Modelling Neutron Stars in Scalar Tensor Theories

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Scalar-Tensor Theories: Beyond General Relativity

General Theory of Relativity

Gravity is described purely by metric $g^{\mu\vartheta}$

$$S_{GR} = \frac{1}{16\pi G}\int R\sqrt{-g}\,d^4x + S_{\text{matter}}$$

Scalar Tensor Theory

Gravity mediated by $g^{\mu\vartheta}$ + scalar field φ

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,[R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)] + S_{\text{matter}}\left(A^2(\varphi)g_{\mu\nu}, \varphi\right)$$

(Einstein Frame)

Motivation? General Relativity may not be the complete theory of gravity; extensions like STT introduce additional degrees of freedom.

Why a scalar field? Appears naturally in many fundamental theories.

Why neutron stars? Neutron stars probe the strong-gravity regime, where deviations from GR could become observable.

Scalar Tensor Theory:

Two ways to write the same theory

Jordan Frame

$$S = \frac{1}{16\pi}\int\left(\varphi\tilde{R} - \frac{\omega(\varphi)}{\varphi}(\tilde{\nabla}\varphi)^2 - V(\varphi)\right)\sqrt{-\tilde{g}}\,d^4x + S_m(\tilde{g}_{\mu\nu},\chi)$$

- Physical frame.
- Test particles follow geodesics.
- Gravity is modified.
- Field equations look complicated.

Conformal Rescaling

$$\tilde{g}_{\mu\nu} = A^2(\varphi)g_{\mu\nu}$$

Einstein Frame

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,[R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)] + S_{\text{matter}}\left(A^2(\varphi)g_{\mu\nu},\chi\right)$$

- Mathematical frame.
- Gravity looks like clean GR.
- Scalar field φ has its kinetic + potential term;

Matter couples to the conformal scaling function.

• Simpler ODEs.

Neutron Stars In Scalar Tensor Theories With A Massive Scalar Field

- Einstein Frame Action:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right] + S_{\text{matter}}(A^2(\varphi)g_{\mu\nu}, \chi)$$

- Potential:

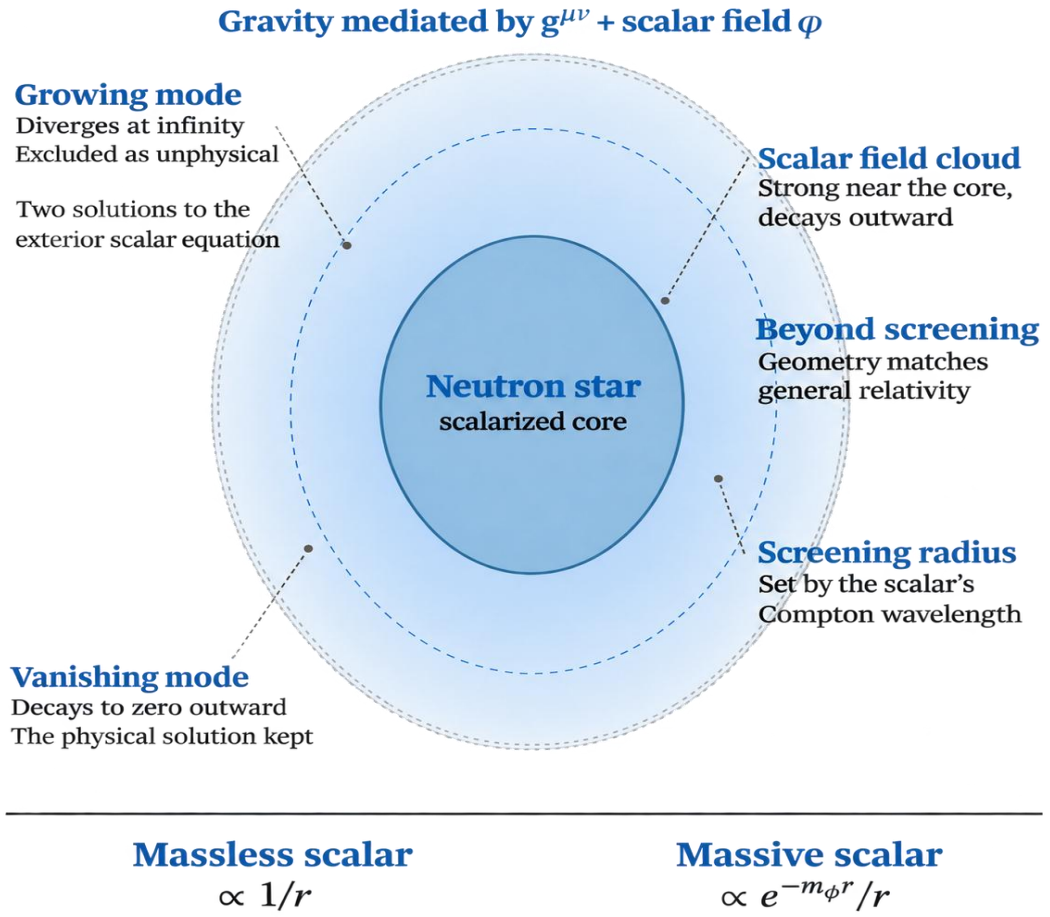
$$V(\varphi) = 2m_\varphi^2\varphi^2$$

- Class of STT: Damour & Esposito-Farèse

$$\alpha(\varphi) = \frac{d\ln A(\varphi)}{d\varphi} \longrightarrow \alpha(\varphi) = \beta\varphi \Leftrightarrow A(\varphi) = \exp(1/2\varphi^2)$$

- Equation of State:

APR (Akmal–Pandharipande–Ravenhall)



Why does Scalarization Occur?

Vary action with respect to φ :

$$\nabla_\mu \nabla^\mu \varphi = -4\pi G \alpha(\varphi) T + \frac{1}{4}\frac{dV}{d\varphi}$$

$$\Box \delta\varphi = m_{eff}^2 \delta\varphi$$

where, $m_{eff}^2 = m_\varphi{}^2 + 4\pi G\beta(\epsilon - 3P)$, $\beta < 0$

(Trivial Solution:GR) $\varphi = 0$ $\varphi = 0 + \delta\varphi$ (Perturbation)

$m_{eff}^2 > 0$

- $\delta\varphi$ oscillates and goes to zero.
- Stable → GR

$\delta\varphi \sim e^{\pm i\omega t}$

$m_{eff}^2 < 0$, $4\pi G\beta(\epsilon - 3P) > m_\varphi{}^2$

$\delta\varphi \sim e^{\pm \omega t}$

- $\delta\varphi$ grows exponentially.
- Scalarization

$4\pi G|\beta|(\varepsilon - 3p) > m_\varphi^2 \quad \beta < 0$ Tachyonic Instability Condition → Spontaneous!

Massless vs Massive Scalar Field

Property	Massless	Massive
V(φ)	0	$2m_{\varphi}{}^{2}\varphi^{2}$
Exterior falloff	1 / r	$e^{(-m_{\varphi})\,/\,r}$
Range	∞ (infinite)	$\lambda\varphi = 1/m_{\varphi}$ (finite)
Instability condition	$4\pi G\lvert\beta\rvert\varepsilon > 0$	$4\pi G\lvert\beta\rvert\varepsilon > m_{\varphi}{}^{2}$
Which stars scalarize	above a critical compactness	only in a limited density window
Observational constraints	Binary pulsars + GW — tightly constrained	Only NS observations — large (β, m_{φ}) space

Why massive STT is physically richer

1. Massless STT is nearly ruled out. Binary pulsar observations constrain $|\beta|$ to sit just above 4.35 — an extremely narrow viable window. Most of the massless parameter space is already excluded.
2. Massive STT has a much larger viable parameter space
3. No scalar field emission detected in binary pulsars. In a binary system, if the scalar field had infinite range it would carry energy away and alter the orbital decay — this is not observed. A massive scalar field decays exponentially beyond Compton Wavelength and never reaches the companion star, so no scalar radiation is emitted. Consistent with observations.

Spacetime Metric and Reduced Einstein Frame Field Equations

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) - 2\omega(r,\theta)\, r^2 \sin^2\theta\, d\phi\, dt$$

Gravitational Mass

$$\frac{1}{r^2}\frac{d}{dr}\left[r\left(1 - e^{-2\Lambda}\right)\right] = 8\pi G\left[A^4(\varphi)\tilde{\epsilon} + e^{-2\Lambda}\left(\frac{d\varphi}{dr}\right)^2 + \frac{1}{2}V(\varphi)\right]$$

conformal Rescaling

Gravitational Potential

$$\frac{2}{r}e^{-2\Lambda}\frac{d\nu}{dr} - \frac{1}{r^2}\left(1 - e^{-2\Lambda}\right) = 8\pi G A^4(\varphi)\tilde{p} + e^{-2\Lambda}\left(\frac{d\varphi}{dr}\right)^2 - \frac{1}{2}V(\varphi)$$

Scalar Field Equation

$$\frac{d^2\varphi}{dr^2} + \left(\frac{d\nu}{dr} - \frac{d\Lambda}{dr} + \frac{2}{r}\right)\frac{d\varphi}{dr} = 4\pi G\alpha(\varphi)A^4(\varphi)(\tilde{\epsilon} - 3\tilde{p})e^{2\Lambda} + \frac{1}{4}\frac{dV(\varphi)}{d\varphi}e^{2\Lambda}$$

Hydrostatic Equilibrium

$$\frac{d\tilde{p}}{dr} = -(\tilde{\epsilon} + \tilde{p})\left(\frac{d\nu}{dr} + \alpha(\varphi)\frac{d\varphi}{dr}\right)$$

Hartle-Thorne approximation at first-order in the star's angular velocity

The first order captures only the effects of frame dragging; star remains a perfect sphere.

$$\frac{e^{\Phi-\Lambda}}{r^4}\frac{d}{dr}\left[e^{-(\Phi+\Lambda)}r^4\frac{d\bar{\omega}(r)}{dr}\right] = 16\pi G A^4(\varphi)(\rho+p)\bar{\omega}(r)$$

where, $\overline{\omega} = \Omega - \omega$.

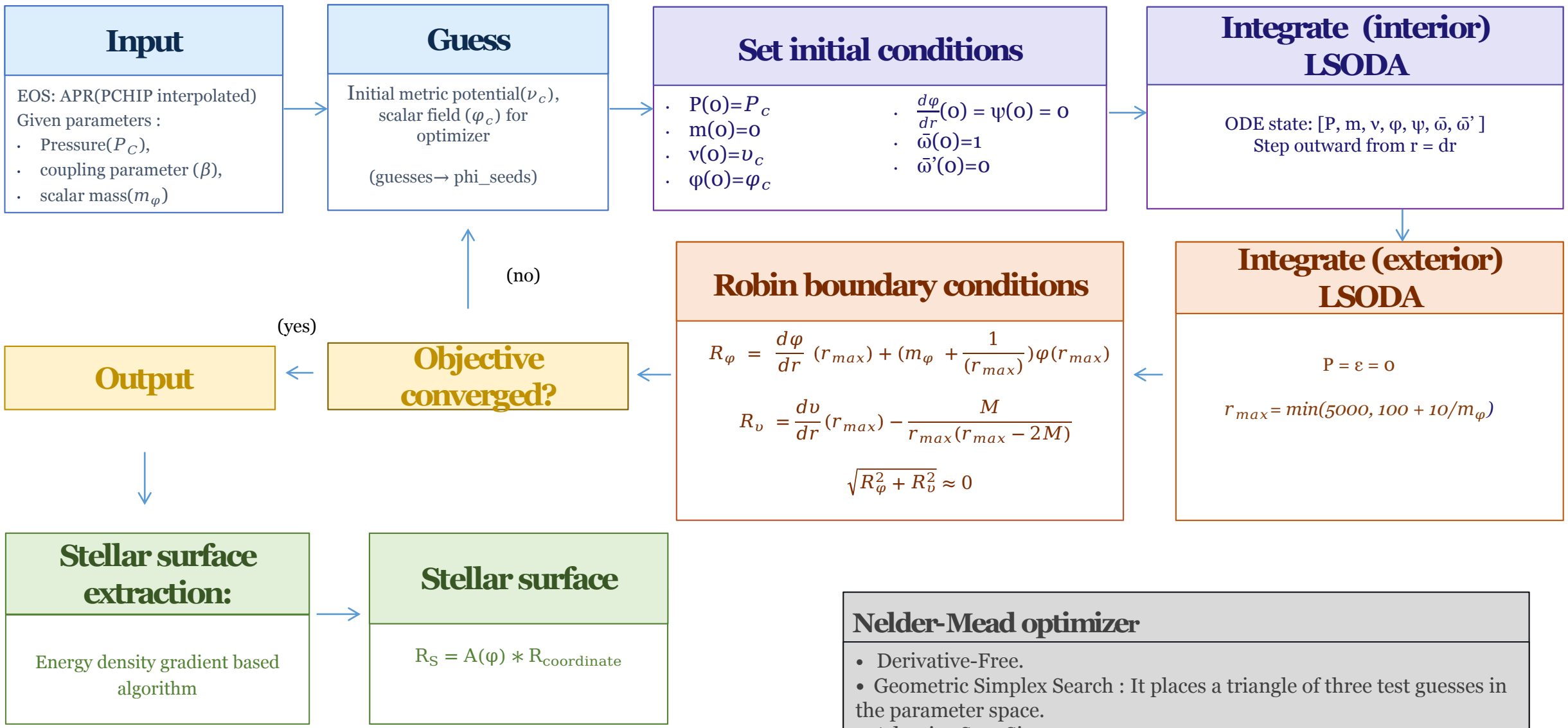
$\overline{\omega}$ →The rotation speed of the stellar fluid relative to space.
ω → The rotation speed of space itself (frame dragging).
Ω →The physical rotation speed of the neutron star.

The asymptotic solution at the boundary(r_{end}) is:
$\overline{\omega}\,(r_{end})=\Omega$ and $\overline{\omega'}\,(r_{end})=\frac{6J}{r^4}$

Moment of Inertia: $I = \frac{J}{\Omega} = \frac{r^4}{6}\frac{\overline{\omega'}\,(r_{end})}{\overline{\omega}\,(r_{end})}$

J → Angular momentum

Modelling: Shooting Method



Nelder-Mead optimizer

- Derivative-Free.
- Geometric Simplex Search : It places a triangle of three test guesses in the parameter space.
- Adaptive Step Sizes

Robin Outer Boundary Condition

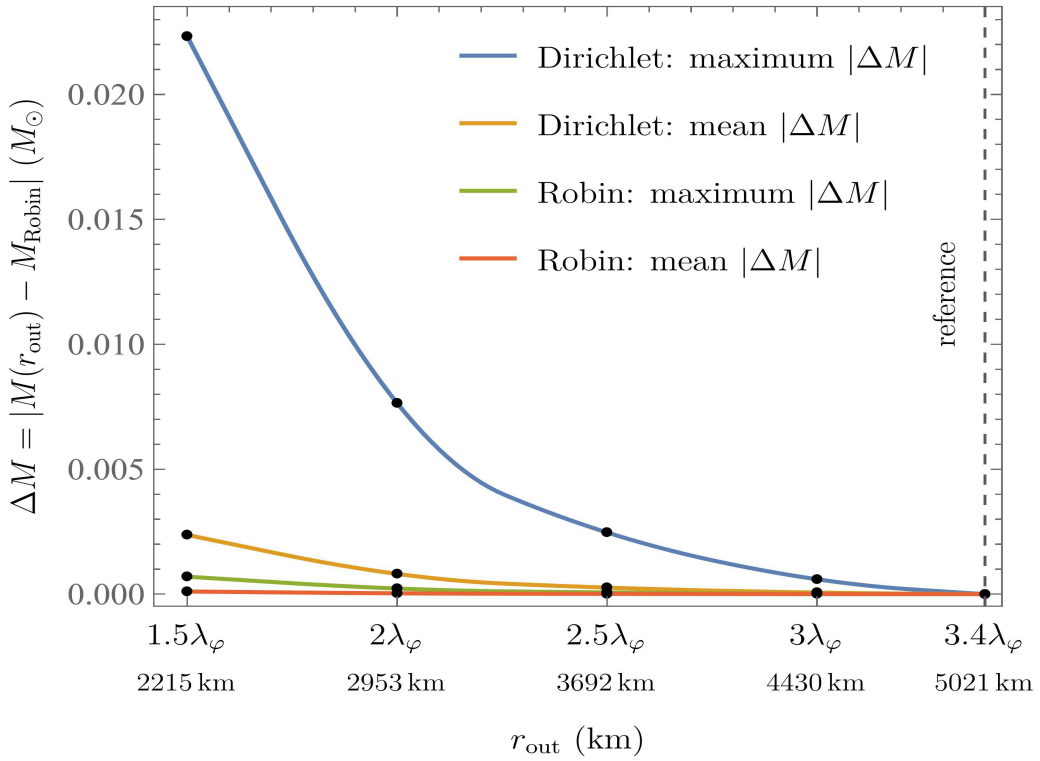
A physical boundary condition for the massive scalar-field tail

DIRICHLET · Artificial truncation

$$\varphi(r_{\text{out}}) = 0$$

ROBIN · Physical Yukawa decay

$$\varphi'(r_{\text{out}}) + \left(m_\varphi + \frac{1}{r_{\text{out}}}\right)\varphi(r_{\text{out}}) = 0$$

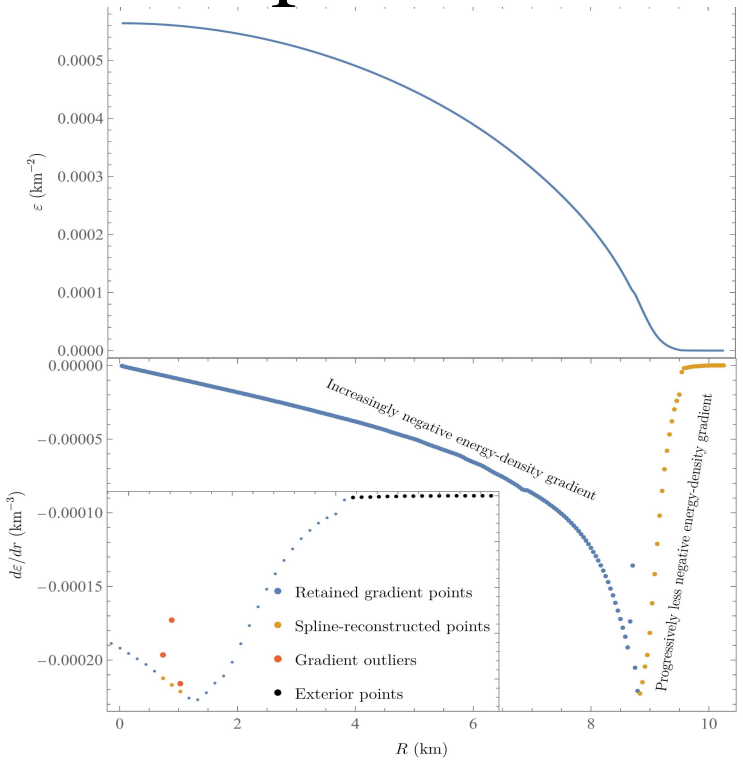


KEY RESULT

Outer-boundary sensitivity reduced by roughly an order of magnitude

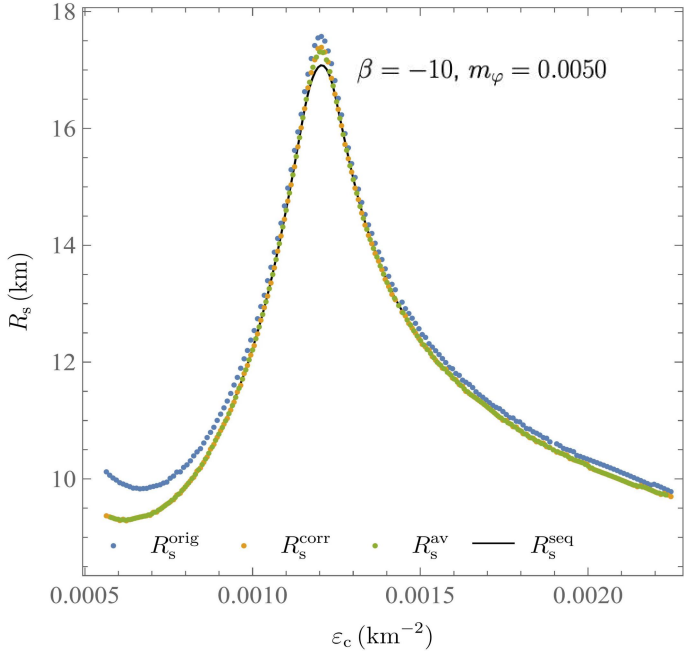
ROBIN < $10^{-3}\,M_\odot$
DIRICHLET~$10^{-2}\,M_\odot$

Improved Stellar Surface Detection: Algorithm Overview



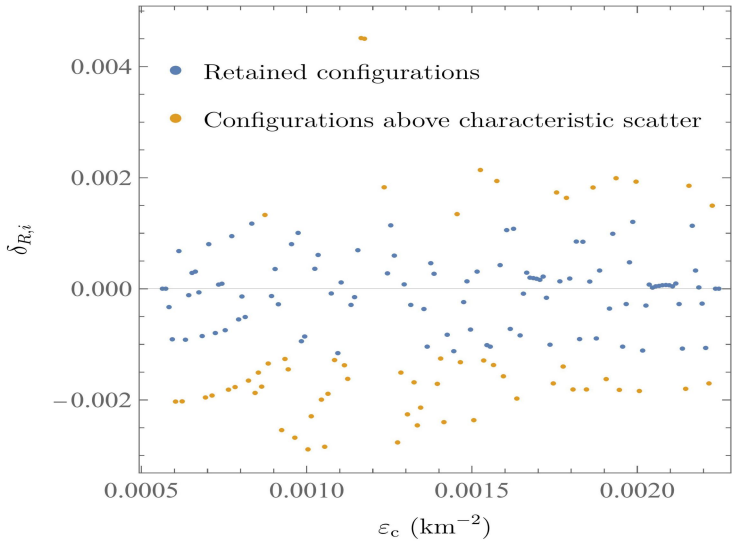
1. Idenitfy Surface

The minimum of the finite difference dε/dr locates the surface.



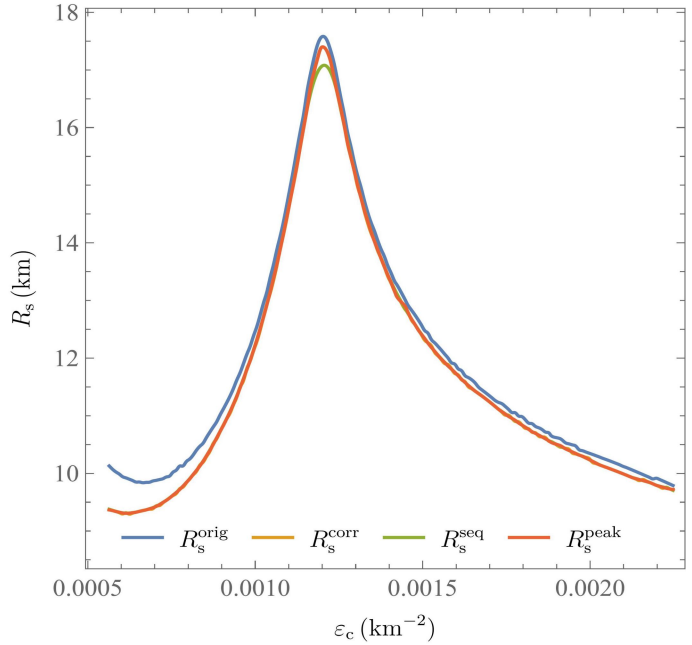
2. Correct Locally

R_s^{orig}, R_s^{corr}, R_s^{av}, R_s^{seq}: original, locally corrected, moving-average and sequence-recalibrated radii.



3. Select & Smooth

Configurations with $|\delta R_i| \leq |\overline{\delta R}|$ are kept for interpolation (blue); larger deviations are excluded (orange).

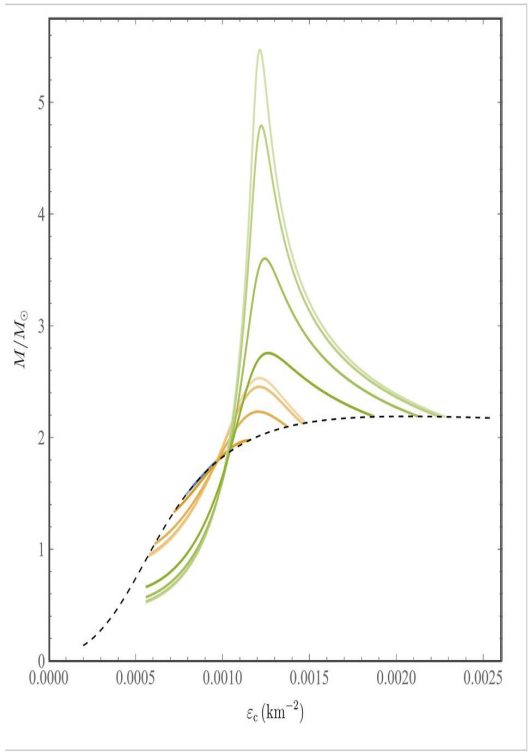


4. Preserve the Peak

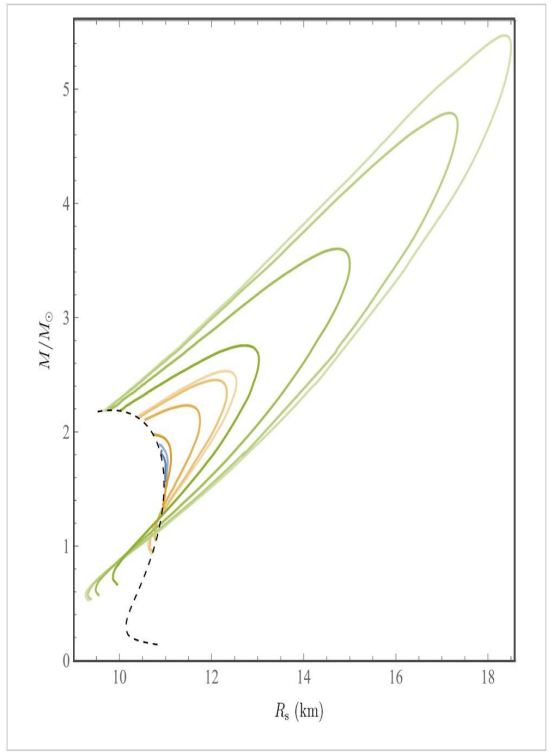
The smoothed sequence sets the baseline; only the smoothed residual is restored near the maximum.

Smooth the residual, not the physical peak.

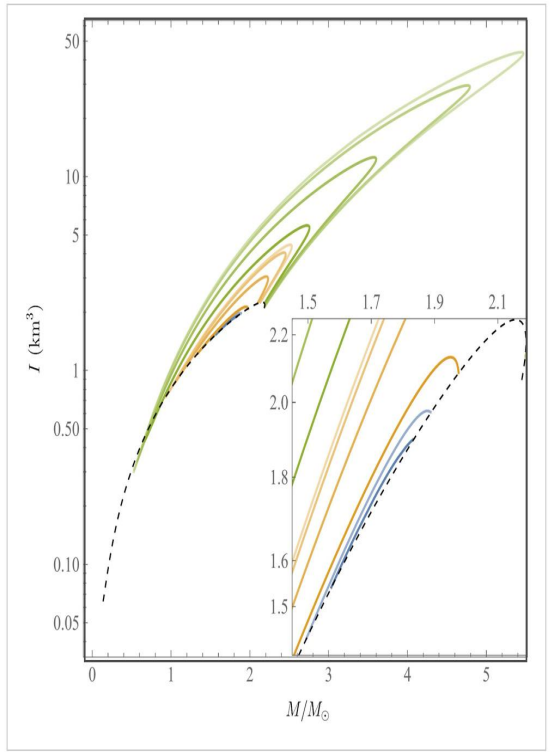
Results



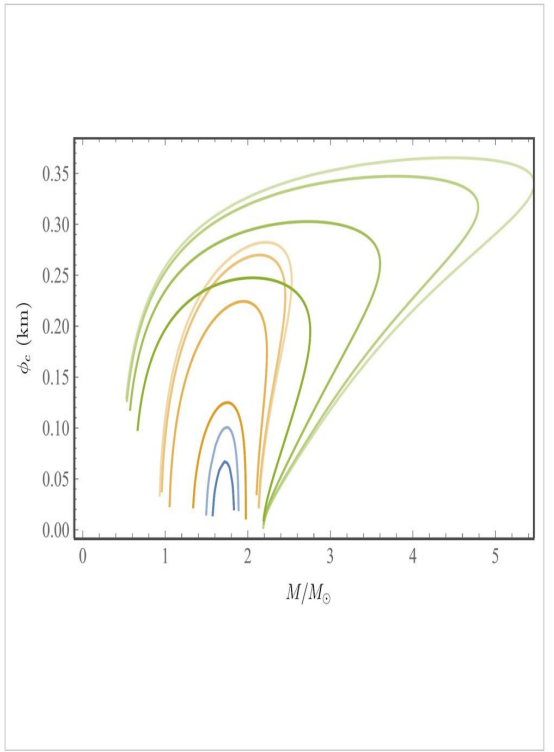
(a)Mass vs Central Energy Density



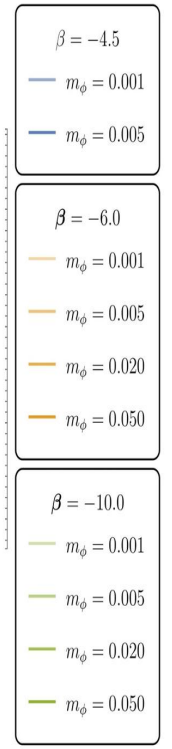
(b)Mass vs Radius



(c)Momentum of Inertia vs Mass



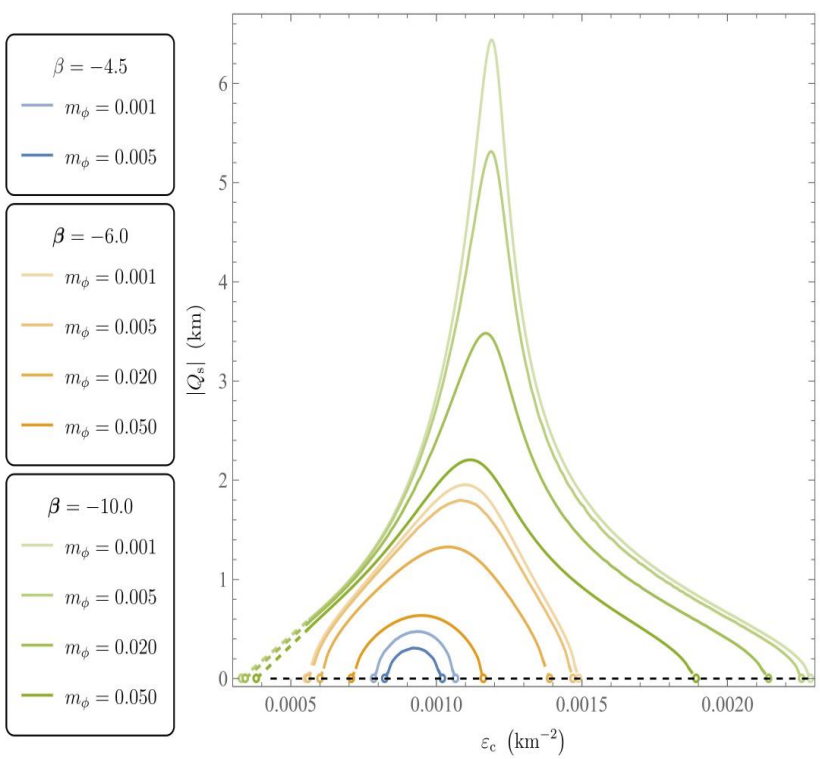
(d)Central Scalar Field vs Mass



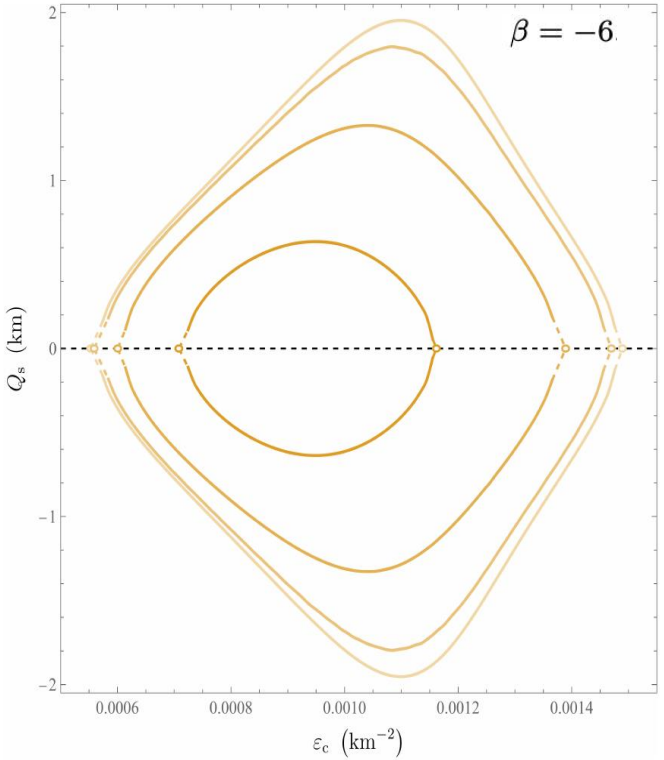
- What enhances scalarization? More negative β.
- What suppresses scalarization? Increasing m_φ.

- What changes? Stellar properties.
- Dashed black curves represent general relativity.

Bifurcation of the scalarized branch



(a) Strength of scalarization along the equilibrium sequences.



(b) Symmetry-completed bifurcation structure.

Continuous branching for GR

Q_s measures the exterior scalar amplitude.
$Q_s = 0$ is GR.
Larger $|Q_s|$ means a stronger exterior field at the same scalar mass.
Bifurcation: nonzero branches meet GR.

Why the branches mirror each other?

$A(\varphi)$ and $V(\varphi)$ depend on φ^2. Reversing the field gives a matching solution with the same mass, radius and moment of inertia.

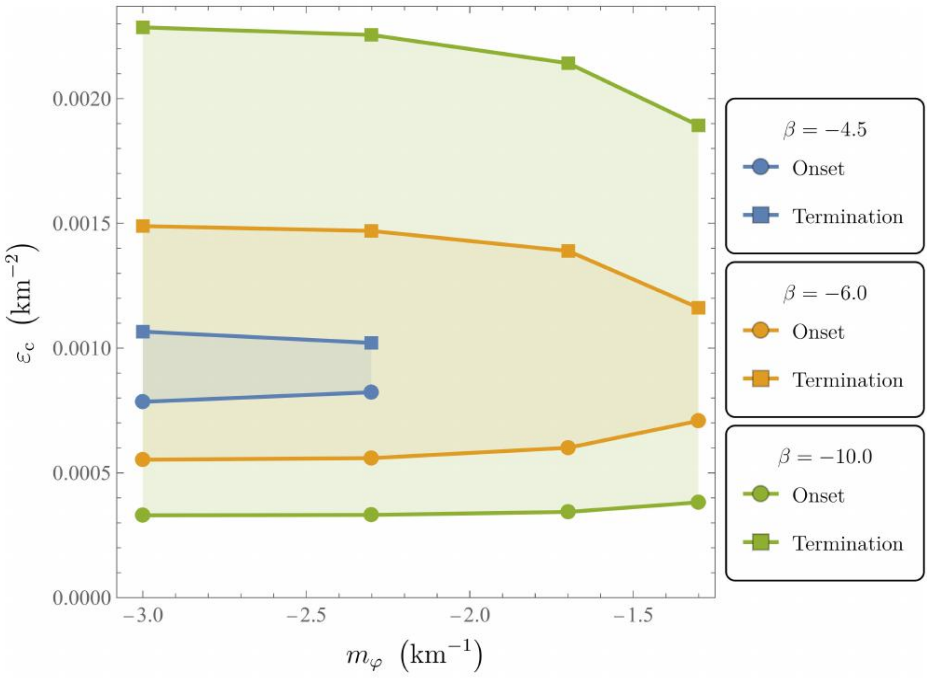
Why the field rises and comes back?

Matter can trigger scalarization, but increasing density also changes pressure and gravity.

Solid curves: computed scalarized sequences|Dashed segments: extrapolation toward $Q_s = 0$|Open circles: Corresponding estimates of the intersections with the GR branch.

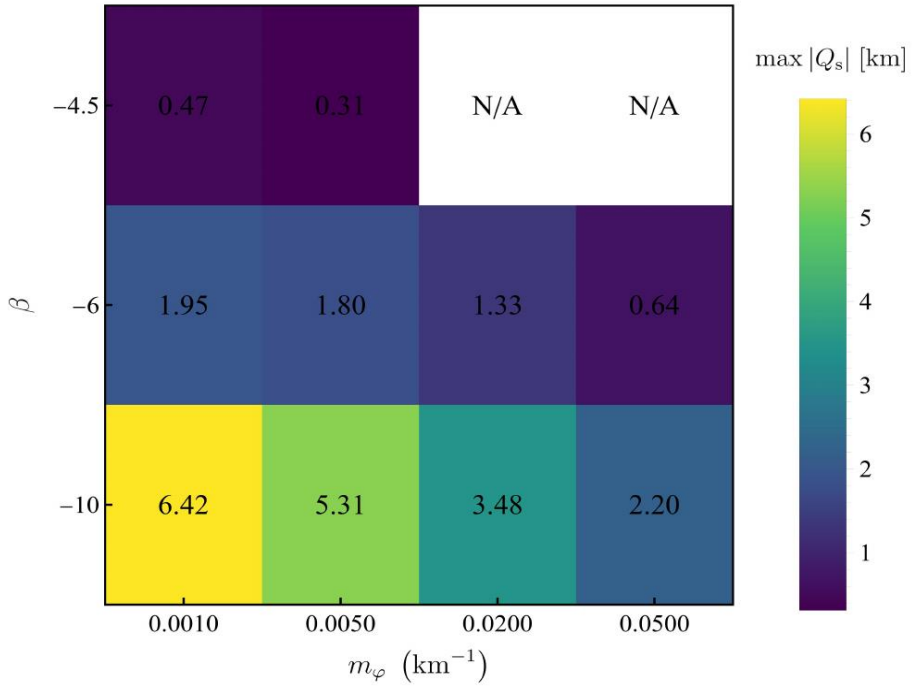
Scalarization domain and strength

Domain: the density interval



- The shaded band shows the density interval containing scalarized solutions.
- A wider band means a wider range of central densities.

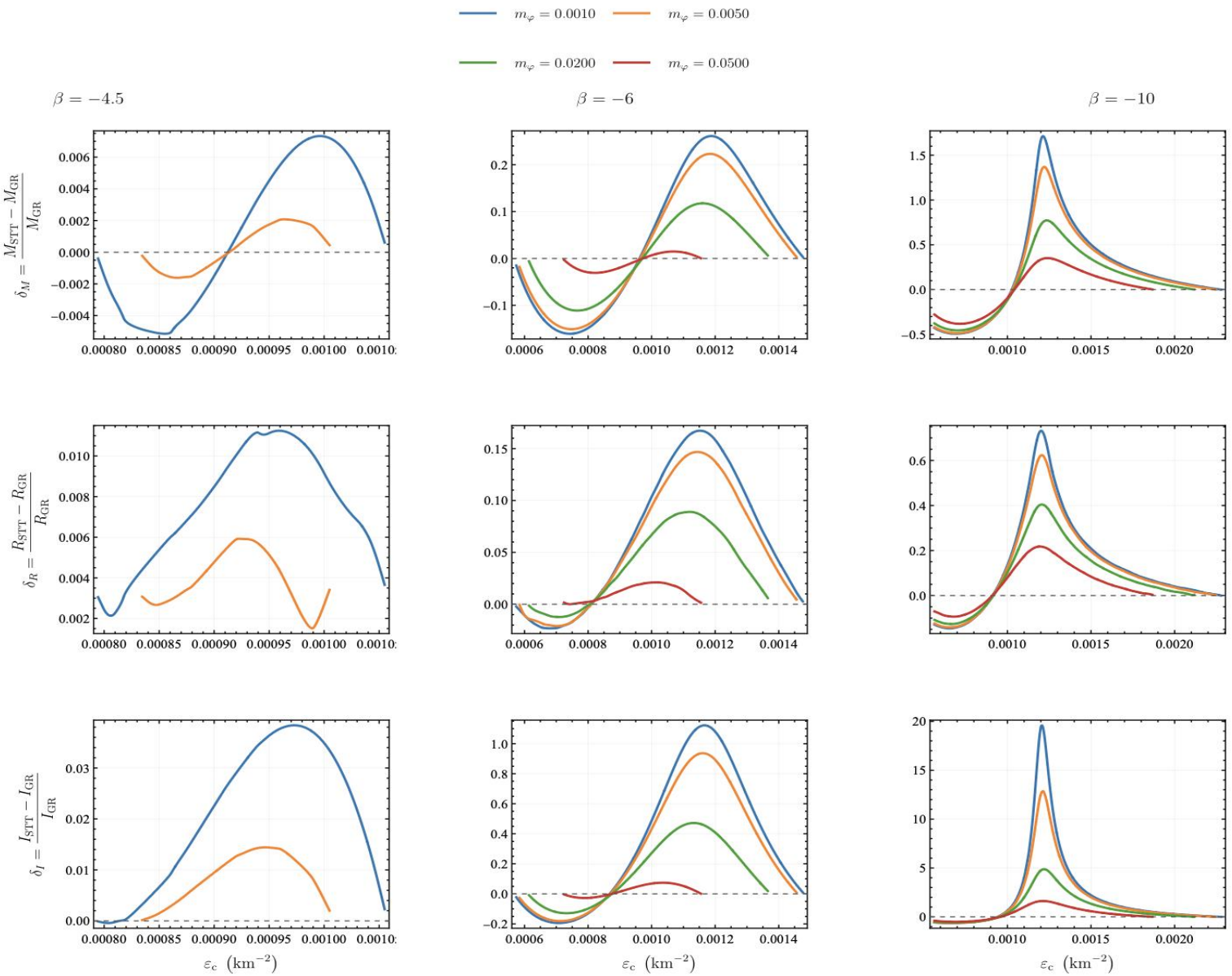
Strength: the peak scalar charge



- Each cell gives max $|Q_s|$ along one stellar sequence.
- N/A means no scalarized sequence was found in the search.

More negative β broadens the interval and raises the peak. A larger m_φ reduces both in these results.

Changes in the stellar bulk properties



Relative changes in the ADM mass, physical radius, and moment of inertia with respect to GR at fixed central energy density.

Tidal deformability and Love numbers in STT

Static, even-parity, quadrupolar perturbations with $\ell = 2$

$$Q_{ij} = -\lambda \varepsilon_{ij}$$

ε_{ij} applied gravitational tide
Q_{ij} induced mass quadrupole
λ tidal deformability

$$k_2 = 3G\lambda R^5/2$$

k_2 electric quadrupolar Love number
R physical stellar radius

General relativity

- The metric perturbation H describes the response of the background star.
- The exterior tidal equation uses a Schwarzschild background.

Massive scalar–tensor gravity

- The tide also perturbs the scalar field, $\delta\varphi$.
- H and φ influence one another at first order.
- Far away, scalar decay permits GR matching.

Hu et al., (2021).

Coupled perturbations: equations and numerical solution

$$H'' + c_1 H' + c_0 H = c_s \delta\varphi$$

$$\delta\varphi'' + d_1 \delta\varphi' + d_0 \delta\varphi = d_s H$$

For $\ell = 2$, both scalar and polar metric perturbations have even parity.
Their parity permits coupling. The scalar–tensor field equations generate the mutual feedback.

H: metric perturbation
δφ: scalar perturbation

1. Integrate twice

$Y = (H, H', \delta\varphi, \delta\varphi')$

$Y^{(1)}(R_0) = (R_0^2, 2R_0, 0, 0)$

$Y^{(2)}(R_0) = (0, 0, R_0^2, 2R_0)$

R_0 is a small radius near the centre.

Each run integrates both coupled equations on the same background.

2. Select the physical combination

$H = aH^{(1)} + bH^{(2)}$

$\delta\varphi = a\delta\varphi^{(1)} + b\delta\varphi^{(2)}$

Choose a and b to cancel the exponentially growing scalar mode.

Use the same a, b for both fields and their derivatives.

3. Extract the tensor response

At r_i, scalar effects are negligible: use GR matching.

$y_i = r_i H'(r_i)/Hr_i)$

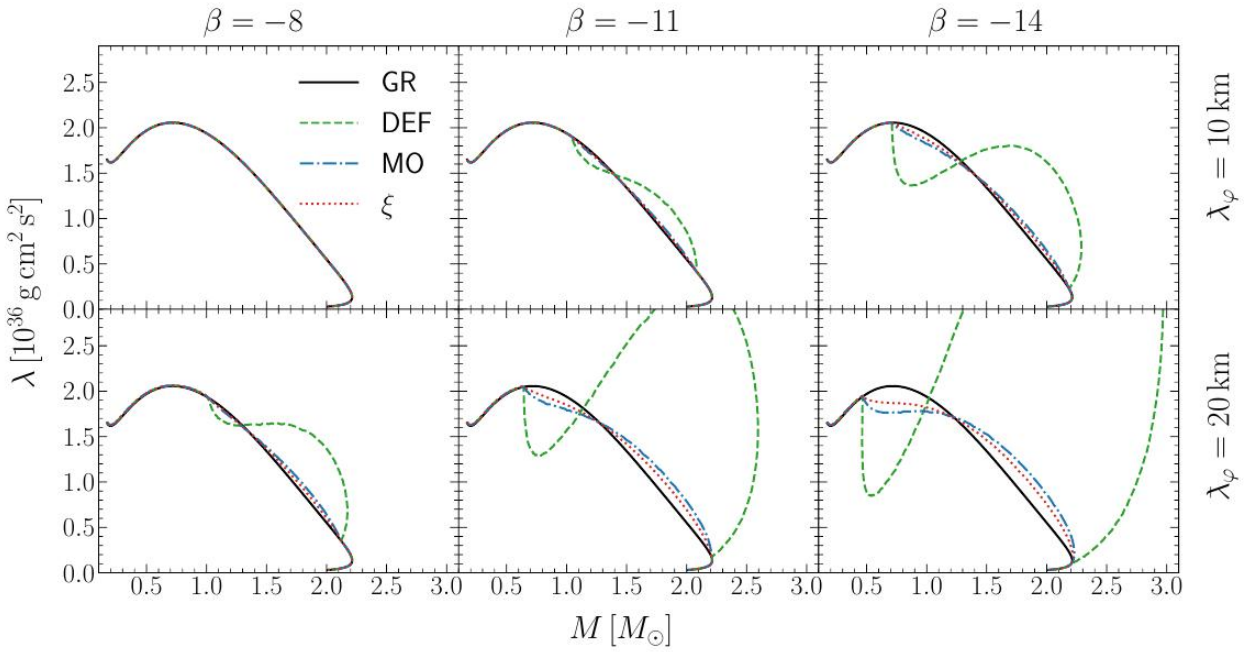
$C_i = m(r_i)/r_i$

$\lambda = (2/3) r_i^5 F(C_i, y_i)$

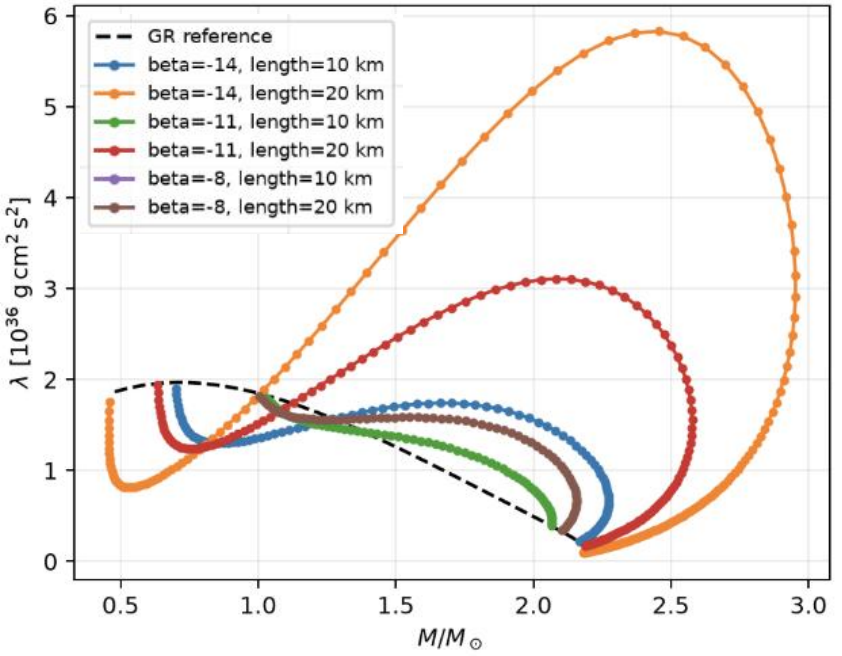
F is the GR matching function.

Tidal Deformability in Massive STT: Comparison with Hu et al.

Hu et al.: AP4 EOS



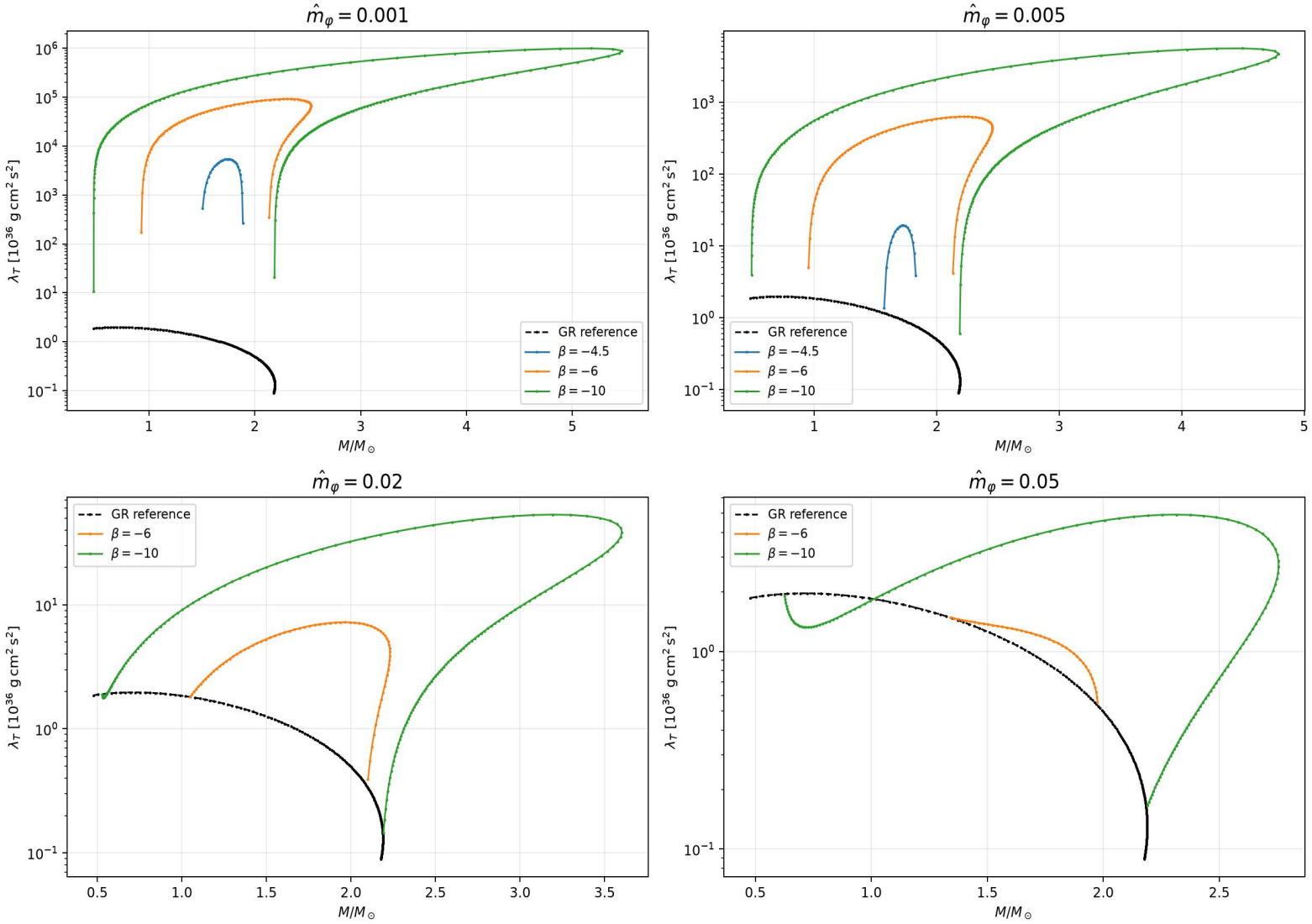
Our graph: APR EOS



Deviation example: DEF, $\beta = -14$, $\lambda_\varphi = 20$ km

Stellar mass	Quantity	AP4: Hu	APR: our results
M = 1 M ⊙	λ_{GR}	1.94	1.84857
	λ_{DEF}	1.85	1.82417
	Deviation from GR	4.37%	1.32%
M = 2 M ⊙	λ_{GR}	0.551	0.49432
	λ_{DEF}	5.29	5.18376
	Deviation from GR	860%	948.67%

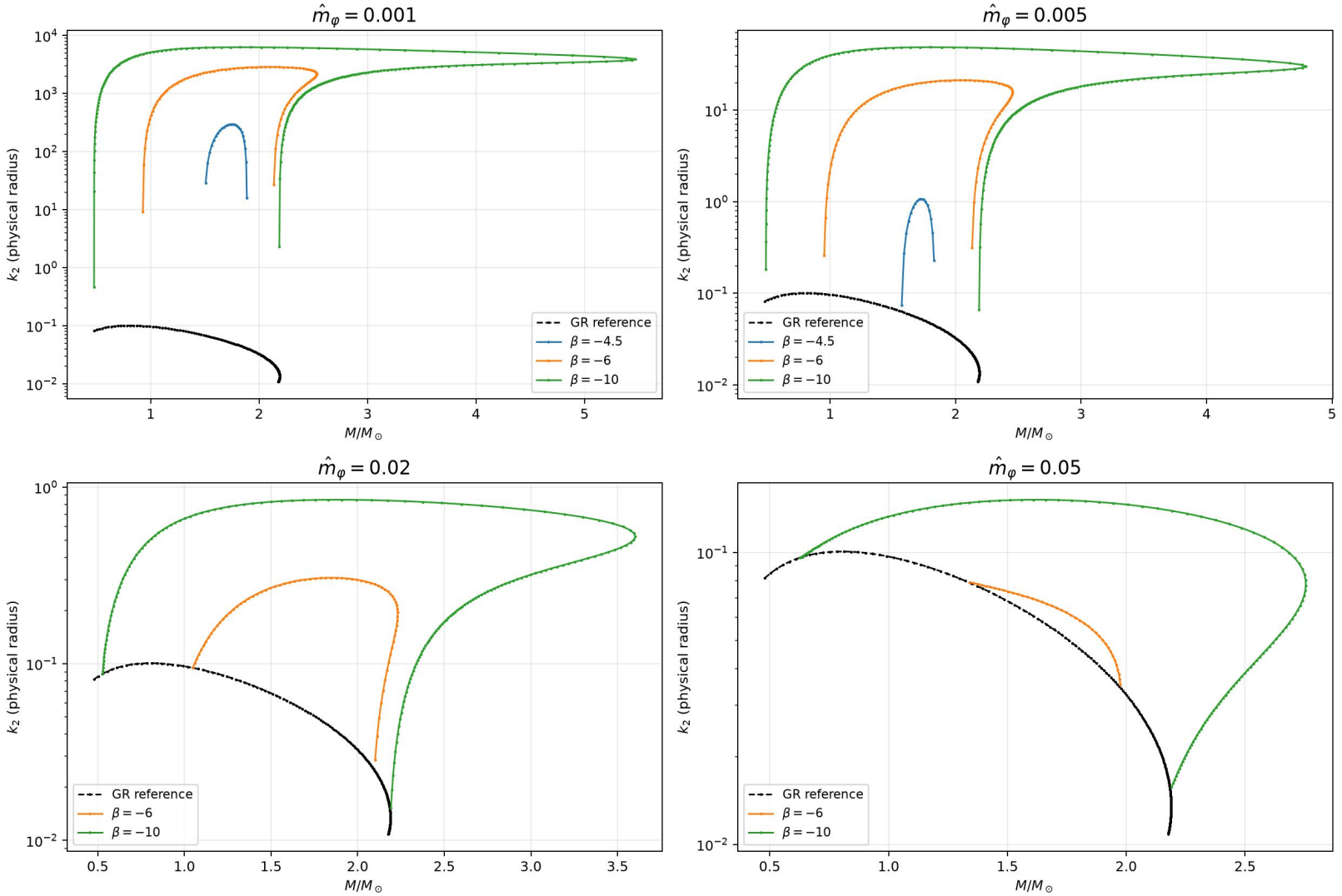
Tidal Deformability for APR EOS



- The dip is associated with the contraction and modified tidal response of lower-mass scalarized configurations.
- At higher masses, scalarization can instead produce larger stars and a strongly enhanced response.
- The R^5 dependence amplifies these structural changes, while the coupled metric–scalar perturbations determine the full result.

Scalar range: $\lambda_\varphi \approx$ 1477, 295, 74, 30 km for the four masses shown.

Electric Love number k_2 for APR EOS



- We use the physical stellar radius because k_2 measures the response relative to the star's size.
- The calculated tensor Love numbers show increasingly large departures from GR for lighter scalar fields and stronger negative DEF couplings.
- These changes include both modifications of the equilibrium star and mutual metric–scalar perturbation feedback.

Future Work

What if the equations are decoupled?

Brown ,S.M(2023): Massless scalar–tensor gravity, static even-parity perturbations

$$H'' + c_1 H' + c_0 H = 0$$

$$\delta\varphi'' + d_1 \delta\varphi' + d_0 \delta\varphi = 0$$

Brown's perturbation prescription

- Brown treats the two perturbations separately to first order, on a fixed scalarized background.
- Each equation evolves one perturbation.
- Neither contains the other as an explicit source.

Distinction from Hu's coupled system

- Hu includes the mutual sources $c_s \delta\varphi$ and $d_s H$.
- These sources are absent from Brown's decoupled equations.

Scalar-background effects remain

- Coefficients depend on the modified matter and metric profiles.
- Decoupled perturbation equations can still describe a background different from GR.

Future Work

Tidal deformability without a scalar feedback

1. Proposed massive extension of Brown's decoupled perturbation equations.
2. Study of tensor tidal deformability with the metric perturbation equation by omitting the scalar feedback. Scalar effect enter only through the background equation.
3. Calculate the electric Love number k_2 either with the stellar surface approximation or exterior series.

Conclusions

- The scalar field contributes extra energy and stiffens the star — scalarized stars can support significantly more mass than GR allows for the same equation of state.
- Massive scalar-tensor theories remain a viable alternative to GR for the strong-field regime.
- Our results successfully match established literature, but achieve higher numerical accuracy. and stability by extracting the stellar surface at the sub-grid point.
- Other possible extensions to this work: Bayesian Analysis, Oscillations

References

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2. Hu, Z., Gao, Y., Xu, R. and Shao, L., 2021. Scalarized neutron stars in massive scalar-tensor gravity: X-ray pulsars and tidal deformability. Physical Review D, 104(10), p.104014.

3. Brown, S. M. (2023). Tidal deformability of neutron stars in scalar-tensor theories of gravity. The Astrophysical Journal, 958(2), 125.

Thank you for your attention