Topological Insulators and Entanglement Spectra

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Is the full set of Schmidt coefficients more useful than just the entanglement entropy?

$$\hat{H} = H_{mn}\hat{c}_m^\dagger \hat{c}_n = \sum_\nu E^{(\nu)} \hat{b}^{(\nu)\dagger}\hat{b}^{(\nu)}$$

$$H_{mn} = \sum_\nu E^{(\nu)} v_m^{(\nu)} v_n^{(\nu)*}$$

$$\hat{b}^{(\nu)} = \sum_n v_n^{(\nu)*}\hat{c}_n \qquad \hat{c}_m = \sum_\nu v_m^{(\nu)}\hat{b}^{(\nu)}$$

$$\rho_A = \mathrm{Tr}_B |GS\rangle\langle GS| \qquad \rho_A = \frac{e^{-H_{(\mathrm{ent})mn}\hat{c}_m^\dagger \hat{c}_n}}{\mathrm{Tr} e^{-H_{(\mathrm{ent})mn}\hat{c}_m^\dagger \hat{c}_n}}$$

$$C_{mn} = \langle GS|\hat{c}_m^\dagger \hat{c}_n|GS\rangle = \sum_{\nu>0} v_m^{\nu*} v_n^{(\nu)}$$

Correlation matrix Eigenvalues λ_j

Entanglement energies, eigvals of H_{ent} $\zeta_j = \ln\frac{1-\lambda_j}{\lambda_j}$

Obtain Schmidt coefficients considering the full RDM

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Trace index and spectral flow in the entanglement spectrum of topological insulators

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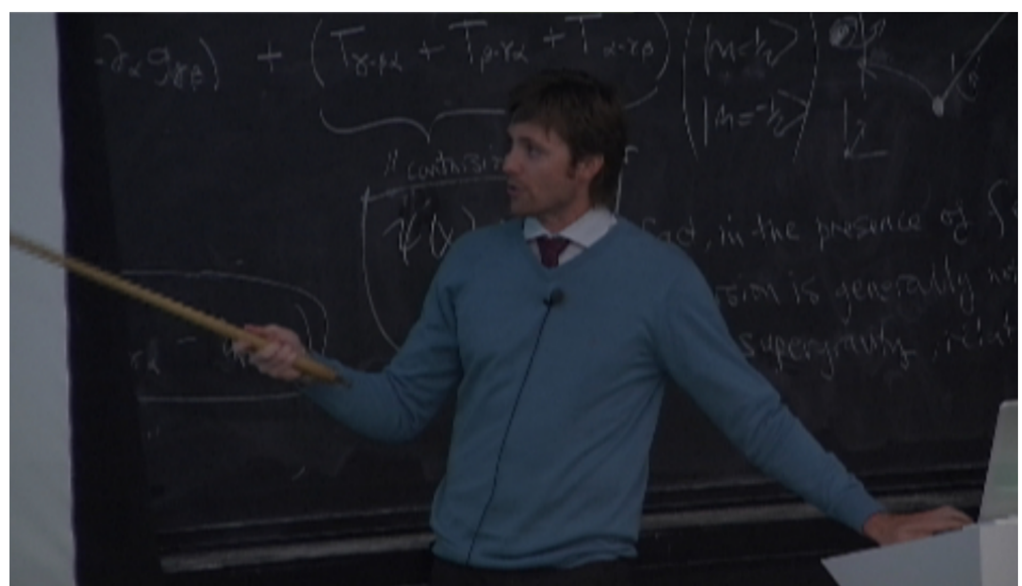
(Developing an idea by Li & Haldane, PRL 2008)

Talk at the Perimeter Institute, full video online



Insulators: low energy states have to be edge states

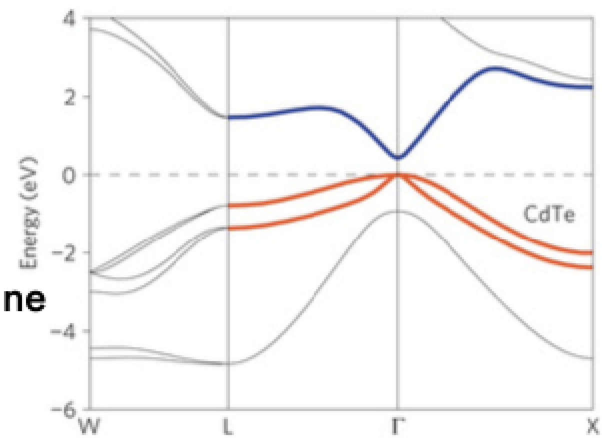
Insulator: - Band insulator, noninteracting electrons on a lattice
- ~~Strongly correlated (Mott Insulator)~~

onsite+hopping $$\hat{H} = \sum_j \sum_l \hat{c}_j^\dagger H_{j,j+l} \hat{c}_{j+l}$$ single-electron Hamiltonian H

edge region: low energy electrons confined here

translation invariant bulk

$H(k)$



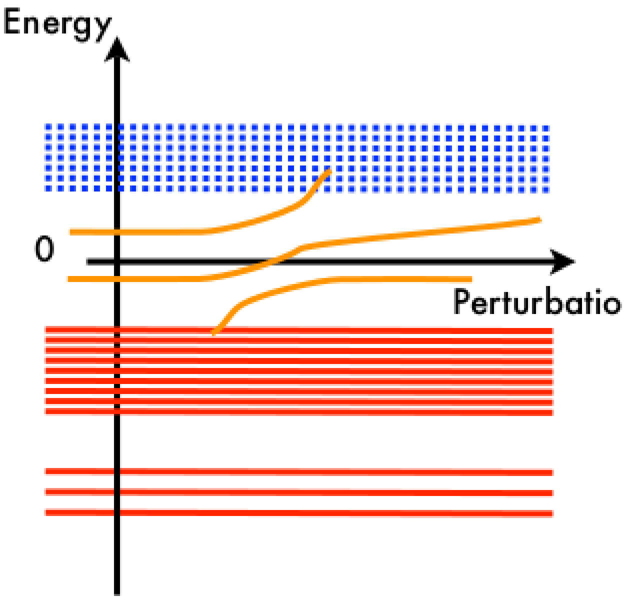
$\Psi_k(x)$ plane waves

$\Psi(x)$ have evanescent tails into the bulk

Conduction band, Unoccupied

Gap, Edge states

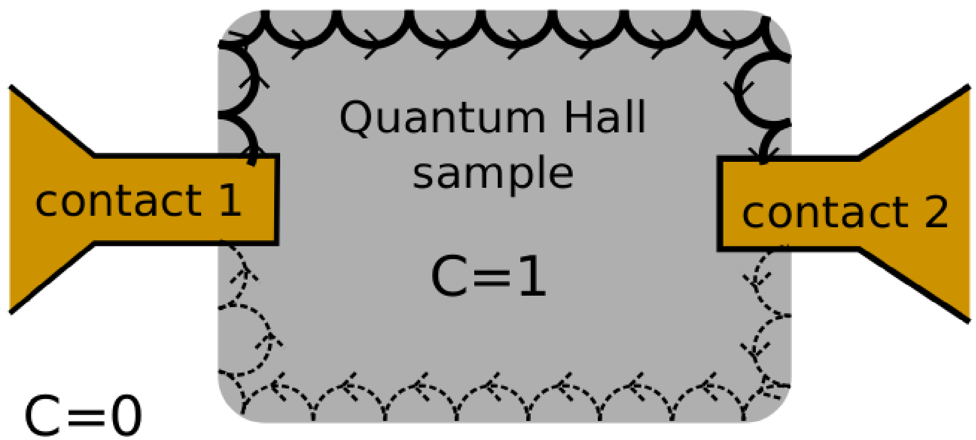
Valence band, Occupied



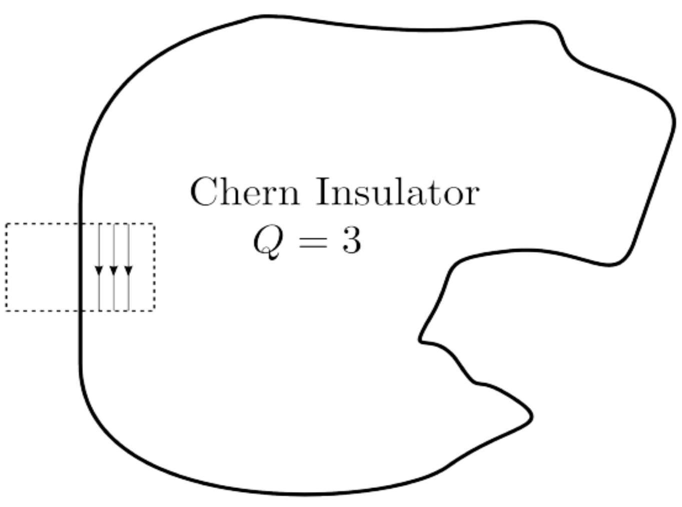
Edge states: defined by position and energy window: high energy states merge with bulk

perturbations change wavefunction energy

Topological Insulators have robust edge states



- Magnetic field: electrons cannot penetrate
- One-way (Chiral) edge states
- Perfect conduction
- Resist localization (Robustness)
- Number predicted by bulk Chern number

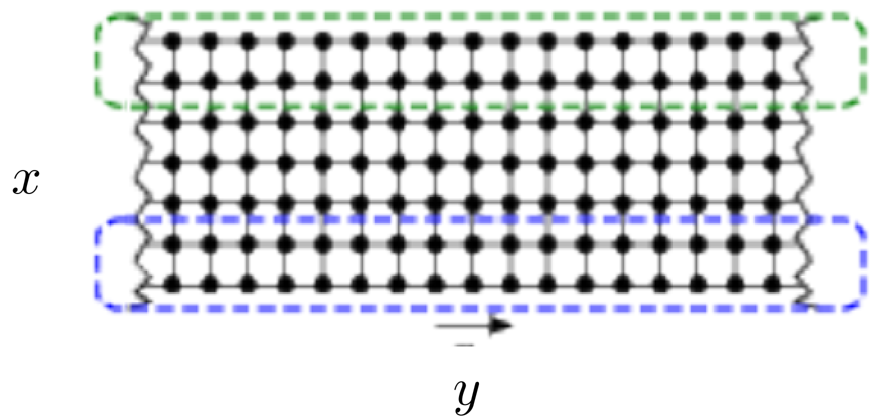


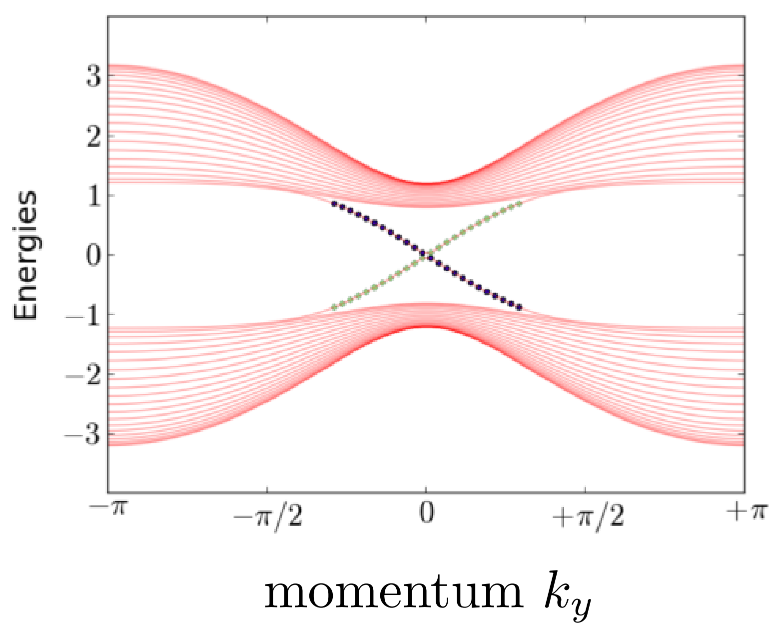
Edge States show up in Edge State Dispersion Relation

- Half BHZ model (Bernevig, Hughes, Zhang, 2006)

$$H(k_x) = \sum_{y=1}^{N} \left[(\Delta + \cos k_x)\, \sigma_z + A \sin k_x \sigma_x \right] \otimes |y\rangle \langle y|$$

$$+ \frac{1}{2} \sum_{y=1}^{N-1} (\sigma_z \;-\; iA\sigma_y) \otimes |y+1\rangle \langle y| + (\sigma_z \;+\; iA\sigma_y) \otimes |y\rangle \langle y+1|$$



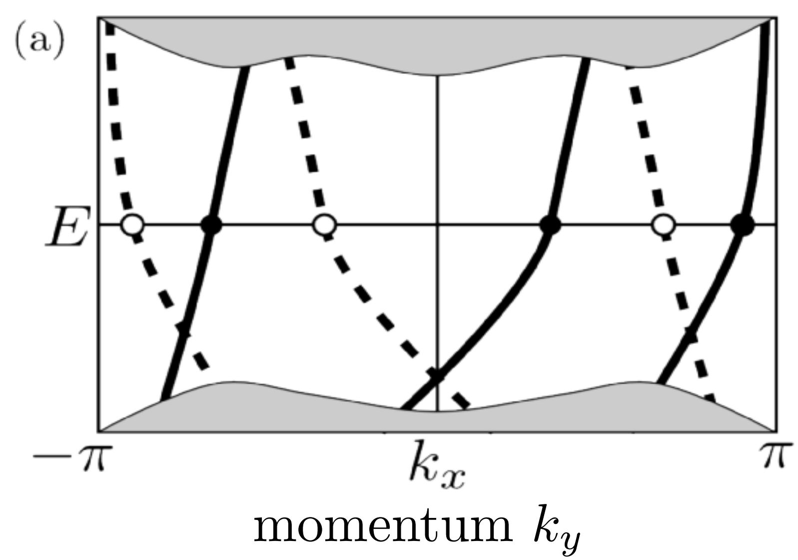


Edge States exist in Time Reversal Symmetric systems

- BHZ model (Bernevig, Hughes, Zhang, 2006)

$$H_{BHZ}(\mathbf{k}) = [(\Delta + \cos k_x + \cos k_y)\sigma_z + A \sin k_y \sigma_y)]\tau_0 + A \sin k_x \sigma_x \tau_z$$

- No scattering allowed between a mode and its time reversed partner (Kramers degeneracy)

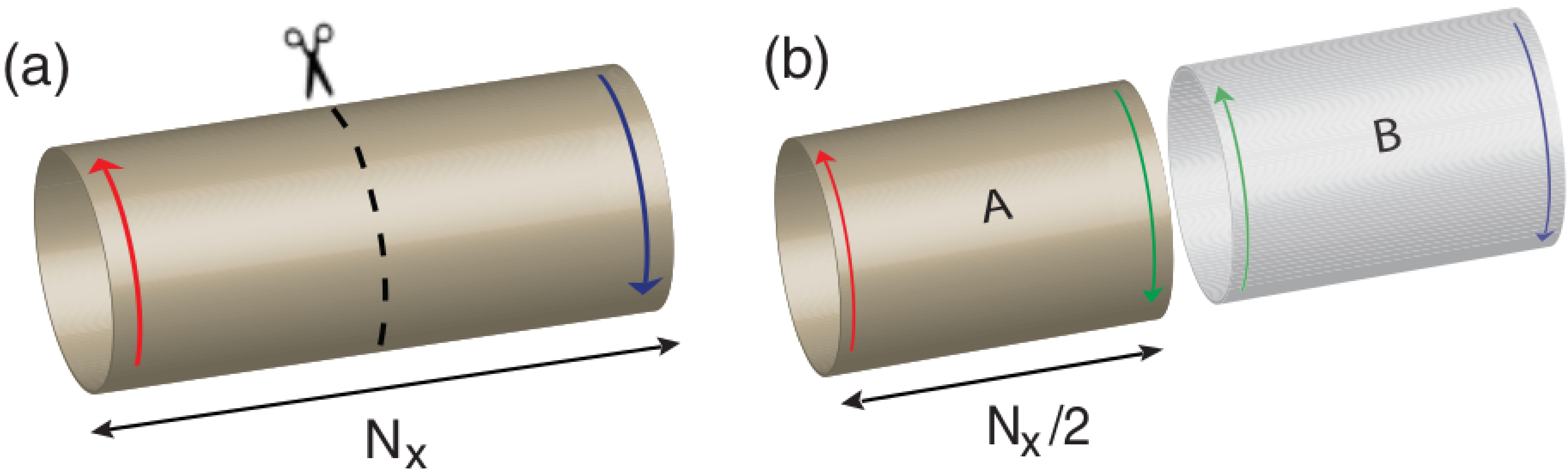


Part of the big family tree of Topological Band Insulators

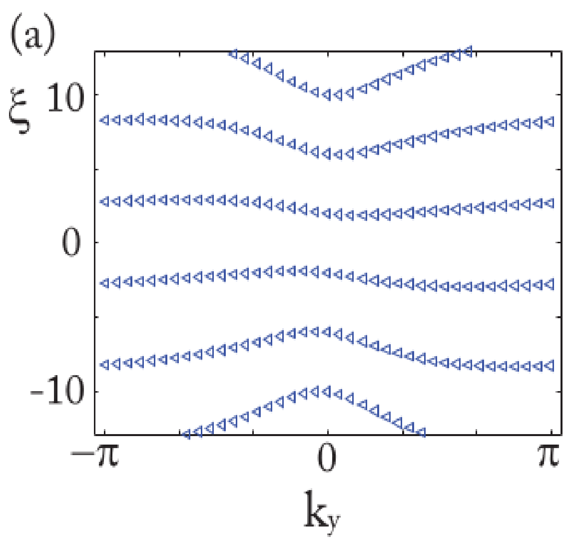
Symmetry				Dimension							
AZ	T	C	S	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	-1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

Different dimensions, symmetry classes
connected by dimensional reduction
(Schnyder et al, 2009; Teo & Kane, 2010)

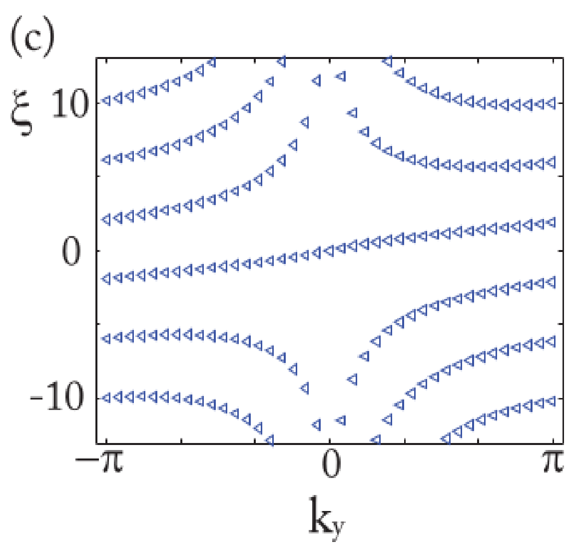
Entanglement Cut in Topological Insulators



Edge modes appear in the entanglement spectrum

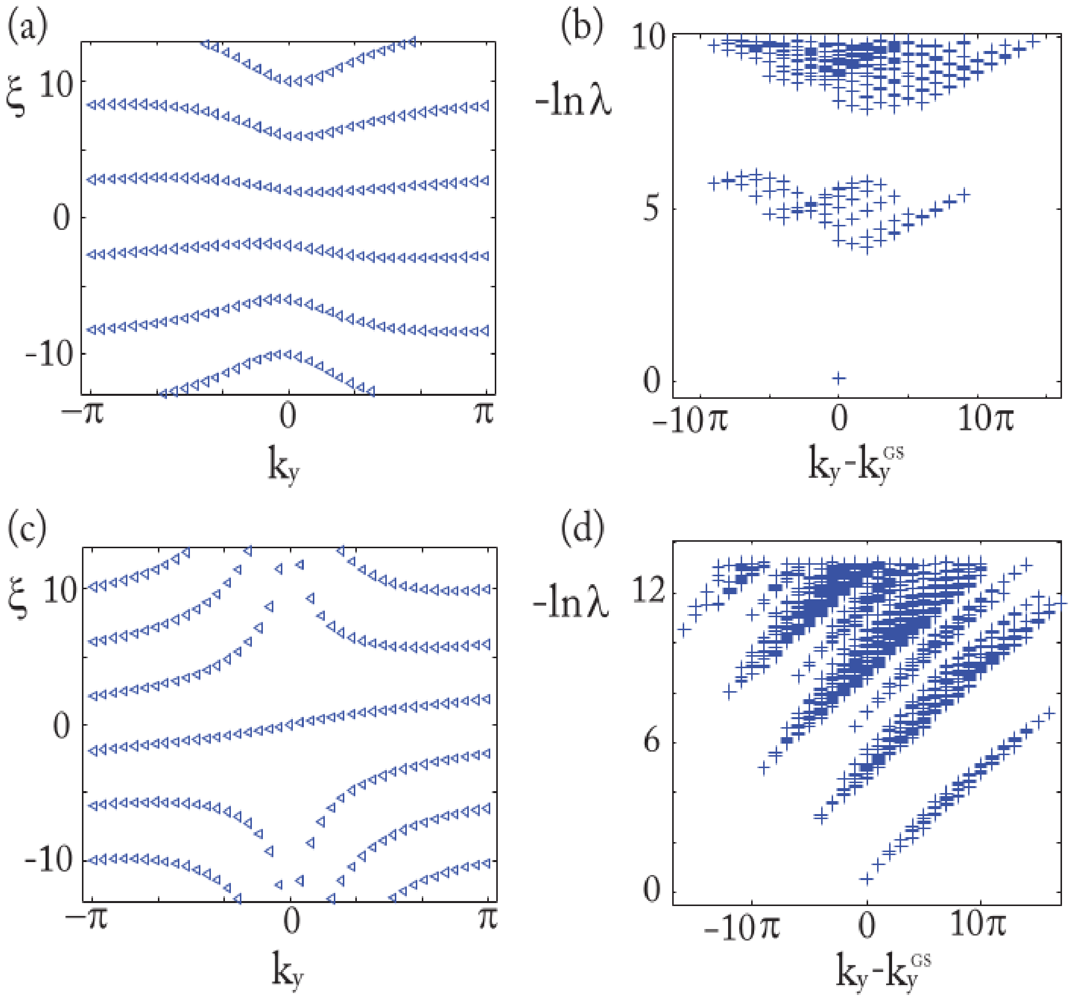


- Trivial insulator (C=0):
  - Fully occupied band, little entanglement
  - Adiabatically connected to atomic limit
  - Gap in Entanglement Spectrum



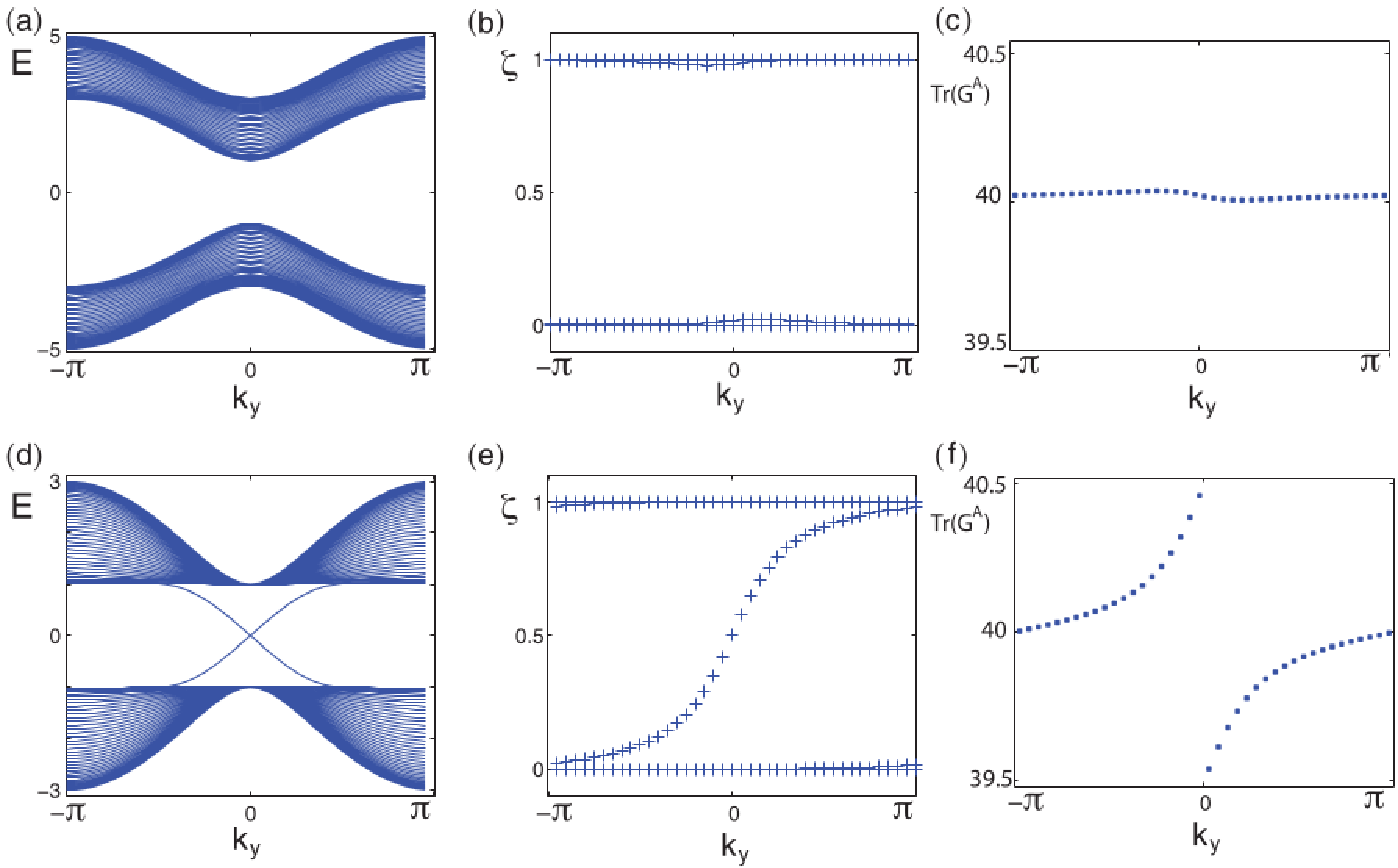
- Chern insulator (C=1):
  - Fully occupied band, why entanglement?
  - No adiabatic connection to atomic limit
  - Gapless Entanglement Spectrum
  - Spectral Flow

Edge modes in the full many-body entanglement spectrum

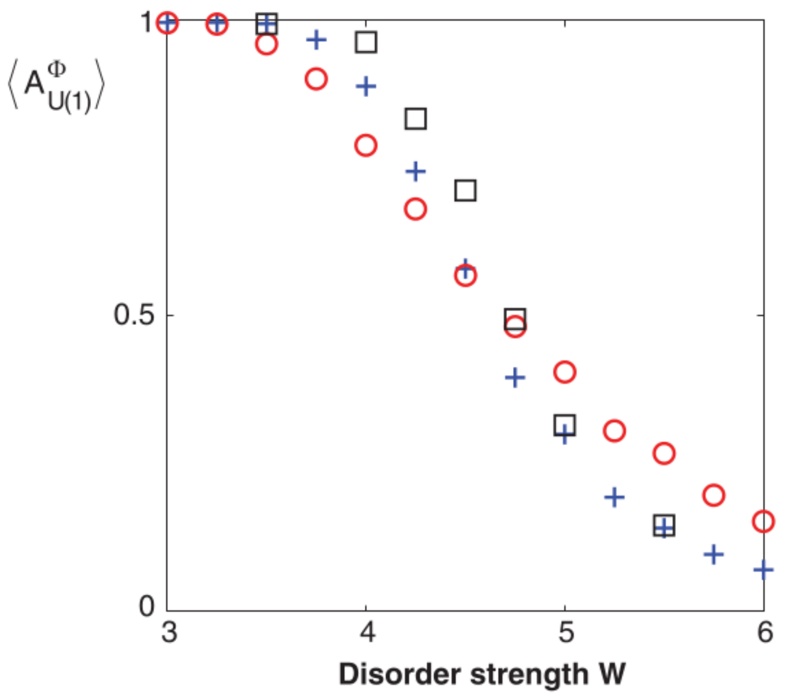


- **Trivial insulator (C=0):**
  - **Gapped Full Entanglement Spectrum**
- **Chern insulator (C=1):**
  - **Gapless Full Entanglement Spectrum**
  - **Reconstruct Edge mode by counting degeneracies**

Trace Index = Tr(C(k)) detects Chern number



Trace Index detects Chern number with disorder



- Alternative to other methods for disorder:
  - Noncommutative Chern number (Prodan, Hughes, Bernevig, PRL 2010)

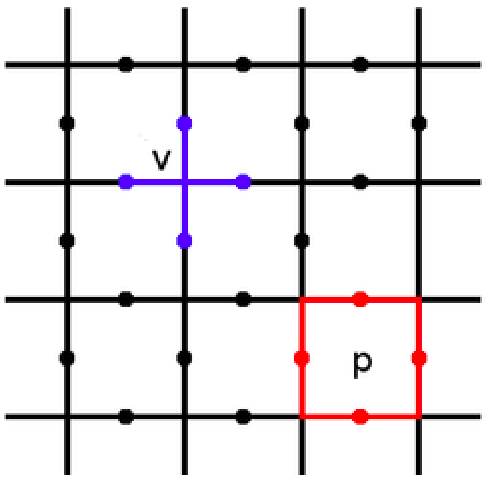
$$C = 2\pi i \sum_{\alpha} \langle 0, \alpha | P[-i[\hat{x}_1, P], -i[\hat{x}_2, P]] | 0, \alpha \rangle$$

  - Scattering Matrix Winding number (Fulga, Hassler, Akhmerov, PRB 2012)

Topological Order and Entanglement

Not the same as Topological Insulators!

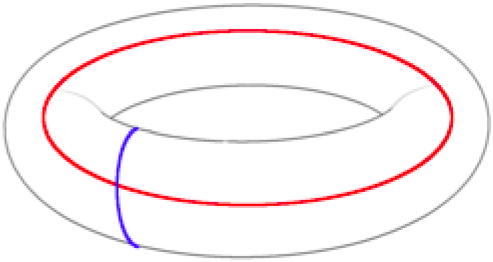
Topological Order = Robust Ground state degeneracy on a torus



Example: Toric Code [Kitaev, Ann Phys, 2006]

$$A_v = \prod_{i \in v} \sigma_i^x, \; B_p = \prod_{i \in p} \sigma_i^z.$$

$$H_T = -J_e \sum_v A_v - J_m \sum_p B_p$$



Strongly correlated Ground State

String operators along great circles commute with H

Degeneracy robust against local operations

Long Range Entanglement detects/defines Topological Order

Topological Entanglement Entropy

$$S(A) = \alpha l - \gamma + \dots$$

Numerics using DMRG in [Jiang, Wang, Balents, NatPhys 2012]

Robustness against Stochastic Local Unitary operations

String operators along great circles commute with H

Degeneracy robust against local operations