τ-Decay and Hadronic Spectral Functions

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H-QM Helmholtz Research School Quark Matter Studies

- eLσM where Scalars, Pseudoscalars, Vectors and Axialvectors are genuine degrees of freedom
- reduce complexity of QCD interaction by effective hadron hadron interaction in a model with hadronic dofs and symmetries known from the QCD Lagrangian.
- however several works suggest that the a_1 is dynamically created from $\rho\pi$ e.g.
  Wagner, Leupold, Phys.Rev.D78:053001,2008
  L. Roca, E. Oset and J. Singh, Phys. Rev. D72:014002,2005
- is a model where the a_1 is chiral partner of ρ and an explicit degree of freedom applicable to describe more complicated systems such as the τ spectral function?
- more elaborate methods for precise determination of masses and decay widths are available.
- We want to construct a phenomenological model that describes the low lying resonances in the vacuum with reasonable precision, therefore we first have to identify the states within the multiplets.

Chiral Symmetry Reminder

- $U(2)_L \times U(2)_R$ Symmetry of the QCD Lagrangian under the transformation for $m_i = 0$

$$\mathscr{L}_{QCD} = \sum_i \bar{q}_i (i\not{D} - m_i) q_i + \frac{1}{4} \sum_a F^a_{\mu\nu} F^{\mu\nu}_a$$

$$q_L \to q_R$$

with $$q_L = \frac{1-\gamma_5}{2} q \,, \quad q_R = \frac{1+\gamma_5}{2} q$$

- explicitly broken

$$\begin{aligned} U(2)_L \times U(2)_R &\simeq U(1)_A \times SU(2)_A \times U(1)_V \times SU(2)_V \\ &\xrightarrow{U(1)_A \text{ - anomaly}} SU(2)_A \times U(1)_V \times SU(2)_V \\ &\xrightarrow{m_i \neq 0} U(1)_V \times SU(2)_V \\ &\xrightarrow{m_u \neq m_d \neq \ldots \neq m_t} U(1)_V \end{aligned}$$

Chiral symmetry and mesons

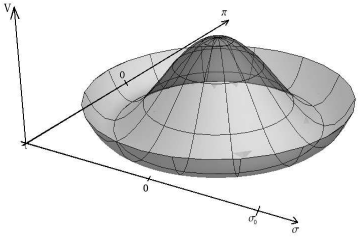
- same symmetry breaking pattern

$$U(1)_A \times SU(2)_A \times U(1)_V \times SU(2)_V \xrightarrow{\text{anomaly,masses}} U(1)_V$$

- chiral partners e.g.

$$a_1 \xrightarrow{SU(2)_A} \rho$$

- Spontaneous Symmetry Breaking generates mass difference of chiral partners



$$\mathscr{L} = \mathscr{L}_{\text{meson}} + \mathscr{L}_{\text{baryon}} + \mathscr{L}_{\text{dilaton}} + \mathscr{L}_{\text{weak}}$$

$$\begin{aligned}\mathscr{L}_{\text{meson}} = {} & \mathrm{Tr}[(D_\mu\Phi)^\dagger(D^\mu\Phi)] - m_0^2\,\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\,\mathrm{Tr}(\Phi^\dagger\Phi)^2 \\ & + c_1(\det\Phi - \det\Phi^\dagger)^2 + \mathrm{Tr}[H(\Phi+\Phi^\dagger)] - \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\ & + \mathrm{Tr}\left[\left(\frac{m_1^2}{2}+\Delta\right)(L_\mu^2 + R_\mu^2)\right] + \frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\ & + \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\,\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2\,\mathrm{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\,\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger) \\ & + \text{chirally invariant vector and axialvector four-point interaction vertices}\end{aligned}$$

$$\begin{aligned}\mathscr{L}_{\text{baryon}} = {} & \bar{\Psi}_{1L} i\gamma_\mu D_{1L}^\mu \Psi_{1L} + \bar{\Psi}_{1R} i\gamma_\mu D_{1R}^\mu \Psi_{1R} + \bar{\Psi}_{2L} i\gamma_\mu D_{2R}^\mu \Psi_{2L} + \bar{\Psi}_{2R} i\gamma_\mu D_{2L}^\mu \Psi_{2R} \\ & - \hat{g}_1(\bar{\Psi}_{1L}\Phi\Psi_{1R} + \bar{\Psi}_{1R}\Phi\Psi_{1L}) - \hat{g}_2(\bar{\Psi}_{2L}\Phi^\dagger\Psi_{2R} + \bar{\Psi}_{2R}\Phi^\dagger\Psi_{2L}) \\ & - M(\bar{\Psi}_{1L}\Psi_{2R} - \bar{\Psi}_{1R}\Psi_{2L} - \bar{\Psi}_{2L}\Psi_{1R} - \bar{\Psi}_{2R}\Psi_{L})\end{aligned}$$

$$\mathscr{L}_{\text{dilaton}} = \frac{1}{2}(\partial^\mu G)^2 - \frac{1}{4}\frac{m_G}{\Lambda^2}\left(G^4 ln\left|\frac{G}{\Lambda}\right| - \frac{G^4}{4}\right)$$

$$\begin{aligned}\mathscr{L}_{\text{weak}} = {} & \delta_w \frac{g\cos\theta_C}{2}\mathrm{Tr}[W_{\mu\nu}L^{\mu\nu}] + \delta_{em}\frac{e}{2}\mathrm{Tr}[B_{\mu\nu}R^{\mu\nu}] + \frac{1}{4}\mathrm{Tr}[(W^{\mu\nu})^2 + (B^{\mu\nu})^2] \\ & + \frac{g}{2\sqrt{2}}\left(W_\mu^- \bar{u}_{\nu_\tau}\gamma_\mu(1-\gamma_5)u_\tau + \text{h.c.}\right)\end{aligned}$$

$N_F = 2$ and $N_F = 3$ meson multiplets:

(Pseudo-)Scalars $\Phi_{ij} \simeq < q_L \bar{q}_R >_{ij} \simeq \frac{1}{\sqrt{2}}(q_i \bar{q}_j - q_i \gamma_5 \bar{q}_j)$

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0)}{\sqrt{2}} + \frac{i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{\star +} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0)}{\sqrt{2}} + \frac{i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{\star 0} + iK^0 \\ K_0^{\star -} + iK^- & \bar{K}_0^{\star 0} + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}$$

Lefthanded $L^\mu_{ij} \simeq < q_L \bar{q}_L >_{ij} \simeq \frac{1}{\sqrt{2}}(q_i \gamma^\mu \bar{q}_j + q_i \gamma_5 \gamma^\mu \bar{q}_j)$

$$L^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} + \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ + a_1^+ & K^{\star +} + K_1^+ \\ \rho^- + a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} + \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{\star 0} + K_1^0 \\ K^{\star -} + K_1^- & \bar{K}^{\star 0} + \bar{K}_1^0 & \omega_S + f_{1S} \end{pmatrix}^\mu$$

Righthanded $R^\mu_{ij} \simeq < q_R \bar{q}_R >_{ij} \simeq \frac{1}{\sqrt{2}}(q_i \gamma^\mu \bar{q}_j - q_i \gamma_5 \gamma^\mu \bar{q}_j)$

$$R^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} - \frac{f_{1N} + a_1^0}{\sqrt{2}} & \rho^+ - a_1^+ & K^{\star +} - K_1^+ \\ \rho^- - a_1^- & \frac{\omega_N - \rho^0}{\sqrt{2}} - \frac{f_{1N} - a_1^0}{\sqrt{2}} & K^{\star 0} - K_1^0 \\ K^{\star -} - K_1^- & \bar{K}^{\star 0} - \bar{K}_1^0 & \omega_S - f_{1S} \end{pmatrix}^\mu$$

Mesonic Lagrangian with Global Chiral Symmetry

D. Parganlija, F. Giacosa and D. H. Rischke, Phys. Rev. D 82 (2010) 054024 arXiv:1003.4934 [hep-ph]
D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, arXiv:1208.0585 [hep-ph]

Global Chiral Symmetry:

$$\begin{aligned}\mathscr{L}_{\rm meson} &= \mathrm{Tr}[(D_\mu\Phi)^\dagger(D^\mu\Phi)] - m_0^2\,\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\,\mathrm{Tr}(\Phi^\dagger\Phi)^2 \\ &+ c_1(\det\Phi - \det\Phi^\dagger)^2 + \mathrm{Tr}[H(\Phi+\Phi^\dagger)] - \frac{1}{4}\mathrm{Tr}({L_{\mu\nu}}^2 + {R_{\mu\nu}}^2) \\ &+ \mathrm{Tr}\left[\left(\frac{m_1^2}{2}+\Delta\right)(L_\mu^2 + R_\mu^2)\right] + \frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\ &+ \text{ch. inv. 4-point interactions among (pseudo-)scalars and (axial-)vectors}\end{aligned}$$

$U(N_F)_L \times U(N_F)_R$ Transformation:

$$\Phi \to U_L \Phi U_R^\dagger\,,\;\; L^\mu \to U_L L^\mu U_L^\dagger\,,\;\; R^\mu \to U_R R^\mu U_R^\dagger$$

Covariant Derivative:

$$D^\mu\Phi = \partial^\mu\Phi - ig_1\,(L^\mu\Phi - \Phi R^\mu)$$

Field Strength Tensors:

$$L^{\mu\nu} = \partial^\mu L^\nu - \partial^\mu L^\nu\,,\;\; R^{\mu\nu} = \partial^\mu R^\nu - \partial^\nu R^\mu$$

Explicit Breaking of Chiral Symmetry

D. Parganlija, F. Giacosa and D. H. Rischke, Phys. Rev. D 82 (2010) 054024 arXiv:1003.4934 [hep-ph]
D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, arXiv:1208.0585 [hep-ph]

Global Chiral Symmetry:

$$\begin{aligned}\mathscr{L}_{\mathrm{meson}} &= \mathrm{Tr}[(D_\mu\Phi)^\dagger(D^\mu\Phi)] - m_0^2\,\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\,\mathrm{Tr}(\Phi^\dagger\Phi)^2 \\ &+ c_1(\det\Phi - \det\Phi^\dagger)^2 + \mathrm{Tr}[H(\Phi+\Phi^\dagger)] - \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\ &+ \mathrm{Tr}\left[\left(\frac{m_1^2}{2}+\Delta\right)(L_\mu^2 + R_\mu^2)\right] + \frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu,L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu,R^\nu]\}) \\ &+ \text{ch. inv. 4-point interactions among (pseudo-)scalars and (axial-)vectors}\end{aligned}$$

$U(1)_A$-Anomaly

$$c_1(\det\Phi - \det\Phi^\dagger)^2$$

non-vanishing quark masses, NO isospin breaking

$$\mathrm{Tr}[H(\Phi+\Phi^\dagger)]\,,\ \ H = h_a t^a \quad (N_f = 3\,,\Delta)$$

remaining symmetry is $U(2)_V$

Spontaneous Breaking of Chiral Symmetry

D. Parganlija, F. Giacosa and D. H. Rischke, Phys. Rev. D 82 (2010) 054024 arXiv:1003.4934 [hep-ph]

D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, arXiv:1208.0585 [hep-ph]

Global Chiral Symmetry:

$$\begin{aligned}\mathscr{L}_{\text{meson}} = & \, \mathrm{Tr}[(D_\mu\Phi)^\dagger(D^\mu\Phi)] - m_0^2\,\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\,\mathrm{Tr}(\Phi^\dagger\Phi)^2 \\ & + c_1(\det\Phi - \det\Phi^\dagger)^2 + \mathrm{Tr}[H(\Phi+\Phi^\dagger)] - \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\ & + \mathrm{Tr}\left[\left(\frac{m_1^2}{2}+\Delta\right)(L_\mu^2 + R_\mu^2)\right] + \frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\ & + \text{ch. inv. 4-point interactions among (pseudo-)scalars and (axial-)vectors}\end{aligned}$$

Spontaneous breaking of global chiral symmetry by non-zero scalar condensate

$$\sigma \to \sigma + \phi\,, \quad \phi = Z_\pi f_\pi$$

i) $m_\rho^2 = m_1^2 + \frac{\phi^2}{2}(h_1 + h_2 + h_3)\,, \quad m_{a_1}^2 = m_1^2 + (g_1\phi)^2 + \frac{\phi^2}{2}(h_1 + h_2 - h_3)$

ii) 3 point interaction vertices and mixing terms in $(D^\mu\Phi)^\dagger D_\mu\Phi$ that are proportional to the VEV ϕ.

iii) unphysical mixing between axial-vector and pseudoscalar fields, diagonalization by shift of axial vectors; $P \to Z_\pi P$

$U(2)_L \times U(2)_R$ Symmetry in the Baryonic Sector

S. Gallas, F. Giacosa and D. H. Rischke, Phys. Rev. D 82 (2010) 014004 arXiv:0907.5084 [hep-ph]

S. Gallas, F. Giacosa and G. Pagliara, Nucl. Phys. A 872 (2011) 13 arXiv:1105.5003 [hep-ph]

Baryons in the mirror assignment:

$$\begin{aligned}\mathscr{L}_{\text{baryon}} &= \bar{\Psi}_{1L} i\gamma_\mu D^\mu_{1L}\Psi_{1L} + \bar{\Psi}_{1R} i\gamma_\mu D^\mu_{1R}\Psi_{1R} + \bar{\Psi}_{2L} i\gamma_\mu D^\mu_{2R}\Psi_{2L} + \bar{\Psi}_{2R} i\gamma_\mu D^\mu_{2L}\Psi_{2R} \\ &- \hat{g}_1(\bar{\Psi}_{1L}\Phi\Psi_{1R} + \bar{\Psi}_{1R}\Phi\Psi_{1L}) - \hat{g}_2(\bar{\Psi}_{2L}\Phi^\dagger\Psi_{2R} + \bar{\Psi}_{2R}\Phi^\dagger\Psi_{2L}) \\ &- M(\bar{\Psi}_{1L}\Psi_{2R} - \bar{\Psi}_{1R}\Psi_{2L} - \bar{\Psi}_{2L}\Psi_{1R} - \bar{\Psi}_{2R}\Psi_{L})\end{aligned}$$

$U(2)_L \times U(2)_R$ Transformation	Covariant Derivative
$\Psi_{\text{1R}} \to U_R\Psi_{\text{1R}}\,,\ \Psi_{\text{1L}} \to U_L\Psi_{\text{1L}}$	$D^\mu_{\text{1R}} = \partial^\mu - ic_1 R^\mu\,,\ D^\mu_{\text{1L}} = \partial^\mu - ic_1 L^\mu$
$\Psi_{\text{2R}} \to U_L\Psi_{\text{2R}}\,,\ \Psi_{\text{2L}} \to U_R\Psi_{\text{2L}}$	$D^\mu_{\text{2R}} = \partial^\mu - ic_2 R^\mu\,,\ D^\mu_{\text{2L}} = \partial^\mu - ic_2 L^\mu$

- allows for chirally invariant mass term generated by the gluon and/or tetraquark condensate
- Nucleons N, N^* are real chiral partners $N(1650)$ is favoured as chiral partner of $N(939)$
- yields correct nuclear matter saturation

Scale Invariance and the Glueball

S. Janowski, D. Parganlija, F. Giacosa and D. H. Rischke, Phys. Rev. D 84 (2011) 054007 arXiv:1103.3238 [hep-ph]

Scale invariance of the QCD Lagrangian is broken on the quantum level

$$\mathscr{L}_{\mathrm{dilaton}} = \frac{1}{2}\left(\partial^{\mu} G\right)^{2} - \frac{1}{4}\frac{m_{\mathrm{G}}}{\Lambda^{2}}\left(G^{4} ln\left|\frac{G}{\Lambda}\right| - \frac{G^{4}}{4}\right)$$

LσM is in principle scale invariant, only mass terms and $U(1)_A$-anomaly break scale invariance

$$x^{\mu} \rightarrow \lambda^{-1} x^{\mu}\,,\ \varphi(x) \rightarrow \lambda\varphi(\lambda^{-1}x)\,,\ \Psi(x) \rightarrow \lambda^{\frac{3}{2}}\Psi(\lambda^{-1}x)$$

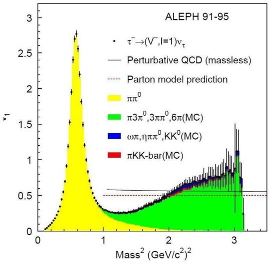
Scalar glueball is associated with fluctuations of the dilaton potential

Ground state of dilaton G_0 is related to the gluon condensate $G_0 = \Lambda = \frac{\sqrt{11}}{2m_G} C^2$

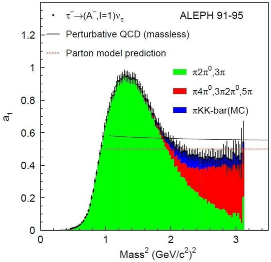
- favours $q\bar{q}$ interpretation of $f_0(1370)$ as chiral partner of the pion ($f_0(500)$ is disfavoured) and $f_0(1500)$ is 75% glueball
- scale invariance extended to $\mathscr{L}_{\mathrm{meson}}, \mathscr{L}_{\mathrm{baryon}}$ by parametrization of meson and baryon masses by G

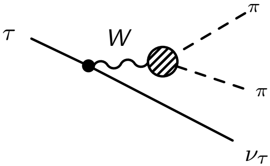
τ-Decay

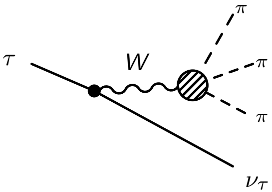
Vector $\tau^- \to \pi^-\pi^0\nu_\tau$



Axial Vector $\tau^- \to 2\pi^{0/-}\pi^{-/+}\nu_\tau$







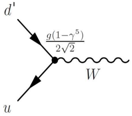
- semileptonic τ-decay involves strong and weak interactions
- effective electroweak interactions of hadrons in the vacuum

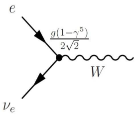
What do we know about weak interaction?

- $SU(2)_L \times U(1)_Y$ gauge symmetry with gauge fields W^μ and B^μ
- Weinberg mixing between $SU(2)_L \times U(1)_Y$ gauge fields B^μ, W_3^μ to physical interaction fields A^μ, Z_0^μ .
- Cabibbo mixing, flavour eigenstates are not weak eigenstates.
- Charged interaction violates symmetry under Charge and Parity transformations but preserves combined CP;
  $W_\pm^\mu$ act on left-handed particles, right-handed antiparticles only

$$P_L = \frac{1-\gamma_5}{2} \qquad P_R = \frac{1+\gamma_5}{2} \, .$$

- Charged bosons induce flavour-changing processes.





Linear Sigma Model with Weak Interaction

i) transformation of composite quarks

$\Phi_{kl} \simeq < \bar{q}_R q_L >_{kl} \,, \quad q_k \xrightarrow{U(1)_Y} e^{i y_k \Theta_Y(X)} q_k \,, \quad y_k = 2(Q_k - I_{3\,k})$

ii) $U(1)_Y$ Transformation:

Scalar and Pseudoscalar Fields:

$$\Phi \xrightarrow{U(1)_Y} \Phi U_Y^\dagger \simeq \Phi + i\Theta_Y \Phi t_3$$

Gauge Field:

$$B^\mu \xrightarrow{U(1)_Y} U_Y B^\mu U_Y^\dagger + \frac{i}{g'} U_Y \partial^\mu U_Y^\dagger$$

Righthanded Fields:

$$R^\mu \xrightarrow{U(1)_Y} U_Y R^\mu U_Y^\dagger$$

iii) Covariant Derivative:

$D_Y^\mu \Phi = \partial^\mu \Phi - i g_1 [L^\mu \Phi - \Phi(R^\mu + \frac{g'}{g_1} B_\mu t_3)]$

iv) Field Strength Tensor:

$R^{\mu\nu} = (\partial^\mu R^\nu - i g' [B^\mu, R^\nu]) - (\partial^\nu R^\mu - i g' [B^\nu, R^\mu])$

Local $SU(2)_L \times U(1)_Y$ Symmetry

$SU(2)_L \times U(1)_Y$ Transformations

$$\Phi \xrightarrow{SU(2)_L \times U(1)_Y} U_L \Phi U_Y^\dagger$$

$$L^\mu \xrightarrow{SU(2)_L} U_L L^\mu U_L^\dagger \qquad W^\mu \xrightarrow{SU(2)_L} U_L W^\mu U_L^\dagger + \frac{i}{g} U_L \partial^\mu U_L^\dagger$$

$$R^\mu \xrightarrow{U(1)_Y} U_Y R^\mu U_Y^\dagger \qquad B^\mu \xrightarrow{U(1)_Y} U_Y B^\mu U_Y^\dagger + \frac{i}{g'} U_Y \partial^\mu U_Y^\dagger$$

$SU(2)_L \times U(1)_Y$ Covariant Derivative:

$$D^\mu \Phi = \partial^\mu \phi - i g_1 [(L^\mu + \frac{g}{g_1} W^\mu)\Phi - \Phi(R^\mu + \frac{g'}{g_1} B_\mu t_3)]$$

Field Strength Tensors:

$$R^{\mu\nu} = (\partial^\mu R^\nu - i g' [B^\mu, R^\nu]) - (\partial^\nu R^\mu - i g' [B^\nu, R^\mu])$$
$$L^{\mu\nu} = (\partial^\mu L^\nu - i g [W^\mu, L^\nu]) - (\partial^\nu L^\mu - i g [W^\nu, L^\mu])$$

Weinberg Mixing and Cabibbo Mixing

Weinberg Mixing:

neutral bare $SU(2)_L \times U(1)_Y$ gauge fields B^μ, W_3^μ are related to the physical fields A^μ, Z^μ by

$$\begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}$$

and

$$e = g' \cos\theta_W = g \sin\theta_W$$

Cabibbo mixing:

strong isospin eigenstates d, s, b are related to the weak eigenstates by the CKM matrix

$N_f = 2$

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Linear Sigma Model with Weak Interaction

$$\mathscr{L}_{\rm weak} = \mathrm{Tr}[(D_\mu\Phi)^\dagger(D^\mu\Phi)] + \delta_{\rm w}\,\frac{g\cos\theta_C}{2}\,\mathrm{Tr}[W_{\mu\nu}L^{\mu\nu}] + \delta_{\rm em}\,\frac{e}{2}\,\mathrm{Tr}[B_{\mu\nu}R^{\mu\nu}]$$
$$+\frac{1}{4}\,\mathrm{Tr}[(W^{\mu\nu})^2 + (B^{\mu\nu})^2] + \frac{g}{2\sqrt{2}}\left(W^-_\mu\,\bar{u}_{\nu_\tau}\gamma_\mu(1-\gamma_5)u_\tau + \text{h.c.}\right)$$

The Decay $\tau \to W\nu_\tau$ is well known from SM.

Gauge invariant mixing term $\simeq \delta_w s\, W_\mu \rho^\mu$.

Covariant Derivative with Physical Interaction Fields:

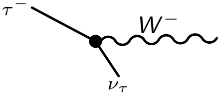
$$D^\mu\Phi \equiv \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ie[A^\mu t_3, \Phi] - ig\cos\theta_C(W_1^\mu t_1 + W_2^\mu t_2)\Phi$$
$$- ig\cos\theta_W(Z^\mu\Phi + \tan^2\theta_W\Phi Z^\mu)$$

Field Strength Tensors:

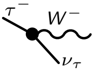
$$L^{\mu\nu} \equiv \{\partial^\mu L^\nu - ie[A^\mu t_3, L^\nu] - ig\cos\theta_C[W_1^\mu t_1 + W_2^\mu t_2, L^\nu] - ig\cos\theta_W[Z^\mu, L^\nu]\}$$
$$- \{\partial^\nu L^\mu - ie[A^\nu t_3, L^\mu] - ig\cos\theta_C[W_1^\nu t_1 + W_2^\nu t_2, L^\mu] - ig\cos\theta_W[Z^\nu, L^\mu]\}$$
$$R^{\mu\nu} \equiv \{\partial^\mu R^\nu - ie[A^\mu t_3, R^\nu] - ig\sin\theta_W[Z^\mu, R^\nu]\}$$
$$- \{\partial^\nu R^\mu - ie[A^\nu t_3, R^\mu] - ig\sin\theta_W[Z^\nu, R^\mu]\}$$

Vector Channel

Common to all channels is the process:

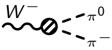


$$\Gamma_{\tau^- \to W^- 2\nu_\tau}(s) \sim \frac{|p(m_\tau^2, s, m_\nu^2)|}{m_\tau^2} \left| \quad \right|^2$$

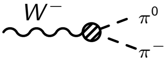


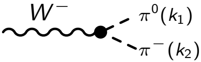
Vector Channel

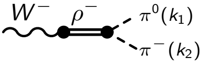
$$\Gamma_{W^- \to \pi^- 2\pi^0}(s) \sim \frac{1}{s} \left| \quad \right|^2 |p(s, m_\pi)|$$



$$\left| \quad \right|^2 = \left| \quad + \quad \right|^2$$



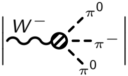


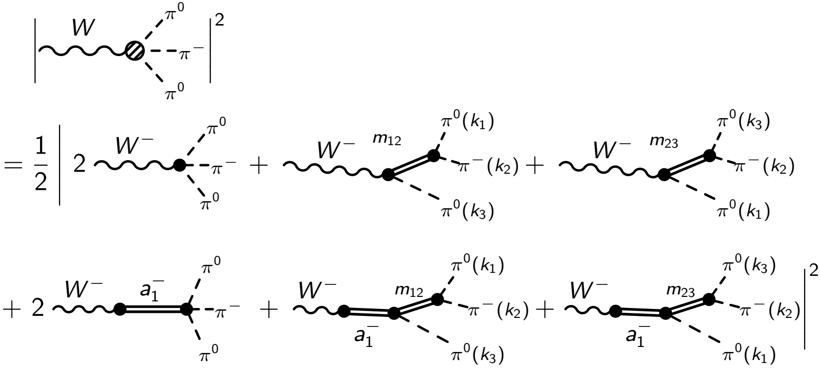


Axial-Vector Channel

Axial-Vector Channel

$$\Gamma_{W^- \to \pi^- 2\pi^0}(s) \sim \frac{1}{s}\frac{2}{2\cdot 3}\int \left| \quad \right|^2 dm_{12}^2\, dm_{23}^2$$





- $\Gamma(W^- \to \pi^- 2\pi^0) \simeq 1\%$ and $\Gamma(a_1^- \to \pi^- 2\pi^0) \simeq 1\%$
- in principle also contributions of σ resonance $\Gamma_{W\to\sigma 2\pi^0\pi^-} \simeq 0$

Spectral Functions

Spectral Density is taken as the Imaginary Part of the Propagator

$$d(s) = -\frac{1}{\pi}\mathrm{Im}[\Delta(s)]\,,\ \ \Delta(s) = \frac{1}{s - m_0^2 + g^2\mathrm{Re}[\Sigma(s)] + g^2 i\,\mathrm{Im}[\Sigma(s)]}$$

Optical Theorem:

$$g^2\mathrm{Im}[\Sigma(s)] = \sqrt{s}\,\Gamma(s)$$

Resonance Mass:

$$m^2_{\mathrm{res.}} = m_0^2 - g^2\mathrm{Re}[\Sigma(m_r)]$$

Sum Rules:

$$\int_0^\infty d_{\rho\to 2\pi}(s, m_\rho, \Gamma_\rho)ds = 1\ , \qquad \int_0^\infty d_{a_1 \xrightarrow{\rho\pi} 3\pi}(s, m_{a_1}, m_\rho)ds = 1$$

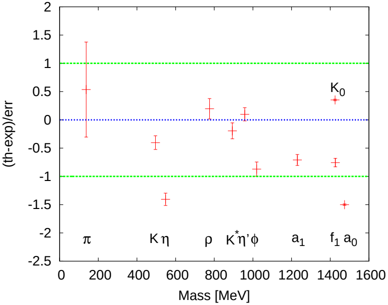
Spectral Functions

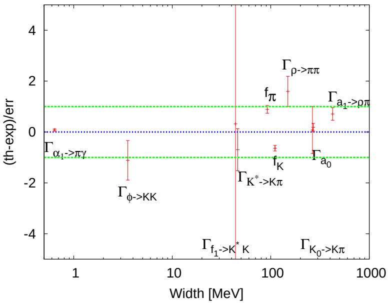
$$d_\rho(s) = \frac{1}{N_\rho}\frac{1}{\pi}\frac{\sqrt{s}\,\Gamma_{\rho\pi\pi}(s)}{(s - m_\rho^2)^2 + (m_\rho^2\,\Gamma_\rho)^2}\ ,\ \ d_{a_1}(s) = \frac{1}{N_{a_1}}\frac{1}{\pi}\frac{\sqrt{s}\,\Gamma_{a_1\rho\pi}(s)}{(s - m_{a_1}^2)^2 + (m_{a_1}^2\,\Gamma_{a_1})^2}$$

$m_\rho, \Gamma_\rho, m_{a_1}, \Gamma_{a_1}$ are now fit parameters

Quest for the Parameters

D. Parganlija, P. Kovacs, Gy. Wolf, F. Giacosa, D.H. Rischke
Scalar mesons in a linear sigma model with (axial-)vector mesons

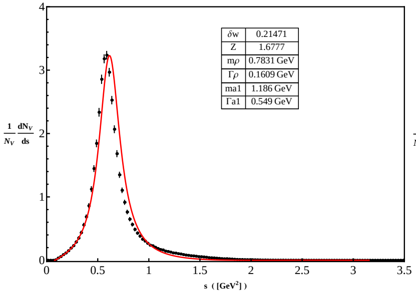


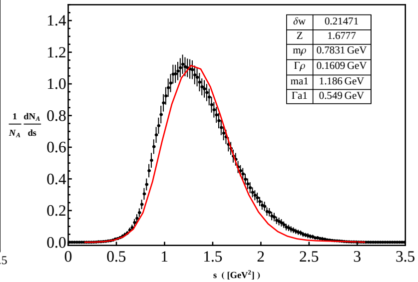


[arXiv:hep-ph/1208.0585]

- global fit of 13 parameters; test model
- 21 decay widths and masses

Results with $m_\rho, \Gamma_\rho, m_{a_1}, \Gamma_{a_1}$ and pion renormalisation constant Z as obtained from $N_f = 3$





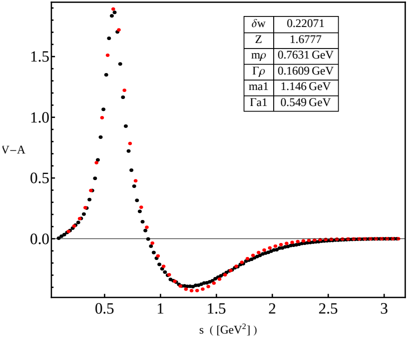
- Only one free parameter δ_w which describes the mixing between the charged weak bosons and the (axial-)vector mesons!
- ρ and a_1 mesons can be considered to be chiral partners

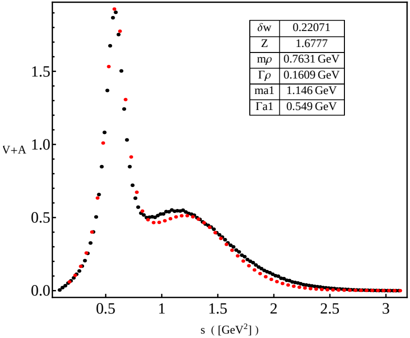
  $W\rho$ mixing $\sim \delta_w s$ $\qquad$ Wa_1 mixing $\sim (\delta_w s + g_1 \phi^2)$
- the parameters have errors within range $\sim 5\%$ therefore we can still improve our results

Inclusive Spectral Functions VMA and VPA

Vector Channel not independent on values of m_{a_1} and Γ_{a_1}.

$\rightarrow$ Inclusive Spectral Functions $V - A$ and $V + A$



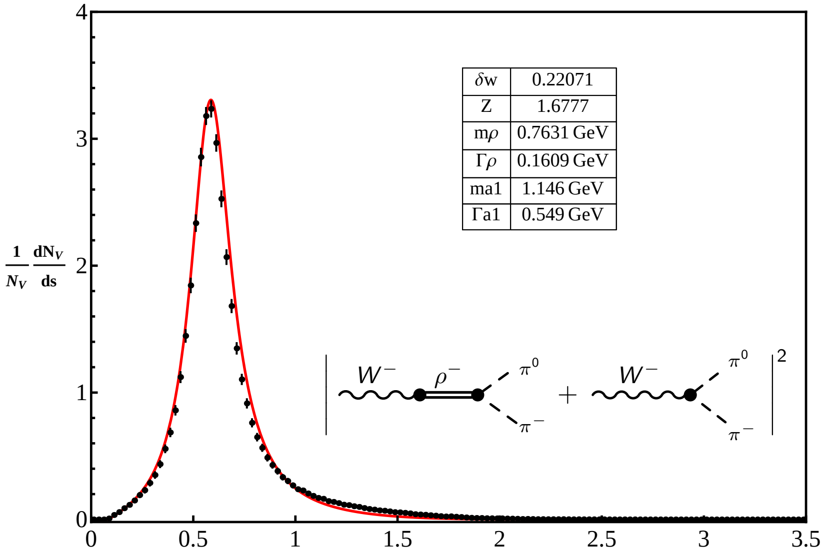


$$V - A = \frac{1}{N_V - N_A} \frac{d(N_V - N_A)}{ds}$$

$$V + A = \frac{1}{N_V + N_A} \frac{d(N_V + N_A)}{ds}$$

Vector Channel Spectral Function $\tau \to 2\pi\nu_\tau$

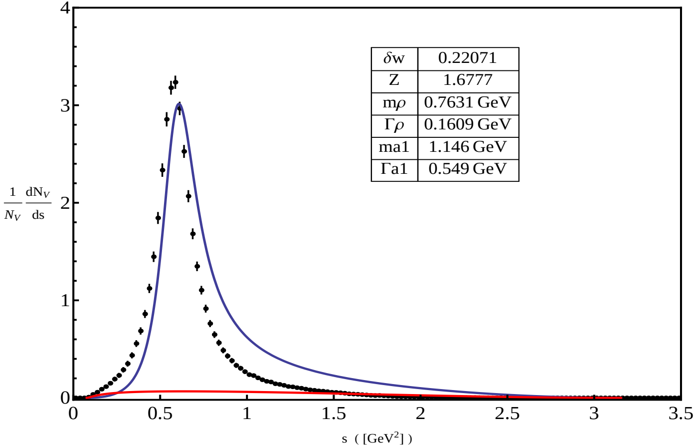
Coherent sum $\left| W \xrightarrow{\text{direct}} 2\pi + W \xrightarrow{\rho} 2\pi \right|^2$



Vector Channel Spectral Function $\tau \to \pi^-\pi^0\nu_\tau$

$W^- \to \pi^-\pi^0$ and $W^- \to \rho^- \to \pi^-\pi^0$

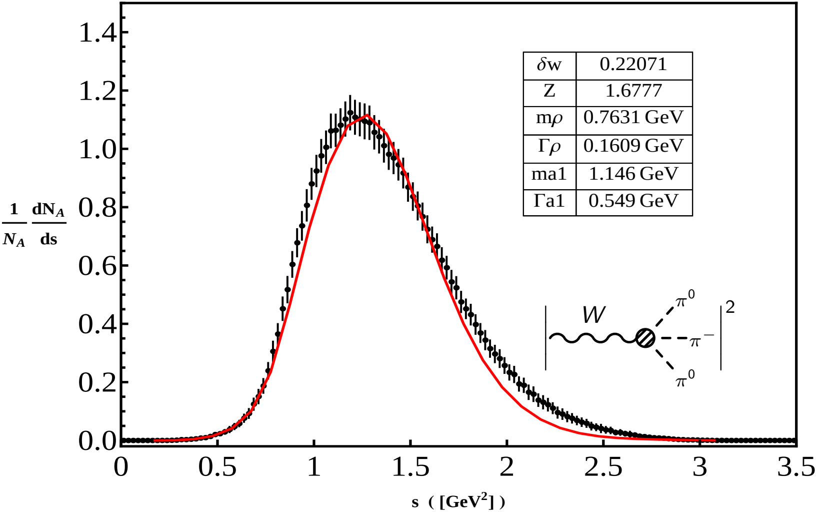
$$\frac{\Gamma(W^- \to \pi^-\pi^0)}{\Gamma(W^- \to \rho^- \to \pi^-\pi^0)} \simeq 0.02$$



δw	0.22071
Z	1.6777
mρ	0.7631 GeV
$\Gamma\rho$	0.1609 GeV
ma1	1.146 GeV
Γa1	0.549 GeV

Axial Vector Channel Spectral Function $\tau^- \to \pi^- 2\pi^0 + \pi^+ 2\pi^- \nu_\tau$

Coherent Sum $|W^- \to \rho\pi \to 3\pi + W \xrightarrow{a_1} \rho\pi \to 3\pi|^2$



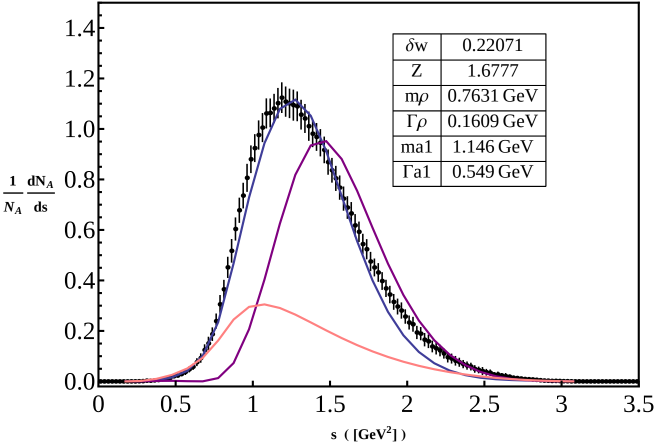
δw	0.22071
Z	1.6777
mρ	0.7631 GeV
$\Gamma\rho$	0.1609 GeV
ma1	1.146 GeV
Γa1	0.549 GeV

Axial Vector Channel Spectral Function

$\tau^- \to \pi^- 2\pi^0 + \pi^+ 2\pi^- \nu_\tau$

$W^- \xrightarrow{\rho^{-/0}\pi^{0/-}} 2\pi^{-/0}\pi^{0/+}$ and $W^- \to a_1^- \xrightarrow{\rho^{-/0}\pi^{0/-}} 2\pi^{-/0}\pi^{0/+}$

$$\frac{\Gamma(W^- \xrightarrow{\rho\pi} \pi^- 2\pi^0)}{\Gamma(W^- \xrightarrow{a_1 \to \rho\pi} \pi^- 2\pi^0)} \simeq 0.25$$



Conclusion

- We described the decay of the τ lepton in an effective hadronic model
- Can we use our effective chiral model to describe the phenomenology of the low energy resonances in the vacuum? Yes!
- Can we consider a_1 to be a $\bar{q}q$ state and obtain a reasonable description of phenomenology? Yes!
- Can we consider ρ and a_1 to be chiral partners? Yes!
- We also have a very nice example of Vector Meson Dominance in the weak hadron sector.