The “Navier–Stokes Equation” Millennium Prize Problem

Physics behind mathematics — or: does the machine understand?

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The Navier–Stokes equation: derived, yet fundamental

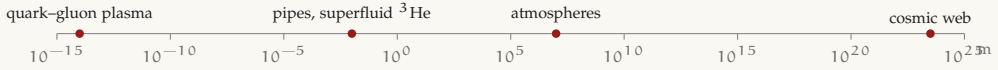
Fluid mysteries: stability and universality

Phenomena that probe the limits of the model

- Turbulence. Nikuradze's pipes (1933): transition, roughness — is the laminar solution *stable*?
- Shock waves, solitons. Nonlinearity versus dissipation: when do solutions stay smooth?
- Extension problems. Burnett order unstable (Bobylev 1982); Eckart instability of relativistic fluids (Hiscock–Lindblom 1985).

Fluids far beyond the conditions of the derivation

- Quark–gluon plasma. 10 fm, 10^{-23} s, yet causal second-order viscous hydrodynamics works; $\eta/s \approx \hbar/(4\pi k_B)$: the most perfect fluid known.
- Cosmic matter. Galaxies as fluid elements: collisionless dust ($p = 0$, local equilibrium at $T \to 0$) up to shell crossing (Zel'dovich 1970); beyond it an effective *viscous* fluid (EFT of large-scale structure). → see backup.
- Superfluid ^{3}He (Volovik 2003): Lorentz invariance, gauge fields, analog gravity emerge at low energies — hydrodynamics vs quantum mechanics.



We derive the Navier–Stokes equation somehow, yet it is valid far more generally than the conditions of its microscopic derivations.

The Boltzmann equation and the closure problem (Hilbert)

$$\partial_t f + \mathbf{c}\cdot\nabla_x f = Q(f,f); \quad \text{invariants } 1,\ m\mathbf{c},\ \tfrac{1}{2}mc^2 \Rightarrow 5 \text{ balances},\ \mathbf{P}, \mathbf{q} \text{ unknown}.$$

Chapman–Enskog: expansion in the Knudsen number

$f = f^{(0)}\left(1 + \mathrm{Kn}\,\varphi_1 + \mathrm{Kn}^2\varphi_2 + \dots\right)$, $f^{(0)}$ local Maxwellian distribution.

- $O(1)$: $\mathbf{P} = p\mathbf{I}$, $\mathbf{q} = 0$ — Euler.
- $O(\mathrm{Kn})$: $\mathbf{P} = p\mathbf{I} - 2\mu\mathbf{D}^0$, $\mathbf{q} = -\lambda\nabla T$ — Navier–Stokes–Fourier; for a monatomic gas $\zeta = 0$; for hard spheres $\mu = \frac{5}{16d^2}\sqrt{\frac{mk_BT}{\pi}}$, $\lambda = \frac{15}{4}\frac{k_B}{m}\mu$, $\mathrm{Pr} = \frac{2}{3}$. $\mu \propto \sqrt{T}$ (Maxwell, 1860).
- $O(\mathrm{Kn}^2)$: Burnett — unstable (Bobylev). The local-equilibrium closure is valid only for $\mathrm{Kn} \to 0$.

Rigorous limits

H-theorem: $s = -k_B\int f \ln f$ increases; here $\mu, \lambda \geq 0$ is a *consequence*, under conditions. Several rigorous results (moment expansion, Grad expansion, entropic closure). E.g. Deng–Hani–Ma (2025): hard spheres → Boltzmann → hydrodynamics. The question of stability is open.

Non-equilibrium thermodynamics: balances

Intuitively: the continuum generalization of the damped Newton equation. Damping proportional to the local velocity difference.

Balances — without constitutive relations

$$\begin{aligned}
\text{mass:} &\quad \dot\rho + \rho\ \operatorname{div}\mathbf{v} = 0,\\
\text{momentum:} &\quad \rho\,\dot{\mathbf{v}} + \operatorname{div}\mathbf{P} = \rho\mathbf{f},\\
\text{total energy:} &\quad \rho\ \tfrac{\mathrm{d}}{\mathrm{d}t}\left(\tfrac{|\mathbf{v}|^2}{2} + e\right) + \operatorname{div}(\mathbf{q} + \mathbf{P}^\top\mathbf{v}) = \rho\mathbf{f}\cdot\mathbf{v}.
\end{aligned}$$

Compressible, heat-conducting, viscous medium. ρ density, $\mathbf{v}$ velocity, $\dot{(\ )} = \partial_t + \mathbf{v}\cdot\nabla$,
$\mathbf{D} = \frac{1}{2}(\nabla\mathbf{v} + \nabla\mathbf{v}^\top)$, $\mathbf{D}^0 = \mathbf{D} - \frac{1}{3}(\operatorname{div}\mathbf{v})\mathbf{I}$, $\mathbf{P}$ pressure tensor, e specific internal energy, $\mathbf{q}$ heat flux.

The balance of internal energy: total minus kinetic

$$\begin{aligned}
\text{kinetic (momentum}\cdot\mathbf{v}\text{):} &\quad \rho\ \tfrac{\mathrm{d}}{\mathrm{d}t}\tfrac{|\mathbf{v}|^2}{2} + \operatorname{div}(\mathbf{P}^\top\mathbf{v}) = \mathbf{P}:\mathbf{D} + \rho\mathbf{f}\cdot\mathbf{v},\\
\text{difference:} &\quad \rho\,\dot e + \operatorname{div}\mathbf{q} = -\mathbf{P}:\mathbf{D}.
\end{aligned}$$

5(3) balances, 5(3) fields $(\rho, \mathbf{v}, e)$ — but $\mathbf{P}$ is symmetric, and $\mathbf{P}$ and $\mathbf{q}$ are *unknown*: the system is not closed. The Clay equation: $\rho = \text{const.}$, $\operatorname{div}\mathbf{u} = 0$, no energy balance.

Non-equilibrium thermodynamics: the second law (Eckart, 1940)

Entropy balance and the Gibbs relation

$$\rho\,\dot{s} + \operatorname{div}\mathbf{J}_s = \sigma_s \;\geq\; 0 \;\; \forall \text{ processes}, \qquad de = T\,ds + \tfrac{p}{\rho^2}\,d\rho.$$

$$\Longrightarrow\; \rho T\dot{s} = \rho\dot{e} + p\,\operatorname{div}\mathbf{v} = -(\mathbf{P} - p\mathbf{I}) : \mathbf{D} - \operatorname{div}\mathbf{q}.$$

The entropy production

Dividing by T and choosing $\mathbf{J}_s = \mathbf{q}/T$ ($\operatorname{div}(\mathbf{q}/T) = \operatorname{div}\mathbf{q}/T - \mathbf{q}\cdot\nabla(1/T)$):

$$\boxed{\sigma_s = -\tfrac{1}{T}\,(\mathbf{P} - p\mathbf{I})^0 : \mathbf{D} + \mathbf{q}\cdot\nabla\frac{1}{T} \;\geq 0.}$$

Solving the inequality: isotropic linear closure

$\sigma_s \geq 0$ for all $\mathbf{D}^0$, $\operatorname{div}\mathbf{v}$, ∇T. Linear isotropic solution: $\mathbf{q} = -\lambda\nabla T$ and $\mathbf{P} - p\mathbf{I} = -2\mu\mathbf{D}^0 - \zeta(\operatorname{div}\mathbf{v})\mathbf{I}$; then

$$\sigma_s = \frac{2\mu}{T}|\mathbf{D}^0|^2 + \frac{\zeta}{T}(\operatorname{div}\mathbf{v})^2 + \frac{\lambda}{T^2}|\nabla T|^2 \geq 0 \Leftrightarrow \mu, \zeta, \lambda \geq 0.$$

Newton's and Fourier's laws, and the positivity of the viscosities, follow from the entropy balance — the energy balance does not contain them. More precise constitutive state space: Coleman–Noll; Liu. T is defined by the entropy ($1/T = \partial s/\partial e$). p is the thermostatic pressure.

The million-dollar simplification/degeneration

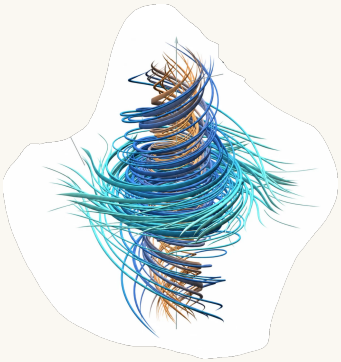
Clay Institute (Fefferman), $\mathbb{R}^3$ or $\mathbb{R}^3/\mathbb{Z}^3$ (torus), $\nu > 0$

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} = \nu \Delta \mathbf{v} - \nabla p + \mathbf{f},$$
$$\operatorname{div} \mathbf{v} = 0, \qquad \mathbf{v}(x, 0) = u^\circ(x) \text{ smooth},$$

Solution: $p, \mathbf{v} \in C^\infty$, and $\int |\mathbf{v}|^2 dx < C$ for all t. Only the kinetic energy is bounded ($\mathbb{R}^3$).

Four branches of the problem

	domain	force	claim
(A)	$\mathbb{R}^3$	$\mathbf{f} \equiv 0$	global smooth solution
(B)	torus	$\mathbf{f} \equiv 0$	global smooth solution
(C)	$\mathbb{R}^3$	smooth, decaying $\mathbf{f}$	finite-time blow-up
(D)	torus	smooth $\mathbf{f}$	finite-time blow-up



Spirals inward and stretches upward.

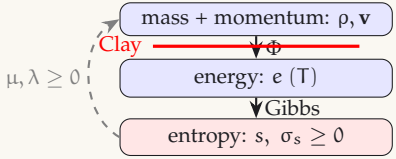
The orange one is faster.

The 2026 results target the (C)/(D) branch. (A)/(B) — what the community regards as "the" problem — is open.

The Clay reduction

$\rho = \text{const.}$, $\operatorname{div} \mathbf{v} = 0$, no energy equation

- the ζ term drops out; heat conduction disappears (T is not a field);
- the entropy production has a single term: $\sigma_s = \frac{2\mu}{T}|\mathbf{D}|^2 = \frac{\nu\rho}{T}|\nabla \mathbf{v}|^2$;
- the pressure comes not from an equation of state but from the constraint $\operatorname{div} \mathbf{v} = 0$: nonlocal, instantaneous.



The part below the red line is absent from the Clay model. The coupling is one-way (Φ: $\mathbf{v} \to T$); of the feedback (dashed) only the sign $\nu \geq 0$ remains.

Isothermal? Adiabatic? — no: athermal

There is no temperature in the model, so nothing can fix T. The dissipated power has nowhere to go. Isothermality would need $\lambda \to \infty$ or a heat bath; $\mathbb{R}^3$ has no boundary.

Degeneration: no energy balance.

What happened — a chronology (until 2026-11-01)

2023–25	Córdoba, Martínez-Zoroa (Zheng): "infinite cascade" technique; unforced Euler blow-up in $C^{1,\alpha}$, forced hypodissipative NS. Buckmaster (+ a DeepMind team) finds unstable singularities numerically.
2026-08-15/22	Alpöge–Buckmaster: blow-up driven by a smooth force for IPM, Boussinesq and 3D Euler; Lean verification. A year of work, with several language models.
2026-09-01/05	OpenAI: prompted by rumors, sets an internal model on all six problems; ~10 000 agents, 88 hours, ~ 130 billion tokens (~$15 million).
2026-09-07/08	Buckmaster's statement (midnight); OpenAI: 166 pages + Lean, forced NS blow-up, (C)/(D).
2026-09-11/13	Clay: "apparently settled", status: active; the procedure is deliberately unhurried. Open letter of 26 Fields medalists. OpenAI denies that user input influenced its system.
2026-09-17	Constantin–Ignatova–Vicol: the force can be neither real-analytic nor vanish near the singularity — no route to the unforced branch.

continued...

What happened — a chronology (until 2026-11-01)

2026-09-20/30	Petrillo–Glimm: a program for the unforced branch; OpenAI's similarity variables matched to an exact 1998 solution; first readable human reconstruction of the AI construction (arXiv:2609.35406).
2026-10-06	OpenAI releases 722 AI-written manuscripts (~372 result families). Bastounis–Circelli–Hansen: the Lean proof does not correspond to the written NS proof.
2026-10	Scientific American: experts see a construction of the force different from Córdoba–Martínez-Zoroa's.
2026-10-26	OpenAI claims the existence half of Yang–Mills (model text 09-09, Lean 10-16; no mass gap), scooping Sussman and Alpöge, who had worked in the open; verification handed to invited physicists.
2026-10-29	Hutchison (MIT): no physical force source examined can supply it; in water the axis pressure falls below the vapor pressure for cores under 9.5 mm.

The question is not whether the machine finds proofs, but who understands and certifies them.

Humans set the frame — the machine found part of the construction

The frame was human

- Target: Clay (2000) allows a smooth force in (C)/(D).
- Direction — forced blow-up via iterated small-scale layers: Córdoba, Martínez-Zoroa; chosen deliberately by Buckmaster and Alpöge.
- Córdoba: real fluids feel external forces, so "to have the force makes complete sense".

The machine's part

- OpenAI: Euler by different means; the NS route close to Buckmaster–Alpöge (Bubeck).
- Experts (Oct.): the force is built differently — a new construction inside the frame.

Limits, understanding: human

- Scope: no analytic or vanishing force (Constantin–Ignatova–Vicol); no physical force source (Hutchison)
- Certification: Lean proof $\neq$ written proof (Bastounis–Circelli–Hansen).
- Understanding: Tao — like seeing only the first and last ten minutes of a film.

The priority dispute

Two OpenAI offers declined; Buckmaster published first. OpenAI cannot rule out a role of de-identified user data, denies seeing the drafts. Neither side independently verified.

Not "we handed over the problem and it solved it".

Why is this interesting at all?

Millennium problems

Two lists, a hundred years apart

Hilbert, Paris, 1900: 23 unsolved problems. Not a forecast, not a prediction — *setting directions*.
Clay Institute, Paris, 2000-05-24: 7 problems, one million dollars each. Only one problem appears on both lists: the Riemann hypothesis (Hilbert's 8th).

The seven Millennium Problems

- Riemann hypothesis
- P vs. NP
- Navier–Stokes: existence and smoothness — 2026: "active"
- Yang–Mills: existence and mass gap — 2026-10: existence claimed, mass gap open
- Hodge conjecture
- Birch–Swinnerton-Dyer conjecture
- Poincaré conjecture — Perelman (2003); he declined the prize

Most of Hilbert's 23 problems are solved, but not all: the 8th, the 12th and the second half of the 16th are open; the 4th, the 6th and the 23rd are *programs* rather than problems.
Hilbert's 6th was essentially disqualified — not mathematics, cannot be closed.

Readings

1. 1900: extending the axiomatic method to physics: probability theory (Kolmogorov, 1933), quantum mechanics (von Neumann, 1932), continuum mechanics (Truesdell–Noll, 1965), thermodynamics (Carathéodory, 1909), ... *The obvious reading: discipline by discipline.*
2. Hydrodynamics as a LIMIT: particles → Boltzmann → Euler/Navier–Stokes.
3. Mathematics in physics: The mathematical formulation of a physical problem is complete if its solution exists, is unique and depends continuously on the initial and boundary data: WELL-POSEDNESS (Hadamard, 1902/1923; Courant–Hilbert II).

Navier–Stokes: Leray (1934) weak solutions, energy inequality; Ladyzhenskaya (1958–59) existence and uniqueness in two dimensions. 3D: open.

2025: DENG–HANI–MA derive the fluid equations from hard-sphere dynamics (a long-time extension of Lanford's theorem (1975)). This is often called "the solution of Hilbert's 6th problem" — a narrowed reinterpretation: it closes a single strand, and the authors themselves put it this way. Deng was awarded a Fields Medal in July 2026.

Reading 3: the Godunov–Friedrichs–Lax–Boillat theorem (1961–1974)

The mathematical formulation of a physical problem is complete if its solution exists and is unique (and possibly depends continuously on the initial and boundary data, if there are any).

Friedrichs–Lax system of balances

$$\partial_t u + \partial_i F^i(u) = 0, \qquad u \in \mathbb{R}^n, A^i = \nabla_u F^i$$

Entropy balance (s, J^i): for every smooth solution $\partial_t s(u) + \partial_i J^i(u) = 0$, i.e. $\nabla s \cdot A^i = \nabla J^i$ (compatibility).

Theorem

If s (the physical entropy density) is strictly concave, $H = -\nabla^2 s \succ 0$, then HA^i is symmetric, hence

$$H\,\partial_t u + HA^i\,\partial_i u = 0$$

is symmetric hyperbolic: symmetric coefficients, positive definite time-derivative matrix.

Proof

In components: $s_{,k}A^i_{kj} = J^i_{,j}$. Differentiate with respect to u_l: $s_{,kl}A^i_{kj} + s_{,k}\,\partial_l A^i_{kj} = J^i_{,jl}$. Here $\partial_l A^i_{kj} = \partial_j \partial_l F^i_k$ is symmetric in (j, l), and so is $J^i_{,jl}$. Hence $(HA^i)_{lj} = s_{,kl}A^i_{kj}$ is symmetric in (j, l). □

For systems with sources ($\partial_t F^0(u) + \partial_i F^i(u) = P$) the same holds, with $-s$ convex in F^0.

Clay degeneration: the Hessian of the dual entropy potential degenerates.

A bit more physics

Blow-up and the domain of validity of the model

Physical objections

- Mach number. Incompressible approximation: $\mathrm{Ma} \lesssim 0.3$. If $\|\mathbf{v}\|_{L^\infty} \to \infty$ at fixed c, then $\mathrm{Ma} \to \infty$. The blow-up leaves the domain of validity of its own model. Sound speed, shock waves and compressibility come first.
- Knudsen number. The size of the singular core is $\ell_r \asymp \tau^{1/2} \to 0$ ($\tau = T^* - t$); once ℓ reaches the mean free path, $\mathrm{Kn} \to \infty$: the Newton–Fourier closure (Chapman–Enskog $O(\mathrm{Kn})$) loses its validity.
- Velocity bounds: speed of sound and light. $|\mathbf{v}| \to \infty$ exceeds c_{light}(in water, at the matching radius, once the core is below 0.12 nm: the last 15 fs); causal viscous hydrodynamics (Israel–Stewart; BDNK) has finite propagation speed.
- Scale. The self-similar profile spans $> 7.2 \times 10^{47}$ core lengths: for any core above the molecular spacing of water it exceeds 10^{12} Hubble radii, and it fits inside the observable universe only for a core $< 1.9 \times 10^{-22}$ m (Hutchison 2026).
- Pressure. In water the pressure on the axis falls below the vapor pressure once the core is under 9.5 mm: cavitation comes first (Hutchison 2026).

The hidden physics in the mathematical problem

All of these concern the *physical* interpretability of the model, not its mathematical well-posedness: the Clay problem is about the idealized PDE. The objections mainly affect the forced (C)/(D) branch; they do not make the unforced (A)/(B) question meaningless — it remains the physically most relevant question: from a faithful model of reality we expect existence and uniqueness.

Variations

- Physical regularization: temperature-dependent density, Müller paradox (Gouin–Ruggeri, 2012)
- Artificial compressibility (Chorin, 1967): $\partial_\tau p + c^2 \operatorname{div} \mathbf{v} = 0$ — finite c, hyperbolic.
- SHTC (Godunov–Romenski; Peshkov–Romenski 2016): first-order, symmetric hyperbolic, thermodynamically compatible; viscosity from a relaxation time, $\mu \simeq \frac{1}{6}\rho c_s^2 \tau$; NS in the limit $\tau \to 0$.
- Hyperbolic relaxation (Ilyin–Rykov–Zelik, 2018, 2D): $\varepsilon \partial_{tt}\mathbf{v} + \partial_t \mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} + \nabla p = \Delta \mathbf{v} + \mathbf{g}$.

“It is nice to know that the computer understands the problem.
But I would like to understand it too.”

Eugene P. Wigner

What is going on?

Understanding ourselves is the key to understanding AI.

We do two things when we do science

Tinkering. Within the existing framework: we compute, check, refine, fit.

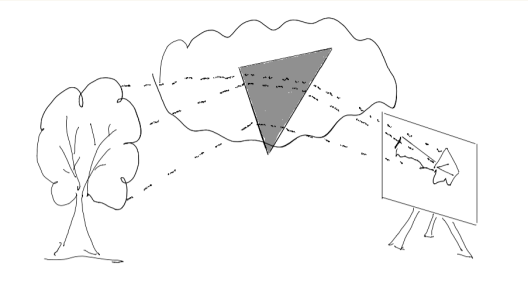
Rebuilding. We replace the framework itself: we start thinking with other concepts, in another language.

The machine is getting ever stronger at tinkering: in a month it does what would otherwise have taken years.

Choosing the criterion and understanding are still ours: tinkering = a fixed, external criterion of success (Clay 2000, Lean); rebuilding = the criterion changes. We have not yet seen a machine rebuild. The Navier–Stokes example showed what this means for a physicist: the framework itself — balances, entropy, stability — was the hidden question.

Cf. Zahavy (2026): induction and deduction are being mastered; the abductive *jump* to new premises is not.

Science as a camera obscura: the *camera scientiae*



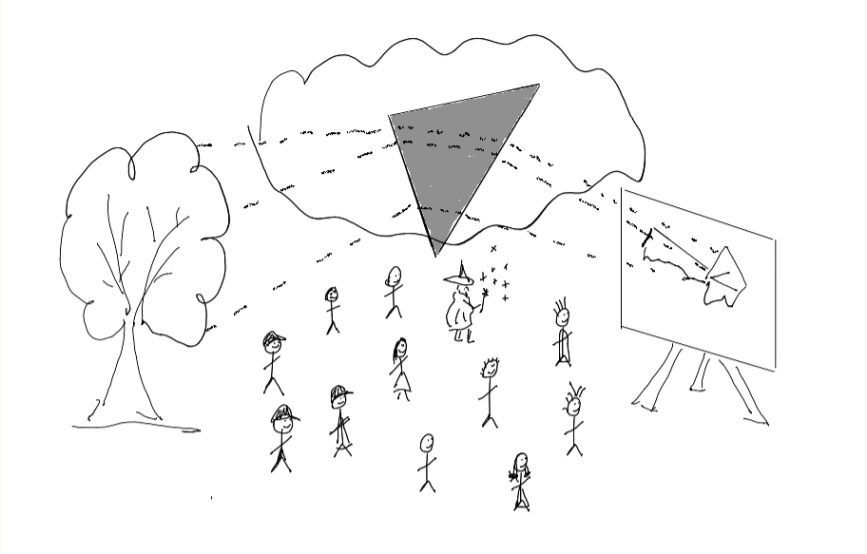
Three functions of the camera

1. Choice of direction — where it turns. 2. Contact with reality — how it images (the slit through which the light enters). 3. Fidelity criterion — what counts as a good image.

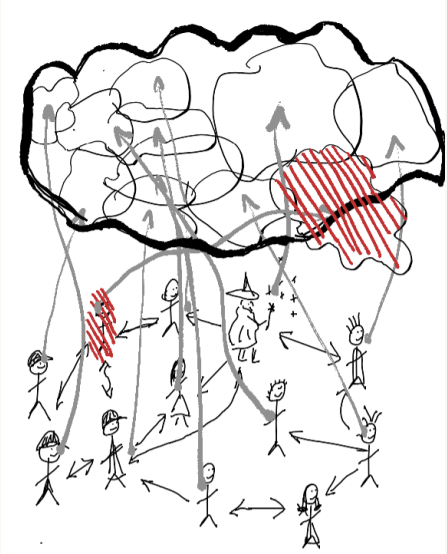
Tinkering retouches the image; rebuilding turns the camera, opens a new slit, or changes the criterion.

The (thinking) machine works inside the camera. It does not turn it; it has no fidelity criterion and no contact with reality.

The image and its viewers: models and modelers



Viewers or pixels? Clouds in the cloud



Handling the images: knowledge technologies

General technologies

- **Transmission.** E.g. speech (human–human), books (human–cloud).
- **Storage.** Delayed transmission: writing, library, data repository.
- **Compression.** Fast transmission, easier storage. E.g. concepts, $F = ma$, $\sigma_s \geq 0$.
- **Self-correction.** Debate, logic, experiment.

In science everything can be disputed, always and everywhere. The aim in principle: truth, i.e. objectivity and faithfulness to reality.

Self-correction requires comparison: the technology of truth

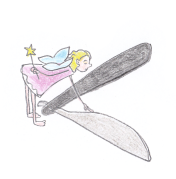
- With each other: debate. Comparing personal and group models. Rules of the game: peer review, conferences. 2026: a press release + Lean does *not* replace them.
- With oneself: logic. Grammar. Finding the errors. The machine is strong here.
- With reality: the scientific method. Model $\to$ experiment $\to$ modification $\to \ldots$ Mach, Knudsen: this is where the blow-up fails.

AI: a combined technology

It stores and transmits. Does it compress? Does it self-correct?

The role of mathematics: Galilean linguistics

- Horizontal. Disciplines. Engineer, scientist, ...
- Vertical. Compression codes: formula, differential equation, geometry, operator algebra. Mathematics: a language and a component.
- Science: a language family of Sapir–Whorf type. Conceptual frameworks determine what you are able to think. Navier–Stokes as a PDE, as a system of balances, as a kinetic limit, as dynamics without an action principle: four dialects, not the same questions.
- NS in mathematics: tool-building? NS in physics: modeling?



And now? Two directions

According to Buckmaster, what matters is not the result itself but that a mathematician and a language model could do this *in one month* — and what this means for how we teach students, how we assign credit, how we review, and what is worth a lifetime of attention. The letter of the 26 Fields medalists (2026-09-11) warns that the fertile ground of conceptual understanding and of training students may erode.

The thinking machine takes over routine work. This can mean two things.

↗ Time is freed up for interesting questions. Less bureaucracy, more thinking. The knowledge network becomes more open and more democratic.

↘ Or: proposal writing and reviewing are mechanized too, and the Mark Gable Foundation finally runs at full capacity. Endless text, endless review, no research. Plus: 15 million dollars for 88 hours — the competition is financial, not intellectual; in October the pattern repeated with Yang–Mills (Scientific American).

This does not depend on the thinking machines. Science is corrupted by its institutions —
and they may now grow stronger, or collapse.

Who points the camera? Leo Szilard's recipe: The Mark Gable Foundation

The task: how could science be *slowed down*?

- Set up a foundation with a lot of money. Anyone may apply, with a convincing justification.
- Have committees, each with at least ten or twelve scientists, to judge the applications.
- Take the most active researchers out of the laboratory and put them on the committees.

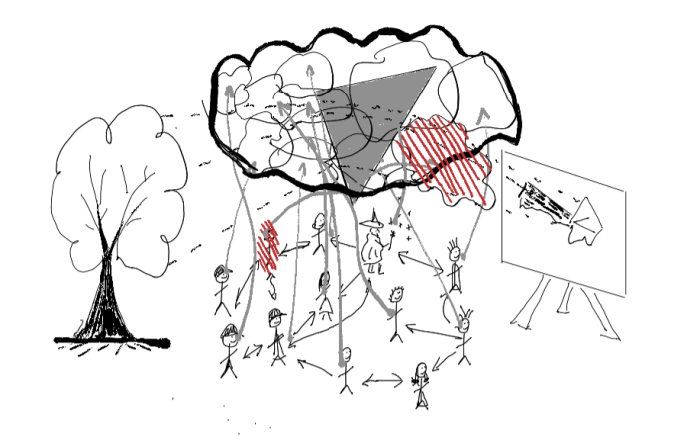
What happens next?

- The best people review instead of doing research.
- Applicants choose topics that will *surely* lead to a publication.
- Fashions emerge. Those who follow them get money. Those who do not, do not.
- Science dries up. It turns into a parlor game.

Szilard wrote this in 1948, shortly before the NSF was established.

Wigner was right: it is nice if the machine understands.

But the consequences depend on how much the work of understanding remains ours.



Thank you for your attention!

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The language of Newton and the language of Fermat — not equivalent

Newton: initial-value problem — $m\ddot{x} = F(x)$

I know where I am and what force acts: I take a small step. Cause → effect, in sequence.

Fermat–Hamilton: boundary-value problem — $\delta S = 0$

I fix the beginning and the end; I look for the path on which the action is stationary. Stationary, *not* minimal.

Why are the two languages not the same?

- The action principle gives more: Noether's theorem (symmetry → conservation law).
- The action principle covers less: friction, heat conduction, the viscous term of Navier–Stokes — *these have no standard Lagrangian*. (The inverse problem of the calculus of variations.)

Navier–Stokes is dissipative: its natural language is not the action principle but balances + the second law. This is the language we looked at in the beginning.

EFT of large-scale structure: a viscous fluid from collisionless dust

- Dust ($p = 0$, single stream) is exact only up to shell crossing. Coarse-grained Vlasov–Poisson dynamics gives a symmetric effective stress $\tau_{ij} = \kappa_{ij} + \Phi_{ij}$: velocity dispersion plus the gravitational stress of the short modes.
- Constitutive form (Carrasco–Hertzberg–Senatore):
$$\tau_{ij} \supset \rho_b \left[c_s^2\, \delta\, \delta_{ij} - \tfrac{c_{bv}^2}{Ha}\, \delta_{ij} \partial_k v^k - \tfrac{3}{4} \tfrac{c_{sv}^2}{Ha} \left(\partial_i v_j + \partial_j v_i - \tfrac{2}{3} \delta_{ij} \partial_k v^k \right) \right] + \Delta\tau_{ij}$$
- No collisions needed: momentum exchange between scales and with the field. Effective mean free path $\nu/H \sim 1/k_{NL}$, Knudsen number k/k_{NL}.
- Memory: local in space, non-local in time, $\tau(\mathbf{x}, t) = \int^t dt'\, K(t, t')\, \tau^{loc}\big(\mathbf{x}_{fl}(t'), t'\big)$; relaxation time $\sim H^{-1}$ — the domain of extended thermodynamics (cf. the quark–gluon plasma).
- Sign: $c_{comb}^2 = c_s^2 + f\, c_v^2 > 0$ is measured. Negative eddy viscosity occurs only in driven flows (Jupiter's jets, soap films); a second-law constraint needs the short-mode energy (velocity dispersion σ_{ij} plus gravitational potential energy: „eddy energy") as a variable.
- Self-gravity: the global negative heat capacity (Lynden-Bell–Wood 1968) is localization (shape) dependent (Giordano et al. 2024).
- Sound speed, temperature: $c_s^2(a)$ is a fitted Wilson coefficient, no polytropic equation of state; temperature only as the dispersion tensor σ_{ij} (Vlasov/cumulant perturbation theory).

Hydrodynamics beyond local equilibrium, thermodynamically unexamined: a test case for extended and weakly nonlocal thermodynamics.

Baumann, Nicolis, Senatore, Zaldarriaga, JCAP 2012 | Carrasco, Hertzberg, Senatore, JHEP 2012 | Carrasco, Foreman, Green, Senatore, JCAP 2014 | Salyk et al. 2006 | Lynden-Bell & Wood, MNRAS 1968 | Giordano et al., Phys. Fluids 36 (2024) 056127

The main field (Godunov–Boillat–Ruggeri) — the dual form of the theorem

$\partial_t \mathbf{F}^0(\mathbf{u}) + \partial_i \mathbf{F}^i(\mathbf{u}) = \mathbf{f}$, supplementary balance $\partial_t s + \partial_i J^i = \Sigma$, s concave.

Main field and Legendre transform

$$\mathbf{u}' = \frac{\partial(-s)}{\partial \mathbf{u}}, \qquad s' = \mathbf{u}' \cdot \mathbf{F}^0 + s, \quad J'^i = \mathbf{u}' \cdot \mathbf{F}^i + J^i, \qquad \mathbf{F}^0 = \frac{\partial s'}{\partial \mathbf{u}'}, \quad \mathbf{F}^i = \frac{\partial J'^i}{\partial \mathbf{u}'}.$$

$$\frac{\partial^2 s'}{\partial \mathbf{u}' \partial \mathbf{u}'}\, \partial_t \mathbf{u}' + \frac{\partial^2 J'^i}{\partial \mathbf{u}' \partial \mathbf{u}'}\, \partial_i \mathbf{u}' = \mathbf{P}, \qquad \frac{\partial^2 s'}{\partial \mathbf{u}' \partial \mathbf{u}'} \succ 0.$$

Euler gas (monatomic): equilibrium components of the main field = Liu multipliers

$$\Lambda_\rho\big|_E = \frac{1}{T}\Big(g - \frac{v^2}{2}\Big), \qquad \Lambda_{v_i}\big|_E = \frac{v_i}{T}, \qquad \Lambda_e\big|_E = -\frac{1}{T}, \qquad g = e + \frac{p}{\rho} - Ts.$$

Signs depend on conventions ($\pm s$, $\pm\Lambda$); the multipliers belong to the balances of $(\rho, \rho\mathbf{v}, \rho(e + v^2/2))$. The component conjugate to the mass balance contains the term $p/(\rho T)$: the pressure is tied to the density through the chemical potential. The proof in conservative variables is in the main text; the two are two coordinatizations of the same symmetry.

Film: *Arrival*, 2016.

The key to understanding

The physicists get through to the heptapods when they realize that they have to start from Fermat's principle, not from $F = ma$. For the aliens the *variational* formulation is the natural one.

Why that one?

- The action principle is *seemingly(?)* teleological: as if the light "knew" where it will arrive.
- The heptapods see time all at once. For them this is not an appearance but the natural reading.
- Their writing is like that too: a semagram is not unfolded in time — the whole sentence is written at once.

Not a difference of language: a difference of conceptual organization. This is the sublinguistic level.



Heptapod semagram

The prehistory left out of the main text

1934	Leray: weak solutions, energy inequality; proposal of a self-similar blow-up.
1982	Caffarelli–Kohn–Nirenberg: the singular set has zero $\mathcal{P}^1$ measure.
1996/2003	Nečas–Růžička–Šverák: no Leray-type self-similar blow-up; Escauriaza–Seregin–Šverák: the L^3 criterion.
2014	Luo–Hou: numerical Euler blow-up with a boundary. 2016: Tao, blow-up of an "averaged" NS (supercriticality as the obstacle).
2019	Buckmaster–Vicol: non-uniqueness of weak NS solutions (convex integration) — outside the admissible class.
2021	Elgindi: unforced $C^{1,\alpha}$ Euler blow-up ($\mathbb{R}^3$, no boundary).
2022	Albritton–Brué–Colombo: Leray–Hopf solutions of the *forced* NS are not unique. Chen–Hou: computer-assisted proof of Euler blow-up with a boundary. MRRS: compressible implosion.
2023–25	Córdoba–Martínez-Zoroa(–Zheng): "infinite cascade". Buckmaster + DeepMind: unstable self-similar profiles (PINN).
2025-09	Hou–Wang–Yang: non-uniqueness of Leray–Hopf solutions of the unforced NS (computer-assisted, not peer-reviewed).
2026-08/09	Alpöge–Buckmaster(–Coiculescu); OpenAI; Clay "active"; Fields letter.

Special-purpose machines and an open space

Chess (10^{47} positions), Go (10^{170}), AlphaFold: a finite space and a clear figure of merit. A language model: no rulebook, no score, no winning condition. Most of the training material is in English.

Probability from distance

From the context the model produces a vector $\mathbf{h}$, and for every word i it takes the scalar product $\mathbf{h} \cdot \mathbf{e}_i$ ($\mathbf{e}_i$: the output vector of the i-th token, a row of the output matrix). *If* the word vectors have roughly the same length,

$$p_i \propto e^{-E_i/T}, \qquad E_i \approx \tfrac{1}{2}\,\|\mathbf{h} - \mathbf{e}_i\|^2.$$

The "energy" is the distance between context and word — hence the Boltzmann form (softmax). In practice, too, T is called the *temperature*.

There are two "distances" here: the distance in meaning space is a *metric*; the cross-entropy minimized in training is a *divergence* between distributions — not symmetric. What is sought is the maximum-entropy distribution at fixed mean energy. A familiar structure.

How does science work?

The backbone (essence) of science is the
intersubjective knowledge network.

- Intersubjective. Fictional and real at once, hence not physically objective. Relativity theory is like money, or like Hungary.
- Knowledge. Personal. The aim is truth, that is, modeling reality.
- Information network. Functions: truth and cooperation.