




Institut d'Astronomie et d'Astrophysique  
Université Libre de Bruxelles

# Unified EoSs for neutron stars within the energy-density functional theory and neutron-star structure

**Anthea F. Fantina** (afantina@ulb.ac.be)

Collaborators: N. Chamel, S. Goriely (IAA, Université Libre de Bruxelles)  
J. M. Pearson (Université de Montréal)  
P. Haensel, J. L. Zdunik (CAMK, Warsaw)

Supernova Remnant 1987A in the Large Magellanic Cloud  HUBBLESITE.org

Annual NewCompStar Conference 2015  
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# Outline

## ❖ EoS

- different models and  $M_{max}$  predictions
- (some) constraints from nuclear and astrophysics

## ❖ Effective nuclear models

- Brussels-Montreal BSk model

## ❖ Equations of state (EoSs) of dense matter

- The model
- Structure and composition of catalysed matter
- Accreted crust

## ❖ Conclusions & Outlook



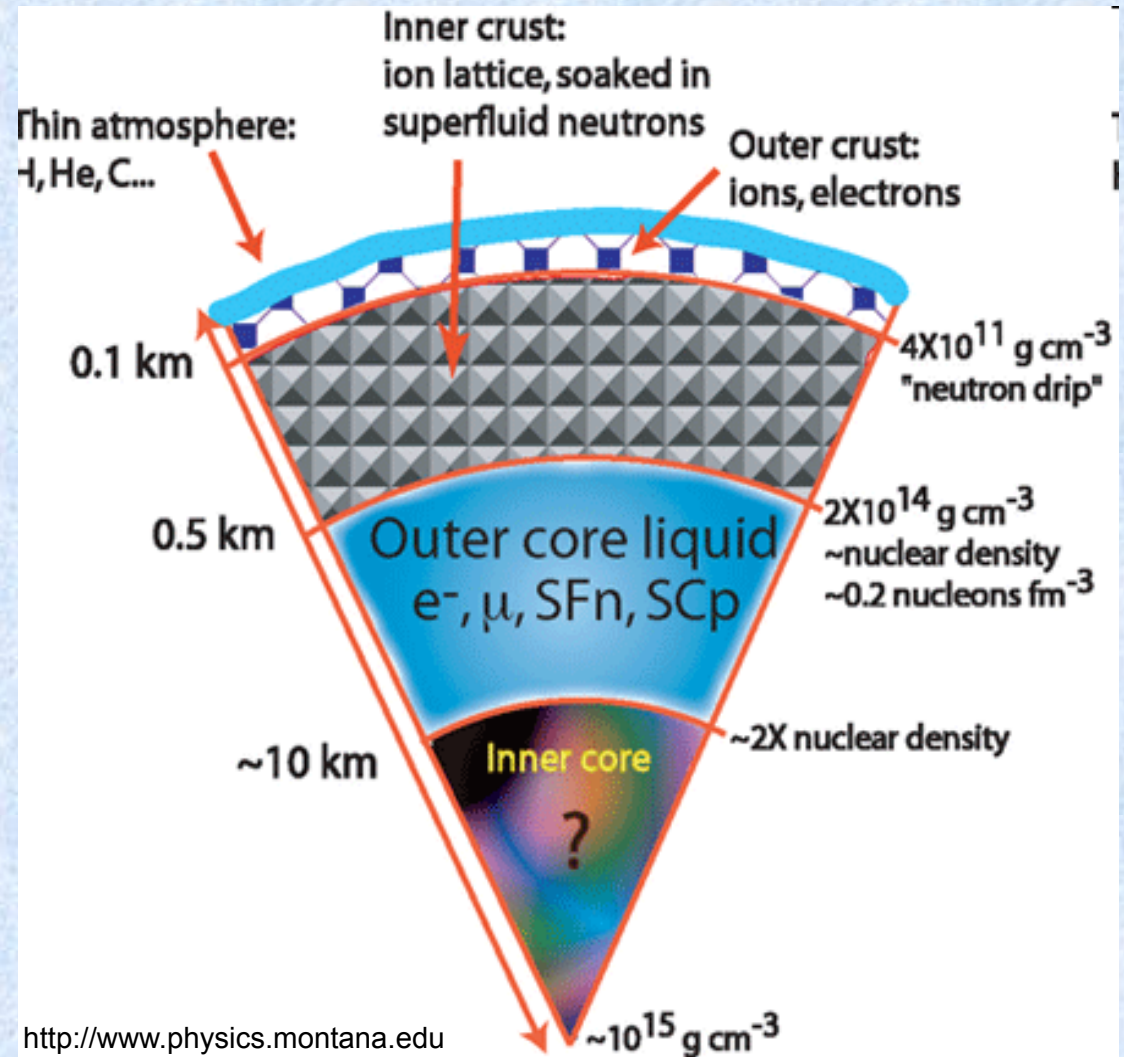
# EoS for NS: the challenge

Contrarily to a normal star, in a NS:

- ✓ **matter is highly degenerate!**  
(  $T = 0$  approximation )
- ✓ **very high density!**  
composition uncertain



different states of matter :  
inhomogeneous, homogeneous,  
exotic particles ?

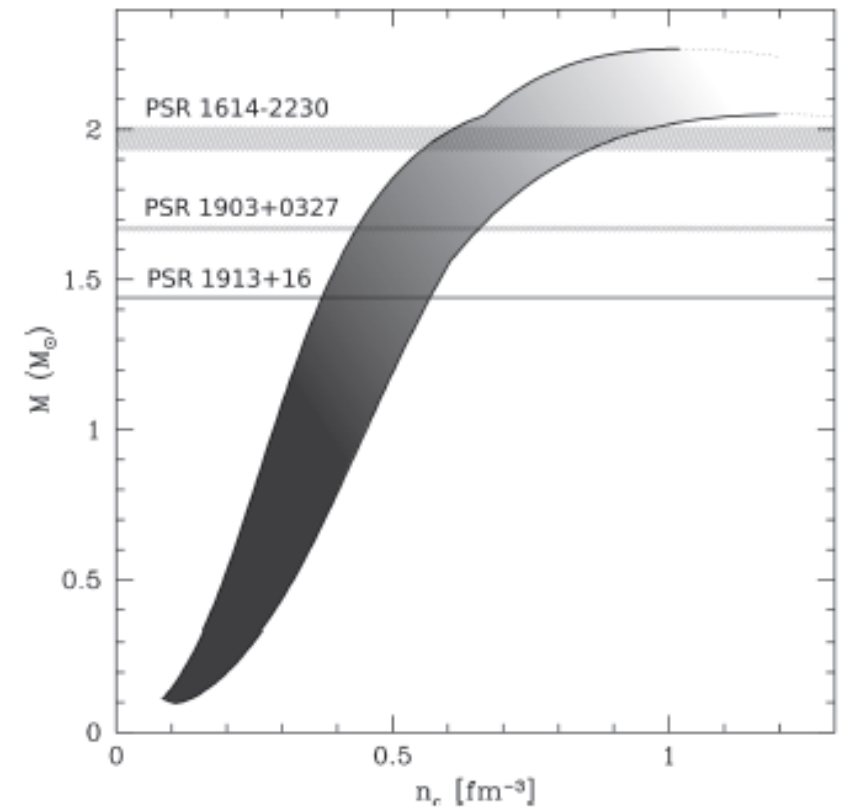
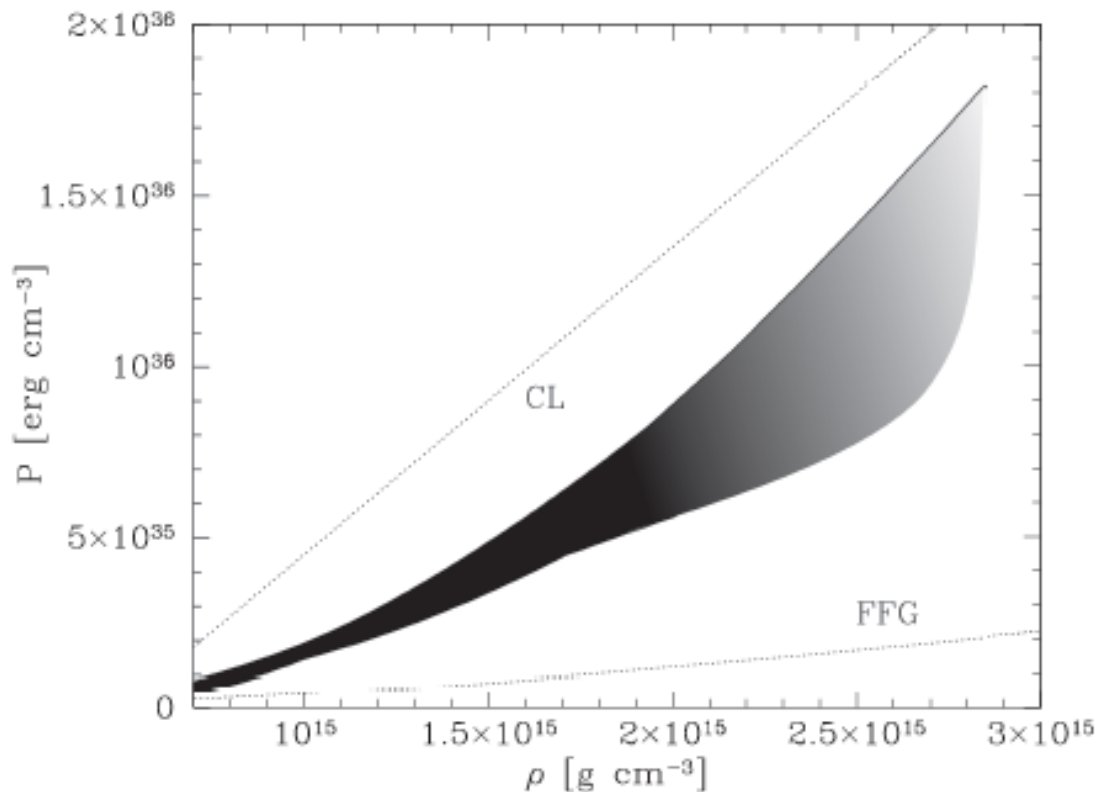


**UNIQUE "LABORATORIES!"** (e.g. superfluidity, deformed nuclei, ... )



# Uncertainties in dense-matter EoS

Pressure  $P$  versus mass-energy density  $\rho$  and corresponding NS mass  $M$  versus central density  $n_c$  relation, as predicted by various models and consistent with the existence of massive NSs.



Chamel, Haensel, Zdunik, Fantina, Int. J. Mod. Phys. E 22, 1330018 (2013); E 22, 1392004 (2013)







# Maximum mass predictions

The core is assumed to contain nucleons (N), nucleons and hyperons (NH), nucleons and quark (NQ). In some cases, to reach  $2 M_{sun} \rightarrow$  *fine tuning of parameters!*

- **Phenomenological models** : start from effective interactions with parameters adjusted on some nuclear properties. E.g. Relativistic Mean Field (RMF), Nambu-Jona-Lasinio (NJL), Modified Bag Model (MBM)

	RMF (N)	RMF (NH)	RMF/NJL (NQ)	RMF/MBM (NQ)
$M_{\max} / M_{\odot}$	2.1-2.8	2.0-2.3	2.0-2.2	2.0-2.5

- **Microscopic models** : start from realistic interaction ( $\rightarrow$  *ab-initio*). E.g. (Dirac) Brueckner Hartree-Fock ((D)BHF), variational chain summation method (VCS), perturbative quantum chromodynamics (pQCD)

	(D)BHF (N)	BHF (NH)	VCS (N)	pQCD (NQ)
$M_{\max} / M_{\odot}$	2.0-2.5	1.3-1.6	2.0-2.2	2.0

hyperon puzzle!

Chamel, Haensel, Zdunik, Fantina, Int. J. Mod. Phys. E 22, 1330018 (2013) ; E 22, 1392004 (2013)



# EoS: properties of nuclear matter

In applying nuclear model in astrophysics → two kinds of parameters :

1. **Thermodynamic variables** → physical conditions in the star (e.g.  $P$ ,  $T$ ,  $B$ , ...)
2. **Nuclear parameters** → properties of nuclear matter around saturation at  $T=0$

- Energy around saturation (in a liquid drop model):

$$E(n, x = Z/A) = E(n_0, x = 1/2) + \frac{1}{2} K_{\infty} \left( \frac{n - n_0}{3n_0} \right)^2 + E_{\text{sym}}$$

- In SN & NS → n-rich matter → symmetry energy important:

$$E_{\text{sym}} = \left[ J + L \left( \frac{n - n_0}{3n_0} \right) + \frac{1}{2} K_{\text{sym}} \left( \frac{n - n_0}{3n_0} \right)^2 \right] (1 - 2x)^2$$

related to NS crust-core boundary (e.g. Vidaña *et al.*, PRC 80, 045806 (2009))



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# Our goal : a unified EoS

- Our goal is to construct a **unified** EoS (till now: SLy4, FPS, BCPM)
  - based on the same nuclear model from energy-density functional theory
  - valid in all regions of NS (and SN) interior
  - outer / inner crust and crust / core transition described consistently
- EoS both at **T = 0** and **finite T**
  - cold non-accreting NS (cold catalysed matter)
  - accreting NS (off-equilibrium) → [see Zdunik's talk!](#)
  - SN cores
- Satisfying:
  - constraints from nuclear physics experiments
  - astrophysical observations
- Direct applicable for astrophysical application





# Brussels-Montreal (BSk) functionals

Mass models based on HFB method with Skyrme type functionals and macroscopically deduced pairing force (→ see N. Chamel's talk).

Fitted to experimental data + N-body calculations with realistic forces.

**BSk19**

**BSk20**

**BSk21**

- fit 2010 AME data (2149 masses, rms = 0.581 MeV)
- different degrees of stiffness (BSk19 softer → BSk21 stiffer)
- constrained to different microscopic
- all have  $J = 30$  MeV,  $K_\infty$  in exper

Goriely *et al.*, PRC 82, 035804 (2010)

**BSk22**

**BSk23**

**BSk24**

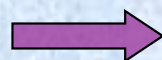
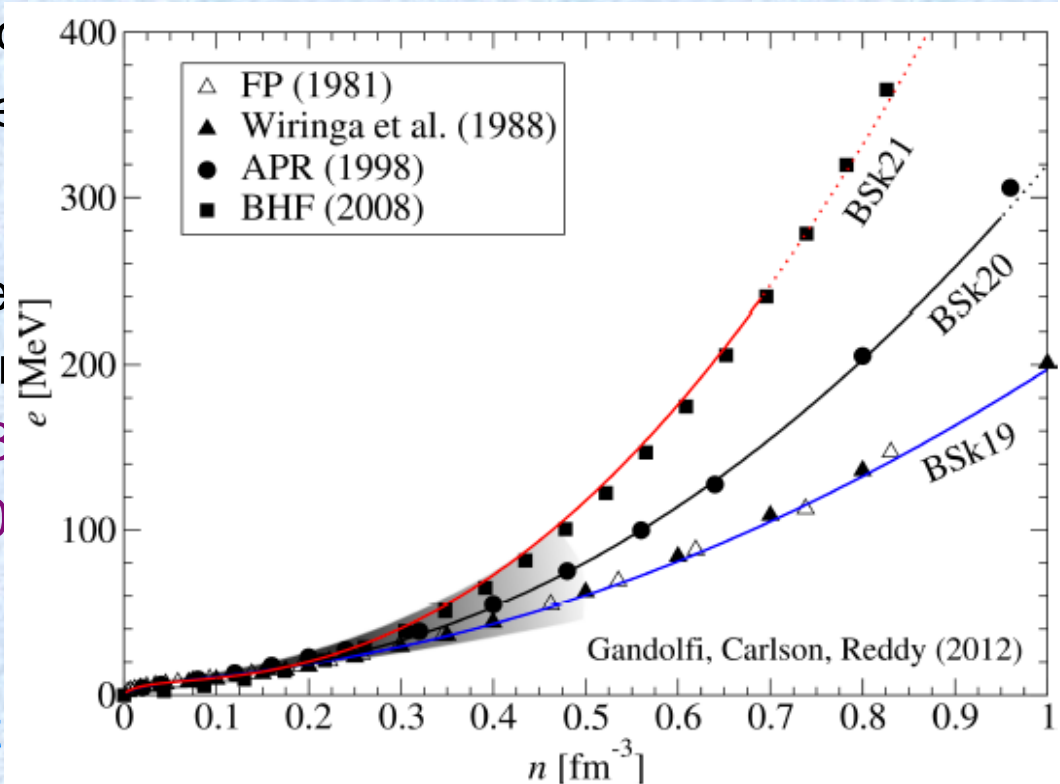
**BSk25**

**BSk26**

- fit 2012 AME data (2353 masses)
- constrained to microscopic neut
- different  $E_{\text{sym}}$  coefficient ( $J = 30$  MeV,  $K_\infty$  in experimental range ( $\approx 240$  MeV))

Goriely *et al.*, PRC 88, 024308 (2013)

**BSk27\*** (2012 AME), rms = 0.5 MeV → most



BSk\*\* suitable to describe all the regions of NS



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- **fit 2010 AME data** (2149 masses, rms = 0.581 MeV)
- **different degrees of stiffness** (BSk19 softer → BSk21 stiffer)  
constrained to different microscopic neutron-matter EoSs at  $T = 0$
- all have  $J = 30$  MeV, ,  $K_\infty$  in experimental range ( $\approx 240$  MeV)

Goriely *et al.*, PRC 82, 035804 (2010)

**BSk22**

**BSk23**

**BSk24**

**BSk25**

**BSk26**

- **fit 2012 AME data** (2353 masses, rms = 0.5-0.6 MeV)
- constrained to microscopic neutron-matter EoSs at  $T = 0$  (rather stiff)
- **different  $E_{\text{sym}}$  coefficient** ( $J = 32, 31, 30, 29, 30$  MeV),  
 $K_\infty$  in experimental range ( $\approx 240$  MeV)

Goriely *et al.*, PRC 88, 024308 (2013)

**BSk27\*** (2012 AME), rms = **0.5 MeV** → most accurate! Goriely *et al.*, PRC 88, 061302 (2013)

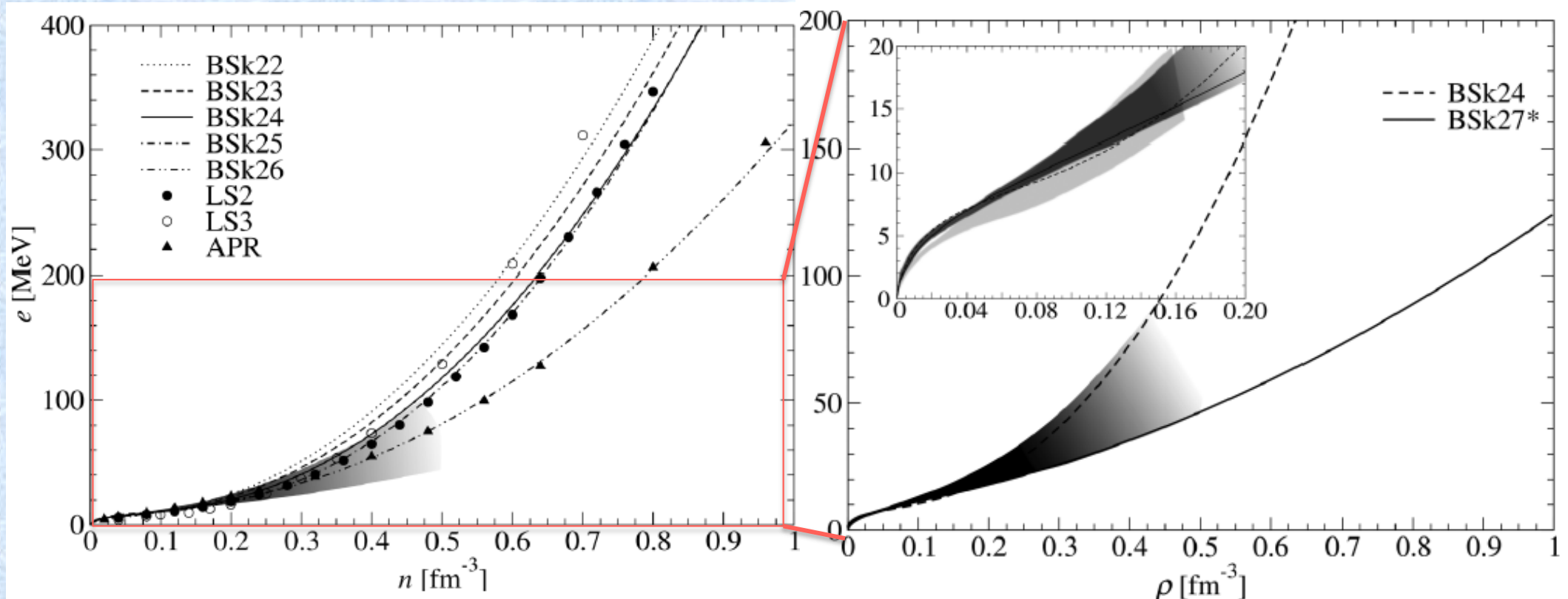


**BSk\*\*** suitable to describe all the regions of NS





# Constraints from nuclear physics: theoretical calculations (neutron matter)



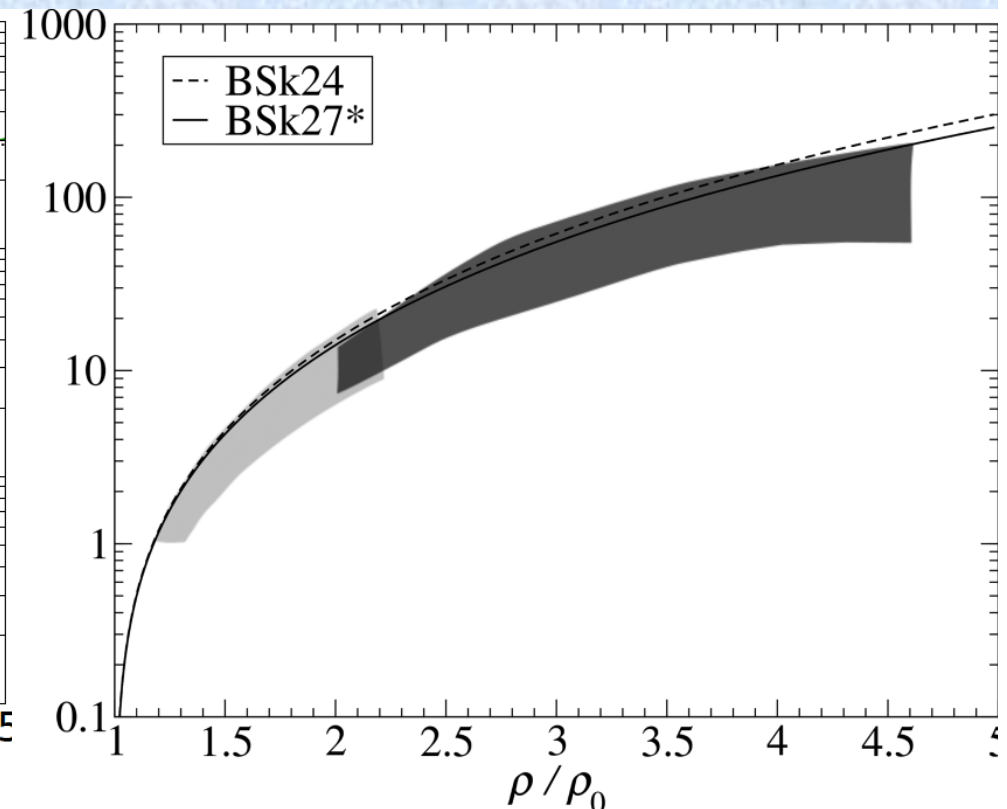
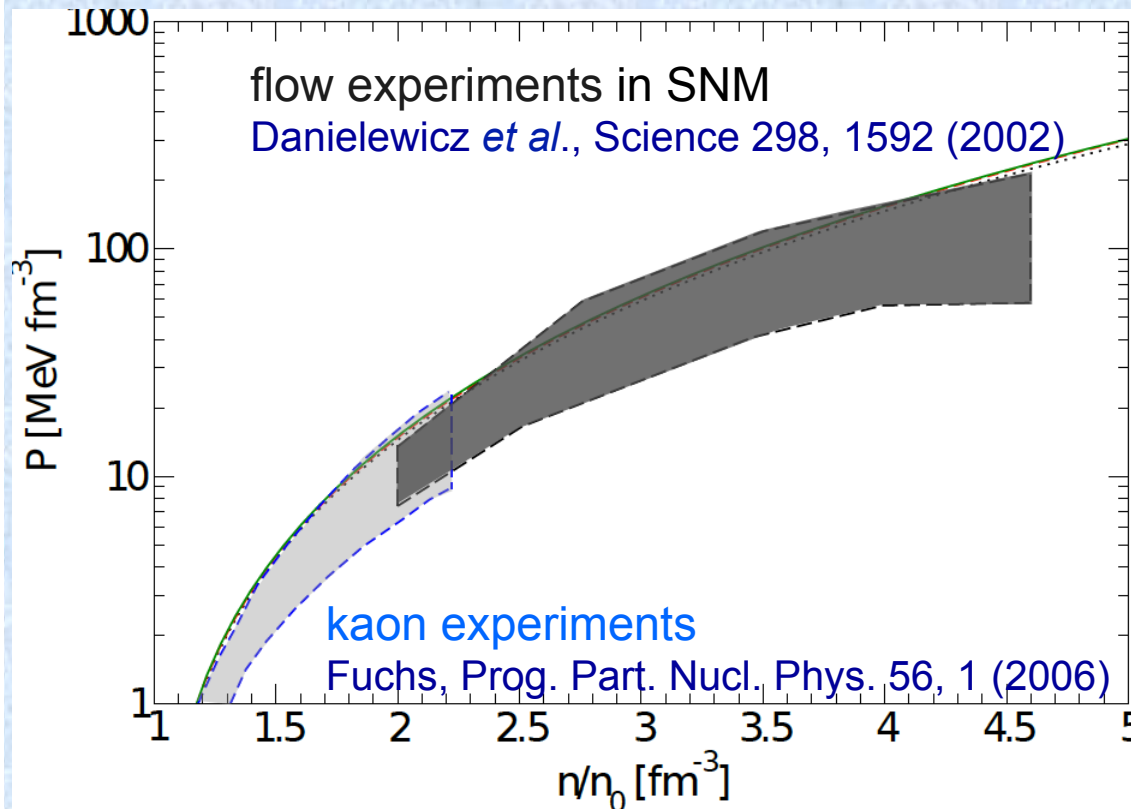
Goriely *et al.*, PRC 88, 024308 (2013)

Goriely *et al.*, PRC 88, 061302 (2013)

BSk\*\* fitted to realistic neutron-matter EoSs with different stiffness  
and agree with more microscopic calculations



# Constraints from nuclear physics: experiments (symmetric nuclear matter)



Goriely *et al.*, PRC 88, 061302 (2013)

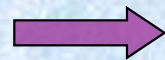
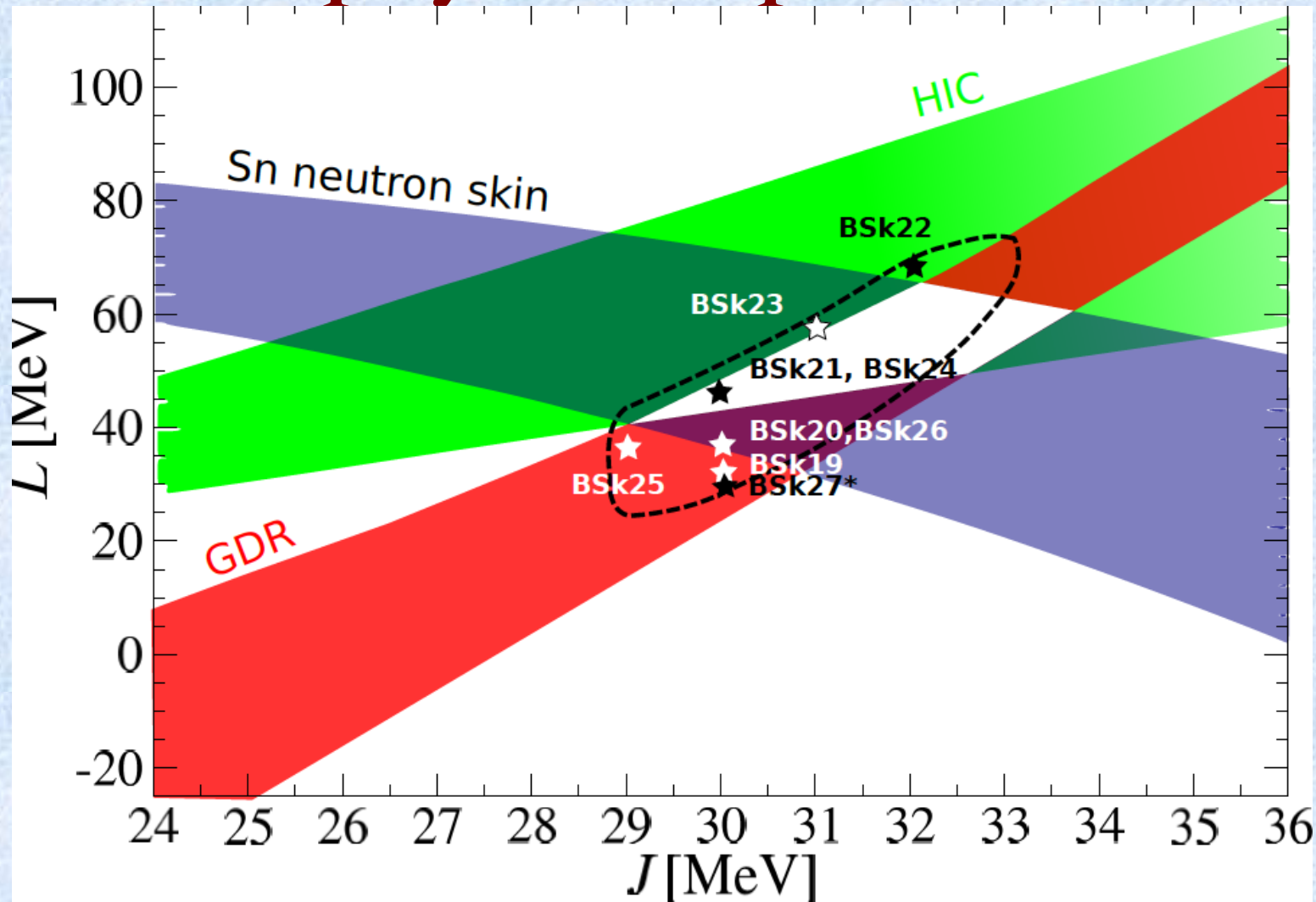
➡ Functionals in good agreement with “experimental” constraints on symm. matter

N.B.: deduced constraints are not direct experimental data, are model dependent!





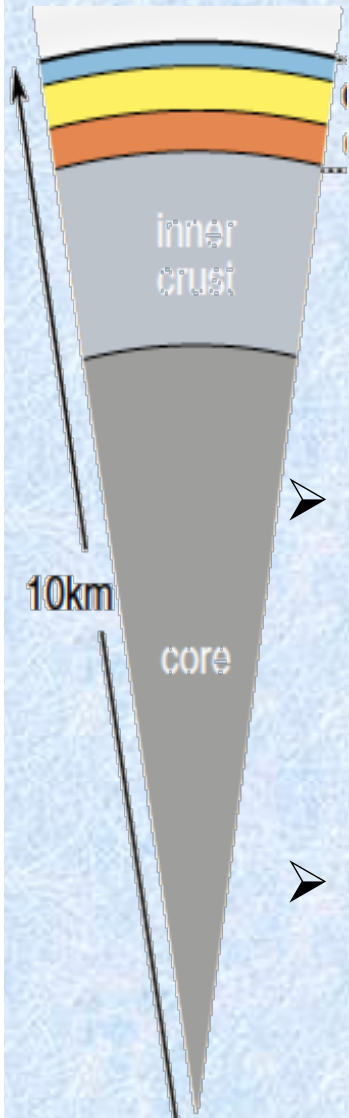
# Comparison with observables from nuclear physics experiments



$J, L$  consistent with experimental constraints



# EoS of neutron star



➤ **OUTER CRUST** (up to neutron drip) (J. M. Pearson *et al.*, PRC83, 065810 (2011))

→ one nucleus (bcc lattice) +  $e^-$  ( $\beta$  equilibrium)

→ minimization of the Gibbs energy per nucleon: BPS model

Only microscopic inputs are nuclear masses

→ Experimental or microscopic mass models HFB19-27

➤ **INNER CRUST** (Pearson *et al.*, PRC85, 065803 (2012))

→ one cluster (spherical) +  $n, e^-$  ( $\beta$  equilibrium)

→ semi-classical model: Extended Thomas Fermi (4th order in  $\hbar$ )  
+ proton shell corrections

➤ **CORE** (Goriely *et al.*, PRC 82, 035804 (2010), Goriely *et al.*, PRC 88, 024308 (2013))

→ homogeneous matter:  $n, p, e^-, \mu$  ( $\beta$  equilibrium) \*

→ same nuclear model to treat the interacting nucleons

A. F. Fantina \* here we do not consider possible phase transition!

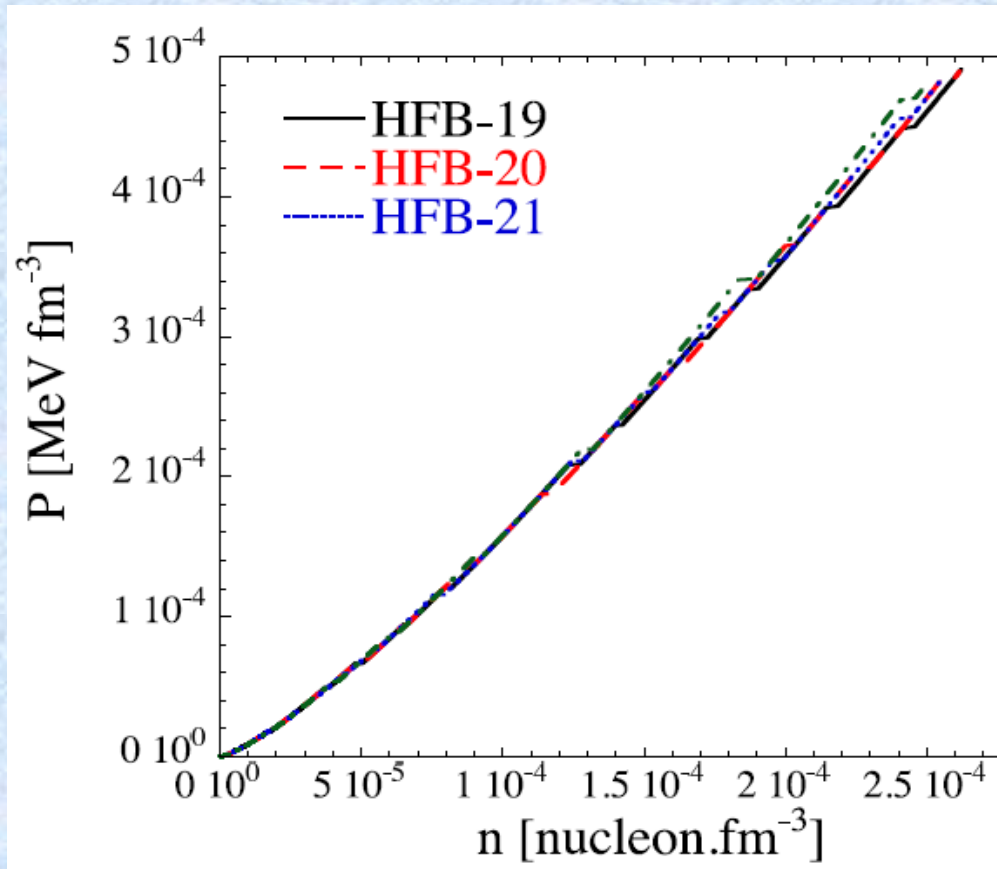
transition to exotic matter in Chamel, Fantina, Pearson, Goriely, A&A 553, A22 (2013)





# EoS: outer crust

$\approx 200$  m

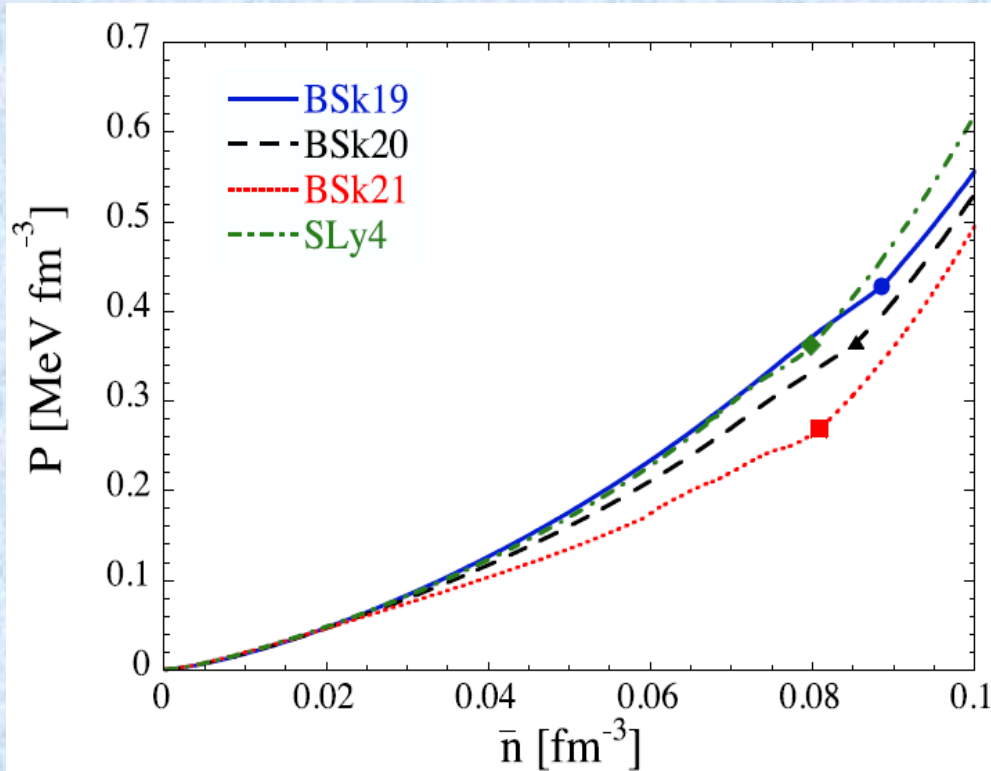


HFB-19	HFB-20	HFB-21	HFB-27*
<sup>56</sup> Fe	<sup>56</sup> Fe	<sup>56</sup> Fe	<sup>56</sup> Fe
<sup>62</sup> Ni	<sup>62</sup> Ni	<sup>62</sup> Ni	<sup>62</sup> Ni
<sup>64</sup> Ni	<sup>64</sup> Ni	<sup>64</sup> Ni	<sup>64</sup> Ni
<sup>66</sup> Ni	<sup>66</sup> Ni	<sup>66</sup> Ni	<sup>66</sup> Ni
<sup>86</sup> Kr	<sup>86</sup> Kr	<sup>86</sup> Kr	<sup>86</sup> Kr
<sup>84</sup> Se	<sup>84</sup> Se	<sup>84</sup> Se	<sup>84</sup> Se
<sup>82</sup> Ge	<sup>82</sup> Ge	<sup>82</sup> Ge	<sup>82</sup> Ge
<sup>80</sup> Zn	<sup>80</sup> Zn	<sup>80</sup> Zn	<sup>80</sup> Zn
<sup>82</sup> Zn	<sup>82</sup> Zn	-	-
-	-	<sup>79</sup> Cu	-
-	<sup>78</sup> Ni	<sup>78</sup> Ni	<sup>78</sup> Ni
<sup>80</sup> Ni	<sup>80</sup> Ni	<sup>80</sup> Ni	-
<sup>126</sup> Ru	<sup>126</sup> Ru	-	<sup>126</sup> Ru
<sup>124</sup> Mo	<sup>124</sup> Mo	<sup>124</sup> Mo	<sup>124</sup> Mo
-	<sup>122</sup> Mo	-	-
<sup>122</sup> Zr	<sup>122</sup> Zr	<sup>122</sup> Zr	<sup>122</sup> Zr
<sup>124</sup> Zr	<sup>124</sup> Zr	-	<sup>124</sup> Zr
-	-	<sup>121</sup> Y	-
<sup>120</sup> Sr	<sup>120</sup> Sr	<sup>120</sup> Sr	<sup>120</sup> Sr
<sup>122</sup> Sr	<sup>122</sup> Sr	<sup>122</sup> Sr	<sup>122</sup> Sr
<sup>124</sup> Sr	<sup>124</sup> Sr	<sup>124</sup> Sr	-
<sup>126</sup> Sr	<sup>126</sup> Sr	-	-

Pearson *et al.*, PRC83, 065810 (2011)

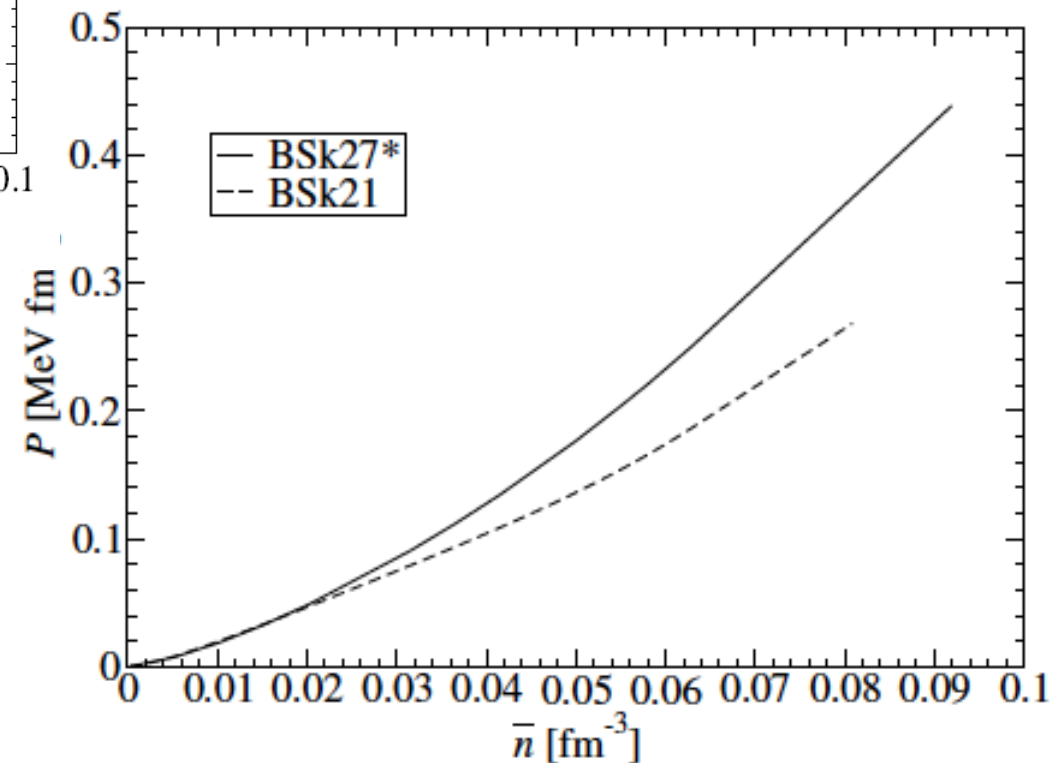


# EoS: inner crust and core



Pearson *et al.*, PRC85, 065803 (2012)

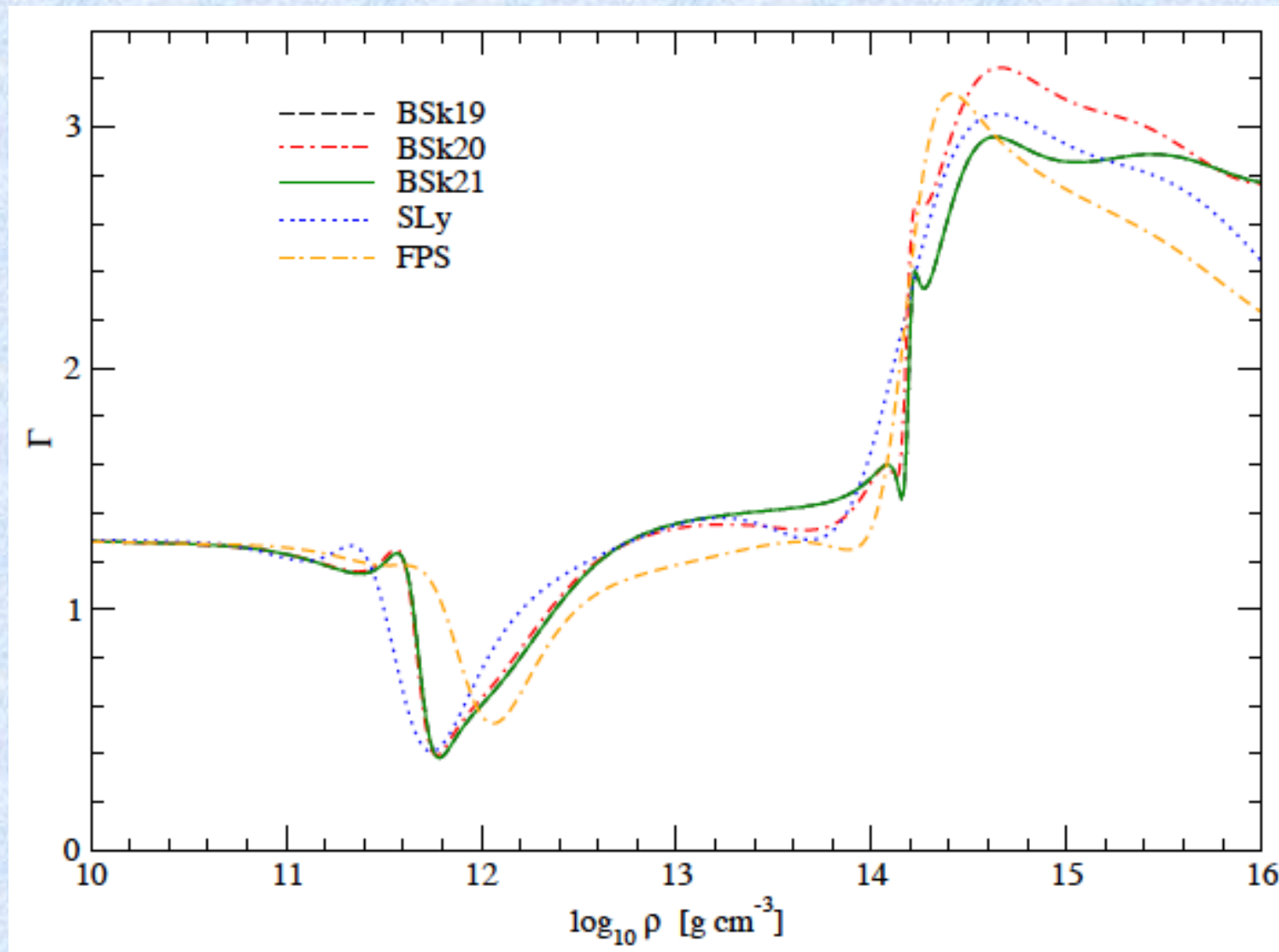
Force	$n_{\text{trans}}$ [fm <sup>-3</sup> ]	$P_{\text{trans}}$ [MeV fm <sup>-3</sup> ]
BSk19	0.0885	0.428
BSk20	0.0854	0.365
BSk21	0.0809	0.268
SLy4	0.0798	0.361
BSk27*	0.0919	0.439



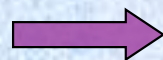




# EoS of NS: adiabatic index



Potekhin, Fantina, Chamel, Pearson, Goriely, A&A 560, A48 (2013)





# Computing the NS structure

## ➤ Nuclear models: **BSk 19-20-21 & BSk 22-23-24-25-26**

→ microscopic mass models that fit:

- ✧ available **nuclear experimental mass data**
- ✧ **nuclear-matter properties** from microscopic calculations

## ➤ Build the NS:

- ✧ **non-rotating NS** → solve Tolman-Oppenheimer-Volkoff (TOV) equations:

$$\frac{dP}{dr} = -\frac{G\rho\mathcal{M}}{r^2} \left(1 + \frac{P}{\rho c^2}\right) \left(1 + \frac{4\pi Pr^3}{\mathcal{M}c^2}\right) \left(1 - \frac{2G\mathcal{M}}{rc^2}\right)^{-1}$$

$$\frac{d\mathcal{M}}{dr} = 4\pi r^2 \rho \quad \rightarrow \text{EoS } P(\rho) \text{ to close the system}$$

- ✧ **rigidly rotating NSs**

*Method:* solve Einstein eqs. in GR for stationary axi-symmetric configurations.

*Code:* **LORENE** library (<http://www.lorene.obspm.fr>)  
developed at Observatoire de Paris-Meudon

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Refs on LORENE: Gourgoulhon, arXiv: 1003.5015 (lectures given at 2010 CompStar school)

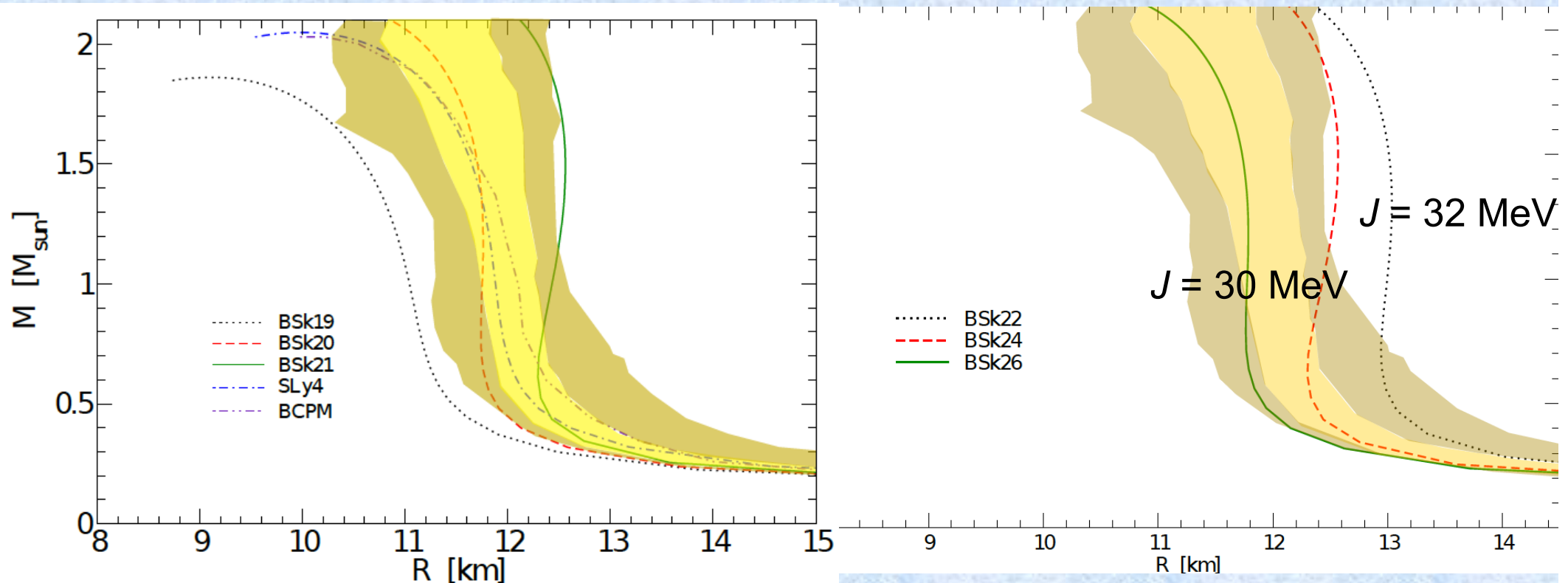
Gourgoulhon *et al.*, A&A 349, 851 (1999)

Granclement & Novak, Liv. Rev. Relativ. 12, 1 (2009)





# NS properties: $M$ - $R$ relation



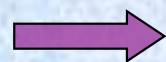
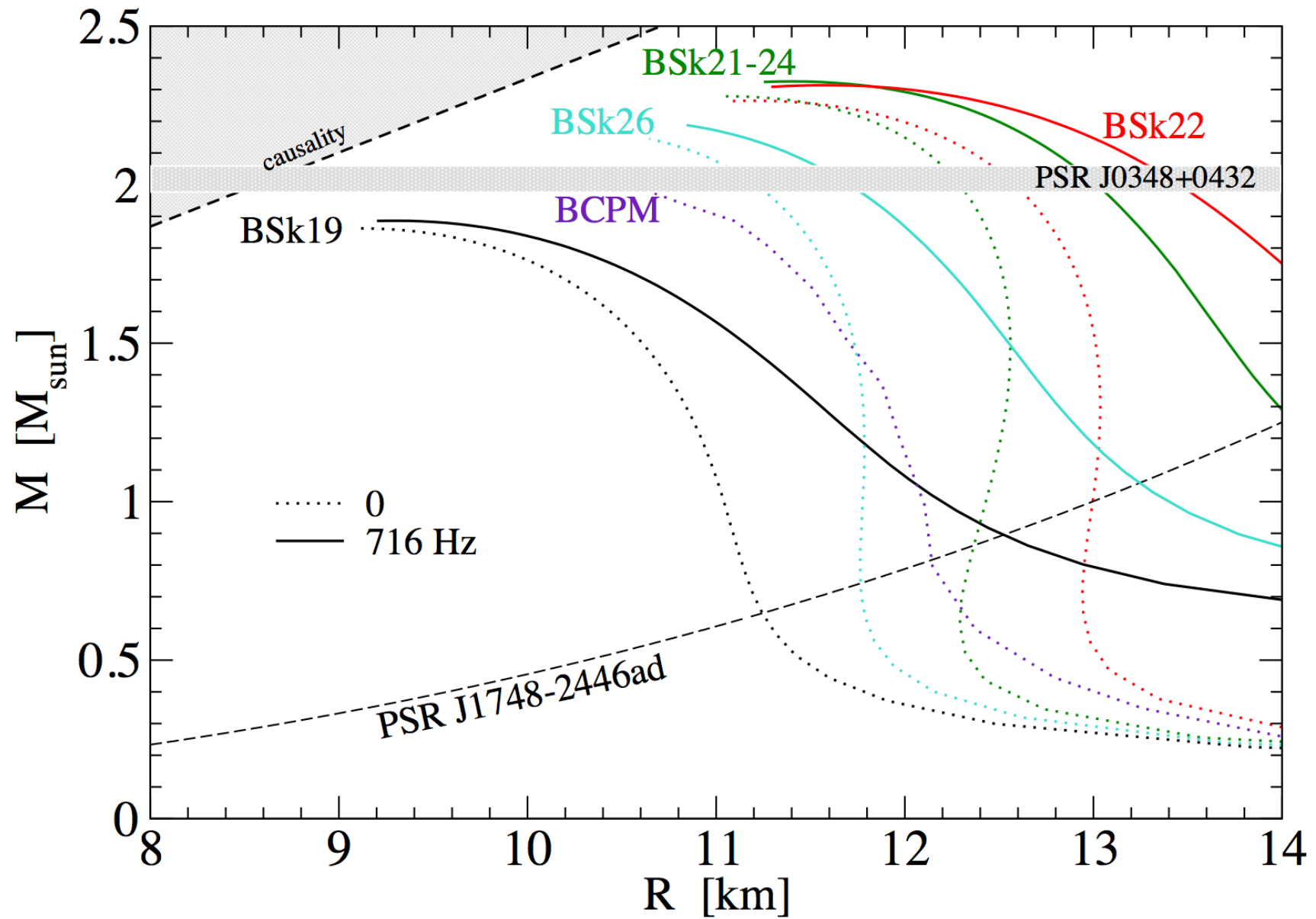
light (dark) shaded area: 1(2)- $\sigma$  contour from [Steiner et al. 2010](#)

[Fantina et al., Astron. Astrophys. 559, A128 \(2013\)](#)





# NS properties: *M-R relation with rotation*



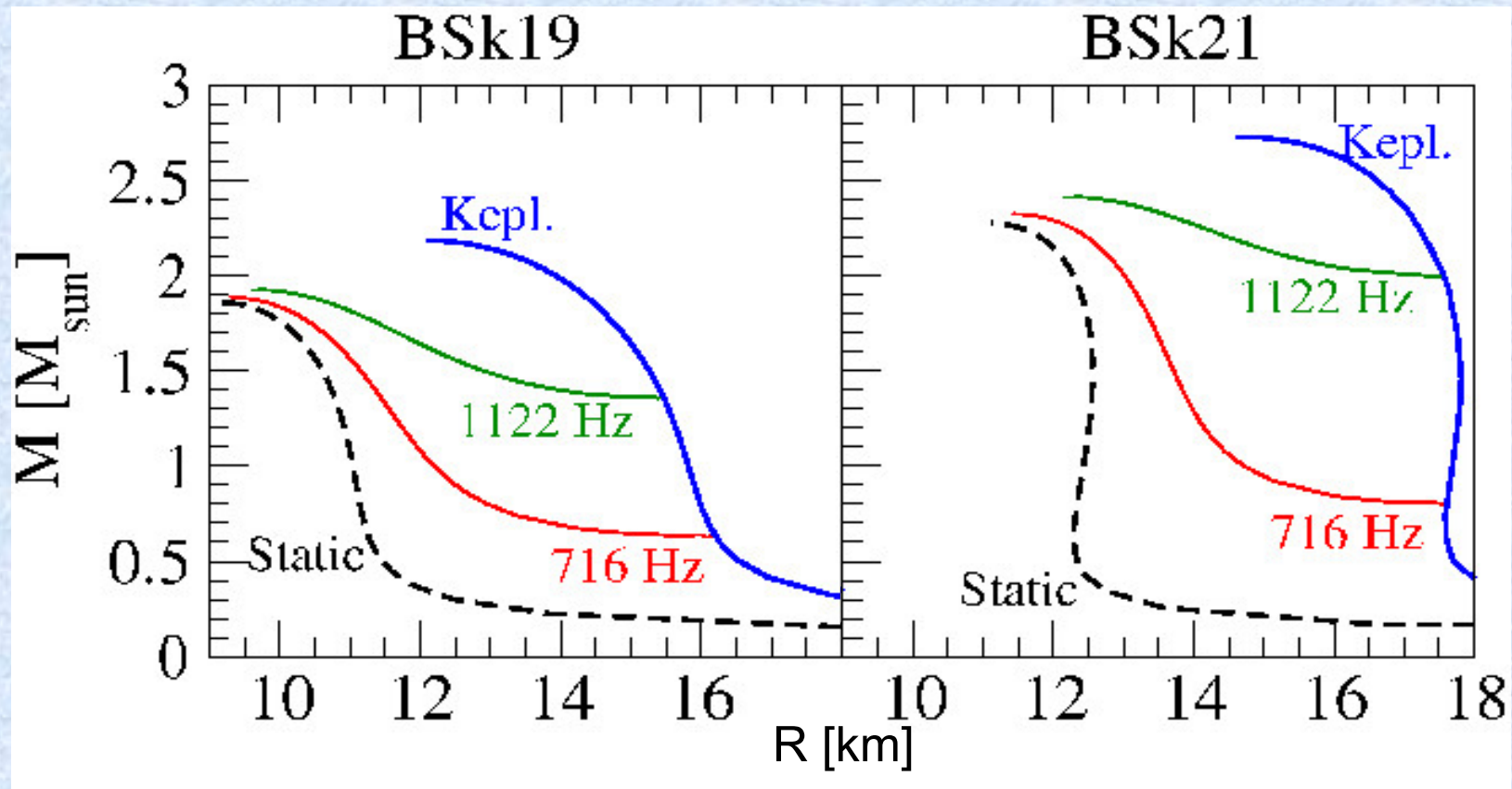
BSk21-22-24-26 compatible with observations





# NS properties: *keplerian velocity*

The rotational frequency of a stable NS is limited by the keplerian frequency above which the NS will be disrupted as a result of mass shedding



Fantina *et al.*, *Astron. Astrophys.* 559, A128 (2013)



only fast rotation increases considerably maximum mass ( $\approx 17\text{-}20\%$ )  
but: rotation can affect structure of low-mass NSs



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- some constraints from nuclear and astrophysics

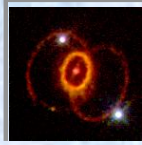
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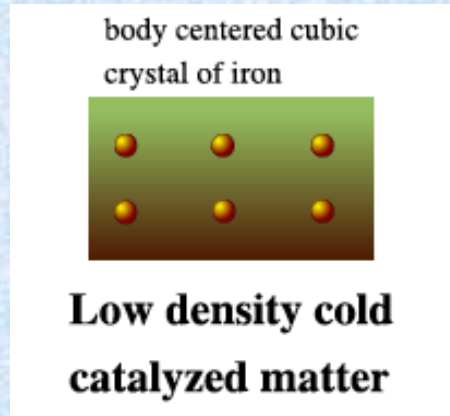
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# NS crust: catalysed vs accreted

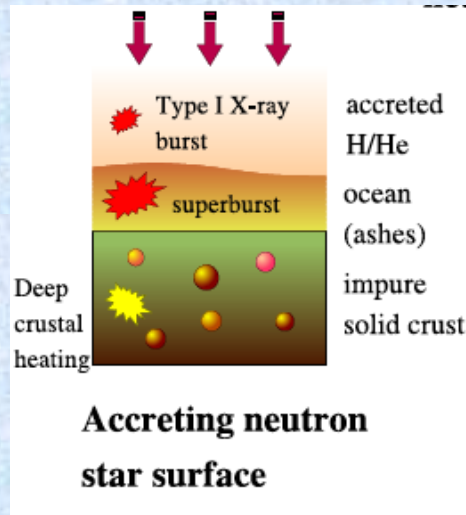
## ➤ Catalysed matter



- NS born a high  $T \approx 10^{11}$  K  $\rightarrow$  “hot” scenario
- **full thermodynamical equilibrium at  $T=0$**
- ground state of matter  
 $\rightarrow$  minimise Gibbs energy wrt  $Z, A$
- no exothermic reactions possible

see e.g. [Baym et al., ApJ 170, 299 \(1971\)](#)

## ➤ Accreted matter



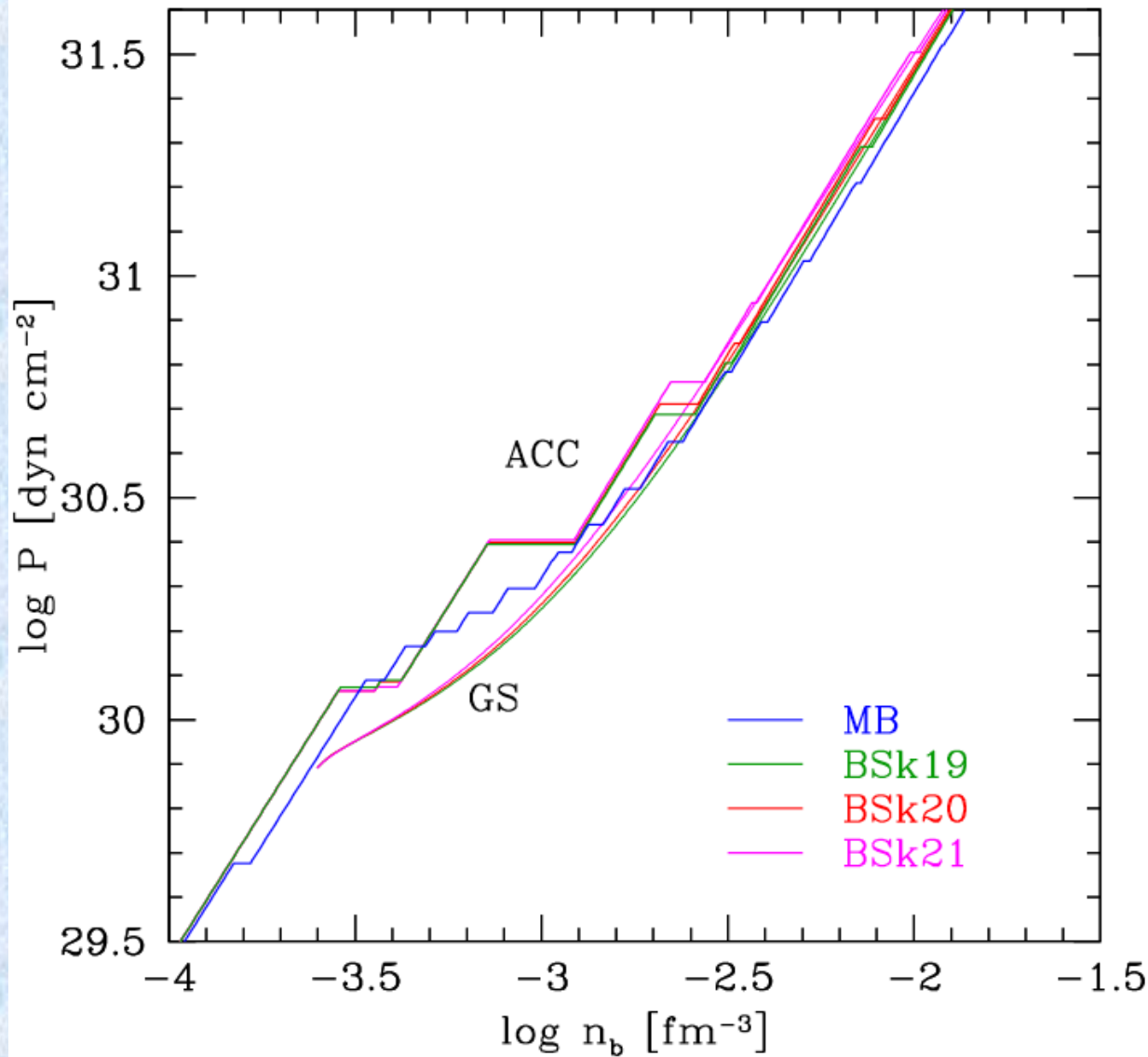
- $T < 10^9$  K  $\rightarrow$  “cold” scenario
- **matter off-equilibrium** (local min of  $E$ )  
 $\rightarrow$  minimum wrt neighbours  $N, Z$  at const.  $A$
- EC,  $n$  emission, pycnonuclear possible
- exothermic reactions possible  $\rightarrow$  energy sources  
 $\rightarrow$  can explain thermal radiation in SXTs in quiescence

see e.g. [Haensel & Zdunik, A&A 227, 431 \(1990\);](#)  
[A&A 229, 117 \(1990\); A&A 404, L33 \(2003\)](#) and Refs. therein





# Accreted NS: EoS

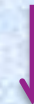


EoS for catalysed (GS) and accreted (ACC) crust.

Initial composition:  $^{56}\text{Fe}$  ashes

**Accreted crust EoS significantly stiffer than GS one for :**

$$\rho = 5 \times 10^{11} - 6 \times 10^{12} \text{ g/cm}^3$$

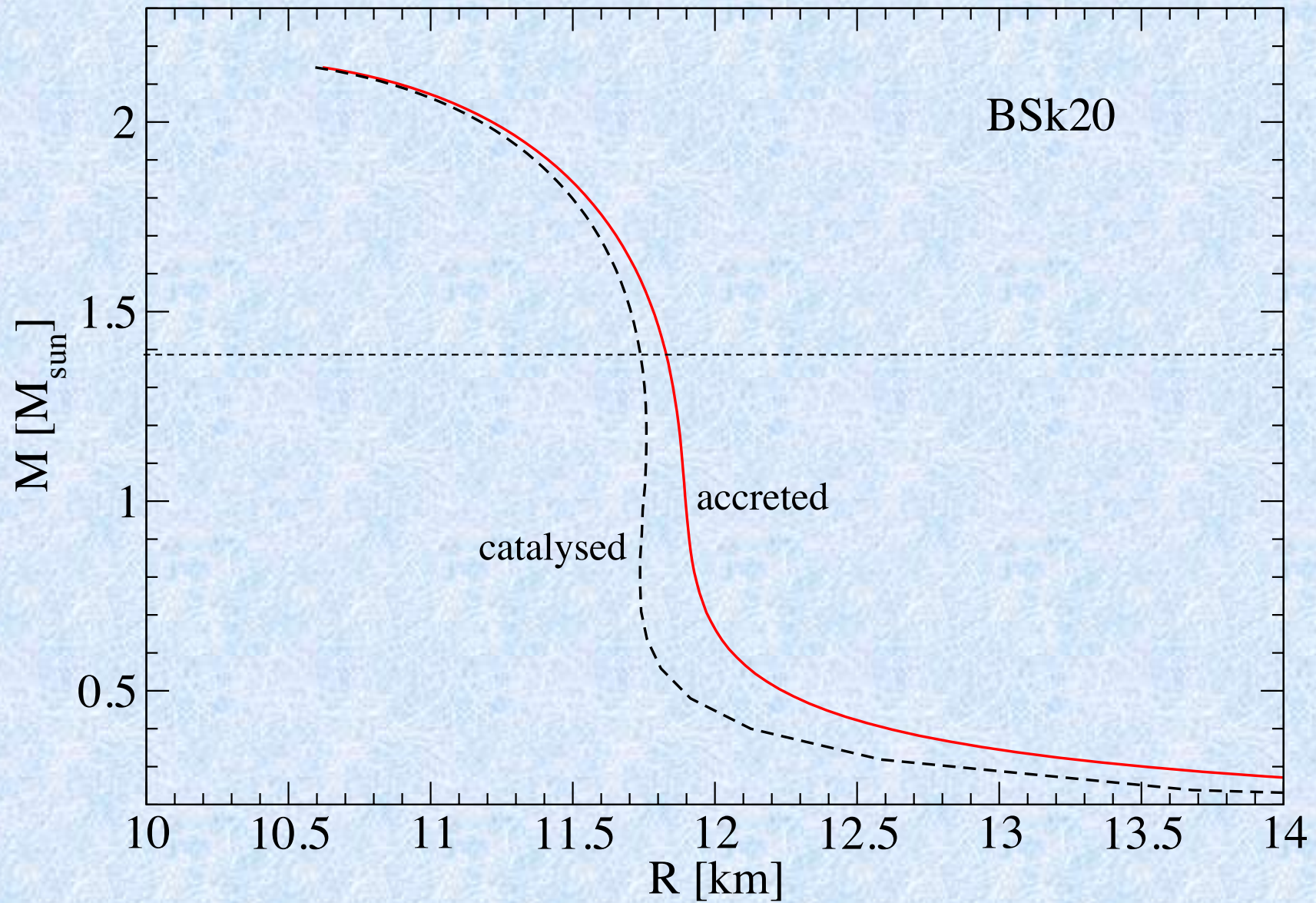


Typically, for  $1.4 M_{\text{sun}}$  NS, one expects :  $R_{\text{ACC}} - R_{\text{GS}} \approx 100 \text{ m}$   
(see e.g. Haensel&Zdunik, A&A, 1990)

PRELIMINARY! (Article in preparation) → see J. L. Zdunik's talk



# Accreted NS: EoS



PRELIMINARY!



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# Conclusions & Outlooks

- ❖ **Nuclear physics experiments + Astrophysical observations can put constraints on the EoS of dense matter!**
- ❖ Unified EoSs for NS matter → same nuclear model to describe all regions of NS fitted on *experimental nuclear data* and *nuclear matter properties*  
EoSs based on BSk 21-24-26 consistent with astrophysical observations!
- ❖ EoSs BSk 19-20-21 at  $T=0$  for catalysed matter available as:
  - **tables** : Fantina *et al.*, A&A 559, A128 (2013), doi: 10.1051/0004-6361/201321884
  - **fit** : Potekhin *et al.*, A&A 560, A48 (2013) at: <http://www.ioffe.ru/astro/NSG/BSk/>  
Fit: EoS, density profiles, electrical conductivities → can be used in NS calculations!
- + **Love number** : Damour, Nagar, Villain, PRD 85, 123007 (2012)
- ❖ **Finite T for SN cores**  
work in progress with J. M. Pearson, N. Chamel, S. Goriely
- ❖ **Accreting NS properties**  
work in progress with N. Chamel, P. Haensel, J. L. Zdunik





*Thank you!*

Supernova Remnant 1987A in the Large Magellanic Cloud  [HUBBLESITE.org](http://HUBBLESITE.org)