Optimization possibilities of inhomogeneous cosmological simulations on massively parallel architectures

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Cosmological principle

On large enough scales, the distribution of matter in the Universe is *homogeneous* and *isotropic*.

- $\Rightarrow$ Friedmann–Lemaître–Robertson–Walker metric
- time evolution is factored into the scale factor $a(t)$
- Einstein's equations with FLRW metric yield the Friedmann equation for $a(t)$:

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \cdot (\Omega_{m,0} a^{-3} + \Omega_{r,0} a^{-4} + \Omega_{\Lambda,0} + \Omega_{k,0} a^{-2})$$

- Ω_i: density of the various components:
  matter, radiation, dark energy, curvature
- Einstein's equation: $\sum \Omega = 1$

Observations make dark energy necessary

Cosmic microwave background

- The space-time of our Universe is flat
- Pythagorean theorem is valid even on very large scales
- $\Rightarrow \Omega_k = 0$

Counting all the matter

- too few to make $\sum \Omega = 1$
- $\Omega_M \approx 0.3$ only
- something is missing

Supernova observations

- "they are too faint for their distance"
- "space expands faster than expected"

Inhomogeneities on all scales

Hubble expansion of the Universe does not affect all scales

- gravitationally bound systems
- Solar System, any other planetary system
- Milky Way galaxy, other galaxies
- galaxy clusters

How do inhomogeneities affect expansion of space?

Cosmological n-body simulations

Structure formation is a non-linear process

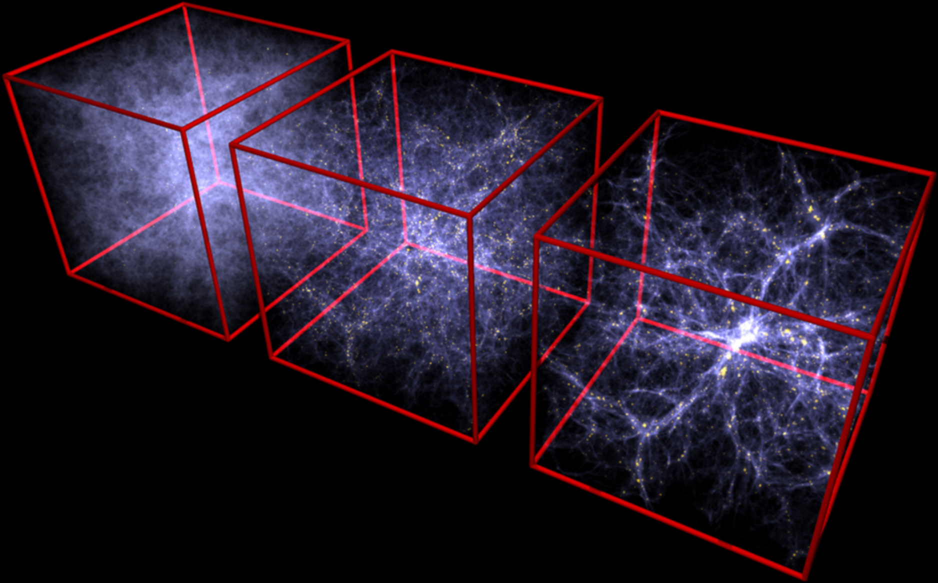
- driven by gravity
- becomes non-linear once clumps in matter distribution appear

Traditional n-body codes

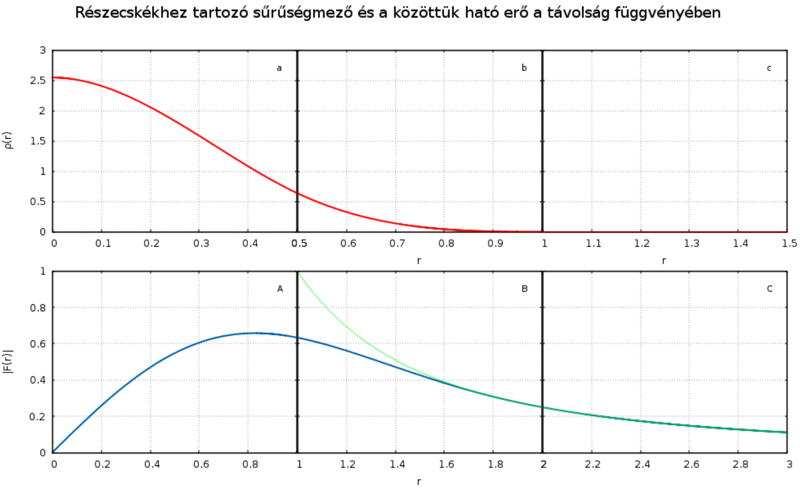
- 10^{10} particles
- periodic boundary conditions
- MPI-based parallelization on CPU clusters

Simplifications

- integrate Newton's law, rescale distances as space expands
- no exact force calculation on large distances
- no explicit time variable, use $a(t)$ instead
- no way to introduce inhomogeneous expansion



Force between smoothed particles



Non-homogeneous expansion of space: backreaction

To take inhomogeneities into account

- would need to solve Einstein's equations explicitly
- not possible

Consider the expansion of a volume $\mathcal{D}$

- as if it was an entire Universe of it's own with local Ω-s
- define a local scale factor $a_{\mathcal{D}}(t)$ as

$$a_{\mathcal{D}}(t) = \left(\frac{V_{\mathcal{D}}(t)}{V_{\mathcal{D}}(t=0)} \right)^{1/3}$$

- volume increment comes from the Friedman equation but with the local Ω-s

Toy model of an inhomogeneous Universe

To account for inhomogeneities

1. tessellate space (cubes, Delaunay, Voronoi etc.)
2. compute density in each space cell
3. determine $a_{\mathcal{D}}(t)$ for each space cell
4. average over simulation box to get $\langle a_{\mathcal{D}} \rangle$
5. rescale distances in *entire volume* using $\langle a_{\mathcal{D}} \rangle$
6. evolve positions and velocities using Newton's law

Due to non-linearity of Einstein's equations

- differentiation by time and averaging over a volume don't commute

$$\frac{d}{dt}\langle \mathcal{A} \rangle_{\mathcal{D}} - \langle \frac{d}{dt}\mathcal{A} \rangle_{\mathcal{D}} \neq 0$$

Parallelization challenge

Traditional n-body codes

- no explicit time variable, use $a(t)$ instead
- no way to introduce inhomogeneity without major changes

Our model

- exact force calculation for $N \times N$ interactions (simpler)
- need to estimate local density
- compute local volume increments from density

To estimate local density

- tessellate space into small volumes
- simple cubes are problematic: shot noise is limiting
- use other density estimators: Delaunay, Voronoi

Force kernel vs. local density estimators

n-body force kernels are easy to implement for the GPU

- as long as memory is accessed in the right order
- particles fit into GPU memory

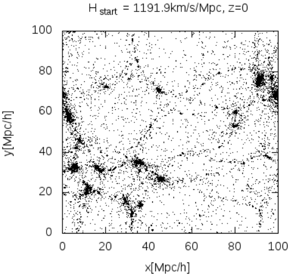
Density estimation on the GPU is a challenge

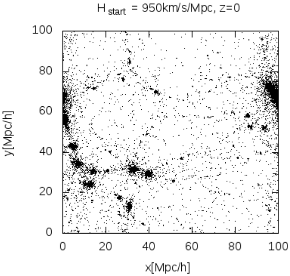
- close to exact Delaunay tessellation is needed
- no out of the box implementation yet in 3D

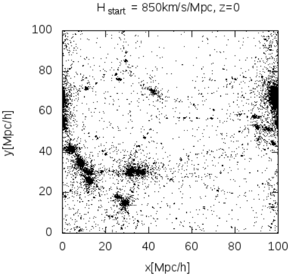
Current implementation

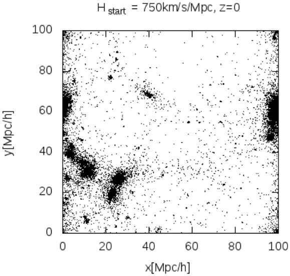
- openMP parallelization over 24 CPU cores (machine on loan)
- single-threaded Voronoi
- limited to a few 100k particles

Preliminary results

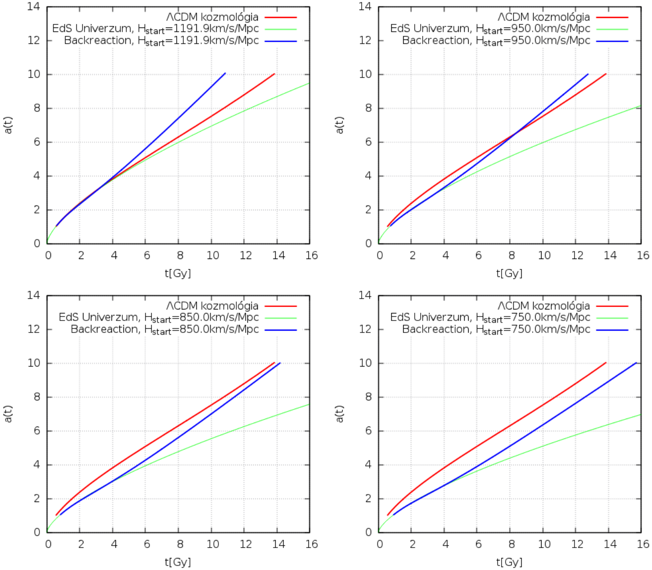








Dark energy can be just an effect of backreaction



Summary

Motivations

- traditional cosmological n-body simulations don't account for inhomogeneity
- non-homogeneous expansion of space due to *backreaction*
- need for a new kind of simulation code

Results

- new code to take inhomogeneity into account written
- n-body force kernel in openMP + Voronoi
- need to move to the GPU

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