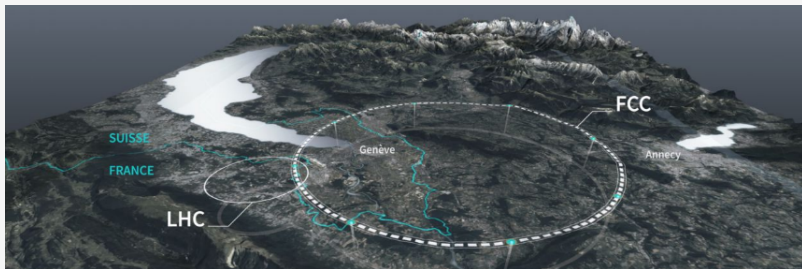


Precision collider and neutrino physics, part I

Janusz Gluza (U. Silesia)

Wigner Institute, Budapest
11 September 2026

The greater danger for most of us lies not in setting our aim too high and falling short; but in setting our aim too low, and achieving our mark Michelangelo, cited by Ian Shipsey in arXiv:1707.03711



Where we are in basic research: half-empty or half-full glass of water?

*"in this field, almost everything is
already discovered, and all that remains
is to fill a few unimportant holes"*



Philipp von Jolly
(1809-1884)

advice to the young Max Planck
not to go into physics, Munich 1878

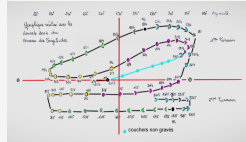
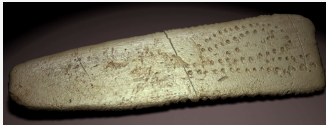
Albert Michelson (1894):

"It seems probable that most of the grand underlying principles have been firmly established (...) the future truths of physical science are to be looked for in the sixth place of decimals"

Q: Dear Albert: What about special and general relativity, and quantum mechanics, particle physics, ...?

Precision, true inception

- ▶ Tycho de Brahe (~ 1601) Mars orbits, Rudolphine tables;
<https://archive.org/details/tabulrudolphinqu00kepl/page/n5/mode/2up>
<https://indico.cern.ch/event/958085/contributions/4329017/attachments/2245318/3807690/Physics>
→ Johannes Kepler (~ 1609) - planets laws of motion;
→ Isaac Newton (~ 1686) - gravitation



Abri Branchard bone ($\sim 30\,000$ years BC),
Alexander Marschack, 'Cognitive Aspects of Upper Paleolithic Engraving'
Current Anthropology (1972),
Interpretation: Chantal Jegues-Wolkiewicz - probably the first Moon calendar,
<https://www.dailymotion.com/video/x8044kz>, Prehistoric Astronomy
Similar paintings, Lascaux caves, $\sim 17\,000$ BC.

PRECISION

In general, and consequences

"Whoever has only a hammer will see nothing but nails."

BBC series, *Precision: The Measure of All Things*

Precision changes history: justice, law, crime, trade, economy, social, ...

To be just was precisely to use balance.



Wisdom 11:20

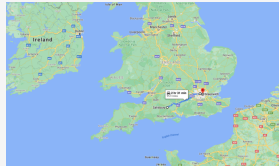
'By weight, measure and number, God made all things'

Code of Hammurabi 1772 BC - any taverner using false weights could be served up with the death penalty

Trade, business.

1. Greenwich time (Airy). Time relativity - **calibration** - 1884 - SI.

1386, Salisbury



2. Time Zones - Steam Engine Time - Start of Globalization

Calibration and standardization



Fire, Baltimore 1904 - incompatible fire brigades' hydraulics from different cities
NIST

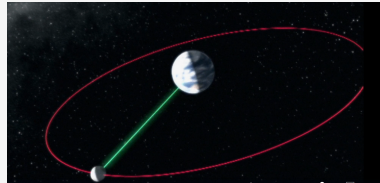
Industrial Revolution



Ford - production line, mass production, strict shifts, ...

Accuracy of making parts.

Politics



"To test our hope in action" [John F. Kennedy Moon Speech \(1962\), youtube](#)

Till 1960 - $\Delta x \sim 10^{-7}$ m $\rightarrow$ ΔX 8000 km in navigation to the Moon
(need for ~ 100 better precision!)

Quantum leap - atom-based measurement

NB: *Maxwell $\sim$ "global sustainable standards should be based not on the dimensions or motion of our planet, but on the length, period, and absolute mass of these imperishable and unalterable and perfectly similar molecules".*

2nd revolution - just like the transition from "cubit rod" and human-based to Earth-based measurements.

Precision, Particle Physics

- ▶ (i) muon discovery, J/Ψ
- (ii) $(g-2)_e$, $(g-2)_\mu$
- (iii) V-A, parity;

EXPERIMENT $\rightarrow$ THEORY

- ▶ (i) $\tau^\pm$ (tau lepton);
- (ii) Tevatron - top quark discovery;
- (iii) H^0 (scalar Higgs-Englert boson)

THEORY $\rightarrow$ EXPERIMENT

SM corrections matters! LEP, SLAC, LHC
M. Veltman (1977) ρ -parametr $\sim m_t^2, \ln(m_H^2)$;
 $\rightarrow$ Acta Physica Polonica B

- ▶ Neutrinos (masses, mixing angles, CP phase(s));
Super-K, Hyper-K, T2K, NOvA, Antares, KM3NeT, Juno, DUNE,
SNO+, Daya Bay, Double Chooz, RENO, ...

Future Colliders; THEORY $\leftrightarrow$ EXPERIMENT

BOOK
Special volume of Acta Physica
Polonica B commemorating
Martinus Veltman

by Jach, Stanisław; Jach, Marek;
Praszkiewicz, Michał

Published by Acta Physica Polonica 2021

This memorial volume of Acta Physica Polonica B is a special tribute to the Nobel Laureate Martinus Veltman. Professor Veltman was a member of the APFB International Editorial Council, lecturer at the Cracow School of Theoretical Physics in 1977 and 1994, and the author of two of the most cited articles ever published in our journal: «Second Threshold in Weak Interactions» [Acta Phys. Pol. B 8, 475 (1977)] and «The Infrared-Ultraviolet Connection» [Acta Phys. Pol. B 12, 437 (1981)].

PRECISION and ENERGY frontiers

(two examples)

Aside: factor-of-2 improvements can matter!

Search for $K_L \rightarrow \pi\pi$

ANNALS OF PHYSICS: 8, 156-181 (1958)

Long-lived Neutral K Mesons*

M. BARDON, K. LANDE, AND L. M. LEDERMAN

*Columbia University, New York, New York, and Brookhaven
National Laboratories, Upton, New York*

AND

WILLIAM CHINOWSKY

Brookhaven National Laboratories, Upton, New York

set an upper limit $<0.6\%$ on the reactions

$<0.6\%$

$$K_S^0 \rightarrow \begin{cases} \mu^+ + e^- \\ e^+ + e^- \\ \mu^+ + \mu^- \end{cases}$$

and on $K_S^0 \rightarrow \pi^+ + \pi^-$.

VOLUME 6, NUMBER 10

PHYSICAL REVIEW LETTERS

MAY 15, 1961

DECAY PROPERTIES OF K_S^0 MESONS*

D. NAGAI, E. O. OIKARINEN, N. I. PETROV, A. M. RUSANOVA, AND V. A. RUSAKOV
Joint Institute of Nuclear Research, Moscow, U.S.S.R.
(Received April 20, 1961)

Combining our data with those obtained in reference 7, we set an upper limit of 0.3% for the relative probability of the decay $K_S^0 \rightarrow \pi^- + \pi^+$. Our

$<0.3\%$

"At that stage the search was terminated by administration of the Lab."

[Okun, hep-ph/0112031]

VOLUME 13, NUMBER 4

PHYSICAL REVIEW LETTERS

27 JULY 1964

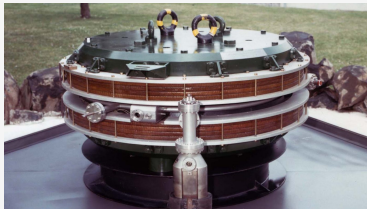
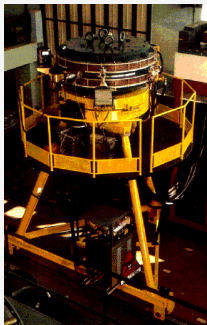
EVIDENCE FOR THE 2π DECAY OF THE K_S^0 MESON*†

J. H. CHRISTENSEN, J. W. CRONIN,[‡] V. L. FITCH,[‡] AND R. TURLAY[§]
Princeton University, Princeton, New Jersey
(Received 10 July 1964)

$= 0.2 \pm 0.04\%$

We would conclude therefore that K_S^0 decays to two pions with a branching ratio $R = (K_S^0 \rightarrow \pi^+ + \pi^-) / (K_S^0 \rightarrow \text{all charged modes}) = (2.0 \pm 0.4) \times 10^{-3}$ where the error is the standard deviation. As empha-

The first circular e^+e^- accelerator



ADA/ADONE, 1969-1993, Frascati, $\sqrt{s} \leq 3$ GeV

To be lucky is an important life/research factor

2 PRLs in 1974 for J/Ψ discovery

SPEAR at SLAC

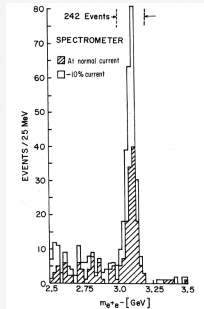


FIG. 2. Mass spectrum showing the existence of J .

"J"

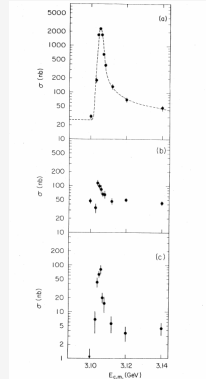
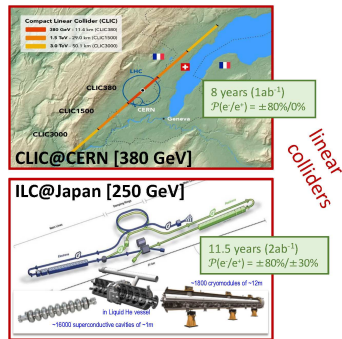
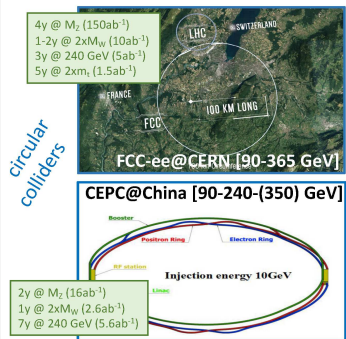


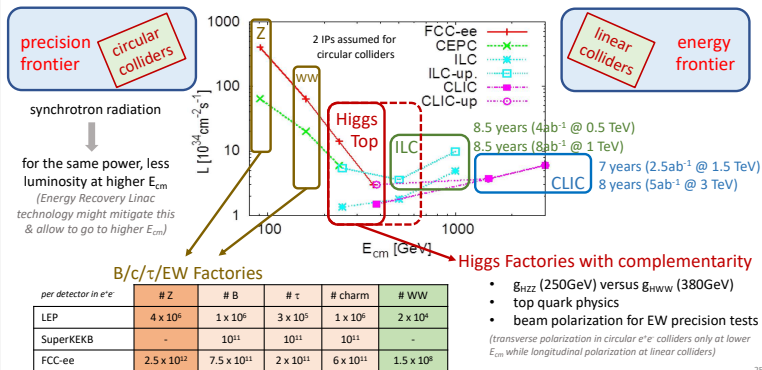
FIG. 1. Cross section versus energy for (a) multi-hadron final states, (b) e^+e^- final states, and (c) $\mu^+\mu^-$, $\tau^+\tau^-$, and K^+K^- final states. The curve in (a) is the ex-

"Psi"

e^+e^- Higgs Factories

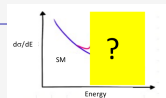


e^+e^- Higgs Factories (incl. B/c/ τ /EW/top factories)



Two discovery strategies at HEP → FCC being superior in many respects

1. Direct searches[†], limitation: energy $M_X \ll \Lambda_{\text{GUT}} \rightarrow \text{FCC-hh}$
2. **Precise predictions** (calculations): SM ↔ BSM, limitation: statistics[†] → FCC-ee



FCC Physics Groups



FCC PED Physics Group meetings

Electroweak physics	21 events	⇒
Higgs physics	67 events	⇒
Top-quark physics	1 event	⇒
Flavour physics at FCC	28 events	⇒
QCD and $\gamma\gamma$ physics	13 events	⇒
BSM physics	75 events	⇒
FCC-hh	41 events	⇒
Global Fits	2 events	⇒
Monte Carlo Event Generators	1 event	⇒
Precision Calculations	4 events	⇒

Link to WG Indices

[†]Non-FCC proponents (LCF, CLIC) emphasize the importance of direct searches for new states at $\sqrt{s} = 0.5(1) \text{ TeV}$ at e^+e^- collisions.

Given the absence of any indication of a new scale beyond the SM, such a scenario is highly constrained and represents a rather idealized limiting case. → LEP3 ...

FCC-ee statistics

Phase	Run duration (years)	Center-of-mass Energies (GeV)	Integrated Luminosity (ab^{-1})	Event Statistics	
FCC-ee-Z	4	88-94	150	$5 \cdot 10^{12}$ Z decays	LEP $\times 10^5$
FCC-ee-W	2	157-163	10	10^8 WW events	LEP $\times 10^3$
FCC-ee-H	3	240	5	10^6 ZH events 25k WW $\rightarrow$ H	Never done
FCC-ee-tt	5	340-365	0.2 $\div$ 1.5	10^6 $t\bar{t}$ events 200k ZH 50k WW $\rightarrow$ H	Never done
s-channel H	?	125		~ 5000	

Precision. FCC-ee is the most demanding HEP project for theoretical SM calculations.

Meeting the required experimental precision for Standard Model observables (1 to 2 orders of magnitude improved measurements) requires solving a wide range of issues across all areas of the program.

Main target defined by the FCC program:

None calculated with required precision!

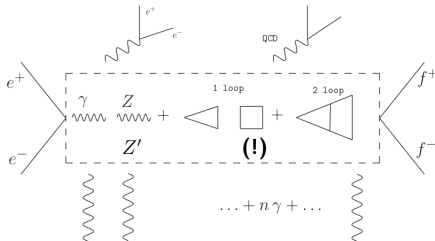
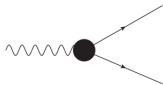
Observable	Now/Future	present value $\pm$ error	FCC-ee Stat.	FCC-ee Syst.	Comment and leading exp. error
m_Z (keV)	22	91186700 ± 2200	4	100	From Z line shape scan Beam energy calibration
Γ_Z (keV)	2505.00272 90 $\rightarrow$ 200	2495200 ± 2300	4	25 12	From Z line shape scan Beam energy calibration
$\sin^2 \theta_W^{\text{eff}} (\times 10^6)$	70	231480 ± 160	2	2.4	from $A_{\text{FB}}^{\mu\mu}$ at Z peak Beam energy calibration
$1/\alpha_{\text{QED}}(m_Z^2) (\times 10^3)$		128952 ± 14	3	small	from $A_{\text{FB}}^{\mu\mu}$ off peak QED&EW errors dominate
$R_\ell^Z (\times 10^3)$		20767 ± 25	0.06	0.2-1	ratio of hadrons to leptons acceptance for leptons
$\alpha_s(m_Z^2) (\times 10^4)$		1196 ± 30	0.1	0.4-1.6	from R_ℓ^Z above
$\sigma_{\text{had}}^0 (\times 10^3)$ (nb)		41541 ± 37	0.1	4	peak hadronic cross section luminosity measurement
$N_\nu (\times 10^3)$		2996 ± 7	0.005	1	Z peak cross sections Luminosity measurement
$R_b (\times 10^6)$	2000	216290 ± 660	0.3	< 60 0.3	ratio of bb to hadrons stat. extrapol. from SLD
$A_{\text{FB},0}^b (\times 10^4)$		992 ± 16	0.02	1-3	b-quark asymmetry at Z pole from jet charge
$A_{\text{FB}}^{\text{pol},\tau} (\times 10^4)$		1498 ± 49	0.15	<2	τ polarization asymmetry τ decay physics
τ lifetime (fs)		290.3 ± 0.5	0.001	0.04	radial alignment
τ mass (MeV)		1776.86 ± 0.12	0.004	0.04	momentum scale
τ leptonic ($\mu\nu_\mu\nu_\tau$) B.R. (%)		17.38 ± 0.04	0.0001	0.003	e/μ /hadron separation
m_W (MeV)	50	80350 ± 15	0.25	0.3	From WW threshold scan Beam energy calibration

MC generators and theory (Z-pole)

Subtraction of γ -exchange, $\gamma - Z$ exchange, box (non-factorizable), non-resonance terms.

How to account for in MC generators (efficiency, particles ID,...)?

Form factors (FF)



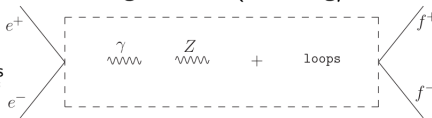
LEP FCC-ee

ISR: ☒ ☒

FSR: ☒ ☐

IFI: ☒ ☐

MC generators (unfolding/deconvolution)



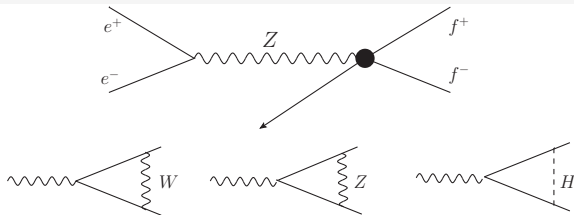
EWPOs

ElectroWeakPseudoObservables

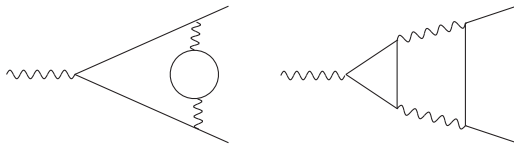
$\Gamma_Z, R_l, A_{FB}, \sin^2 \theta_{\text{eff}}^b, \sin^2 \theta_{\text{eff}}^{\text{lept}}$

Rough scheme for extracting the $Zf\bar{f}$ vertex and EW corrections

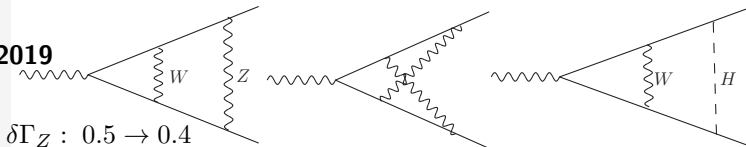
1986



1993-2014



2018-2019



To get to the experimental precision, we must improve very much!

Expected precision in 2040

Conclusion of the 2018 Workshop

J. Gluza

"We anticipate that, at the beginning of the FCC-ee campaign of precision measurements, the theory will be precise enough not to limit their physics interpretation. This statement is however conditional to sufficiently strong support by the physics community and the funding agencies, including strong training programmes".

Numerical evaluation with three-loops calculations:

arXiv:1901.02648

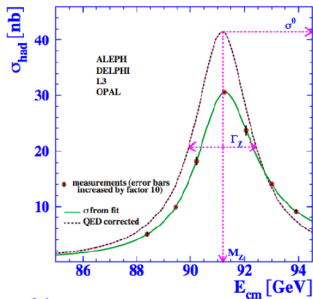
0.5 → 0.4
Five years!

	$\delta\Gamma_Z$ [MeV]	δR_l [10^{-4}]	δR_b [10^{-5}]	$\delta \sin_{eff}^{2,l} \theta$ [10^{-6}]
Present EWPO theoretical uncertainties				
EXP-2018	2.3	250	66	160
TH-2018	0.4	60	10	45
EWPO theoretical uncertainties when FCC-ee will start				
EXP-FCC-ee	0.4 0.025	10	2 ÷ 6	6 3
TH-FCC-ee	0.07	7	3	7

- 500 person-years needed over 20 years – **Recognized as strategic priority.**

To get to the experimental precision, we must improve very much!

□ Cross section : Z mass and width



♦ ~30% QED corrections (ISR)

CERN Yellow Reports:
Monographs

CERN-2019-003

Standard Model Theory for
the FCC-ee Tera-Z stage

A. Blondel
J. Gluza
S. Jadach
P. Janot
T. Riemann



$$\frac{\delta\Gamma_Z}{\Gamma_Z} = \frac{.025}{2495} \simeq 10^{-5}!$$

Luminosity and matching between fixed-order and resummed calculations

ISR with perturbative series for an EW-mediated cross section $2 \rightarrow n$

Issue 1

My choices, for more complete

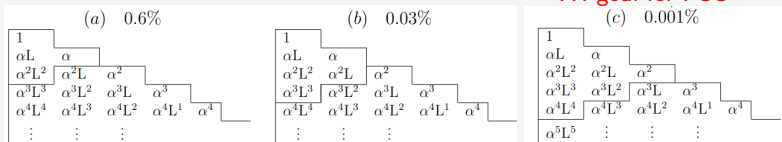
→ Pier Monni's FCC Week 2026 plenary talk

$$\sigma \sim \alpha^n \sum_{m=0}^{\infty} \alpha^m \sum_{i=0}^m \sum_{j=0}^m c_m^{(i,j)} L^i \ell^j$$

$$L = \ln\left(\frac{Q^2}{m_e^2}\right) \quad (\text{collinear log}) \sim 25$$

$$\ell = \ln\left(\frac{Q^2}{(\Delta E_\gamma)^2}\right) \quad (\text{IR log}) \sim 15 \quad (\text{small angle scattering})$$

TH goal for FCC



EPJC'19; Jadach, Skrzypek [1903.09895](#)

Getting 10^{-5} precision regime, see M. Skrzypek et al '24, HOW WELL COULD WE CALCULATE LUMINOSITY AT FCCee? [pdf](#), also F. Piccinini'26: [pdf](#)

A. Freitas et al., "Theoretical uncertainties for electroweak and Higgs-boson precision measurements at FCC-ee", <https://arxiv.org/abs/1906.05379>

Quantity	FCC-ee	Current intrinsic error	Projected intrinsic error (at start of FCC-ee)
M_W [MeV]	$0.5-1^{\ddagger}$	4 ($\alpha^3, \alpha^2 \alpha_s$)	1
$\sin^2 \theta_{\text{eff}}^{\ell}$ [10^{-5}]	0.6	4.5 ($\alpha^3, \alpha^2 \alpha_s$)	1.5
Γ_Z [MeV]	0.1	0.4 ($\alpha^3, \alpha^2 \alpha_s, \alpha \alpha_s^2$)	0.15
R_b [10^{-5}]	6	11 ($\alpha^3, \alpha^2 \alpha_s$)	5
R_l [10^{-3}]	1	6 ($\alpha^3, \alpha^2 \alpha_s$)	1.5

$\ddagger$ The pure experimental precision on M_W is ~ 0.5 MeV.

Quantity	FCC-ee	future parametric unc.	Main source
M_W [MeV]	0.5 – 1	1 (0.6)	$\delta(\Delta\alpha)$
$\sin^2 \theta_{\text{eff}}^{\ell}$ [10^{-5}]	0.6	2 (1)	$\delta(\Delta\alpha)$
Γ_Z [MeV]	0.1	0.1 (0.06)	$\delta\alpha_s$
R_b [10^{-5}]	6	< 1	$\delta\alpha_s$
R_{ℓ} [10^{-3}]	1	1.3 (0.7)	$\delta\alpha_s$

Important input parameter errors are $\delta(\Delta\alpha) = 3 \cdot 10^{-5}$, $\delta\alpha_s = 0.00015$.

Wider perspective: Existing - global - wishlists

ESPPU briefing book <https://arxiv.org/abs/2511.03883>

Table 4.2: Wish-list for calculations of missing higher-order perturbative QCD $\mathcal{O}(\alpha_s^n)$ and/or EW $\mathcal{O}(\alpha^n)$ to match the expected experimental uncertainty at future e^+e^- and ep colliders.

Observable	Missing higher-order & power-suppressed corrections
Hadronic Z width	$\mathcal{O}(\alpha_s^5)$, $\mathcal{O}(\alpha_s^6)$, $\mathcal{O}(\alpha^3)$, $\mathcal{O}(\alpha_s \alpha^3)$, $\mathcal{O}(\alpha_s^2 \alpha^2)$
Hadronic W width	$\mathcal{O}(\alpha_s^5)$, $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha_s^2 \alpha)$
Hadronic τ width	$\mathcal{O}(\alpha_s^5)$
Hadronic event shapes (Z, W, H decays)	N ³ LO differential, N ^{3,4} LL resummation, power corrections
Inclusive jet rates	3-jet cross-sections at N ³ LO, 4-jets at N ² LO, 5-jets at NLO
Lattice QCD results	$\mathcal{O}(\alpha_s^6)$ β -function; $\mathcal{O}(\alpha_s^5)$ heavy quark decoupling; $\mathcal{O}(\alpha_s^4)$ static potential
(α_s extr.; quark masses m_c, m_b)	$\mathcal{O}(\alpha^3)$ lattice perturbation theory matching (lattice coupling to $\alpha_s^{\overline{MS}}$ etc.)
$\sigma(e^+e^- \rightarrow W^+W^-)$ vs. $\sqrt{s}$	EW N ² LO: $\mathcal{O}(\alpha^2)$, Mixed EW-QCD: $\mathcal{O}(\alpha_s \alpha^2)$, $\mathcal{O}(\alpha_s^2 \alpha)$
$\sigma(e^+e^- \rightarrow t\bar{t})$ vs. $\sqrt{s}$	NRQCD: $\mathcal{O}(\alpha_s^5)$, Non-resonant: $\mathcal{O}(\alpha_s^5)$, $\mathcal{O}(\alpha_s^3)$ differential; QED: $\mathcal{O}(\alpha^3)$ at NNLL
$H \rightarrow b\bar{b}$ width	N ⁴ LO ($m_b \neq 0$); N ⁴ LO differential ($m_b = 0$)
$H \rightarrow gg$ width	N ⁵ LO (heavy-top limit), N ⁴ LO ($m_t \neq 0$); N ⁴ LO differential, N ³ LO differential ($m_t \neq 0$)
MC simulations for $e^+e^- \rightarrow X$ processes	N ^{2,3} LO matched to N ^{2,3} LL PS. Per mille control of non-pQCD effects (hadronization, CR, ...)
$ep \rightarrow$ hadrons (PDF and α_s determ.)	N ^{3,4} LO evolution equations and inclusive cross-sections
$ep \rightarrow$ jets (α_s determ.)	N ³ LO cross-sections

Future Circular Collider Feasibility Study Report Volume 1 Physics, Experiments, Detectors, Eur. Phys. J. C (2025) 85:1468, [link](#)

Table 10 A few sample precision quantities of interest for the FCC-ee programme, their current and projected experimental uncertainties, and the required theory input for their *extraction* from the data. The last

two columns show the current state of the art for calculations of this theory input and higher-order calculations needed to reach the FCC-ee precision target. More details can be found in Ref. [40]

Quantity	Current precision	FCC-ee stat. (syst.) precision	Required theory input	Theory status as of today	Needed theory improvement ^a
m_Z (MeV)	2.0	0.004 (0.1)	Non-resonant $e^+e^- \rightarrow f\bar{f}$, initial-state radiation (ISR)	NLO, ISR logarithms up to 6th order	NNLO for $e^+e^- \rightarrow f\bar{f}$
Γ_Z (MeV)	2.3	0.004 (0.012)			
$\sin^2 \theta_{\text{eff}}^\ell$	1.6×10^{-4}	$1.2 (1.2) \times 10^{-6}$			
m_W (MeV)	9.9	0.18 (0.16)	Lineshape of $e^+e^- \rightarrow WW$ near threshold	NLO ($e^+e^- \rightarrow 4f$ or EFT framework)	NNLO for $e^+e^- \rightarrow WW, W \rightarrow f\bar{f}'$ in EFT setup
HZZ coupling ^{-b}		0.1%	Cross section for $e^+e^- \rightarrow ZH$	NLO EW plus partial NNLO QCD/EW	Full NNLO EW
m_{top} (MeV)	290	4.2 (4.9)	Threshold scan $e^+e^- \rightarrow t\bar{t}$	N ³ LO QCD, NNLO EW, resummations up to NNLL, $\mathcal{O}(30 \text{ MeV})$ scale uncert.	Matching fixed orders with resummations, merging with MC, α_s (input)

^aThe necessary theory calculations mentioned are a minimum baseline; additional partial higher-order contributions may also be required

^bNo absolute value for the HZZ coupling can be extracted from the LHC data without additional assumptions

Table 11 Required theory calculations for the *prediction* of the listed precision quantities within the SM. These predictions are needed for comparison with the quantities extracted from data (Table 10), for the

purpose of testing the validity of the SM and probing BSM physics. More details in Ref. [40]

Quantity	Required theory input	Theory status as of today	Needed theory improvement ^a
Γ_Z	vertex corrections for $Z \rightarrow f\bar{f}$	NNLO + partial higher orders	N ³ LO EW + partial higher orders
$\sin^2 \theta_{\text{eff}}^\ell$			
m_W	SM corrections to the muon decay rate	NNLO + partial higher orders	N ³ LO EW + partial higher orders

^aThe mentioned needed theory calculations are a minimum baseline; additional partial higher-order contributions may also be required

Issue 3 Higgs boson decays: theoretical status, M. Spira (2019), pdf

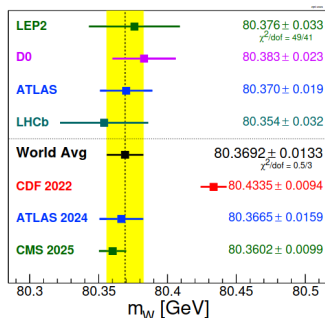
Estimated theoretical uncertainties from missing higher orders and the perturbative orders (QCD/EWK.) of the results included in the analysis.

Partial Width	QCD	Electroweak	Total	on-shell Higgs
$H \rightarrow b\bar{b}/c\bar{c}$	$\sim 0.2\%$	$\sim 0.5\%$	$\sim 0.5\%$	N ⁴ LO / NLO
$H \rightarrow \tau^+\tau^-/\mu^+\mu^-$	—	$\sim 0.5\%$	$\sim 0.5\%$	— / NLO
$H \rightarrow gg$	$\sim 3\%$	$\sim 1\%$	$\sim 3\%$	N ³ LO / NLO
$H \rightarrow \gamma\gamma$	$< 1\%$	$< 1\%$	$\sim 1\%$	NLO / NLO
$H \rightarrow Z\gamma$	$< 1\%$	$\sim 5\%$	$\sim 5\%$	LO / LO
$H \rightarrow WW/ZZ \rightarrow 4f$	$< 0.5\%$	$\sim 0.5\%$	$\sim 0.5\%$	NLO/NLO

- ▶ Decay widths and BRs known with sufficient accuracy for the LHC [FCC-ee: more to be done]
- ▶ sizeable corrections from QCD
- ▶ EWK corrections required for high precision

Recent progress (2026): J. Wang, Partial QCD N4LO result $\mathcal{O}(\alpha_s^4)$, pdf

Issue 4



Science 376 (2022) 6589, 170-176

$$\text{SM EWK FIT} : M_W = 80356 \pm 6 \text{ MeV, (PDG2025)}$$

$$\text{Global AVG, no CDF} : M_W = 80369 \pm 13 \text{ MeV, (PDG2025)}$$

$$\text{CDFII} : M_W = 80433.5 \pm 9.4 \text{ MeV}$$

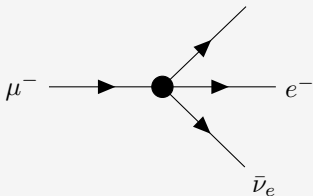
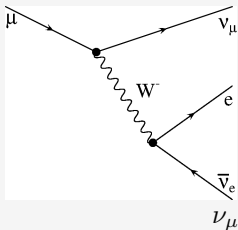
$$\text{ATLAS 2023}^* : M_W = 80360 \pm 16 \text{ MeV}$$

$$\text{FCC-ee forecast} : M_W = X \pm \mathbf{0.4 - 1 \text{ MeV!}}$$

* CDF-II data on W mass are in contradiction with global electroweak e^+e^- fits and recent ATLAS LHC analysis, with systematic uncertainty improved by 15% [ATLAS:2023fsi](#) and optimised reconstruction of the W -boson transverse momentum [ATLAS:2023lhf](#).

Muon decay, SM-QED matching

$$\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e$$



$$\tau_\mu^{-1} = \frac{G_\mu^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right) (1 + \Delta q),$$

where

$$F(x) = 1 - 8x - 12x^2 \ln x + 8x^3 - x^4$$

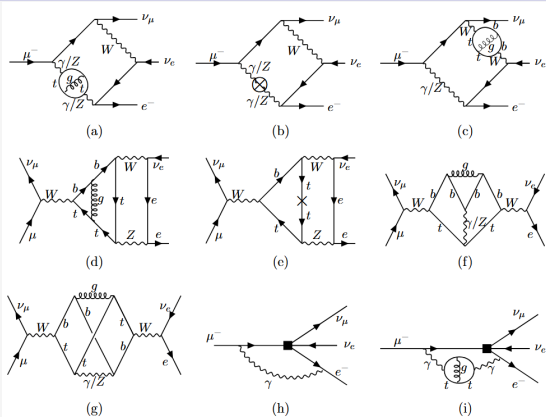
$$M_W^2 \sin^2 \theta_W = \frac{\pi\alpha}{\sqrt{2}G_\mu} (1 + \Delta r),$$

Δq and

$\Delta r = \Delta r(\alpha, M_W, M_Z, M_H, m_f)$
gathers QED and SM corrections

A. Freitas FCC WG precision talk: [pdf-link](#) (+video, for WG members)

Third order mixed EWK-QCD $\mathcal{O}(N_f \alpha^2 \alpha_s)$ corrections



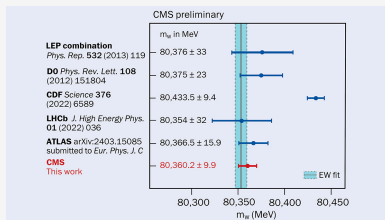
$$\Delta M_W = \frac{\pi \alpha M_Z^2}{2\sqrt{2} G_\mu M_W (M_Z^2 - 2M_W^2)} \Delta r^*$$

I. Dubovyk, A. Freitas, JG, J. Usovitsch, [2603.06571](#)

Muon decay, substantial $\mathcal{O}(N_f \alpha^2 \alpha_s)$ correction

	$\Delta r [10^{-4}]$	$\Delta M_W [\text{MeV}]$
$\mathcal{O}(\alpha)$	+296.8	-441.5
$\mathcal{O}(\alpha\alpha_s, \alpha\alpha_s^2)$	+42.2	-73.0
$\mathcal{O}(\alpha^2)$	+29.0	-49.4
$\mathcal{O}(\alpha_t^2\alpha_s, \alpha_t^3, \alpha_t\alpha_s^3)$	-0.30	+0.50
$\mathcal{O}(N_f^3\alpha^3, N_f^2\alpha^2\alpha_s)$	-0.89	+1.46
$\mathcal{O}(N_f\alpha^2\alpha_s)$	-2.01	+3.14

Today, integrals can be calculated with > 10 -digit precision using many methods and tools (see Issue 6 below).



G. Degrossi, talk in Mainz: [pdf](#):

"A prediction of m_W with a precision 1 MeV seems not feasible in the near future"

- ▶ A solid prediction of m_W with a precision below 10 MeV requires a better understanding of the top pole mass (and an improvement in $\Delta\alpha_{\text{had}}^{(5)}(m_Z^2)$) [$\delta m_W \sim 10$ MeV]

and there are more ingredients, e.g.

$$\text{Z boson mass: } \frac{\Delta m_Z}{m_Z} = 2 \cdot 10^{-5} \quad \Longrightarrow \quad \delta m_W \sim 2.5 \text{ MeV}$$

or (4-loop estimate)

$$\mathcal{O}(\alpha\alpha_s^3) \rightarrow \delta m_W \sim 2 \text{ MeV}$$

The electromagnetic gauge coupling is a fundamental parameter in the Standard Model

$$\alpha(q^2) = \frac{\alpha}{1 - \Delta\alpha(q^2)}, \quad \Delta\alpha(q^2) = \Delta\alpha_{\text{lep}}(q^2) + \Delta\alpha_{\text{had}}^{(5)}(q^2) + \Delta\alpha_{\text{top}}(q^2).$$

Uncertainty on $\alpha(M_Z^2)$ dominated by non-perturbative QCD effects in $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$.

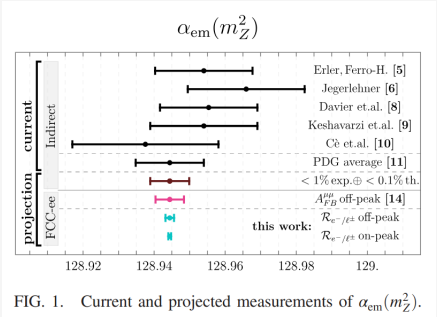


FIG. 1. Current and projected measurements of $\alpha_{\text{em}}(m_Z^2)$.

- ▶ The electroweak precision program at FCC-ee requires $\delta\Delta\alpha_{\text{had}}^{(5)} \sim 3 \times 10^{-5}$, based on FB asymmetry muon production measurements.

P. Janot, *JHEP* **02** (2016) 053

- ▶ A novel method based on Z -pole measurements could reach a statistical sensitivity of $\delta\Delta\alpha_{\text{had}}^{(5)} \sim 1 \times 10^{-5}$.

M. Riemann, arXiv:2501.05508

A. Conigli, talk in Mainz (lattice): [pdf](#)

Outlook: path to FCC-ee precision

- ▶ The FCC-ee EW program requires $\delta\Delta\alpha_{\text{had}} \sim 3 \times 10^{-5}$, and possibly $\delta\Delta\alpha_{\text{had}} \sim 1 \times 10^{-5}$
- ▶ Increasing the matching scale Q_0^2 alone is insufficient, but combining this with moderate lattice improvements ($\times 2 - \times 4$) yields substantial gains
- ▶ Significant progress in pQCD inputs needed for the 1×10^{-5} regime, and LQCD can play a crucial role

Issue 6

- ▶ Sector decomposition (SD) method:
 - ▶ FIESTA [2016], [A.V.Smirnov]
 - ▶ pySecDec, [Expansion by regions with pySecDec], 2022
 - ▶ The Mellin-Barnes (MB) method:
 - ▶ MB [M.Czakon, 2006]
 - ▶ MBnumerics [J.Usovitsch, I.Dubovyk, T.Riemann, 2015] – Minkowskian kinematics
 - ▶ Differential equations (DEs) method:
 - ▶ DiffExp [F. Moriello, 2019; M. Hidding], 2021
 - ▶ AMFlow [X. Liu, Y.-Q. Ma] 2022
 - ▶ SeaSyde [T. Armadillo, R. Bonciani, S. Devoto, N. Rana, A. Vi] 2022
 - ▶ AmpRed [W. Chen] 2025
 - ▶ Integration-By-Parts (IBPs) - crucial for DEs
 - ▶ Kira [F. Lange, P. Maierhöffer, J. Usovitsch] 2021
 - ▶ Reduze [C. Studerus, 2010]
 - ▶ FIRE [A. Smirnov, 2008]
 - ▶ LiteRed [R. Lee, 2014]
- plus fast progress in analytical approaches**

Precision calculations: Methods and tools for higher order SM corrections

A. Vicini talk in Mainz: [pdf](#)

Challenges in the Master Integrals evaluation

courtesy of T.Armadillo

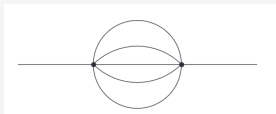
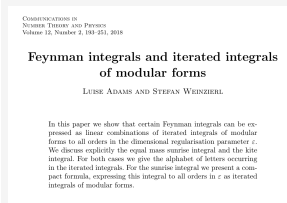
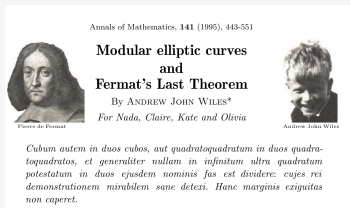
	NCDY - 2L Mixed	CCDY - 2L Mixed	Z on shell - 2L EW	NCDY - 2L EW	NCDY - 2L EW	NCDY - 2L EW	NCDY - 2L EW
Example Topology							
Number of masters	36	56	51	47	104	126	140
Reduction Kira + Firefly	12 hours (32 core)	16 hours (32 core)	1 day (32 core)	1.5 m (96 core + Ratracer)	30 m (120 core + Ratracer)	8 h (120 core + Ratracer)	54 h (120 core + Ratracer)
AMFlow 1 point	50 min (32 core)	75 min (32 core)	6 h 45 m (32 core)	15 min (96 core)	1 hour (120 core)	4 hours (120 core)	X
Dimension equations	700 Kb	2.1 Mb	/	4 Mb	45 Mb	350 Mb	??
SeaSyde 3250 points	5 days	10 days	/	??	??	??	??

key to overcome these problems: the improvement in the choice of the Feynman Integrals
systematic usage of polynomial reconstruction

Collider physics ... magic of the math world!

$$T = 2\pi\sqrt{\frac{l}{g}} \times {}_2F_1\left[\frac{1}{2}, \frac{1}{2}; 1; \sin^2\theta\right]$$

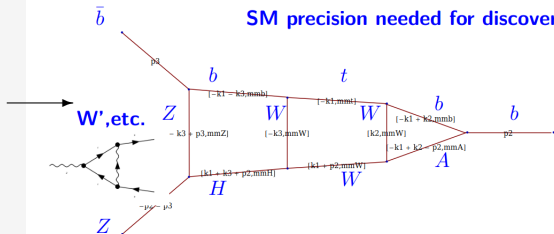
Analytic solutions for massive multiloop integrals, which describe scattering processes/decays, go beyond elliptic functions - how far? 😊



"Epsilon-factorized form" of banana 4-loop integrals → **fast evaluations to nearly arbitrary precision.**

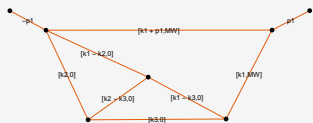
'Taming Calabi-Yau Feynman Integrals: The Four-Loop Equal-Mass Banana Integral', S. Pögel, X. Wang, S. Weinzierl,

SM precision needed for discovery studies (indirect effects)

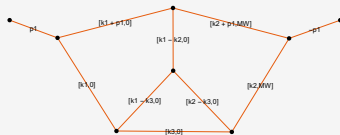


Four scales :

$$\left\{ \frac{M_H^2}{M_Z^2}, \frac{M_W^2}{M_Z^2}, \frac{m_t^2}{M_Z^2}, \frac{s+i\epsilon}{M_Z^2} \right\}$$



1-dim



4-dim

$$-18.779406962 - 6.390785027i \quad -22.5213 + 4.74442i \pm (0.001 + 0.001i)$$

$$I = -\frac{1}{(-s)^{1+3\epsilon}} \int_{-i\infty}^{+i\infty} \prod_{i=1}^4 dz_i \left(-\frac{M_W^2}{s} \right)^{z_3} \frac{\Gamma(-\epsilon - z_1) \Gamma(-z_1) \Gamma(1 + 2\epsilon + z_1)}{\Gamma(1 - 2\epsilon) \Gamma(1 - 3\epsilon - z_1)} \times \frac{\Gamma(-2\epsilon - z_{12}) \Gamma(1 - \epsilon + z_2) \Gamma(1 + z_{12}) \Gamma(1 + \epsilon + z_{12}) \Gamma(1 + 3\epsilon + z_3) \Gamma(1 - \epsilon - z_4)}{\Gamma(1 - z_2) \Gamma(2 + \epsilon + z_{12})} \dots$$



+ differential equations, numerical methods, ...

To collimate the focus, a new FCC WG

Precision Calculations, talks, Indico link

<https://indico.cern.ch/category/21380/>

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Precision Calculations

 [Create event](#) [Navigate](#) [Parent category](#)

The "Precision calculations" subgroup is connected to the next step in the development of the FCC project: the Reference Design Phase and the assembly of the required materials, in preparation for a CERN Council decision on the future HEP machine, foreseen in 2028.

For a full list of working groups, see <https://gms.web.cern.ch/>, where you can join our group (fcc-ped-physicsgroup-precisioncalculations).

Direct link to join: <https://gms.web.cern.ch/group/FCC-PED-PhysicsGroup-PrecisionCalculations>

There is one event in the future. [Hide](#)

July 2026

 08 Jul [FCC precision calculation WG meeting: talks S. Jones & S. Banik](#)

June 2026

 17 Jun [FCC precision calculation WG meeting: talks A. Freitas & J. Usovitsch](#)

May 2026

 27 May [FCC precision calculation WG meeting talks: A. Denner & J. Lindert](#)

 13 May [FCC precision calculation WG kick-off meeting](#)

Managers

 [fcc-ped-physicsgroup-precisioncalculation-admin](#)

Materials

 [PED_egroups_chart_2026_03_21.pdf](#)

 [PED_OrgChart_CloseUp_2026_04_15.pdf](#)

Precision-calculation targets for FCC-ee in the coming years

- ▶ *Improved unfolding techniques to go from observables to pseudo-observables* (QED interference effects, non-factorizable corrections, matching between EW corrections and radiated/resummed photons, ...).
- ▶ *Developing tools*: MC generators including NLO QCD and EW corrections (Bhabha, exclusive NNLO $e^+e^- \rightarrow f\bar{f}$), multiloop numerical, analytical programs.
- ▶ *Improved input parameters by roughly one order of magnitude* ($\alpha, \alpha_s, m_W, m_H, m_t, \Delta\alpha_{\text{had}}, \dots$).
- ▶ *Full two-loop corrections* for $e^+e^- \rightarrow W^+W^-$, $e^+e^- \rightarrow ZH$ and $e^+e^- \rightarrow VVH$, $H \rightarrow f\bar{f}(+\gamma)$.
- ▶ *Going beyond 2-loop corrections* (3-loop EW and mixed EW-QCD, leading 4-loop corrections for $Z \rightarrow 2f$ vertices).

No fundamental show-stopper has been identified. With a concerted theory effort, these goals appear achievable, paving the way for FCC to become a flagship facility of the 21st century.

Thank you for your attention (part I)

SPEECH DELIVERED BY PROFESSOR NIELS BOHR
ON THE OCCASION OF THE INAUGURATION OF THE CERN PROTON SYNCHROTRON

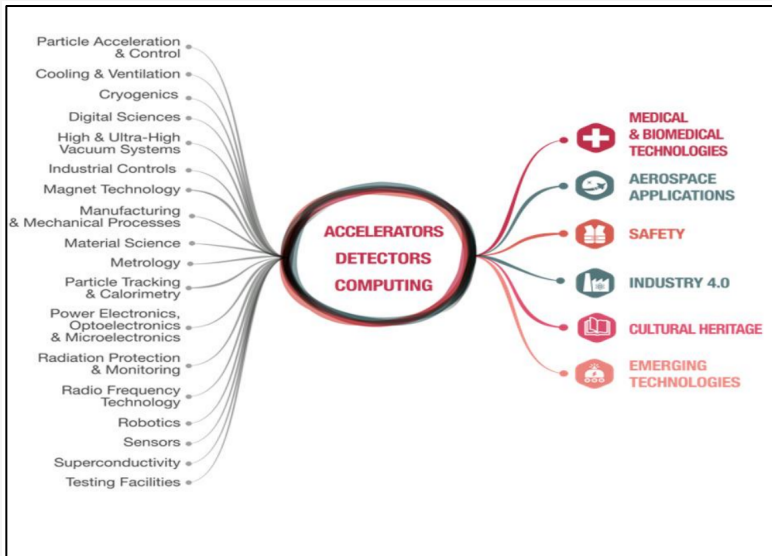
ON 5 FEBRUARY, 1960

Press Release PR/56
12 February, 1960

It may perhaps seem odd that apparatus as big and as complex as our gigantic proton synchrotron is needed for the investigation of the smallest objects we know about. However, just as the wave features of light propagation make huge telescopes necessary for the measurement of small angles between rays from distant stars, so the very character of the laws governing the properties of the many new elementary particles which have been discovered in recent years, and especially their transmutations in violent collisions, can only be studied by using atomic particles accelerated to immense energies. Actually we are here confronted with most challenging problems at the border of physical knowledge, the exploration of which promises to give us a deeper understanding of the laws responsible for the very existence and stability of matter.



DG, Fabiola Gianotti, CERN vision and goals until next strategy update, → pdf



Usefulness of basic science, some links

- ▶ Eugene Wigner (!), [The Unreasonable Effectiveness of Mathematics in the Natural Sciences](#)
- ▶ What's the use of basic science? by Llewellyn Smith, [vidyo](#) and [pdf](#)
- ▶ Alexander Flexner, 1939 [The usefulness of useless knowledge](#), see comment by Paul Kirschner [blog](#)
- ▶ Essay in Nature: [No final frontier](#)
- ▶ 4 essays on FCC usefulness discussion, extract from one of them: [The Mirage Of Freed Resources](#) "When the Superconducting Super Collider was abandoned in the United States, the project simply ceased to exist. Teams dispersed, and world leadership in particle physics gradually shifted towards CERN. The resources supposedly "freed" did not give rise to a new major international scientific programme. They were not redirected towards other large-scale fundamental research projects. Nor did they lead to any substantial increase in CERN's funding. The SSC was not replaced. It was abandoned. The closure of Superphénix in France did not trigger a broad revival of French nuclear research. Instead, it gradually led to the dispersal of teams, the erosion of rare expertise in fast-neutron reactor technologies, and the weakening of a field that several countries are now attempting to rebuild. Whatever one's views of Superphénix itself, the lesson is straightforward. History did not record the emergence of a major replacement programme. What it remembers above all is the disappearance of a capability. JG in red: [Can anyone point to an example of a major scientific project whose cancellation led to a lasting strengthening of other disciplines? We are aware of none.](#)"
- ▶ Tadeusz Lesiak lecture for Polish Physical Society with gathered many arguments: [talk](#)

BACKUP



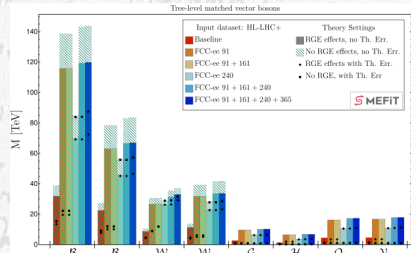
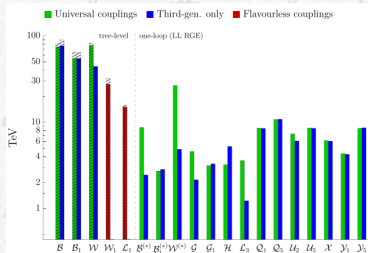
HL-LHC

with Current EW measurements:

— Correlation < 50%

● Correlation > 50%

Power of Tera-Z: Vectors



Allwicher, MM, Renner: 2408.03992

E. Celada, L. Mantani, J. Rojo,
A. N. Rossia, M. Thomas, J. ter Hoeve,
B. E. Vryonidou

Huge work on BSM effects within FCC feasibility study, e.g. talks at 8th FCC Physics Workshops, 2025, 2026 and S. Dawson talk in Mainz: [pdf](#)

Future: W, t, H

- ▶ $e^+e^- \rightarrow W^+W^-$ at 161 GeV: $\delta m_W^{exp} = 0.5 \div 1$ MeV.
Challenge to get the same TH error:
NNLO $e^+e^- \rightarrow 4f$.
- ▶ $e^+e^- \rightarrow t\bar{t}$ at 350 GeV: $\delta m_t^{exp} = 17$ MeV
Big challenge for theory, today > 100 MeV, future projection ≤ 50 MeV:
 ~ 10 MeV unc. from mass def.;
 ~ 15 MeV from α_s unc. to threshold mass def.;
 ~ 30 MeV - h. orders resummation
- ▶ $e^+e^- \rightarrow HZ$ at 240 GeV: Kinematic constraint fits with $Z \rightarrow ll$ and $H \rightarrow b\bar{b}$, ...,
 $m_H = 125.35 \text{ GeV} \pm 150 \text{ MeV}$ [[link CMS](#)], $\Gamma_H = 4.1_{4.0}^{5.1} \text{ MeV}$, $\Gamma_H < 13 \text{ MeV}$ at 95 % C.L., [1901.00174](#)
 $\delta m_H^{exp} = 10 \text{ MeV}$; Theory errors subdominant.

Higgs boson decays: theoretical status

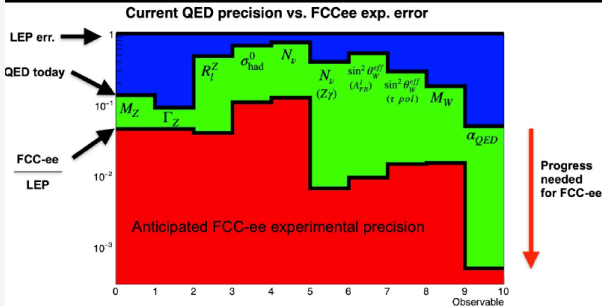
Projected intrinsic and parametric uncertainties for the partial and total Higgs-boson decay width predictions. The last column: the target of FCC-ee precisions.

decay	intrinsic	para. m_q	para. α_s	para. M_H	FCC-ee prec. on g_{HXX}^2
$H \rightarrow b\bar{b}$	$\sim 0.2\%$	0.6%	$< 0.1\%$	—	$\sim 0.8\%$
$H \rightarrow c\bar{c}$	$\sim 0.2\%$	$\sim 1\%$	$< 0.1\%$	—	$\sim 1.4\%$
$H \rightarrow \tau^+\tau^-$	$< 0.1\%$	—	—	—	$\sim 1.1\%$
$H \rightarrow \mu^+\mu^-$	$< 0.1\%$	—	—	—	$\sim 12\%$
$H \rightarrow gg$	$\sim 1\%$	0.5% (0.3%)		—	$\sim 1.6\%$
$H \rightarrow \gamma\gamma$	$< 1\%$	—	—	—	$\sim 3.0\%$
$H \rightarrow Z\gamma$	$\sim 1\%$	—	—	$\sim 0.1\%$	
$H \rightarrow WW$	$\lesssim 0.3\%$	—	—	$\sim 0.1\%$	$\sim 0.4\%$
$H \rightarrow ZZ$	$\lesssim 0.3\%^\dagger$	—	—	$\sim 0.1\%$	$\sim 0.3\%$
Γ_{tot}	$\sim 0.3\%$	$\sim 0.4\%$	$< 0.1\%$	$< 0.1\%$	$\sim 1\%$

† From $e^+e^- \rightarrow HZ$ production

Sven Heinemeyer, Stanislaw Jadach, Jürgen Reuter, <https://arxiv.org/abs/2106.11802>

QED challenges beyond LEP: the FCC-ee example



The present precision of QED theoretical predictions would severely limit the analysis of precise measurements at FCC-ee.

To properly confront the data with theoretical predictions of similar accuracy demands a huge progress in precision calculations!

Needed factor 6-200 improvement with respect to LEP.

arXiv:1903.09895

(Jadach&Skrzypek)

EW pseudo-observables
EWPOs

Observable EWPO	Source LEP	Err.{QED} LEP	Stat{Syst} FCC-ee	LEP FCC-ee	main development to be done
M_Z [MeV]	Z linesh.	2.1{0.3}	0.005[0.1]	$3 \times 3^*$	light fermion pairs
Γ_Z [MeV]	Z linesh.	2.1{0.2}	0.008[0.1]	$2 \times 3^*$	fermion pairs
σ_{had}^0 [pb]	σ_{had}^0	37{25}	0.1[4.0]	$6 \times 3^*$	better lumi MC
$R_l^Z \times 10^3$	$\sigma(M_Z)$	25{12}	0.06[1.0]	$12 \times 3^{**}$	better FSR
$N_\nu \times 10^3$	$\sigma(M_Z)$	8{6}	0.005[1.0]	$6 \times 3^{**}$	CEEX in lumi MC
$N_\nu \times 10^3$	$Z\gamma$	150{60}	0.8[< 1]	$60 \times 3^{**}$	$\mathcal{O}(\alpha^2)$ for $Z\gamma$
$\sin^2 \theta_W^{\text{eff}} \times 10^5$	$A_{\text{FB}}^{\text{lept.}}$	53{28}	0.3[0.5]	$55 \times 3^{**}$	h.o. and EWPOs
$\sin^2 \theta_W^{\text{eff}} \times 10^5$	$\langle \mathcal{P}_\tau \rangle, A_{\text{FB}}^{\text{pol}, \tau}$	41{12}	0.6[< 0.6]	$20 \times 3^{**}$	better τ decay MC
M_W [MeV]	mass rec.	33{6}	0.3[?.?]	$20 \times 3^{**}$	$\mathcal{O}(\alpha)$, FSR _{exp}
M_W [MeV]	threshold	200{30}	0.5[0.3]	$100 \times 3^{***}$	$\mathcal{O}(\alpha^2)$ at thresh.
$A_{\text{FB}, \mu}^{M_Z \pm 3.5 \text{ GeV}} \times 10^5$	$\frac{d\sigma}{d \cos \theta}$	2000{100}	1.0[0.3]	$100 \times 3^{***}$	improved IFI

Scheme of construction and the use of EWPO/EWPP at FCC-ee

EXPERIMENT

(A)
Raw experimental **DATA**
including
cut-offs, efficiencies, QED

Removing detector
inefficiencies,
(simplifying cut-offs)

(B)
Experimental **DATA**
with idealised cut-offs
QED still present
(realistic observables)

Predicting realistic distributions

Fitting with MC, WT-diffs

Fitting using WT diffs.
MC programs
of KKMC class

(C)
EWPO's
or **EWPP's**
Parameters in
the effective Born,
QED subtracted

THEORY

BSM Physics Models
+SM without QED

(D)
SM calculations
1-2-3 EW loops
QED subtracted

non-MC fitters
like ZFITTER/TOPAZ0

Scheme DATA $\rightarrow$ Born $\otimes$ QED
 $\rightarrow \mathcal{O}(\alpha^1)_{\text{noQED}} \otimes \text{QED}$

Published results on EWPOs in the SM @NNLO

Complete corrections	Freitas, Hollik, Walter, Weiglein: '00
$\Delta r, \sin^2 \theta_{\text{eff}}^l$:	Awramik, Czakon: '02, Onishchenko, Veretin: '02
	Awramik, Czakon, Freitas, Weiglein: '04
	Awramik, Czakon, Freitas: '06
	Hollik, Meier, Uccirati: '05, '07
	Degrassi, Gambino, Giardino: '14
Fermionic corrections	Awramik, Czakon, Freitas, Kniehl: '09
$\sin^2 \theta_{\text{eff}}^b, a_f, v_f$:	Czarnecki, Kühn: '96
	Harlander, Seidensticker, Steinhauser: '98
	Freitas: '13, '14
Bosonic corrections	
$\sin^2 \theta_{\text{eff}}^b$:	Dubovyk, Freitas, JG, Riemann, Usovitsch '16
Bosonic corrections	
$\Gamma_Z, R_l, \dots$:	Dubovyk, Freitas, JG, Riemann, Usovitsch '18, '19

→ 1-loop results: Many seminal works by Veltman, Passarino, Sirlin, Marciano, Bardin, Hollik, Jegerlehner, Riemann, Degrassi, Kniehl, ...

> 2020 results on EWPOs in the SM @NNNLO and beyond

- ▶ L. Chen and A. Freitas, *JHEP* **07**, 210 (2020), "Leading fermionic three-loop corrections to electroweak precision observables", <https://arxiv.org/abs/2002.05845>
- ▶ L. Chen and A. Freitas, "Mixed EW-QCD leading fermionic three-loop corrections at $\mathcal{O}(\alpha_s\alpha^2)$ to electroweak precision observables" *JHEP* **03**, 215 (2021), arXiv:2012.08605.
- ▶ L. Chen and A. Freitas, "GRIFFIN: A C++ library for electroweak radiative corrections in fermion scattering and decay processes" *SciPost Phys. Codebases* **2023**, 18 (2023), arXiv:2211.16272.
- . P. Martin, "Three-loop corrections to the Fermi decay constant in the $\overline{MS}$ scheme" *Phys. Rev. D* **112**, 073002 (2025), arXiv:2507.15946.

For earlier approximate 3- and 4-loop results, (enhanced by y_t and/or N_f), see references in "Third-order mixed electroweak-QCD corrections to the W-boson mass prediction from the muon lifetime", I. Dubovyk, A. Freitas, JG, J. Usovitsch, [2603.06571](#)

Input and renormalization schemes

E.g. the bosonic 2-loop corrections shift the value of Γ_Z by 0.51 MeV when using M_W as input and 0.34 MeV when using G_μ as input.

Reminder: $\delta\Gamma_{Z,\text{FCC-ee}} = 0.1 \text{ MeV}$

Dubovyk et al, <https://doi.org/10.1016/j.physletb.2018.06.037>

Γ_i [MeV]	$\Gamma_e, \Gamma_\mu, \Gamma_\tau$	$\Gamma_{\nu_e}, \Gamma_{\nu_\mu}, \Gamma_{\nu_\tau}$	Γ_d, Γ_s	Γ_u, Γ_c	Γ_b	Γ_Z
Born	81.142	160.096	371.141	292.445	369.56	2420.2
$\mathcal{O}(\alpha)$	2.273	6.174	9.717	5.799	3.857	60.22
$\mathcal{O}(\alpha\alpha_s)$	0.288	0.458	1.276	1.156	2.006	9.11
$\mathcal{O}(N_f^2\alpha^2)$	0.244	0.416	0.698	0.528	0.694	5.13
$\mathcal{O}(N_f\alpha^2)$	0.120	0.185	0.493	0.494	0.144	3.04
$\mathcal{O}(\alpha_{\text{bos}}^2)$	0.017	0.019	0.058	0.057	0.167	0.505
$\mathcal{O}(\alpha_t^2\alpha_s^2, \alpha_t\alpha_s^3, \alpha_t^2\alpha_s, \alpha_t^3)$	0.038	0.059	0.191	0.170	0.190	1.20

* Fixed values of M_W

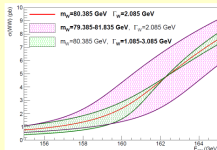
W-mass, slide by A.Freitas, Snowmass 2020, pdf

- High-precision measurement of M_W from $e^+e^- \rightarrow W^+W^-$ at threshold

- a) Corrections near threshold enhanced by $1/\beta$ and $\ln \beta$

$$\beta \sim \sqrt{1 - 4 \frac{M_W^2 - i M_W \Gamma_W}{s}} \sim \sqrt{\Gamma_W / M_W}$$

- b) Non-resonant contributions are important



- Full $\mathcal{O}(\alpha)$ calculation of $e^+e^- \rightarrow 4f$

Denner, Dittmaier, Roth, Wieders '05

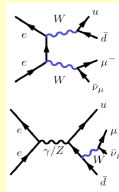
- EFT expansion in $\alpha \sim \Gamma_W / M_W \sim \beta^2$

Beneke, Falgari, Schwinn, Signer, Zanderighi '07

- NLO corrections with NNLO Coulomb correction ($\propto 1/\beta^n$): $\delta_{\text{th}} M_W \sim 3 \text{ MeV}$

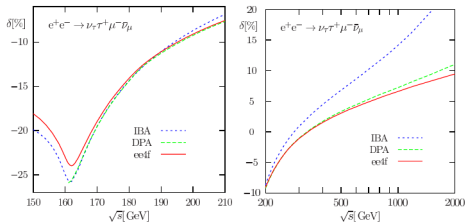
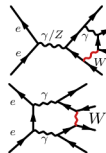
Actis, Beneke, Falgari, Schwinn '08

- Adding NNLO corrections to $ee \rightarrow WW$ and $W \rightarrow f\bar{f}$ and NNLO ISR: $\delta_{\text{th}} M_W \lesssim 0.6 \text{ MeV}$



Full NLO calculation for $e^+e^- \rightarrow 4f$ (Denner, Dittmaier, Roth, Wieders 05)

- More than 1000 1-loop diagrams, 5, 6-point loop integrals
- ⇒ pioneering methods for six-point diagrams
now automated for LHC: RECOLA, OpenLoops, MadLoops
- **complex mass scheme** for W decay width
- fully differential calculation
- not easy to incorporate higher-order effects
- DPA not sufficient at threshold and for $\sqrt{s} > 500$ GeV



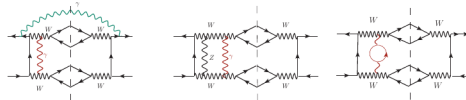
SM W-physics, FCC-ee-W

EFT expansion in $\alpha \sim \frac{\Gamma_W}{M_W} \sim \beta^2$ (Beneke/Falgari/CS/Signer/Zanderighi 07)

- systematically possible to include higher-order corrections
- limited to total cross section near threshold

Leading NNLO corrections

- 2nd Coulomb correction $\sim \alpha^2/\beta^2 \sim \alpha$ (Fadin et al. 95)
- Coulomb-enhanced corrections $\sim \alpha^2/\beta \sim \alpha^{3/2}$ (Actis et al. 08)



- Numerical effect: $\Delta\sigma_{WW} \sim 5\text{‰}$; $[\delta M_W] \lesssim 3 \text{ MeV}$

$\sqrt{s} [\text{GeV}]$	$\sigma(e^-e^+ \rightarrow \mu^- \nu_\mu u \bar{d})(\text{fb})$			
	NLO _{EFT}	NLO _{ee4f} [DDRW]	$\Delta_{\text{NNLO}}(\alpha^2/\beta^2)$	$\Delta_{\text{NNLO}}(\alpha^2/\beta)$
161	117.5	118.77	0.44 (3.7‰)	0.15 (1.3‰)
170	397.8	404.5	0.25 (0.6‰)	1.6 (3.9‰)

Future improvements of theory predictions?

Implementation of state-of-the art calculations in public tools?

- **NLO-EW** $e^-e^+ \rightarrow 4f$ now possible with standard tools
(RECOLA, OpenLoops, MadLoops + SHERPA, MadGraph, WHIZARD...)
but not (yet) optimized for e^-e^+ (ISR, Beamstrahlung)
- **Two-loop Coulomb-enhanced** corrections for differential observables doable; (related: $t\bar{t}$ with Coulomb resummation in WHIZARD)
(no guarantee of formal accuracy for general distributions)

Full NNLO in EFT for total cross section

- Soft $\log \beta$ terms can be adapted from QCD results
- NNLO $\log(m_e/M_W)$ terms doable (c.f. Bhabha scattering)
- two-loop hard non-logarithmic corrections
(from amplitudes for $e^+e^- \rightarrow W^+W^-$ at threshold: border of current capabilities)
resulting uncertainty from cross-section calculation

$$\Delta\sigma_{\text{hard}}^{(2)} = \left(\frac{\alpha}{2\pi}\right)^2 c^{(2)}\sigma^{(0)} \sim (1-2)\% \text{ for estimate } c^{(2)} = (c^{(1)})^2$$

Full NNLO for $e^+e^- \rightarrow 4f$: completely new methods needed

Conclusions and outlook



- ▶ KoralW+YFSWW3: LEP2 precision is 0.5%.
Factor of $20 \div 50$ improvement is needed for FCCee
- ▶ Lesson from LEP2: be pragmatic, split into Double- and Single-Pole, pick only numerically dominant terms:
 - ▶ $\mathcal{O}(\alpha^1)$ for $e^-e^+ \rightarrow 4f$ must be implemented in MC with explicit split into Double Pole and Single Pole. Calculations exist
 - ▶ $\mathcal{O}(\alpha^2)_{DP}$ calculations for the Double-Pole production and decay parts are needed! Feasible?
 - ▶ $\mathcal{O}(\alpha^2)_{SP}$ and $\mathcal{O}(\alpha^3)$ seem to be negligible
- ▶ More detailed analysis at the threshold may be instrumental
 - ▶ EFT methods promising, but for now inclusive results only
 - ▶ Non-factorizable soft interferences can be exponentiated within YFS scheme. How much of the higher order corrs. would be reproduced this way?

The overall precision tag $\sim 2 \times 10^{-4}$ feasible (?)

YFSWW3 $\oplus$ KoralW with new exponentiation
look like a good starting point

- M_Z, Γ_Z : From $\sigma(\sqrt{s})$ lineshape
 - Main uncertainties: B -field calibration, QED
 - $\delta M_Z, \delta \Gamma_Z \sim 0.1$ MeV could be achievable

- m_t : Current status $\delta m_t \sim 0.4$ GeV at LHC
 - Additional theory uncertainties?

PDG '18

Butenschoen et al. '16

Ferrario Ravasio, Nason, Oleari '18

From $e^+e^- \rightarrow t\bar{t}$ at $\sqrt{s} \sim 350$ GeV

today:

$$\begin{aligned}\delta m_t^{\overline{\text{MS}}} &= [\quad]_{\text{exp}} \\ &\oplus [50 \text{ MeV}]_{\text{QCD}} \\ &\oplus [10 \text{ MeV}]_{\text{mass def.}} \\ &\oplus [70 \text{ MeV}]_{\alpha_s} \\ &> 100 \text{ MeV}\end{aligned}$$

future:

$$\begin{aligned}&[20 \text{ MeV}]_{\text{exp}} \\ &\oplus [30 \text{ MeV}]_{\text{QCD}} \quad (\text{h.o. resummation}) \\ &\oplus [10 \text{ MeV}]_{\text{mass def.}} \\ &\oplus [15 \text{ MeV}]_{\alpha_s} \quad (\delta \alpha_s \lesssim 0.0002) \\ &\lesssim 50 \text{ MeV}\end{aligned}$$

Conclusions

- Top pair threshold scan allows precise mass determination

$$\Delta m_t < 100 \text{ MeV}$$

- Theory-dominated error, $\sim 3\%$ QCD scale uncertainty
- Known corrections:
 - N³LO QCD + Higgs
 - N²LO electroweak + non-resonant
 - LL initial state radiation
- All corrections included in version 2 of QQbar_threshold
<https://qqbarthreshold.hepforge.org/>

MC Top Quark Mass Parameter

Why is there a non-trivial issue in the interpretation of m_t^{MC} ?

- picture of “top quark particle” does not apply (non-zero color charge)
- m_t is a scheme-dependent parameter of a perturbative computation
→ in which scheme do MC event generators calculate?
- relation of m_t^{MC} to any field theory mass definition can be affected by different contributions (let's consider pole mass just for convention)

$$m_t^{\text{MC}} = m_t^{\text{pole}} + \Delta_m^{\text{pert}} + \Delta_m^{\text{non-pert}} + \Delta_m^{\text{MC}}$$

pQCD contribution:

- perturbative corrections
- depends on MC parton shower setup

non-perturbative contribution:

- effects of hadronization model
- may depend on parton shower setup

Monte Carlo shift:

- contribution arising from systematic MC uncertainties
- e.g. color reconnection, b-jet modelling, finite width,...

To get to the experimental precision, we must improve very much!

I. Dubovyk et al [JHEP 2019 \(1906.08815\)](#)

Present theory uncertainty for Γ_Z : 0.4 MeV

Observable	max. dev.	EXP now	FCC-ee	CEPC	GigaZ
Γ_Z [MeV]	0.04	2.3	0.1	0.5	0.8
$\sin^2 \theta_{\text{eff}}^\ell \times 10^4$	0.056	1.6	0.06	0.23	0.1
$\sin^2 \theta_{\text{eff}}^b \times 10^4$	0.025	160	9	9	15

Table 4. Goodness of fit for some chosen EWPOs, compared with the envisaged precision measurements for Γ_Z and $\sin^2 \theta_{\text{eff}}^\ell$ (statistical errors), and $\sin^2 \theta_{\text{eff}}^b$ (systematic errors) at the collider projects FCC-ee Tera-Z [84], CEPC [85] and ILC/GigaZ [86]. The values of maximal deviations are taken from tables 1 and 3. The entry “EXP now” gives the present experimental precision, as known since LEP 1 [44].

Present theoretical calculations are far behind the FCC-ee Tera-Z demands.

And enough for GigaZ LC.