Introduction to the dynamics of Haumea's ring

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Size, shape, and gravitational field

- Triaxial ellipsoid

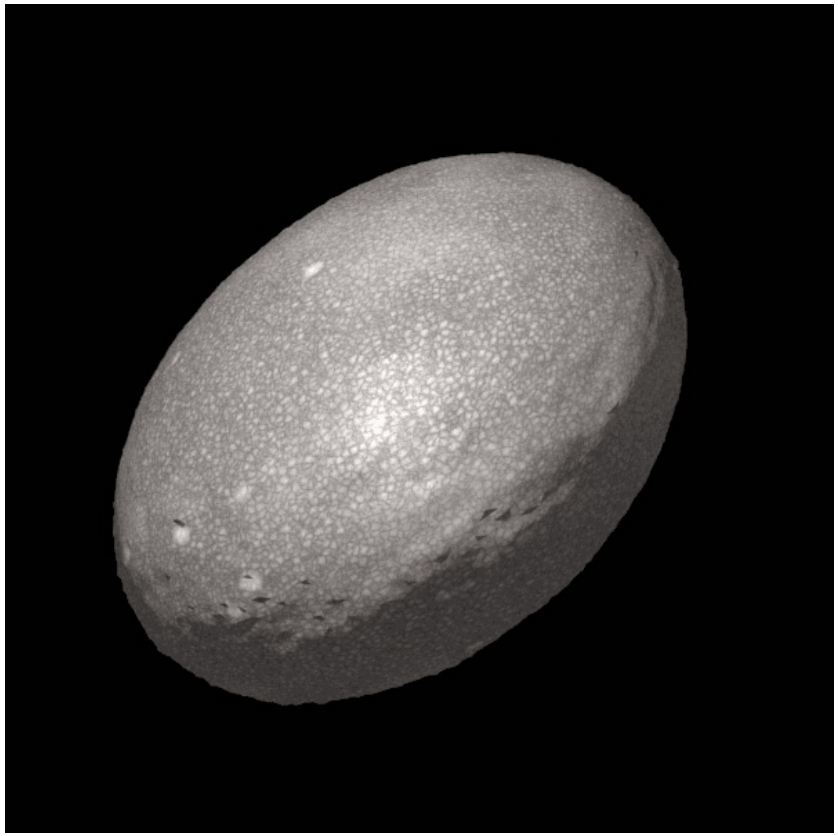


Figure: https://en.wikipedia.org/wiki/Haumea, last opened on 2026.09.16.

Size, shape, and gravitational field

- The gravitational potential in spherical coordinates, in the body-fixed frame of Haumea, is

$$\Phi(r,\theta,\phi) = -\frac{\mu}{r} + \frac{J_2 \mu R_e^2}{2r^3}(3\cos^2\theta - 1) + \frac{3C_{22}\mu R_e^2}{r^3}(1-\cos^2(\theta))\cos(2\phi), \quad (1)$$

where $\mu = GM$, R_e is the volume equivalent radius.

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- Its ring was discovered in 2017.(Ortiz, J. 2017)
- We can ask questions about
  - orbit shapes,
  - chaotic/stable regions of phase space,
  - resonances,
  - kinetic behavior, etc.

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- Orbits are non-Keplerian $\implies$ easier to think about orbits in phase space.
- Consider a "slice" of phase space (Poincaré surface of section):

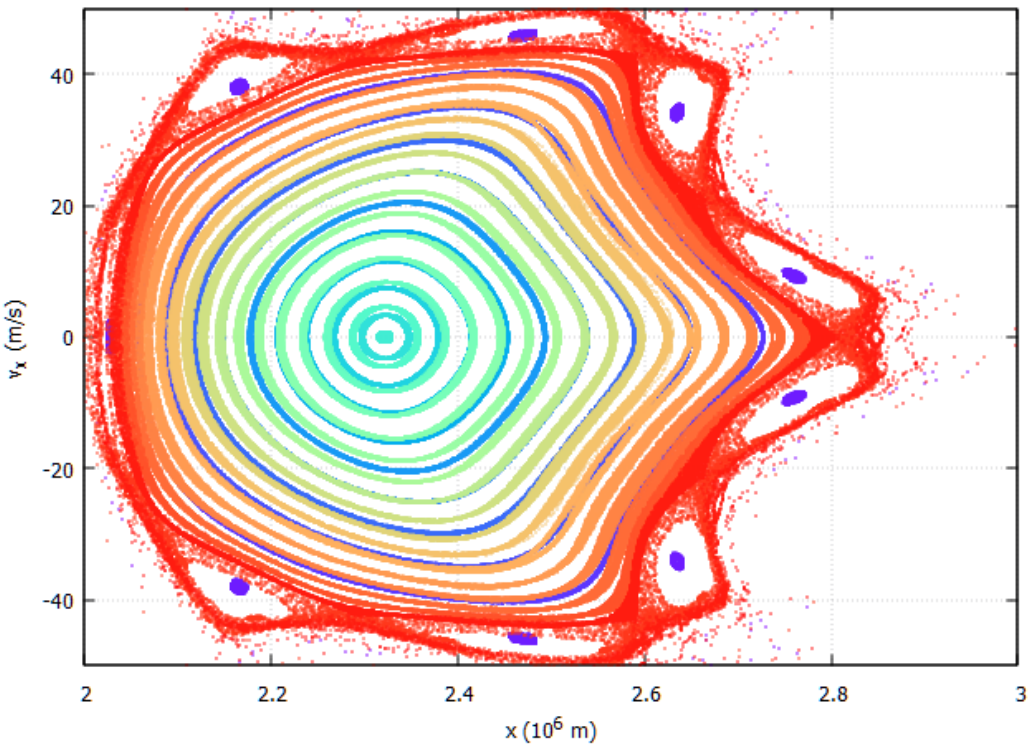


Figure: This figure was created using REBOUND (Rein, H. and Liu, S.-F. 2012). The plot was motivated by (Winter, Borderes-Motta, and Ribeiro 2019)

Simulation of the ring on a computer

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- For a Keplerian ring, it is common to use the *shearing sheet model*.

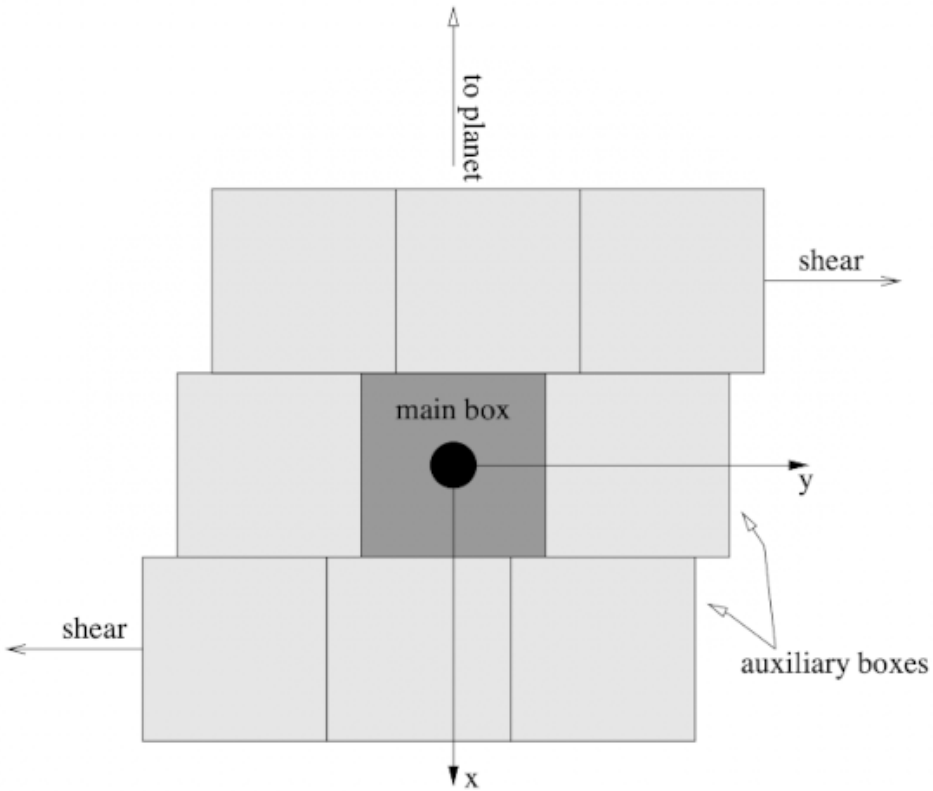


Figure: https://rebound.hanno-rein.de/boundaryconditions/, last opened on 2026.09.16.

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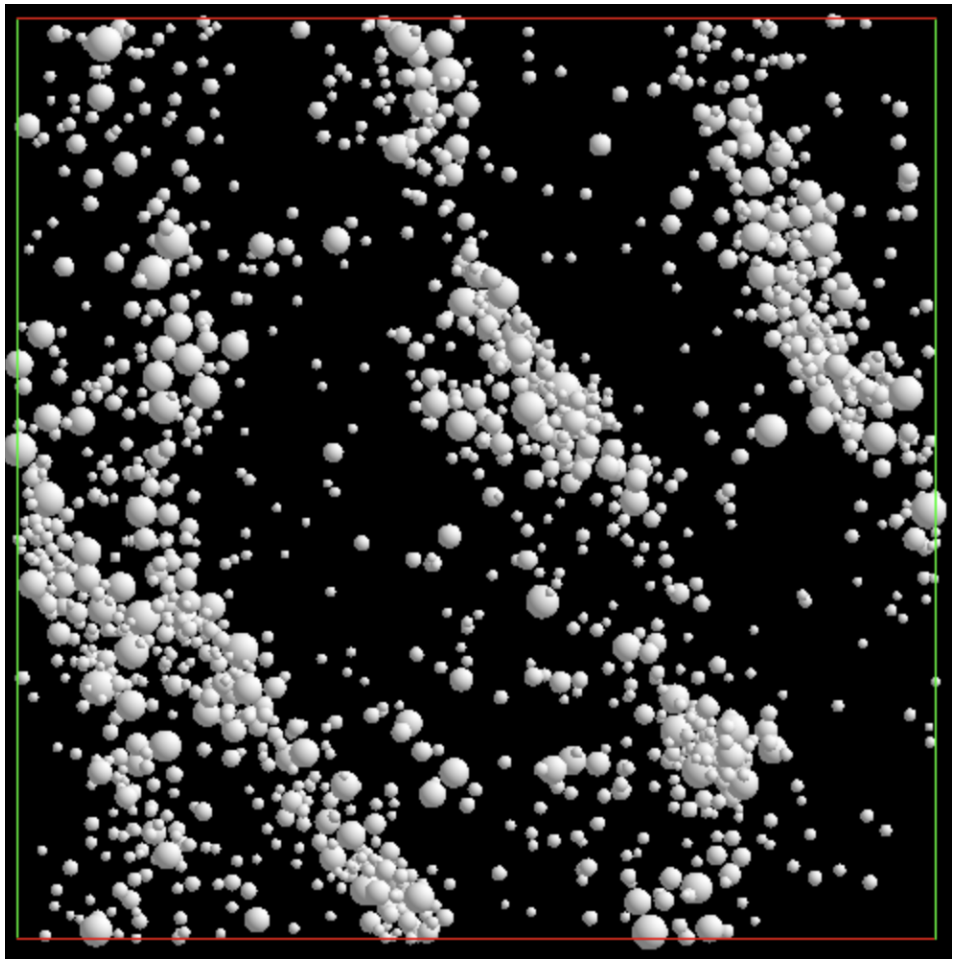


Figure: Shearing sheet simulation in REBOUND.

Simulation of the ring on a computer

- In a shearing sheet, the equations of motion are linearized about a guiding center, resulting in Hill's equations:

$$\ddot{x} - 2\Omega\dot{y} - 3\Omega^2 x = 0 \tag{2}$$

$$\ddot{y} + 2\Omega^2\dot{x} = 0, \tag{3}$$

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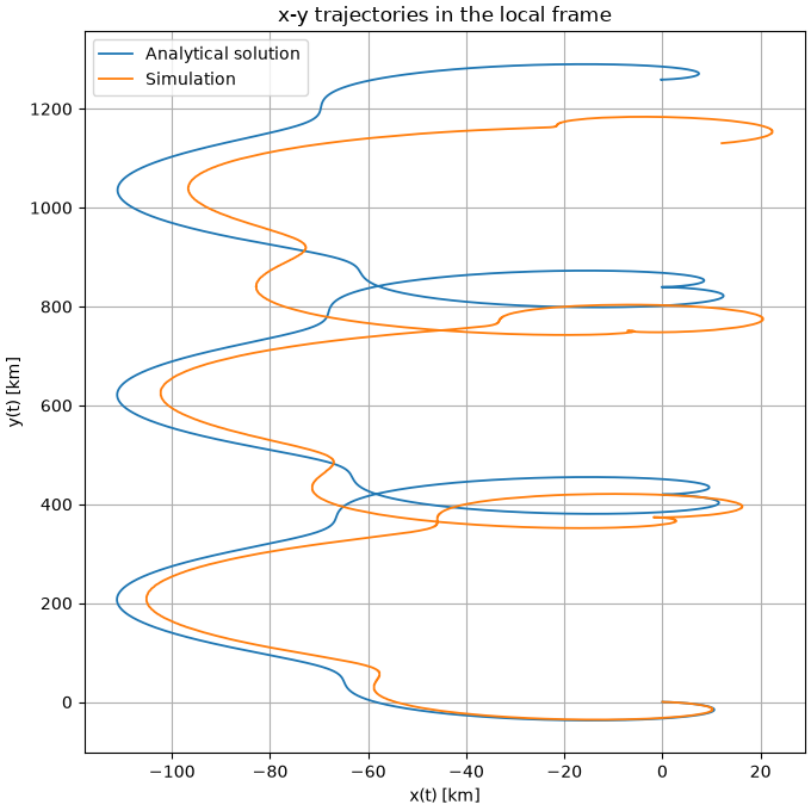
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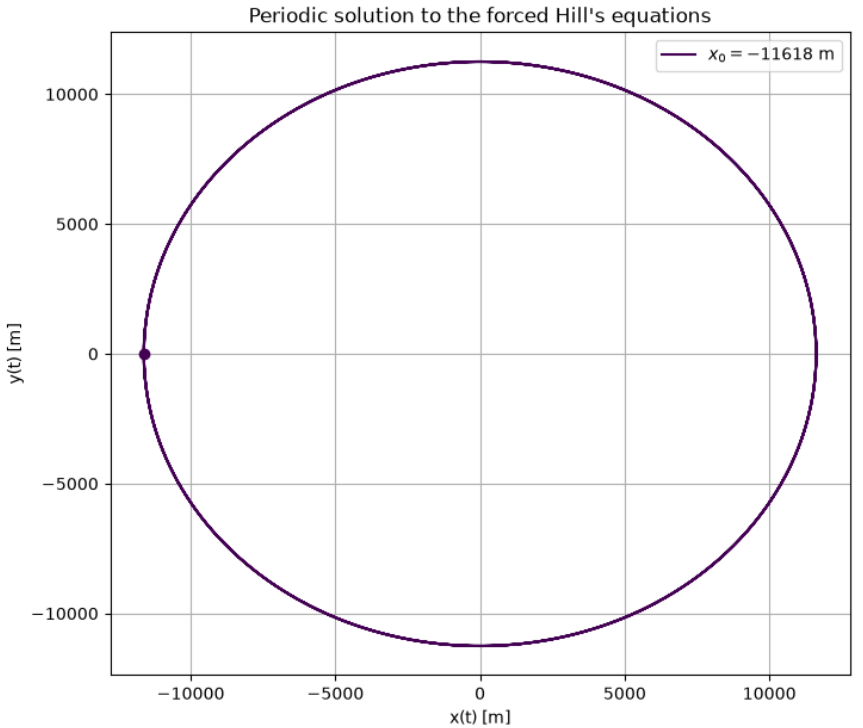
  where Ω is the guiding center's orbital frequency.
- We add the non-axisymmetric perturbation of Haumea's C_{22} term to Eq. (2)-(3) as forcing terms.

Simulation of the ring on a computer

- Periodic solution $\Longrightarrow$ reference orbit for the shearing sheet.



(a) Comparison of the analytically derived trajectory and the simulated one.



(b) Periodic solution.

Figure: Trajectories in the local frame.

References

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Winter, O C, G Borderes-Motta, and T Ribeiro (Feb. 2019). "On the location of the ring around the dwarf planet Haumea". In: *Monthly Notices of the Royal Astronomical Society* 484.3, pp. 3765–3771. ISSN: 1365-2966. DOI: 10.1093/mnras/stz246. URL: http://dx.doi.org/10.1093/mnras/stz246.