The non-extensivity of the Quark-Gluon Plasma and the meaning of the parameter q

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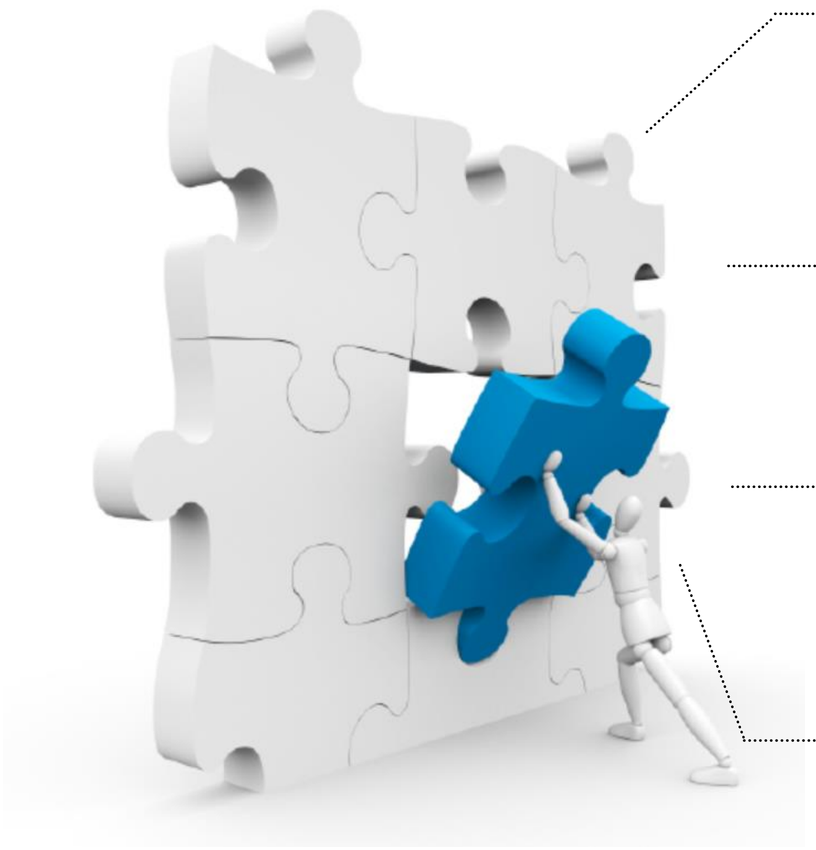


Background Knowledge

- **Theoretically**

    dynamically B-G statistical mechanics not yet fully established, the concept of entropy one step further [1], …

- **Experimentally**

    power tailed pT distributions, solar neutrino problem [2], …

1: C. Tsallis, Introduction to Nonextensive Statistical Mechanis
2: D.C. Clayton, Nature 249, 131 (1974); Kaniadakis,G.*et al.* Astrophys.Space Sci. 258 (1998) 145-162 astro-ph/9701118

Background Knowledge

Tsallis (1988), Daroczy (1963), Renyi* (1959)

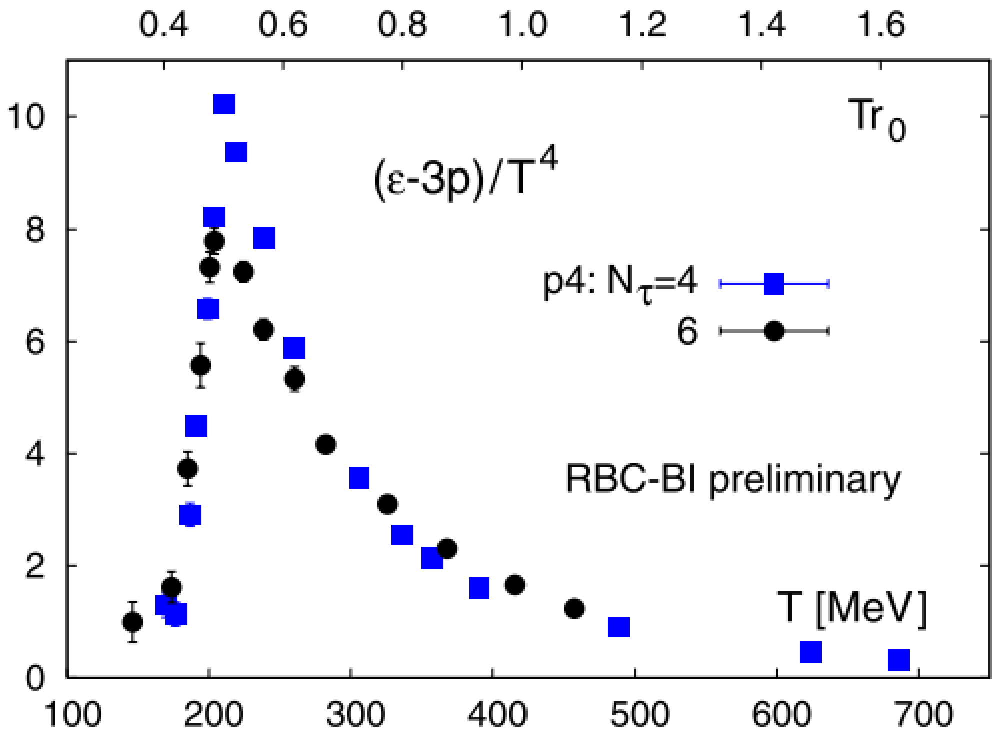
$$S_q = k\frac{1-\sum_{i=1}^{W} P_i^q}{q-1}$$

$$S_q = k\frac{1}{q-1}(1-\int[f(x)]^q d\Omega)$$

Where k is a positive constant (from now on set equal to 1), W is the number of microstates in the system, P_i are the associated normalized probabilities and the Tsallis parameter q is a real number. Similar to the integral form.

C. Tsallis, J. Stat. Phy. **52** (1988) 479

The Quark-Gluon Plasma close to the critical Temperature:



The trace anomaly, $(\varepsilon - 3P)/T^4$, calculated in (2+1)-flavor QCD on lattices with temporal extent $N_\tau = 4$ and 6, respectively.

Frithjof Karsch (for the RBC-Bielefeld Collaboration) 2007 *J. Phys. G: Nucl. Part. Phys.* **34** S627

Nonextensive Effects in the Quark-Gluon Plasma

The Quark-Gluon Plasma close to the critical Temperature:

- A strongly interacting system
- Long-range color interactions
- Memory effects are not negligible
- The ordinary mean field approximation of the plasma is no longer correct
- …

W.M.Alberico, A.Lavagno , Eur.Phys.J. A40 (2009) 313-323 arXiv:0901.4952 [nucl-th]

Nonextensive Effects in the Quark-Gluon Plasma

- The nonextensive entropy for fermions proposed as,

$$S_q = \sum_i \{ \left(\frac{n_i - n_i^q}{q-1} \right) + [\frac{(1-n_i)-(1-n_i)^q}{q-1}] \}$$

- The extremization of it under the constraints imposed by the total number of particles and the total energy of the system leads to the distributions,

$$n_i = \frac{1}{[1+(q-1)\beta(\varepsilon_i-\mu)]^{1/(q-1)}+1}$$

Similarly, for bosons,

$$n'_i = \frac{1}{[1+(q-1)\beta(\varepsilon_i-\mu)]^{1/(q-1)}-1}$$

A. M. Teweldeberhan, H. G. Miller, and R. Tegen, *Int. J. Mod. Phys. E* **12**, 395 (2003)

Nonextensive Effects in the Quark-Gluon Plasma

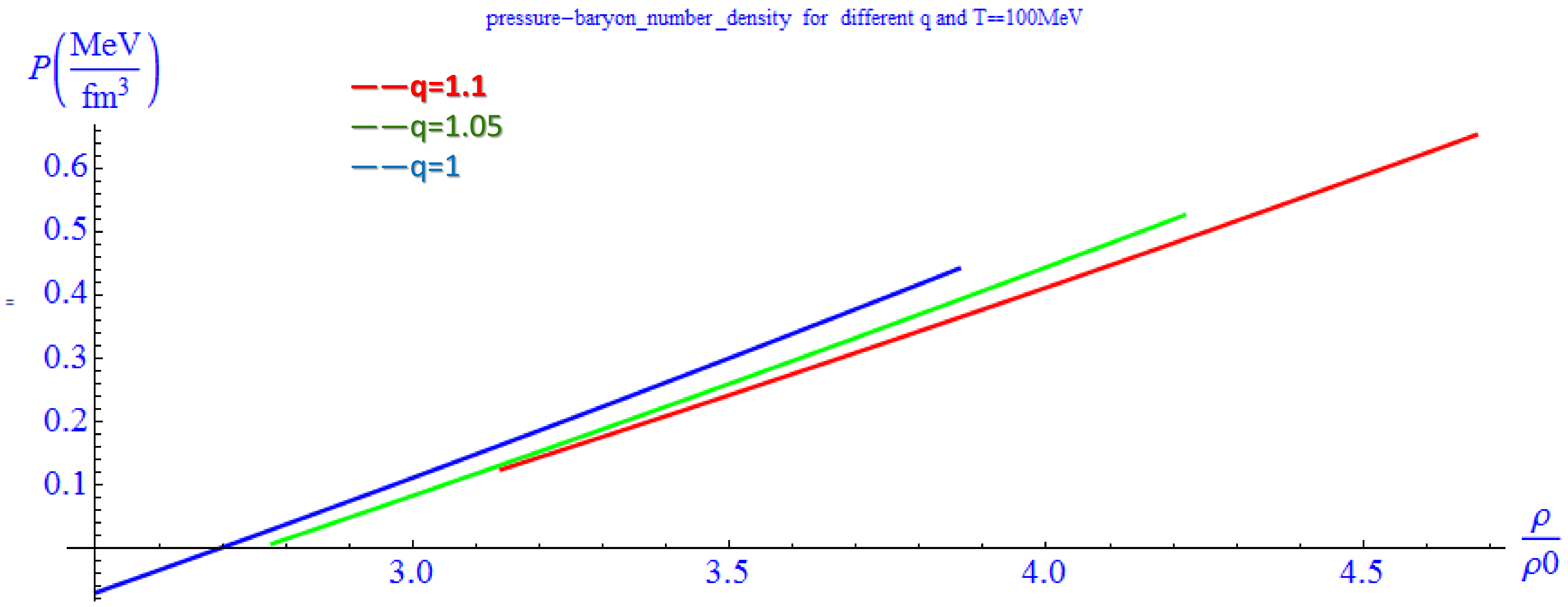
- Nonextensive QGP Equation of State

$$P_q = \sum_{f=u,d} \frac{1}{3} \frac{r_f}{2\pi^2} \int_0^\infty k \frac{\partial \varepsilon_k}{\partial k} \left[n_f^q(k, \mu_f) + n_f^q(k, -\mu_f) \right] k^2 dk - B,$$

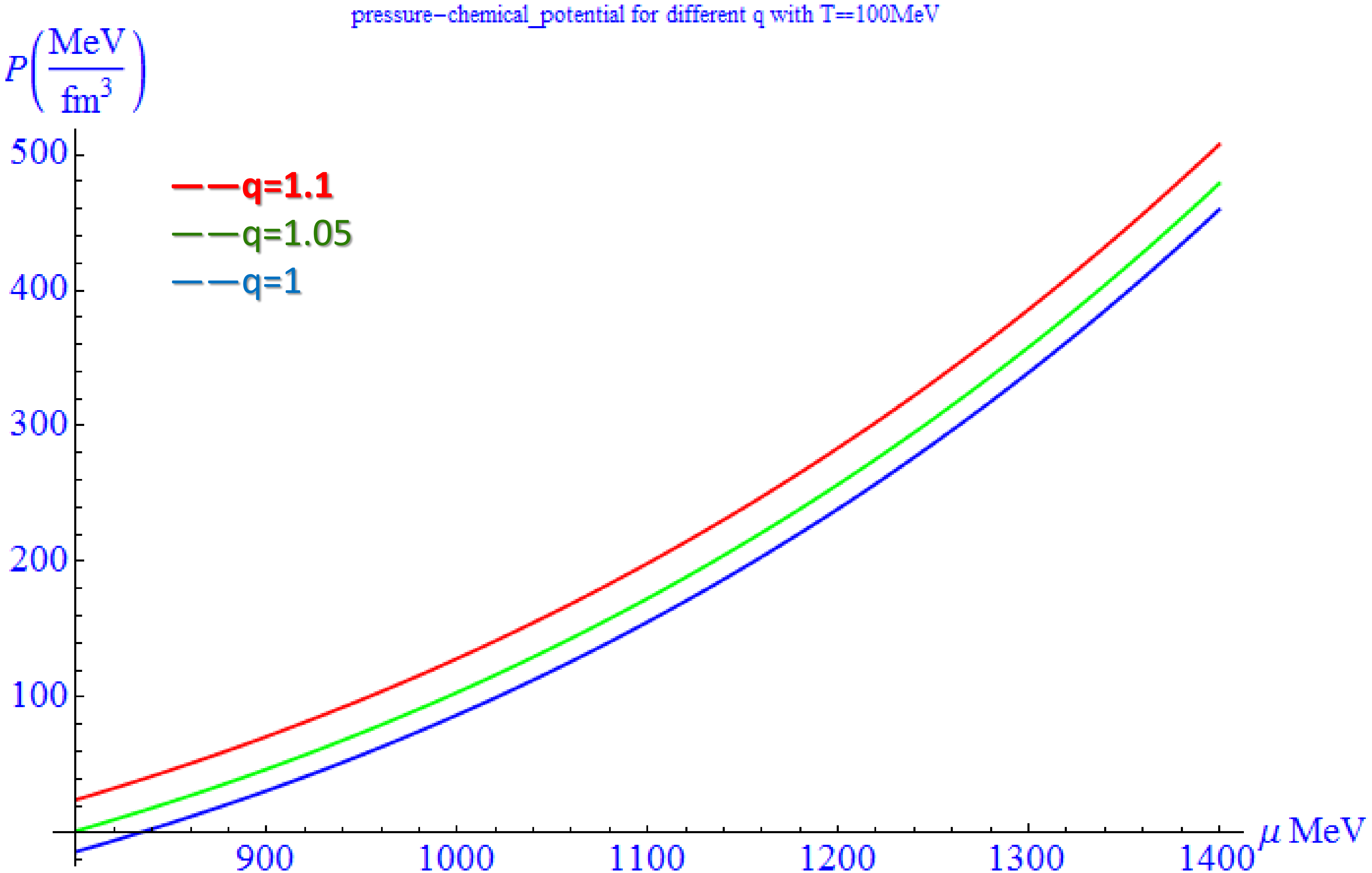
$$\varepsilon_q = \sum_{f=u,d} \frac{r_f}{2\pi^2} \int_0^\infty \varepsilon_k \left[n_f^q(k, \mu_f) + n_f^q(k, -\mu_f) \right] k^2 dk + B,$$

$$\rho_q = \sum_{f=u,d} \frac{1}{3} \frac{r_f}{2\pi^2} \int_0^\infty [n_f(k, \mu_f) - n_f(k, -\mu_f)] \, k^2 dk,$$

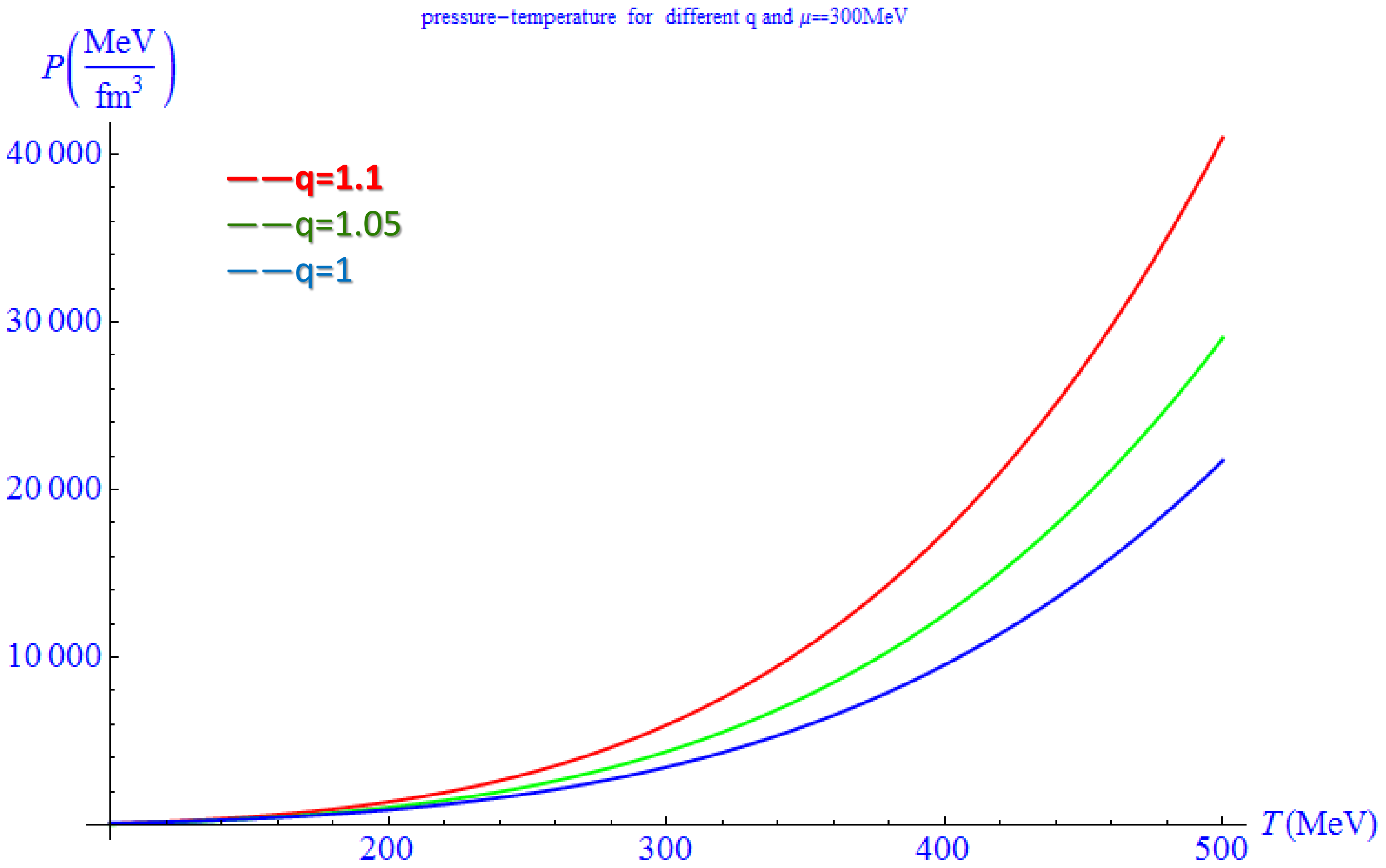
Nonextensive Effects in the Quark-Gluon Plasma



Nonextensive Effects in the Quark-Gluon Plasma



Nonextensive Effects in the Quark-Gluon Plasma



Nonextensive Effects in the Quark-Gluon Plasma

- Transverse momentum spectrum and *q*-blue shift

$$E\frac{d^3N}{d^3p}=\frac{dN}{dy m_\perp dm_\perp d\phi}=\frac{g}{(2\pi)^3}\int p^\mu d\sigma_\mu(x) f(x,p)$$

W.M.Alberico, A.Lavagno and P.Quarati, Nucl.Phys. A680 (2000) 94-97 nucl-th/0005010

Nonextensive Effects in the Quark-Gluon Plasma

- Transverse momentum spectrum and *q*-blue shift

Boltzmann distribution

- $\frac{dN}{m_\perp dm_\perp} = A m_\perp K_1(z)$
- $\frac{dN}{m_\perp dm_\perp} = B\sqrt{m_\perp} e^{-z}$, $(z \gg 1)$

Tsallis distribution

- $\frac{dN}{m_\perp dm_\perp} = C m_\perp \{K_1(z) + \frac{q-1}{8} z^2 [3K_1(z) + K_3(z)]\} + \ldots$ $(q \rightarrow 1)$
- $\frac{dN}{m_\perp dm_\perp} = D\sqrt{m_\perp} e^{-z+\frac{q-1}{2}z^2}$, $(z \gg 1)$

$z = \frac{m_\perp}{T}, m_\perp = \sqrt{{p_\perp}^2 + m^2}$

Nonextensive Effects in the Quark-Gluon Plasma

- Transverse momentum spectrum and *q*-blue shift

$$T_q = T + (q-1)m_\perp \quad , \text{ (q>1)}$$

W.M.Alberico, A.Lavagno and P.Quarati, Nucl.Phys. A680 (2000) 94-97 nucl-th/0005010

Nonextensive Effects in the Quark-Gluon Plasma

- Transverse momentum spectrum and *q*-blue shift

$$\frac{d^2N}{2\pi p_\perp dp_\perp dy} = Cm_\perp[1-(1-q)\frac{m_\perp}{T}]^{1/(1-q)}$$

W.M.Alberico, A.Lavagno , Eur.Phys.J. A40 (2009) 313-323 arXiv:0901.4952 [nucl-th]

Nonextensive Effects in the Quark-Gluon Plasma

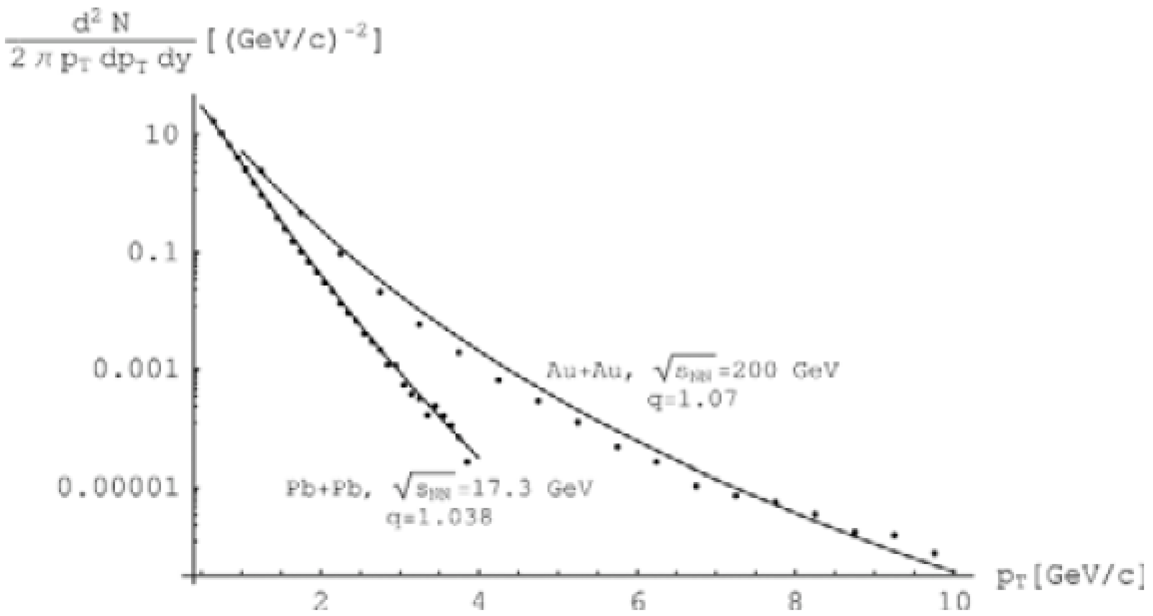
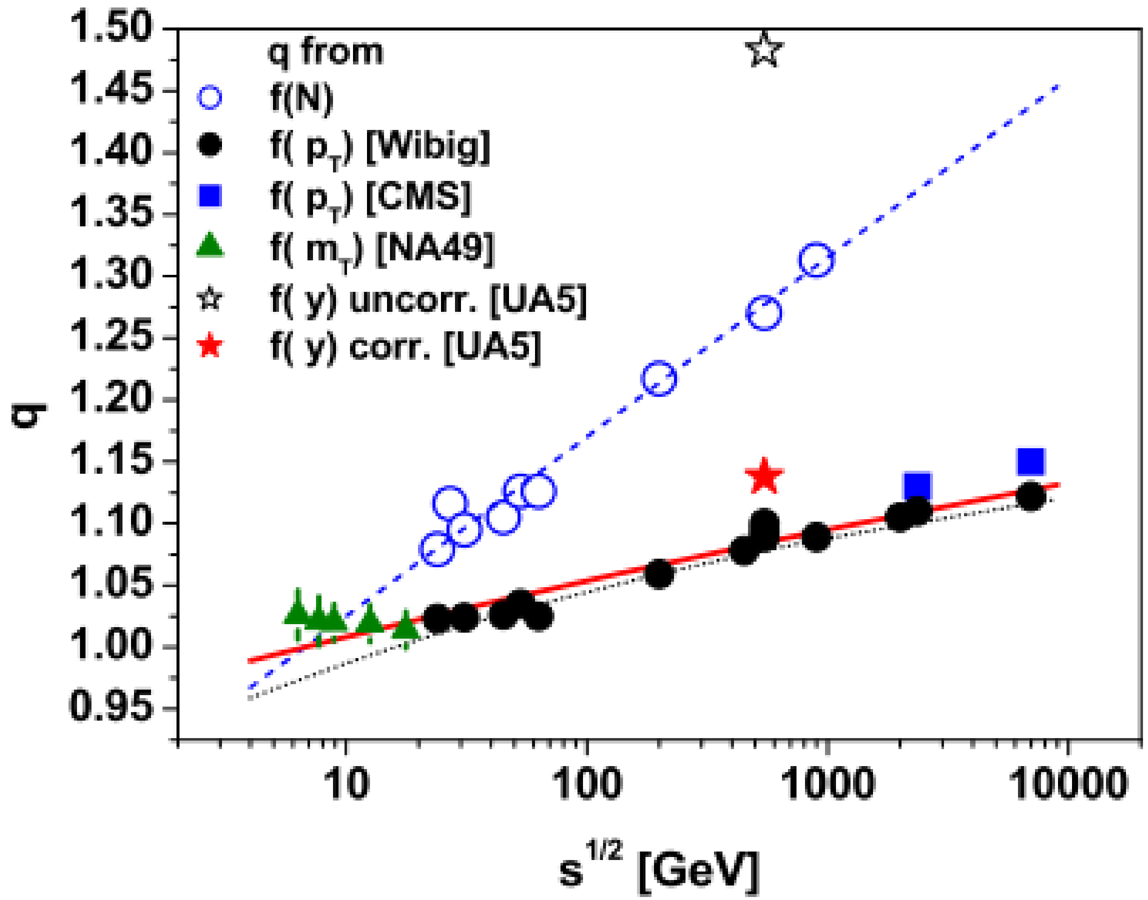


Fig. 4. Experimental neutral pion invariant yields in central Pb+Pb collisions at $\sqrt{s_{NN}} = 17.3\,\text{GeV}$ [39] and in central Au+Au collisions at $\sqrt{s_{NN}} = 200\,\text{GeV}$ [40] compared with the modified thermal distribution shape by using non-extensive statistics ($q = 1.038$ for Pb+Pb and $q = 1.07$ for Au+Au collisions.)

W.M.Alberico, A.Lavagno , Eur.Phys.J. A40 (2009) 313-323 arXiv:0901.4952 [nucl-th]

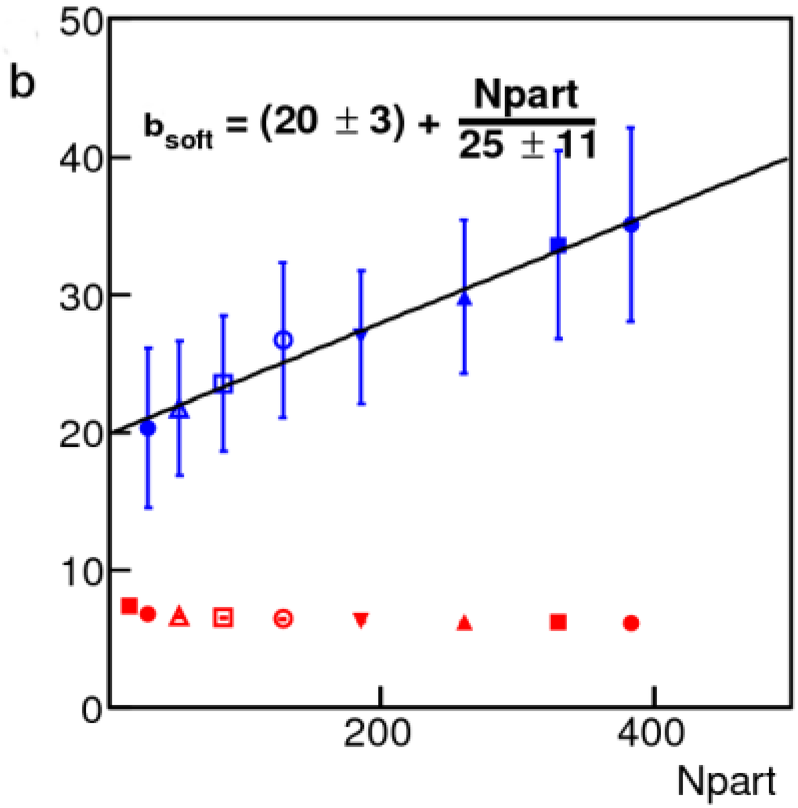
What's the parameter *q*



Examples of energy dependence of the nonextensivity parameter q as obtained from different observables. Open symbols show q obtained from multiplicity distributions. Solid symbols show q=qT obtained from a different analysis of transverse momentum distributions. Data points are taken from different experimental groups. The dotted line represents a fit and the full line comes from another one. Stars show q=qL obtained from dN/dy.

G.Wlik and Z.Wlodarczyk, Correlations from generalized thermodynamic .PoS WPCF2011 (2011) 041
arXiv:1202.2711 [hep-ph]

Nonextensive Effects in the Quark-Gluon Plasma



Powers in the power law, b=1/(q-1), follow a statistical trend for the soft spectra (upper symbols), while remain nearly constant for the hard spectra (lower symbols). The results belong to the participant numbers, N_{part}.

T.S.Biro, G.G.Barnafoldi, P.Van and K.Urmossy, Statistical Power-Law Spectra due to Reservoir Fluctuations, arXiv:1404.1256 [hep-ph]

What's the parameter *q*

What is the parameter q ?

What's the parameter *q*

- The physical origin and quantitative determination of such parameters from microscopic theories are longstanding but still actual and intriguing questions.
- Its universal origin is best understood by the study of finite reservoir effects, usually neglected in the classical thermodynamical limit.

T.S.Biro, Physica A392 (2013) 3132-3139 arXiv:1211.5284 [cond-mat.stat-mech]

What's the parameter q

- In high energy physics it is widespread to formulate that the ideal gas has no interaction. This is not precise, since even elastic (kinetic energy conserving but momentum changing) collisions represent short term, but violent interactions.

$$I_{12} = S_1(E_1) + S_2(E_2) - S_{12}(E_1 + E_2) \neq 0$$

$$L(S_{12}) = L(S_1) + L(S_2)$$

T.S.Biro, Physica A392 (2013) 3132-3139 arXiv:1211.5284 [cond-mat.stat-mech]

What's the parameter *q*

- The thermal equilibrium of two systems, one with energy E1 (subsystem) and the other with energy E-E1 (reservoir). The micro-canonical condition for a maximal entropy state then defines the thermo-dynamical inverse temperature.

$$L\big(S(E_1)\big) + L\big(S(E - E_1)\big) = max$$

T.S.Biro, G.G.Barnafoldi and P.Van, Derivation of Tsallis Entropy and the Quark-Gluon-Plasma Temperature, arXiv: 1208.2533v2 [hep-ph] 21 Sep 2012

What's the parameter q

$$\beta_1 = L'\big(S(E_1)\big) \cdot S'(E_1) = L'\big(S(E - E_1)\big) \cdot S'(E - E_1)$$
$$= L'\big(S(E)\big) \cdot S'(E) - \left[S'(E)^2 L''\big(S(E)\big) + S''(E) L'\big(S(E)\big)\right] \cdot E_1 + \cdots$$

$$\beta = L'\big(S(E)\big) \cdot S'(E) = L'(S) \cdot \frac{1}{T}$$

What's the parameter *q*

$\beta_1 = \beta \quad \rightarrow$

$$\frac{L''(S)}{L'(S)} = -\frac{S''(E)}{S'(E)^2}$$

$$\frac{L''(S)}{L'(S)} = a = \frac{1}{C}$$

——Universal Thermostat Independence (UTI) Principle

$\frac{1}{C} = -\frac{S''(E)}{S'(E)^2} = \frac{d}{dE}\left(\frac{1}{S'(E)}\right) = \frac{dT}{dE}$ is the inverse heat capacity

What's the parameter *q*

- With finite constant heat capacity C,

$$L(S) = C(e^{\frac{S}{C}} - 1).$$

With generalizing the classical entropy formula, $S = -\sum_i P_i lnP_i$,

$$L(S) = \sum_i P_i L(-lnP_i) = C \sum_i \left(P_i^{1-\frac{1}{C}} - P_i\right) = S_{Tsallis},$$

——where $q = 1 - \frac{1}{C}$

By the way, $S_L = \frac{1}{1-q} ln \sum_i P_i^q = S_{Renyi}$

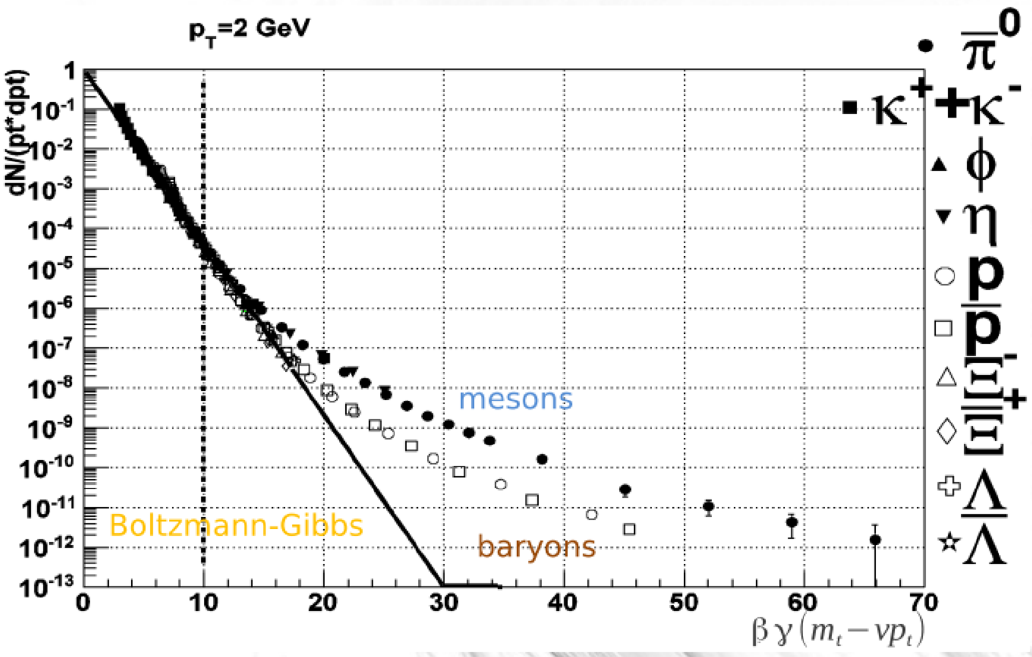
What’s the parameter *q*

◆Negative heat capacity problem

Beyond exponential momentum distributions

Boltzmann-Gibbs Fits to AuAu → h X at s = (200 GeV)2

T = 100 ± 20 MeV, v = 0.5 ± 0.1

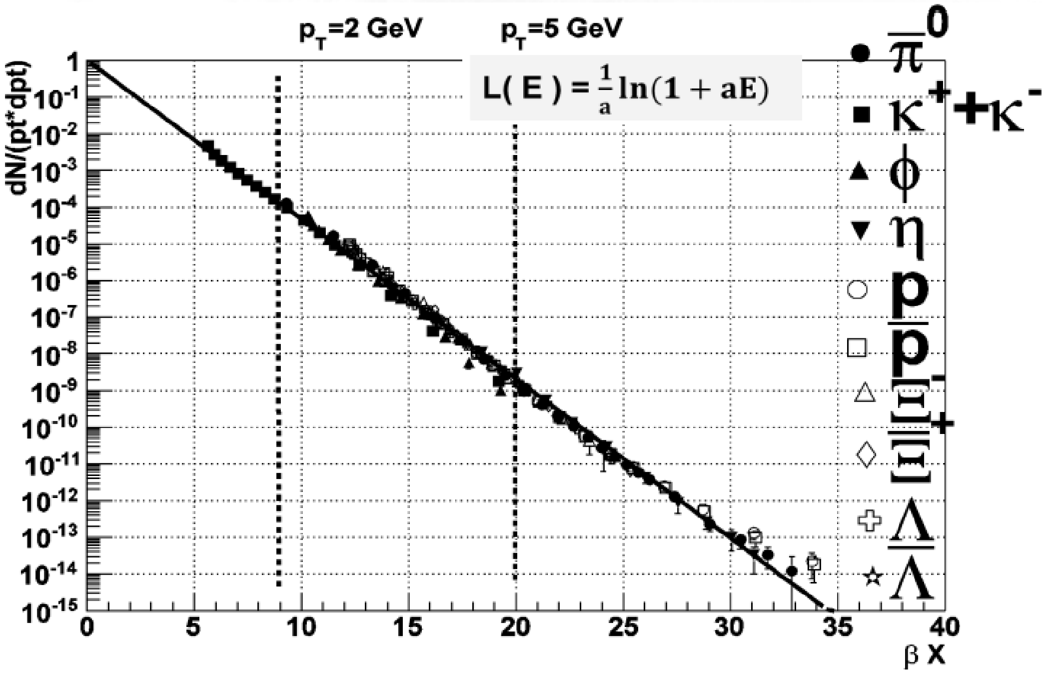


Biró T.S., Ürmössy K. and Schram Zs., J. Phys. G 37, 094027, (2010).

Rescaling: formal logarithms

Tsallis Fits to AuAu → h X at s = (200 GeV)2

T= 51 ±10 MeV, q= 1.062 ±7.65×10^{-3}, v = 0.5 ±0.1



What's the parameter q

- Consider a general system with general reservoir fluctuations, and the probability factor is

$$\left\langle e^{S(E-\omega)-S(E)} \right\rangle_{\omega \ll E} = \left\langle e^{-\omega S'(E) + \frac{\omega^2 S''(E)}{2} - \cdots} \right\rangle$$

$$= 1 - \omega \langle S'(E) \rangle + \frac{\omega^2}{2} \langle S'(E)^2 + S''(E) \rangle - \cdots$$

- Compare this with the expansion of the Tsallis-Pareto distribution

$$\left(1 + (q-1)\frac{\omega}{T}\right)^{-1/(q-1)} = 1 - \frac{\omega}{T} + q\,\frac{\omega^2}{2T^2} - \cdots$$

$$q = 1 + \frac{\Delta T^2}{T^2} - \frac{1}{C}$$

T.S.Biro, G.G.Barnafoldi, P.Van and K.Urmossy, Statistical Power-Law Spectra due to Reservoir Fluctuations, arXiv:1404.1256 [hep-ph]

Summary & Outlook

- The nonextensive effects in the Quark-Gluon Plasma: EOS, high energy transverse momentum spectra, …
- A subsystem-reservoir couple leads to the Tsallis distribution with $q=1-1/C$ for ideal gas reservoirs, with C being the heat capacity of the total system.
- The parameter q and the generalized entropy L(S) are investigated with considering the fluctuations of temperatures.
- Next I will consider the application of it in QGP and study the effects of the generalized q and L(S) with discussing fluctuations in the reservoir further.


If the accumulation of false beliefs is cleared away, Enlightenment will appear. But, strange enough, when people attain Enlightenment, they will realize that without false beliefs there could be no Enlightement.
(The Teaching of Buddha)