Pasta phases in core-collapse supernova matter

Helena Pais, S. Chiacchiera, C. Providência

University of Coimbra, Portugal

Annual NewCompStar conference
Budapest, Hungary, June 15-19, 2015

Acknowledgments:
NewCompStar COST Action MP1304

Outline

Motivation

The pasta phases can appear in

Core-collapse supernova matter

- Neutrino opacity is thought to be affected by these heavy inhomogeneities, and also by light clusters.
- These geometrical configurations can modify the neutrino transport, affecting the cooling of PNS.

and in the inner crust of

Neutron stars

- Neutron stars are to date believed to be made of inner layers enclosed in a crust and possibly, a shallow atmosphere.
- Will these structures modify the M(R) relation of the star?

The pasta phase of nuclear matter

- It is the result of a frustrated system. At low densities a competition between the strong and the electromagnetic interactions takes place leading to a frustrated system.
- Under laboratory conditions the short and large distance scales related to the nuclear and Coulomb interactions are well separated so that nucleons bind into nuclei but at densities of the order of $10^{13} - 10^{14}$ g/cm^3 these length scales are comparable.
- The pasta phase is the ground state configuration if its free energy is lower than the corresponding homogeneous phase.

RMF Lagrangian for npe matter

- The force between protons and neutrons is mediated by exchange of mesons (σ, ω, ρ).
- Lagrangian density

$$\mathcal{L}_{NLWM} = \sum_{i=p,n} \mathcal{L}_i + \mathcal{L}_e + \mathcal{L}_{mesons} + \mathcal{L}_\gamma + \mathcal{L}_{\omega\rho},$$

  - Nucleon contribution: $\mathcal{L}_i = \mathcal{L}_p + \mathcal{L}_n$
  - Electron contribution: $\mathcal{L}_e$
  - Meson contribution: $\mathcal{L}_{mesons} = \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho$
  - Electromagnetic contribution: $\mathcal{L}_\gamma$
  - nonlinear $\omega\rho$ coupling contribution: $\mathcal{L}_{\omega\rho}$

These terms are given by

$$
\begin{aligned}
\mathcal{L}_i &= \bar{\psi}_i \left[\gamma_\mu i D^\mu - M^*\right] \psi_i, \\
\mathcal{L}_e &= \bar{\psi}_e \left[\gamma_\mu \left(i\partial^\mu + eA^\mu\right) - m_e\right] \psi_e, \\
\mathcal{L}_\sigma &= \frac{1}{2}\left(\partial_\mu \phi \partial^\mu \phi - m_s^2 \phi^2 - \frac{1}{3}\kappa \phi^3 - \frac{1}{12}\lambda \phi^4\right) \\
\mathcal{L}_\omega &= -\frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2} m_v^2 V_\mu V^\mu + \frac{1}{4!}\xi g_v^4 (V_\mu V^\mu)^2 \\
\mathcal{L}_\rho &= -\frac{1}{4}\mathbf{B}_{\mu\nu} \cdot \mathbf{B}^{\mu\nu} + \frac{1}{2} m_\rho^2 \boldsymbol{b}_\mu \cdot \boldsymbol{b}^\mu \\
\mathcal{L}_\gamma &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
\mathcal{L}_{\omega\rho} &= \Lambda_v g_v^2 g_\rho^2 \mathbf{b}_\mu \cdot \mathbf{b}^\mu \, V_\mu V^\mu .
\end{aligned}
$$

with

$$
\begin{aligned}
iD^\mu &= i\partial^\mu - g_v V^\mu - \frac{g_\rho}{2}\boldsymbol{\tau} \cdot \boldsymbol{b}^\mu - e\frac{1+\tau_3}{2} A^\mu, \\
M^* &= M - g_s \phi, \quad \Omega_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu \\
\mathbf{B}_{\mu\nu} &= \partial_\mu \boldsymbol{b}_\nu - \partial_\nu \boldsymbol{b}_\mu - g_\rho (\boldsymbol{b}_\mu \times \boldsymbol{b}_\nu), \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu
\end{aligned}
$$

The state that minimizes the energy of asymmetric nuclear matter is characterized by the distribution functions, $f_{0k\pm}$, of particles $(+)$ and antiparticles $(-)$ $k = p, n, e$, given by:

$$f_{0j\pm} = \frac{1}{1 + e^{(\epsilon_{0j} \mp \nu_j)/T}}, \quad j = p, n$$

with

$$\epsilon_{0j} = \sqrt{p^2 + M^{*2}}, \quad \nu_j = \mu_j - g_v V_0^{(0)} - \frac{g_\rho}{2} \tau_j b_0^{(0)}$$

and

$$f_{0e\pm} = \frac{1}{1 + e^{(\epsilon_{0e} \mp \mu_e)/T}},$$

with

$$\epsilon_{0e} = \sqrt{p^2 + m_e^2},$$

where μ_k is the chemical potential of particle $k = p, n, e$.

In the mean field approximation, the thermodynamic quantities of interest are given in terms of the meson fields, which are replaced by their constant expectation values. For homogeneous stellar matter, we have

$$
\begin{aligned}
\varepsilon &= \frac{1}{\pi^2}\sum_{j=p,n,e}\int dp\, p^2 \epsilon_{0j}\left(f_{0j+}+f_{0j-}\right)+\frac{m_v^2}{2}V_0^2+\frac{\xi g_v^4}{8}V_0^4+\frac{m_\rho^2}{2}b_0^2 \\
&+ \frac{m_s^2}{2}\phi_0^2+\frac{k}{6}\phi_0^3+\frac{\lambda}{24}\phi_0^4+3\Lambda_v g_\rho^2 g_v^2 V_0^2 b_0^2, \\
\mathcal{S} &= -\frac{1}{\pi^2}\sum_{j=p,n,e}\int dp\, p^2\left[f_{0j+}\ln f_{0j+}+(1-f_{0j+})\ln(1-f_{0j+})\right. \\
&+ \left. f_{0j-}\ln f_{0j-}+(1-f_{0j-})\ln(1-f_{0j-})\right], \\
\mathcal{F} &= \varepsilon - TS, \\
P &= \mu_p\rho_p+\mu_n\rho_n+\mu_e\rho_e-\mathcal{F}
\end{aligned}
$$

Light clusters ($d \equiv {}^2$H, $t \equiv {}^3$H, $\alpha \equiv {}^4$He, $h \equiv {}^3$He)

The Lagrangian density becomes

$$\mathcal{L}_{NLWM} = \sum_{i=p,n,t,h} \mathcal{L}_i + \mathcal{L}_\alpha + \mathcal{L}_d + \mathcal{L}_e + \mathcal{L}_{mesons} + \mathcal{L}_\gamma + \mathcal{L}_{\omega\rho},$$

where

$$\begin{aligned}
\mathcal{L}_\alpha &= \frac{1}{2}(iD^\mu_\alpha \phi_\alpha)^*(iD_{\mu\alpha}\phi_\alpha) - \frac{1}{2}\phi^*_\alpha \left(M^*_\alpha\right)^2 \phi_\alpha, \\
\mathcal{L}_d &= \frac{1}{4}(iD^\mu_d \phi^\nu_d - iD^\nu_d \phi^\mu_d)^*(iD_{d\mu}\phi_{d\nu} - iD_{d\nu}\phi_{d\mu}) \\
&- \frac{1}{2}\phi^{\mu *}_d \left(M^*_d\right)^2 \phi_{d\mu},
\end{aligned}$$

with

$$\begin{aligned}
iD^\mu_j &= i\partial^\mu - g_{vj}V^\mu - \frac{g_{\rho j}}{2}\boldsymbol{\tau}\cdot\boldsymbol{b}^\mu - e\frac{1+\tau_3}{2}A^\mu, \quad j = t, h, \alpha, d, \\
g_{vj} &= A_j g_v, \quad g_{\rho j} = |Z_j - N_j| g_\rho, \quad \mu_j = N_j\mu_n + Z_j\mu_p, \\
M^*_j &= A_j M - B_j
\end{aligned}$$

The Thomas-Fermi approximation

- Nonuniform *npe* matter system described inside **Wigner-Seitz cell**:
  - Sphere, cilinder or slab in 3D (spherical symmetry), 2D (axial symmetry around z axis) and 1D (reflexion symmetry).
- Matter is assumed locally homogeneous and, at each point, its density is determined by the corresponding local Fermi momenta.
- Fields are assumed to vary slowly so that baryons can be treated as moving in locally constant fields at each point.
- Surface effects are treated self-consistently.
- Quantities such as the energy and entropy densities are averaged over the cells. The free energy density and pressure are calculated from these two thermodynamical functions.

The Coexisting phases approximation

- Matter is organized into separated regions of higher and lower density, the higher ones being the pasta phases, and the lower ones a background nucleon gas. The interface between these regions is sharp.

- **Gibbs equilibrium conditions** are used to get the lowest energy state, and, for a temperature $T = T^{I} = T^{II}$, are written as:
  - $\mu_n^{I} = \mu_n^{II}$
  - $\mu_p^{I} = \mu_p^{II}$
  - $P^{I} = P^{II}$

- Finite size effects are taken into account by a surface and a Coulomb terms in the energy density, **after the coexisting phases are achieved**.

- Total $\mathcal{F}$ and total ρ_p of the system:

$$\begin{aligned} \mathcal{F} &= f\mathcal{F}^I + (1-f)\mathcal{F}^{II} + \mathcal{F}_e + \varepsilon_{surf} + \varepsilon_{Coul}, \\ \rho_p &= \rho_e = y_p\rho = f\rho_p^I + (1-f)\rho_p^{II}, \end{aligned}$$

- Minimizing $\varepsilon_{surf} + \varepsilon_{Coul}$ wrt r, one gets $\varepsilon_{surf} = 2\varepsilon_{Coul}$ with

$$\varepsilon_{Coul} = \frac{2\alpha}{4^{2/3}} \left(e^2\pi\Phi\right)^{1/3} \left[\sigma D(\rho_p^I - \rho_p^{II})\right]^{2/3},$$

- $\alpha = f$ for droplets, rods, slabs; $\alpha = 1 - f$ for tubes and bubbles
- f is the volume fraction of phase I; σ is the surface energy coefficient

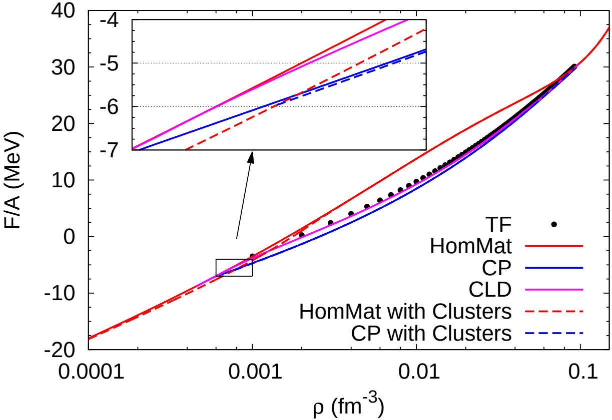
The Compressible Liquid Drop approximation

- The total free energy density is minimized, **including the surface and Coulomb terms**.
- This minimization is done with respect to four variables:
  - r_d, the size of the geometric configuration, which gives $\varepsilon_{surf} = 2\varepsilon_{Coul}$,
  - ρ^I, the baryonic density in the high-density phase,
  - ρ_p^I, the proton density in the high-density phase,
  - f, the volume fraction.
- The equilibrium conditions become:
  - $\mu_n^I = \mu_n^{II}$
  - $\mu_p^I = \mu_p^{II} - \frac{\varepsilon_{surf}}{f(1-f)(\rho_p^I - \rho_p^{II})}$
  - $P^I = P^{II} - \varepsilon_{surf}\left(\frac{1}{2\alpha} + \frac{1}{2\Phi}\frac{\partial\Phi}{\partial f} - \frac{\rho_p^{II}}{f(1-f)(\rho_p^I - \rho_p^{II})}\right)$
- Total P and the total μ_p of the system:

$$P_{tot} = \mu_p\rho_p + \mu_n\rho_n + \mu_e\rho_e - \mathcal{F}, \quad \mu_p = f\mu_p^I + (1-f)\mu_p^{II},$$

Free energy per particle

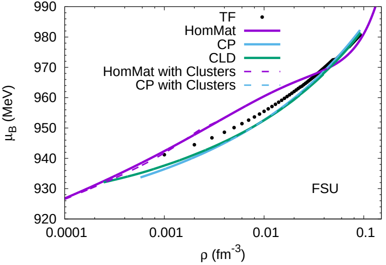
FSU interaction, $T = 4$ MeV, $y_p = 0.3$

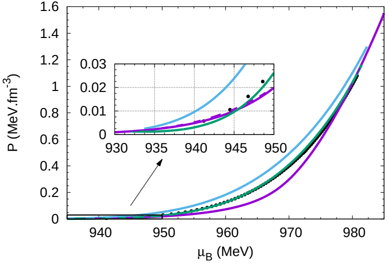


F is lowered when pasta is present. The effect of light clusters is only seen at small densities.

Chemical potential and pressure

$T = 4$ MeV, $y_p = 0.3$





$\mu_B = (1 - y_p)\mu_n + y_p(\mu_p + \mu_e)$

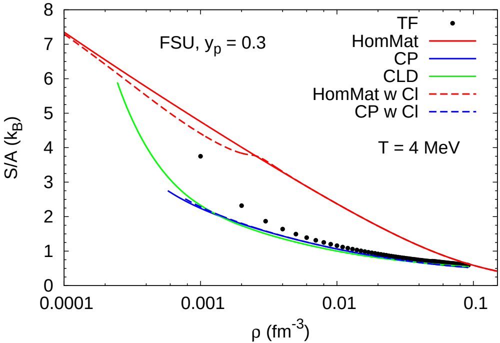
CP, CLD, TF $\Rightarrow$ the negative curvature of μ_B is removed; at the crust-core transition, they give similar results.

CLD and TF $\Rightarrow$ P does not show any discontinuity.

CP presents a very large discontinuity at the onset of the pasta phase (left) and at both the onset and the crust-core transition (right), due to the non-consistent treatment of the surface energy.

Entropy per particle

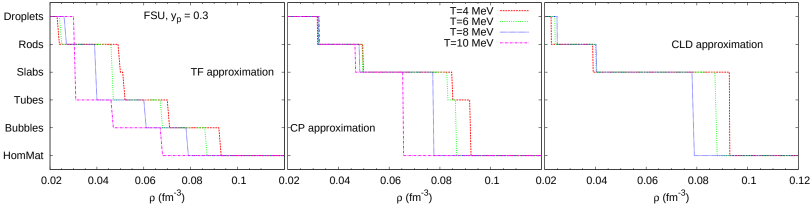
$T = 4$ MeV, $y_p = 0.3$



CP, CLD, TF $\Rightarrow S/A$ is lowered with the inclusion of pasta. At low densities, the same effect is seen in HMwC.

Transition densities between pasta formations - FSU

$y_p = 0.3$

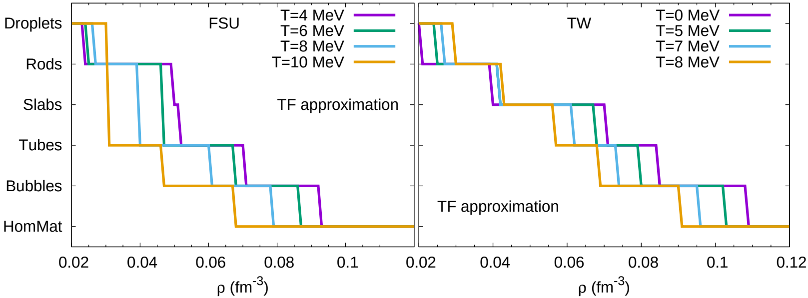


Slabs are omitted (except for $T = 4$) in TF (left) and occupy the widest density range in CP (middle) and CLD (right).
The density range of each shape decreases with increasing T.
At $T = 10$ MeV, CLD no longer has pasta.

Transition densities between pasta formations

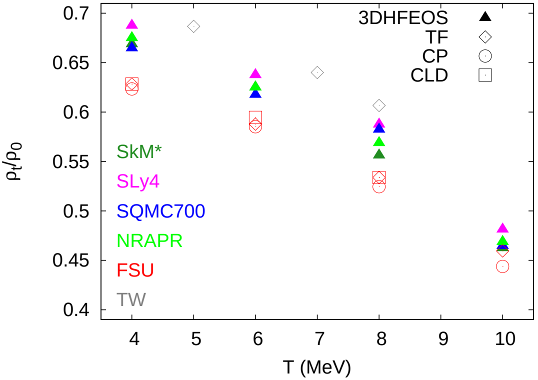
$y_p = 0.3$

However, the slab geometry is present in other parametrizations:



In FSU, the difference between slabs and tubes for F is $< 10^{-3}$.
Stable geometries depend on parametrizations: which properties influence them should be investigated.

Transition densities to uniform matter



TF, CP and CLD calculations are almost coincident. The difference to 3DHFEOS is ~ 0.015 fm^{-3} and decreases with increasing T.

Summary

- The effect of light clusters is very weak and only noticeable at very low densities.
- The density range of the pasta phase decreases with increasing T.
- Crust-core transition density decreases with increasing T.
- The jumps in the pressure and chemical potential, as a function of the density, indicate a first order phase transition to uniform matter.

- Stable geometries depend on the parametrizations: which properties influence them should be investigated.
- CP method gives a larger correction than TF, and not so realistic, though it predicts concordant transition densities to uniform matter.
- TF and CLD calculations give very similar results in the whole range of densities and temperatures considered.
- All the methods considered show a very good agreement with respect to the transition density to homogeneous matter.

THANK YOU!