Evolving Dark Energy: Past, Future Fates, and Physical Bounds

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Introduction & Motivation

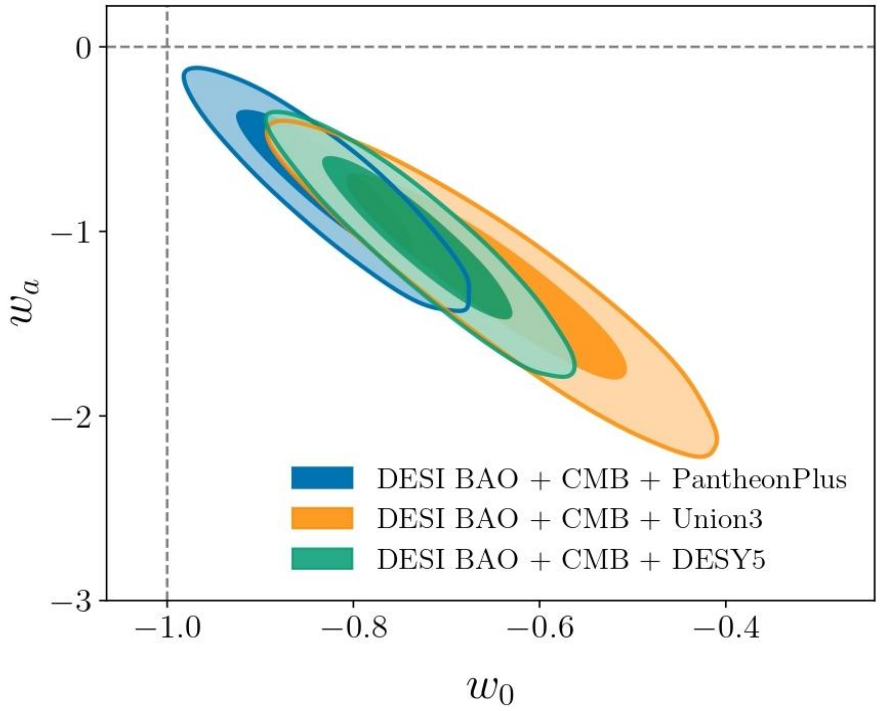
- Equation of state: $w_i = p_i/\rho_i$
- Standard ΛCDM: the dark energy equation of state is constant ($w_{DE} = -1$)

Why dynamical dark energy?

- Cosmological tensions (Hubble tension)
- Recent measurements (DESI, DES Y6)
- Easiest choice: The Chevallier–Polarski–Linder (CPL) parametrization

$$w_{DE} = w_0 + w_a(1 - a)$$

Introduction & Motivation



- Lighter region: 2σ (95%) confidence
- Dark region: 1σ (68%) confidence

Desi collaboration (2024), arXiv:2404.03002

Problems with CPL

Future problem

- It’s a linear Taylor expansion around today (a = 1)
  - Consequence: The usage is limited
- Most studies and cosmological packages deal with the past (a ≤ 1)
  - What about the future?
- We can’t use CPL to describe the late future: EoS (ω) diverges

Solution: Introducing other, non diverging parametrisations, to describe the future

How w(a) Dictates the Cosmic Fate

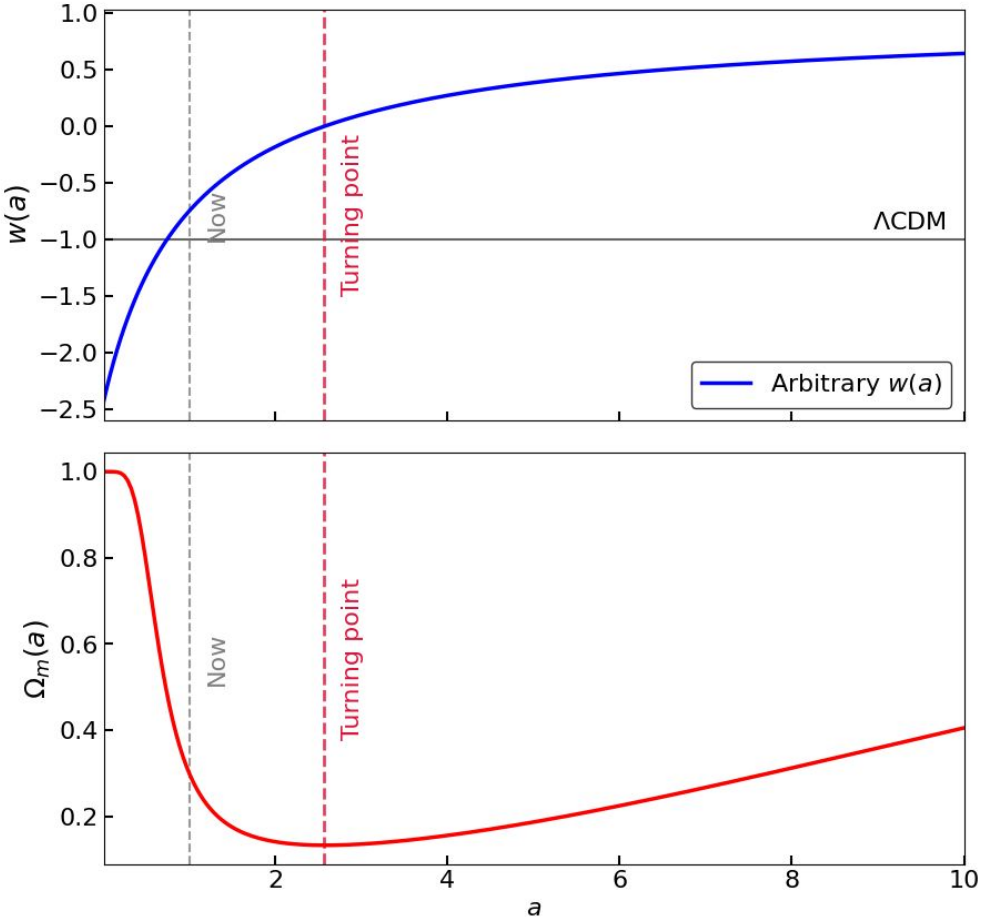
- Continuity equation:

$$\dot{\rho} + 3H(\rho + p) = 0 \quad \xrightarrow{p=w\rho} \quad \frac{d\rho}{da} + \frac{3}{a}\rho(1 + w(a)) = 0$$

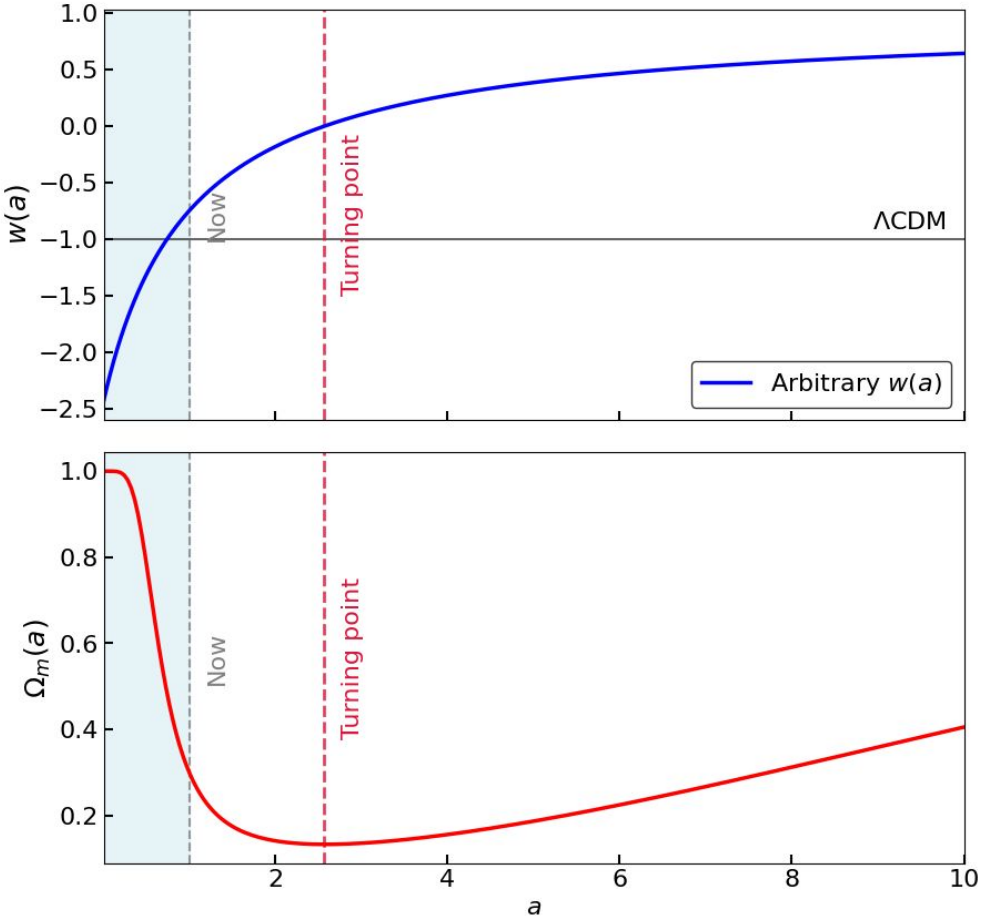
- Behaviour of Ω_m:

$$\Omega_m(a) = \frac{\rho_m(a)}{\rho_m(a) + \rho_{DE}(a)} \quad , \quad \frac{d\Omega_m}{da} = 0 \quad \longrightarrow \quad w(a) = 0$$

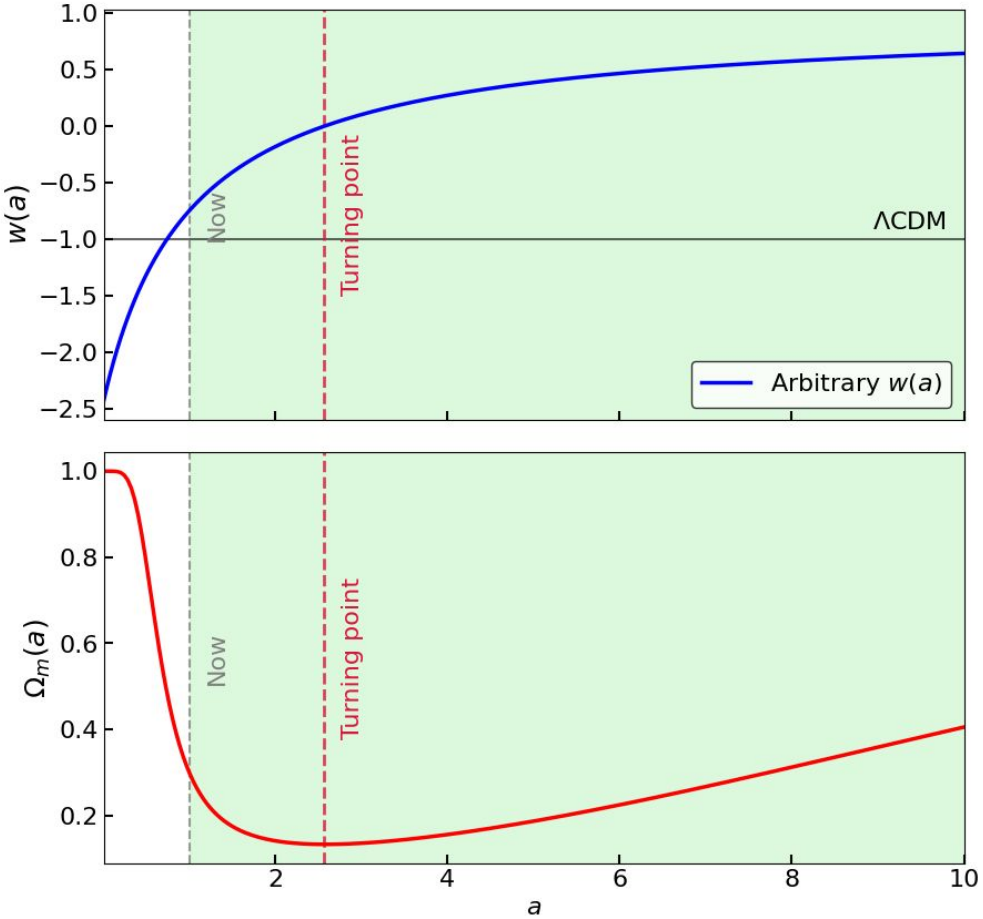
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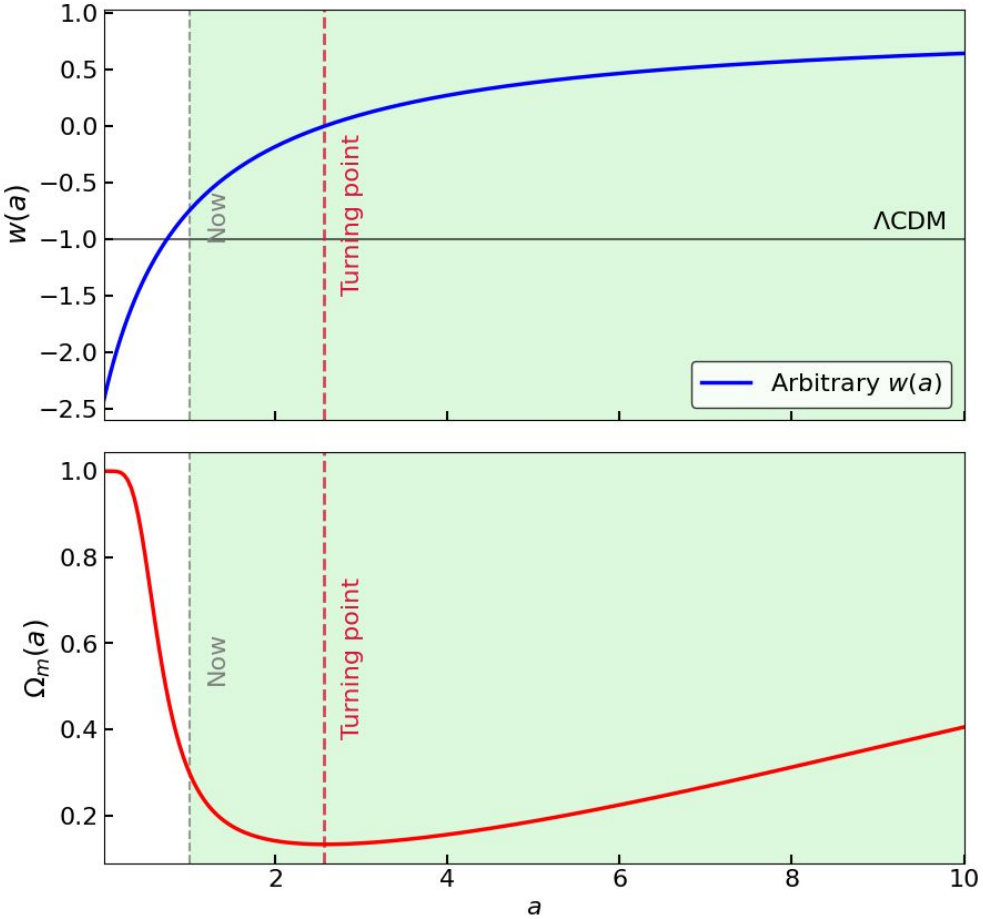


How w(a) Dictates the Cosmic Fate



How w(a) Dictates the Cosmic Fate

- ***Matter dominance again??***
- ***How?***



Some Alternative Parametrisations

Normalised-CPL

- Easiest solution to the divergence: add a denominator:

$$w(a) = w_0 + 2w_a \frac{1-a}{1+a} \qquad \lim_{a\to\infty} w(a) = w_0 - 2w_a$$

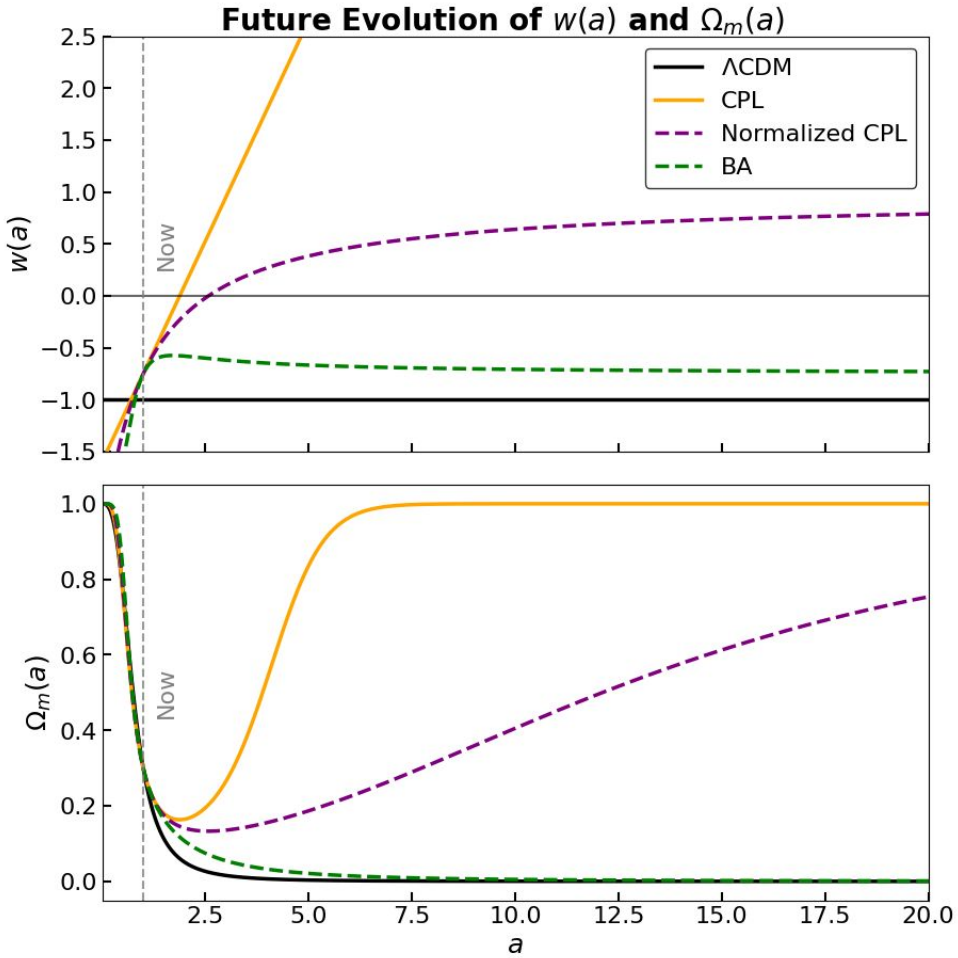
Barboza–Alcaniz (BA) parametrisation

- BA is a widely used bounded parametrisation to describe the future:

$$w_0 + w_a \frac{1-a}{1-2a+2a^2} \qquad \lim_{a\to\infty} w(a) = w_0$$

Some Alternative Parametrisations

Used values:
$w_0 = -0.75$
$w_a = -0.85$

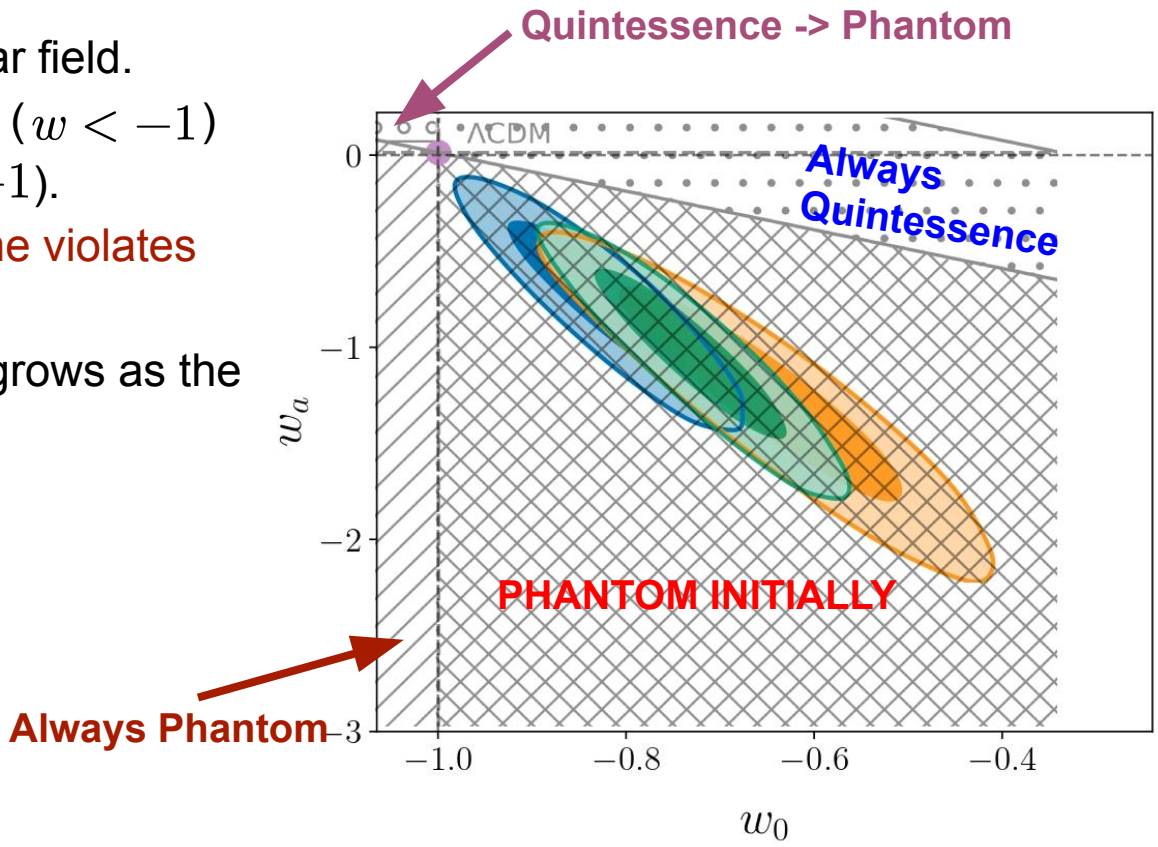


The Phantom Divide Line: A Challenge for Scalar Fields

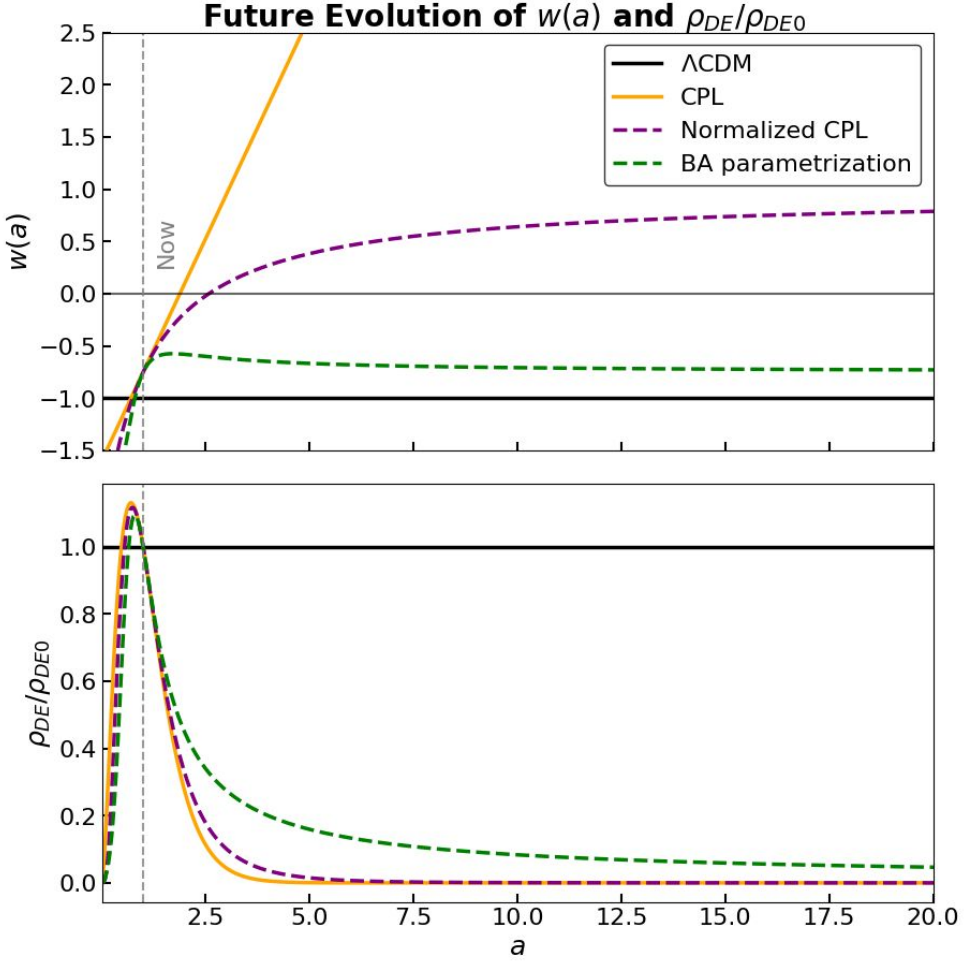
- **Model:** Dark Energy as a scalar field.
- **Observations:** Early Phantom ($w < -1$)

→ Present Quintessence ($w > -1$).

- **Theory Clash:** Phantom regime violates energy conditions (NEC).
- **Non-physical behavior:** ρ_{DE} grows as the universe expands.



The Phantom Divide: A Challenge for Scalar Fields



The Phantom Divide: A Challenge for Scalar Fields

The Jassal–Bagla–Padmanabhan (JBP) parametrisation

$$w(a) = w_0 + w_a a(1-a)$$

- Functional form reduces the probability of phantom crossing.
- Problem: Diverges in the future

Designing an Alternative Parametrization

Goal:

- Unifying the suppression of past phantom crossing with future stability.
- Strict bounds: $-1 < w(a) < 0$
- Recover CPL behavior near the present.

Method

- Symbolic regression (Machine Learning)
- Training the algorithm with CPL ($0.5 < a < 1.1$)
- Loss function: penalty if $w(a)$ crosses -1 or 0

Physics-Informed Functional Search

The Base Structure:

- Exact CPL match at *a=1* (value and 1st derivative):

$$w(a) = w_{\mathrm{CPL}}(a) + (a-1)^2 \cdot f(a)$$

- ML task: find $f(a)$ with the given conditions.

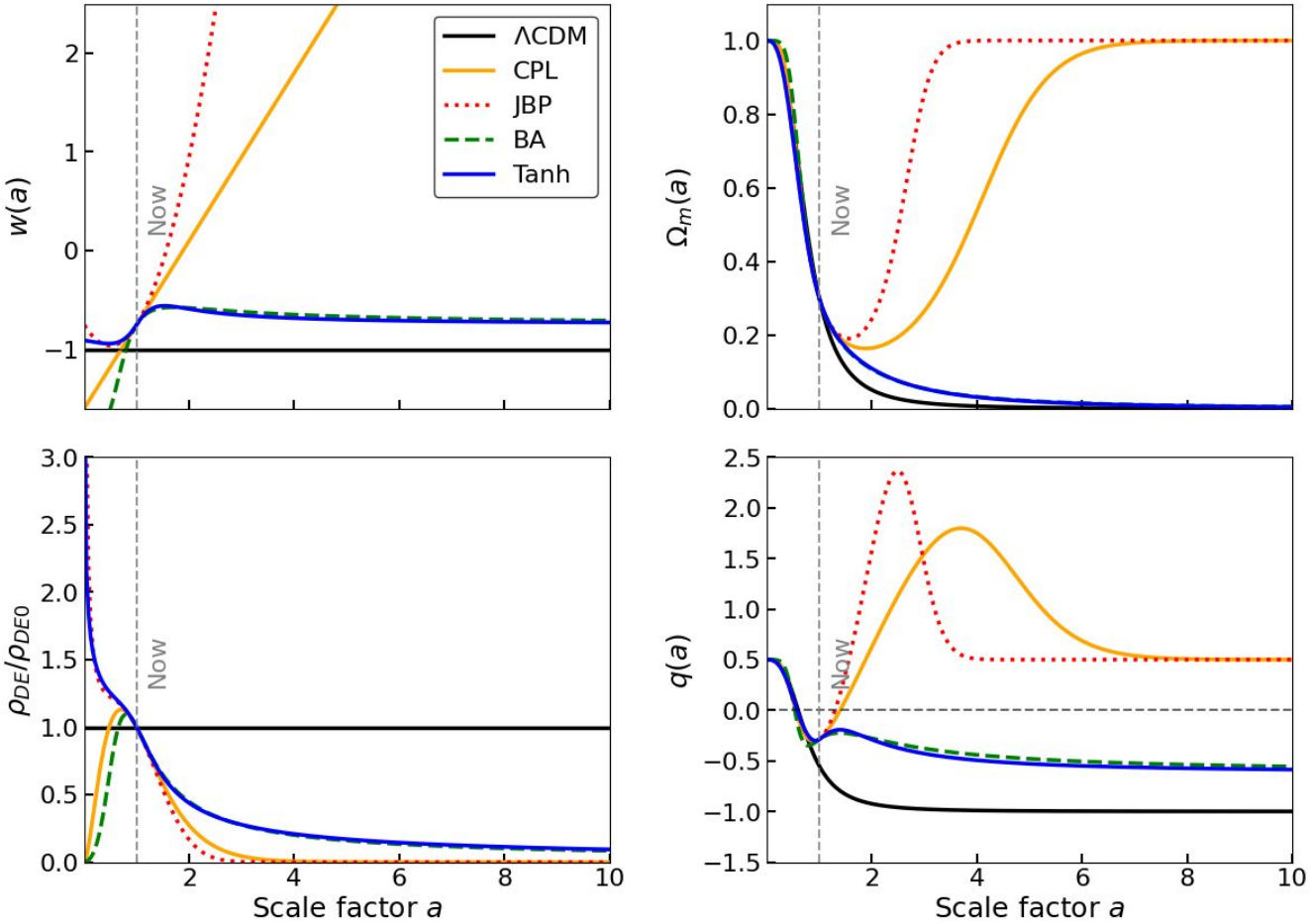
Result parametrization:

$$w(a) = w_0 + w_a(1-a) + (a-1)^2 \tanh\left(\frac{w_a}{a-1}\right)$$

$$\lim_{a\to\infty} w(a) = w_0$$

Cosmological Evolution of Models

Used values:
$w_0 = -0.75$
$w_a = -0.85$



Observational Constraints & Statistical Analysis

- Fitting the parameters to actual observational data

Still Phantom???

Model	$\chi^2_{\rm best}$	Ω_{m0}	h	w_0	w_a
Free BA	1399.23	$0.308^{+0.007}_{-0.007}$	$0.682^{+0.007}_{-0.008}$	$-0.873^{+0.055}_{-0.053}$	$-0.285^{+0.129}_{-0.147}$
Free CPL	1399.45	$0.308^{+0.007}_{-0.007}$	$0.681^{+0.007}_{-0.007}$	$-0.854^{+0.063}_{-0.063}$	$-0.549^{+0.272}_{-0.289}$
Free Tanh	1400.19	$0.309^{+0.007}_{-0.007}$	$0.680^{+0.007}_{-0.007}$	$-0.820^{+0.073}_{-0.089}$	$-1.035^{+0.413}_{-0.325}$
Free JBP	1400.48	$0.309^{+0.007}_{-0.007}$	$0.679^{+0.007}_{-0.007}$	$-0.819^{+0.089}_{-0.090}$	$-1.068^{+0.582}_{-0.589}$

Strict Model	$\chi^2_{\rm best}$	$\Delta\chi^2$ (Strict - Free)	w_0	w_a
Strict BA	1402.38	$+3.15$	$-0.952^{+0.035}_{-0.030}$	$-0.015^{+0.038}_{-0.036}$
Strict CPL	1403.70	$+4.25$	$-0.965^{+0.030}_{-0.023}$	$+0.008^{+0.068}_{-0.048}$
Strict Tanh	1402.00	$+1.81$	$-0.948^{+0.064}_{-0.034}$	$-0.235^{+0.389}_{-0.368}$
Strict JBP	1402.15	$+1.67$	$-0.919^{+0.055}_{-0.048}$	$-0.222^{+0.235}_{-0.249}$

Conclusion

Observational Phantom Preference

- Observations prefer early phantom crossing
- The parametrization form can reduce it
- Alternative solutions: Modified Gravity, Interacting Dark Energy

Our cosmic fate is heavily tied to how we mathematically describe dark energy today.

Thanks for your attention!

Backup slides

Technical details

Used datasets:

- Pantheon+ (1580 SNe, z > 0.01)
- DESI 2024 DR1 BAO
- Compressed CMB data: Shift Parameter & Acoustic Scale (R, l_α)

Method:

- Markov Chain Monte Carlo (MCMC)
- Fitting $\Omega_{m0}, h, w_0, w_a, M$ for different parametrizations

Non Phantom vs Phantom Posterior

Model	χ^2_{best}	Ω_{m0}	h	w_0	w_a
Free BA	1399.23	$0.308^{+0.007}_{-0.007}$	$0.682^{+0.007}_{-0.008}$	$-0.873^{+0.055}_{-0.053}$	$-0.285^{+0.129}_{-0.147}$
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Still Phantom???

Model	Non-Phantom Posterior / Total	Probability
Free CPL	1,499 / 80,000	1.87%
Free BA	1,637 / 80,000	2.05%
Free JBP	6,528 / 80,000	8.16%
Free Tanh	10,724 / 80,000	13.41%