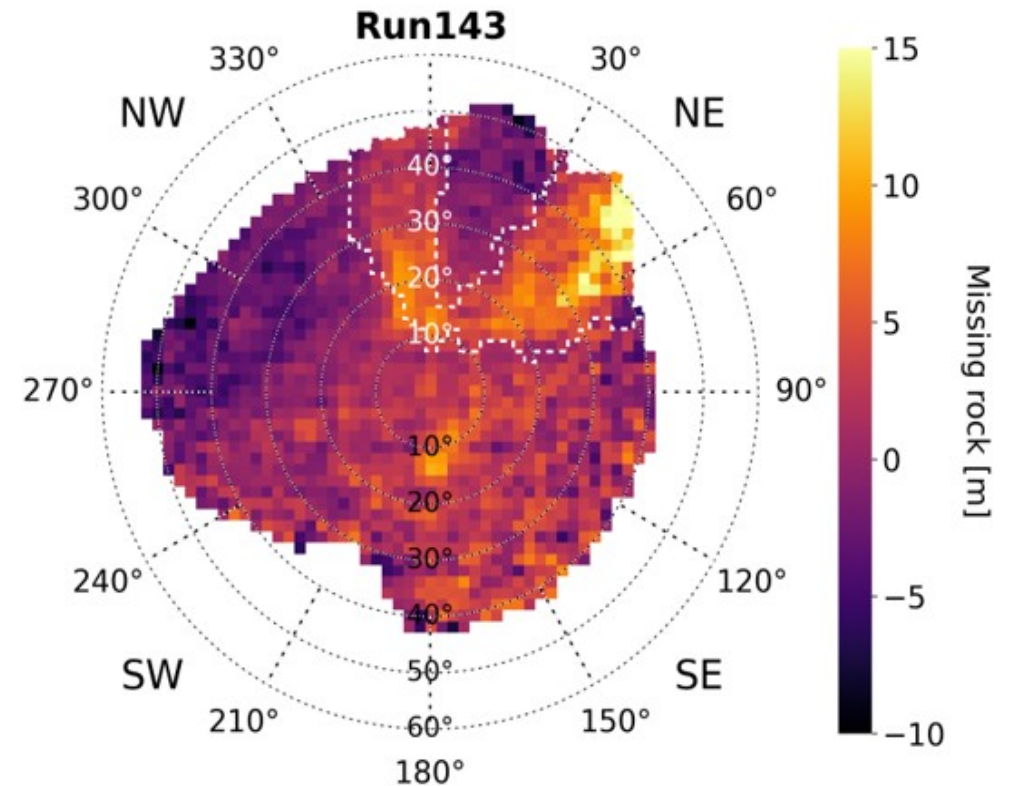
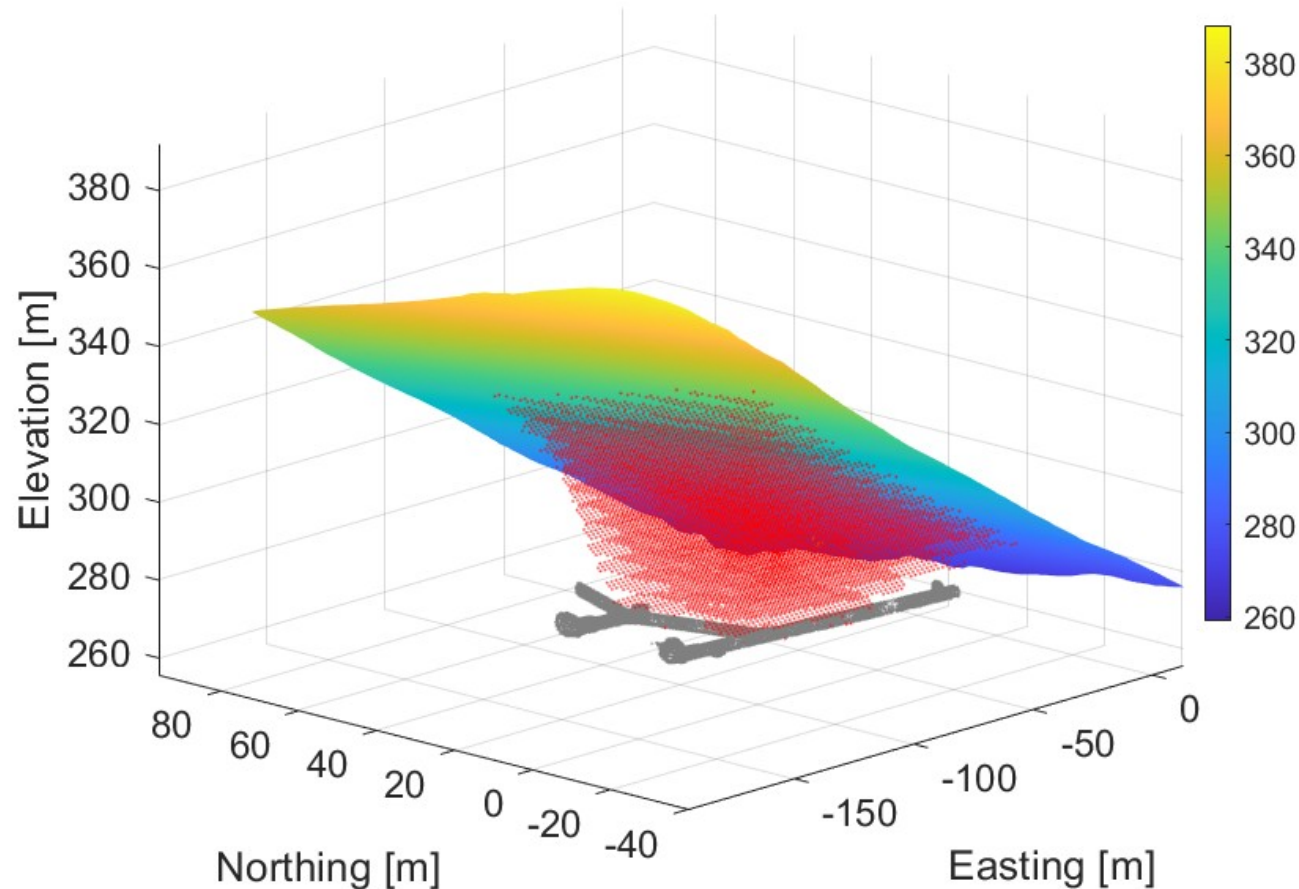
Thank you for your attention!



3D: Limited-Angle Tomographic Inversion

Model and measurement discretization:

Equidistant voxel grid



3D: Limited-Angle Tomographic Inversion

Why does it work?

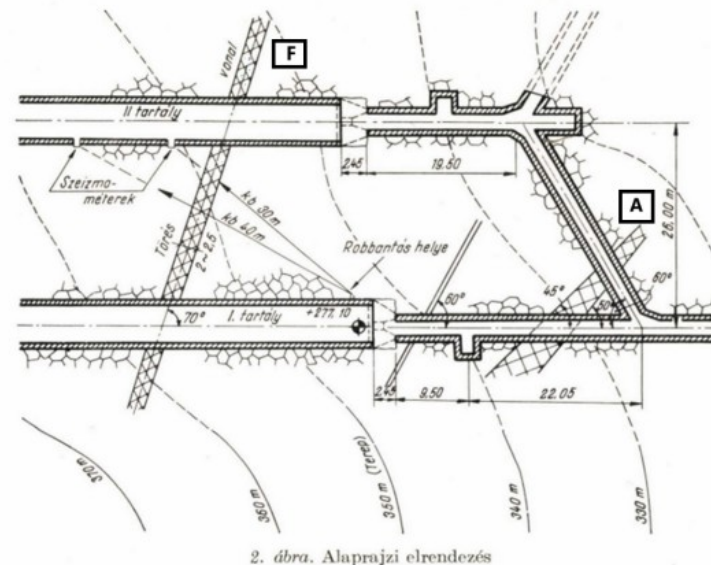
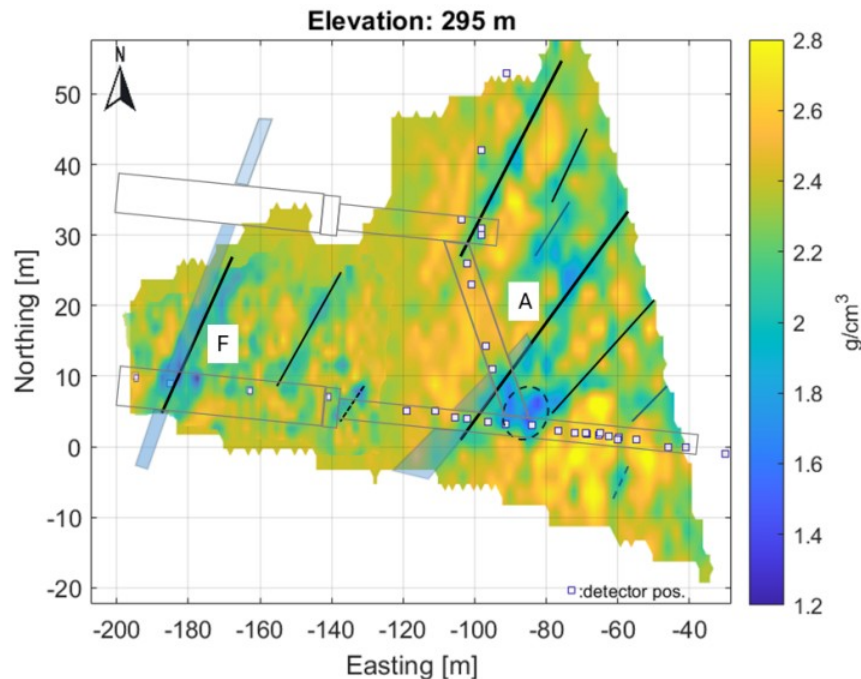
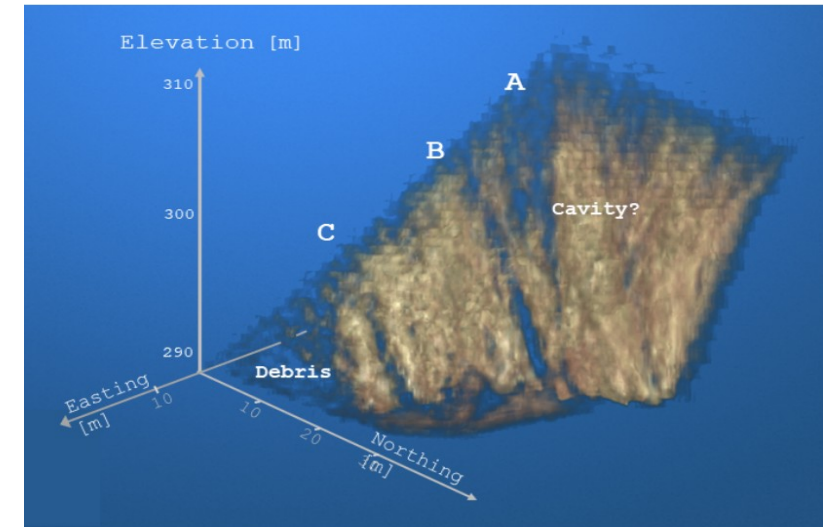
With what bias, and what artifacts appear?

$$\delta \hat{\rho} = (R + C_B^{-1})^{-1} C_B^{-1} (\rho_B - \rho)$$

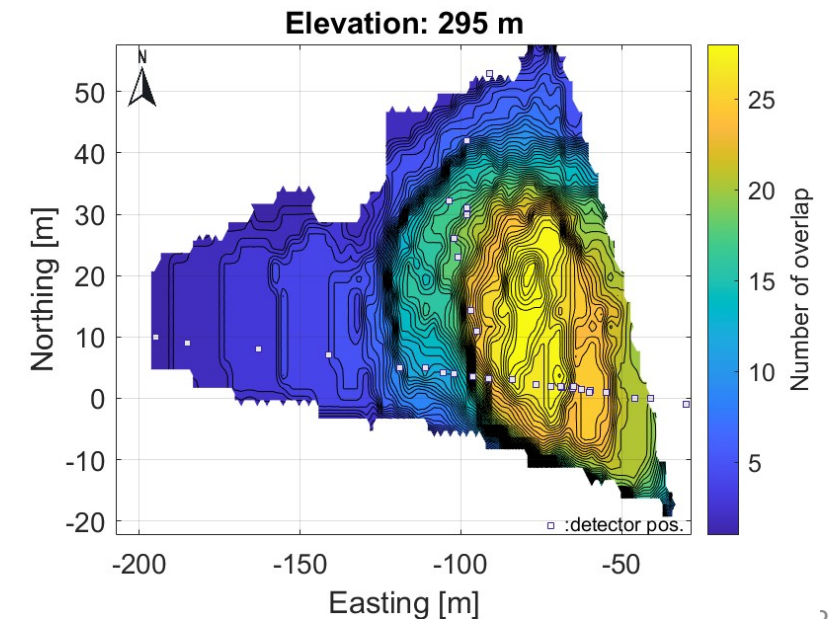
$$C_{\hat{\rho}} = (R + C_B^{-1})^{-1}$$

Bias

Parameter covariance matrix

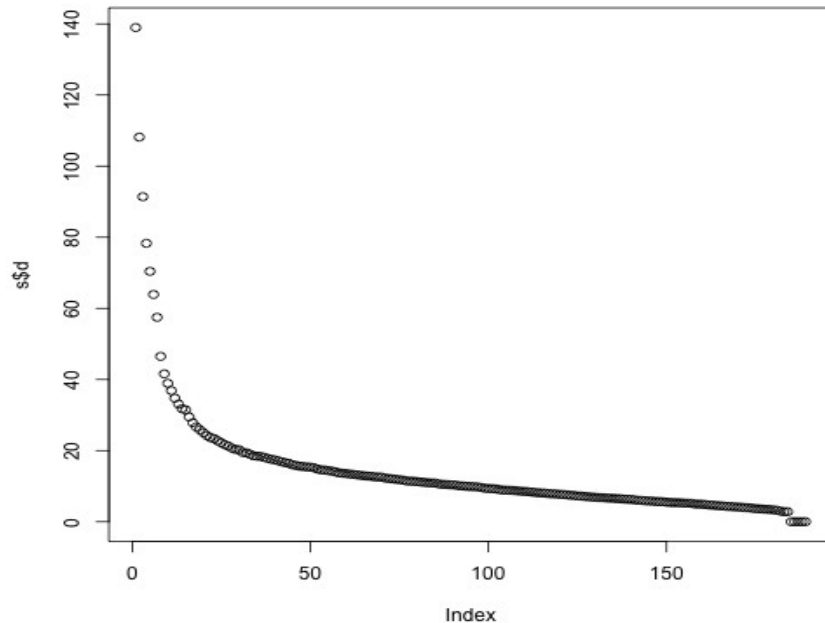


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Structure and symmetries of the null space

Spectral decomposition → null-space dimension and basis



3D Muon Tomography:

Typically: 20–50,000 measurements – 100,000–150,000 voxels

Rank-deficient Fisher matrix, not invertible without regularization

SVD: high-multiplicity zero eigenvalue (“blind spot”)

Null-space basis

part of it consists of random-like vectors

the other part reflects the symmetries of the mapping

$$F \rho_1 = F \rho_2$$

Equivalences – symmetry – null space

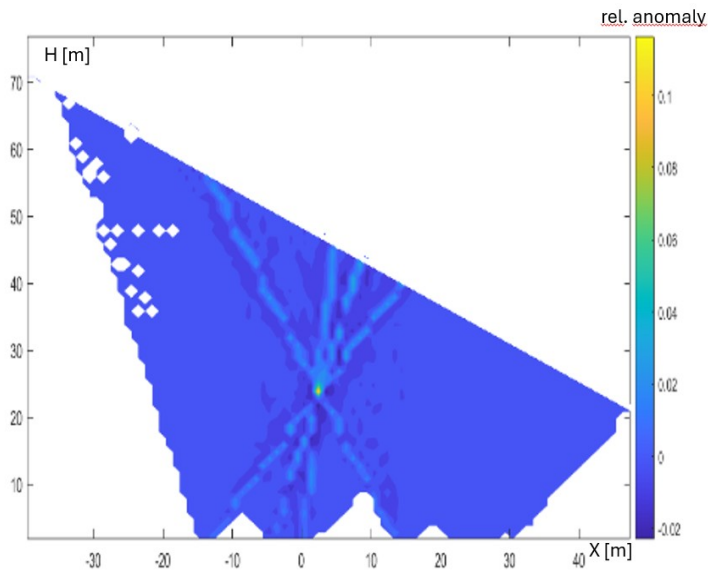
Symmetry arises from the narrow angular range of the mapping. The Jacobian matrix's symmetry, which the Fisher matrix also inherits

Unitary, permutation symmetry

Z-direction interchangeability along muon-trajectory bundles

The regularized solution's biases

The blurring of information can again be examined using the Fisher matrix (parameter resolution matrix)



$$\mathbf{P} = \left(\mathbf{R} + \mathbf{C}_B^{-1} \right)^{-1} \mathbf{R}$$

$$\hat{\boldsymbol{\rho}} = \mathbf{P} \boldsymbol{\rho} \quad \square$$

A finer **voxel subdivision** increases the correlation among the unknowns and increases the variance of the parameters.

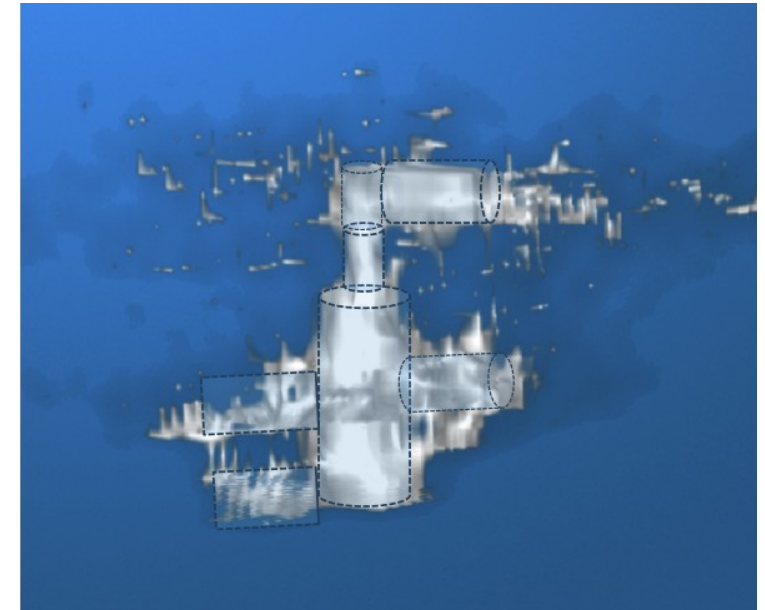
Increasing the number of overlapping measurements localizes the effect

Blurring is oriented along the mapping (directions of correlation)

$$\delta \hat{\boldsymbol{\rho}} = \left(\mathbf{R} + \mathbf{C}_B^{-1} \right)^{-1} \mathbf{C}_B^{-1} (\boldsymbol{\rho}_B - \boldsymbol{\rho})$$

The source of bias is the difference between the prior density and the true density

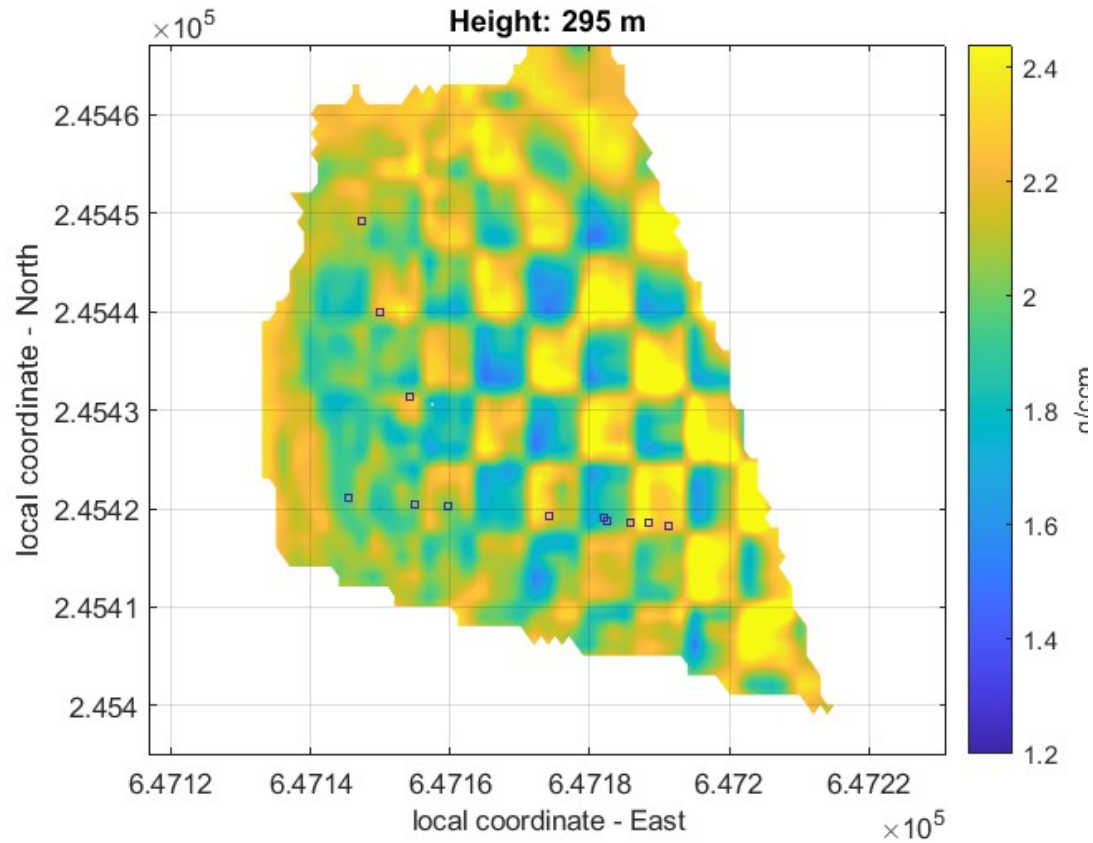
It must be examined which objects, for a given measurement layout, have a significant **projection onto the null space** — detecting these is less certain



3D: Limited-Angle Tomographic Inversion

Simulation studies preceding the inversion:

Checkerboard tests:



Specific tests

