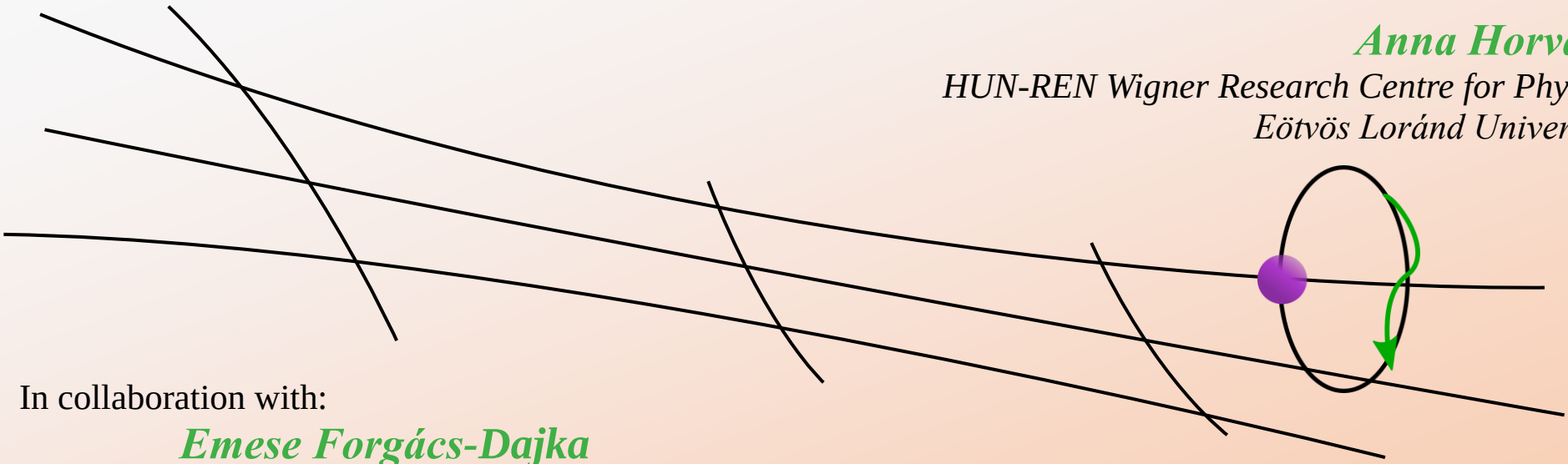
Searching for **extra dimensions** in **neutron stars** and **black holes**

Anna Horváth

HUN-REN Wigner Research Centre for Physics

Eötvös Loránd University



In collaboration with:

Emese Forgács-Dajka

Eötvös Loránd University

Gergely Gábor Barnaföldi

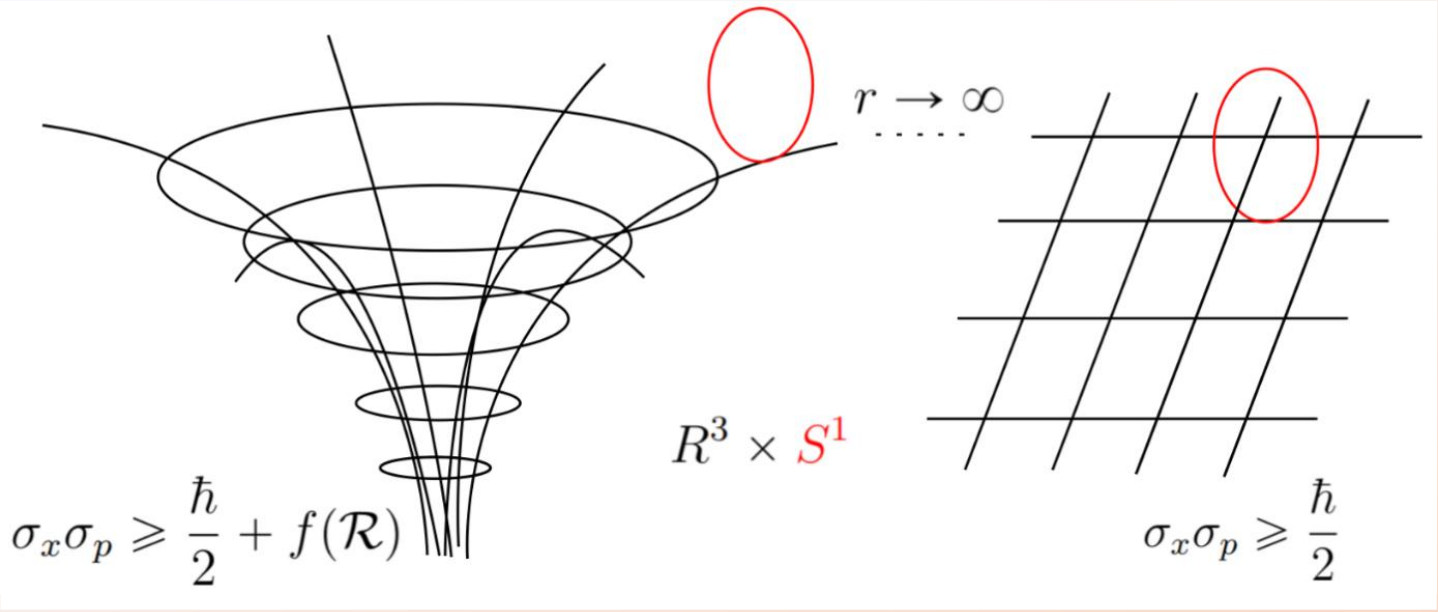
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Aneta Wojnar

University of Wrocław

Support: NKFIH OTKA K147131
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TKP2021-NKTA-64
ELTE DKÖP
WSCLAB
KMP-2023/101, KMP-2024/31, KMP-2025-I/45
BridgeQG (CA23130), FuSe (CA24101)

What I’m going to talk about:



Extra dimensional spacetimes with different **curvature**

How do **massive particles** behave?

What can we **see**?

[1] A. Horváth, E. Forgács-Dajka, G.G. Barnaföldi: "Application of Kaluza-Klein Theory in Modeling Compact Stars: Exploring Extra Dimensions", Mon.Not.Roy.Astron.Soc. 536 (2024) 1, 816-826

[2] A. Horváth, E. Forgács-Dajka, G.G. Barnaföldi: "The effect of multiple extra dimensions on the maximal mass of compact stars in Kaluza-Klein space-time", Int.J.Mod.Phys.A 40 (2025) 21, 2542004

[3] A. Horváth, E. Forgács-Dajka, G.G. Barnaföldi: “Speed of sound in Kaluza-Klein Fermi gas", Accepted in APP-B (2025)

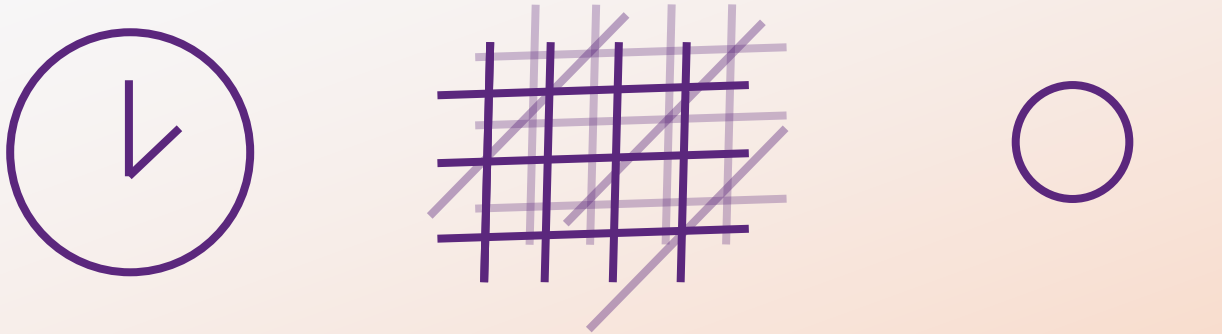
[4] A. Horváth, A. Wojnar, G.G. Barnaföldi: “The effects of strong gravity on the dispersion relation of massive particles in the Kaluza–Klein theory”, arXiv:2510.16631 (2025)

[2] A. Horváth, A. Wojnar, GG. Barnaföldi: "Modified Dispersion Relation in Kaluza–Klein Theory". Particles. 2026; 9(3):87.

What I mean by extra dimensions:

$x = (t, x_1, x_2, x_3, x_5)$

1 **3** **1_C**



time **space** **extra dimension**

spacetime

5-dimensional spacetime

Spatial dimension

Compactified: curled up into a microscopic **circle**

Too small to see

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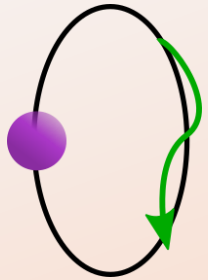
What happens to particles?

Effects of spacetime **curvature**

- Modified **uncertainty principle**

$$\sigma_x \sigma_p \geqslant \frac{\hbar}{2} + f(\mathcal{R})$$

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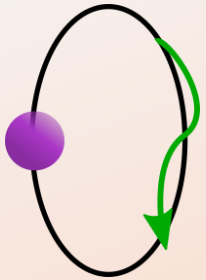
- Modified **uncertainty principle**

Moving in the extra dimension

- Particles gain **excitations** in the extra dimension

$$E = \sqrt{\mathbf{k}^2 + \left(\frac{N_{\text{exc}}}{r_{\text{c}}}\right)^2 + m^2}$$

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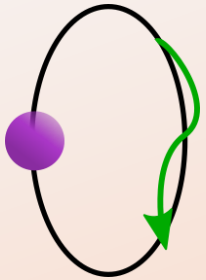
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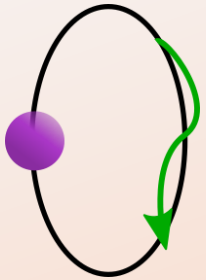
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- **Effective mass**

$$m_{\text{eff}}^2 = m^2 + \hbar^2 \frac{N_{\text{exc}}^2}{c^2 r_{\text{c}}^2}$$

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- Modified **thermodynamics**

Could lead to **observables** in **compact objects** – neutron stars and black holes

Why though?

Extra dimensions could **solve** certain **problems** in modern physics

- Unifying fundamental interactions
  - Electromagnetism
  - Weak force
  - Strong force

} **Q**uantum **F**ield **T**heory description

Why though?

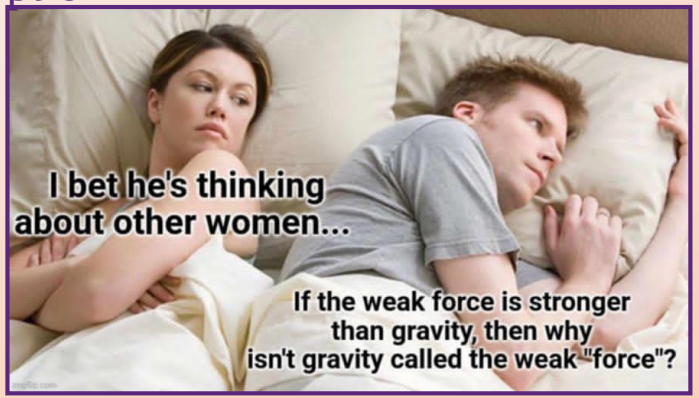
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  Gravity — **Geometric** description



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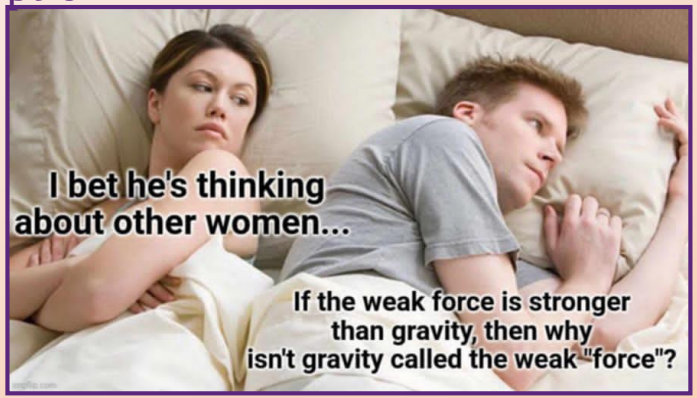
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- Dark matter, dark energy?

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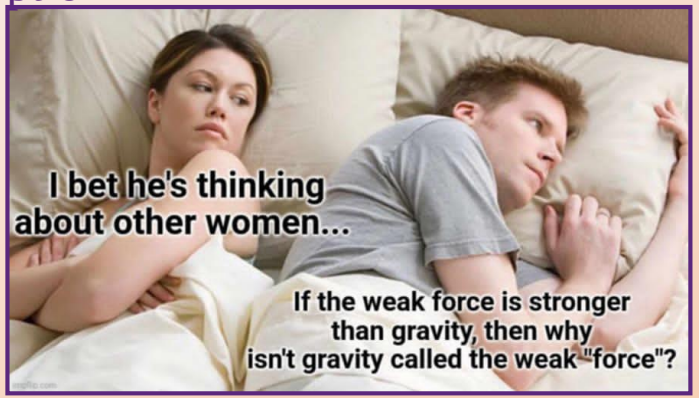
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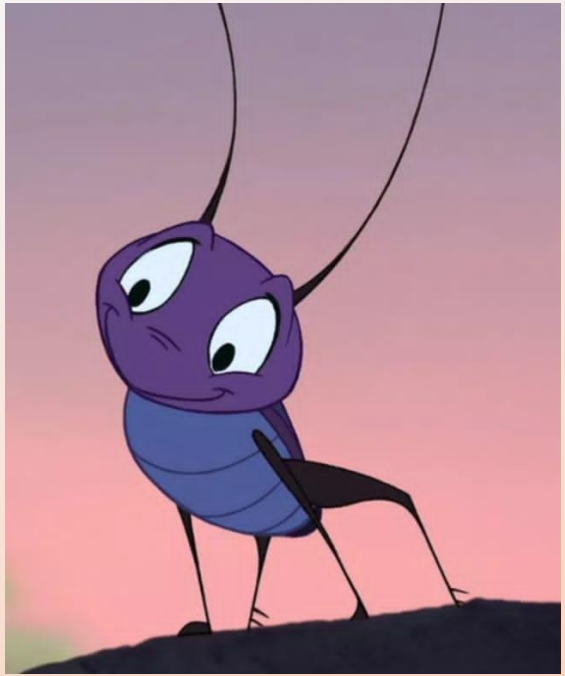


- Hierarchy problem – why is gravity so weak?
- Dark matter, dark energy?

- Many theories exist
- Phenomenological predictions often overlap

→ Let's use a **simple** model!

Let’s get down to business...



The Kaluza–Klein theory



Theodor Kaluza (1885-1954)

1921 Theodor Kaluza

- **Unify gravity** with **electromagnetism** in a 5D spacetime
- Cylinder condition – no metric component depends on the fifth dimension

Base of: scalar-tensor (Brans-Dicke) and string theories

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Theodor Kaluza (1885-1954)

1926 Oskar Klein

- The fifth **dimension** is a **microscopic** one, curled up into a circle



Oskar Klein (1894-1977)

↓

QM interpretation

Base of: scalar-tensor (Brans-Dicke) and string theories

Metric tensor

15 independent components:

10 + **4** + **1** extra

Usual 4D metric describing gravity

$$g_{AB} = \begin{bmatrix} g_{\alpha\beta} + \kappa^2 \Phi^2 A_\alpha A_\beta & \kappa \Phi^2 A_\alpha \\ \kappa \Phi^2 A_\beta & \Phi^2 \end{bmatrix}$$

Scalar field

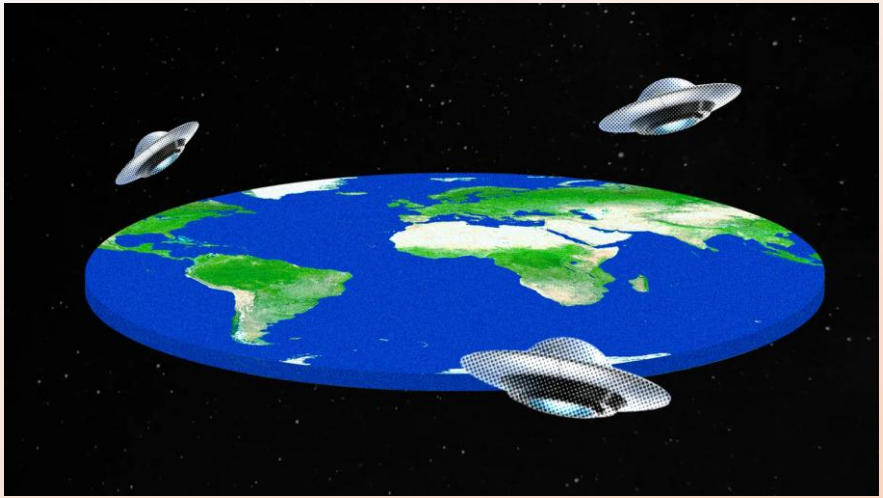
5D metric

Vector potential of electromagnetism

Overduin, J. M., & Wesson, P. S. 1997, Phys. Rept., 283, 303, doi: arXiv:gr-qc/9805018

But I’m only 4-dimensional...

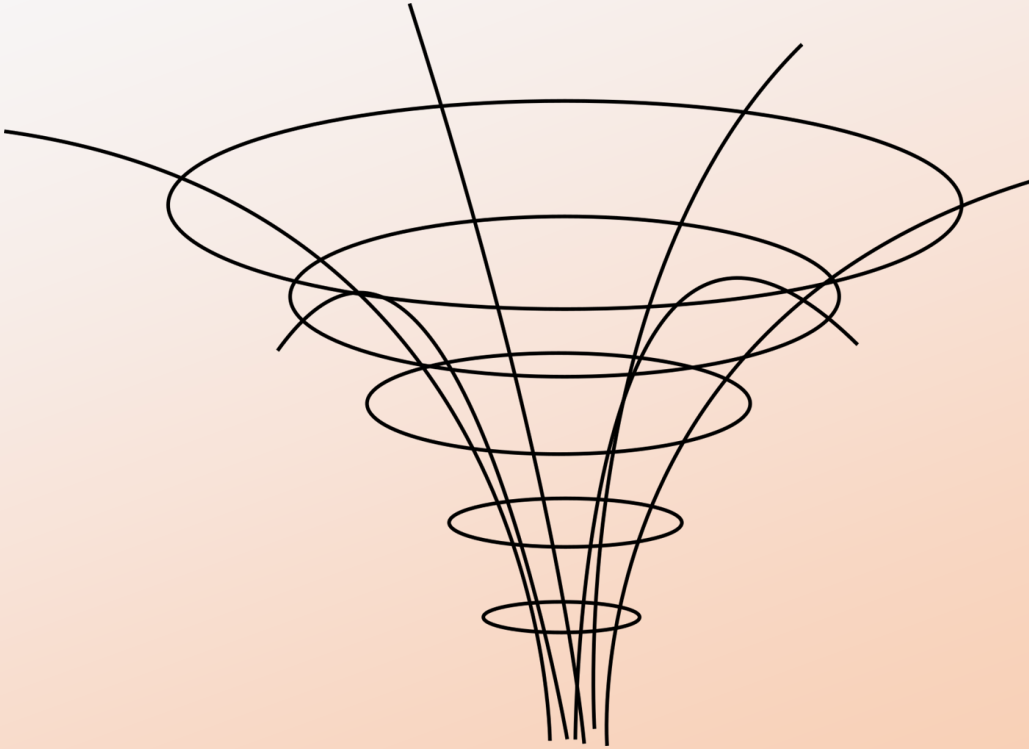
Look at things from a 4D point of view



Black holes

- Spherically symmetric
- Static
- Empty

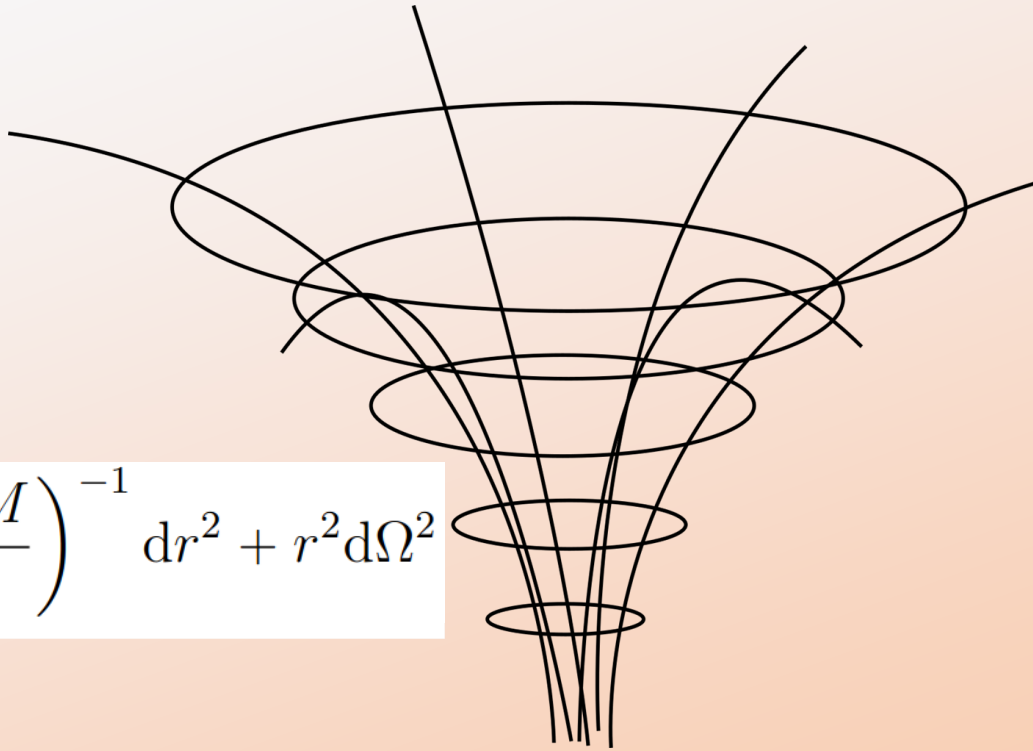
Looking for **solutions t**o the field equations in this **form**



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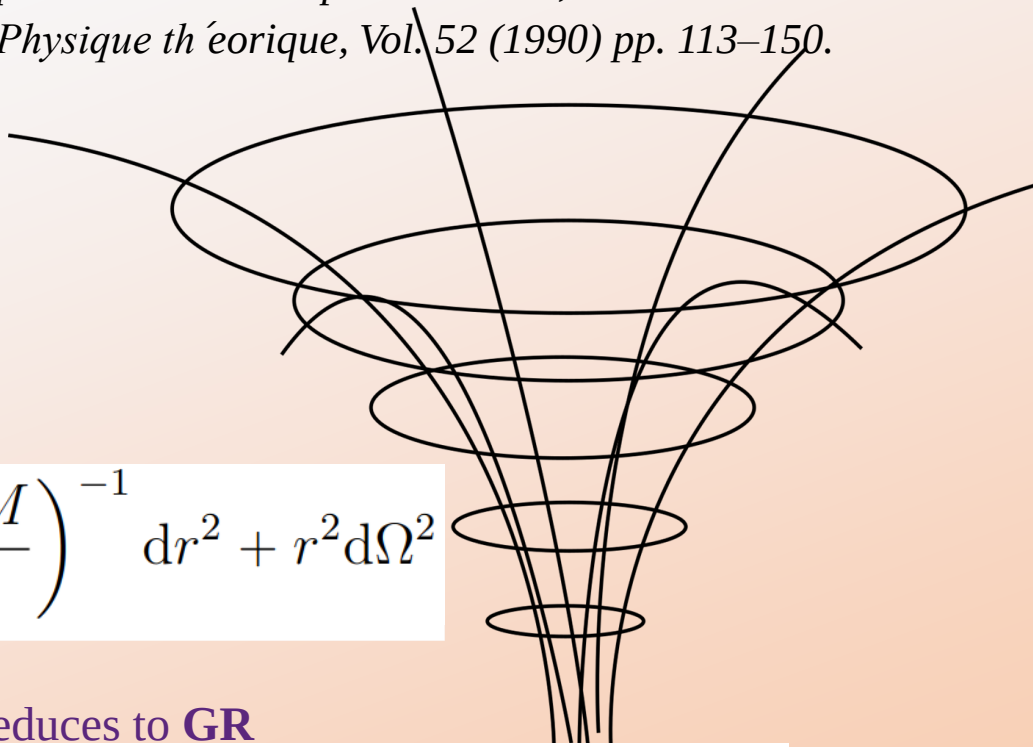


- GR:

$$\mathrm{d}s^2 = -\left(1-\frac{2M}{r}\right)\mathrm{d}t^2 + \left(1-\frac{2M}{r}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

Black holes

R. Coquereaux and G. Esposito-Farese, in Annales de l'IHP Physique th ́eorique, Vol. 52 (1990) pp. 113–150.



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- KK:

a* = *b reduces to **GR**

$$\mathrm{d}s^2 = -\left(1-\frac{a}{r}\right)^{\frac{b-d}{a}}\mathrm{d}t^2 + \left(1-\frac{a}{r}\right)^{-\frac{b+d}{a}}\mathrm{d}r^2 + r^2\left(1-\frac{a}{r}\right)^{1-\frac{b+d}{a}}\mathrm{d}\Omega^2$$

$$a^2 = b^2 + 3d^2$$

d is **proportional** to the **scalar field**

$$\sigma(r) = \frac{d}{a}\ln\left(1-\frac{a}{r}\right)$$

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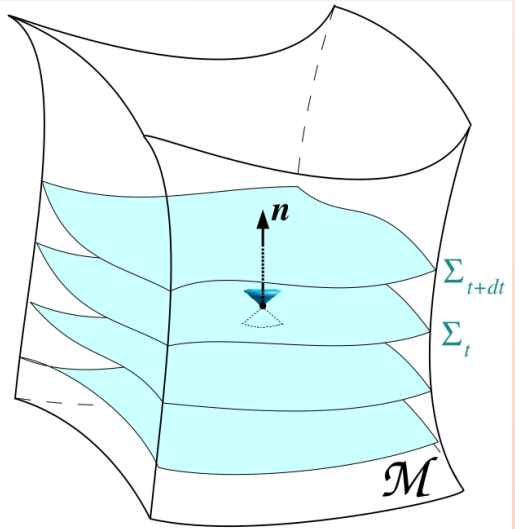
Particles

$$L = -m\sqrt{-g_{\mu\nu}(x)\dot{x}^{\mu}\dot{x}^{\nu}}$$

- We would like to describe time-evolution
- In quantum mechanics time is a parameter

↓

- Splits 4D spacetime into **3D foliations**



$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

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L. Petruzziello and F. Wagner, Physical Review D 103, 104061 (2021).

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$$\sigma_{\mathrm{p}}\rho \gtrsim \pi\hbar\left[1 - \frac{\rho^2\mathcal{R}|_{x_0}}{12\pi^2} + \xi\frac{\rho^4}{\lambda_C^2}\bar{\nabla}_j N_i \bar{\nabla}^j N^i\Big|_{x_0}\right]$$

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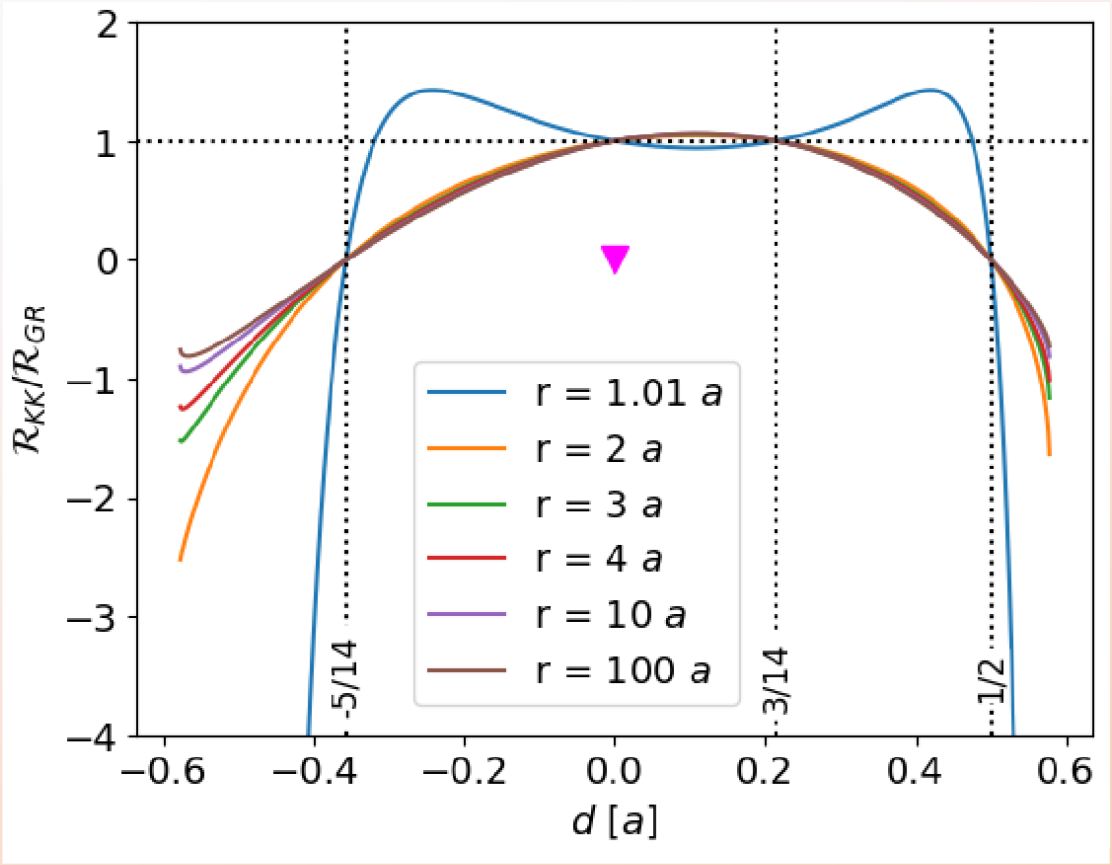
Modification caused by curvature

Ricci scalar curvature of phase space

Cat Figure

Calculated from G_{ij}

$N^i = 0$



$$\mathcal{R} = \frac{4a^2 - (3b+d)^2}{8r^4}\left(1 - \frac{a}{r}\right)^{\frac{3b+d}{2a}-2}$$

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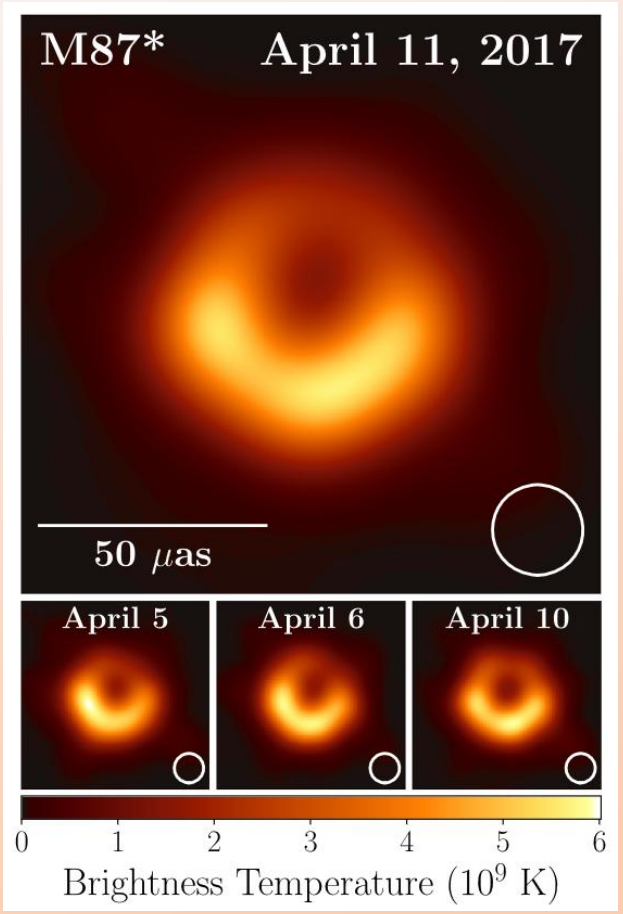
Effective mass near black holes

- Extended uncertainty principle

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EHT Collaboration, Akiyama, K., et al.. Astrophys. J. Lett., 875:L1, 2019.

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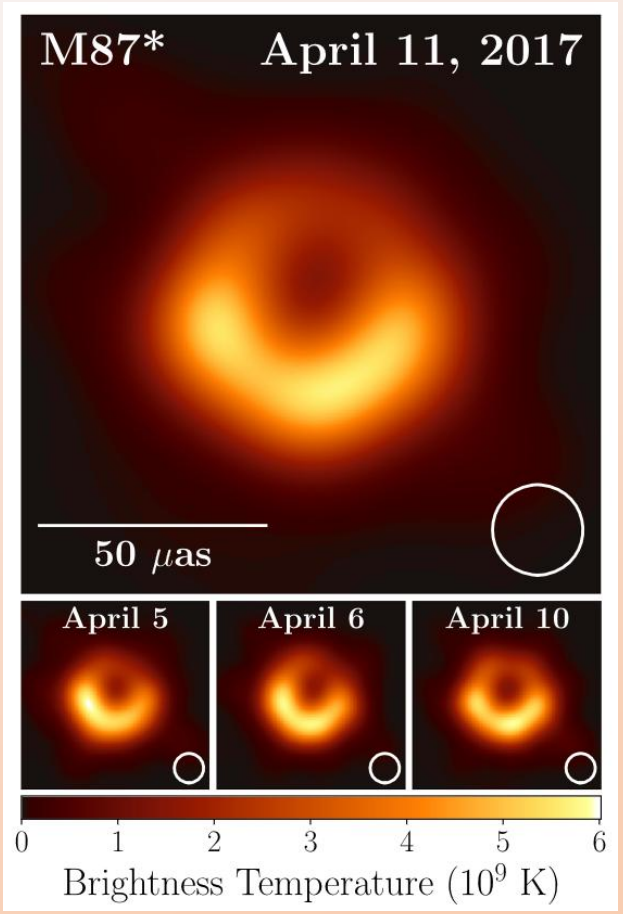
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Modified photon ring?



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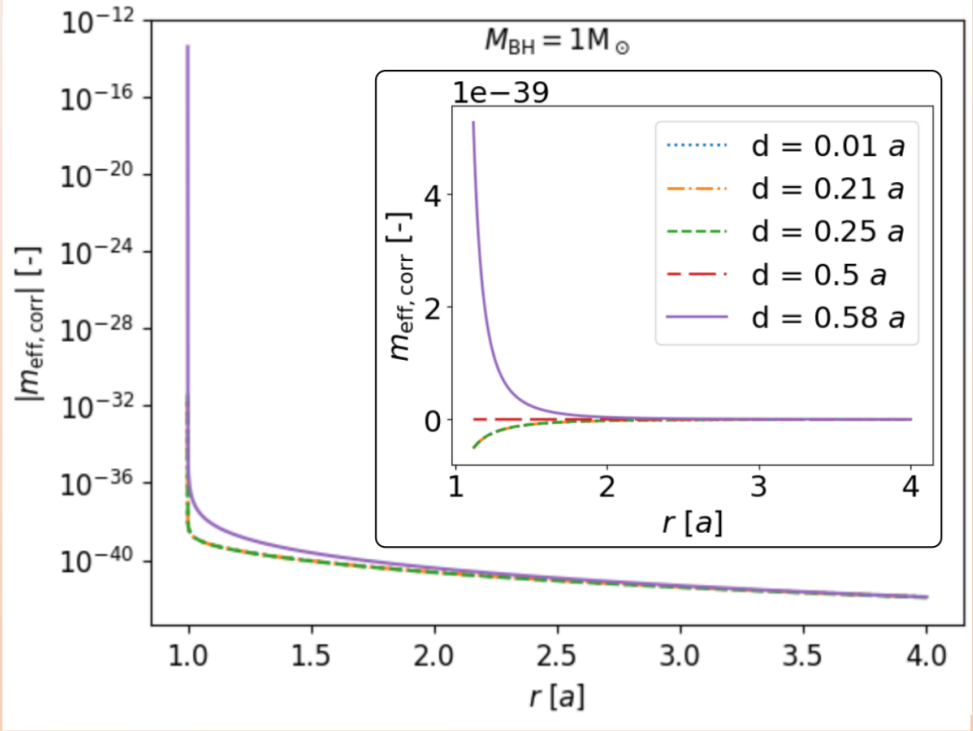
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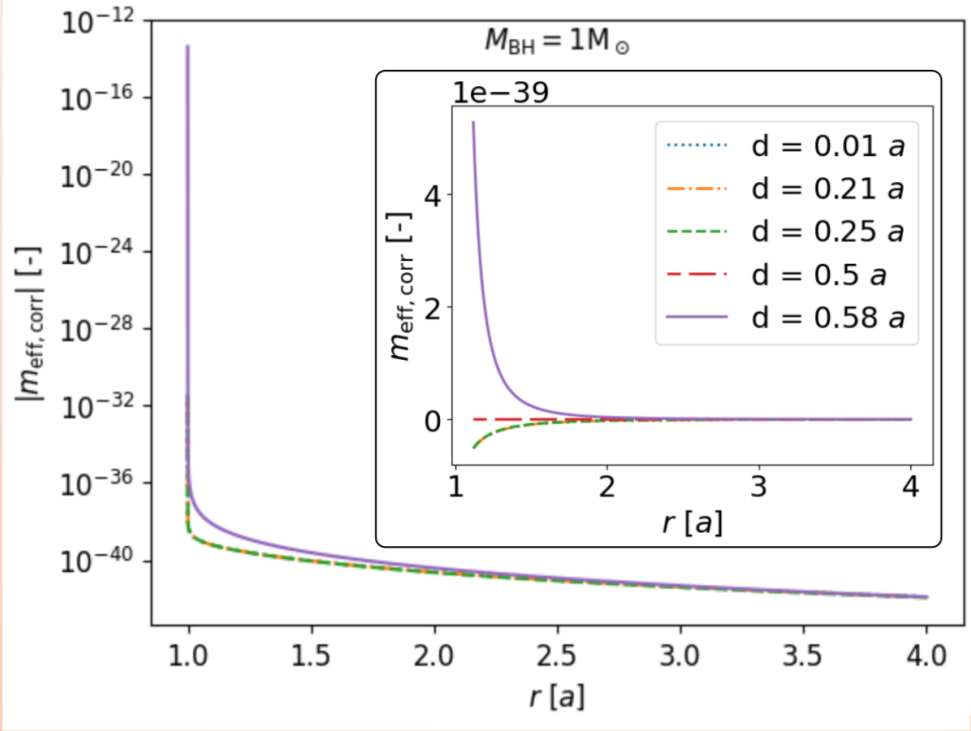
Particle decay?
Hawking radiation?

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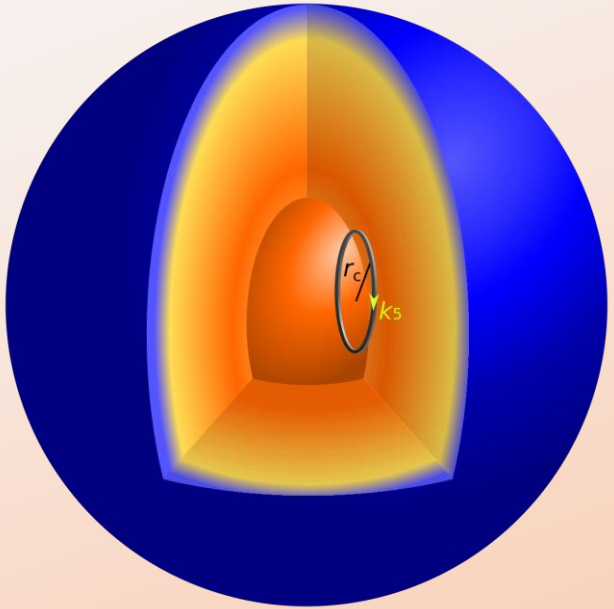
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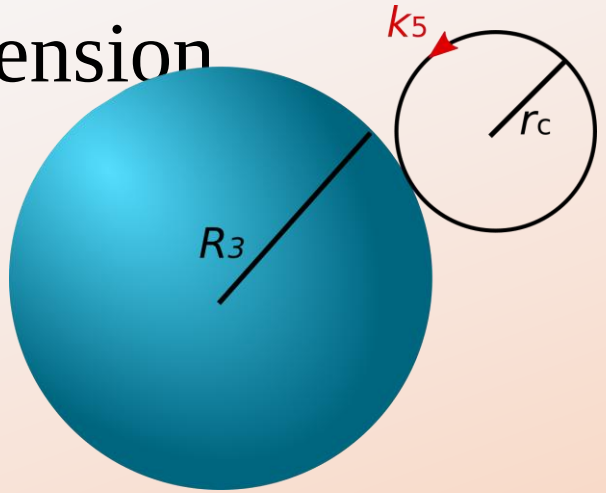
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Let’s move on to neutron stars!



A. Horváth, E. Forgács-Dajka, G.G. Barnaföldi: "Application of Kaluza-Klein Theory in Modeling Compact Stars: Exploring Extra Dimensions" Mon.Not.Roy.Astron.Soc. 536 (2024) 1, 816-826

Effective mass from movement in 5th dimension

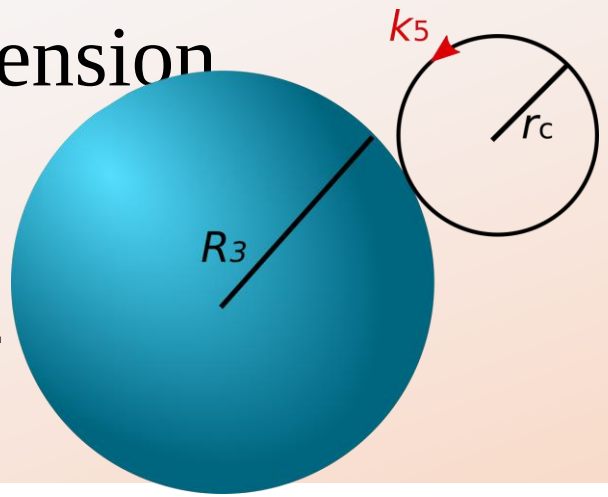


Particles moving in the extra dimension possess a modified effective mass from a 3-dimensional point of view.

$$E = \sqrt{\mathbf{k}^2 + k_5^2 + m^2} = \sqrt{\mathbf{k}^2 + \left(\frac{N_{\mathrm{exc}}}{r_{\mathrm{c}}}\right)^2 + m^2}$$

Here the ground state of the Kaluza–
–Klein ladder is the neutron.

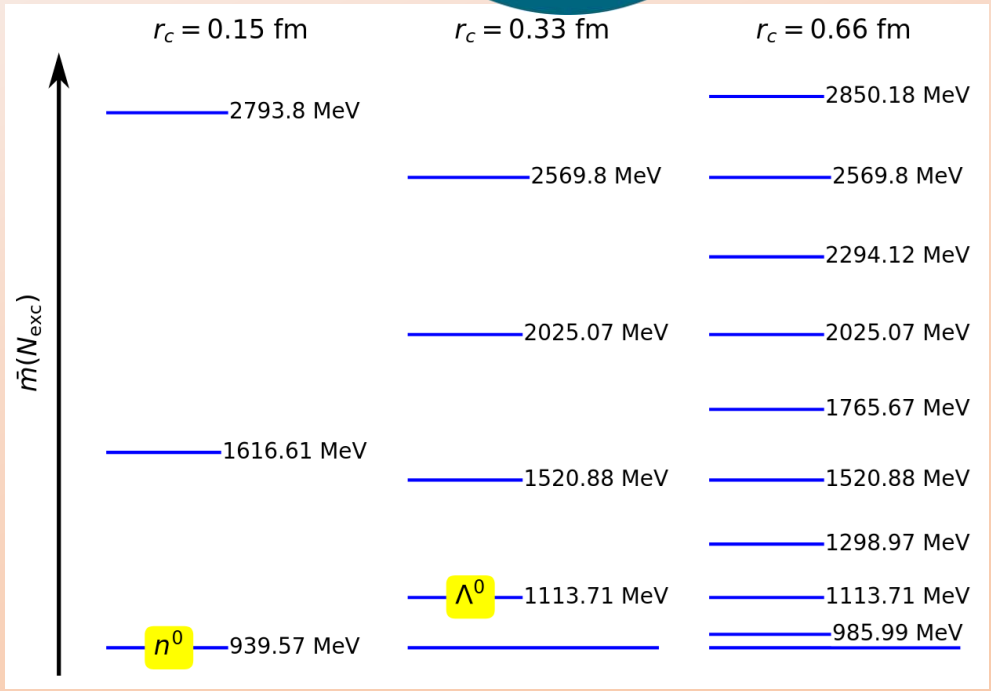
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$$\bar{m}^2(N_{\text{exc}}) = m^2 + k_5^2 \qquad k_5 = \frac{N_{\text{exc}}}{r_c}$$



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Modelling neutron stars

Microscopic properties of spacetime and nuclear matter are connected to the mass-radius relation via

- the Tolman–Oppenheimer–Volkoff (TOV) equation

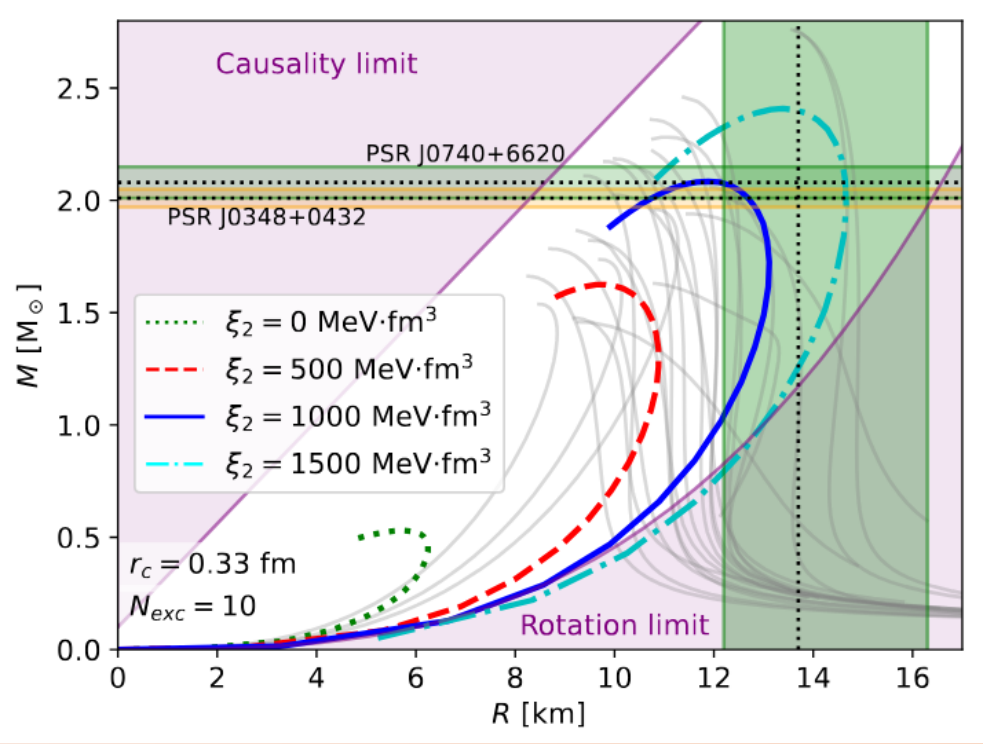
$$\frac{\mathrm{d}p(r)}{\mathrm{d}r} = -\frac{GM(r)\varepsilon(r)}{r^2} \times \left[1+\frac{p(r)}{\varepsilon(r)}\right]\left[1+\frac{4\pi r^3 p(r)}{M(r)}\right]\left[1-\frac{GM(r)}{r}\right]^{-1}$$

$$M(r) = \int_0^r \mathrm{d}r' 4\pi r'^2 \varepsilon(r')$$

Boundary conditions: central energy density ε_c, surface pressure $p(R)$

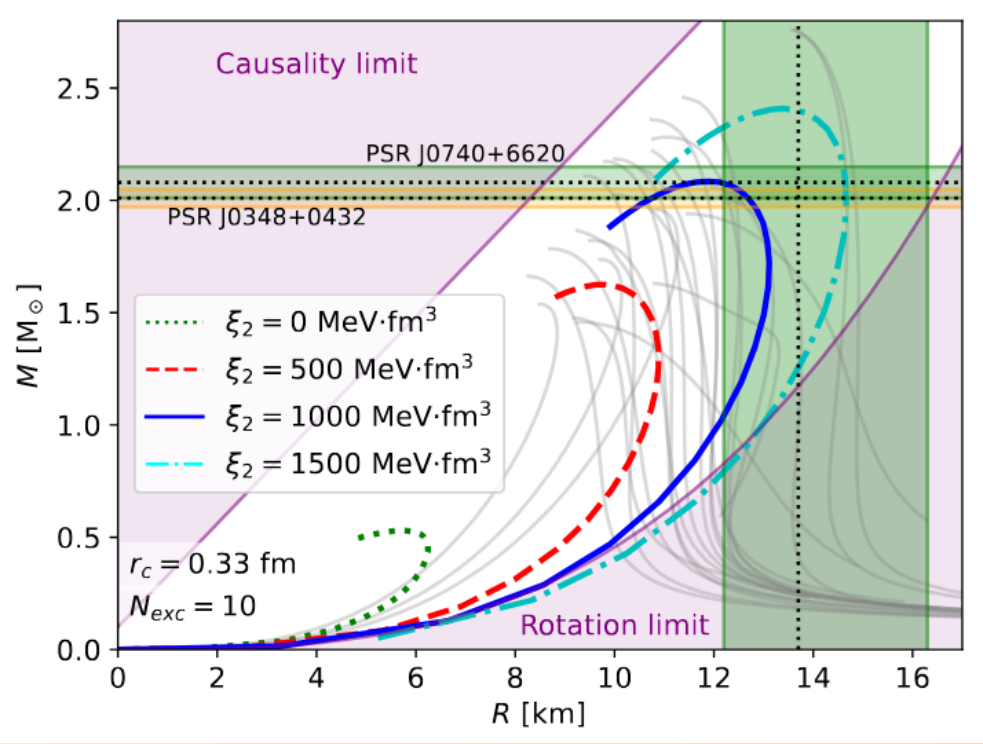
- and the equation of state (EoS)
  Connects pressure (p) and energy density (ε)
  Encodes microscopic properties of the theory

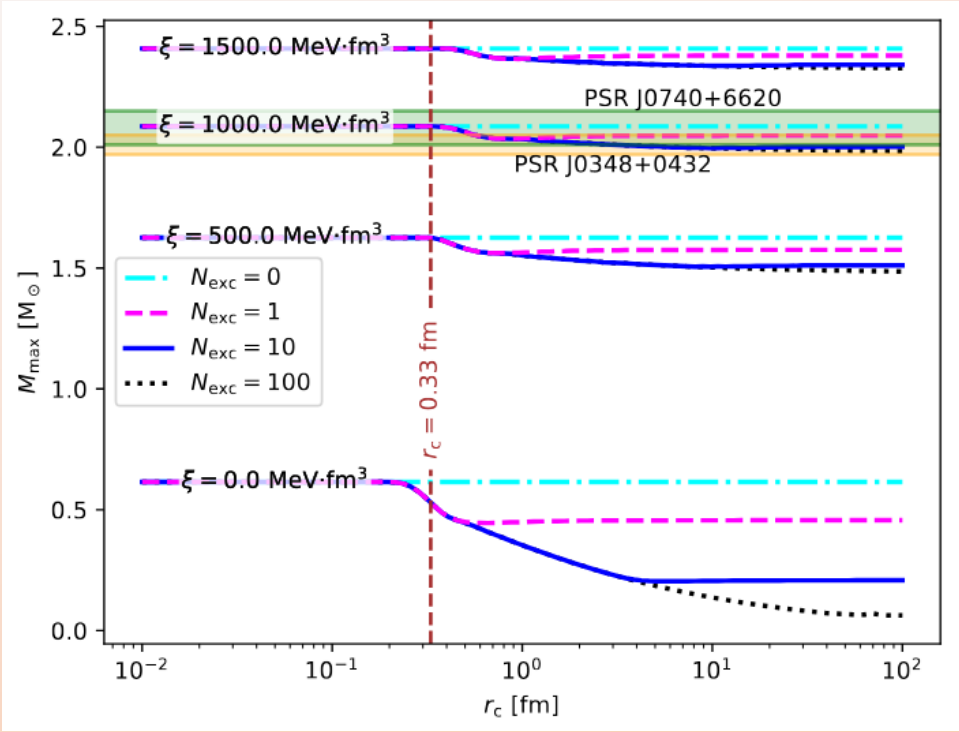
Modelling neutron stars



Fonseca E., et al., 2021, The Astrophysical Journal Letters, 915, L12
Miller M. C., et al., 2021, Astrophys. J. Lett., 918, L28
Antoniadis J., et al., 2013, Science, 340, 6131

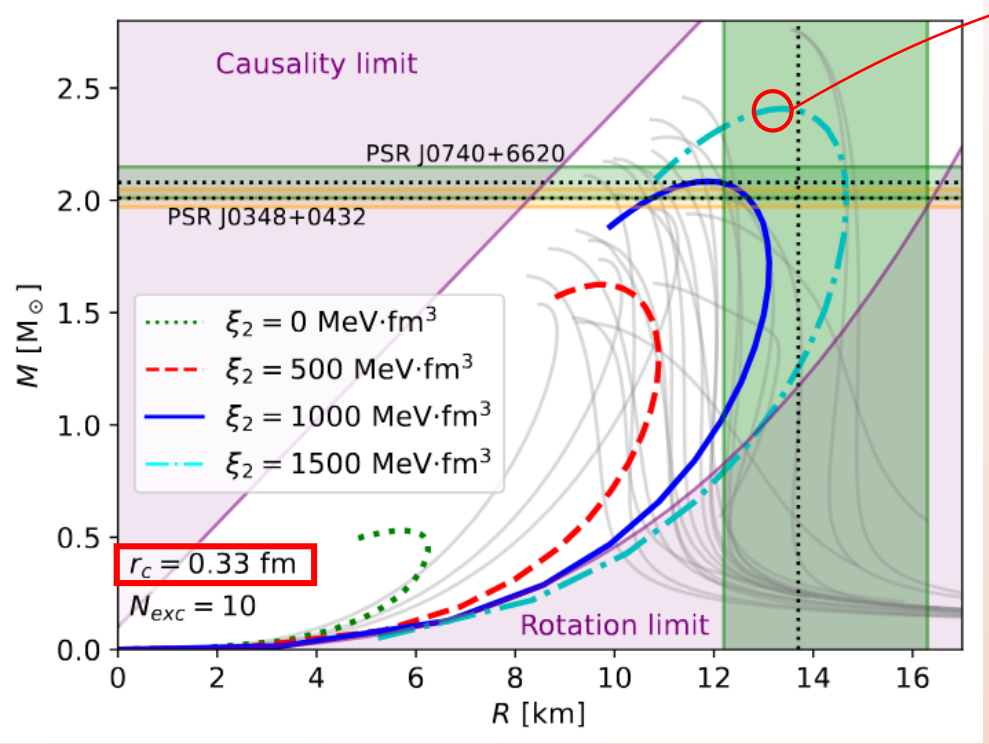
Modelling neutron stars

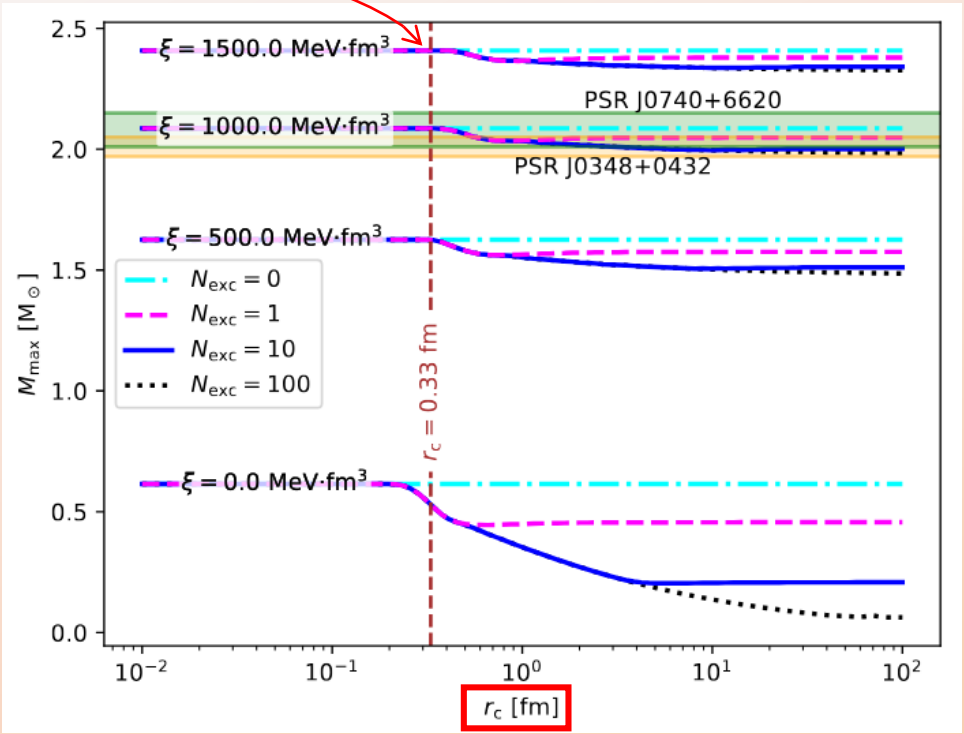




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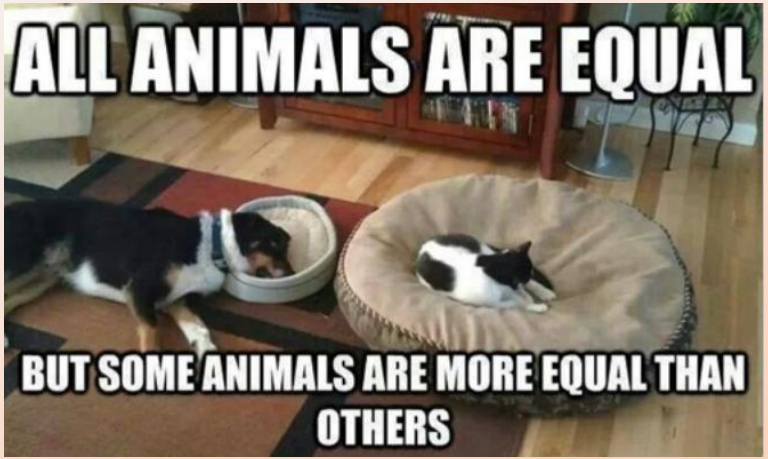
Modelling neutron stars





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Comparison...


ALL ANIMALS ARE EQUAL
BUT SOME ANIMALS ARE MORE EQUAL THAN OTHERS

What happens to particles?

Effects of spacetime **curvature**

- Modified **uncertainty principle**

$$m_{\text{eff}} = \sqrt{m^2 + \frac{\hbar^2 \mathcal{R}}{6}}$$

Moving in the extra dimension

- Particles gain **excitations** in the extra dimension

$$m_{\text{eff}}^2 = m^2 + \hbar^2 \frac{N_{\text{exc}}^2}{c^2 r_{\text{c}}^2}$$

What happens to particles?

Effects of spacetime **curvature**

- Modified **uncertainty principle**

$$m_{\mathrm{eff}} = \sqrt{m^2 + \frac{\hbar^2 \mathcal{R}}{6}}$$

Effect at black hole horizon – particle decay?

$$\sim \mathrm{e}^{iEt} \approx \mathrm{e}^{imt}$$

Moving in the extra dimension

- Particles gain **excitations** in the extra dimension

$$m_{\mathrm{eff}}^2 = m^2 + \hbar^2 \frac{N_{\mathrm{exc}}^2}{c^2 r_{\mathrm{c}}^2}$$

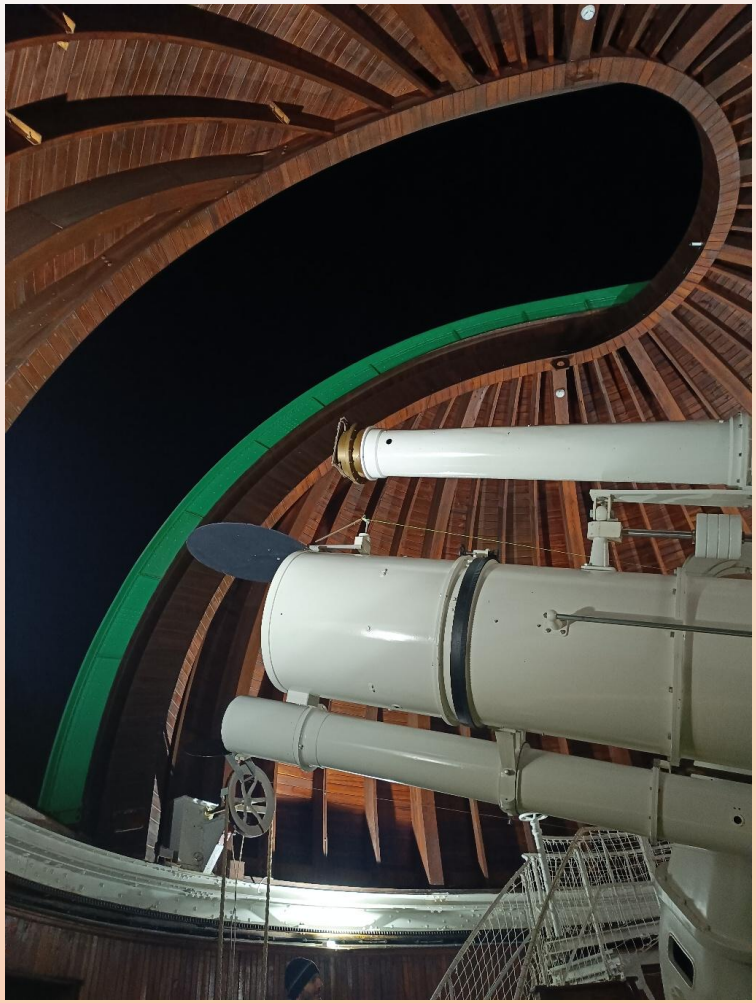
Neutron star observables are modified

Summary

Kaluza–Klein theory with one **extra** spatial **dimension**

Two cases studied:

- Modification to phase space due to spacetime **curvature**
  - Effect at **black hole** horizon
  - Particle decay
- Particles **moving in extra dimension** gaining excitations
  - Effect on **neutron star** observables
  - Maximal mass

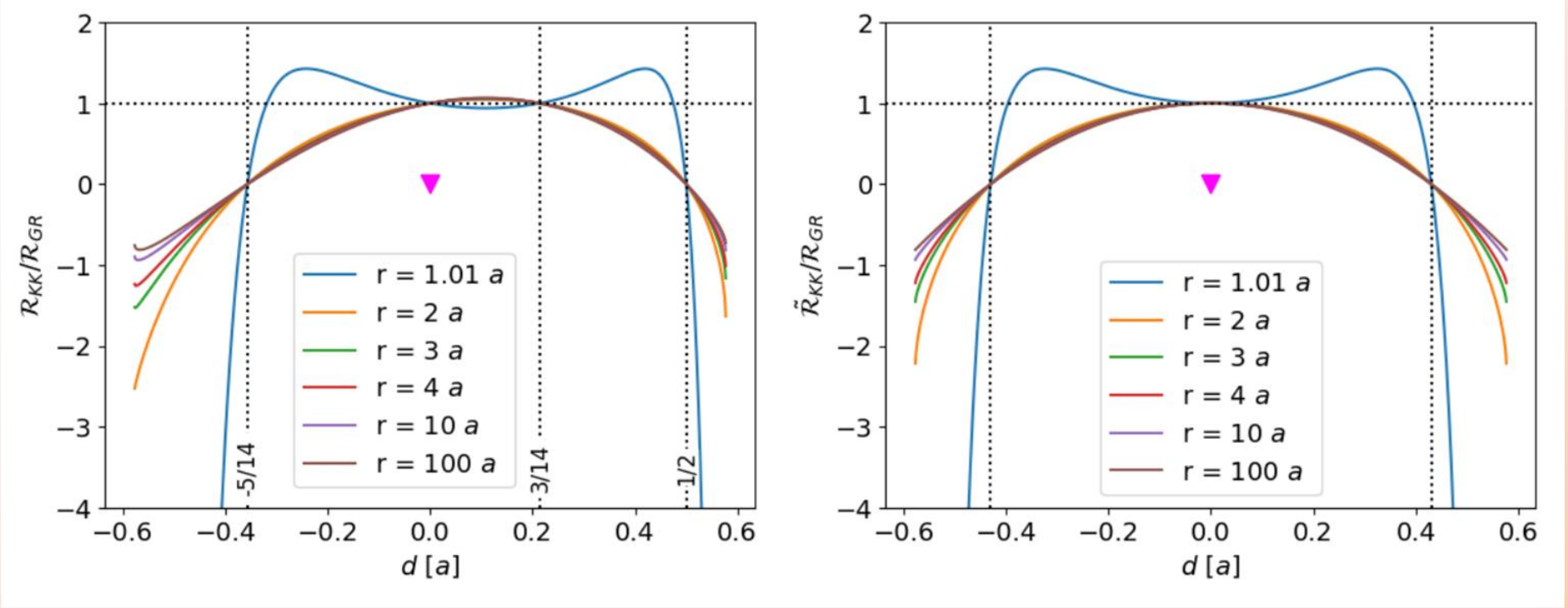


Thank you for your attention!

Ricci scalar curvature of phase space

Cat FIgure

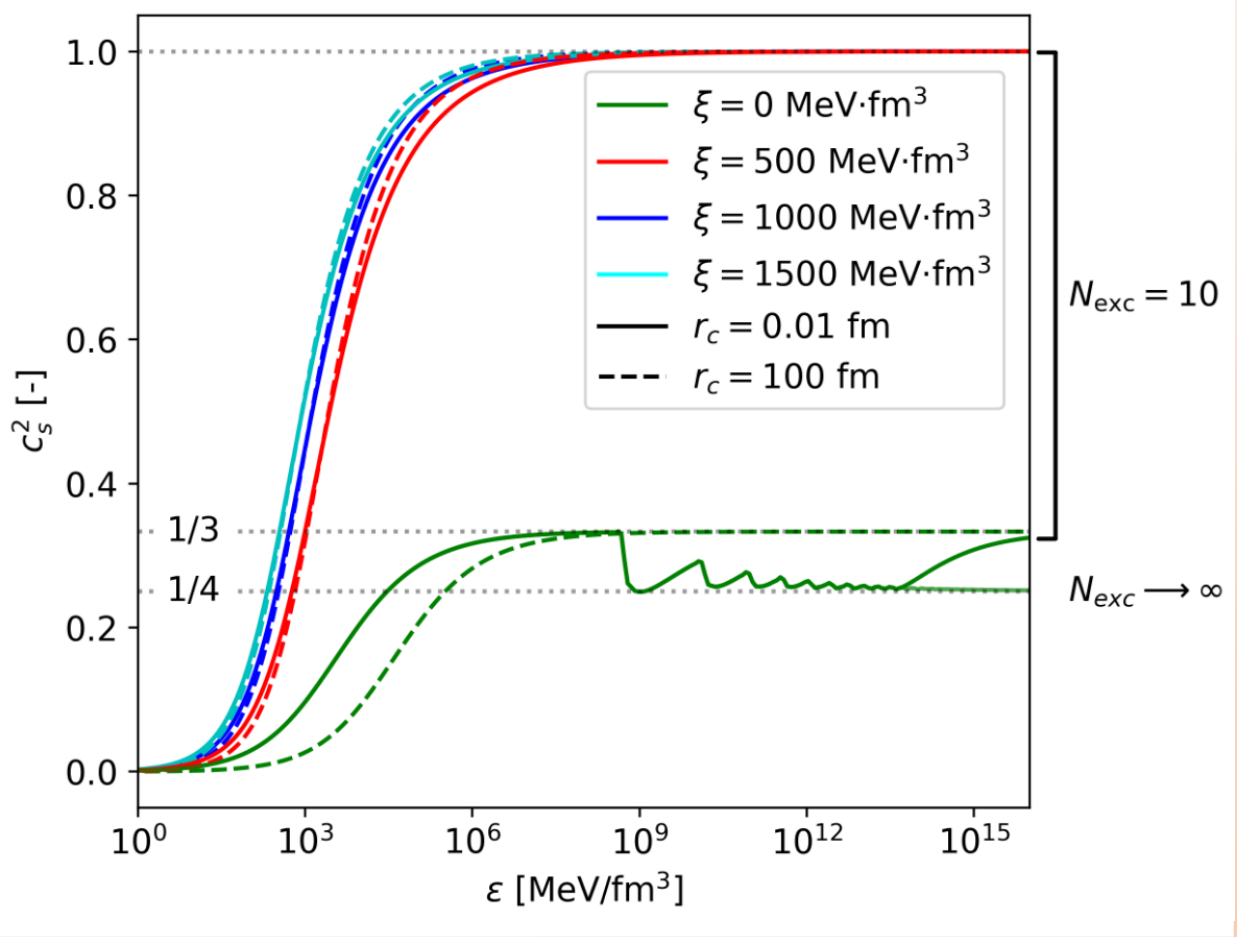
Calculated from G_{ij}



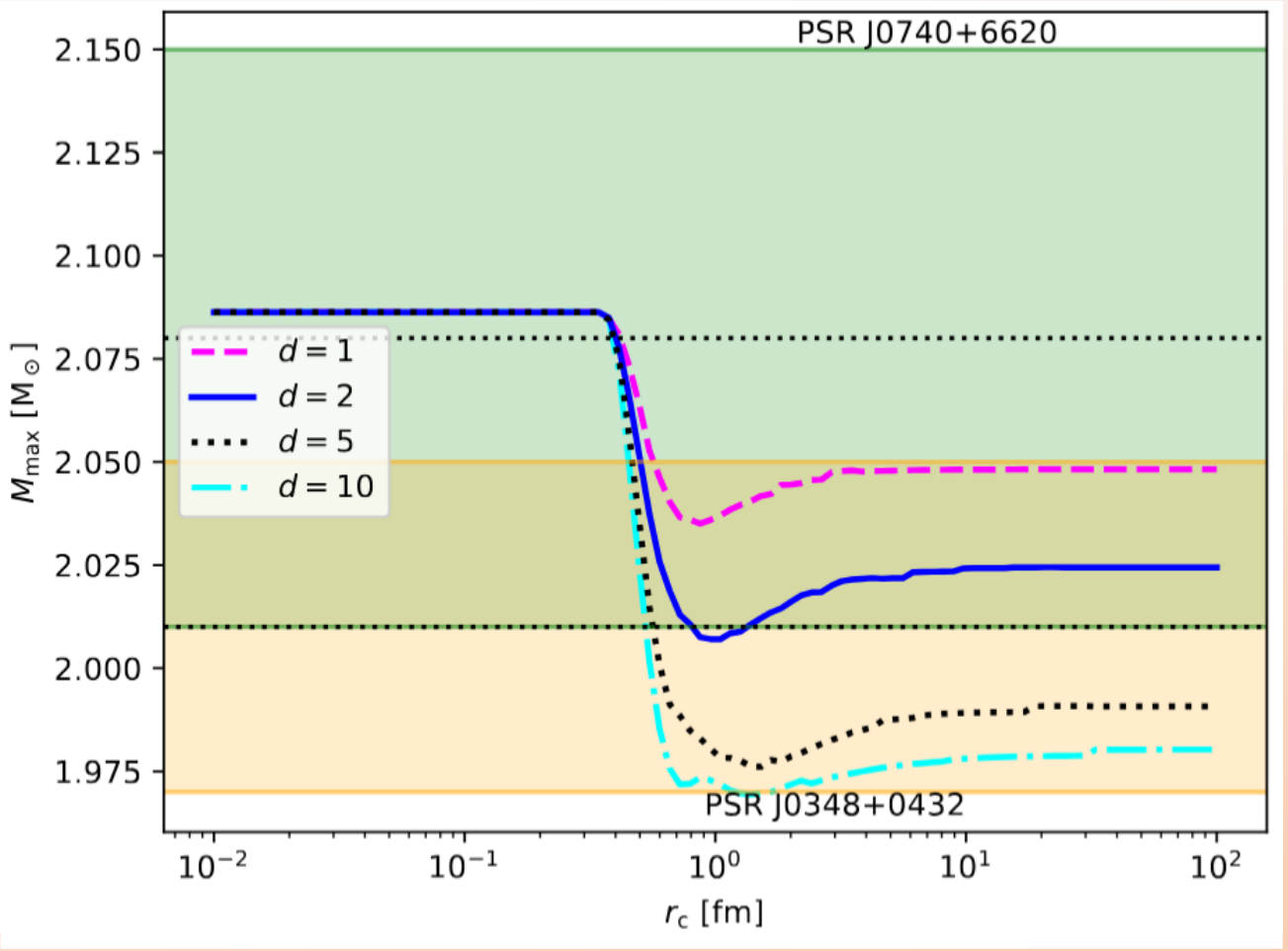
$$\mathcal{R} = \frac{4a^2 - (3b+d)^2}{8r^4}\left(1 - \frac{a}{r}\right)^{\frac{3b+d}{2a}-2}$$

$$\tilde{\mathcal{R}} = \frac{4a^2 - 9b^2}{8r^4}\left(1 - \frac{a}{r}\right)^{\frac{3b}{2a}-2}$$

Speed of sound



Multiple extra dimensions



Different observers

- Flat spacetime – Rindler coordinates

$$\mathrm{d}s^2 = -(\alpha x)^2 \mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$$

- Schwarzschild spacetime – Painlevé–Gullstrand coordinates

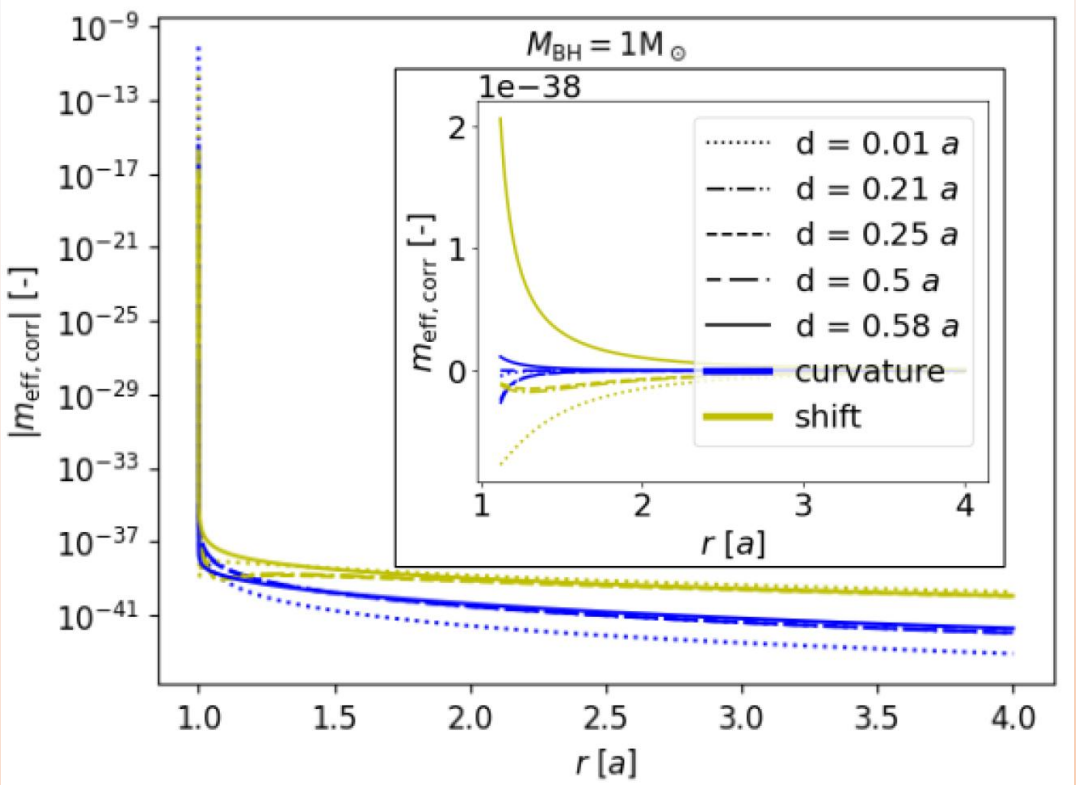
$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r}\right)\mathrm{d}t_{\mathrm{PG}}^2 + 2\sqrt{\frac{2M}{r}}\,\mathrm{d}t_{\mathrm{PG}}\,\mathrm{d}r + \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

- Kaluza–Klein Schw. spacetime – Painlevé–Gullstrand coordinates

$$\begin{aligned}\mathrm{d}s^2 = &-\left(1 - \frac{a}{r}\right)^{\frac{b-d}{a}}\mathrm{d}T^2 + \\ &+ 2\left[\left(1 - \frac{a}{r}\right)^{-\frac{2d}{a}} - \left(1 - \frac{a}{r}\right)^{\frac{b-3d}{a}}\right]^{\frac{1}{2}}\mathrm{d}T\mathrm{d}r + \\ &+ \left(1 - \frac{a}{r}\right)^{-\frac{2d}{a}}\mathrm{d}r^2 + r^2\left(1 - \frac{a}{r}\right)^{1-\frac{b+d}{a}}\mathrm{d}\Omega^2\end{aligned}$$

Free-falling observer

- Non-zero shift vector, N^i



$$m_{\mathrm{eff}} = \sqrt{m^2 + \frac{\hbar^2 \mathcal{R}}{6} - \hbar^2 \xi \frac{\rho^2}{2\lambda_{\mathrm{C}}^2} \bar{\nabla}_j N_i \bar{\nabla}^j N^i}$$

But I’m only 4-dimensional...

Nothing’s 5D here...



- Dimensional reduction

Scalar field

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{e}^{2\sigma}\left(\mathrm{d}x^5 + \kappa A_\mu \mathrm{d}x^\mu\right)^2$$

- General relativistic action (Einstein–Hilbert)

$$S = \frac{1}{2\kappa}\int R\sqrt{-g}\,\mathrm{d}^4x$$

- Kaluza–Klein action

$$S = \int \frac{\sqrt{-g}}{16\pi\mathbb{G}\mathrm{e}^{-\sigma}}\left[R - \frac{\kappa^2}{4}\mathrm{e}^{2\sigma}F_{\mu\nu}F^{\mu\nu} - 2\mathrm{e}^{-\sigma}\Box\mathrm{e}^{\sigma}\right]\mathrm{d}^4x$$

From here the **field equations** can be **derived**

R. Coquereaux and G. Esposito-Farese, in Annales de l’IHP Physique th ́eorique, Vol. 52 (1990) pp. 113–150.

Boson stars

- Assume existence of an additinal scalar field
- Supported by the uncertainty principle

- For the 5D Kaluza–Klein:

$$S_5 = \frac{1}{16\pi G_5}\int \mathrm{d}^5x\sqrt{-\gamma}\,\left(R_5 - \frac{1}{2}\gamma^{MN}\left(\Psi^*_{,M}\Psi_{,N} + \Psi^*_{,N}\Psi_{,M}\right) - m^2\Psi^*\Psi\right)$$

with Betti Hartmann